

Consider the sequence of intervals $A_n = (0, \frac{1}{n})$

Claim 1: $A_{n+1} \subset A_n$

Proof:

Let $x \in A_{n+1}$

$$\implies 0 < x < \frac{1}{n+1}$$

$$\implies 0 < x < \frac{1}{n} \text{ as } \frac{1}{n} > \frac{1}{n+1}$$

$$\implies x \in A_n$$

$$\implies A_{n+1} \subset A_n$$

Claim2:- $\bigcap_{n=1}^{\infty} A_n = \phi$

proof :-

Let $\bigcap_{n=1}^{\infty} A_n \neq \phi$

$$\implies \exists x \in S = \bigcap_{n=1}^{\infty} A_n$$

Since $x \in A_n$ $x > 0$

$$\implies \exists n \in \mathbb{N} \text{ such that } x > \frac{1}{n} \text{ Archimedian Property}$$

$$\implies x \notin A_{n+1}$$

$$\implies x \notin S$$

$$\implies x \in S \text{ and } x \notin S.$$

Which is a contradiction.

Hence $\nexists x$ such that $x \in \bigcap_{n=1}^{\infty} A_n$

$$\implies \bigcap_{n=1}^{\infty} A_n = \phi$$