# Trabajo Autónomo 2.14 - Cálculo I

Primer Ciclo "A" - Ingeniería de Software

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1. **Presentar una tabla con las fórmulas de las derivadas de las funciones básicas:**

| Función | Derivada |
|---|---|
| $(k)$ (constante) | $0$ |
| $x^n$ | $nx^{n-1}$ |
| $x^{\frac{m}{n}}$ | $\frac{m}{n}x^{\frac{m}{n}-1}$ |
| $\sin(x)$ | $\cos(x)$ |
| $\cos(x)$ | $-\sin(x)$ |
| $\tan(x)$ | $\sec^2(x)$ |
| $\arcsin(x)$ | $\dfrac{1}{\sqrt{1-x^2}}$ |
| $\arccos(x)$ | $-\dfrac{1}{\sqrt{1-x^2}}$ |
| $\arctan(x)$ | $\dfrac{1}{1+x^2}$ |
| $e^x$ | $e^x$ |
| $\ln(x)$ | $\dfrac{1}{x}$ |

| | |
|---|---|
| $a^x$ | $a^x\ln(a)$ |
| $\log_b x$ | $\dfrac{1}{x\ln(b)}$ |
| $\sin^{-1}(x)$ | $\dfrac{1}{\sqrt{1-x^2}}$ |
| $\cos^{-1}(x)$ | $-\dfrac{1}{\sqrt{1-x^2}}$ |
| $\tan^{-1}(x)$ | $\dfrac{1}{1+x^2}$ |
| $\cot^{-1}(x)$ | $-\dfrac{1}{1+x^2}$ |
| $\sec^{-1}(x)$ | $\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| $\csc^{-1}(x)$ | $-\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| $ax^n$ | $anx^{n-1}$ |
| $ax^{\frac{m}{n}}$ | $\frac{m}{n}ax^{\frac{m}{n}-1}$ |
| $ax^{bx}$ | $abx^{bx}\ln(a)$ |

2. **Resolver los siguientes ejercicios de derivadas:**

a. $f(x) = (\sin(x) + x)^2$

$$f'(x) = 2(\sin(x) + x)(\cos(x) + 1)$$

b. $f(x) = \log(x^2 + 2x^4)$

$$f'(x) = \frac{1}{x^2 + 2x^4} \cdot (2x + 8x^3) = \frac{2x(1 + 4x^2)}{x^2 + 2x^4}$$

c. $f(x) = a^{x^2}, a$ constante

$$f'(x) = a^{x^2} \cdot \ln(a) \cdot 2x$$

d. $f(x) = a^{\tan(nx)}, a$ constante

$$f'(x) = a^{\tan(nx)} \cdot \ln(a) \cdot \sec^2(nx) \cdot n$$

e. $f(x) = \arcsin\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} \cdot \left(\frac{\sqrt{1+x^2} - \frac{x \cdot x}{\sqrt{1+x^2}}}{(1+x^2)}\right) = \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \cdot \frac{1}{\sqrt{1+x^2}}$$

$$= \frac{1+x^2}{\sqrt{1+x^2-x^2}} \cdot \frac{1}{\sqrt{1+x^2}} = \frac{1}{1+x^2}$$

f. $f(x) = x^x$

$$f'(x) = x^x(\ln(x)+1)$$

g. $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{2x}{1+(x^2)^2} = \frac{2x}{1+x^4}$$

3. **Resolver los siguientes ejercicios de derivadas:**

a. $2x^3 - 4x^2y + y^2 = 0$

$$\frac{d}{dx}(2x^3 - 4x^2y + y^2) = 0 \implies 6x^2 - 4x^2\frac{dy}{dx} - 8xy + 2y\frac{dy}{dx} = 0$$

$$\implies 6x^2 - 8xy = (4x^2 - 2y)\frac{dy}{dx}$$

$$\implies \frac{dy}{dx} = \frac{6x^2 - 8xy}{4x^2 - 2y}$$

b. $e^x \sin(y) + e^y \cos(x) = 1$

$$\frac{d}{dx}(e^x \sin(y) + e^y \cos(x)) = 0 \implies e^x \sin(y) + e^y(-\sin(x))\frac{dy}{dx} + e^y \cos(x) + e^x \cos(x)\frac{dy}{dx} = 0$$

$$\implies e^x \sin(y) + e^y \cos(x) = -e^y \sin(x)\frac{dy}{dx} - e^x \cos(x)\frac{dy}{dx}$$

$$\implies \frac{dy}{dx} = \frac{e^x \sin(y) + e^y \cos(x)}{-e^y \sin(x) - e^x \cos(x)}$$

4. **Encontrar las derivadas de orden superior que se indican:**

a. $y = 10x^2 - 3x + 1,\ y''$

$$y' = 20x - 3$$

$$y'' = 20$$

b. $y = \sin(7x),\ y''$

$$y' = 7\cos(7x)$$

$$y'' = -49\sin(7x)$$

c. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 1, A, B, C, D, E, F$ son constantes. $y''$

$$\frac{d}{dx}(2Ax + By + D) + \frac{d}{dy}(2Cy + Bx + E) = 0$$

$$y'' = -\frac{2A + By'}{B + 2Cy'}$$

5. **Encontrar las derivadas parciales que se indican:**

a. $f(x, y) = -x^2 + 2xy - y$ ,encontrar $f'_x$ y $f'_y$

$$f'_x = \frac{d}{dx}(-x^2 + 2xy - y) = -2x + 2y$$

$$f'_y = \frac{d}{dy}(-x^2 + 2xy - y) = 2x - 1$$

b. $f(x, y) = \sqrt{x^3 + y^2}$, calcular $f'_x(1, 1)$

$$f'_x = \frac{d}{dx}(\sqrt{x^3 + y^2}) = \frac{1}{2\sqrt{x^3 + y^2}} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3 + y^2}}$$

$$f'_x(1, 1) = \frac{3 \cdot 1^2}{2\sqrt{1^3 + 1^2}} = \frac{3}{2\sqrt{2}}$$

c. $f(x, y) = \dfrac{2xy - y}{x^2 + y}$, encontrar $f'_x$ y $f'_y$

$$\begin{aligned} f'_x = \frac{d}{dx}\left(\frac{2xy - y}{x^2 + y}\right) &= \frac{(2y)(x^2 + y) - (2xy - y)(2x)}{(x^2 + y)^2} \\ &= \frac{2yx^2 + 2y^2 - 4x^2y + 2xy}{(x^2 + y)^2} \\ &= \frac{-2yx^2 + 2y^2 + 2xy}{(x^2 + y)^2} \end{aligned}$$

$$\begin{aligned} f'_y = \frac{d}{dy}\left(\frac{2xy - y}{x^2 + y}\right) &= \frac{(2x - 1)(x^2 + y) - (2xy - y)}{(x^2 + y)^2} \\ &= \frac{2xx^2 + 2xy - x^2 - y - 2xy + y}{(x^2 + y)^2} \\ &= \frac{2xx^2 - x^2}{(x^2 + y)^2} \\ &= \frac{x(2x - 1)}{(x^2 + y)^2} \end{aligned}$$

6. **Encontrar lo que se indica:**

a. $f'\left(\frac{\pi}{4}\right)$ si $f(x) = \sin(x) + x$

$$f'(x) = \cos(x) + 1$$

$$f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) + 1 = \frac{\sqrt{2}}{2} + 1$$

b. $f'(2)$ si $f(x) = x^x$

$$f(x) = e^{x\ln(x)}$$
$$f'(x) = e^{x\ln(x)}\left(\ln(x) + 1\right) = x^x\left(\ln(x) + 1\right)$$
$$f'(2) = 2^2\left(\ln(2) + 1\right) = 4\left(\ln(2) + 1\right)$$

c. Encontrar la ecuación de la recta tangente a $f(x) = x^2 - 2x + 2$ en $x = 2$

$$f'(x) = 2x - 2$$

$$f'(2) = 2(2) - 2 = 2$$

La pendiente de la recta tangente es $2$ y el punto tangente es:

$$(2, f(2)) = (2, 2^2 - 2(2) + 2) = (2, 2)$$

La ecuación de la recta tangente es:

$$y - 2 = 2(x - 2) \implies y = 2x - 2$$