Histogram Processing:

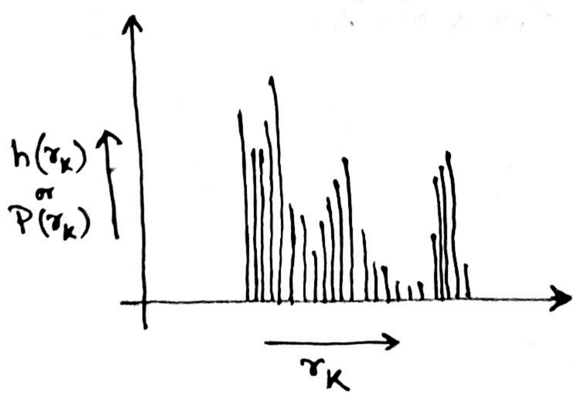
The histogram of a digital image with intensity levels in the range $[0, L-1]$ is a discrete function $h(r_k) = n_k$ where r_k is the k^{th} intensity value and n_k is the number of pixels in the image with intensity r_k

To normalize a histogram each of its components is divided by the total number of pixels in the image, denoted by the product MN, where M and N are the row and column dimensions of the image respectively.

Thus a normalized histogram is given by,

$$P(r_k) = n_k / MN \quad \text{for } k = 0, 1, 2, \ldots, L-1$$

The horizontal axis of each histogram plot corresponds to intensity values r_k and the vertical axis corresponds to the values of $h(r_k) = n_k$ or $p(r_k) = n_k / MN$. if the values are normalized. Thus, histograms may be viewed graphically as plots of $h(r_k) = n_k$ ~~vs~~ versus r_k or $P(r_k) = n_k / MN$ versus r_k



Histogram Equalization :

Let the variable 'r' denote the intensities of an image to be processed. We assume that 'r' is in the range $[0, L-1]$ with $r=0$ represting black and $r=L-1$ representing white.

For 'r' satisfying these conditions, the transformations (intensity mapping) is of the form,

$$S = T(r) \quad 0 \leq r \leq L-1$$

that produces an output intensity level s for every pixel in the input image having intensity 'r'. We assume that,

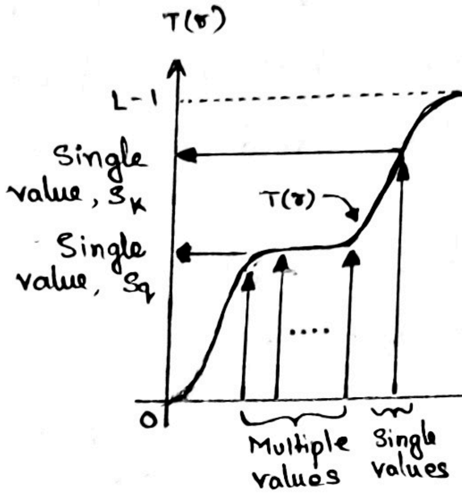
i) $T(r)$ is monotonically increasing function in the interval $0 \leq r \leq L-1$

ii) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

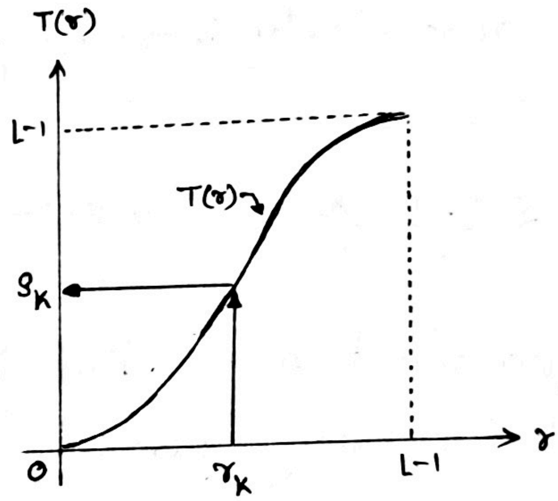
The requirement in condition (i) that $T(r)$ is monotonically increasing guarantees that output intensity values will never be less than corresponding input values.

Condition (ii) garuntees that the range of output intensities is the same as the input.

The condition that $T(r)$ is strictly monotonically increasing function in the interval $0 \leq r \leq L-1$ garuntees that the mappings from s back to r will be one-to-one, thus preventing ambiguities.



A function $T(r)$ is monotonically increasing if $T(r_2) \geq T(r_1)$ for $r_2 > r_1$.



A function $T(r)$ is strictly monotonically increasing function if $T(r_2) > T(r_1)$ for $r_2 > r_1$

** Similar defination is applied to monotonically decreasing function.

The intensity levels in an image may be viewed as random variables in the interval $[0, L-1]$. Let $p(r)$ and $p(s)$ denote the probability density functions of r and s respectively.

A fundamental result from basic probability theory is that if $P(r)$ and $T(r)$ are known and $T(r)$ is continuous and differentiable over the range of values, then the probability density function of the transformed variable s can be obtained using the simple formula,

$$P(s) = P(r) \left| \frac{dr}{ds} \right|$$

Thus we see that the probability density function of the output variable s is determined by the probability density function of the input intensities and the transformation function used.

A transformation function of particular importance in image processing has the form,

$$s = T(r) = (L-1) \int_0^r P(w)\, dw$$

Here w is the dummy variable of integration. The right side of this equation is recognized as the cumulative distribution function (CDF) of the random variable r.