PROBLEMS FOR SECTION 4.1

Compute the following finite Riemann sums. If a hand calculator is available, the Riemann sums can also be computed with $\Delta x = \frac{1}{10}$.

1	$\sum_0^1 (3x + 1)\,\Delta x,$	$\Delta x = \frac{1}{3}$	2	$\sum_0^1 (3x + 1)\,\Delta x,$	$\Delta x = \frac{2}{5}$
3	$\sum_{-1}^1 (3x + 1)\,\Delta x,$	$\Delta x = \frac{1}{4}$	4	$\sum_0^1 2x^2\,\Delta x,$	$\Delta x = \frac{1}{4}$
5	$\sum_{-1}^1 2x^2\,\Delta x,$	$\Delta x = \frac{1}{4}$	6	$\sum_0^5 (2x - 1)\,\Delta x,$	$\Delta x = 1$
7	$\sum_0^5 (2x - 1)\,\Delta x,$	$\Delta x = 2$	8	$\sum_{-1}^1 (x^2 - 1)\,\Delta x,$	$\Delta x = \frac{1}{2}$
9	$\sum_0^2 (x^2 - 1)\,\Delta x,$	$\Delta x = \frac{1}{2}$	10	$\sum_{-1}^1 (x^2 - 1)\,\Delta x,$	$\Delta x = \frac{3}{10}$
11	$\sum_{-4}^3 (5x^2 - 12)\,\Delta x,$	$\Delta x = 2$	12	$\sum_{-4}^3 (5x^2 - 12)\,\Delta x,$	$\Delta x = 1$
13	$\sum_1^3 (1 + 1/x)\,\Delta x,$	$\Delta x = \frac{1}{3}$	14	$\sum_0^5 10^{-2x}\,\Delta x.$	$\Delta x = \frac{1}{2}$
15	$\sum_{-1}^0 x^4\,\Delta x,$	$\Delta x = \frac{1}{4}$	16	$\sum_{-1}^1 2x^3\,\Delta x,$	$\Delta x = \frac{1}{2}$
17	$\sum_0^\pi \sqrt{x}\,\Delta x,$	$\Delta x = 1$	18	$\sum_{-2}^9 \lvert x - 4\rvert\,\Delta x,$	$\Delta x = 2$
19	$\sum_0^\pi \sin x\,\Delta x,$	$\Delta x = \pi/4$	20	$\sum_0^\pi \sin^2 x\,\Delta x,$	$\Delta x = \pi/4$
21	$\sum_0^1 e^x\,\Delta x,$	$\Delta x = 1/5$	22	$\sum_0^1 xe^x\,\Delta x,$	$\Delta x = 1/5$
23	$\sum_1^5 \ln x\,\Delta x,$	$\Delta x = 1$	24	$\sum_1^5 \frac{\ln x}{x}\,\Delta x,$	$\Delta x = 1$

□ **25** Let b be a positive real number and n a positive integer. Prove that if $\Delta x = b/n$,

$$\sum_0^b x\,\Delta x = (1 + 2 + \cdots + (n - 1))\,\Delta x^2.$$

Using the formula $1 + 2 + \cdots + (n - 1) = \frac{n(n-1)}{2}$, prove that

$$\sum_0^b x\,\Delta x = (1 - 1/n)b^2/2.$$

□ **26** Let H be a positive infinite hyperinteger and $dx = b/H$. Using the Transfer Principle and Problem 25, prove that $\int_0^b x\,dx = b^2/2$.

□ **27** Let b be a positive real number, n a positive integer, and $\Delta x = b/n$. Using the formula

$$1^2 + 2^2 + 3^2 + \cdots + (n - 1)^2 = \frac{n(n - 1)(2n - 1)}{6},$$

prove that

$$\sum_0^b x^2\,\Delta x = \frac{n(n - 1)(2n - 1)}{6}\,\frac{b^3}{n^3}.$$

□ **28** Use Problem 27 to show that $\int_0^b x^2\,dx = b^3/3$.

4.2 FUNDAMENTAL THEOREM OF CALCULUS

In this section we shall state five basic theorems about the integral, culminating in the Fundamental Theorem of Calculus. Right now we can only approximate a definite integral by the laborious computation of a finite Riemann sum. At the end of this section we will be in a position easily to compute exact values for many definite integrals. The key to the method is the Fundamental Theorem. Our first theorem shows that we are free to choose any positive infinitesimal we wish for dx in the definite integral.