❖ Properties of Fourier Series

A Fourier series is a technique for decomposing a periodic signal into a sum of sine and cosine terms. It is used to represent periodic signals with discrete spectral components.

A periodic signal **x[n]** with DFS coefficients $\mathbf{X_k}$ is represented as

$$x[n] \leftrightarrow X_k$$

1. Linearity

If x[n] and y[n] are two periodic signals with period N, and their corresponding DFS coefficients are $\mathbf{X_k}$ and Y_k . i.e.

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{And } y[n] \leftrightarrow Y_k$$
$$\text{Then, } Ax[n] + By[n] \leftrightarrow AX_k + BY_k$$

2. Time — shifting

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{then } x[n - n_o] \leftrightarrow e^{-jkw_o n_o} X_k$$

Here $x[n - n_o]$ signal is the time-shifted signal.

3. **Frequency-shifting**

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{Then } e^{jn\omega_0 k_0} x[n] \leftrightarrow X_{k-k_0}$$

K_o is a real constant.

4. Time-Reversal

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{Then } x[-n] \leftrightarrow X_{-k}$$

Reversing the time sequence corresponds to complex conjugation of the frequency sequence.

5. Periodic Convolution

$$\text{If } x[n] \leftrightarrow X_k$$

$$\text{And } y[n] \leftrightarrow Y_k$$

$$\text{Then, } \sum_{r=\langle N \rangle} x[r]y[n-r] \leftrightarrow NX_kY_k$$

6. Multiplication

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{And } y[n] \leftrightarrow Y_k$$

$$\text{Then } x[n]y[n] \leftrightarrow \sum_{r=\langle N \rangle} X_r Y_{k-r}$$

7. **Complex Conjugation**

$$\text{If } x[n] \leftrightarrow X_k$$
$$\text{Then } x^*[n] \leftrightarrow X^*_{-k}$$

8. **Perseval`s Relation**

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |X_k|^2$$

❖ Properties of Fourier Transform

A Fourier transform is a mathematical operation for converting a signal from time domain into its frequency domain. It is a mathematical tool that transforms a function of time (or space) into a function of frequency. A Fourier transform is used when the signal is aperiodic or non-periodic and requires a continuous frequency spectrum for representation

1. **Linearity :**

The FT of a linear combination of the signals is equal to the linear combination of their Fourier transforms.

$$\text{If } x_1[n] \overset{FT}{\leftrightarrow} X_1(k)$$
$$\text{And } x_2[n] \overset{FT}{\leftrightarrow} X_2(k)$$

$$\text{Then } Ax_1[n] + Bx_2[n] \overset{FT}{\leftrightarrow} AX_1(k) + BX_2(k)$$

2. **Time Shifting:**

The time-shifting property states that if x(t) and X(f) form a Fourier transform pair then,

$$x(t - t_d) \overset{FT}{\longleftrightarrow} e^{-j2\pi f t_d} X(f)$$

Hence, A shift of 't_d' in the time domain is equivalent to introducing a phase shift of **$-2\pi f t_d$**. But amplitude remains the same.

3. **Frequency Shifting**

$$\textbf{If } x(t) \overset{FT}{\leftrightarrow} X(\omega)$$

$$\textbf{Then } e^{j\omega_0 t}.\, x(t) \overset{FT}{\leftrightarrow} X(\omega - \omega_0)$$

Hence, Shifting the frequency by ω_0 in the frequency domain is equivalent to multiplying the time domain signal by $e^{j\omega_0 t}$.

4. **Circular Time Reversal**

If x[n] is plotted on a circle in an anti-clockwise direction, time reversal is equivalent to plotting the sequence in a clockwise direction. This is called circular time reversal.

Conjugate symmetry

$$\text{If } x[n] \xleftrightarrow{N\text{-point DFT}} X[k]$$
$$\text{Then } x[\langle -n\rangle_N] \xleftrightarrow{N\text{-point DFT}} X[\langle -k\rangle_N]$$

5. **Circular Even Sequence**

An N – point sequence x[n] is called circularly even if it is symmetric about the point zero on the circle. This implies that

$$x[n] = x[\langle -n\rangle_N] = x[N-n], 1 \le n \le N-1$$

6. **Circular Odd Sequence**

An N – point sequence x[n] is called circularly odd if it is antisymmetric about the point zero on the circle. This implies that

$$x[n] = -x[\langle -n\rangle_N] = -x[N-n], 1 \le n \le N-1$$

7. **Circular convolution**

The circular convolution of two finite-length sequences in the time domain is equivalent to the multiplication of their DFTs in the frequency domain.

$$\text{If } x_1[n] \xleftrightarrow{N\text{-point DFT}} X_1(k)$$
$$\text{And } x_2[n] \xleftrightarrow{N\text{-point DFT}} X_2(k)$$

$$\text{Then} x_1[n] \otimes x_2[n] \xleftrightarrow{N\text{-point DFT}} X_1(k)X_2(k)$$

8. **Conjugate Symmetry**

The DFT of a complex conjugate of any sequence (let x_n) is equal to the complex conjugate of the DFT of that sequence; with the sequence delayed by k samples in the frequency domain.

If $x(\mathrm{n}) \leftrightarrow X(\mathrm{k})$

Then x*(n) ↔ X* (N-K)

9. **Periodicity**

The Periodicity Property of the Discrete Fourier Transform (DFT) states that if the time-domain sequence ***x[n]*** is periodic with period ***N***, then the corresponding DFT ***X[k]*** will also be periodic with the same period ***N.***

$$\text{If } x[n] \leftrightarrow X(k)$$

then,

$$X(k+mN) = X(k), m = integer$$
$$and, x[n+mN] = x[n], m = integer$$