AP Calculus

Riemann Sum to Integral Worksheet

Convert each limit of a Riemann sum to a definite integral, and evaluate.

1.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{k}{n}\right) \frac{1}{n}$$

2.
$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{1 + \frac{k}{n}}$$

3.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \left(2 + \frac{k}{n}\right)^2$$

4.
$$\lim_{n \to \infty} \frac{\frac{\pi}{2}}{n} \sum_{k=1}^{n} \sin\left(\frac{k\pi}{2n}\right)$$

5.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(1 + \frac{3k}{n}\right)^3 \frac{3}{n}$$

6.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \left(\left(\frac{k}{n}\right)^3 + 1\right)$$

7.
$$\lim_{n \to \infty} \frac{3}{n} \sum_{k=1}^{n} \left(\left(2 + \frac{3k}{n}\right)^2 - 2\left(2 + \frac{3k}{n}\right)\right)$$

8.
$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^3 + 5\left(\frac{2i}{n}\right)\right) \frac{1}{n}$$

Challenge:

9.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n + k}$$

10.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{n + k}}\right)$$