Zubin Kadva 902772316
MS Computer Science
CSE 5400 Functional Programming

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Assignment lambda

1. Define the functions "less than" and "greater than" of two numerical arguments.

T = λ*xy.x*

F = λ*xy.y*

NOT = λ*x.x* F T

PRE = λ*nfx.n (*λ*gh.h (g f))* (λ*u.x*) (λ*u.u)*

ZERO = λ*x.x*FT

GT = λ*xy*.NOT (ZERO (SUB *x y*))

LT = λ*xy*.GT *y x*

2. Define the positive and negative integers using pairs of natural numbers

CHECK = λ*n*.IF (ZERO (PRE n)) T F

3. Define addition and subtraction of integers

ADD = λ*abfx.af (bfx)*

SUB = λ*ab.b* PRE *a*

4. Define the division of positive integers recursively

IF = λ*fab.fab*

0 = λ*sz.z*

1 = λ*sz.s (z)*

DIVIDE = λ*ab*.IF (ZERO *b*) 0 IF (SUB *a b*) 1 (DIVIDE (SUB *a b*) *b*)

5. Define the function n! = n * (n - 1) … 1 recursively

MUL = λ*xyz.x (yz)*

FACT = λ*n*.IF (ZERO *n*) 1 MUL (*n* (FACT (PRE *n*)))

6. Define the rational numbers as pairs of integers

CONS = λ*pqf.(fp) q*

CAR = λ*p.p* T

CDR = λ*p*.*p* F

MAKE = λ*nd*.(CONS *n*) *d*

7. Define functions for the addition, subtraction, multiplication and division of rationals.

NUMERATOR = λ*x*.CAR *x*

DENOMINATOR = λ*x*.CDR *x*

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RADD = λxy.MAKE (ADD (
                        MUL ((NUMERATOR x) (DENOMINATOR y))
                        MUL ((NUMERATOR y) (DENOMINATOR x)) )
                MUL ((DENOMINATOR x) (DENOMINATOR y)) )

RSUB = λxy.MAKE (SUB (
                        MUL ((NUMERATOR x) (DENOMINATOR y))
                        MUL ((NUMERATOR y) (DENOMINATOR x)) )
                MUL ((DENOMINATOR x) (DENOMINATOR y)) )

RADD = λxy.MAKE (MUL ((NUMERATOR x) (NUMERATOR y))
                MUL ((DENOMINATOR x) (DENOMINATOR y)) )

RDIV = λxy.MAKE (MUL ((NUMERATOR x) (DENOMINATOR y))
                MUL ((NUMERATOR y) (DENOMINATOR x)) )
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8. Define a data structure to represent a list of numbers.

[] := λcn. n

[1, 2, 3] := λcn. c 1 (c 2 (c 3 n))

9. Define a function which extracts the first element from a list

HEAD = λ*l*.*l* (λ*ab*.*a*)

10. Define a recursive function which counts the number of elements in a list

LENGTH = λ*l*. IF (null *l*) 0 ADD (1 (LENGTH *l*))