problem 6 solution

Proof

Any set of prime triples is of the form $\{p, p+2, p+4\}$

Case I :- p=2

We have p+2 = 4 = 2x2 which is not prime.Hence $p \neq 2$

Case II p = 3

p+2 = 5

p+4 = 7

(3,5,7) form a prime triplet.

Case III: p$\geq$ 4

Since p is prime > 3

So p = 3k+1 or 3k+2 $k \in \mathbb{N}$

Case I: p = 3k+1

p+2 = 3k+1+2 = 3k+3 = 3(k+1) where 3$\geq$ 2 and k+1 $\geq$ 2

p+2 is not prime. Hence (p,p+2,p+4) do not form a prime triple.

Case II: p = 3k+2

p+4 = 3k+2+4 = 3k+6 = 3(k+2) where 3 $\geq$ 2 and k+2 $\geq$ 2

p+4 is not prime and hence (p,p+2,p+4) do not form a prime triple.

Hence the only case in which (p,p+2,p+4) form a prime triple is when p = 3.

Hence the only prime triple is (3,5,7).

Hence $\nexists$ prime triple besides (3,5,7)