2) $\|X\|_2 \leq \sqrt{m}\,\|X\|_\infty$

$\circ$ $\|X\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$

$$\|X\|_\infty = \lim_{p\to\infty} \|X\|_p =$$

$$= \lim_{p\to\infty} \left(\sum_{i=1}^{m} |x_i|^p\right)^{1/p} =$$

$$= \lim_{p\to\infty} \left(|X_{max}|^p \left(1 + \left(\left(\frac{x_1}{x_{max}}\right)^p + \ldots + \left(\left(\frac{x_n}{x_{max}}\right)^p\right)\right)\right)^{1/p} =$$

$$= |X_{max}| \longrightarrow$$

$$\longrightarrow \sqrt{\sum_{i=1}^{m} |x_i|^2} \leq \sqrt{m}\,|X_{max}| \longrightarrow$$

$$\longrightarrow \sum_{i=1}^{m} |x_i|^2 \leq m X_{max}^2 \longrightarrow$$

$$\longrightarrow X_{max}^2 + \sum_{\substack{i=1 \\ i\neq i_{max}}}^{m} |x_i|^2 \leq m X_{max}^2 \longrightarrow$$

$$\sum_{\substack{i=1 \\ i\neq i_{max}}}^{m} |x_i|^2 \leq X_{max}^2 (m-1) \longrightarrow \bullet$$

Пример равенства, когда $X_{max} = x_i, \forall i$:

$$\sum_{i=1}^{m} |X_{max}|^2 \leq X_{max}^2 (m-1) \longrightarrow |X_{max}|^2 (m-1) \leq$$
$$\leq X_{max}^2 (m-1)$$

4) $\|UA\|_F = \|AU\|_F = \|A\|_F$

○ Пусть $\bar{y}, \bar{x}$ – произвольные вектора

Тогда:

$$\langle U\bar{x}, U\bar{y} \rangle = \bar{x}^+ \underbrace{U^+ U}_{=1} \bar{y} = \bar{x}^+ y \longrightarrow \|U\bar{x}\|_F^2 = \|\bar{x}\|_F^2 \quad (1)$$

$A = (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n)$

$$\|UA\|_F^2 = \|U\bar{a}_1, U\bar{a}_2, \ldots U\bar{a}_n\| = \sum_{i=1}^{n} (Ua_i, Ua_i) =$$

$$= \| \text{используя (1)} \| = \sum_{i=1}^{n} \|a_i\|_F^2 = \|A\|_F^2$$

А также:

$$\|AU\|_F = \|(AU)^T\|_F = \|U^T A^T\|_F = \|A^T\|_F = \|A\|_F =$$

$$= \|UA\|_F,$$

т.к. U – унитарна ⊙