

We use induction to prove the statement

Let $P(n)$ denote the statement $2 + 2^2 + \dots + 2^n = 2^{n+1} - 2$

Base case :- For $n = 1$

LHS = 2

RHS = $2^2 - 2 = 2$

LHS = RHS

Hence $P(1)$ is True.

Induction Hypothesis :- Let the statement be true for $k, k \in \mathbb{N}, k > 1$

$$\implies 2 + 2^2 + \dots + 2^k = 2^{k+1} - 2$$

for $n=k+1$ we have

$$\text{LHS} = 2 + 2^2 + \dots + 2^k + 2^{k+1}$$

$= 2^{k+1} - 2 + 2^{k+1}$ from induction hypothesis.

$$= 2(2^{k+1}) - 2$$

$$= 2^{k+2} - 2$$

LHS = RHS

Hence $P(n)$ is true for $n = k+1$

Hence by induction $P(n)$ is true $\forall n \in \mathbb{N}$