Apuntes de Derivadas

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1 Derivadas

1.1 Ejercicios

Hallar la derivada de $f(x) = x^3$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \frac{x^3 - x^3}{0} = \frac{0}{0} \\
&= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2 \\
&= \lim_{h\to 0}(3x^2 + 3xh + h^2) = 3x^2
\end{aligned}$$

Hallar la derivada de $f(x) = \sqrt[3]{x^2}$

$$f(x) = x^{\frac{2}{3}} \implies f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} \qquad = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3}\cdot\frac{1}{\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{x}}$$

Hallar la derivada de $f(x) = \sin x$

$$\boxed{\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h\to 0} \frac{2\sin(\frac{h}{2})\cos(2+\frac{h}{2})}{h} \\
&= \lim_{h\to 0} \frac{\sin(\frac{h}{2})\cos(2+\frac{h}{2})}{\frac{h}{2}} \\
&= \lim_{h\to 0} 1\cdot\cos(x+\frac{h}{2}) \\
&= \cos x
\end{aligned}$$

Hallar la derivada de $f(x) = e^x$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x\cdot e^h - e^x}{h} \\
&= \lim_{h\to 0} e^x\cdot\frac{e^h - 1}{h} \\
&= e^x\cdot\lim_{h\to 0}\frac{e^h - 1}{h} \\
&= e^x\cdot 1 \; * \\
&= e^x
\end{aligned}$$

Hallar la derivada de $f(x) = \ln x$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h} \\ &= \lim_{h\to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} \\ &= \frac{1}{x} \lim_{u\to 0} \frac{\ln(1+u)}{u} \quad (u = \frac{h}{x}) \\ &= \frac{1}{x} \cdot 1 = \frac{1}{x} \end{aligned}$$

Teoremas de Álgebra de Derivadas

$$(f \pm g)'(x) = f'(x) \pm g'(x)$$

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}, \quad g(x) \neq 0$$

Hallar la derivada de $f(x) = 2x^3 + x + 3$

$$\begin{aligned} f'(x) &= (2x^3)' + (x)' - (3)' = (2)'x^3 + 2(x^3)' + (x)' - (3)' \\ &= (0) \cdot x^3 + 2(3x^2) + 1 - (0) = 6x^2 + 1 \end{aligned}$$

Hallar la derivada de $f(x) = k \cdot x^n$

$$\begin{aligned} f'(x) &= k' \cdot x^n + k \cdot (x^n)' = (0) \cdot x^n + k \cdot nx^{n-1} \\ &= k \cdot nx^{n-1} \end{aligned}$$

Hallar la derivada de $f(x) = \left(\frac{2x+3}{x}\right)$

$$\begin{aligned} f(x) &= \frac{2x+3}{x} = 2 + \frac{3}{x} \\ f'(x) &= 0 - \frac{3}{x^2} = -\frac{3}{x^2} \end{aligned}$$

Hallar la derivada de $f(x) = \frac{1}{x}$

$$\begin{aligned} f'(x) &= \frac{(1)' \cdot x - 1 \cdot (x)'}{x^2} \\ &= \frac{-1}{x^2} \end{aligned}$$

Hallar la derivada de $f(x) = \tan x$

$$\begin{aligned} f'(x) &= \frac{\sin x}{\cos x} \\ &= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{1}{\cos x} = \sec^2 x \end{aligned}$$

Hallar la derivada de $f(x) = \cot x$

$$\begin{aligned} f'(x) &= \frac{-(\sin^2 + \cos^2)}{\sin^2 x} \\ &= \frac{-1}{\sin^2 x} = \frac{-1}{\sin x} \cdot \frac{-1}{\sin x} \\ &= -\csc^2 x \end{aligned}$$

Hallar la derivada de $f(x) = \sec x$

$$f'(x) = \sec x \tan x$$

Hallar la derivada de $f(x) = \csc x$

$$f'(x) = -\csc x \cot x$$

Regla de la cadena

Si$f(x) = u(v(x))$ y existe $u'(x)$ y $v'(x)$, entonces:

$$f'(x) = u'(v(x)) \cdot v'(x)$$

Hallar la derivada de $f(x) = \sin^2 x$

$$f(x) = \sin x^2 \begin{cases} \sin x \\ x^2 \end{cases} \implies f'(x) = \cos(x^2) \cdot 2x = 2x \cdot \cos(x^2)$$

Hallar la derivada de $f(x) = \sqrt{\sqrt{x^4}}$

$$f(x) = \begin{cases} u(x) = x^4 \\ v(x) = x^{\frac{1}{2}} \\ w(x) = x^{\frac{1}{2}} \end{cases} \implies f'(x) = w'(v(u(x))) \cdot v'(u(x)) \cdot u'(x)$$

$$f'(x) =$$