2022

COMPUTER SCIENCE AND ENGINEERING

Paper : CSCL-0901

(Topics in Algorithms)

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1, 2** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) Suppose you have algorithms with average time complexities n^2, n log n and 2^n. How much slower does each of these algorithms get when you

(i) double the input size

(ii) increase the input size by one?

(b) Define NP and NP-hard complexity classes.

(c) Give example of a flow network where Ford Fulkerson algorithm takes the maximum time.

(d) Give the node structure of a skip list.

(e) What is the significance of approximation ratio in approximation algorithm?

(f) Let A[1..95] be a sorted array of integers. In order to find the 17th smallest integer, how many calls to Select algorithm will be required?

2. Answer ***any five*** questions : 4×5

(a) Define universal hash function and explain how it is used in randomized closest pair algorithm.

(b) In the combined step of divide and conquer closest pair algorithm, let $Y' = \{y_i\}$ represent the sorted y coordinates of the points that lie in 2δ region of the middle point 1 on x-axis, where δ is the minimum distance obtained between points either in the Left side L or Right side R of the middle point 1. Explain why the algorithm needs only to check the distance between each y_i and the next eight points in the sorted sequence of Y' in order to obtain the closest pair.

(c) Draw the state transition diagram for a string matching automata for the pattern ABABBABBABABBABABBABB.

(d) Show that the first fit strategy to solve bin packing problem gives an approximation factor of 2.

(e) Show that on expectation, the search time in a randomized skip list is O(n).

(f) Give an algorithm to compute only the prefix function in KMP string matching along with time complexity.

(g) Prove the maximum flow min-cut theorem.

Please Turn Over

3. Suppose a pattern P to be searched in a text string T, P has gap character ϕ to indicate zero or more length set of characters that can be ignored in T in order to match P. For example, let P = ab ϕba ϕc and T = cabccbacbacab. Then P occurs in T as **cabccbac**bac**a**b, i.e., **cabϕbaϕc**ab. Bold characters show the match of P in T. Write a polynomial time algorithm to obtain such a match and analyze the time complexity. 7+3

4. You are asked to find all the first k smallest elements from a given array A of n distinct integers. Note that k is an input.

   (a) Comment on the time complexity if Sorting is used.

   (b) Give a $\theta(n)$ algorithm to solve this problem along with time complexity analysis. 3+(4+3)

5. (a) Given vertices u and v in a flow network, where capacity c(u, v) = 5 and c(v, u) = 8, suppose that 3 units of flow are shipped from u to v and 4 units are shipped from v to u. Compute the net flow from u to v by giving a diagram.

   (b) After a devastative flood in a region, the food needs to be supplied to m different locations $R = \{r_1, r_2, r_3,...r_m\}$. The food distribution centers have been opened in a n number of places. Let the locations are represented by $L = \{l_1, l_2, l_3,...l_n\}$. The organizer knows how much supply of food can be passed through the road network N = (V, E), which are still accessible. Here V is the set of road junctions and E is the set of road segment between junctions. The amount of food supplies at every l_i, and the food requirement in every affected location r_j is also known. Design a flow network along with the algorithm to help the organizer to decide supply from which location should go to which affected location and whether the amount of food supplies is enough to satisfy the demands at all locations. 3+7

6. (a) Give the randomized algorithm for finding the *k*th smallest element.

   (b) Derive the expected time complexity. 4+6

7. (a) Define the 3SAT and Vertex cover decision problem.

   (b) Show that 3SAT is polynomial time reducible to Vertex cover. 3+7