2024

COMPUTER SCIENCE AND ENGINEERING

Paper : CSC-902

(Advanced Algorithms)

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1 & 2**, and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Define the Universal hash function.

(b) Find the number of possible parenthesization of matrix chain multiplication for the sequence of matrices $<M_1, M_2, M_3, M_4>$.

(c) Let L_1 and L_2 be two problems such that $L_1 \leq_p L_2$. Suppose that problem L_2 can be solved in $O(n^k)$ time and the reduction can be done in $O(n^j)$ time. Derive the time complexity of solving L_1.

(d) Prove the approximation bound of the bin-packing problem.

(e) Define P, NP complexity class.

(f) Derive the time complexity of the divide and conquer strategy-based algorithm to solve the closest pair algorithm.

(g) Define flow in flow network.

2. Answer ***any five*** questions : 4×5

(a) Given a flow network $G = (V, E)$, where $V = \{s, u, t, v\}$ and $E = \{(s, u, 2), (u, t, 4), (s, v, 4), (v, t, 2)\}$. Note that $(s, u, 2)$ means the capacity of edge (s, u) is 2. What is the maximum flow in the flow network G ? Show the steps of your solution.

(b) Give the dynamic programming formulation of all pair shortest path problems. State the time complexity of this algorithm.

(c) Suppose that flow network G contains an edge (u, v) and we create a new flow network G' by creating a new vertex x and replacing (u, v) with new edges (u, x) and (x, v) with capacity $c(u, x) = c(x, v) = c(u, v)$. Does a maximum flow in G' have the same value as the maximum flow in G? Justify your answer.

(d) Differentiate between the Finite state machine-based string matching algorithm and the KMP string matching algorithm.

(e) Give a polynomial time reduction algorithm to reduce the maximum bipartite matching problem to the maximum flow problem.

(f) Derive the expected search time complexity of the skip list.

(g) Derive the time complexity of randomized quicksort.

3. (a) Derive the expected running time of RANDOMIZED-SELECT.

(b) Suppose we use RANDOMIZED-SELECT to select the minimum element of the array $A = \{3, 2, 9, 0, 7, 5, 4, 8, 6, 1\}$. Describe a sequence of partitions that results in a worst-case performance of RANDOMIZED-SELECT. 5+5

4. (a) State and prove the Max-flow Min-cut Theorem.

(b) Give the time complexity of the Ford-Fulkerson Algorithm that finds the maximum flow in a flow network with justification.

(c) Give an instance of a flow network where the Ford-Fulkerson Algorithm yields the maximum number of iterations. 4+3+3

5. (a) Define the approximation ratio for a minimization problem for an approximation algorithm.

(b) Define k-center problem.

(c) Give an approximation algorithm to solve the k-center problem and derive the approximation bound. 2+2+(2+4)

6. (a) Define the closest pair problem.

(b) Briefly discuss how randomization is used in the closest pair problem.

(c) Derive the time complexity of the randomized closest pair algorithm. 2+4+4

7. (a) Generate the state-transition table for a string-matching automaton for the pattern *ababbabbababbababbabb* over the alphabet $\{a,b\}$.

(b) Generate the prefix function used in KMP Algorithm for the pattern *ababbabbababbababbabb* over the alphabet $\{a,b\}$. 5+5