Apuntes de Derivadas

Ariel Alejandro Calderón

July 25, 2024

Contents

1 Derivadas

1.1 Ejercicios

Hallar la derivada de $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \frac{x^3 - x^3}{0} = \frac{0}{0}$$

$$f'(x) = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$$

$$f'(x) = \lim_{h \to 0}(3x^2 + 3xh + h^2) = 3x^2$$

Hallar la derivada de $f(x) = \sqrt[3]{x^2}$

$$f(x) = x^{\frac{2}{3}} \implies f'(x) = \frac{2}{3}x^{\frac{2}{3}-1}$$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{x}}$$

Hallar la derivada de $f(x) = \sin x$

$$\boxed{\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$$

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h \to 0} \frac{2\sin(\frac{h}{2})\cos(2 + \frac{h}{2})}{h} \\
&= \lim_{h \to 0} \frac{\sin(\frac{h}{2})\cos(2 + \frac{h}{2})}{\frac{h}{2}} \\
&= \lim_{h \to 0} 1 \cdot \cos(x + \frac{h}{2}) \\
&= \cos x
\end{aligned}$$

Hallar la derivada de $f(x) = e^x$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} \\
&= \lim_{h\to 0} e^x \cdot \frac{e^h - 1}{h} \\
&= e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h} \\
&= e^x \cdot 1 \; * \\
&= e^x
\end{aligned}
$$

Hallar la derivada de $f(x) = \ln x$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\ln x + h - \ln x}{h} \\
&= \lim_{h\to 0} \frac{\ln \frac{x+h}{x}}{h} \\
\frac{h}{x} &= u \implies h = ux \\
&= \lim_{u\to 0} \frac{\ln(1+u)}{ux} \\
&= \frac{1}{x} \lim_{u\to 0} \frac{\ln(1+u)}{u} \\
&= \frac{1}{x} \cdot 1 = \frac{1}{x}
\end{aligned}
$$

Teoremas de Álgebra de Derivadas

$$(f \pm g)'(x) = f'(x) \pm g'(x)$$

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}, \quad g(x) \neq 0$$