Digital Image Processing

Digital image:-

A digital image is an image composed of picture elements also known as pixels, each with finite discrete qualities of numeric representation for its intensity or gray level that is an output from its 2D function fed as input by its spatial coordinates denoted by x and y on the x-axis and y-axis ~~represent~~ respectively.

- The 2D function can be written as $f(x, y)$ where x and y are spatial coordinates.
- The amplitude of f is called the intensity or gray level of the point (x, y).

Digital Image Processing:-

It is the use of a digital computer to process digital image through an algorithm. It covers low, mid and high level processes.

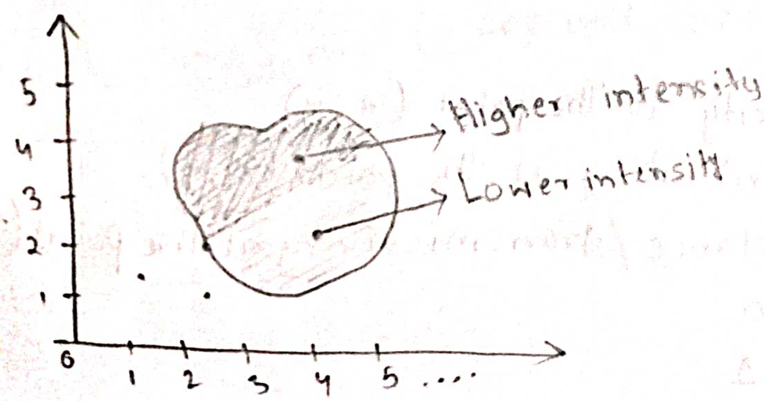
→ low-level: inputs and outputs are image. (noise removal/ image shapening)

(object recognizion) or Segmentation) → mid-level: Output attributes extracted from an input image.

→ high-level: an asemble of recognition of individual objects (understang and autonomous navigation)

Pixels: It is the smallest controllable element of a picture represented on the screen.

$$f(x,y) = \begin{bmatrix} f(0,0), & (0,1), & (0,2) & \cdots \\ f(1,0), & (1,1), & (1,2) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$



intensity ranges from 0 to 255 for a 8 bit gray scale img.

Image Formation model:-

→ We know that an object is represented in the form of a 2-D image.

→ For an image to be produced, there should be a light source illuminating the object.

→ When an image is generated from ~~a~~ a physical process, its values are proportional to the energy radiated by a ~~p~~ physical source (ex. electromagnetic waves).

→ Also, the intensity / amplitude of f at spacial coordinates is a positive scalar quantity whose physical meaning is determined by the source of the image.

→ Therefore the function $f(x, y)$ must be non-zero and finite.

i.e., $0 < f(x, y) < \infty$

→ The function may be characterized by 2 components:-

- amount of source illumination incedent on the scene being viewed.
- The illumination reflected by the objects in the scene.

→ These illumination and reflectance components can be denoted by $i(x,y)$ and $r(x,y)$ respectively.

This 2 functions combine to form $f(x,y)$,

$$f(x,y) = i(x,y) * r(x,y)$$

Where,
$f(x,y)$: intensity at the point (x,y).
$i(x,y)$: illumination at the point (x,y).
$r(x,y)$: reflectance / transmissivity at the point (x,y)

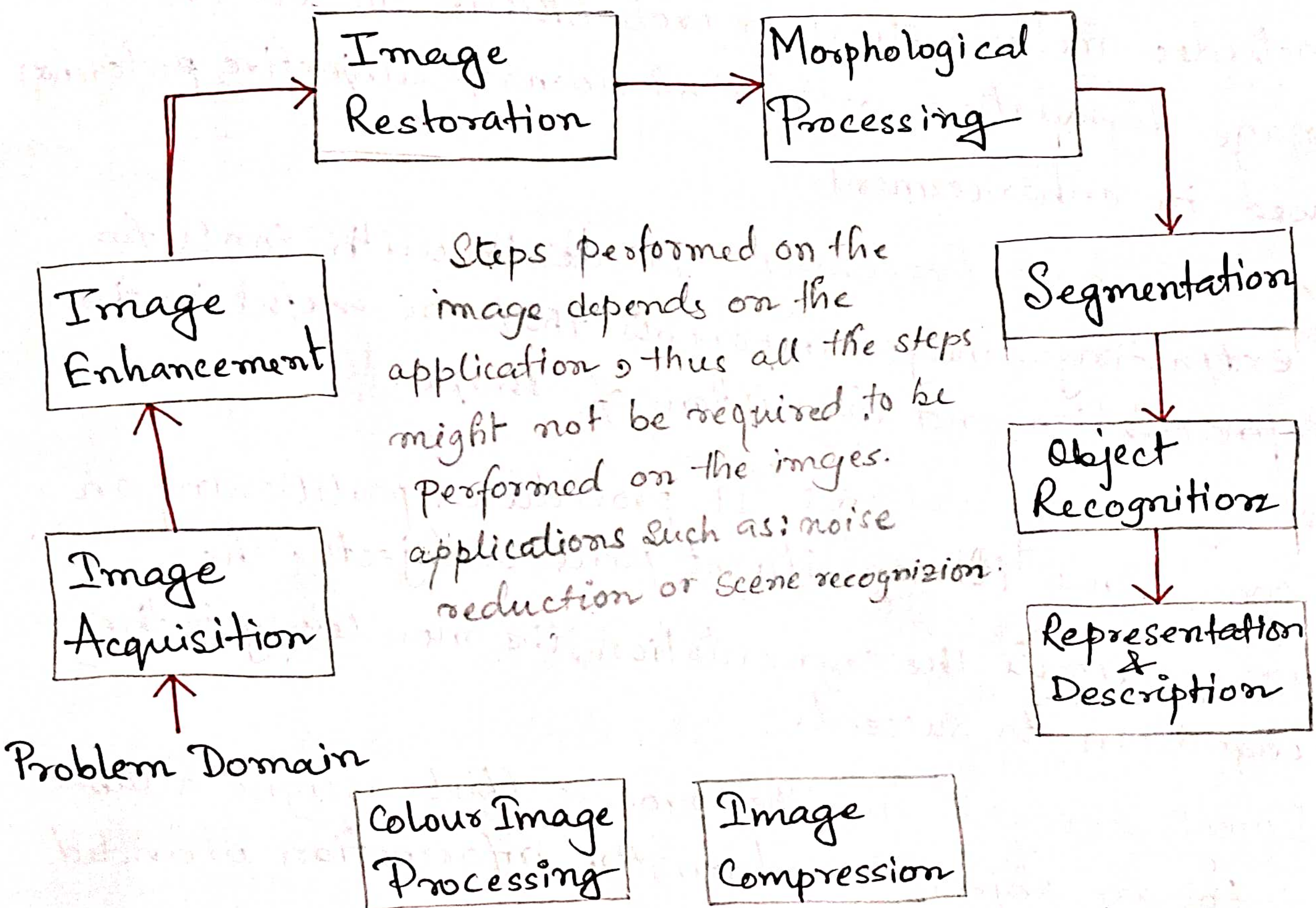
$$0 \leq i(x,y) < \infty$$
$$0 \leq r(x,y) \leq 1$$

→ This reflectance is bounded by 0 (total absorption) and 1 (total reflectance).

→ The nature of $i(x,y)$ is determined by the illuminating source. (very high or very little light).

→ The nature of $r(x,y)$ is determined by characteristics of the imaged objects. (Lighter colored objects or fully dark objects, shinny or dull surface. etc)

Key Stages (or steps) in Digital Image Processing :-



* image Acquisition: The image is captured by sensor (eg. camera) and digitized if the output of the camera or the sensor is not already in digital form, using analog to digital convertor.

(subjective).

* image Enhancement: The process of manipulating an image so that the result is more suitable than the original image, for a specific applications.

Enhancing an image brings out the hidden details of an ~~img~~ image and highlights certain features that may be important.

* Image Restoration (objective): The process of improving the appearance of an image. This mainly includes mathematical or probabilistic models of image degradation instead of human subjective preferences used in enhancement.

* Morphological Processing: It deals with tools for extracting image components that are useful in the representation and description of shape. (Eg:- Fingerprint recognition)

* image Segmentation: It procedures partitioning an image into its constituent parts or objects. The more accurate the segmentation, the more likely is the recognition to succeed.

* Object Recognition: The process that assigns a label to an object based on the information provided by the description.

* Representation and Description: Choosing a representation is only a part of the solution for transforming raw data into a form suitable for subsequent computer processing (mainly recognition)

The two types of image representations are :-

they tells what parts of the image we want to and represent.

- boundary representation : if we want to emphasize on the corners then we use boundary representation
- Regional representation : to emphasize on the texture and skeletal shape of the image we use regional representation.

Description : also called feature selection, deals with extracting attributes that results in some information of interest.

* Image compression :- It includes the techniques for reducing the storage required to save an image or the bandwidth required to transmit it.

* Color image Processing : Its an area that has been gaining importance because of the significant increase in the use of digital image over the internet. It includes the use of colour of the image to extract feature of interest in the image.

Applications in different fields like : computer vision, bio medicines, telecommunications etc.

Sampling and Quantization.

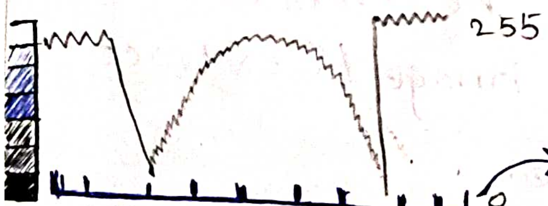
In order to become suitable for digital processing an image function must be digitized both spatially and in amplitude. This digitization process involves two main processes called.

i) Sampling : digitizing the coordinate value is called sampling.

ii) Quantization: digitizing the amplitude value is called quantization.

Example:-

Black - low intensity (0)
Gray - slightly higher intensity
Blue - medium intensity
lightblue - high intensity
white - very high intensity. (255)

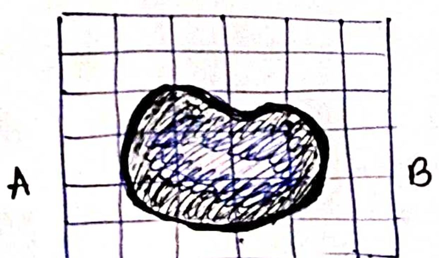


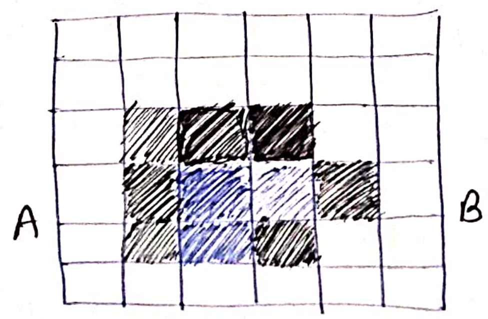
→ dividing the image into equal parts. known as sampling

The intensity graph
→ quantization

Sampling → digitizing the coordinates

Quantization → digitizing the amplitude.





Analog Image → Digital image.

Relationship between pixels, Neighbourhood and Adjacency of pixels:

1) Neighbourhood of pixels can be represented as the following :-

$(y+1)$	$(x-1, y+1)$	$(x, y+1)$	$(x+1, y+1)$
y	$(x-1, y)$	(x, y)	$(x+1, y)$
$(y-1)$	$(x-1, y-1)$	$(x, y-1)$	$(x+1, y-1)$
	$(x-1)$	x	$(x+1)$

(a) $N_4(P)$ 4-neighbourhood : The set of horizontal and vertical neighbours~~hood~~.

	$(x, y+1)$	
$(x-1, y)$	(x, y)	$(x+1, y)$
	$(x, y-1)$	

(b) $N_D(P)$ diagonal neighbours : The set of 4 diagonal neighbours.

$(x-1, y+1)$		$(x+1, y+1)$
	(x, y)	
$(x-1, y-1)$		$(x+1, y-1)$

c) $N_8(P)$: 8 neighbours - Union of 4 diagonal neighbor and 4-neighbours.

$(x-1, y+)$	$(x, y+1)$	$(x+1, y+)$
$(x-1, y)$	(x, y)	$(x+1, y)$
$(x-1, y)$	$(x, y-1)$	$(x+1, y)$

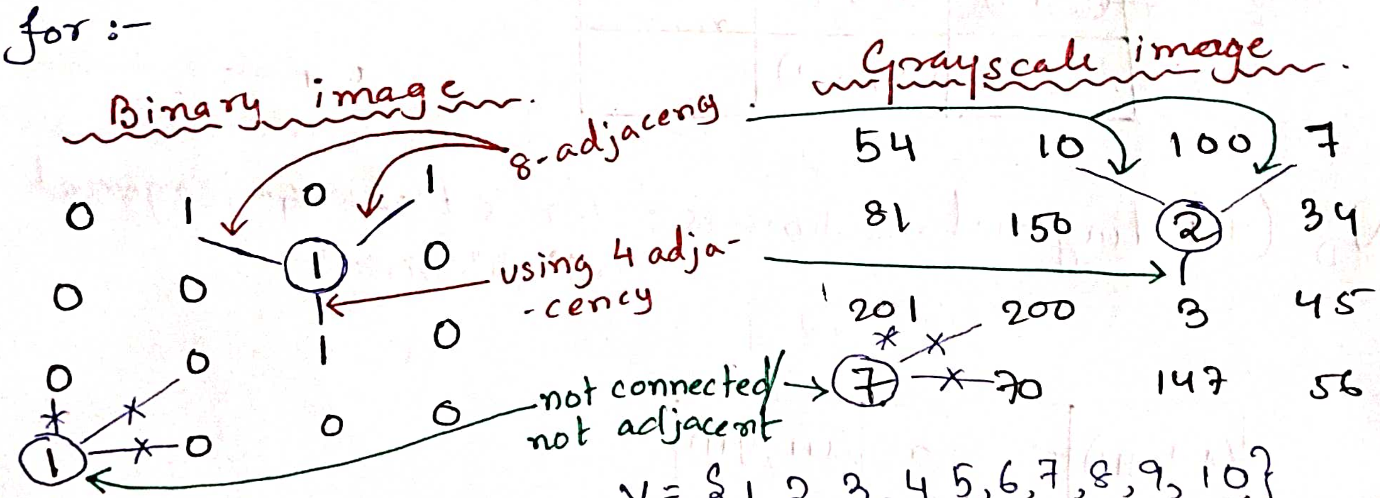
$$\boxed{N_8(P) = N_4(P) + N_D(P)}$$

2) Connectivity / Adjacency

Two pixels that are neighbours and have the same gray level are adjacent.

(a) 4-adjacency and (b) 8 adjacency. (c) m-adjacency. Paths forming using the adjacencies are shown below.

for :-



$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

let $v = \{1\}$

only the elements which belongs to this set can be used to make connections.

(c) m-adjacency (mixed adjacency).

Let $V = \{1\}$.

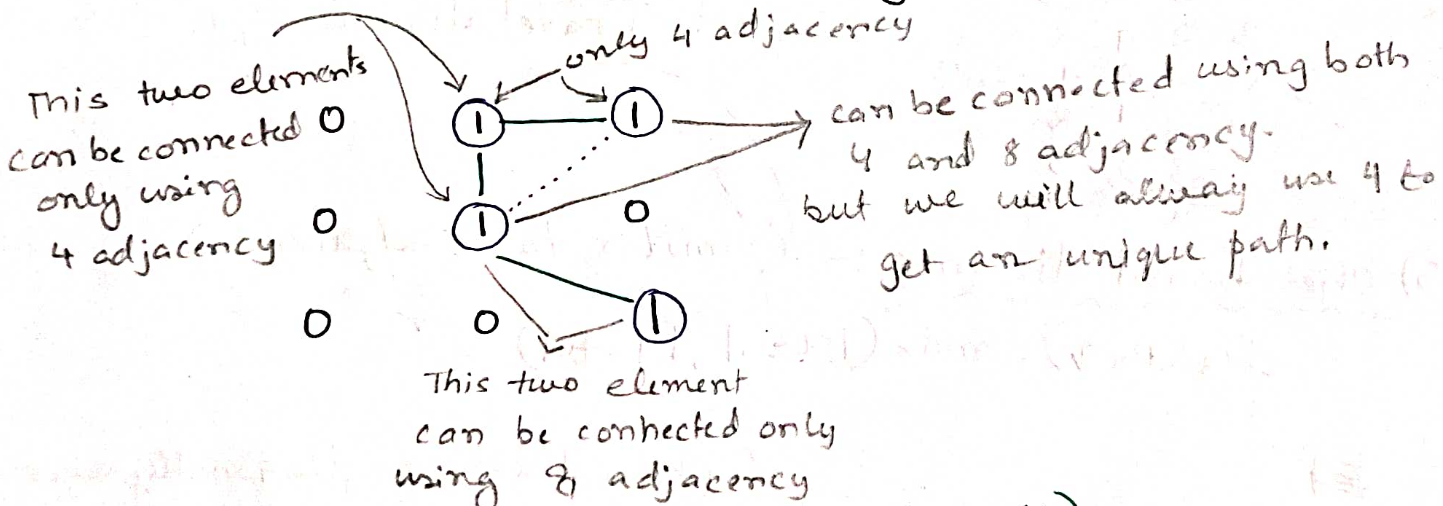
Let the graphs and their different paths are shown below :-

[# Thus there are many paths for a single image which gives rise to ambiguity, but when we make paths we have to make sure that the path is unique.] This is done using the m-adjacency.

To do that, we have to follow a rule, that is;

if two elements can be connected using 4 adjacency and 8 adjacency then always use 4-adjacency and not 8.

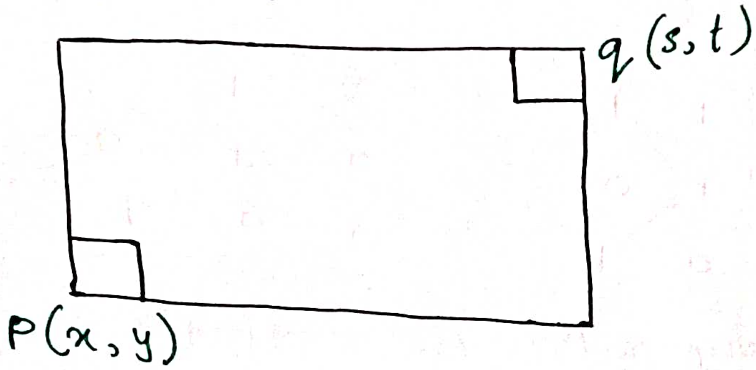
Rule:- ✓ 4 adjacency
✗ 8 adjacency



M-adjacency (unique path).

Distance Measures between Pixels.

Let p and q be pixels with coordinates (x, y) and (s, t) respectively.



The distance between the two points p and q using the different methods are given below:-

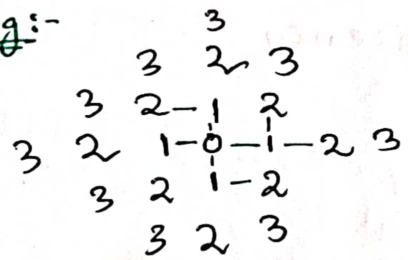
1) Euclidean distance:-

$$D_e(p, q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$

2) City block distance:- (similar to 4 adjacency).

$$D_4(p, q) = |x-s| + |y-t|$$

Eg:-

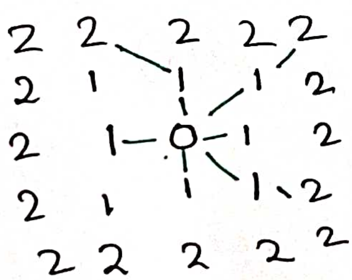


(can't Travel diagonally).

3) Chess board distance:- (Similar to 8 adjacency).

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

Eg:-



(can traverse diagonally also)

4) DM distance :-

It is defined as the shortest ~~path~~ m - path between the points. This distance between 2 pixels depends on the value of the pixels along the path as well as the values of their neighbours.

[# Note : The distance measurement D_4 and D_8 depends only on the starting and the ending point's coordinate and is independent of the path or their ~~&~~ neighbours. Where as the Dm distance measurement depends on both the co-ordinates as well as the paths and their neighbouring co-ordinates.]

Example Question :-

1. An image segment is shown below. Let v be the set of gray level values used to define connectivity in the image. Compute D_4, D_8, and Dm distance between pixels 'p' and 'q' for :-
   i) $V = \{2,3\}$ and ii) $V = \{2,6\}$

(0,0)(P)	2	3	2	6	1
	6	2	3	6	2
	5	3	2	3	5
	2	4	3	5	2
	4	5	2	3	6 (q) (4,4)

Soln :-

Coordinates of $P(x,y) = (0,0)$

Co-ordinates of $q(s,t) = (4,4)$

(city block)

$\therefore D_4(P,q) = |x-s| + |y-t|$

$= |0-4| + |0-4|$

$= 8$ units

(chess board)

$\therefore D_8(P,q) = max(|x-s|, |y-t|)$

$= max(|0-4|, |0-4|)$

$= 4$ units

now for Dm distance, consider :-

i) $V = \{2,3\}$ there is no path from (p) to (q) as (q) doesn't even belong to the set $v = \{2,3\}$

ii) $V = \{2, 6\}$

There is no path between (p) and (q)

(p)	2	3	2	6	1
	6	2	3	6	2 } no path
	5	3	2	3	5
	2	4	3	5	2
	4	5	2	3	6 (q)

Question 2:-

Consider the following image segment. Compute D_m, D_4 and D_8 distance between pixels 'p' and 'q' for $V = \{0, 1\}$ under V is the set of gray levels values used to define connectivity. Repeat for $V = \{1, 2\}$.

	3	1	2	1 (q)
	0	2	0	2
	1	2	1	1
(p)	1	0	1	2

(3,0)

(0,3)

Solun:-

Co-ordinate of $p(x, y) = (0, 3)$

Co-ordinate of $q(s, t) = (3, 0)$

(city block)

$\therefore D_4 = |x - s| + |y - t|$

$= |0 - 3| + |3 - 0|$

$= 3 + 3 = 6$ units

(chess board)

$D_8 = \max(|x - s|, |y - t|)$

$= \max(|0 - 3|, |3 - 0|)$

$= \max(3, 3) = 3$ units

for D_M distance, consider:-

i) $V = \{0, 1\} = 5$ units.

ii) $V = \{1, 2\} = 6$ units

Arithmetic Operations Between Images.

There are array operations which are carried out between corresponding pixels pairs. The four arithmetic operations are denoted as :-

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y).$$

Few important points:-

→ If the result is ~~fo~~ floating point numbers, round off its values.

→ If the result is above the pixel range, select the max range value. (i.e., 255)

→ If the result is below the pixel range select the min range value (i.e, 0).

→ If the result is Infinity, write it as ~~zero~~ zero.

Addition:-

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 200 & 15 \\ 6 & 0 & 10 \\ 8 & 10 & 15 \end{bmatrix}$$

uses :-

- Addition of noisy images for noise reduction.
- Image averaging in the field of astronomy.

Subtraction :-

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 5 \\ 2 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix}$$

Uses:-

- Enhancement of differences between images.
- Mask mode radiography in medical imaging.

Multiplication :-

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 255 & 50 \\ 8 & 0 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

uses:-

- Shading correction
- Masking (or) Region of Interest (ROI) operations.

Division :-

$$\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 5 \end{bmatrix} / \begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

use:-

- Shading correction.

Logical Operation on Images.

AND operation :-

Truth Table :-

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$\overset{X}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}} \text{ AND } \overset{Y}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \overset{Z}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

OR Operation :-

Truth table :-

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$$\overset{X}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}} \text{ OR } \overset{Y}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \overset{Z}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}$$

NOT operation :-

Truth table

A	B
0	1
1	0

$$\overset{X}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}} \text{ NOT } \overset{Y}{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$$

Point Operations

They are a method of image processing in which each pixels in the output image is only dependent upon the corresponding pixel in the i/p image and is ~~tha~~ independent of its location on neighbouring pixels.

Let 'r' be the gray value at a point (x, y) of the input image $f(x, y)$ and 's' be the gray value at a point (x, y) of the output image $g(x, y)$, then

The point output can be defined as :-

$$\underbrace{S}_{\substack{\text{Gray value}\\ \text{of the o/p}\\ \text{Pixels}}} = \underbrace{T}_{\substack{\text{Transformation}\\ \text{function}}} \underbrace{(r)}_{\substack{\text{gray value}\\ \text{of the input}\\ \text{pixels.}}}$$

where T is the point operation of a certain gray-level mapping relationship between the original image and output image.

Different point operations :-

1) Digital negative :-

here,

$$\underbrace{S}_{\substack{\text{output}\\ \text{gray level}}} = \underbrace{(L-1)}_{\substack{\text{highest}\\ \text{intensity}\\ \text{value.}\\ [0 \text{ to } (L-1)]}} - \underbrace{r}_{\substack{\text{input gray}\\ \text{level.}}}$$

input image

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

highest pixels value, require 2^3 numbers to represent, thus 3 bits.

Thus we require highest 8 levels that is 2^{3} and 3 bits to represent the intensity values of the image. Thus here $L = 8$.

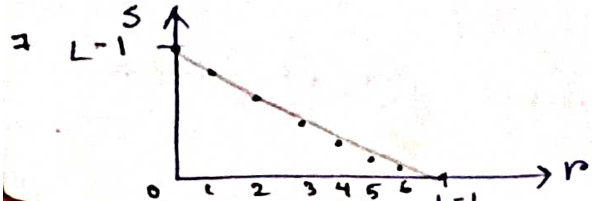
now we calculate the output images intensity correspond. to the input image intensity.

for the corresponding r values, s values will be.

$r = 0$, $s = (8-1) - 0 \Rightarrow 7 - 0 = 7$.
$r = 1$, $s = (8-1) - 1 \Rightarrow 7 - 1 = 6$
$r = 2$, $s = 7 - 2 \Rightarrow 5$
$r = 3$, $s = 7 - 3 = 4$
$r = 4$, $s = 7 - 4 = 3$
$r = 5$, $s = 7 - 5 = 2$
$r = 6$, $s = 7 - 6 = 1$
$r = 7$, $s = 7 - 7 = 0$.

Thus the output Image will be,

3	4	2	5
4	1	3	1
5	5	1	2
0	1	3	6



2) Thresholding with T = 4.

Given input image.

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$$S = \begin{cases} L-1 = 7 & ; r \geq 4. \\ 0 & ; r < 4 \end{cases}$$

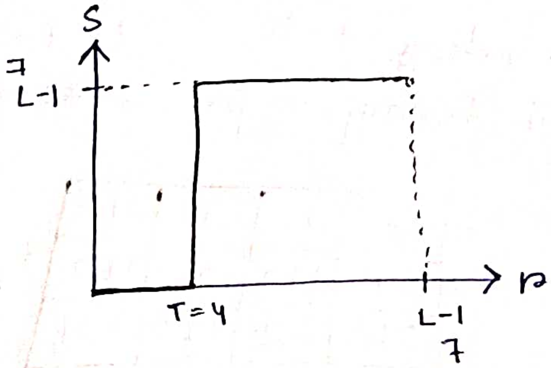
for examples,

for r = 1, s will be 0

r = 5, s will be 7.

Thus the output image will be

7	0	7	0
0	7	7	7
0	0	7	7
7	7	7	0



3) Clipping with $r_1 = 2$ and $r_2 = 5$.

here L = 8 ∴ L-1 = 7.

input image

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

$$\text{clipping function } (s) = \begin{cases} L-1 & ; 2 \leq r \leq 5 \\ 0 & ; \text{otherwise} \end{cases}$$

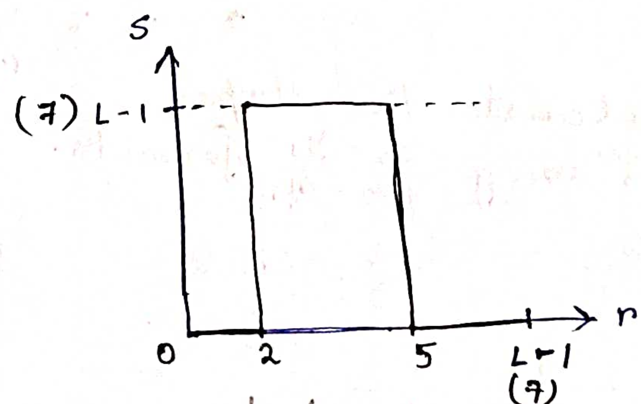
thus, for, r = 2, 3, 4, 5 → s = 7

r = 0, 1, 6, 7 → s = 0

Thus the output image will be:-

7	7	7	7
7	0	7	0
7	7	0	7
0	0	7	0

output image



graph for clipping.

4) Bit-plane slicing :-

$n = 3$

$L = 8$

3-bit plane

MSB plane | Center bit plane | LSB plane

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

input image

Converting it to 3 bit plane, we just use the 3 bit binary representation of ~~the~~ each point.

100	011	101	010
011	110	100	110
010	010	110	101
111	110	100	001

output image

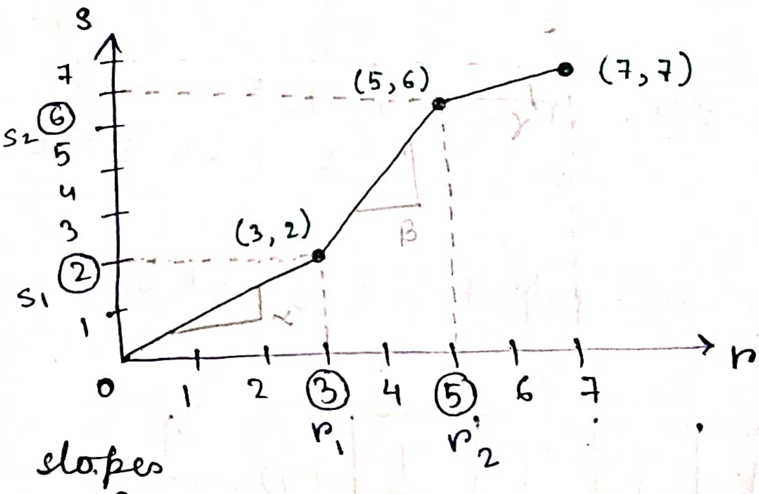
5) Contrast Stretching

Given :- $r_1 = 3$ and $r_2 = 5$

$s_1 = 2$ and $s_2 = 6$.

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

input image



now calculate the slopes α, β, γ using $\frac{y_2 - y_1}{x_2 - x_1}$ formula.

for, $\alpha = \frac{2-0}{3-0} = \frac{2}{3} = 0.66$

$\beta = \frac{6-2}{5-3} = \frac{4}{2} = 2$

$\gamma = \frac{7-6}{7-5} = \frac{1}{2} = 0.5$

The function for contrast streching are :-

$$S = \begin{cases} \alpha . r & 0 \leq r < 3 \\ \beta . (r - r_1) + s_1 & 3 \leq r < 5 \\ \gamma . (r - r_2) + s_2 & 5 \leq r \leq 7 \end{cases}$$

(input gray level) r	(output gray values) S
0	$S = \alpha . r = 0.66 \times 0 = 0$
1	$S = \alpha . r = 0.66 \times 1 = 0.66 \approx 1$
2	$S = \alpha . r = 0.66 \times 2 = 1.32 \approx 1$
3	$S = \beta . (r - r_1) + s_1 = 2 \times (3-3) + 2 = 2$
4	$S = \beta . (r - r_1) + s_1 = 2 \times (4-3) + 2 = 4$
5	$S = \gamma . (r - r_2) + s_2 = 0.5 \times (5-5) + 6 = 6$
6	$S = \gamma . (r - r_2) + s_2 = 0.5 \times (6-5) + 6 = 6.5 \approx 7$
7	$S = \gamma . (r - r_2) + s_2 = 0.5 \times (7-5) + 6 = 7$

Thus the output image will be :-

4	2	6	1
2	7	4	7
1	1	7	6
7	7	4	1

output image

6) Intensity level slicing :-

intensity level slicing with $r_1 = 3$ and $r_2 = 5$

1) Without background → same as clipping.

2) With background.

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

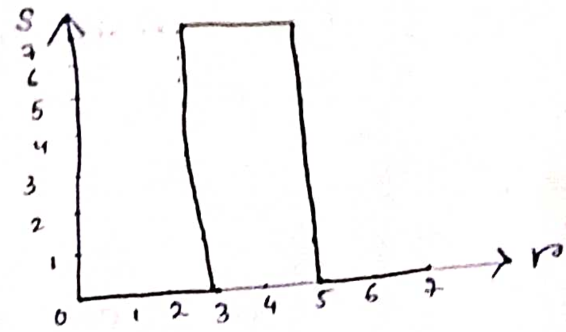
1) without ~~clipping~~ background, slicing function (s) be,

$$S = \begin{cases} L-1 = 7 & 3 \leq r \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

Thus the output image without background will be,

7	7	7	0
7	0	7	0
0	0	0	7
0	0	7	0

output Image.

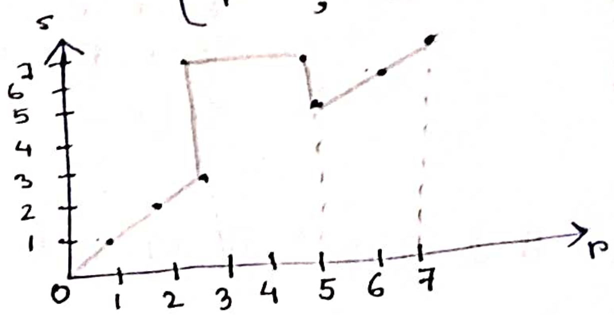


2) With background :- for $r_1 = 3$, $r_2 = 5$

$$S = \begin{cases} L-1 = 7 \ ; & 3 \leq r \leq 5 \\ r \ ; & \text{otherwise} \end{cases}$$

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

input image.



output image →

7	7	7	2
7	6	7	6
2	2	6	7
7	6	7	1

7) Logarithmic Transformation :-

$$S = c \log(1 + r)$$

output gray level ; constant ; input gray level

where c is a constant and it is assumed that $r \geq 0$

110	120	90	90
91	94	98	98
90	91	99	99
90	91	99	99

input image.

i) $c = 1$

ii) $c = \dfrac{L}{\log_{10}(1+L)}$

here We will first calculate L,

highest pixel value is 120, which lies beween 0 to 128

thus, $2^7 = 128 = \cancel{\phi} L$

$\therefore n = 7$. and $L = 128$ thus all the pixels values lies between 0 to 128.

(i) for given $c = 1$, lets calculate the value of s correspondion to the value of r.

r	s.
110	$s = 1 \cdot \log(1+r) = \log(111) = 2.04 \approx 2$
120	$s = 1 \cdot \log(1+r) = \log(121) = 2.08 \approx 2$
90	$s = 1 \cdot \log(1+r) = \log(91) = 1.95 \approx 2$
91	$s = 1 \cdot \log(1+r) = \log(92) = 1.96 \approx 2$
94	$s = 1 \cdot \log(1+r) = \log(95) = 1.97 \approx 2$
98	$s = 1 \cdot \log(1+r) = \log(99) = 1.99 \approx 2$
99	$s = 1 \cdot \log(1+r) = \log(100) = 2$

thus output image will be

output image →

2	2	2	2
2	2	2	2
2	2	2	2
2	2	2	2

ii) for $c = \dfrac{L}{\log_{10}(1+L)} = \dfrac{128}{\log_{10}(129)} = 60.66 \approx 61$ $\quad \because L = 128$

$\therefore$ r	s
110	$2.04 \times 61 = 124.44 \approx 124$
120	$2.08 \times 61 = 126.88 \approx 127$
90	$1.95 \times 61 = 118.95 \approx 119$
91	$1.96 \times 61 = 119.56 \approx 120$
94	$1.97 \times 61 = 120.17 \approx 120$
98	$1.99 \times 61 = 121.39 \approx 121$
99	$2 \times 61 = 122$

124	127	119	119
120	120	121	121
119	120	122	122
119	120	122	122

output image.

8) Power-law transformation :-

$S = cr^{\gamma}$

where c and γ are the positive constant.

Given c = 1 and γ = 0.2. Calculate for :-

$S = Cr^{\gamma}$

$= 1 \cdot r^{0.2}$

$= (r)^{0.2}$

110	120	90
91	94	98
90	91	99

Input image

r	S
110	$S = 1 \cdot (110)^{0.2} = 2.56 \approx 3$
120	$S = 1 \cdot (120)^{0.2} = 2.605 \approx 3$
90	$S = 1 \cdot (90)^{0.2} = 2.45 \approx 2$
91	$S = 1 \cdot (91)^{0.2} = 2.464 \approx 2$
94	$S = 1 (94)^{0.2} = 2.480 \approx 2$
98	$S = 1 (98)^{0.2} = 2.501 \approx 3$
90	$S = 1 (90)^{0.2} = 2.45 \approx 2$
91	$S = 1 \cdot (91)^{0.2} = 2.46 \approx 2$
99	$S = 1 \cdot (99)^{0.2} = 2.506 \approx 3$

∴ Output image :-

3	3	2
2	2	3
2	2	3

Image Enhancement

The process of manipulating the image so that the result is more ~~simple~~ suitable than the original for specific application. Thus image enhancement is the process of emphasizing specific details within an images, while ~~Teach~~ simultaneously reducing or removing any unnecessary elements. This can ~~incl~~ include removing noise, revealing obscured details and adjusting image levels to bring attention to particular features

Image Enhancement Techniques

Spatial domain
involves direct manipulation of individual pixels based on their spatial coordinates at a specific resolution.

Frequency domain
involves modifying the Fourier transform of an image by manipulating pixels in groups indirectly.

Combination
involves a combination of spatial and frequency domains

Point operation
involves applying the same operation to each pixels in grayscale images, based on its original pixel values and independent of its location or neighbouring pixels value.

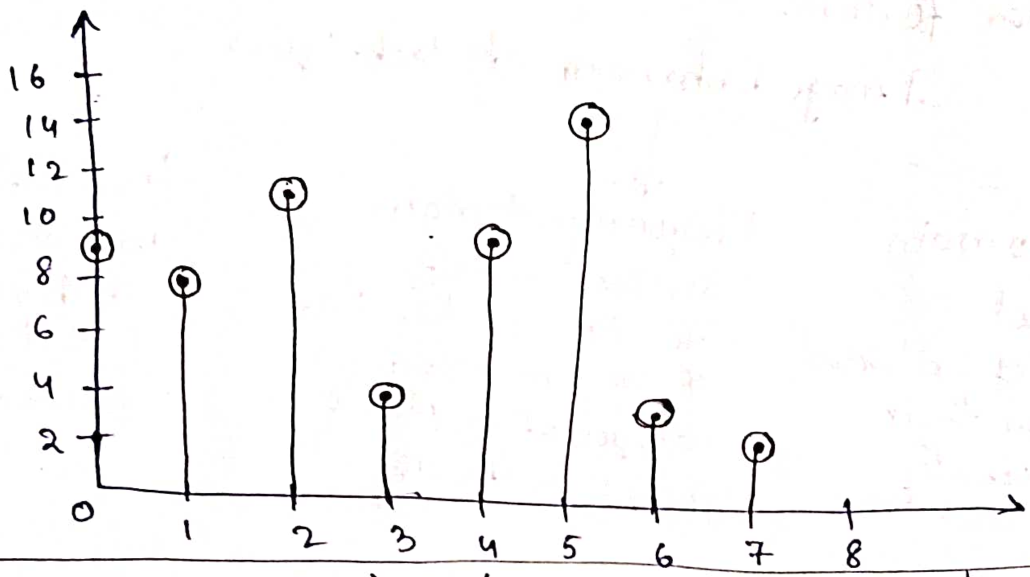
Spatial filter
output of these operations is dependent on ~~its original~~ the value of the function $f(x, y)$ and ~~pixel values and independent~~ is dependent on its neighbourhood.

Spatial Domain

1) Histogram Equalization.

Perform the histogram equalization for an 8x8 image as given below.

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	9	8	11	4	10	15	4	3



Gray Levels (r_k)	no. of Pixels (n_k)	$P(r_k) = n_k/n$ (PDF)	S_k (CDF)	$S_k \times 7$	Histogram Equilization Level
0	9	0.141	0.141	0.987	1
1	8	0.125	0.266	1.862	2
2	11	0.172	0.438	3.066	3
3	4	0.0625	0.5005	3.5035	4
4	10	0.156	0.6565	4.5955	5
5	15	0.234	0.8905	6.2335	6
6	4	0.0625	0.953	6.671	7
7	3	0.047	1	7	7
	64				

new pixels values

new Gray Levels

*PDF = Probability distribution function

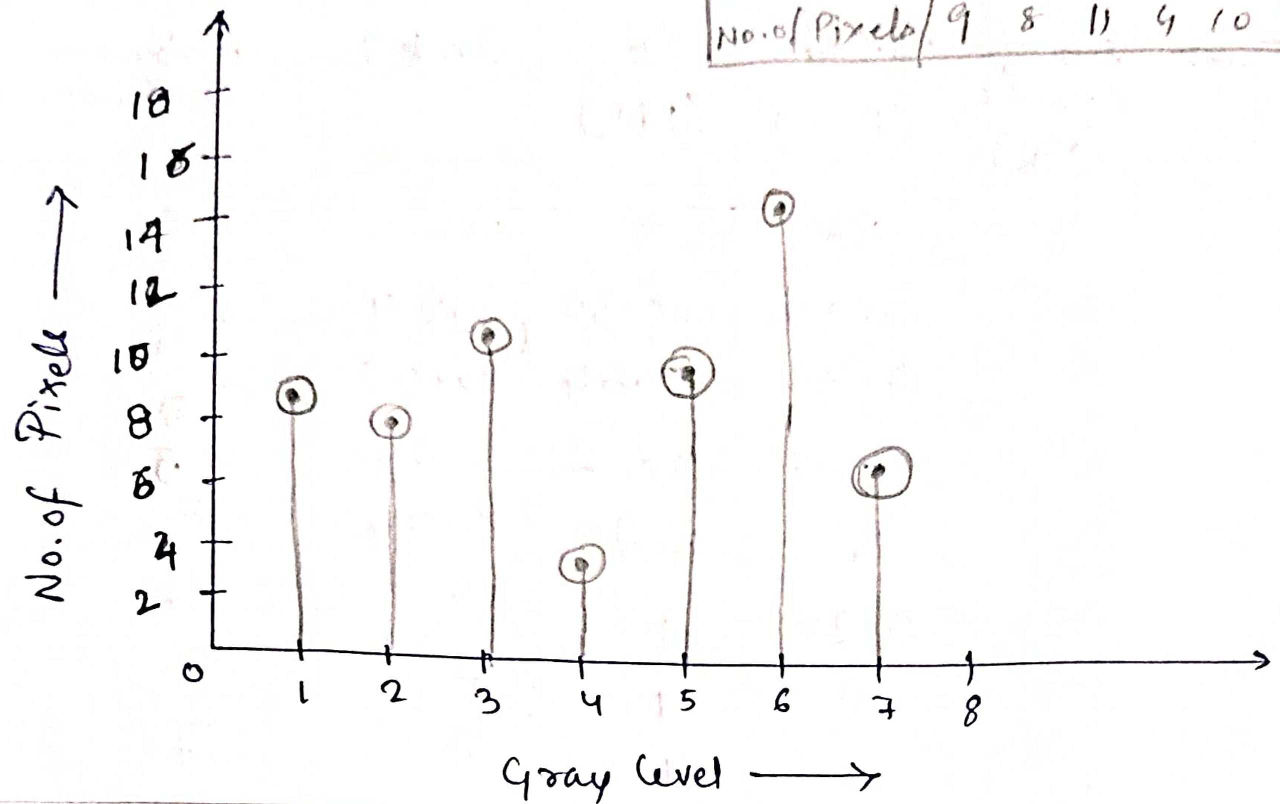
Thus,

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	9	8	11	4	5	6	4	4

* CDF = Cumulative Distribution func.

Equalized Histogram

Gray Level	1	2	3	4	5	6	7
No. of Pixels	9	8	11	4	10	15	7



Q2.) Perform histogram equalization for the following image

$$f(x, y) = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 2 & 5 & 3 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

input image

max value = 5

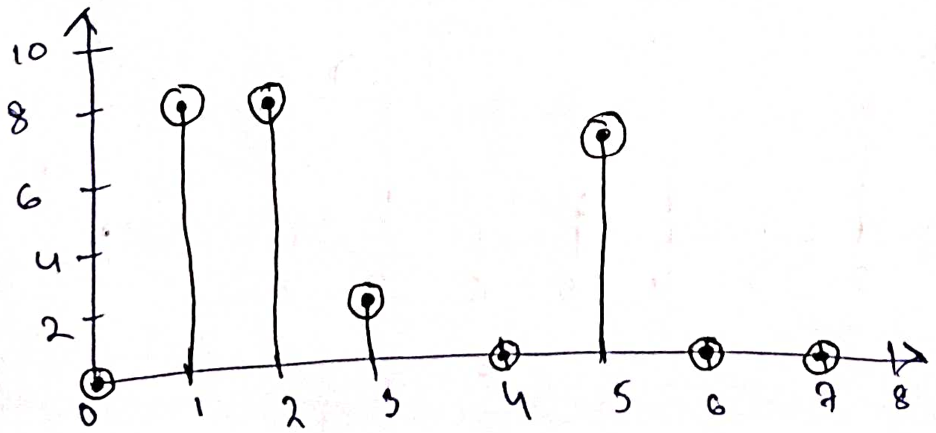
needs 3 bits to represent

$\therefore L = 2^3 = 8$

0 to 7.

Solun:-

Gray Levels (r_k)	0	1	2	3	4	5	6	7
No. of pixels (n_k)	0	8	8	2	0	7	0	0



∴ From the above histogram we get the following values,

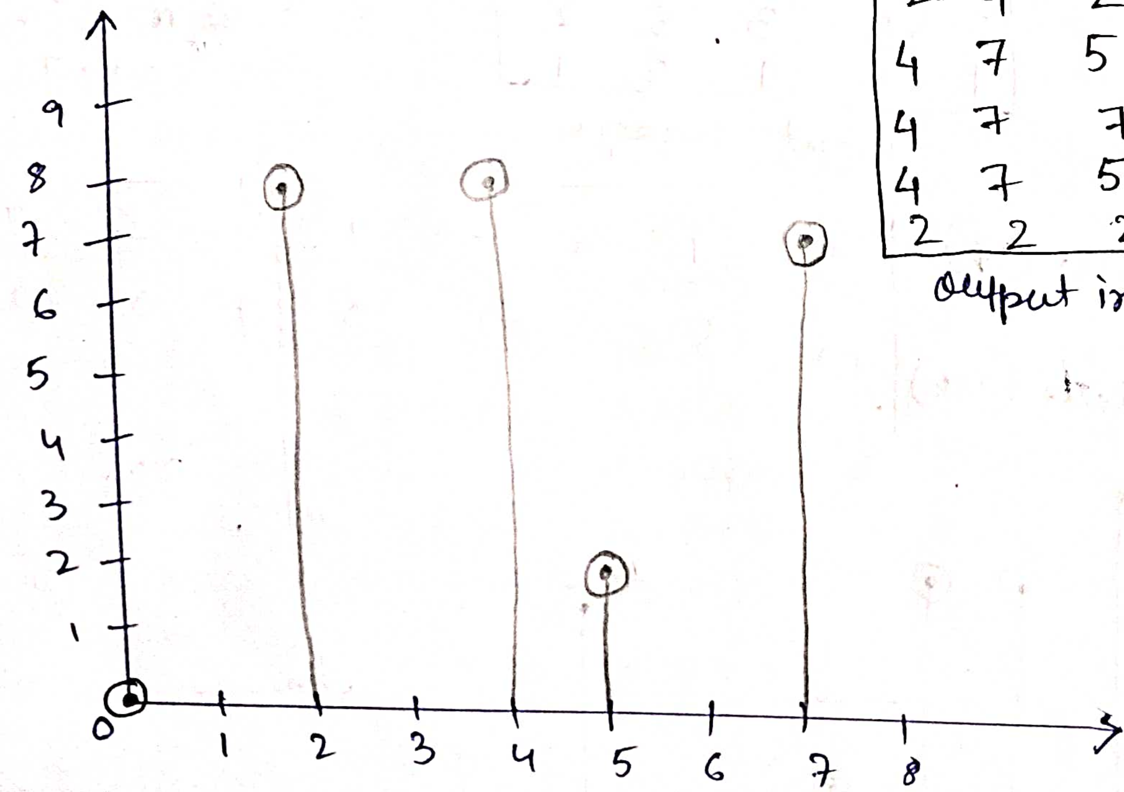
Gray level (r_k)	No. of Pixels (n_k)	$P(r_k)=n_k/n$ (PDF)	S_k (CDF)	$S_k * 7$	Histogram Equilization level
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1	7	7
6	0	0	1	7	7
7	0	0	1	7	7
	25				

no Pixels value

new Gray levels

Thus,

Gray level	0	1	2	3	4	5	6	7
No. of Pixels.	0	0	8	0	8	2	0	7



2	4	2	2	2
4	7	5	7	4
4	7	7	7	4
4	7	5	7	4
2	2	2	4	2

Output image

Histogram matching (Specification).

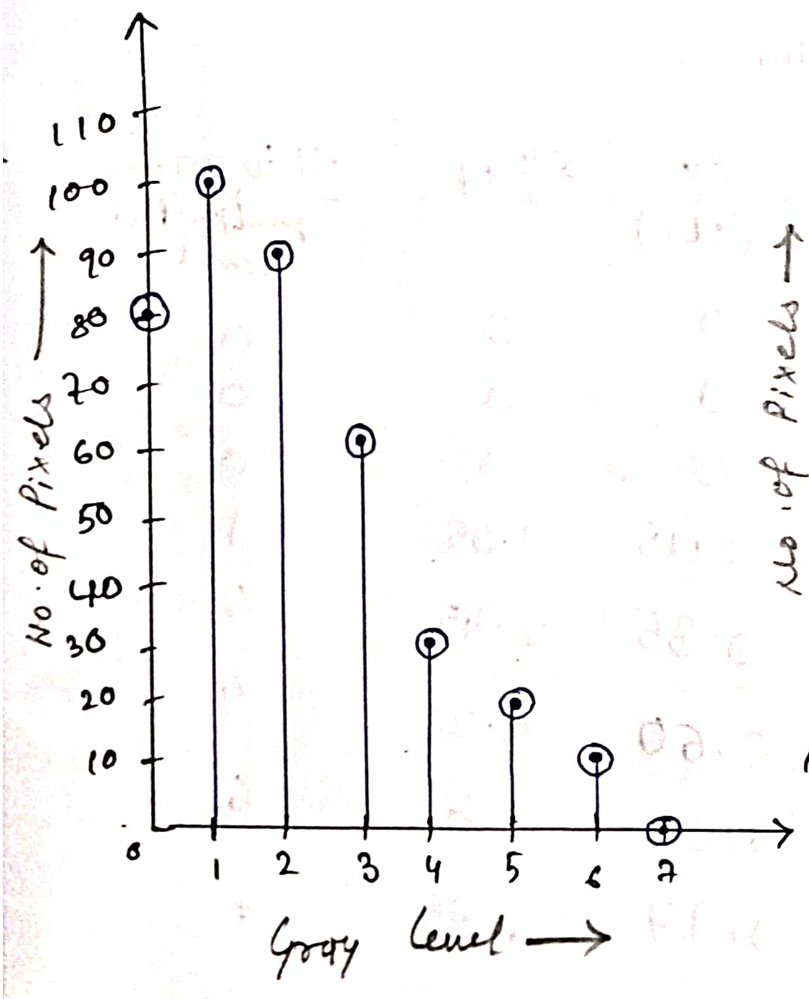
Q) Given below are 2 histograms (i) and (ii). Modify the histogram (i) as given by histogram (ii).

(i)

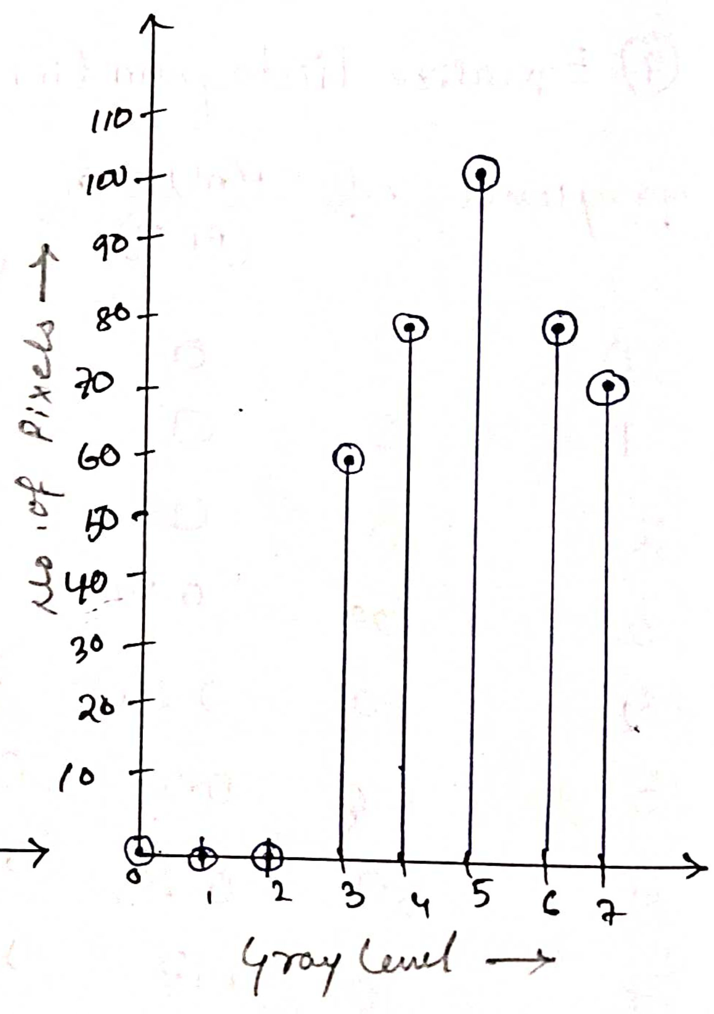
Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	80	100	90	60	30	20	10	0

(ii)

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	60	80	100	80	70



Histogram (i)



Histogram (ii).

① Equalize Histogram (i)

Gray level	nk (no. of pixels)	P(nk)=nk/n (PDF)	Sk (CDF)	Sk×7	Histogram Equalization level.	No. nk
0	80	0.20	0.20	1.4	1	80
1	100	0.25	0.45	3.15	3	100
2	90	0.23	0.68	4.76	5	90
3	60	0.15	0.83	5.81	6	60 + 30 = 90
4	30	0.07	0.90	6.3	6	
5	20	0.05	0.95	6.65	7	20 + 10 + 0 = 30
6	10	0.02	0.97	6.79	7	
7	0	0	0.97	6.79	7	

n = 390

② Equalize Histogram (ii)

Gray level	nk	P(nk)=nk/n (PDF)	Sk (CDF)	Sk×7	Histogram Equalization level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	60	0.15	0.15	1.05	1
4	80	0.20	0.35	2.45	2
5	100	0.25	0.60	4.2	4
6	80	0.20	0.80	5.6	6
7	70	0.17	0.97	6.79	7

n = 390

③ Mapping

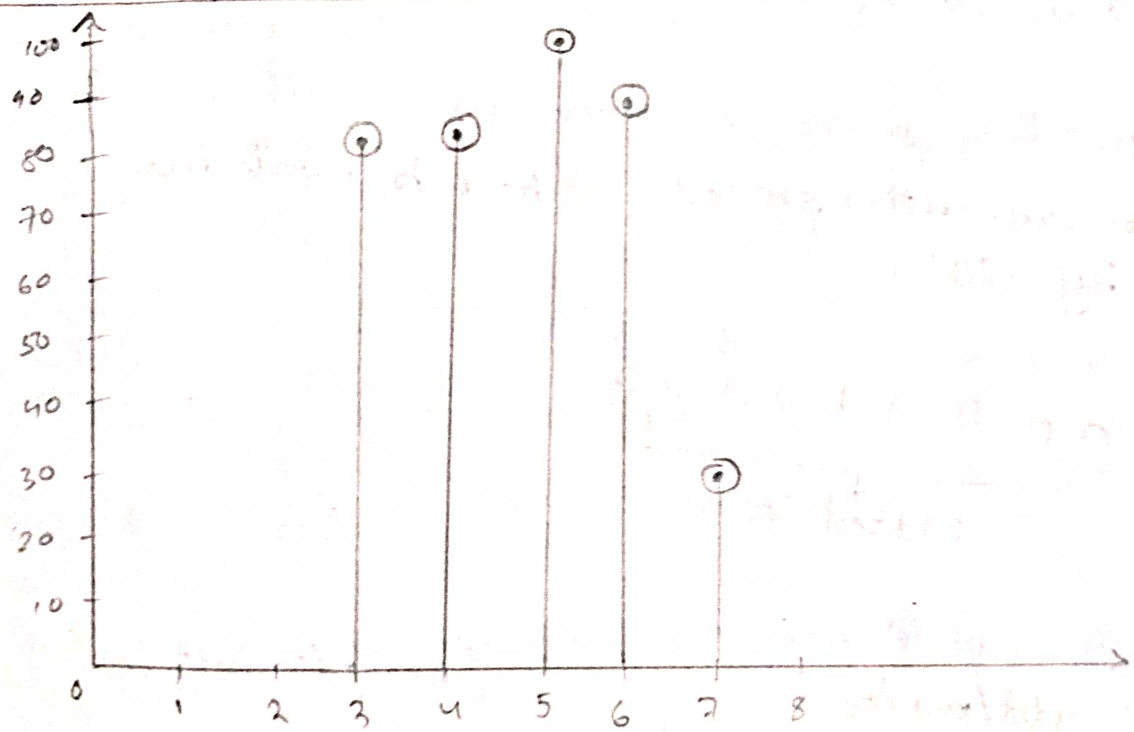
→ Take the first and last columns of histogram (ii).

→ Take last two columns of histogram (i).

Gray level	Histogram Equilization level
0	0
1	0
2	0
3	1
4	2
5	4
6	6
7	7

Histogram Equilization Level		new no. of Pixels (n_k)	
1		80	
3		100	
5		90	
6	6	60	90
6		30	
7	7	20	30
7		10	
7		0	

Gray level	0	1	2	3	4	5	6	7
No. of Pixels	0	0	0	80	80	100	90	30



Fundamentals of
SPATIAL Filtering

The name 'filter' is borrowed from frequency domain processing. It basically refers to accepting (passing) or rejecting certain frequency components.

Eg:- A filter that passes low frequencies is called Low pass filter, inverse is called high pass filter.

We can accomplish a similar smoothing directly on the image itself using spatial filters (also called masks, kernels, templates, and windows).

Linear Spatial Filtering

Convolution　　　　Corelation

Convolution :-

Let $I = \{0, 0, 1, 0, 0\}$ be an image. Using the mask $k = \{3, 2, 8\}$ perform the convolution.

$I = \{0, 0, 1, 0, 0\}$ and $k = \{3, 2, 8\}$

i) zero padding process for convolution
　to convolution process, we have to rotate the kernel by 180°.

8 2 3　　　　　　8 2 3

$\underbrace{0\ 0}$ 0 0 1 0 0 $\underbrace{0\ 0}$

Padded zeros.

(ii) Initial position
Templet/mask

8 2 3
0 0 0 0 1 0 0 0 0
0

$(8 \times 0) + (2 \times 0) + (3 \times 0)$
$= 0$

∴ The ouput is 0 located at the center bit

(iii) Position after one shift.

mask is shifted by one bit.

```
    8 2 3
0 0 0 0 1 0 0 0 0
    0 0
```

$\therefore (8\times0)+(2\times0)+(3\times0)$
$=0$

The output is 0 located at the center bit.

(iv) position after 2 shifts.

mask is shifted again

```
     8 2 3
0 0 0 0 1 0 0 0 0
     0 0 3
```

$(8\times0)+(2\times0)+(3\times1)$
$=3$

$\therefore$ The output is 3 located at the center bit.

(v) Position after 3 shifts.

mask is shifted again

```
       8 2 3
0 0 0 0 1 0 0 0 0
    0 0 3 2
```

$(8\times0)+(2\times1)+(3\times0)$
output $=2$

The output is 2 located at the center bit.

(vi) Position after 4 shifts.

mask is shifed again

```
         8 2 3
0 0 0 0 1 0 0 0 0
  0 0 3 2 8
```

$(8\times1)+(2\times0)+(3\times0)$
output $=8$

(vii) Position after 5 shifts.

mask is shifted again

```
           8 2 3
0 0 0 0 1 0 0 0 0
  0 0 3 2 8 0
```

(viii) Position after 6 shifts. (final position)

```
             8 2 3
0 0 0 0 1 0 0 0 0
  0 0 3 2 8 0 0
```

$(8\times0)+(2\times0)+(3\times0)$
output $=0$

$\therefore$ Further shifts exceeds the range.