Find Inverse Z Transform if a polynomial is Given

1)

Find the Signal $x(n)$ for which the z transform is

$$X(z) = 4z^4 - z^3 - 3z + 4z^{-1} + 3z^{-2}$$

- $x(n) = Z^{-1}[x(z)] = \frac{1}{2\pi j}\int x(z)\, z^{n-1}\, dz$

$$x(z) = \underset{x(-4)}{4z^4} - \underset{x(-3)}{1z^3} + \underset{x(-2)}{0z^2} - \underset{x(-1)}{3z} + \underset{x(0)}{0z^0} + \underset{x(1)}{4z^{-1}} + \underset{x(2)}{3z^{-2}}$$

- $$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-4}^{2} x(n) z^{-n}$$

- $x(n) = \{4, -1, 0, -3, \underset{\uparrow}{0}, 4, 3\}$

2)

If $x(z) = 2z^2 + 3$, find $x(n)$

- $$x(z) = \underset{-2}{2z^2} + \underset{-1}{0 \times z^1} + \underset{0}{3 \times z^0}$$

- $$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-2}^{0} x(n) z^{-n}$$

- $x(n) = \{2, 0, \underset{\uparrow}{3}\}$

3)

If $x(z) = 3 + 2z^{-1} + 3z^{-3}$, find $x(n)$

- $$x(z) = \underset{x(0)}{3 \times z^0} + \underset{x(1)}{2 \times z^{-1}} + \underset{x(2)}{0 \times z^{-2}} + \underset{x(3)}{3 \times z^{-3}}$$

- $$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{3} x(n) z^{-n}$$

- $x(n) = \{\underset{\uparrow}{3}, 2, 0, 3\}$

Z transform of standard basic signals.

1) Find the Z transform of the sequence

$$x(n) = a^{-n} u(-n-1)$$

Ans =

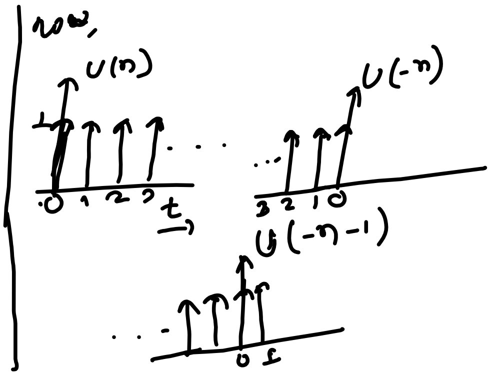
$$x(n) = a^{-n} \cdot u(-n-1)$$

$$\therefore x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^{-n} u(-n-1$$

$$= \sum_{n=-\infty}^{1} a^{-n} z^{-n}$$

$$= \sum_{n=-\infty}^{1} (az)^{-n}$$



$$X(z) = \sum_{n=1}^{\infty} (az)^{n} = \sum_{n=0}^{\infty} (az)^{n} - (az)^{0} = \sum_{n=0}^{\infty} (az)^{n} - 1$$

we know,

$$\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}$$

$$\therefore x(z) = \frac{1}{1-az} - 1$$

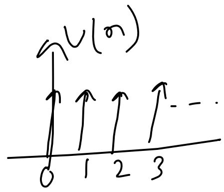
$$= \frac{1-1+az}{1-az} = \frac{az}{1-az}$$

● Z transform of unit step function.

$$x(n) = u(n)$$

$$\therefore x(z) = Z.T\,[x(n)]$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=-\infty}^{\infty} u(n)\, z^{-n} =$$



$$x(z) = \sum_{n=0}^{\infty} z^{-n}$$

$$\boxed{\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}}$$

$$= \frac{1}{1-z^{-1}} = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$$