QUICKSORT

Worst Case Analysis

Recurrence Relation:
$T(0) = T(1) = 0$ *(base case)*
$T(N) = N + T(N-1)$

Solving the RR:

$T(N) = N + T(N-1)$
$T(N-1) = (N-1) + T(N-2)$
$T(N-2) = (N-2) + T(N-3)$

...

$T(3) = 3 + T(2)$
$T(2) = 2 + T(1)$
$T(1) = 0$

Hence,

$$T(N) = N + (N-1) + (N-2) \ldots + 3 + 2$$

$$\approx \frac{N^2}{2}$$

which is $O(N^2)$

QUICKSORT

Best Case Analysis

Recurrence Relation:

T(0) = T(1) = 0 (base case)
T(N) = 2T(N/2) + N

Solving the RR:

$$\frac{T(N)}{N} = \frac{N}{N} + \frac{2T(N/2)}{N}$$

Note: Divide both side of recurrence relation by N

$$\frac{T(N)}{N} = 1 + \frac{T(N/2)}{N/2}$$

$$\frac{T(N/2)}{N/2} = 1 + \frac{T(N/4)}{N/4}$$

$$\frac{T(N/4)}{N/4} = 1 + \frac{T(N/8)}{N/8}$$

...

$$\frac{T(\frac{N}{N/2})}{\frac{N}{N/2}} = 1 + \frac{T(\frac{N}{N})}{\frac{N}{N}} = 1 + \frac{T(1)}{1}$$

same as

$$\frac{T(2)}{2} = 1 + \frac{T(1)}{1}$$

Note: T(1) = 0

Hence,

$$\frac{T(N)}{N} = 1 + 1 + 1 + \ldots 1$$

Note: log(N) terms

$$\frac{T(N)}{N} = \log N$$

$T(N) = N \log N$ *which is O(N logN)*