Тригонометрия

$$\sin^2 x + \cos^2 x = 1 \qquad \operatorname{tg} x \cdot \operatorname{ctg} x = 1 \qquad 1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x} \qquad 1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}$$

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} \qquad \operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \qquad \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\operatorname{tg} 3x = \frac{3 \operatorname{tg} x - \operatorname{tg}^3 x}{1 - 3 \operatorname{tg}^2 x} \qquad \operatorname{ctg} 3x = \frac{3 \operatorname{ctg} x - \operatorname{ctg}^3 x}{1 - 3 \operatorname{ctg}^2 x}$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \qquad \sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \qquad \cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\operatorname{tg}(x + y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y} \qquad \operatorname{tg}(x - y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y}$$

$$\operatorname{ctg}(x + y) = \frac{\operatorname{ctg} x \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y} \qquad \operatorname{ctg}(x - y) = \frac{\operatorname{ctg} x \operatorname{ctg} y + 1}{\operatorname{ctg} x - \operatorname{ctg} y}$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} \qquad \sin x - \sin y = 2 \sin \frac{x - y}{2} \cos \frac{x + y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} \qquad \cos x + \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin x = \frac{2 \operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} \qquad \cos x = \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} \qquad \operatorname{tg} x = \frac{2 \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg}^2 \frac{x}{2}} \qquad \operatorname{tg} x = \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{2 \operatorname{tg} \frac{x}{2}}$$

arc функции

$\arcsin(-m) = -\arcsin m \qquad \arcsin m \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\arccos(-m) = \pi - \arccos m \qquad \arccos m \in [0, \pi]$
$\operatorname{arctg}(-m) = -\operatorname{arctg} m \qquad \operatorname{arctg} m \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\operatorname{arcctg}(-m) = \pi - \operatorname{arcctg} m \qquad \operatorname{arcctg} m \in (0, \pi)$

$\arcsin x + \arccos x = \frac{\pi}{2}$
$\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}$

$$\arcsin x = \begin{cases} \arccos\sqrt{1-x^2}, x \in [0,1] \\ -\arccos\sqrt{1-x^2}, x \in [-1,0] \end{cases}$$

$$\arccos x = \begin{cases} \arcsin\sqrt{1-x^2}, x \in [0,1] \\ -\arcsin\sqrt{1-x^2}, x \in [-1,0] \end{cases}$$

$$\arcsin x = \begin{cases} \operatorname{arcctg}\frac{\sqrt{1-x^2}}{x}, x \in (0,1] \\ \operatorname{arcctg}\frac{\sqrt{1-x^2}}{x} - \pi, x \in [-1,0) \end{cases}$$

$$\arccos x = \begin{cases} \operatorname{arctg}\frac{\sqrt{1-x^2}}{x}, x \in (0,1] \\ \pi + \operatorname{arctg}\frac{\sqrt{1-x^2}}{x}, x \in [-1,0) \end{cases}$$

$$\operatorname{arctg} x = \begin{cases} \arccos\frac{1}{\sqrt{1+x^2}}, x \geqslant 0 \\ -\arccos\frac{1}{\sqrt{1+x^2}}, x \leqslant 0 \end{cases}$$

$$\operatorname{arcctg} x = \begin{cases} \arcsin\frac{1}{\sqrt{1+x^2}}, x \geqslant 0 \\ \pi - \arcsin\frac{1}{\sqrt{1+x^2}}, x \leqslant 0 \end{cases}$$

$$\operatorname{arctg} x = \begin{cases} \operatorname{arcctg}\frac{1}{x}, x > 0 \\ \operatorname{arcctg}\frac{1}{x} - \pi, x < 0 \end{cases}$$

$\sin(\arcsin x) = x, x \in [-1,1]$
$\sin(\arccos x) = \sqrt{1-x^2}, x \in [-1,1]$
$\sin(\operatorname{arctg} x) = \frac{x}{\sqrt{1+x^2}}, x \in (-\infty, \infty)$
$\sin(\operatorname{arcctg} x) = \frac{1}{\sqrt{1+x^2}}, x \in (-\infty, \infty)$

$\cos(\arccos x) = x, x \in [-1,1]$
$\cos(\arcsin x) = \sqrt{1-x^2}, x \in [-1,1]$

$\cos(\operatorname{arctg} x) = \dfrac{1}{\sqrt{1+x^2}}, x \in (-\infty, \infty)$
$\cos(\operatorname{arcctg} x) = \dfrac{x}{\sqrt{1+x^2}}, x \in (-\infty, \infty)$

$\operatorname{tg}(\operatorname{arctg} x) = x, x \in (-\infty, \infty)$
$\operatorname{tg}(\operatorname{arcctg} x) = \dfrac{1}{x}, x \neq 0$
$\operatorname{tg}(\arcsin x) = \dfrac{x}{\sqrt{1-x^2}}, x \in (-1, 1)$
$\operatorname{tg}(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}, x \in (-1, 1), x \neq 0$

$\operatorname{ctg}(\operatorname{arcctg} x) = x, x \in (-\infty, \infty)$
$\operatorname{ctg}(\operatorname{arctg} x) = \dfrac{1}{x}, x \neq 0$
$\operatorname{ctg}(\arcsin x) = \dfrac{\sqrt{1-x^2}}{x}, x \in (-1, 1), x \neq 0$
$\operatorname{ctg}(\arccos x) = \dfrac{x}{\sqrt{1-x^2}}, x \in (-1, 1)$

Геометрия

$S = \sqrt{p(p-a)(p-b)(p-c)}, p = \dfrac{a+b+c}{2}$

$S = \dfrac{1}{3}\sqrt{M(M-m_a)(M-m_b)(M-m_c)}, M = \dfrac{m_a+m_b+m_c}{2}$

$S = \dfrac{1}{\sqrt{H(H-\frac{2}{h_a})(H-\frac{2}{h_b})(H-\frac{2}{h_c})}}, H = \dfrac{1}{h_a} + \dfrac{1}{h_b} + \dfrac{1}{h_c}$

$\lambda = \dfrac{m}{n}(\dfrac{p}{q}+1)$

$S = \dfrac{abc}{4R} = \dfrac{a^2 \sin B \sin C}{\sin A} = 2R^2 \sin A \sin B \sin C$

$m_a = \dfrac{1}{2}\sqrt{2c^2+2b^2-a^2}$

$l_a = \dfrac{bc \sin A}{(b+c)\sin\frac{A}{2}} = \sqrt{bc - a_b a_c}$

$r = \dfrac{S}{p-d}$(вневписанная)

$a^2 = b^2 + c^2 - 2bc\cos A$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

$\cos\alpha = \dfrac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$

Неравенства

Неравенсво Коши: $x_1, x_2, \dots x_n \in \mathbb{R}^+$

$$\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}} \leqslant (x_1 \cdot x_2 \cdot \ldots \cdot x_n)^{1/n} \leqslant \frac{x_1+x_2+\cdots+x_n}{n} \leqslant \sqrt{\frac{x_1^2+x_2^2+\cdots+x_n^2}{n}}$$

Неравенство Бернулли: $x > -1, n \in \mathbb{N}$; $(1+x)^n \geqslant 1+nx$

Замечательные пределы

$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ $\lim\limits_{x\to 0}\frac{\operatorname{tg} x}{x}=1$ $\lim\limits_{x\to 0}\frac{\operatorname{sh} x}{x}=1$ $\lim\limits_{x\to 0}\frac{\operatorname{th} x}{x}=1$

$\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

$\lim\limits_{x\to 0}\frac{\arcsin x}{x}=1$ $\lim\limits_{x\to 0}\frac{\operatorname{arctg} x}{x}=1$

$\lim\limits_{x\to 0}(1+x)^{1/x}=e$ $\lim\limits_{x\to +\infty}(1+\frac{1}{x})^x=e$

$\lim\limits_{x\to 0}\frac{\ln x}{x}=1$ $\lim\limits_{x\to 0}\frac{a^x-1}{x}=\ln a$

$\lim\limits_{x\to 0}\frac{(1+x)^p-1}{x}=p$

Гиперболические функции

$\operatorname{sh} x=\frac{e^x-e^{-x}}{2}$ $\operatorname{ch} x=\frac{e^x+e^{-x}}{2}$ $\operatorname{ch}^2 x - sh^2 x = 1$

Ряды Тейлора

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... + \frac{x^n}{n!} + o(x^n)$$

$$\operatorname{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ... + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ... + \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1}\frac{x^n}{n} + o(x^n)$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots + \frac{p(p-1)\dots(p-n+1)}{n!}x^n + o(x^n)$$

$$\operatorname{arctg} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ... + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+1})$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + o(x^7)$$

Производные

$$(x^n)' = nx^{n-1} \qquad (\sin x)' = \cos x \qquad (\cos x)' = -\sin x \qquad (\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x} \qquad (e^x)' = e^x \qquad (a^x)' = a^x \ln a \qquad (\ln|x|)' = \frac{1}{x} \qquad (\log_a|x|)' = \frac{1}{x \ln a}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2} \qquad (\operatorname{sh} x)' = \operatorname{ch} x \qquad (\operatorname{ch} x)' = \operatorname{sh} x \qquad (\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$$

$$(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$$

$$(af(x) + bg(x))' = af'(x) + bg'(x) \qquad (f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \qquad (f(g(x)))' = f'(g(x))g'(x)$$

$$(u(x)v(x))^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)}(x) v^{(n-k)}(x)$$

Интегралы

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$

$\int \dfrac{dx}{x} = \ln|x| + C$

$\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a} \operatorname{arctg} \dfrac{x}{a} + C$

$\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a} \ln\left|\dfrac{x-a}{x+a}\right| + C$

$\int \dfrac{dx}{\sqrt{a^2-x^2}} = \arcsin \dfrac{x}{a} + C$

$\int \dfrac{dx}{\sqrt{x^2-a^2}} = \ln|x+\sqrt{x^2-a^2}| + C$

$\int \dfrac{dx}{\sqrt{x^2+a}} = \ln|x+\sqrt{x^2+a}| + C$

$\int \sin x dx = -\cos x + C$

$\int \cos x dx = \sin x + C$

$\int \dfrac{dx}{\sin^2 x} = -ctgx + C$

$\int \dfrac{dx}{\cos^2 x} = tgx + C$

$\int \dfrac{dx}{\sin x} = \ln|\operatorname{tg} \dfrac{x}{2}| + C$

$\int \dfrac{dx}{\cos x} = -\ln|\operatorname{tg}(\dfrac{x}{2} - \dfrac{\pi}{4})| + C$

$\int \operatorname{tg} x dx = -\ln|\cos x| + C$

$\int \operatorname{ctg} x dx = \ln|\sin x| + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \dfrac{a^x}{\ln a} + C$

$\int \operatorname{sh} x dx = \operatorname{ch} x + C$

$\int \operatorname{ch} x dx = \operatorname{sh} x + C$

$\int \ln x dx = x \ln x - x + C$

$\int \sqrt{x^2-a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{4} \ln|\frac{\sqrt{x^2-a^2}-x}{\sqrt{x^2-a^2}+x}| + C$

$\int \sqrt{a^2-x^2} dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \operatorname{arctg} \frac{x}{\sqrt{a^2-x^2}} + C$

$\int \sqrt{x^2+a^2} dx = \frac{a^2}{2} \ln|x+\sqrt{x^2+a^2}| + \frac{x}{2}\sqrt{x^2+a^2} + C$