

Assignment 3

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1. Schrödinger Equation

(a) Separation of Variables

We begin with the time-dependent Schrödinger equation in one dimension:

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x, t) \quad (1)$$

Assume a separable solution:

$$\Psi(x, t) = \psi(x)\phi(t) \quad (2)$$

Substituting into the Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} [\psi(x)\phi(t)] = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] [\psi(x)\phi(t)] \quad (3)$$

$$i\hbar \psi(x) \frac{d\phi(t)}{dt} = \phi(t) \left[-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) \right] \quad (4)$$

Divide both sides by $\psi(x)\phi(t)$:

$$i\hbar \frac{1}{\phi(t)} \frac{d\phi(t)}{dt} = \frac{1}{\psi(x)} \left[-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) \right] \quad (5)$$

Since the left-hand side depends only on t and the right-hand side only on x , both sides must be equal to a separation constant, which we denote by E :

$$i\hbar \frac{1}{\phi(t)} \frac{d\phi(t)}{dt} = E = \frac{1}{\psi(x)} \left[-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) \right] \quad (6)$$

(b) Time-Independent and Time-Dependent Equations

We now have two differential equations:

Time-dependent equation:

$$i\hbar \frac{d\phi(t)}{dt} = E\phi(t) \quad (7)$$

Time-independent Schrödinger equation (TISE):

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad (8)$$

(c) Solving the Time-Dependent Equation

We solve the time-dependent equation:

$$\frac{d\phi(t)}{dt} = -\frac{iE}{\hbar}\phi(t) \quad (9)$$

Integrate both sides:

$$\int \frac{1}{\phi(t)} d\phi(t) = -\frac{iE}{\hbar} \int dt \quad (10)$$

$$\ln \phi(t) = -\frac{iE}{\hbar}t + C \quad (11)$$

$$\phi(t) = Ae^{-iEt/\hbar} \quad (12)$$

where $A = e^C$ is an integration constant.