湖南师范大学数达学院
数学物理方法课后习题集解答

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摘要

此系四川大学数学学院高等数学、微分方程教研室编《高等数学--物理类专用（第四册）（第四版）》所有课后习题的详细答案。未完成草稿，如有错误，欢迎纠正。

目录

1 复数与复变函数

1.1 计算:

1.1.1 $(\sqrt{2}-i)-i(1-i\sqrt{2})$

解:

$$(\sqrt{2}-i)-i(1-i\sqrt{2}) \tag{1.1}$$
$$=\sqrt{2}-i-i+i^2\sqrt{2} \tag{1.2}$$
$$=\sqrt{2}-2i-\sqrt{2} \tag{1.3}$$
$$=-2i \tag{1.4}$$

1.1.2 $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$

解:

$$\frac{1+2i}{3-4i}+\frac{2-i}{5i} \tag{1.5}$$
$$=\frac{(1+2i)5i+(2-i)(3-4i)}{(3-4i)5i} \tag{1.6}$$
$$=\frac{5i+10i^2+6-8i-3i+4i^2}{5(3i-4i^2)} \tag{1.7}$$
$$=\frac{5i-10+6-11i-4}{5(3i+4)} \tag{1.8}$$
$$=\frac{-6i-8}{5(3i+4)} \tag{1.9}$$
$$=-\frac{2(4+3i)}{5(4+3i)} \tag{1.10}$$
$$=-\frac{2}{5} \tag{1.11}$$

1.1.3 $\frac{5}{(1-i)(2-i)(3-i)}$

$$\frac{5}{(1-i)(2-i)(3-i)} \tag{1.12}$$
$$=\frac{5(1+i)(2+i)(3+i)}{(1-i)(2-i)(3-i)(1+i)(2+i)(3+i)} \tag{1.13}$$
$$=\frac{5(2+i+2i-1)(3+i)}{(1^2-i^2)\times(2^2-i^2)\times(3^2-i^2)} \tag{1.14}$$
$$=\frac{5(1+3i)(3+i)}{2\times5\times10} \tag{1.15}$$
$$=\frac{(1+3i)(3+i)}{20} \tag{1.16}$$
$$=\frac{3+i+9i-3}{20} \tag{1.17}$$
$$=\frac{10i}{20} \tag{1.18}$$
$$=\frac{i}{2} \tag{1.19}$$

1.1.4 $(1-i)^4$

$$
\begin{aligned}
&(1-i)^4 && (1.20)\\
=&\left[(1-i)^2\right]^2 && (1.21)\\
=&[1-2i-1]^2 && (1.22)\\
=&4i^2 && (1.23)\\
=&-4 && (1.24)
\end{aligned}
$$

1.2 求下列复数的实部 u 与虚部 v，模 r 与辐角 θ：

1.2.1 $\frac{1-2i}{3-4i}-\frac{2-i}{5i}$

$$
\begin{aligned}
&\frac{1-2i}{3-4i}-\frac{2-i}{5i} && (2.1)\\
=&\frac{(1-2i)5i-(2-i)(3-4i)}{(3-4i)5i} && (2.2)\\
=&\frac{5i+10-(6-8i-3i-4)}{5(3i+4)} && (2.3)\\
=&\frac{5i+10-2+11i}{5(4+3i)} && (2.4)\\
=&\frac{8+16i}{5(4+3i)} && (2.5)\\
=&\frac{8(1+2i)(4-3i)}{5(4+3i)(4-3i)} && (2.6)\\
=&\frac{8(4-3i+8i+6)}{5\times 25} && (2.7)\\
=&\frac{8(10+5i)}{5\times 25} && (2.8)\\
=&\frac{8(2+i)}{25} && (2.9)\\
=&\frac{16}{25}+\frac{8}{25}i && (2.10)\\
=&\frac{8}{25}\sqrt{5}e^{i(\arctan\frac{1}{2}+2k\pi)} && (2.11)
\end{aligned}
$$

所以 $u=\frac{16}{25}$，$v=\frac{8}{25}$，$r=\frac{8}{25}\sqrt{5}$，$\theta=\arctan\frac{1}{2}+2k\pi$。

1.2.2 $(\frac{1+\sqrt{3}i}{2})^n, n=2,3,4$

首先 $\frac{1+\sqrt{3}i}{2}=e^{i(\frac{\pi}{3}+2m\pi)}$

- n=2 时

$$
\begin{aligned}
&\left(\frac{1+\sqrt{3}i}{2}\right)^2 && (2.12)\\
=&(e^{i(\frac{\pi}{3}+2m\pi)})^2 && (2.13)\\
=&e^{i(\frac{2\pi}{3}+4m\pi)} && (2.14)\\
=&e^{i(\frac{2\pi}{3}+4m\pi+2k\pi)} && (2.15)\\
=&e^{i(\frac{2\pi}{3}+2k\pi)} && (2.16)\\
=&-\frac{1}{2}+\frac{\sqrt{3}}{2}i && (2.17)
\end{aligned}
$$

所以 $u=-\frac{1}{2}$，$v=\frac{\sqrt{3}}{2}$，$r=1$，$\theta=\frac{2\pi}{3}+2k\pi$。

- n=3 时

$$
\begin{aligned}
&\left(\frac{1+\sqrt{3}i}{2}\right)^3 && (2.18)\\
=&(e^{i(\frac{\pi}{3}+2m\pi)})^3 && (2.19)\\
=&e^{i(\pi+6m\pi)} && (2.20)\\
=&e^{i(\pi+2k\pi)} && (2.21)\\
=&-1 && (2.22)
\end{aligned}
$$

所以 $u=-1$，$v=0$，$r=1$，$\theta=\pi+2k\pi$。

- n=4 时

$$
\begin{aligned}
&\left(\frac{1+\sqrt{3}i}{2}\right)^4 && (2.23)\\
=&(e^{i(\frac{\pi}{3}+2m\pi)})^4 && (2.24)\\
=&e^{i(\frac{4\pi}{3}+8m\pi)} && (2.25)\\
=&e^{i(-\frac{2\pi}{3}+2k\pi)} && (2.26)\\
=&-\frac{1}{2}-\frac{\sqrt{3}}{2}i && (2.27)
\end{aligned}
$$

所以 $u=-\frac{1}{2}$，$v=-\frac{\sqrt{3}}{2}$，$r=1$，$\theta=-\frac{2\pi}{3}+2k\pi$。

1.2.3 $\sqrt{1+i}$

$$
\begin{aligned}
&\sqrt{1+i} && (2.28)\\
=&(1+i)^{\frac{1}{2}} && (2.29)\\
=&\left[\sqrt{2}e^{i\left(\frac{\pi}{4}+2k\pi\right)}\right]^{\frac{1}{2}} && (2.30)\\
=&2^{\frac{1}{4}}e^{i\left(\frac{\pi}{8}+k\pi\right)} && (2.31)\\
=&2^{\frac{1}{4}}\left[\cos\left(\frac{\pi}{8}+k\pi\right)+i\sin\left(\frac{\pi}{8}+k\pi\right)\right] && (2.32)
\end{aligned}
$$

所以 $u=2^{\frac{1}{4}}\cos\left(\frac{\pi}{8}+k\pi\right)$，$v=2^{\frac{1}{4}}\sin\left(\frac{\pi}{8}+k\pi\right)$，$r=2^{\frac{1}{4}}$，$\theta=\frac{\pi}{8}+k\pi$。