

Problem 1 Solution

The statement is incorrect.

Proof:-

Let $\exists m \in \mathbb{N}$ and $n \in \mathbb{N}$ such that $3m + 5n = 12$

Since $m \in \mathbb{N}$ we have $m > 0$.

Also $3m = 12 - 5n$

Case I: $n = 1$

$$\implies 3m = 12 - 5$$

$$\implies 3m = 7$$

$$\implies 3|7$$

which is incorrect. Hence $n \neq 1$

Case II: $n = 2$

$$\implies 3m = 12 - 10$$

$$\implies 3m = 2$$

$$\implies 3|2$$

which is incorrect. Hence $n \neq 2$

Case III: $n \geq 3$

So we have $5n \geq 15$

$$\implies 12 - 5n \leq 12 - 15$$

$$\text{implies } 12 - 5n \leq -3$$

$$\implies 3m \leq -3$$

$$m \leq -1$$

But since $m \in \mathbb{N}$ $m > 0$. Hence $\forall n \in \mathbb{N}; n \geq 2, 3m + 5n = 12 \implies m \notin \mathbb{N}$

From case I, II and III we conclude that our assumption is incorrect. Hence

$\forall m, n \in \mathbb{N} \ 3m + 5n \neq 12$