

### Problem 1 Solution

The statement is incorrect.

Proof:-

Let  $\exists m \in \mathbb{N}$  and  $n \in \mathbb{N}$  such that  $3m + 5n = 12$

Since  $m \in \mathbb{N}$  we have  $m > 0$ .

Also  $3m = 12 - 5n$

Case I:  $n = 1$

$$\Rightarrow 3m = 12 - 5$$

$$\Rightarrow 3m = 7$$

$$\Rightarrow 3|7$$

which is incorrect. Hence  $n \neq 1$

Case II:  $n = 2$

$$\Rightarrow 3m = 12 - 10$$

$$\Rightarrow 3m = 2$$

$$\Rightarrow 3|2$$

which is incorrect. Hence  $n \neq 2$

Case III:  $n \geq 3$

So we have  $5n \geq 15$

$$\Rightarrow 12 - 5n \leq 12 - 15$$

implies  $12 - 5n \leq -3$

$$\Rightarrow 3m \leq -3$$

$$m \leq -1$$

But since  $m \in \mathbb{N}$   $m > 0$ . Hence  $\forall n \in \mathbb{N}; n \geq 2, 3m + 5n = 12 \Rightarrow m \notin \mathbb{N}$

From case I ,II and II we conclude that our assumption is incorrect. Hence  $\forall m, n \in \mathbb{N} 3m + 5n \neq 12$