problem 5 solution

Proof:-

Let $n \in \mathbb{N}$. Using Euclid's Division lemma we have $n = 3q + r$ where $r \in 0, 1, 2$.

Hence every integer is of one of the forms 3q or 3q+1 or 3q+2 $q \in \mathbb{Z}$

Case I: $n = 3q$

Hence $3|q$ and the statement is correct in this case

Case II:- $n = 3q + 1$

$\implies n + 2 = 3q + 1 + 2 = 3q + 3 = 3(q + 1)$

$\implies n + 2 = 3(q + 1)$

$\implies 3|n + 2$

Hence the above statement is correct in this case

Case III:- $n = 3q + 2$

$\implies n + 4 = 3q + 2 + 4 = 3q + 6$

$\implies n + 4 = 3(q + 2)$

$\implies 3|n + 4$

Hence the statement is correct in this case

Hence the statement is correct in all cases and hence is correct $\forall n \in \mathbb{Z}$