Electric Circuits

Lecture Notes

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Chapter 1

Basic Concepts

1.1 Introduction

Electrical engineering discipline involves many branches such as transfer of electric power, electric machines, electronic circuits, computing systems, control systems, communication systems, instrumentation, and many more. All of these disciplines rely on the concepts of electric circuit theory. Therefore, it is very important for electrical engineering students to study and understand how to analyze electric circuit. This is also important to non-electrical engineering students as well since they are expected to operate and maintain various electrical engineering systems in their own field.

The main purpose of electrical engineering systems are transferring energy from one place to another, communicating information, and processing information for better presentation and analysis. This is achieved by connecting various electrical and electronic devices that can be modeled by electric circuits. An electric circuit consists of various basic circuit elements connected in a closed path by conductors. Figure 1.1 shows an example of an electric circuit that represents a simple electrical headlights system of a car.

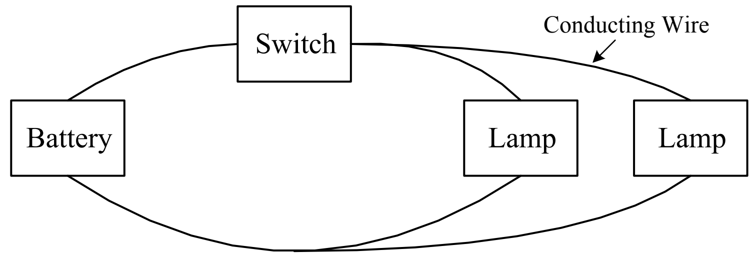


Figure 1.1: An electrical headlights system of a car.

The battery converts the chemical energy into electrical energy that forces the electrical charges (electrons) to flow around the circuit to transfer the energy. The wires are made from excellent conducting material (copper). They connect the elements such that charges can easily move between the elements. The switch is used to control the operation of the system. When the switch is closed, electric current (moving charges) can flow through the circuit. When it is open, the current cannot flow. The headlamps contain special tungsten wires that convert the electric energy of the charges into heat and light.

The physical headlights system is modeled by ideal circuit elements for simplicity of analysis and design, as shown in Figure 1.2.

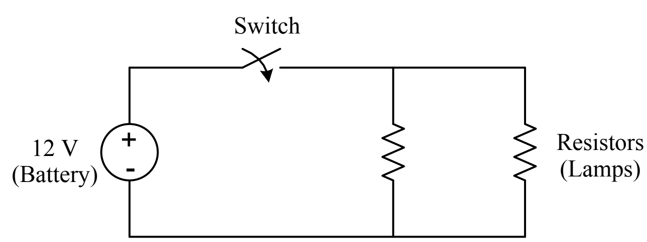


Figure 1.2: Electric circuit model of the headlights system.

The real physical battery is modeled as an ideal voltage source that establishes a specified voltage (say 12 Volts) regardless of the current. The switch acts as a perfect connection when it is closed and no connection when it is open. The physical lamps are modeled as pure ideal resistors that convert electric energy into heat and light. It should be mentioned that these ideal circuit elements are approximations of the real physical elements. This modeling is necessary to be able to analyze and design real-world electrical and electronic systems.

Each circuit element is usually characterised by electrical quantities, namely electric current, electric voltage, and power or energy, which are discussed next.

1.2 Electric Current

When electric charges move through an element or a wire in a circuit, they create an electric current, denoted by i. Since there are both positive and negative charges, the actual direction of the current i is taken, by convention, to be the direction of movement of positive charges. For moving negative charges, the direction of the current i is the opposite direction. This is illustrated in Figure 1.3.

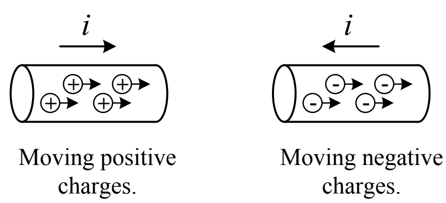


Figure 1.3: Electric current due to moving charges.

Mathematically, the current $i(t)$ is defined as the time rate of change of the charge $q(t)$, i.e.

$$i(t) = \frac{dq(t)}{dt} \tag{1.1}$$

where the current $i(t)$ is measured in Amperes (A), the charge $q(t)$ is in Coulombs (C), and the time t is in seconds (s).

Multiplying Eq. (1.1) by dt and integrating both sides gives the charge in terms of the current.

$$q(t) = \int_{t_0}^{t} i(\tau)\, d\tau + q(t_0) \qquad (1.2)$$

Example 1.1

The charge flowing through an electric element is given by $q(t) = 3t^2 - 12t$. Find the current $i(t)$ at $t = 0$ s and $t = 3$ s.

Solution:

$$i(t) = \frac{dq(t)}{dt} = 6t - 12$$

$$i(0) = -12 \text{ A}$$

$$i(3) = 6 \text{ A}$$

Example 1.2

The current through an element is given by $i(t) = 0$ for $t < 0$ and $i(t) = e^{-2t}$ for $t \geq 0$. Find the charge for $t \geq 0$ given $q(0) = 0$.

Solution:

$$q(t) = \int_{t_0}^{t} i(\tau)\, d\tau + q(t_0) = \int_{0}^{t} e^{-2\tau}\, d\tau + 0$$

or
$$q(t) = \frac{1}{-2}(e^{-2t} - e^{-0}) = \frac{1}{2}(1 - e^{-2t}), \qquad t \geq 0$$

Reference Direction of Flow of Current

The **actual directions** of currents for the elements in a circuit are usually unknown before analysing the circuit. In addition, the value of a current for the same element can be position or negative for different times, as seen in Example 1.1. Therefore, **reference directions** are assigned arbitrarily to the elements before analyzing the circuit. After analyzing the circuit, if the value of the current is positive, then the actual direction is the same as the reference. Otherwise, it is in the opposite direction. This is illustrated in Figure 1.4.

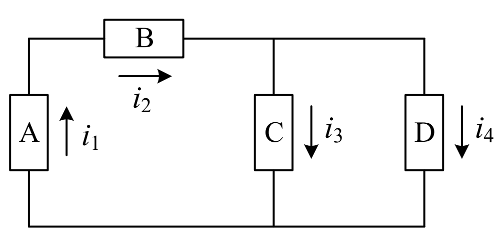


Figure 1.4: Reference direction of currents.

All currents in the circuit are assigned names and directions arbitrarily as references. Let us say, after analyzing the circuit, it is found $i_1 = +2$ A (positive), then the actual direction of i_1 is the same as the indicated reference. If $i_3 = -5$ A (negative), then the actual direction of i_3 is the opposite of the indicated reference.

Classification of Current

In circuit analysis, currents are classified based on how the current changes with time, as follows:

- **Direct Current (DC):** The current flows in one direction over time such as a constant current $i = 3$ A.
- **Alternating Current (AC):** The current reverses direction over time such as a sinusoidal current $i(t) = 2\sin(5t)$.
- **Time Varying:** Any other type of current such as $i(t) = e^{-t}$.

1.3 Electric Voltage

The motion of charge (current) in a circuit is created by a force called **electromotive force (emf)**, **potential difference**, or **electric voltage**. In every circuit, there should be at least one element that provides this force. Also, the moving charges have the ability to transfer energy from one point to another. The **electric voltage** is defined as the amount of energy required to move a unit charge from one point to another. Mathematically, the electric voltage is defined by:

$$v = \frac{dw}{dq} \tag{1.3}$$

where w is the energy measured in joules (J), q is the charge in coulombs (C), and v is the voltage in volts (V), where 1 V = 1 J/C. It should be emphasized that a voltage is established between two points in a circuit, regardless of the circuit elements connected between them.

When charges move through an element, they establish a voltage difference across it since energy is absorbed or supplied by the element. Polarities, plus (+) and minus (-), are used to distinguish the higher potential (+) from the lower (-).

Reference Polarities of Voltage

The **actual polarities** of voltages for the elements in a circuit are usually unknown before analysing the circuit. Therefore, **reference polarities** are assigned arbitrarily to the elements before analyzing the circuit. After analyzing the circuit, if the value of the voltage is positive, then the actual polarities are the same as the reference. Otherwise, they are the opposite. This is illustrated in Figure 1.5.

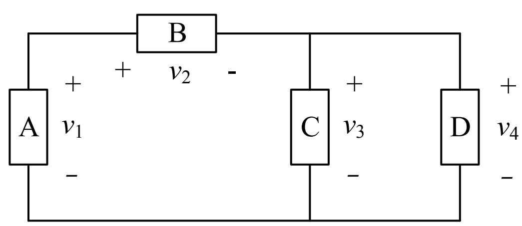


Figure 1.5: Reference polarities of voltages.

All voltages in the circuit are assigned names and polarities arbitrarily as references. Let us say, after analyzing the circuit, it is found $v_1 = +5$ V (positive), then the actual polarities of v_1 are the same as the indicated reference. If $v_3 =$

-3 V (negative), then the actual polarities of v_3 are the opposite of the indicated reference.

Voltages are classified similar to currents. For example, a voltage that is constant over time such as $v = 2$ V is classified as a **DC voltage**, while the voltage $v = 3\cos(2t)$ is an **AC voltage**.

1.4 Power and Energy

Current and voltage are the two basic variables that are computed for every element in a circuit. However, power and/or energy are also equally important. For example, we know a 100-watt bulb provides more light than a 60-watt bulb. In addition, the proper operation of an electric element depends on a maximum power value, after which the element can be damaged. Moreover, electric utility companies charge their customers based on the energy they consume over a certain period of time.

We know from physics that power is defined to be the time rate of change of energy, i.e.

$$p = \frac{dw}{dt} \tag{1.4}$$

where w is the energy measured in Joules (J) and p is the power measured in Watts (W). Equation (1.4) is rarely used. Instead, power can be computed from voltage and current as follows:

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \tag{1.5}$$

which says that power is the product of voltage and current. In electric circuits, we distinguish between power/energy absorbed by an element and power/energy supplied (or delivered) by the element, as defined below.

Absorbed Power

The convention or reference for an element to absorb power is when the direction of the reference current i is from the positive sign (+) to the negative sign (**-**) of the reference voltage v, as shown in Figure 1.6, and the reference absorbed power is $p_{abs} = v \cdot i$. This convention is also called **passive sign convention** (PSC). The convention makes sense because if both voltage and current have positive values, then positive charges move from higher potential (+) to lower potential (**-**). This implies the charges lose energy/power to the element that absorbs it and converts it to another form, such as heat.

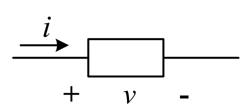


Figure 1.6: Passive sign convention (PSC).

Since the values of the voltage and current can be positive or negative, the absorbed power $p_{abs} = v \cdot i$ can have a positive or negative value. If $p_{abs} > 0$, then the power is **actually absorbed**. On the other hand, if $p_{abs} < 0$, then the power is **actually supplied** by the element.

➢ **Remark**: If the absorbed power is computed for an element that does not comply with the passive sign convention, as shown in Figure 1.7, then $p_{abs} = -v \cdot i$ is used.

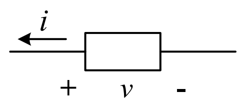


Figure 1.7: An element that does not comply with the passive sign convention.

Supplied/Delivered Power

The convention or reference for an element to supply or deliver power is when the direction of the reference current i is from the negative sign (**-**) to the positive sign (+) of the reference voltage v, as shown in Figure 1.7, and the reference supplied power is $p_{sup} = v \cdot i$. This convention makes sense because if both voltage and current have positive values, then positive charges move from lower potential (**-**) to higher potential (+). This implies the charges gain energy/power from the element which is supplied to the rest of the circuit.

Again, since the values of the voltage and current can be positive or negative, then the supplied power $p_{sup} = v \cdot i$ can have a positive or negative value. If $p_{sup} > 0$, then the power is **actually supplied** by the element. On the other hand, if $p_{sup} < 0$, then the power is **actually absorbed** by the element.

➢ **Remark**: If the supplied power is computed for an element that complies with the passive sign convention, as shown in Figure 1.6, then $p_{sup} = -v \cdot i$ is used. Notice that for every element $p_{abs} = -p_{sup}$.

Example 1.3

For the circuit shown in Figure 1.8, the values of voltages and currents have been found to be $i_1 = i_2 = 5$ A, $i_3 = -2$ A, $i_4 = 3$ A, $v_1 = 8$ V, $v_2 = -2$ V, $v_3 = -10$ V, $v_4 = 10$ V.

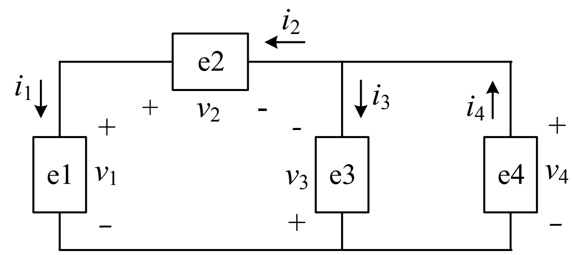


Figure 1.8: Circuit for Example 1.3.

(a) Compute the power for element e1.

(b) Compute the absorbed power by the element e2.

(c) Compute the supplied power by the element e3.

(d) Compute the absorbed power by the element e4.

Solution:

(a) For e1, using PSC (absorbed power reference), $p_1 = v_1 \cdot i_1 = (8\text{ V}) \cdot (5\text{ A}) = 40$ W. Since p_1 is positive, then the power is actually absorbed.

(b) For e2, the absorbed power is $p_2 = -v_2 \cdot i_2 = -(-2\text{ V}) \cdot (5\text{ A}) = 10$ W. Again, since p_2 is positive, then the power is actually absorbed.

(c) For e3, the supplied power is $p_3 = v_3 \cdot i_3 = (-10\text{ V}) \cdot (-2\text{ A}) = 20$ W. Since p_3 is positive, then the power is actually supplied.

(d) For e4, the absorbed power is $p_4 = -\, v_4 \cdot i_4 = -(10\text{ V}) \cdot (3\text{ A}) = -30\text{ W}$. Since p_4 is negative, then the power is actually supplied.

Principle of Conservation of Power/Energy

For every circuit, the sum of all supplied powers is equal to the sum of all absorbed powers; i.e.

$$\sum absorbed\ powers = \sum supplied\ powers \tag{1.6}$$

For Example 1.3, we should have $p_1 + p_2 + p_4 = p_3$ which can be verified from $40 + 10 + (-30) = 20$.

Another equivalent statement of Eq. (1.6) is that *the algebraic sum of all powers in a circuit is zero*; i.e.

$$\sum_n p_n = 0 \tag{1.7}$$

In this equation, algebraic means that if an absorbed power is assigned a positive sign, then a supplied power should be assigned a negative sign. For Example 1.3, this can be written $p_1 + p_2 - p_3 + p_4 = 0$.

Computation of Energy

Multiplying Eq. (1.4) by dt and integrating both sides gives the absorbed/supplied energy:

$$w(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_{-\infty}^{t} v(\tau)\, i(\tau)\, d\tau \tag{1.8}$$

And the energy from time t_1 to time t_2 is given by:

$$w = w(t_2) - w(t_1) = \int_{t_1}^{t_2} p(\tau)\, d\tau \tag{1.9}$$

1.5 Circuit Elements

Ideal circuit elements are used to model *real physical components* in an electric circuit. This modeling is necessary to simplify the analysis and design of real physical circuits. It should be emphasized that a real physical element can be approximated by one or more ideal elements. Therefore, the accuracy of an electric circuit model depends on the goodness of approximation of the real components by their ideal circuit elements. In this course, we analyze electric circuits with the *ideal models* described below, regardless of the original real physical components.

Ideal circuit elements are classified as *active* or *passive* elements. An active element can generate or supply power or energy to the rest of the circuit, such as a *voltage source* or a *current source*. A passive element can not supply power or energy to the rest of the circuit, such as a *resistor*, a *capacitor*, or an *inductor*. These passive elements can absorb energy only, in the case of a resistor, or store and later release the stored energy, in the case of a capacitor or inductor. In addition, the active elements are sub-classified into *independent* and *dependent* sources. In this section, we define these elements except for the resistor, capacitor and inductor, which are defined in later chapters.

Remark: Circuit elements are also classified as *linear* or *nonlinear* elements. A linear element has its voltage and current related by a linear relation, such as $v = 5\,i$. A nonlinear element has a nonlinear relation, such as $v = 3\,i^2$. In this course, we consider linear elements only, since nonlinear circuits are more difficult to analyze and most of the time the nonlinear elements are approximated by linear elements.

Ideal Independent Voltage Source

This source is defined to have a specified voltage across its terminals independent of the current through it or other circuit elements. The current is determined by the rest of the circuit. Physical sources such as batteries and generators can be approximated by ideal independent voltage sources. Figure 1.9 shows the two common symbols used for this element.

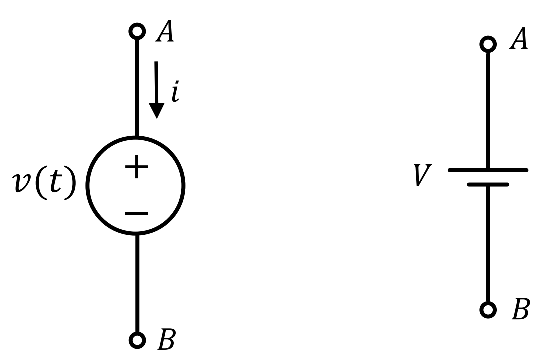


Figure 1.9: Independent voltage source.

Ideal Independent Current Source

This source is defined to have a specified current independent of the voltage across its terminals or other circuit elements. The voltage is determined by the rest of the circuit. A good approximation to an ideal current source can be implemented using electronic amplifiers. Figure 1.10 shows the common symbol used for this element.

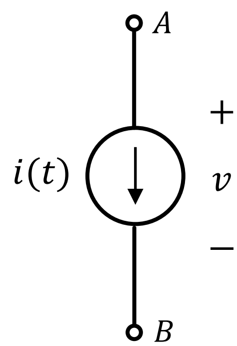


Figure 1:10: Independent current source.

<u>Remark</u>: Keep in mind, above models are *<u>ideal</u>*, which means they do not exist as separate physical elements. However, these models with the other circuit models that we will discuss later will help us in modeling real physical elements more accurately and in analysing circuits. For example, a real physical battery can be modeled more accurately by an ideal constant voltage source with a resistor.

<u>Definition</u> (Short Circuit): Two points (nodes) in a circuit are called a *<u>short circuit</u>* if the voltage between the two nodes is zero. An ideal voltage source with a zero voltage is equivalent to a short circuit. A line, as shown in Figure 1.11 represents the short circuit.

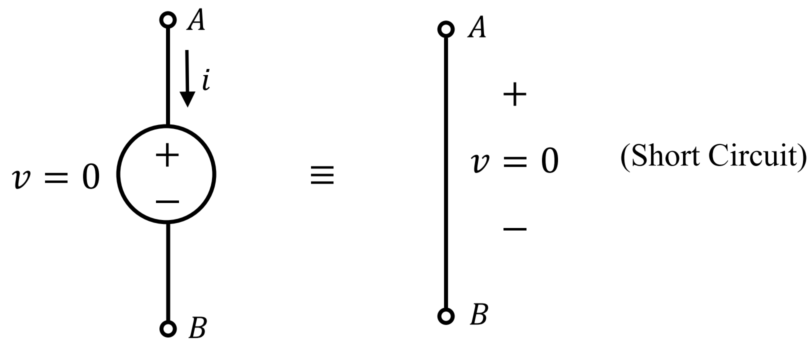


Figure 1.11: Short circuit.

<u>Definition</u> (Open Circuit): Two points (nodes) in a circuit are called an *<u>open circuit</u>* if the current between the two nodes is zero. An ideal current source with a zero current is equivalent to an open circuit. The open circuit is represented by no connection between the two points, as shown in Figure 1.12.

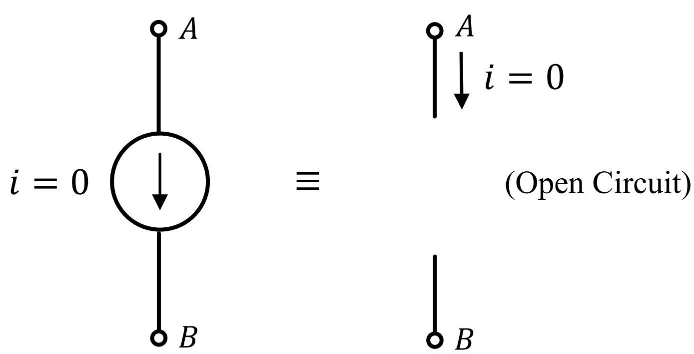
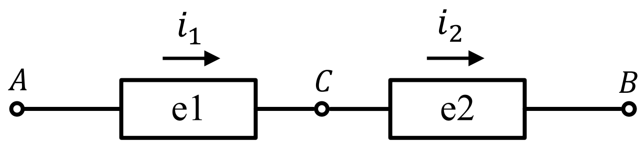


Figure 1.12: Open circuit.

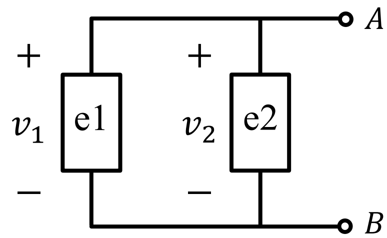
<u>Definition</u> (Series Connection): Two elements are called *<u>connected in series</u>* if they are the only two elements connected to a node. Series connected elements have the same current. This is shown in Figure 1.13 where the elements e1 and e2 are the only two elements connected to the node *C*.



$i_1 = i_2$

Figure 1.13: Series connection.

<u>Definition</u> (Parallel Connection): Two or more elements are called *<u>connected in parallel</u>* if they are connected between the same two nodes. Parallel-connected elements have the same voltage. This is shown in Figure 1.14.



$v_1 = v_2$

Figure 1.14: Parallel connection.

<u>Remark</u>: Ideal source elements have limitations in regard to series and parallel connections, as we see next.

Series Connection of Sources

The series connection of ideal voltage sources is always valid. While, the series connection of current sources is not valid since in general $i_1 \neq i_2$, which violates the series connection condition. This is seen in Figure 1.15.

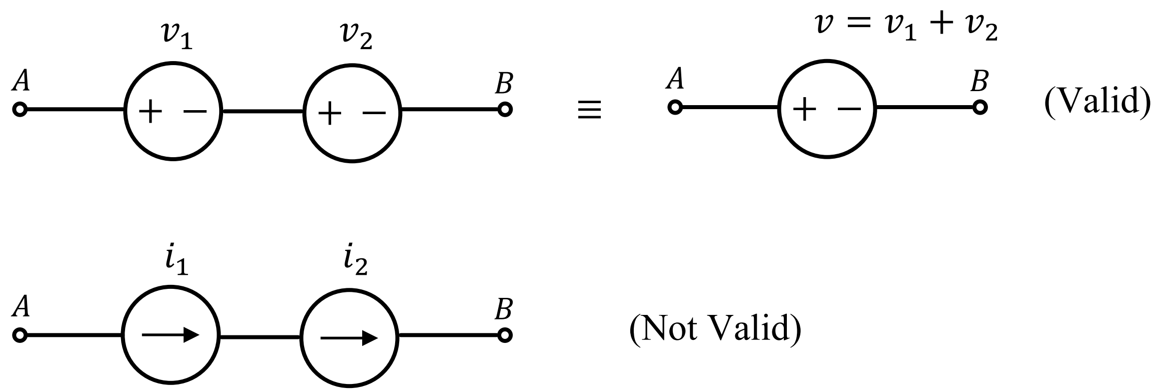


Figure 1.15: Series connection of sources.

Parallel Connection of Sources

The parallel connection of ideal current sources is always valid. While, the parallel connection of voltage sources is not valid since in general $v_1 \neq v_2$, which violates the parallel connection condition. This is seen in Figure 1.16.

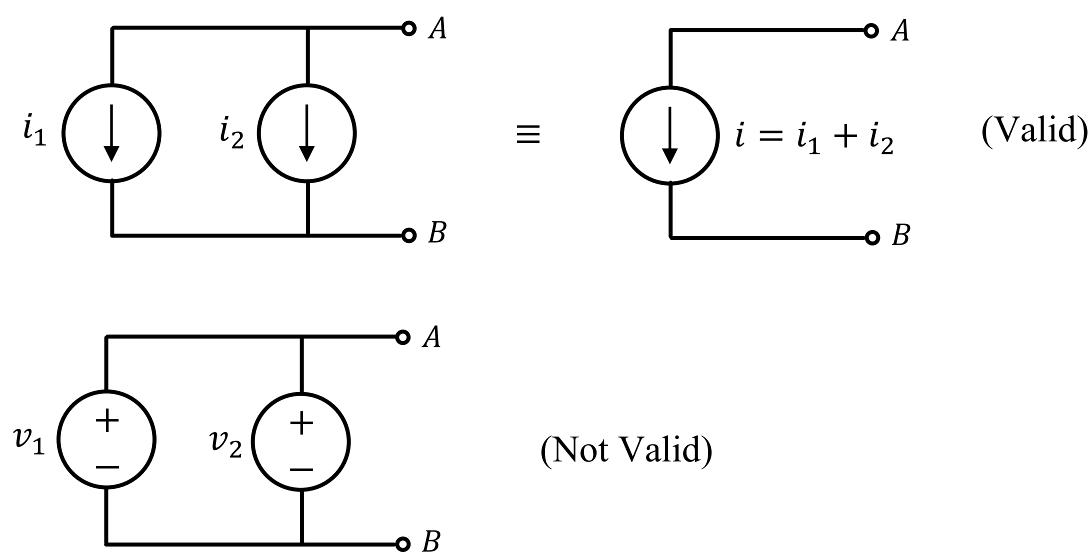


Figure 1.16: Parallel connection of sources.

Dependent Sources

Electronic devices, such as a transistor or an amplifier, are modeled by a dependent voltage source or a dependent current source, which are also called *controlled sources*. The dependent voltage source can be controlled by a voltage or a current in another circuit element, and the dependent current source can be controlled by a voltage or a current in another circuit element. Therefore, there are four models for the dependent sources as shown in Figure 1.17 to Figure 1.20. Notice that a *diamond shape* is used to indicate a dependent source.

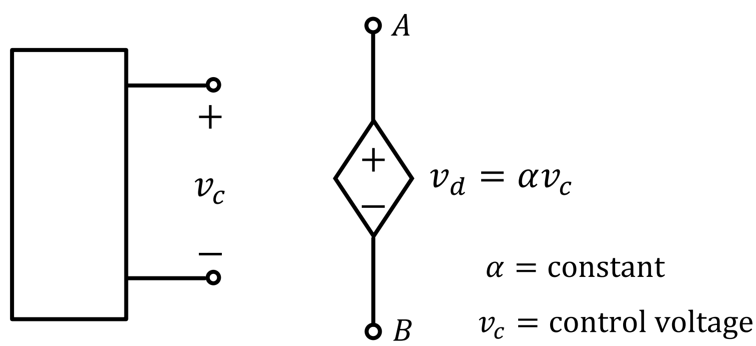


Figure 1.17: Voltage-Controlled Voltage Source (VCVS).

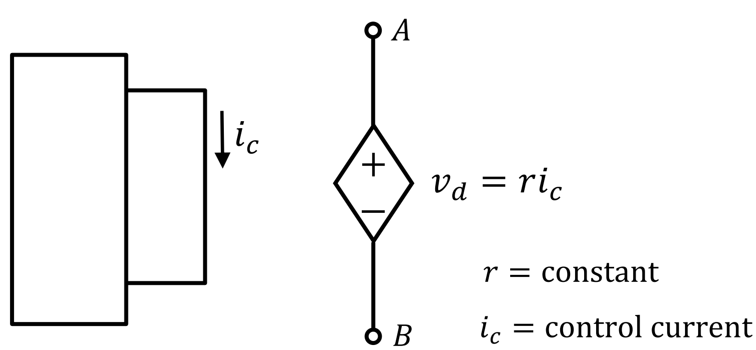


Figure 1.18: Current-Controlled Voltage Source (CCVS).

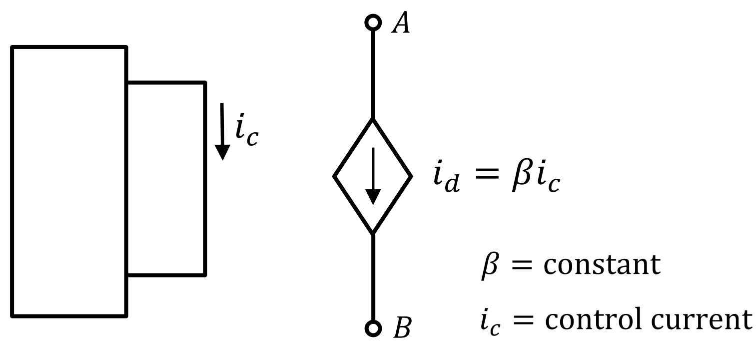


Figure 1.19: Current-Controlled Current Source (CCCS).

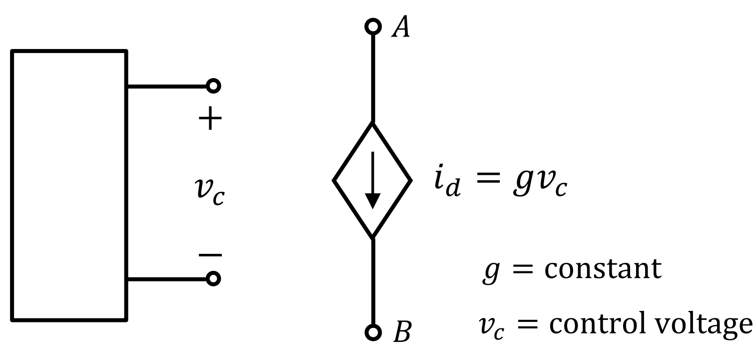
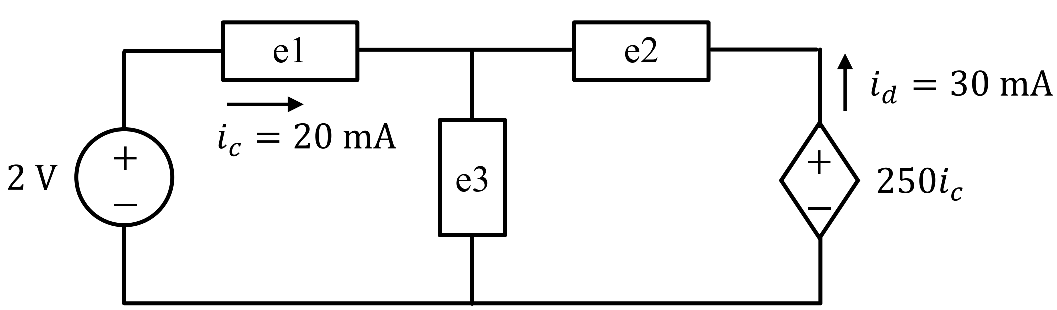


Figure 1.20: Voltage-Controlled Current Source (VCCS).

Example 1.4

Find the supplied power by the controlled voltage source.



$$p_d = (250 i_c) i_d = (250 \times 20 \times 10^{-3})(30 \times 10^{-3}) = 150 \text{ mW}$$