Fundamentals of Spatial Filtering :

Spatial filtering is one of the principle tools used in the field of digital image processing for a broad spectrum of applications. The name 'filter' is borrowed from frequency domain processing, where 'filtering' refers to accepting or rejecting certain frequency components.

For example, a filter that passes low frequencies is called lowpass filter. The effect produced by low pass filter is to ~~smooth~~ blur or smooth an image.

The mechanics of spatial filtering :

A spatial filter consists of (i) a neighbourhood pixel and (ii) a predefined operation that is performed on the image pixels encompassed or mapped by the neighborhood.

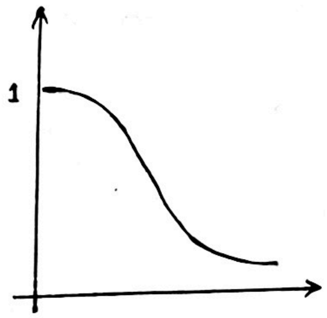
If the operation performed on the image pixels is linear, then the filter is called a linear spatial filter. Otherwise the filter is non-linear.

Filtering creates a new pixel at the same location as of original image but in the new image. A filtered image is generated as the filter visits each pixel in the input image.

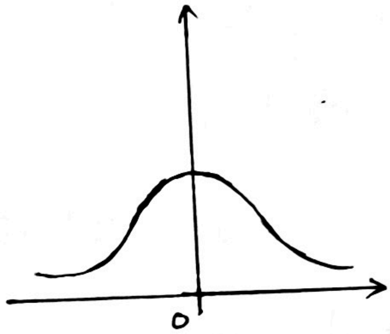
Linear filters:

Linear filters are based on the concepts of impulse response function and transfer function. There are mainly three types of linear filters,

i) Lowpass filter: These type of filters eleminate high frequencies, resulting in the removal of edges and sharpness in images, thus blurring or smoothing the overall image.

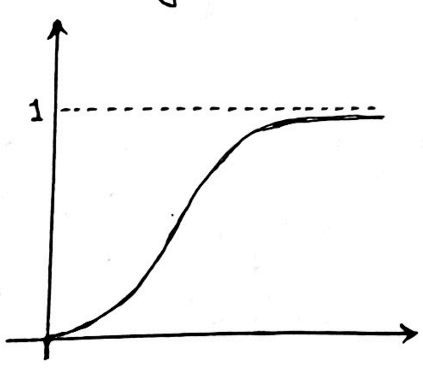


Frequency domain low-pass filter

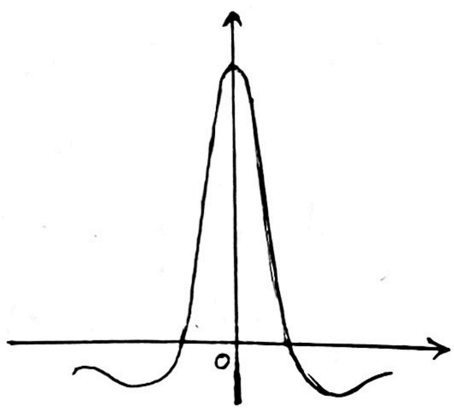


Spatial domain lowpass filter

ii) Highpass filter: These type of filters eleminate low frequencies, resulting in sharper images (i.e the edges of the image are more pronounced.

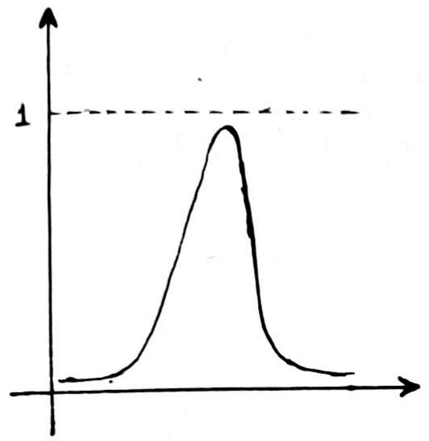


Frequency domain Highpass filter

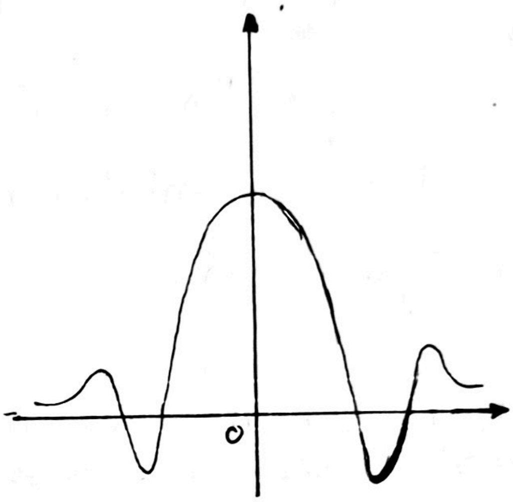


Spatial domain Highpass filter

(iii) Bandpass filter: These type of filters are a combination of lowpass and highpass filters. They are mostly used for image restoration rather than image enhancement.



Frequency Domain bandpass filter



Spatial Domain bandpass filter

A linear filter operates on an M×N neighborhood, defined by M×N mask co-efficient. These co-efficients are used in a sum that is calculated for each pixel at location (x, y) in the input image.

$$\boxed{R = w_1 z_1 + w_2 z_2 + w_3 z_3 + \cdots\cdots + w_n z_n}$$

where, n is the number of co-efficients in the mask, $z_1, z_2, z_3, \ldots, z_n$ are the gray-level of the pixels under the mask, $w_1, w_2, w_3, \ldots, w_n$ are the co-efficients (also called weights) of the mask and R is the response of the linear mask. This is done for each pixel in the input image, thus the mask is moved from pixel-to-pixel.

Nonlinear filters:

These filters operates also on some neighborhood of every pixel at location (x, y), however, the non-linearity is expressed in the computation of R which can now be for example the maximum gray-level, the median gray-level or the minimum gray-level of all the pixel in the neighborhood.

Vector Representation of Linear Filtering:

While using the characteristic response R, of a mask either for corelation or convolution, it is convenient sometimes to write the sum of product as,

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_{mn} z_{mn}$$

$$= \sum_{k=1}^{mn} w_k z_k \qquad \text{———} \quad (1)$$

$$= w^T z$$

where the ws are the co-efficient of an $m \times n$ filter and zs are the corresponding image intensities encompassed by the filter.

If we use equation (1) for correlation we use the mask as given.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

In this case eq^n (1) becomes,

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9$$

$$= \sum_{k=1}^{9} w_k z_k$$

$$= w^T z$$

To use the same equation for convolution, we simply rotate the mask by 180°

Generating Spatial Filter Masks:

Generating an m×n linear spatial filter requires specifying m×n mast co-efficients. These co-efficients are selected based on what the filter is supposed to do.

For example, suppose that we want to replace the pixel in an image by the average intensity of a 3×3 filter mask centered on the pixel. The average value at any location (x, y) in the image is the sum of the nine intensity values of the 3×3 neighbours divided by 9.

$$\therefore R = \frac{1}{9} \sum_{i=1}^{9} z_i$$

In some applications, we have a continuous function of two variables and the objective is to obtain a spatial filter mask based on that function. For example, a Gaussian function of two variable has the basic form

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

where σ is the standard deviation.

To generate a 3×3 filter mask from this function we sample it about its center. Thus $w_1 = h(-1,-1)$, $w_2 = (-1, 0)$, ..., $w_9 = (1, 1)$.

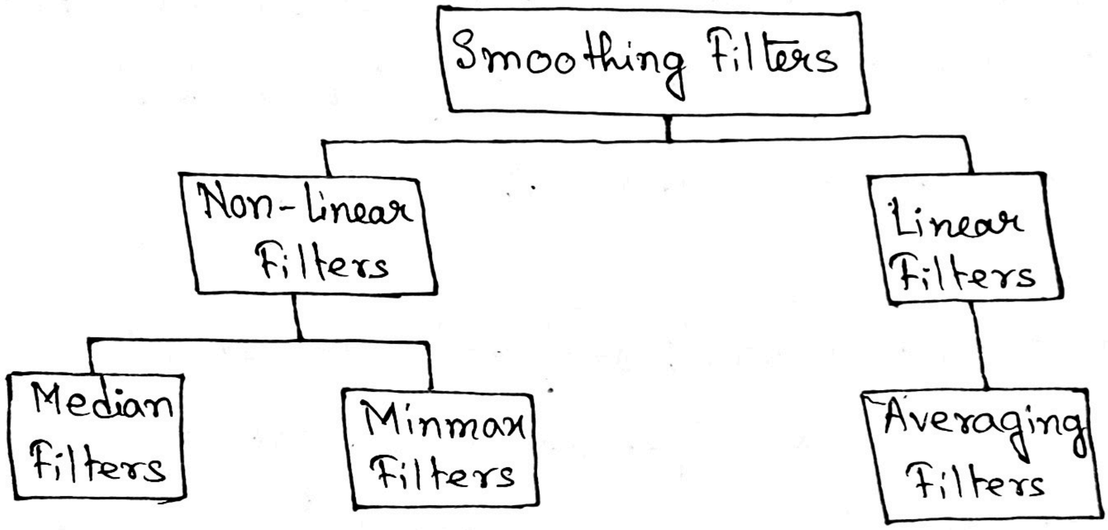
$h(-1,-1)$ w_1	$h(-1,0)$ w_2	$h(-1,1)$ w_3
$h(0,-1)$ w_4	$h(0,0)$ w_5	$h(0,1)$ w_6
$h(1,-1)$ w_7	$h(1,0)$ w_8	$h(1,1)$ w_9

A $m \times n$ filter mask is generated in the similar way.

Generating a non-linear filter requires that we specify the size of the neighbourhood and the operation(s) to be performed on the image pixel contained in the neighbourhood. For example, max operation is non-linear. A 5×5 max filter centered at an arbitary point (x, y) of an image gives the maximum intensity value of the 25 pixels and assigns the value to location (x, y) in the processed image.

Smoothing Spatial Filters:

- Smoothing filters are used for blurring and for noise reduction
- Noise reduction can be accomplished by blurring with a linear filter and also by non-linear filtering.
- Blurring is used in preprocessing tasks, such as removal of small details from an image prior to object extraction and bridging of small gaps in lines or curves.



Smoothing linear Filters:

- The output of a smoothing linear filter is simply the average of the pixels contained in the neighborhood of the filter mask.
- These filters are sometimes called averaging filters or a lowpass filters.

Order-statistic (Nonlinear) Filters :

Order-statistic filters are non-linear spatial filters whose response is based on ordering the pixels contained in the image area encompassed by the filter and then replacing the value of the center pixel with the value determined by the ranking result.

i) Median filter :

- Median filters are one of the best known filter among non-linear filters.
- The median filter replaces the value of a pixel by the median of the intensity values in the neighborhood of that pixel.
- Median filter provide excellent noise reduction capabilities, with considerably less blurring.
- Median filters are particularly effective in the presence of impulse noise, also called salt and pepper noise.

ii) Max filter :

- The max filter is a type of non-linear spatial filtering.
- The max filter is used for finding the brightest points of an image.
- The response of a 3x3 max filter is given by,

$$R = \max\{z_k \mid k = 1, 2, \ldots, 9\}$$

iii) Min filter:

- The min filter is a type of non-linear spatial filtering.
- The min filter is used to find the dull points of an image.
- The response of a 3x3 min filter is given by,

$$R = \min \{ z_K \mid K = 1, 2, 3, \ldots, 9 \}$$

Using the second order derivative for image sharpening:

- This approach uses the laplacian for image sharpening.
- The laplacian for a function $f(x,y)$ of two variables is defined as,

$$\nabla^2 f = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

- In x-direction we have,

$$\frac{d^2 f}{dx^2} = f(x+1, y) + f(x-1, y) - 2f(x,y)$$

- In y-direction we have,

$$\frac{d^2 f}{dy^2} = f(x, y+1) + f(x, y-1) - 2f(x,y)$$

- By combining the above three equations we have,

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x,y)$$

Arithmatic Mean Filter:

- An arithmatic mean filter operation on an image removes short tailed noise such as uniform and Gaussian type noise.
- The arithmatic mean filter is defined as the average of all pixels within a ~~log~~ local region or a filter mask of an image.
- The arithmatic mean is defined as,

$$x' = \frac{1}{n} \sum_{i=0}^{n} z_n$$

- The larger the filter mask becomes the blurring of the image becomes predominent and less high spatial frequency details remains in the image.

Geometric Mean Filter:

- In the geometric mean filter, the value of each pixel is replaced with the geometric mean of the values of the pixels surrounding the region or the filter mask.
- A larger filter mask yields a stronger filter effect with the drawback of some bluring.
- The geometric mean is defined as,

$$G = \sqrt[n]{a_1 . a_2 \ldots a_n}$$

- The geometric mean filter is better at removing Gaussian type noise and preserving the edge features than the arithmatic mean filter.

Harmonic Mean Filter:

- In harmonic mean filter method, the value of each pixel is replaced with the harmonic mean of the values of the pixels in the surrounding region.
- The harmonic mean filter is defined as,

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

- A larger filter mask size yields a stronger filter effect with the drawback of some blurring.
- The harmonic mean filter is very good at removing positive outliers.

Contraharmonic Mean Filter:

- In contraharmonic mean filter, the value of each pixel is replaced with the contraharmonic mean of values of pixel in the surrounding region.
- The contraharmonic mean with order Q is defined as,

$$C_Q = \frac{x_1^{Q+1} + x_2^{Q+1} + \cdots + x_n^{Q+1}}{x_1^{Q} + x_2^{Q} + \cdots + x_n^{Q}}$$

- A contraharmonic mean filter reduces or virtually eliminates the effects of salt-and-pepper noise.
- For positive value Q it eleminates the pepper noise and for negative value of Q it eleminates the salt noise