The merger history of primordial-black-hole binaries implied by GWTC-3

Primordial black holes (PBHs), as dark matter candidates, have attracted more and more attentions as they could be possible progenitors of the heavy binary black holes (BBHs) observed by LIGO/Virgo. Accurately estimating the merger rate of PBH binaries will be crucial to reconstruct the mass distribution of PBHs. It was pointed out the merger history of PBHs may shift the merger rate distribution depending on the mass function of PBHs. In this paper, we use BBH events from LIGO/Virgo GWTC-3 data release to constrain the merger rate distribution of PBHs by taking into account the effect of merger history. It is found that the second merger process makes subdominant contribution to the total merger rate, and hence the merger history effect can be safely neglected. We also confirm that the main components of dark matter should not be made of stellar mass PBHs.

I. INTRODUCTION

The direct detection of gravitational wave (GW) from a binary black hole (BBH) coalescence [?] has opened a new window into the universe. Over the past few years, ten BBH mergers have been reported by LIGO/Virgo during the O1 and O2 observing runs [? ? ? ? ? ? ?]. The progenitors of these BBHs are still unknown and under intensively investigation (see *e.g.* [?]). These LIGO/Virgo BBHs present a much heavier mass distribution (in particular the source-frame primary mass of GW170729 event can be as heavy as $50.2^{+16.2}_{-10.2} M_\odot$ [?]) than that inferred from X-ray observations [? ? ? ?], which would challenge the formation and evolution mechanisms of astrophysical black holes. One possible explanation for LIGO/Virgo BBHs is the primordial black holes (PBHs) [? ? ?] formed through the gravitational collapse of the primordial density fluctuations [? ?], which may accompany the induced GWs [? ? ? ? ? ? ? ? ? ?]. On the other hand, PBHs can also be a candidate of cold dark matter (CDM), and the abundance of PBHs in CDM has been constrained by various experiments [?].

In order to account for the LIGO/Virgo BBHs, the merger rate of PBH binaries has been estimated to be $17 \sim 288\ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ [?]. One should notice that theoretically there exist some uncertainties in estimating the merger rate distribution of PBH binaries, and the accuracy of the estimation has been continuously improved. The merger rate of PBH binaries with monochromatic mass function has already been given in [? ?] for the case where two neighboring PBHs having sufficiently small separation can form a binary in the early Universe due to the torque from the third nearest PBH. These binaries would then evolve and coalesce within the age of the Universe and finally explain the merger events observed by LIGO/Virgo [?]. Later, the merger rate estimation is improved in [?] by taking into account the torques exerted by all CDM (including all the PBHs and linear density perturbations), but it is also assumed that all PBHs have the same mass. It is also pointed out in [?] that the effects such as encountering with other PBHs, tidal field from the smooth halo, and the baryon accretion are subdominant and can be neglected when estimating the merger rate.

Various attempts have been made to estimate the merger rate distribution of PBH binaries when PBHs have an extended mass function [? ? ? ? ? ? ?]. In particular, a formalism to estimate the effect of merger history of PBHs on merger rate distribution has been developed in [?], and it is argued that the multiple-merger effect may not be ignored if PBHs have a power-law or a log-normal mass function by choosing some specific parameter values of the mass function. An accurate estimation of the merger rate will be crucial to infer the event rate of LIGO/Virgo BBHs and constrain the abundance of PBHs in CDM either through null detection of sub-solar mass PBH binaries or the null detection of stochastic GW background (SGWB) from PBH binaries.

In this paper, we will use the publicly available GW data from LIGO/Virgo O1 and O2 observations to estimate the merger rate distribution of PBH binaries with a general mass function assuming all of LIGO/Virgo BBHs are of primordial origin. We find that the merger history effect makes no significant contribution to the merger rate of PBHs and can be safely ignored. The rest of this paper is organized as follows. In Sec. II, we review the calculation of merger rate distribution accounting for the merger history effect. In Sec. III, we elaborate the data analysis method used to infer the PBH populations from LIGO/Virgo data. In Sec. IV, we present the results for PBHs with a power-law and a log-normal mass function respectively. Finally, we summarize and discuss our results in Sec. V.

II. MERGER RATE DISTRIBUTION OF PBHS

In this section, we will briefly review the calculation of merger rate density by closely following [?]. We denote the probability distribution function (PDF) of PBH masses by $P(m)$ which satisfies the following normalization condition

$$\int_0^\infty P(m)\,\mathrm{d}m = 1. \tag{1}$$

Consequently the abundance of PBHs in the mass interval $(m, m + \mathrm{d}m)$ is given by [?]

$$0.85 f_{\mathrm{pbh}}\, P(m)\,\mathrm{d}m, \tag{2}$$

where f_{pbh} is the fraction of PBHs in CDM, and the coefficient 0.85 is roughly the fraction of CDM in nonrelativistic matter. Similar to [?], one may define a quantity $m_{\rm pbh}$ as

$$\frac{1}{m_{\rm pbh}} = \int \frac{P(m)}{m} {\rm d}m. \tag{3}$$

Furthermore, the present average number density of PBHs with mass m in the present total average number density of PBHs, $F(m)$, can be obtained by [?]

$$F(m) = P(m)\frac{m_{\rm pbh}}{m}. \tag{4}$$

With the above definitions at hand, one can go directly to estimate the merger rate densities of PBH binaries contributed from different merger processes. We assume that PBHs are randomly distributed in the early Universe when they decouple from the cosmic background evolution [? ? ?]. Due to gravitational interactions, two nearest PBHs would attract each other and form a bound system if they get the angular momentum from the torque of the third closest PBH. Because of gravitational radiations, PBH binaries would eventually merge and may be detected by LIGO/Virgo. Let us consider the first-merger process now. Assuming PBHs posses a random distribution in the early Universe, the differential distribution, for three PBHs with masses (m_i, m_i+dm_i), (m_j, m_j+dm_j) and (m_l, m_l+dm_l), is given by [?]

$$\begin{aligned} dP_1 =& 16\pi^2 n_T^2 x^2 y^2 F(m_i)F(m_j)F(m_l) \\ & \times e^{-4\pi y^3 n_T/3} {\rm d}m_i {\rm d}m_j {\rm d}m_l {\rm d}x {\rm d}y \Theta(y-x), \end{aligned} \tag{5}$$

where Θ is the Heaviside step function, and x is the comoving distance between two closest PBHs with masses m_i and m_j. Note that these two nearest PBHs will form a binary due to the torque of third PBH with mass m_l. In Eq. (5), y is the comoving distance from the binary to the third PBH, and n_T is the average number density of PBHs defined by

$$n_T = \frac{f_{\rm pbh}\rho_{\rm CDM}}{m_{\rm pbh}}, \tag{6}$$

where $\rho_{\rm CDM}$ is the energy density of CDM. Notice that the e-factor in Eq. (5) guarantees that there are no other PBHs within the distance y. It is now straightforward to obtain the fraction of PBHs that have merged before cosmic time t [?]

$$G_1(t) = \int {\rm d}x {\rm d}y {\rm d}m_l \frac{{\rm d}P_1}{{\rm d}x{\rm d}y{\rm d}m_i{\rm d}m_j{\rm d}m_l}\Theta(t-\tau), \tag{7}$$

where τ is the cosmic time at which the PBH binary with masses of m_i and m_j merges. Therefore the merger rate density of first-merger process, $\mathcal{R}_1(t, m_i, m_j)$, is given by [?]

$$\mathcal{R}_1(t, m_i, m_j) = \frac{1}{2} n_T \frac{{\rm d}G_1(t, m_i, m_j)}{{\rm d}t} = \int \hat{\mathcal{R}}_1 {\rm d}m_l, \tag{8}$$

where the factor $1/2$ accounts for the symmetry of m_i and m_j, and

$$\begin{aligned} \hat{\mathcal{R}}_1(t, m_i, m_j, m_l) =& 1.32\times 10^6 \times \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{f_{\rm pbh}}{m_{\rm pbh}}\right)^{\frac{53}{37}} \\ & \times m_l^{-\frac{21}{37}} (m_i m_j)^{\frac{3}{37}} (m_i+m_j)^{\frac{36}{37}} F(m_i)F(m_j)F(m_l). \end{aligned} \tag{9}$$

After the first-merger process, the mass of the remnant BH will approximately be m_i+m_j if the two initial PBHs have masses of m_i and m_j. A second-merger process would happen if this remnant BH and the nearest PBH with mass m_k form a new binary by the torque from another PBH with mass m_l. Similarly, the merger rate density of second-merger process, $\mathcal{R}_2(t, m_i, m_j)$, is given by [?]

$$\begin{aligned} \mathcal{R}_2(t, m_i, m_j) =& \frac{1}{2}\int \hat{\mathcal{R}}_2(t, m_i-m_e, m_e, m_j, m_l)\ {\rm d}m_l {\rm d}m_e \\ & + \frac{1}{2}\int \hat{\mathcal{R}}_2(t, m_j-m_e, m_e, m_i, m_l)\ {\rm d}m_l {\rm d}m_e, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \hat{\mathcal{R}}_2(t, m_i, m_j, m_k, m_l) =& 1.59\times 10^4 \times \left(\frac{t}{t_0}\right)^{-\frac{31}{37}} \left(\frac{f_{\rm pbh}}{m_{\rm pbh}}\right)^{\frac{69}{37}} \\ & \times m_k^{\frac{6}{37}} m_l^{-\frac{42}{37}} (m_i+m_j)^{\frac{6}{37}} (m_i+m_j+m_k)^{\frac{72}{37}} \\ & \times F(m_i)F(m_j)F(m_k)F(m_l). \end{aligned} \tag{11}$$

Following the same spirit, in principle one can derive the merger rate density $\mathcal{R}_n(t, m_i, m_j)$ contributed from the n-th merger process. Therefore the total merger rate density, $\mathcal{R}(t, m_i, m_j)$, of PBHs at cosmic time t with masses m_i and m_j is

$$\mathcal{R}(t, m_i, m_j) = \sum_{n=1} \mathcal{R}_n(t, m_i, m_j), \tag{12}$$

and the total merger rate can be directly obtained by

$$R(t) = \int \mathcal{R}(t, m_i, m_j) {\rm d}m_i {\rm d}m_j = \sum_{n=1} R_n(t), \tag{13}$$

where

$$R_n(t) = \int \mathcal{R}_n(t, m_i, m_j) {\rm d}m_i {\rm d}m_j. \tag{14}$$

As demonstrated in [?], $\mathcal{R}_{n+1}(t, m_i, m_j)$ is not necessarily be smaller than $\mathcal{R}_n(t, m_i, m_j)$ (see Fig. 7 and Fig. 8 in [?]). However, $R_{n+1}(t)$ should be smaller than $R_n(t)$ as expected [?]. Here, we only consider the merger history up to second-merger process.

III. INFERENCE ON PBH MASS DISTRIBUTION FROM GW DATA

Given a general mass function of PBHs $P(m|\boldsymbol{\theta})$ which satisfy the normalization condition of Eq. (1), the time (or redshift) dependent merger rate can be obtained by Eq. (13), namely

$$R(t|\boldsymbol{\theta}) = \int \mathcal{R}(t, \boldsymbol{\lambda}|\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\lambda}, \tag{15}$$

where $\boldsymbol{\lambda} \equiv \{m_1, m_2\}$, and $\boldsymbol{\theta}$ are the parameters that characterize the mass function and will be inferred from GW data. For instance, $\boldsymbol{\theta} = \{\alpha, M\}$ for the power-law PDF (see Eq. (26)) and $\boldsymbol{\theta} = \{m_c, \sigma\}$ for the log-normal PDF (see Eq. (23)). The local merger rate density distribution then reads [?]

$$\mathcal{R}(t_0, \boldsymbol{\lambda}|\boldsymbol{\theta}) = R_0\, p(\boldsymbol{\lambda}|\boldsymbol{\theta}), \tag{16}$$

where $R_0 \equiv R(t_0|\boldsymbol{\theta})$ is the local merger rate, and $p(m_1, m_2|\boldsymbol{\theta})$ is the population distribution of BBH mergers. Note that Eq. (16) guarantees $p(m_1, m_2|\boldsymbol{\theta})$ is normalized, namely

$$\int p(\boldsymbol{\lambda}|\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\lambda} = 1. \tag{17}$$

Given the GW data, $\boldsymbol{d} = (d_1, \ldots, d_N)$, which consist of N BBH merger events, we aim to extract the population parameters $\{\boldsymbol{\theta}, R_0\}$ from $\boldsymbol{d}$. In order to do that, it is necessary to perform the hierarchical Bayesian inference on the BBHs' mass distribution [? ? ? ? ? ? ?]. In this work, we will use the data of ten BBHs [? ?] reported by LIGO/Virgo O1 and O2 observations, and hence $N = 10$. The posterior samples of these BBHs are publicly available from [?]. Because the standard priors on masses for each event in LIGO/Virgo analysis are taken to be uniform [? ?], the likelihood of an individual event $p(d_i|\boldsymbol{\lambda})$ is proportional to the posterior of that event $p(\boldsymbol{\lambda}|d_i)$. The total likelihood for an inhomogeneous Poisson process can be evaluated as [? ? ? ?]

$$p(\boldsymbol{d}|\boldsymbol{\theta}, R_0) \propto R_0^N e^{-R_0\, \beta(\boldsymbol{\theta})} \prod_i^N \int \mathrm{d}\boldsymbol{\lambda}\, p(d_i|\boldsymbol{\lambda})\, p(\boldsymbol{\lambda}|\boldsymbol{\theta}), \tag{18}$$

where $\beta(\boldsymbol{\theta})$ is defined as

$$\beta(\boldsymbol{\theta}) \equiv \int \mathrm{d}\boldsymbol{\lambda}\; VT(\boldsymbol{\lambda})\; p(\boldsymbol{\lambda}|\boldsymbol{\theta}), \tag{19}$$

in which $VT(\boldsymbol{\lambda})$ is the sensitive spacetime volume [? ?] of LIGO. We adopt the semi-analytical approximation from [? ?] to estimate VT, where we use the "IMRPhenomPv2" waveform to simulate the BBH templates and neglect the effect of spins for BHs. Furthermore, the threshold signal-to-noise ratio (SNR) of detection for a single-detector is set to 8, which corresponds to a network SNR threshold of around 12.

Assuming the prior distributions $p(\boldsymbol{\theta}, R_0)$ are uniform for $\boldsymbol{\theta}$ parameters and log-uniform for local merger rate R_0 [? ?], namely

$$p(\boldsymbol{\theta}, R_0) \propto \frac{1}{R_0}, \tag{20}$$

the posterior probability distribution $p(\boldsymbol{\theta}, R_0|\boldsymbol{d})$ can be directly calculated by

$$p(\boldsymbol{\theta}, R_0|\boldsymbol{d}) \propto p(\boldsymbol{d}|\boldsymbol{\theta}, R_0)\; p(\boldsymbol{\theta}, R_0). \tag{21}$$

The marginalized posterior $p(\boldsymbol{\theta}|\boldsymbol{d})$ can then be readily obtained by integrating over R_0 in Eq. (21), namely

$$p(\boldsymbol{\theta}|\boldsymbol{d}) \propto [\beta(\boldsymbol{\theta})]^{-N} \prod_i^N \int \mathrm{d}\boldsymbol{\lambda}\; p(d_i|\boldsymbol{\lambda})\; p(\boldsymbol{\lambda}|\boldsymbol{\theta}). \tag{22}$$

This marginalized posterior has been widely used in previous population inferences [? ? ? ? ? ?]. In the following section, we will utilize the posterior (21) to infer the population parameters $\{\boldsymbol{\theta}, R_0\}$ by considering two concrete mass distributions, a power-law PDF and a log-normal PDF, respectively.

IV. RESULTS

A. Log-normal mass function

We now consider a log-normal mass function of PBHs as [?]

$$P(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left(-\frac{\ln^2(m/m_c)}{2\sigma^2}\right), \tag{23}$$

where m_c presents the peak mass of $mP(m)$, and σ denotes the width of the mass spectrum. The log-normal mass function is a good approximation to a large class of extended mass distributions if PBHs are formed from smooth symmetric peaks in the inflationary power spectrum [?], as demonstrated in [? ?]. Note that $\boldsymbol{\theta} = \{m_c, \sigma\}$, and the free parameters are $\{\boldsymbol{\theta}, R_0\} = \{m_c, \sigma, R_0\}$ in this case. Using Eq. (3) and Eq. (4), it is easily to get

$$m_{\rm pbh} = m_c \exp\left(-\frac{\sigma^2}{2}\right), \tag{24}$$

$$F(m) = \frac{m_c}{\sqrt{2\pi}\sigma m^2} \exp\left(-\frac{\sigma^2}{2} - \frac{\ln^2(m/m_c)}{2\sigma^2}\right). \tag{25}$$

Using data of 10 BBHs observed by LIGO/Virgo O1 and O2 observations and performing the hierarchical Bayesian inference, we obtain $m_c = 8.9^{+7.8}_{-7.3} M_\odot$, $\sigma = 0.91^{+0.50}_{-0.42}$, and $R_0 = 55^{+42}_{-27} \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$. It is then easy to infer the abundance of PBHs in CDM to be $f_{\rm pbh} = 2.6^{+6.8}_{-1.4} \times 10^{-3}$ from the posterior distribution of local merger rate R_0. The results of local merger rate and abundance of PBHs are consistent with the previous estimations, confirming that the main components of CDM should not be made of stellar mass PBHs. The posteriors of parameters $\{\boldsymbol{\theta}, R_0\} = \{m_c, \sigma, R_0\}$ are shown in Fig. 1.

posterior-log.pdf

FIG. 1. The marginalized one- and two-dimensional posterior distributions for parameters $\{\boldsymbol{\theta}, R_0\} = \{m_c, \sigma, R_0\}$ in the log-normal mass function of PBHs, by using 10 BBH events from LIGO/Virgo O1 and O2 observing runs. The contours are at the 68% and 95% credible levels, respectively.

Fig. 2 shows the ratio of merger rate density from second-merger history to the one from first-merger history, namely $\mathcal{R}_2(t_0, m_1, m_2)/\mathcal{R}_1(t_0, m_1, m_2)$, by fixing $\{\boldsymbol{\theta}, R_0\}$ to their best-fit values. The correction to total merger rate density from second-merger history is larger as component masses are heavier. However, the ratio of merger rate from second-merger history to the one from first-merger history is negligible, namely $R_2(t_0)/R_1(t_0) = 3.0\%$. This is because the major contribution to the merger rate comes from the binaries with masses less than $50 M_\odot$. Therefore the merger history effect can be safely ignored when estimating the merger rate of PBHs.

B. Power-law mass function

We now consider a power-law mass function of PBHs as [?]

$$P(m) = \frac{\alpha - 1}{M} \left(\frac{m}{M}\right)^{-\alpha}, \tag{26}$$

where $m > M$, and $\alpha > 1$ is the power-law index. The power-law form is a commonly used mass function as

ratio-log.pdf

FIG. 2. The ratio of merger rate density from second-merger history to that from first-merger history, $\mathcal{R}_2(t_0, m_1, m_2)/\mathcal{R}_1(t_0, m_1, m_2)$. Note that the parameters $\{m_c, \sigma\}$ in Eq. (23) are fixed to their best-fit values.

it naturally arises if PBHs are formed from primordial energy density fluctuations due to a scale-invariant primordial power spectrum [? ?]. Note that $\boldsymbol{\theta} = \{\alpha, M\}$ and the free parameters are $\{\boldsymbol{\theta}, R_0\} = \{\alpha, M, R_0\}$ in this case. Using Eq. (3) and Eq. (4), it is easily to get

$$m_{\mathrm{pbh}} = M \frac{\alpha}{\alpha - 1}, \tag{27}$$

$$F(m) = \frac{\alpha}{m} \left(\frac{m}{M}\right)^{-\alpha}. \tag{28}$$

Using data of 10 BBHs observed by LIGO/Virgo O1 and O2 observations and performing the hierarchical Bayesian inference, we obtain $\alpha = 2.41^{+1.00}_{-0.87}$, $M = 7.4^{+1.4}_{-3.3} M_\odot$, and $R_0 = 48^{+37}_{-24} \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$. It is then easy to infer the abundance of PBHs in CDM to be $f_{\mathrm{pbh}} = 2.8^{+1.8}_{-1.2} \times 10^{-3}$ from the posterior distribution of local merger rate R_0. The results of local merger rate and abundance of PBHs are consistent with the previous estimations, confirming that the main components of CDM should not be made of stellar mass PBHs. The posteriors of parameters $\{\boldsymbol{\theta}, R_0\} = \{\alpha, M, R_0\}$ are shown in Fig. 3.

Fig. 4 shows the ratio of merger rate density from second-merger history to the one from first-merger history, namely $\mathcal{R}_2(t_0, m_1, m_2)/\mathcal{R}_1(t_0, m_1, m_2)$, by fixing $\{\boldsymbol{\theta}, R_0\}$ to their best-fit values. It is clearly that the correction of total merger rate density from second-merger history is less than 10%. It is then readily to calculate the ratio of merger rate from second-merger history to the one from first-merger history, $R_2(t_0)/R_1(t_0) = 0.5\%$.

posterior-power.pdf

FIG. 3. The marginalized one- and two-dimensional posterior distributions for parameters $\{\boldsymbol{\theta}, R_0\} = \{\alpha, M, R_0\}$ in the power-law mass function of PBHs, by using 10 BBH events from LIGO/Virgo O1 and O2 observing runs. The contours are at the 68% and 95% credible levels, respectively.

ratio-power.pdf

FIG. 4. The ratio of merger rate density from second-merger history to that from first-merger history, $\mathcal{R}_2(t_0, m_1, m_2)/\mathcal{R}_1(t_0, m_1, m_2)$. Note that the parameters $\{\alpha, M\}$ in Eq. (26) are fixed to their best-fit values.

We therefore conclude that the merger history effect can be safely ignored when estimating the merger rate (density) of PBHs.

V. CONCLUSION

In this paper, we use the publicly available GW data of 10 BBH events from LIGO/Virgo O1 and O2 observing runs to constrain the merger rate distribution of PBHs by taking into account the effect of merger history. Considering two concrete mass functions of PBHs, a power-law PDF and a log-normal one, we demonstrate that the contribution of merger rate (density) from second-merger history to total merger rate (density) is subdominant, and hence the second-merger history effect can be safely ignored. As third-merger (and later merger) history will make even less contribution to the total merger rate (density), we conclude that the effect of merger history is subdominant and can be neglected when evaluating the merger rate of PBH binaries.

Furthermore, the results of local merger rate and abundance of PBHs inferred from the updated analysis are consistent with the previous estimations, confirming that the main components of CDM should not be made of stellar mass PBHs.

ACKNOWLEDGMENTS

We thank the anonymous referee for providing constructive comments and suggestions to improve the quality of this paper. We would also like to thank Zu-Cheng Chen and Lang Liu for useful conversations. This work is supported by grants from National Natural Science Foundation of China (grant No.11847107). This research has made use of data, software and/or web tools obtained from the Gravitational Wave Open Science Center [**?**] (https://www.gw-openscience.org), a service of LIGO