2023

COMPUTER SCIENCE AND ENGINEERING

Paper : CSCL-0901

(Topics in Algorithms)

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Define the approximation ratio for a maximization problem.

(b) Compute the prefix function for the pattern ABBBACABB in a string-matching algorithm.

(c) State and prove the time complexity of a range search [a:b] in a randomized skip list.

(d) How to use randomization to find the k-th smallest element from an array with linear expected time complexity?

(e) Why do we compute expected case time complexity for randomized algorithms?

(f) Define the universal hash function.

(g) Define complexity classes NP and NP-complete.

2. Answer ***any five*** questions : 4×5

(a) Let n be a positive integer. Design an efficient randomized algorithm that generates a random permutation of the integers 1, 2, ..., n. Assume that you have access to a fair coin. Analyze the time complexity of your algorithm.

(b) Derive the expected case time complexity of the search in the Skip list.

(c) Let f and f′ be two feasible (s, t)-flows in a flow network G, such that $|f'| > |f|$. Prove that there is a feasible (s. t)-flow with value $|f'|-|f|$ in the residual network Gf.

(d) Prove that any comparison-based sorting algorithm in an array A of n numbers must take Ω (n log n) running time in the worst case.

(e) Show that the satisfiability problem, 3SAT is polynomial time reducible to the Clique problem.

(f) Prove the approximation bound for the k-centre problem.

(g) Prove that if problem P1 is reducible to P2 in polynomial time, and P2 is reducible to P3 in polynomial time, then P1 is reducible to P3 in polynomial time.

3. (a) Define the String Matching Problem.

   (b) Give the naïve string matching algorithm and find its time complexity.

   (c) Suppose that all characters in the pattern P are different. Show how to accelerate the naïve-string matching algorithm to run in time O(n) on an n-character text T. 2+4+4

4. (a) Define flow network.

   (b) In a hospital, there are 'n' doctors and 'm' patients. Each doctor can examine a certain number of patients in a day. The hospital management would like to serve a maximum number of patients, utilizing the capacity of the doctors. Give an efficient algorithm to serve a maximum number of patients with time complexity analysis. 3+7

5. (a) State the vertex cover problem and comment on the complexity class of this problem.

   (b) Give an approximation algorithm to solve the vertex cover problem and derive the approximation bound. 3+(3+4)

6. (a) Define the closest pair problem.

   (b) Briefly discuss how randomization is used in the closest pair problem.

   (c) Derive the time complexity of the randomized closest pair algorithm. 2+4+4

7. Suppose we allow a pattern P to contain occurrences of a gap character '<>' that can match an arbitrary string of characters (one of zero length). For example, the pattern ab<>ba<> c occurs in the text 'cabccbacbacab' as 'cab<>ba<>cab'.

   Give a polynomial-time algorithm to determine whether such a pattern P occurs in a given text T, and analyze the running time of your algorithm. 10

8. (a) Define the bin packing problem.

   (b) Give the first-fit approximation algorithm for it.

   (c) State the approximation ratio for first-fit to solve a bin-packing problem and prove the approximation bound. 2+4+4
