2019

COMPUTER SCIENCE AND ENGINEERING

Paper : CSCL-0901

(Topics in Algorithms)

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** from the following : 2×5

(a) Why do we analyze the expected running time of a randomized algorithm and not its worst-case running time?

(b) Can quicksort be made to run in O(n log n) time in the worst case, assuming that all elements are distinct?

(c) Draw the decision tree for linear search on four elements.

(d) Define PSPACE.

(e) Construct an I-D range tree for the following key values: 34, 90, 12, 32, 54, 73, 29.

(f) State the properties to be satisfied by a flow in a flow network.

(g) What is the time complexity of KMP string matching algorithm?

2. Answer ***any five*** from the following : 4×5

(a) Show that the approximation algorithm has a ratio bound of 2 for the k-center problem.

(b) Let L1, L2, L3 are decision problems. Show that if $L1 \leq_p L2$ and $L2 \leq_p L3$, then $L1 \leq_p L3$.

(c) Given that all characters in a pattern P are different, write an algorithm to accelerate the naive string matching algorithm to run in O(n) time. Justify the time complexity.

(d) State and prove the maxflow-mincut theorem.

(e) Prove that the number of tosses required to insert in a skip list is bounded by O(log n) with high probability.

(f) Prove or disprove: if we order the characters in an alphabet set so that their frequencies are monotonically decreasing, then there exists an optimal code whose codeword lengths are monotonically increasing.

(g) Derive the average case lower bound of comparison based sorting.

(h) Let Al and A2 be two sorted arrays of n elements each. Show that at least 2n-l comparisons are required to merge Al and A2 in the worst case. Give an example of such a scenario.

Please Turn Over

3. Answer ***any four*** from the following :

(a) Give a pseudocode for the randomized quicksort algorithm. When randomized quicksort runs, how many calls are made to the random number generator in the (i) worst case and (ii) best case? Give your answer in terms of θ notation. 6+2+2

(b) Give a pseudocode for the randomized selection algorithm. Suppose we use randomized selection algorithm to select the minimum element of the array A = {3; 2; 9; 0; 7; 5; 4; 8; 6; 1}. Describe a sequence of partitions that results in a worst-case performance. 5+5

(c) Define a 3 dimensional range tree with illustration. What are the time complexities of

(i) building a 3D range tree

(ii) querying in a 3D range tree? Justify the time complexity. 10

(d) Describe the randomized closest pair algorithm. Derive a bound on the total number of insert operations. 6+4

(e) Define (i) general matching problem (ii) bipartite matching problem. Give a polynomial time reduction algorithm to reduce the bipartite matching problem to a network flow problem justifying the time complexity. 2+2+6

(f) Draw a state-transition diagram for a string-matching automation for the pattern 'ababbabbababbababbabb' over the alphabet {a,b}. Compute the prefix function for the same pattern. 5+5

(g) Define the following : 3+3+2+2

(i) approximation algorithm

(ii) approximation scheme

(iii) PTAS

(iv) FPTAS.