

We use induction to prove the statement
 Let $P(n)$ denote the statement $2 + 2^2 + \dots + 2^n = 2^{n+1} - 2$
 Base case :- For $n = 1$
 $LHS = 2$
 $RHS = 2^2 - 2 = 2$
 $LHS = RHS$
 Hence $P(1)$ is True.
 Induction Hypothesis :- Let the statement be true for $k, k \in \mathbb{N}, k > 1$
 $\implies 2 + 2^2 + \dots + 2^k = 2^{k+1} - 2$
 for $n=k+1$ we have
 $LHS = 2 + 2^2 + \dots + 2^k + 2^{k+1}$
 $= 2^{k+1} - 2 + 2^{k+1}$ from induction hypothesis.
 $= 2(2^{k+1}) - 2$
 $= 2^{k+2} - 2$
 $LHS = RHS$
 Hence $P(n)$ is true for $n = k+1$
 Hence by induction $P(n)$ is true $\forall n \in \mathbb{N}$