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Тригонометрия

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$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 & \operatorname{tg} x \cdot \operatorname{ctg} x &= 1 & 1 + \operatorname{tg}^2 x &= \frac{1}{\cos^2 x} & 1 + \operatorname{ctg}^2 x &= \frac{1}{\sin^2 x} \\ \sin 2x &= 2 \sin x \cos x & \cos 2x &= \cos^2 x - \sin^2 x & \operatorname{tg} 2x &= \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} & \operatorname{ctg} 2x &= \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x} \\ \sin 3x &= 3 \sin x - 4 \sin^3 x & \cos 3x &= 4 \cos^3 x - 3 \cos x \\ \operatorname{tg} 3x &= \frac{3 \operatorname{tg} x - \operatorname{tg}^3 x}{1 - 3 \operatorname{tg}^2 x} & \operatorname{ctg} 3x &= \frac{3 \operatorname{ctg} x - \operatorname{ctg}^3 x}{1 - 3 \operatorname{ctg}^2 x} \\ \sin(x + y) &= \sin x \cos y + \cos x \sin y & \sin(x - y) &= \sin x \cos y - \cos x \sin y \\ \cos(x + y) &= \cos x \cos y - \sin x \sin y & \cos(x - y) &= \cos x \cos y + \sin x \sin y \\ \operatorname{tg}(x + y) &= \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y} & \operatorname{tg}(x - y) &= \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y} \\ \operatorname{ctg}(x + y) &= \frac{\operatorname{ctg} x \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y} & \operatorname{ctg}(x - y) &= \frac{\operatorname{ctg} x \operatorname{ctg} y + 1}{\operatorname{ctg} x - \operatorname{ctg} y} \\ \sin x + \sin y &= 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} & \sin x - \sin y &= 2 \sin \frac{x - y}{2} \cos \frac{x + y}{2} \\ \cos x + \cos y &= 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} & \cos x - \cos y &= -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2} \\ \sin x \sin y &= \frac{1}{2} [\cos(x - y) - \cos(x + y)] \\ \sin x \cos y &= \frac{1}{2} [\sin(x + y) + \sin(x - y)] \\ \cos x \cos y &= \frac{1}{2} [\cos(x + y) + \cos(x - y)] \\ \sin^2 x &= \frac{1 - \cos 2x}{2} & \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \sin x &= \frac{2 \operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} & \cos x &= \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} & \operatorname{tg} x &= \frac{2 \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg}^2 \frac{x}{2}} & \operatorname{ctg} x &= \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{2 \operatorname{tg} \frac{x}{2}}\end{aligned}$$

# Геометрия

$$S = \sqrt{p(p-a)(p-b)(p-c)}, p = \frac{a+b+c}{2}$$

$$S = \frac{1}{3}\sqrt{M(M-m_a)(M-m_b)(M-m_c)}, M = \frac{m_a+m_b+m_c}{2}$$

$$S = \frac{1}{\sqrt{H(H-\frac{2}{h_a})(H-\frac{2}{h_b})(H-\frac{2}{h_c})}}, H = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}$$

$$\lambda = \frac{m}{n}(\frac{p}{q} + 1)$$

$$S = \frac{abc}{4R} = \frac{a^2 \sin B \sin C}{\sin A} = 2R^2 \sin A \sin B \sin C$$

$$m_a = \frac{1}{2}\sqrt{2c^2 + 2b^2 - a^2}$$

$$l_a = \frac{bc \sin A}{(b+c) \sin \frac{A}{2}} = \sqrt{bc - a_b a_c}$$

$$r = \frac{S}{p-d} \text{ (внеписанная)}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\cos \alpha = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2}}$$

## Неравенства

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Неравенство Коши:  $x_1, x_2, \dots, x_n \in \mathbb{R}^+$

$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} \leqslant (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{1/n} \leqslant \frac{x_1 + x_2 + \dots + x_n}{n} \leqslant \sqrt[n]{\frac{x_1^2 + x_2^2 + \dots + x_n^2}{n}}$$

Неравенство Бернулли:  $x > -1, n \in \mathbb{N}; (1+x)^n \geqslant 1+nx$

## Замечательные пределы

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$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{sh} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{th} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x} = 1$$

$$\lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$\lim_{x \rightarrow +\infty} (1 + \frac{1}{x})^x = e$$

$$\lim_{x \rightarrow 0} \frac{\ln x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^p - 1}{x} = p$$

## Гиперболические функции

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$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

## Ряды Тейлора

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\operatorname{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots + \frac{p(p-1)\dots(p-n+1)}{n!}x^n + o(x^n)$$

$$\operatorname{arctg} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+1})$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + o(x^7)$$

## Производные

$$(x^n)' = nx^{n-1} \quad (\sin x)' = \cos x \quad (\cos x)' = -\sin x \quad (\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x} \quad (e^x)' = e^x \quad (a^x)' = a^x \ln a \quad (\ln |x|)' = \frac{1}{x} \quad (\log_a |x|)' = \frac{1}{x \ln a}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2} \quad (\operatorname{sh} x)' = \operatorname{ch} x \quad (\operatorname{ch} x)' = \operatorname{sh} x \quad (\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$$

$$(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$$

$$(af(x) + bg(x))' = af'(x) + bg'(x) \quad (f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (f(g(x)))' = f'(g(x))g'(x)$$

$$(u(x)v(x))^{(n)} = \sum_{k=0}^n \binom{n}{k} u^{(k)}(x)v^{(n-k)}(x)$$

# Интегралы

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{dx}{x} = \ln |x| + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln |x + \sqrt{x^2 - a^2}| + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln |x + \sqrt{x^2 + a^2}| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C$$

$$\int \frac{dx}{\cos x} = -\ln \left| \operatorname{tg} \left( \frac{x}{2} - \frac{\pi}{4} \right) \right| + C$$

$$\int \operatorname{tg} x dx = -\ln |\cos x| + C$$

$$\int \operatorname{ctg} x dx = \ln |\sin x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \operatorname{sh} x dx = \operatorname{ch} x + C$$

$$\int \operatorname{ch} x dx = \operatorname{sh} x + C$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{4} \ln \left| \frac{\sqrt{x^2 - a^2} - x}{\sqrt{x^2 - a^2} + x} \right| + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arctg} \frac{x}{\sqrt{a^2 - x^2}} + C$$

$$\int \sqrt{x^2 + a^2} dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 + a^2}| + \frac{x}{2} \sqrt{x^2 + a^2} + C$$