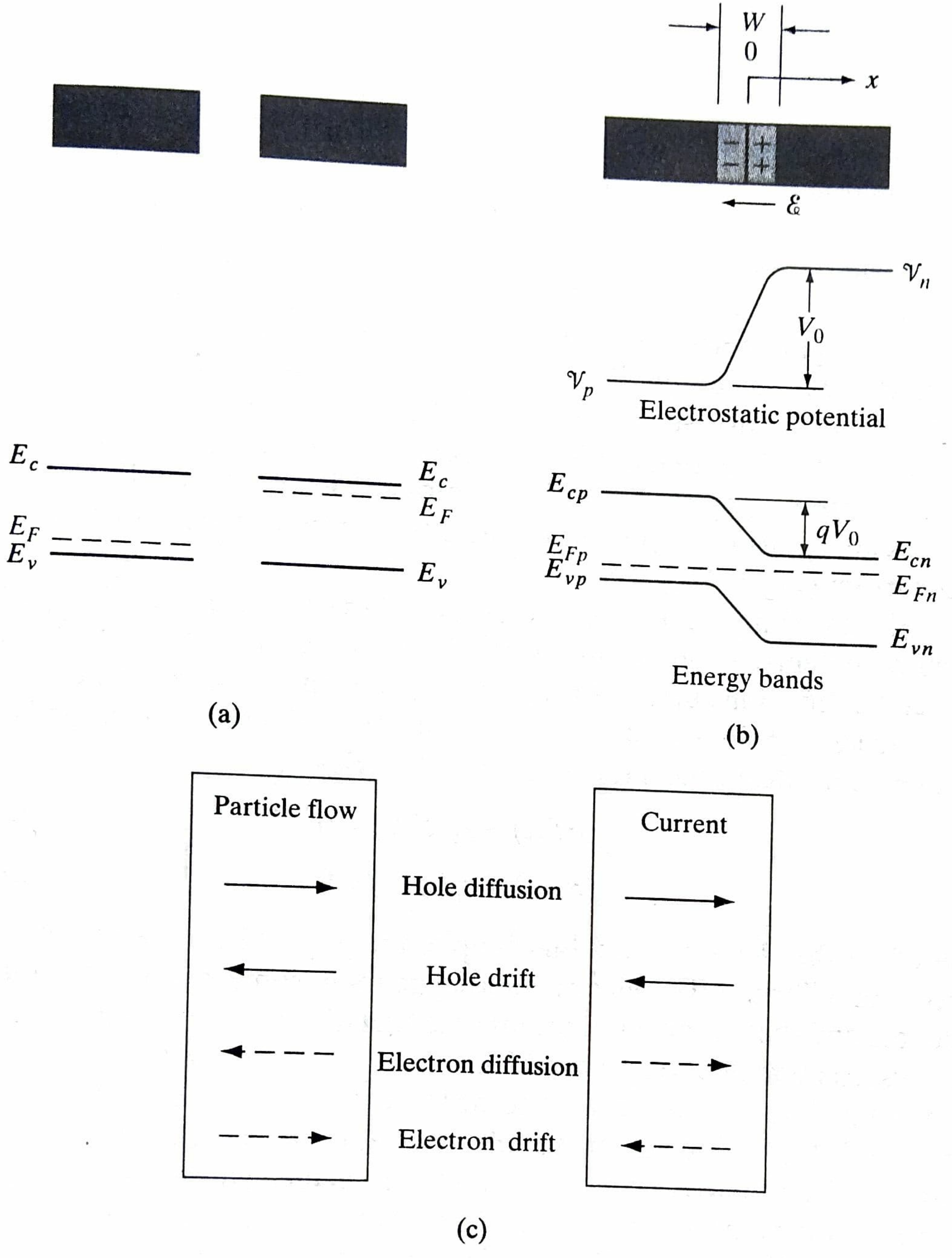


Figure 5–11 Properties of an equilibrium p-n junction: (a) isolated, neutral regions of p-type and n-type material and energy bands for the isolated regions; (b) junction, showing space charge in the transition region W, the resulting electric field $\mathscr{E}$ and contact potential V_0, and the separation of the energy bands; (c) directions of the four components of particle flow within the transition region, and the resulting current directions.

Expression for Contact Potential

In equilibrium, $J_p = J_n = 0$.

$$\Rightarrow J_p = q\mu_p p \vec{\xi} - qD_p \frac{dp}{dx} = 0$$

$$\Rightarrow q\mu_p p \vec{\xi} = qD_p \frac{dp}{dx} \quad \text{[drift current = diffusion current]}$$

$$\Rightarrow -\frac{\mu_p}{D_p} dV(x) = \frac{dP(x)}{P(x)} \quad \left[\because \vec{\xi}(x) = -\frac{dV(x)}{dx}\right]$$

$$\Rightarrow -\frac{q}{KT}\int_{V_p}^{V_n} dV(x) = \int_{p_{po}}^{p_{no}} \frac{dP(x)}{P(x)} \quad \left[\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{KT}{q}\right]$$

$$\Rightarrow -\frac{q}{KT}(V_n - V_p) = \ln \frac{p_{po}}{p_{no}}.$$

$$\Rightarrow V_0 \text{ (contact potential)} \equiv V_n - V_p = \frac{KT}{q} \ln \frac{p_{po}}{p_{no}}$$

① $\Rightarrow \boxed{p_{po} = p_{no}\, e^{\frac{qV_0}{KT}}}$

Similarly, $\boxed{n_{po} = n_{no}\, e^{-\frac{qV_0}{KT}}}$

② $p_{po} = N_A$ and $p_{no} = \frac{n_i^2}{N_D}$

$$\therefore \boxed{V_0 = \frac{KT}{q} \ln \frac{N_A N_D}{n_i^2}}$$

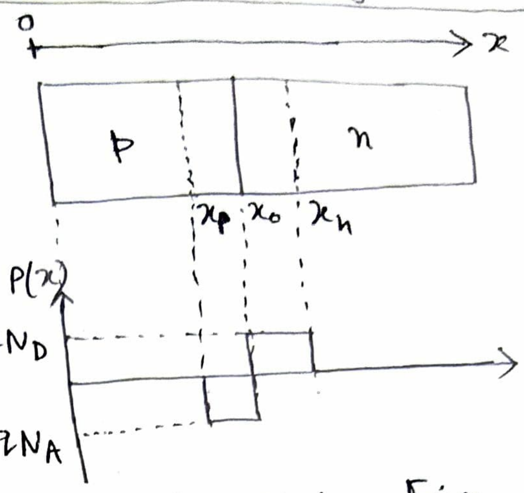
③ $$\frac{p_{po}}{p_{no}} = e^{\frac{qV_0}{KT}} = \frac{N_v e^{-(E_F - E_{vp})/KT}}{N_v e^{-(E_F - E_{vn})/KT}}$$

$$\Rightarrow e^{\frac{qV_0}{KT}} = e^{(E_{vp} - E_{vn})/KT}$$

$$\Rightarrow \boxed{qV_0 = E_{vp} - E_{vn}}$$

Electric-field and potential distribution in a p-n junction diode

$$\rho(x) = q(N_D - N_A + p - n)$$



Abrupt junction

$$\rho(x) = 0 \quad \text{for } 0 \le x \le x_p$$
$$= -qN_A \quad \text{for } x_p < x < x_0$$
$$= qN_D \quad \text{for } x_0 < x < x_n$$
$$= 0 \quad \text{for } x \ge x_n$$

Poisson's equation : $\dfrac{d^2\varphi(x)}{dx^2} = -\dfrac{\rho(x)}{\epsilon}$

$$\Rightarrow \quad \frac{d^2\varphi_p(x)}{dx^2} = \frac{qN_A}{\epsilon_s} \quad \text{for } x_p < x < x_0 \quad \cdots (1)$$

$$\& \quad \frac{d^2\varphi_n(x)}{dx} = -\frac{qN_D}{\epsilon_s} \quad \text{for } x_0 < x < x_n \quad \cdots (2)$$

Boundary conditions :

$\varphi_p(x_0) = \varphi_n(x_0) \cdots (3)$

$\varphi_p(x) = 0$ for $x \le x_p \cdots (4)$

$\varphi_n(x) = V_0 - V$ for $x \ge x_n \cdots (5)$

considering applied potential $-V$ to the n-side

$\dfrac{d\varphi_p(x_0)}{dx} = \dfrac{d\varphi_n(x_0)}{dx} \cdots (6)$

$\dfrac{d\varphi_p(x)}{dx} = 0$ for $x \le x_p \cdots (7)$

& $\dfrac{d\varphi_n(x)}{dx} = 0$ for $x \ge x_n \cdots (8)$

Integrating eq. (1) and applying eq. (7), we get

$$\boxed{\frac{d\varphi_p(x)}{dx} = \frac{qN_A}{\epsilon_s}(x - x_p) \quad \cdots (9) \text{ for } x_p < x < x_0}$$

Similarly, integrating eq. (2) and applying eq. (8),

we get

$$\boxed{\frac{d\varphi_n(x)}{dx} = \frac{qN_D}{\epsilon_s}(x_n - x) \quad \cdots (10) \text{ for } x_0 < x < x_n}$$

$$* \quad \vec{\mathcal{E}}(x) = -\frac{d\varphi(x)}{dx}$$

Integrating eq.(9) and applying eq.( ), we get

$$\boxed{\varphi_p(x) = \frac{qN_A}{2\epsilon_s}(x-x_p)^2} \quad \cdots (11) \text{ for } x_p < x < x_o$$

similarly, integrating eq.(10) and applying eq.(5),

we get
$$\boxed{\varphi_n(x) = -\frac{qN_D}{2\epsilon_s}(x_n - x)^2 + (V_o - V) \quad \cdots (12) \quad \text{for } x_o < x < x_n}$$

Applying eq.(6), one obtains $N_A(x_o - x_p) = N_D(x_n - x_o) \cdots (13)$

& applying eq.(6); $\frac{qN_A}{2\epsilon_s}(x_o - x_p)^2 = (V_o - V) - \frac{qN_D}{2\epsilon_s}(x_n - x_o)^2 \cdots (14)$

From (13), $x_o - x_p = \frac{N_D}{N_A}(x_n - x_o)$

$$\Rightarrow (x_n - x_o) + (x_o - x_p) = (x_n - x_o) + \frac{N_D}{N_A}(x_n - x_o)$$

$$\Rightarrow x_n - x_p = \left(1 + \frac{N_D}{N_A}\right)(x_n - x_o)$$

$$\Rightarrow x_n - x_o = \frac{N_A}{N_A + N_D} \cdot W \quad \cdots (15)$$

similarly, $x_o - x_p = \frac{N_D}{N_A + N_D} \cdot W \quad \cdots (16)$

From (14); $V_o - V = \frac{qN_A}{2\epsilon_s}(x_o - x_p)^2 + \frac{qN_D}{2\epsilon_s}(x_n - x_o)^2$

Using (15) & (16); $V_o - V = \frac{q}{2\epsilon_s}\left[N_A \cdot \frac{N_D^2}{(N_A+N_D)^2} \cdot W + N_D \cdot \frac{N_A^2}{(N_A+N_D)^2} \cdot W\right]$

$$\Rightarrow V_o - V = \frac{q}{2\epsilon_s} \cdot \frac{N_A N_D}{(N_A + N_D)} \cdot W^2$$

$$\Rightarrow \boxed{W = \left[\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o - V)\right]^{1/2}} \quad \cdots (17)$$

* $V_o - V$ is the potential difference across the junction

Now $|Q^+| = qA(x_n - x_0)\cdot N_D$ & $|Q^-| = qA(x_0 - x_p)\cdot N_A$

$$\therefore Q = |Q^+| = |Q^-| \quad [\text{by eq. (13)}]$$

$$= qA(x_n - x_0)N_D = qAN_D \cdot \frac{N_A}{N_A+N_D}\cdot W \quad [\text{by eq. (15)}]$$

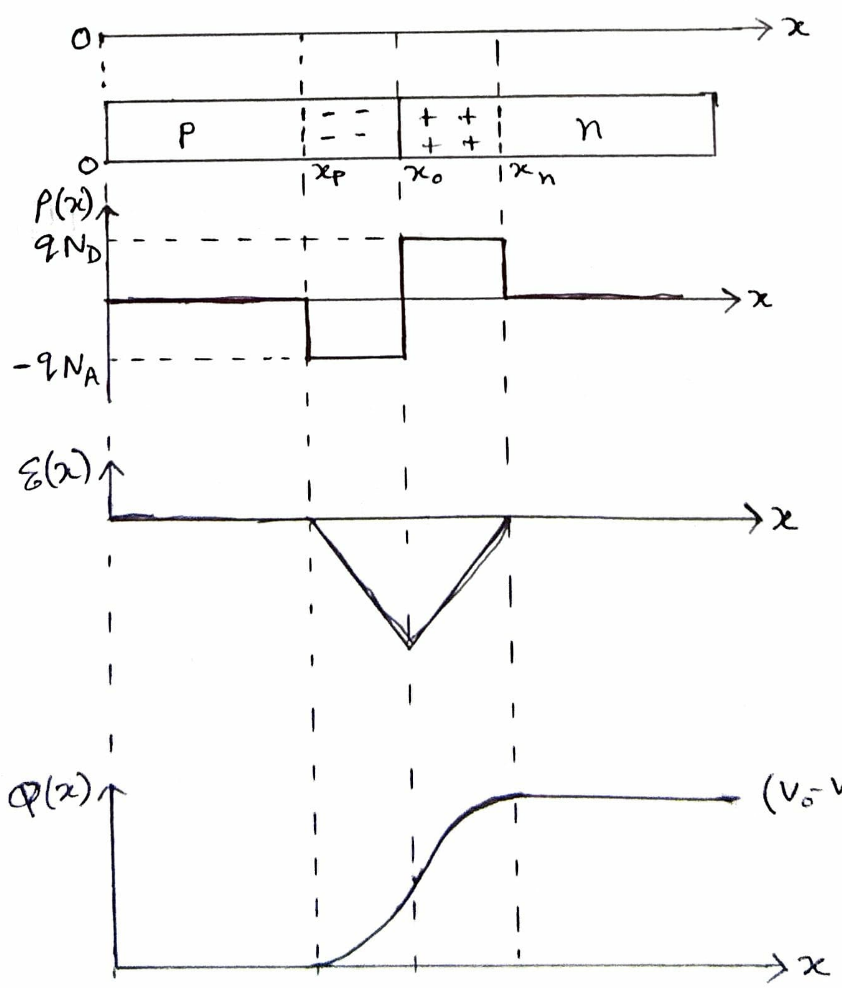
$$= qA\frac{N_AN_D}{N_A+N_D}\left[\frac{2\epsilon_s}{q}\cdot\frac{N_A+N_D}{N_AN_D}\cdot(V_0-V)\right]^{1/2} \quad [\text{by (17)}]$$

$$= A\left[2\epsilon_s q\frac{N_AN_D}{N_A+N_D}\right]^{1/2}(V_0-V)^{1/2}$$

$$\therefore C_j = \left|\frac{dQ}{dV}\right| = \left|A\left(2\epsilon_s q\frac{N_AN_D}{N_A+N_D}\right)^{1/2}\cdot\frac{1}{2}(V_0-V)^{-1/2}\cdot(-1)\right|$$

$$= A\left[\frac{\epsilon_s q}{2}\left(\frac{N_AN_D}{N_A+N_D}\right)\right]^{1/2}(V_0-V)^{-1/2}$$

$$\Rightarrow \boxed{C_j = \frac{\epsilon_s A}{W}}$$



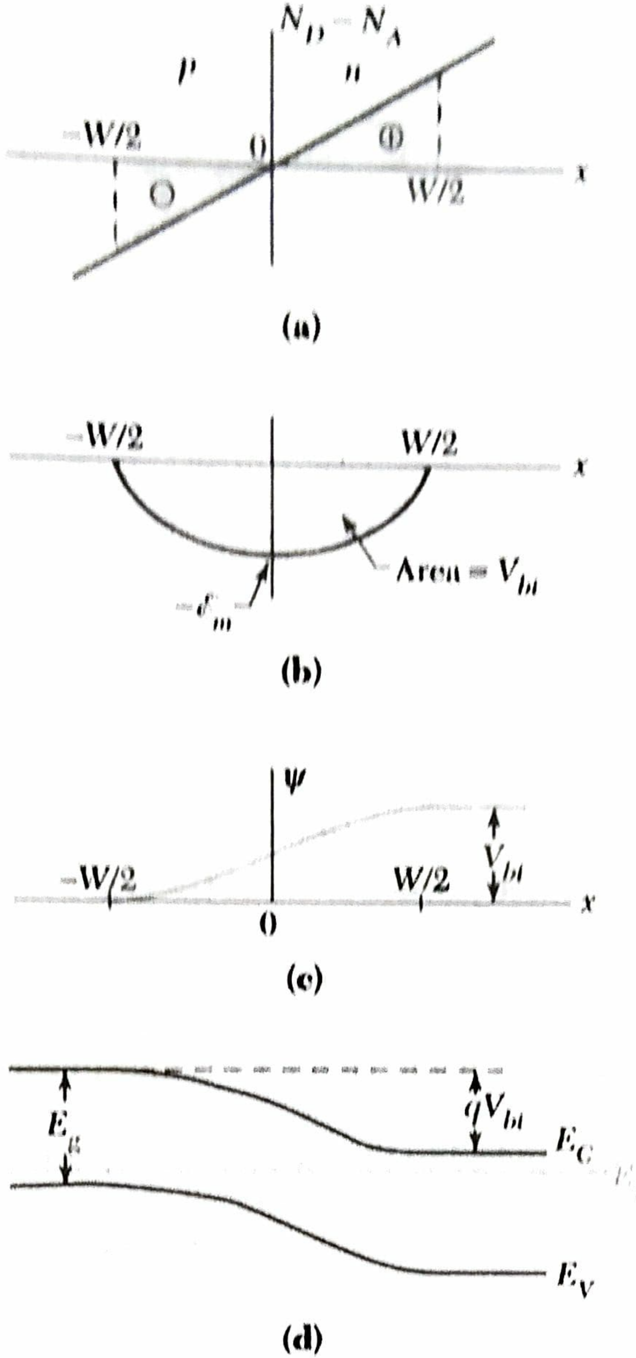


Fig. 9 Linearly graded junction in thermal equilibrium. (*a*) Impurity distribution. (*b*) Electric field distribution. (*c*) Potential distribution. (*d*) Energy band diagram.

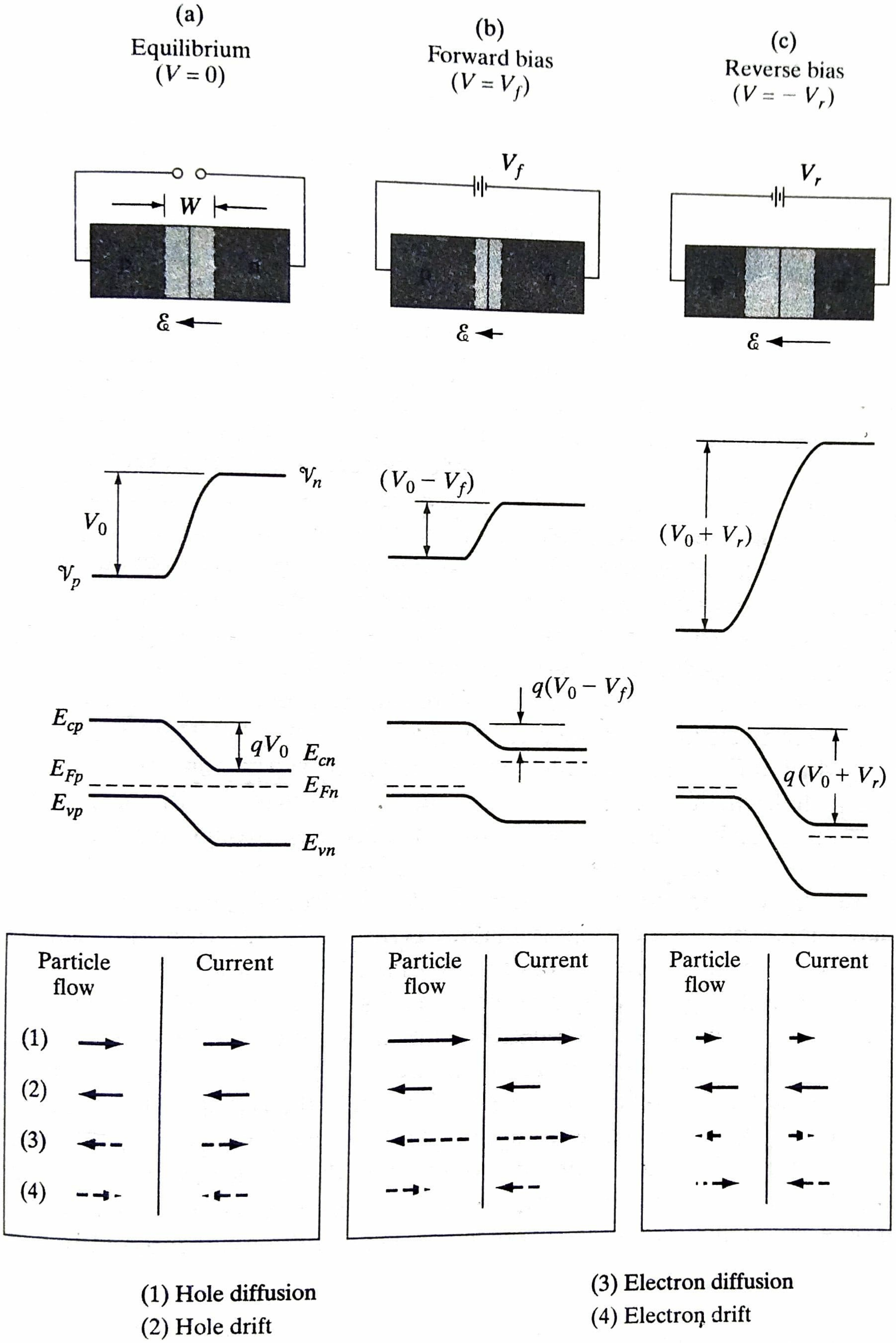
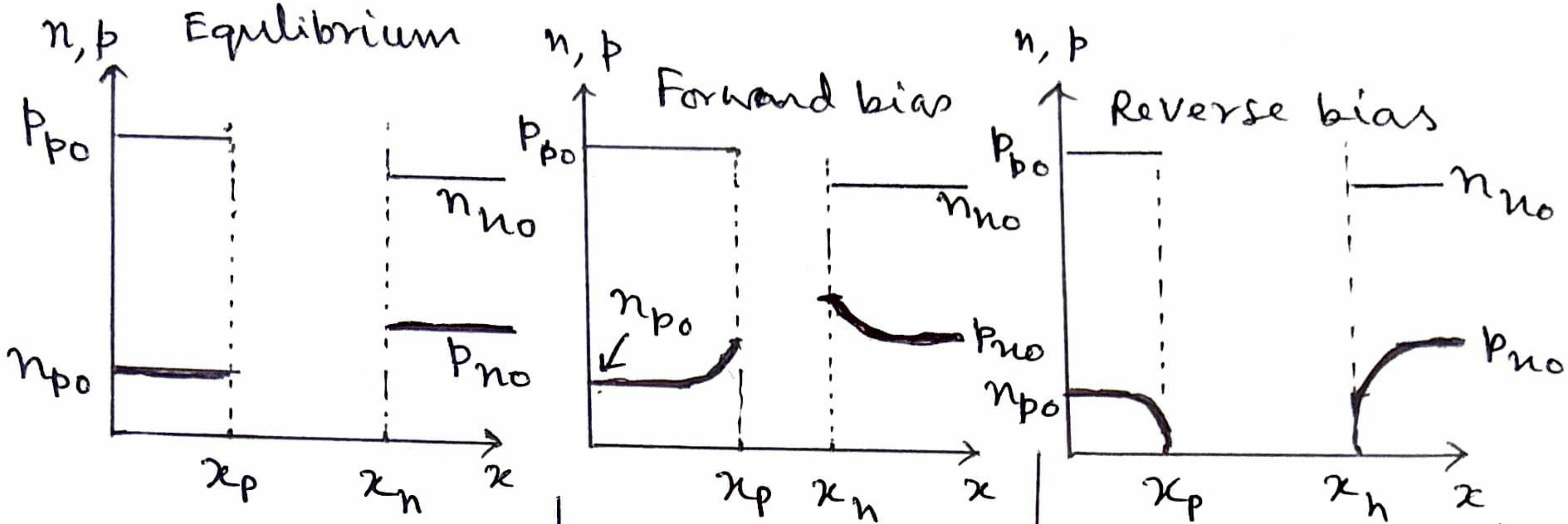


Figure 5–13 Effects of a bias at a p-n junction; transition region width and electric field, electrostatic potential, energy band diagram, and particle flow and current directions within W for (a) equilibrium, (b) forward bias, and (c) reverse bias.

The *diffusion current* is composed of majority carrier electrons on the n side surmounting the potential energy barrier to diffuse to the p side, and

Carrier Distribution



Equilibrium

$$p_{po} = p_{no}\, e^{\frac{qV_o}{kT}}$$

$$n_{no} = n_{po}\, e^{\frac{qV_o}{kT}}$$

Forward bias

$$p_p(x_p) = p_n(x_n)\, e^{\frac{q(V_o-V)}{kT}}$$

$$\Rightarrow p_{po} = p_n(x_n)\, e^{\frac{q(V_o-V)}{kT}}$$

$$\Rightarrow p_n(x_n) = p_{po}\, e^{-\frac{qV_o}{kT}}\, e^{\frac{qV}{kT}}$$

$$\Rightarrow p_n(x_n) = p_{no}\, e^{\frac{qV}{kT}}$$

$$\& \; n_p(x_p) = n_{po}\, e^{qV/kT}$$

Reverse bias

$$p_p(x_p) = p_n(x_n)\, e^{\frac{q(V_o+V)}{kT}}$$

$$\Rightarrow p_{po} = p_n(x_n)\, e^{q(V_o+V)/kT}$$

$$\Rightarrow p_n(x_n) = p_{po}\, e^{-\frac{qV_o}{kT}}\, e^{-\frac{qV}{kT}}$$

$$\Rightarrow p_n(x_n) = p_{no}\, e^{-\frac{qV}{kT}}$$

$$\& \; n_p(x_p) = n_{po}\, e^{-\frac{qV}{kT}}$$

IV characteristics of a p-n junction diode

From continuity equation;

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{p-p_0}{\tau_p}$$

& $$\frac{\partial n}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{n-n_0}{\tau_n}$$

Outside the depletion region, $\vec{\mathcal{E}} = 0$

$$\therefore \frac{\partial J_p}{\partial x} = \frac{\partial}{\partial x}\left(p q \mu_p \vec{\mathcal{E}} - qD_p\frac{\partial p}{\partial x}\right) = -qD_p\frac{\partial^2 p}{\partial x^2}$$

Similarly, $\frac{\partial J_n}{\partial x} = qD_n\frac{\partial^2 n}{\partial x^2}$

Hence, $$\frac{\partial p}{\partial t} = D_p\frac{\partial^2 p}{\partial x^2} - \frac{p-p_0}{\tau_p}$$

& $$\frac{\partial n}{\partial t} = D_n\frac{\partial^2 n}{\partial x^2} - \frac{n-n_0}{\tau_n}$$

In steady state, $\frac{\partial p}{\partial t} = \frac{\partial n}{\partial t} = 0$.

$$\therefore \frac{d^2 p}{dx^2} = \frac{p-p_{no}}{D_p\tau_p}$$ [as $p_0 = p_{no}$ in n-side]

& $$\frac{d^2 n}{dx^2} = \frac{n-n_{po}}{D_n\tau_n}$$ [as $n_0 = n_{po}$ in p-side]

$$\Rightarrow \frac{d^2(p-p_{no})}{dx^2} = \frac{p-p_{no}}{L_p^2}$$ [where $L_p = \sqrt{D_p\tau_p}$ is called diffusion length of holes]

& $$\frac{d^2(n-n_{po})}{dx^2} = \frac{n-n_{po}}{L_n^2}$$ [$L_n = \sqrt{D_n\tau_n}$, diffusion length of electrons in p-side]

Solution: $p - p_{no} = A e^{x/L_p} + B e^{-x/L_p}$

since $p = p_{no}$ at $x = \alpha$, $A = 0$

$$\therefore p - p_{no} = B e^{-x/L_p}$$

Again $p = p_n(x_n)$ at $x = x_n$

$$\Rightarrow B = \left[p_n(x_n) - p_{no}\right] e^{\frac{x_n}{L_p}}$$

$$= \left[p_{no} e^{qV/kT} - p_{no}\right] e^{x_n/L_p}$$

$$= p_{no}\left(e^{qV/kT} - 1\right) e^{x_n/L_p}.$$

$$\therefore p - p_{no} = p_{no}\left(e^{qV/kT} - 1\right) e^{-(x-x_n)/L_p}$$

$$\boxed{\delta p(x) = \Delta P_{x_n} e^{-(x-x_n)/L_p}}$$ where $\delta p = p - p_{no}$ & $\Delta P_{x_n} = p_{no}\left(e^{qV/kT} - 1\right)$

$$\therefore J_p = -qD_p \frac{\partial p}{\partial x} = -qD_p\, p_{no}\left(e^{qV/kT} - 1\right) e^{-(x-x_n)/L_p} \cdot \left(-\frac{1}{L_p}\right)$$

$$= q\frac{D_p}{L_p} p_{no}\left(e^{qV/kT} - 1\right) \cdot e^{-(x-x_n)/L_p}$$

$$\therefore J_p\Big|_{x=x_n} = q\frac{D_p}{L_p} p_{no}\left(e^{qV/kT} - 1\right)$$

similarly, in the p-region;

$$n - n_{po} = n_{po}\left(e^{\frac{qV}{kT}} - 1\right) e^{-(x_p - x)/L_n}$$

& $J_n(x) = qD_n \frac{\partial n}{\partial x} = q\frac{D_n}{L_n} n_{po}\left(e^{\frac{qV}{kT}} - 1\right) \cdot e^{-(x_p - x)/L_n}$

$$\therefore J_n\Big|_{x=x_p} = q\frac{D_n}{L_n} n_{po}\left(e^{qV/kT} - 1\right)$$

Considering no generation/recombination in the depletion region, both J_p & J_n are constant in the depletion region.

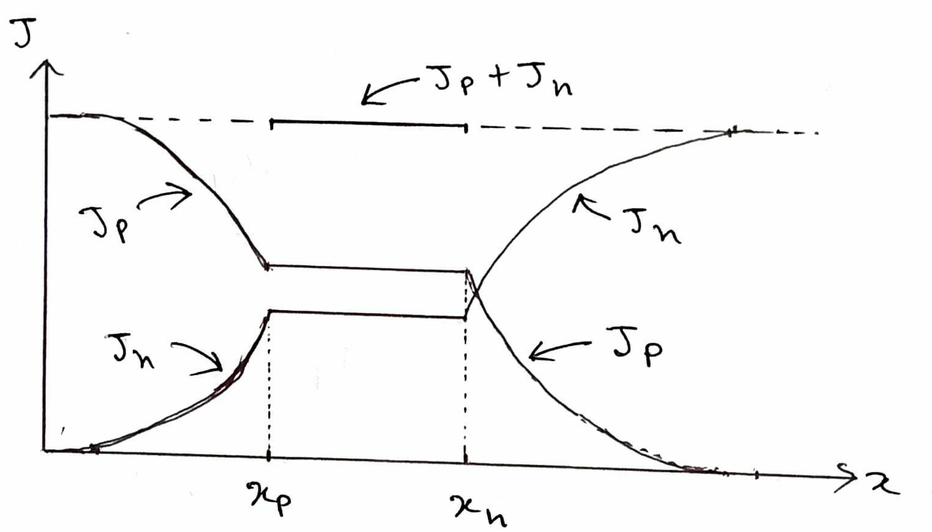
$\therefore$ Total current in the depletion region,

$$J = J_p + J_n = q\left(\frac{D_p}{L_p} p_{no} + \frac{D_n}{L_n} n_{po}\right)\left(e^{\frac{qV}{kT}} - 1\right)$$

$$\Rightarrow J = J_o\left(e^{qV/kT} - 1\right)$$

$$\boxed{\therefore\ I = J \times A = I_o\left(e^{qV/kT} - 1\right)}$$

As current is continuous, J is constant everywhere.



For p^+-n junction, i.e., $N_A \gg N_D$

$$J = q\left(\frac{D_p}{L_p}\cdot\frac{n_i^2}{N_D} + \frac{D_n}{L_n}\cdot\frac{n_i^2}{N_A}\right)\left(e^{qV/kT} - 1\right)$$

$$= q\frac{D_p}{L_p}\cdot\frac{n_i^2}{N_D}\left(e^{qV/kT} - 1\right) = q\frac{D_p}{L_p} p_{no}\left(e^{qV/kT} - 1\right)$$

$$= q\frac{D_p}{L_p} N_v\, e^{-(E_F - E_{vn})/kT}\cdot e^{qV/kT} \quad \text{(neg. 1 in forward bias)}$$

$$= \frac{qD_p}{L_p} N_v\, e^{[qV - (E_F - E_{vn})]/kT}$$

$\therefore$ Current is small if the forward bias is less than $(E_F - E_{vn})/q$. The current increases rapidly if the forward bias is slightly higher than $(E_F - E_{vn})/q$. This is the cut-in voltage, which is slightly less than bandgap value. For heavily doped p-side $E_F = E_{vp} \Rightarrow E_F - E_{vn} \simeq E_{vp} - E_{vn} = qV_o$

Diffusion Capacitance

Considering p^+-n junction, total excess hole charge in the n-side, $Q_p = qA\int_0^\infty \delta p(x)\,dx = qA\Delta p_{x_n}\int_0^\infty e^{-(x-x_n)/L_p}\,dx$

$$= qAL_p\,\Delta p_{x_n} \cong qAL_p\,p_{no}\,e^{qV/kT}$$

$$\therefore C_s \equiv \frac{dQ_p}{dV} = \frac{q^2}{kT}AL_p\,p_{no}\,e^{qV/kT} = \frac{q}{kT}\cdot Q_p = \frac{q}{kT}\cdot I\,\tau_p$$

& $G_s = \dfrac{dI}{dV} = \dfrac{q}{kT}\cdot I$

Temperature Dependance of IV characteristics

$$I = I_0\left(e^{qV/kT}-1\right)\;;\; I_0 \propto n_i^2 = AT^3 e^{-E_g/kT}$$

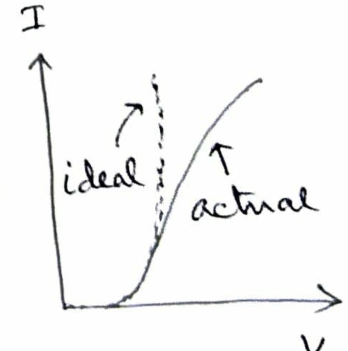
$$\left.\begin{aligned}\frac{dI_0}{dT} &\simeq \cdot 08 \text{ for Si}\\ &\simeq \cdot 11 \text{ for Ge}\end{aligned}\right\}$$

I_0 gets doubled for every 10°c rise in temperature for Si

Also, $I \propto e^{qV/kT}$ & $\dfrac{dV}{dT} \simeq -2\cdot 5$ mV/°c for Si.

Effects of Ohmic Loss

We assumed that the applied voltage appears entirely across the junction (i.e., $V_a = V_j$). This is valid in most cases as doping is fairly high (i.e., small ρ) and area A is large compared to length l. For low doping, device may exhibit Ohmic effects, as



$$V_j = V_a - I\left[R_p(I) + R_n(I)\right]$$

R_p & R_n are function of I because of conductivity modulation.

Effects of Generation - Recombination in the depletion region

$$J = \underbrace{q \frac{D_p}{L_p} \cdot \frac{n_i^2}{N_D} \cdot e^{qV/kT}}_{\text{diffusion current}} + \underbrace{\frac{q W n_i}{2\tau_r} \cdot e^{qV/2kT}}_{\text{recombination current}}$$

τ_r is the effective recombination lifetime (indirect)

Overall $J \propto e^{qV/\eta kT}$; η = ideality factor

$1 < \eta < 2$; $\eta = 1$ when diffusion current dominates

$\phantom{1 < \eta < 2 ;\ \eta} = 2$ " recombination " "

Value of η (2) depends upon temperature (low), bandgap (wide) and applied bias (low).

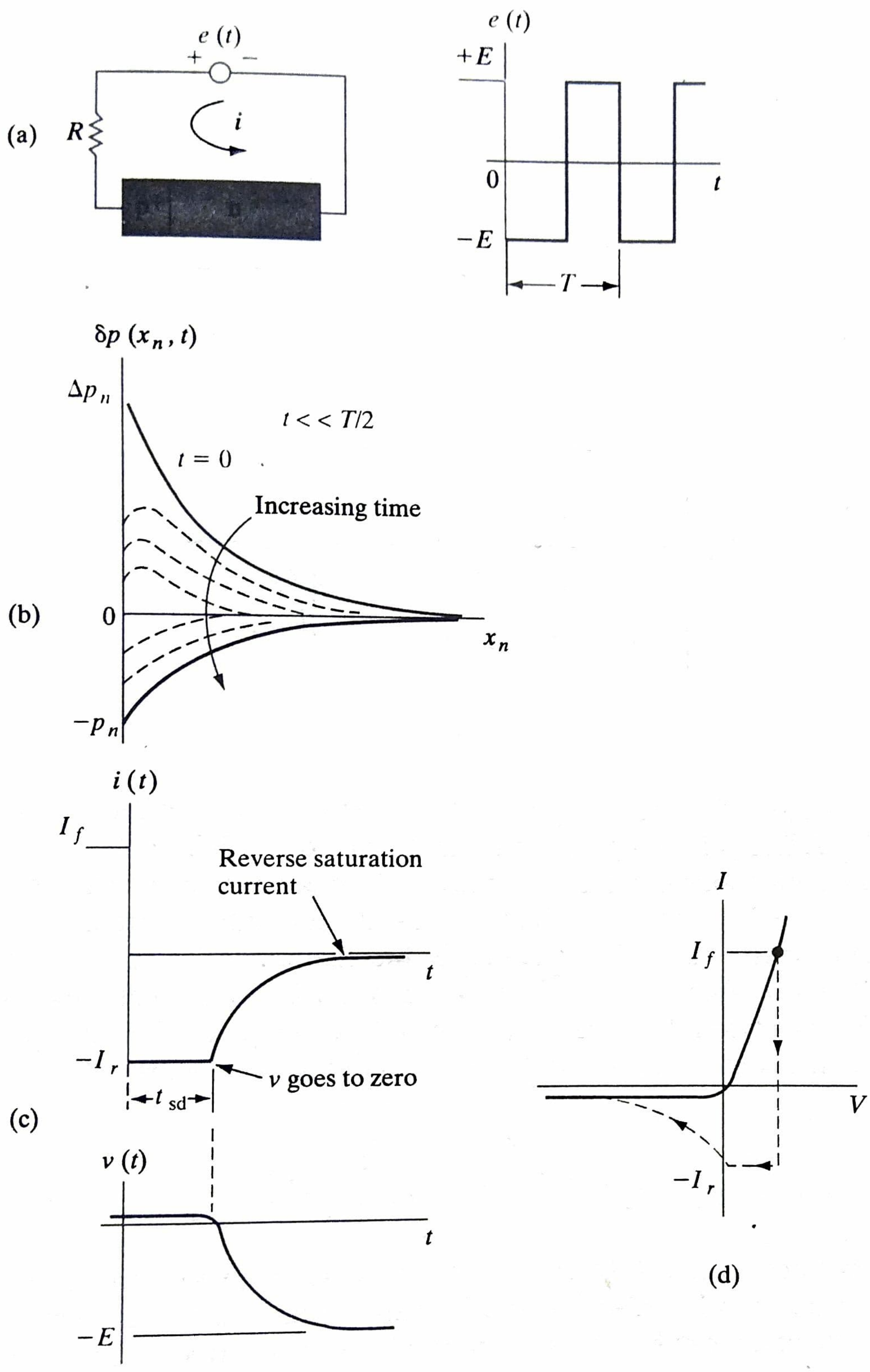


Figure 5–28
Storage delay time in a p⁺-n diode: (a) circuit and input square wave; (b) hole distribution in the n-region as a function of time during the transient; (c) variation of current and voltage with time; (d) sketch of transient current and voltage on the device *I–V* characteristic.

comes negative, the junction exhibits a negative voltage. Since the reverse-bias voltage of a junction can be large, the source voltage begins to divide between R and the junction. As time proceeds, the magnitude of the reverse current becomes smaller as more of $-E$ appears across the reverse-biased junction, until finally the only current is the small reverse saturation current which is characteristic of the diode. The time t_{sd} required for the stored charge

Figure 5–29 Effects of storage delay time on switching signal: (a) switching voltage; (b) diode current.