

Assignment

1. The post system has variables $\{x, y, z\}$ and signs $\{N, |, -, =\}$

The productions are:

$$\frac{}{\overline{N}} \quad \frac{Nx}{\overline{Nx|}} \quad \frac{Ny}{\overline{-y = y}} \quad \frac{-y = y \quad y + z = w}{\overline{w - y = z}}$$

Eg.

$$\frac{\frac{\frac{N}{\overline{N|}}}{\overline{N||}}}{\overline{-|=|}} \quad \frac{\frac{N}{\overline{N|}}}{\overline{+||=||}} \quad \frac{\frac{\frac{N}{\overline{N|}}}{\overline{+||=||}}}{\overline{|||-|=||}}$$

2. The post system has variables $\{x, y, z\}$ and signs $\{N, |, \cdot, !, =\}$

The productions are:

$$\frac{}{\overline{N}} \quad \frac{Nx}{\overline{Nx|}} \quad \frac{Nx}{\overline{x \cdot =}} \quad \frac{x \cdot y = z}{\overline{x \cdot y| = xz}} \quad \frac{x \cdot y = z}{\overline{xy! = z|}} \quad \frac{x! = z}{\overline{x! = xz}}$$

Proof: $|||!=|||$

$$\frac{\frac{\frac{N}{\overline{N|}}}{\overline{N||}}}{\overline{|| \cdot =}} \quad \frac{\frac{\frac{N}{\overline{N|}}}{\overline{|| \cdot =}}}{\overline{|| \cdot |=||}} \quad \frac{\frac{\frac{N}{\overline{N|}}}{\overline{|| \cdot |=||}}}{\overline{|||!=|||}} \quad \frac{\frac{\frac{N}{\overline{N|}}}{\overline{|||!=|||}}}{\overline{|||!=|||}}$$

3. This post system produces multiples of x where $x = \{ |, ||, ||| \dots \}$

Eg. $x = |||$ produces $|||||||$ ($3 * 3 = 9$)

$$\begin{array}{r}
 N \\
 \hline
 N| \\
 \hline
 N|| \\
 \hline
 ||| \cdot = \\
 \hline
 ||| \cdot | = ||| \\
 \hline
 ||| \cdot || = ||||| \\
 \hline
 ||| \cdot ||| = ||||| \\
 \hline
 |||||
 \end{array}$$

6. Given the **MIU** post system

$$\frac{MI}{MI} \quad \frac{xI}{xIU} \quad \frac{Mx}{Mxx} \quad \frac{xIIIy}{xUy} \quad \frac{xUUy}{xy}$$

MU	Begins with M	Ends in U	2 characters
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Production	Beginning with	Ending with	Total characters
1	M	I	2
2	M / I / U	U	> 2
3	M	M / I / U	> 1
4	M / I / U	M / I / U	> 1
5	M / I / U	M / I / U	> 0

Eg.

$$\begin{array}{r}
 MI \\
 \hline
 MIU \\
 \hline
 MIUIU
 \end{array}
 \quad
 \begin{array}{r}
 MI \\
 \hline
 MII \\
 \hline
 MIIU \\
 \hline
 MUU
 \end{array}$$

According to production 2, for a string ending in **I**, a **U** can be appended.

According to production 3, a string occurring after **M** can be doubled.

According to production 4, a string containing **III** can be replaced by a **U**.

According to production 5, a string containing **UU** can be removed.

Rule 3, can double the number of strings and rule 4 can reduce it by 3.

Eg.

$$\begin{array}{c} \text{MI} \\ \hline \text{MII} \\ \hline \text{MIII} \\ \hline \text{MUI} \end{array}$$

This means that the number of **I**'s should be divisible by 3.

But **MI** cannot be replaced by **MU** since **MI** contains only 1 **I** and 1 is not divisible by 3.

This justifies that **MU** is not derivable in the MIU post system.

9.

$$\begin{array}{c} \text{P} \\ \hline \text{FP} \\ \hline \begin{array}{ccccc} \text{P} & \text{FNP} & \text{P} & \text{P} & \text{P} \\ \hline \text{FP} & \text{FCNPN} & \text{FP} & \text{FP} & \text{FP} \end{array} \\ \hline \begin{array}{ccccc} \text{ThCCPCNPPCCCNPPPCPP} & & \text{ThCPCNPP} & & \end{array} \\ \hline \text{ThCCCNPPPCPP} & & \text{ThCCNPPP} & & \\ \hline \text{ThCPP} \end{array}$$

X. Consider the following post canonical system:

$$\overline{()} \quad \frac{N}{[N]} \quad \frac{N}{\{N\}}$$

Eg.

$$\begin{array}{c} () \\ \hline [()] \\ \hline \{ [()] \} \end{array}$$