

Cuestionario

Resuelva las siguientes derivadas:

1. $f(x) = (\sin x + x)^2$

$$f'(x) = 2(\sin x + x)(\cos x + 1)$$

2. $f(x) = \log(x^2 + 2x^4)$

$$f'(x) = \frac{1}{x^2 + 2x^4} \cdot (2x + 8x^3) = \frac{2x(1 + 4x^2)}{x^2 + 2x^4}$$

3. $f(x) = \sqrt{\frac{x^2 + 3x}{2x + 1}}$

$$f'(x) = \frac{(2x + 3)(2x + 1) - (x^2 + 3x)2}{2(2x + 1)^2 \sqrt{\frac{x^2 + 3x}{2x + 1}}}$$

4. $f(x) = \frac{ax^2}{\sqrt[3]{x}} + \frac{b}{x \cdot \sqrt{x}} - \frac{\sqrt[3]{x}}{\sqrt{x}}$

$$f'(x) = \frac{2ax^{\frac{5}{3}}}{3} - \frac{b}{2x^{\frac{3}{2}}} - \frac{1}{6x^{\frac{5}{6}}}$$

5. $f(x) = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}}$

$$f'(x) = \frac{1}{2} \cdot \frac{d}{dx} \left[\ln \left(\frac{1 + \sin x}{1 - \sin x} \right) \right] = \frac{\cos x}{\sqrt{1 - \sin^2 x}}$$

6. $f(x) = a^{x^2}$

$$f'(x) = 2x \cdot a^{x^2} \ln a$$

7. $f(x) = a^{\tan(nx)}$

$$f'(x) = a^{\tan(nx)} \ln a \cdot n \sec^2(nx)$$

8. $f(x) = \arcsin \left(\frac{x}{\sqrt{1 + x^2}} \right)$

$$f'(x) = \frac{1}{\sqrt{1 + x^2} \sqrt{1 - \left(\frac{x}{\sqrt{1 + x^2}} \right)^2}} = \frac{1}{1 + x^2}$$

9. $f(x) = x^x$

$$f'(x) = x^x (\ln x + 1)$$

10. $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{2x}{1+x^4}$$

11. $2x^3 - 4x^2y + y^2 = 0$

$$\frac{dy}{dx} = \frac{6x^2 - 4xy}{2y - 4x^2}$$

12. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 1$

$$\frac{dy}{dx} = -\frac{2Ax + By + D}{Bx + 2Cy + E}$$

13. $e^x \sin y + e^y \cos x = 1$

$$\frac{dy}{dx} = -\frac{e^x \cos y + e^y \sin x}{e^y \cos x - e^x \sin y}$$

14. $y = 10x^2 - 3x + 1$, encontrar y'' .

$$y'' = 20$$

15. $y = \sin(7x)$, encontrar y''' .

$$y''' = -343 \sin(7x)$$

16. $y = \sqrt{1+2t}$, encontrar y'' .

$$y'' = -\frac{1}{(1+2t)^{3/2}}$$

17. $y = \ln(\cos(2x))$, encontrar y''' .

$$y''' = -8 \sec^2(2x) \tan(2x)$$

18. $f(x, y) = -x^2 + 2xy - y$, encontrar $f'_x y f'_y$.

$$f'_x = -2x + 2y, \quad f'_y = 2x - 1$$

19. $f(x, y) = \sqrt{x^3 + y^2}$, encontrar $f'_x(1, 1)$.

$$f'_x(1, 1) = \frac{3\sqrt{1}}{2\sqrt{1+1}} = \frac{3}{2\sqrt{2}}$$

20. $f(x, y) = \frac{2xy - y}{x^2 + y}$, calcular $f'_x y f'_y$.

$$f'_x = \frac{2y(x^2 + y) - 4x^2y + y^2}{(x^2 + y)^2}, \quad f'_y = \frac{2x(x^2 + y) - y(2xy - 1)}{(x^2 + y)^2}$$

21. Encontrar $f'(\frac{\pi}{4})$ si $f(x) = \sin(x) + x$

$$f'(x) = \cos(x) + 1$$

$$f'(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) + 1 = \frac{\sqrt{2}}{2} + 1$$

22. Encontrar $f'(2)$ si $f(x) = x^x$

$$f'(x) = x^x(\ln x + 1)$$

$$f'(2) = 2^2(\ln 2 + 1) = 4(\ln 2 + 1)$$

23. Encontrar la ecuación de la recta tangente a $f(x) = x^2 - 2x + 2$ en $x = 7$

$$f'(x) = 2x - 2$$

$$f'(7) = 2(7) - 2 = 12$$

$$f(7) = 7^2 - 2 \cdot 7 + 2 = 49 - 14 + 2 = 37$$

La ecuación de la recta tangente es:

$$y - 37 = 12(x - 7)$$

Simplificando:

$$y = 12x - 84 + 37 = 12x - 47$$