

The Measurement of Water Salinity for Antarctic Research



Presented by:
Zuhayr Loonat

Prepared for:
Justin Pead
Dept. of Electrical Engineering
University of Cape Town

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Acknowledgments

I would like to express my sincere gratitude to my supervisor, Justin Pead, for his invaluable guidance, support, and expertise throughout this research. His insights and encouragement were instrumental in bringing this work to fruition.

I am also deeply grateful to my friends for their unwavering moral support during this journey. Their encouragement and understanding made the challenges of this process far more manageable.

Lastly, I would like to thank my family for their support during my studies.

Abstract

Ice shelves form when glaciers flow from the Antarctic out over the ocean. This ice, which is originally freshwater ice is formed from accumulated snow, extends over the water and floats, because ice is less dense than water. When saltwater freezes at the bottom of the ice shelves, salt is expelled, forming a brine like solution, with a high salt density, directly under the ice shelves.

The salinity of this brine solution and the seawater underneath need to be measured to aid in Antarctic research. This project documents the design and testing of a conductivity-based salinity measuring device to measure the salt content in these areas. The prototype was designed in two parts, a probe module, which would be lowered down through a hole drilled into the ice shelves to measure the salinity, and a controller, which would be used to send instructions and receive data from the probe. The probe utilised gold electrodes to measure and analyse the salinity through conductivity. Two methods were investigated, Direct Current (DC), where point voltage measurements and voltage sweeps were conducted, and Alternating Current (AC) where a machine learning model was used to map properties of the input and output waves to a salinity value. [Talk about accuracy TODO]

Contents

Chapter 1

Introduction

1.1 Background to the study

A very brief background to your area of research. Start off with a general introduction to the area and then narrow it down to your focus area. Used to set the scene[?].

1.2 Objectives of this study

1.2.1 Problems to be investigated

Description of the main questions to be investigated in this study.

1.2.2 Purpose of the study

Give the significance of investigating these problems. It must be obvious why you are doing this study and why it is relevant.

1.3 Scope and Limitations

Scope indicates to the reader what has and has not been included in the study. Limitations tell the reader what factors influenced the study such as sample size, time etc. It is not a section for excuses as to why your project may or may not have worked.

1.4 Plan of development

Here you tell the reader how your report has been organised and what is included in each chapter.

I recommend that you write this section last. You can then tailor it to your report.

Chapter 2

Literature Review

2.1 Introduction

Accurate salinity measurement is fundamental to oceanographic research. Traditional measurement techniques have evolved from labour-intensive chemical titration methods to modern electronic sensors, with electrical conductivity emerging as the predominant approach due to its combination of accuracy, speed, and practical deployability. This literature review examines the current state of salinity measurement technology with particular emphasis on conductivity-based methods and emerging machine learning approaches for electrochemical data interpretation. The review is organised into three main sections. First, we establish the fundamental concepts of salinity and provide a comprehensive comparison of available measurement techniques. Second, we examine the theoretical foundations and practical implementation of electrical conductivity measurements for salinity determination, including instrumentation, calibration procedures, and current limitations. Finally, we explore the application of Electrical Impedance Spectroscopy (EIS) and Machine Learning (ML) as advanced approaches for enhanced salinity analysis, examining how frequency-domain measurements and intelligent data processing can overcome limitations of traditional single-frequency conductivity methods. This comprehensive review provides the theoretical and methodological foundation for developing a machine learning-enhanced impedance spectroscopy approach for salinity determination.

2.2 Salinity: Definition

Salinity is a fundamental characteristic of water, and is most commonly defined as the total amount of dissolved salts in water, and in the context of oceanography, seawater. It is typically expressed in Parts per thousand (‰) or Practical Salinity Unit (PSU) [?]. The concept of salinity has evolved significantly from its early definition, which was based on chlorinity measurements. Modern salinity is defined through the Practical Salinity Scale 1978 (PSS-78), where salinity is based on the conductivity ratio of standard seawater solutions, to a standard Potassium Chloride solution, and is dimensionless [?]. The salinity-conductivity relationship is however, quite complex, requiring corrections and calibrations needed for depth and temperature, as these both play a factor in the conductivity of the water.

2.3 Overview of Salinity Measurement Methods

There are a multitude of methods which can be used to measure salinity, each with their own advantages, limitations and levels of accuracy. Traditional methods include gravimetric analysis, chemical titration (such as the Mohr-Knudsen method for chlorinity), and refractometry. While these techniques can provide accurate results, they are often time-consuming, require skilled operators, and are not easily adaptable to in-situ or automated measurements. Modern approaches predominantly rely on electrical conductivity sensors, which offer rapid, repeatable, and automated salinity determination. Other techniques, such as optical methods and ion-selective electrodes, have also been explored, but are less commonly used in oceanographic applications due to issues with robustness, calibration, or specificity. The choice of method depends on the required accuracy, operational environment, and available resources.

2.3.1 Historical Methods

Chlorinity Titration

Early salinity measurements relied on chemical titration methods, in particular the Mohr-Knudsen chlorinity titration, which used silver nitrate. The chlorinity of a solution has the definition ‘the mass of silver required to precipitate completely the halogens

in 0.3285234kg of the ocean-water sample' This method was highly accurate, with results within (± 0.001 PSU). However, it relied heavily on toxic chemicals, and was a time-consuming laboratory procedure, with limited practical application in the field.

Gravimetric Methods

Gravimetric analysis, a technique used to determine an amount of a substance, by measuring its change in mass, involves evaporation and the weighing of dissolved solids. This method directly provided measurements of the salt content, within accuracies of (± 0.001 PSU), under controlled laboratory conditions. This method remains the reference standard for calibration processes, but is however, extremely slow.

2.3.2 Physical Property Based Methods

There are several methods that utilise the relationship between salinity and the physical properties of water.

Hydrometric and Density Methods

Hydrometric methods using density measurements via hydrometers, offer salinity measurements that are low-cost, and electronics free. However, they are limited in precision with accuracies of $\pm 1 - 2$ PSU, and require large sample volumes. The hydrometer is a floating instrument, that sinks to different depths depending on the density of the solution, and by measuring how high or low it floats, the density of the solution can be determined. The following equation is used to map the relationship between salinity and density.

$$\rho = \rho_0(1 + kS) \quad (2.1)$$

where ρ is the density, ρ_0 is the density of fresh water, S is the salinity and k is the proportionality constant.

This can then be inverted to give Salinity from Density:

$$S = \frac{\frac{\rho}{\rho_0} - 1}{k} \quad (2.2)$$

This however, does not include temperature correction.

Refractometric Thechniques

Refractometric techniques measure the refractive index changes cuased by the dissovled salts. The refractive index of seawater is influenced by wavelength, temperature, salinity, and pressure. Within the range of 500-700 nm wavelength, 0-30°C temperature, 0-40 PSU salinity, and 0-11000 dbar pressure, the refractive index equation provides an accuracy of 0.4-80 ppm PSU, with accuracy decreasing as pressure increases. Refractometers, which require only a small sample volume, are compact devices, making them suitable for portable field measurements. Fibre optic refractometers have improved portability and reduced temperature sensitivity, with moderate accuracy (± 0.5 -1 PSU), making them increasingly popular in aquaculture applications.

Freezing Point Osmometry

Freezing point depression osmometry exploits the colligative (i.e. relating to the binding together of molecules) properties of dissolved salts. The main principle relies on freezing point depression, which is the phenomenon where a solvents freeving point is lowered when a solute is added to it. To perform the measurement, the water is cooled till its freezing point and the temperature drop is measured, which is then used to calculate the osmolality. This method can achieve accuracies as high as (± 0.001 PSU), however its requirement for precise temperature control limits its usage to laboratory applications.

Interferometry

Interferometry is a measurement technique which measures how electro-magnetic waves are affected by changes in a solution. Two identical light waves are passed through two solutions, one benchmark and one test sample solution. The gain and phase shift between the waves is then used to calculate the salinity. This method requires precisely aligned mirrors to direct the light waves, causing it to be relatively large.

Electromagnetic Induction and Magnetic Permeability

2.3.3 Advanced Analytical Methods

2.3.4 Remote Autonomous Sensing

2.4 Conductivity-Based Salinity Measurements

2.4.1 Theoretical Foundation

Electrical conductivity has emerged as the predominant method for salinity measurement due to its practical implementation, high accuracy and fast response time. The technique utilises the strong correlation between dissolved ionic content and electrical conductivity.

The conductivity of a liquid is measured by its ability to conduct electrical current. The relationship between conductivity and salinity is based on the concentration of dissolved ions in seawater. The main ions found in sea water (Na^+ , Cl^- , Mg^{2+} , SO_4^{2-} , Ca^{2+} , K^+) maintain a relatively constant proportional relationship, in ocean waters [?]. This enables robust corrections between conductivity and total dissolved salt content. Unlike other measurement techniques, conductivity accounts for all the ions in the water, not only chlorine, which is why it is considered a more accurate measure of salinity [?].

The Practical Salinity Scale 1978 (PSS-78) defines Practical Salinity S_p through the conductivity ratio K_{15} , as shown below [?]:

$$K_{15} = \frac{C(S_p, 15, 0)}{C(KCl, 15, 0)} \quad (2.3)$$

where the numerator, $C(S, 15, 0)$ represents the conductivity of seawater sample at 15°C and standard atmospheric pressure ($1\text{atm}/101.325\text{kPa}/0\text{dbar}$), and the denominator, $C(KCl, 15, 0)$ is the conductivity of a standard KCl (Potassium Chloride) solution under identical temperature and pressure. The standard KCl solution consists of $32.4356 \times 10^{-3}\text{kg}$ of KCl dissolved in 1kg of water [?]. When the ratio between the water sample and the KCl solution is 1, i.e. $K_{15} = 1$, then the Practical Salinity S_p is, according to the definition, 35 [?].

It is important to note that Practical Salinity is a unit-less quantity, and though it may be convenient, it would be incorrect to quote it in PSU. Practical salinity should rather be quoted as a certain Practical Salinity ‘on the Practical Salinity Scale PSS-78’ [?].

When K_{15} does not equal 1, Practical Salinity, S_p can be calculated using the equation below [?]:

$$S_p = \sum_{i=0}^5 a_i (K_{15})^{i/2} \quad (2.4)$$

where K_{15} is the equation defined above (Equation ??), and the coefficients a_i are given in Table (??).

2.4.2 Temperature and Pressure Compensation

When calculating salinity at conditions other than 15°C , and 0dbar , the conductivity ratio R is expanded to the product of three ratios R_p , R_t and r_t as follows [?]:

$$R = \frac{C(S_p, t, p)}{C(35, 15, 0)} = R_p R_t r_t \quad (2.5)$$

where t , and p are the temperature and pressure valid over the ranges $-2^\circ\text{C} \leq t \leq 35^\circ\text{C}$ and $0 \leq p \leq 10000\text{dbar}$ respectively.

These ratios can be expanded as follows:

$$R = \frac{C(S_p, t, p)}{C(35, 15^\circ\text{C}, 0)} = \frac{C(S_p, t, p)}{C(S_p, t, 0)} \cdot \frac{C(S_p, t, 0)}{C(35, t, 0)} \cdot \frac{C(35, t, 0)}{C(35, 15^\circ\text{C}, 0)} = R_p R_t r_t \quad (2.6)$$

This equation represents the ratio between the conductivity measurement of a sample $C(S_p, t, p)$ and the conductivity of the standard solution $C(35, 15^\circ, 0)$ [?]. In order to find the salinity, R_p , R_t and r_t need to be calculated. First, r_t is calculated using the temperature of the sample:

$$r_t = \sum_{i=0}^4 c_i t_i \quad (2.7)$$

R_p is then calculated as a function of the temperature t , pressure p , and conductivity

ratio R :

$$R_p = 1 + \frac{\sum_{i=1}^3 e_i p^i}{1 + d_1 t + d_2 t^2 + R[d_3 + d_4 t]} \quad (2.8)$$

Finally, R_t can be evaluated using R , R_p and r_t :

$$R_t = \frac{R}{R_p r_t} \quad (2.9)$$

At standard conditions, i.e., temperature $t = 15^\circ\text{C}$, R_t is equal to K_{15} and therefore Practical salinity S_p can be calculated from Equation ?? . For cases where the temperature is not $t = 15^\circ\text{C}$, Practical Salinity S_p is given as a function of R_t , with $k = 0.0162$ [?]:

$$S_p = \sum_{i=0}^5 a_i (R_t)^{i/2} + \frac{t - 15}{1 + k(t - 15)} \sum_{i=0}^5 b_i (R_t)^{i/2} \quad (2.10)$$

Note that Equations (??) to (??) are only valid in the range $2 < S_p < 42$, $-2^\circ\text{C} \leq t \leq 35^\circ\text{C}$ and $0 \leq p \leq 10000\text{dbar}$.

i	a_i	b_i	c_i	d_i	e_i
0	0.0080	0.0005	$6.766\,097 \times 10^{-1}$		
1	-0.1692	-0.0056	$2.005\,64 \times 10^{-2}$	3.426×10^{-2}	2.070×10^{-5}
2	25.3851	-0.0066	$1.104\,259 \times 10^{-4}$	4.464×10^{-4}	-6.370×10^{-10}
3	14.0941	-0.0375	-6.9698×10^{-7}	4.215×10^{-1}	3.989×10^{-15}
4	-7.0261	0.0636	1.0031×10^{-9}	-3.107×10^{-3}	
5	2.7081	-0.0144			

Table 2.1: Table of Coefficients for PSS-78 Equations [?]

It must be noted that the PSS-78 equations use the IPTS-68 temperature scale and in order for them to work with the current ITS-90 scale, must be converted using the equation below [?]:

$$t_{68}^\circ\text{C} = 1.00024 \times t_{90}^\circ\text{C} \quad (2.11)$$

2.4.3 Instrumentation and Technology

The most common method for measuring salinity is by using a Conductivity, Temperature, Depth (CTD) device. The fundamental concept of these devices involves placing two electrodes in a sample of water, applying a voltage across them and measuring the water's response. This is then paired with a temperature and depth correction, allowing for an accurate salinity measurement. The depth value for these calculations is taken from the pressure at which the measurement is taken. This pressure is then translated to depth using the standard depth to pressure equation. Modern CTD systems achieve salinity accuracies better than ± 0.005 PSU, with some instruments like the Sea-Bird 911 Plus demonstrating historical accuracies of ± 0.002 PSU or ± 0.0002 PSU.

2.4.4 Applications and Limitations

Conductivity-based salinity measurements excel in most oceanographic and water quality applications due to its accuracy, speed, and reliability. The conductivity method allows for real-time data capture, continuous monitoring, and easy integration with autonomous devices.

However, this method does face some limitations. Due to its dependence on the water's capacity to conduct electricity, freshwater applications require specialised low-conductivity sensors, while hyper-saline environments could exceed the standard calibration range.

The method's reliance on empirical correlations derived from typical seawater compositions can introduce errors in waters ocean waters affected by external factors such as pollution or freshwater inflow from connecting rivers, which can alter ionic composition and introduce variability not captured by standard seawater-based calibrations. In such environments supplementary practices may be necessary for accurate salinity measurements.

2.5 Machine Learning Applications in Electrical Impedance Spectroscopy

2.5.1 Electrical Impedance Spectroscopy

Chapter 3

Methodology

3.1 Salinity Measurement Method

A CTD sensor, which measures salinity using conductivity, temperature and depth, was chosen as the salinity measurement device. When choosing a measurement technique multiple factors needed to be considered. Firstly, the salinity measurements are to be conducted in the Antarctic, where the environment, and remote nature of the area, make majority of the measurement methods unusable. Secondly, the device would need to fit through an ice core hole with a diameter of $100mm$, and lastly, the device would need to be able to take continuous measurements.

CTD sensors do not require sample collection, unlike chlorinity titration, gravimetric analysis and refractometry. This removes both the need for sample collection and the challenges of sample degradation, storage and transport logistics.

Modern CTD sensors are compact, and can easily be designed for specific space constraints. This coupled with its deployments flexibility make it the preferred choice over methods, such as laboratory methods, which suffer from deployment constraints. CTD sensors allow for continuous realtime monitoring, a characteristic none of the the alternative methods provide. The alternative methods either require sample collection, or cannot measure continuously.

CTD instruments inherently measure conductivity, temperature and pressure simultaneously, providing salinity measurements with temperature and depth compensation, whereas laboratory methods measure salinity only, and require seperate temperature measurements.

These factors coupled with the researcher's significant experience with PCB design and electronics influenced the choice for a CTD sensor.

3.2 Electrode Design

When measuring conductivity, choosing an electrode material plays a significant role in the accuracy of the measurements. To get an accurate measurement of the resistance of the water, ideally, a electrode resistance of zero is required. This would allow the resistance measurement to be entirely due to the resistance of the water. Most conductive materials have conductivities of order $10^6 - 10^8 S/m$, which is negligible compared to sea (salt) water, which has an average conductivity of $3.31 S/m$ [?][?]. Preferably, the material with the highest conductivity, silver, would be used. However, conductivity is not the only factor considered when designing an electrode. The electrodes will be submerged in saltwater, which is highly corrosive. The material chosen will require high corrosive resistance. Silver, though having the highest conductivity, has a low corrosion resistance, and therefore cannot be used in this application [?].

Titanium is the material of choice for ocean-use [?]. It is essentially corrosion-free, and offers a conductivity of 2.68×10^6 [?]. However, titanium is expensive and fell out of the budget of this project. Gold boasts both a high conductivity of 4.10×10^7 , higher than titanium but lower than silver, and a high corrosion resistance, making it an ideal choice. Gold is also a commonly used material in electronic design, with it being used in Printed Circuit Board (PCB) manufacturing, to protect copper pads from corrosion. This is done through a Electroless Nickel Immersion Gold (ENIG) plating process, where a layer of nickel is chemically deposited onto the exposed copper traces, to prevent the copper from oxidizing, and then a layer of gold is applied over the nickel through an emmersion process, to protect the nickel. This process is significantly more expensive compared to standard PCB manufacturing, however, it allowed for the use of gold electrodes, and therefore was factored into the budget.

In order to utilise the ENIG process a PCB was used to design the gold electrodes. This allowed the electrodes to be designed with a known area and seperation distance, allowing for accurate conductivity calculations. A solder pad was used to design the portion of the PCB that would act as the electrode, since it allowed the copper/gold to be exposed. Then during manufacturing ENIG was chosen as the surface finish, to achieve the gold finish.

The PCB was designed to allow for easy calculation of the conductivity σ , using the equation below:

$$\sigma = \frac{L}{RA} \quad (3.1)$$

where L is the distance between the electrodes, R is the resistance of the water, and A is the cross-sectional area of the electrodes. A square face of $20mm \times 20mm$ was chosen to allow for easy cross-sectional area calculations, and a distance of $10mm$ was chosen as the separation distance. This distance was chosen as it is close enough to reduce current spreading, but not too small to where the water could not flow easily between the electrodes. A $2mm$ fringe guard was added around the main electrode area to reduce current fringing, which is an effect that causes the current to spread beyond the edges of the gap [?]. The fringe guards counteract this by saturating the area surrounding the main pads with current, preventing them from fringing.

The resistance of the electrodes was calculated using the Equation ?? and was found to have an approximate resistance of 7.55Ω

The electrode PCB was designed with the consideration of mounting to the probe PCB. To accomodate this, solder pads were added to allow the electrodes to be soldered to the probe PCB. Supports were also factored into the design to ensure that the electrodes stayed straight and secure. The design can be seen in Figure [insert figure].

3.3 Resistance Measurement

There are multiple ways to measure resistance, however most rely on the same principle, which is the voltage divider principle. This principle works by using a series circuit with two resistors, and a constant known input voltage. The voltage over each of the resistors will be proportional to their resistance, and therefore, if the resistance of one resistor is known, the resistance of the other can be calculated. A simple voltage divider circuit can be seen in Figure ?. For this application the electrodes were chosen as the R2 resistor, with R1 being a large resistor of known resistance.

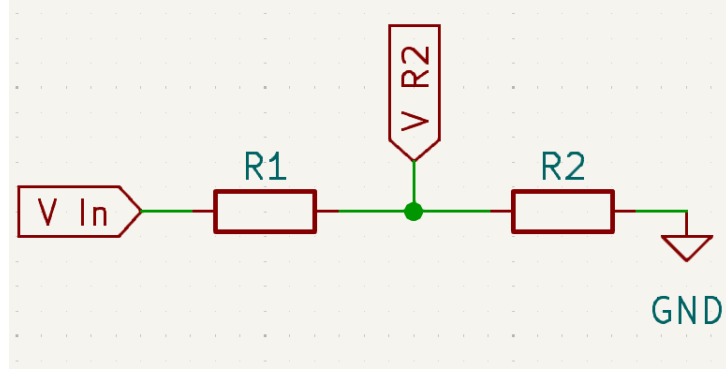


Figure 3.1: Simple voltage divider circuit used for resistance measurement.

Equations ?? and ?? are used to calculate the resistance from the voltage divider equation.

$$V_{R2} = V_{In} \times \frac{R_2}{R_1 + R_2} \quad (3.2)$$

$$R_2 = \frac{R_1 \times V_{R2}}{V_{In} - V_{R2}} \quad (3.3)$$

3.4 Circuit Design

3.4.1 Probe Circuit

The probe circuit is the circuit which contains the resistor divider, was designed to be printed onto a PCB. This design was influenced by Reference [?], where a similar device was designed for salinity measurements in ice columns. A PCB was chosen for this circuit as the researcher had significant experience with PCB design, and the manufacturing process offered higher precision than hand soldering, and is relatively cost-effective. Significant improvements and modifications were made to the resistor divider circuit, to allow for a wider range of testing.

For input power, a Digital to Analogue Converter (DAC) was used to drive the circuit. This allowed to the input voltage to be varied between 0V and the referencevoltage, which was chosen to be 5V. This allowed for a range of voltages to be applied, which allowed for the measurement of the water's voltage-resistance relationship, and the creation of AC signals. A function generator was considered for generating the AC signal, as it would allow for signals of a wider frequency and and high precision, however the price could not be accomodated by the budget. The choice of DAC, and all following components,

was first influenced by availability on JLCPCB, the PCB manufacturing house. The MCP4725 was chosen for its high resolution of 12-bits, offering a digital range of 0-4095, fast update time of $6\mu s$, and interface speed of 3.4MHz. These features allow for both DC and AC signal analysis.

An op-amp with unity gain was the connected to the output of the DAC. This is because DACs have limited output drive capabilities, and the op-amp would allow for heavier loads to be driven. Additionally the op-amp offers improved output stability, introduces impedance isolation, which protects the DAC from load variations and feedback effects, and allows for better sine wave quality.

As mentioned in Section ??, for the resistor divider circuit, the electrodes would serve as R2 and a known resistor as R1. Three alternative values of R1 were chosen, to accomodate for any circuit errors. These could be switched between using the TS3A4751 multiplexer Integrated Circuit (IC). This switching multiplexer was chosen, for its low on-state resistance of 0.9Ω , and fast switching speed of $4 - 5ns$ [?].

The R1 resistor values were chosen to be 100Ω , $1K\Omega$ and $10K\Omega$. These values would be used when the resistance between the probes was $1 - 10\Omega$, $10 - 100\Omega$, and $100 - 1K\Omega$ respectively. Each IC contained 4 switches.

For measuring the output resistor, the voltage over it was directed into a multiplying op-amp with a gain of 11. This increases the resolution for the Analogue to Digital Converter (ADC) readings, as low voltages may be hard to differentiate between when converted to digital data.

This configuration would allow for a minumum resolution of 11% of V_{DAC} and maximum of 100% of V_{DAC} , for the voltage measurement by the ADC, as shown in Equations ?? and ?? [?]. Equations ?? and ?? show for the expected resistance of 7.55Ω falling into the $1 - 10\Omega$ range. However, if the resistance falls into the $10 - 100\Omega$, or $100 - 1K\Omega$, the respective R1 resistors would be used and the maximum and minimum DAC resolutions would be the same.

$$\frac{1\Omega}{1\Omega + 100\Omega} \times V_{DAC} \times 11 = 11\%V_{DAC} \quad (3.4)$$

$$\frac{10\Omega}{10\Omega + 100\Omega} \times V_{DAC} \times 11 = 100\%V_{DAC} \quad (3.5)$$

The accuracy of the R1 resistor is integral to achieving an accurate R2 measurement. The resistors available on JLCPCB had an accuracy of $\pm 1\%$. To increase the accuracy 3 equal resistors were put in parallel. This decreases the uncertainty of the total equivalent resistance [?]. This is shown in Equations ?? to ??.

$$R_{Equivalent} = \left[\sum_{i=1}^n \frac{1}{R_n} \right]^{-1} \quad (3.6)$$

If all the Resistors are equal this simplifies to:

$$R_{Equivalent} = \left(\frac{n}{R} \right)^{-1} = \frac{1}{n} \times R \quad (3.7)$$

To propagate uncertainty the standard equation for combined uncertainty can be used:

If a quantity y depends on several independent variables x_1, x_2, \dots, x_n :

$$y = f(x_1, x_2, \dots, x_n)$$

and each x_i has a standard uncertainty $u(x_i)$ then the combined standard uncertainty of y , denoted $u_c(y)$, is:

$$u_c(y) =$$

Chapter 4

Results

These are the results I found from my investigation.

Present your results in a suitable format using tables and graphs where necessary. Remember to refer to them in text and caption them properly.

4.1 Simulation Results

4.2 Experimental Results

Chapter 5

Discussion

Here is what the results mean and how they tie to existing literature...

Discuss the relevance of your results and how they fit into the theoretical work you described in your literature review.

Chapter 6

Conclusions

These are the conclusions from the investigation and how the investigation changes things in this field or contributes to current knowledge...

Draw suitable and intelligent conclusions from your results and subsequent discussion.

Chapter 7

Recommendations

Make sensible recommendations for further work.

Appendix A

Additional Files and Schematics

Add any information here that you would like to have in your project but is not necessary in the main text. Remember to refer to it in the main text. Separate your appendices based on what they are for example. Equation derivations in Appendix A and code in Appendix B etc.

IMPORTANT: Appendix A (see the table below) should provide a summary of how you have met the GAs associated with the course, and where in the report the evidence can be found.

If you have used AI in writing your report, you need to provide details in one of the appendices of how you used it.

If appropriate, in a subsequent appendix, provide a link to any GitHub repository where the code and additional materials for your project can be found.

GA	Requirement	Justification and section in the report
1	Problem-solving	Description here
4	Investigations, experiments and data analysis	Description here
5	Use of engineering tools	Description here
6	Professional and technical communication (Long report)	Description here
8	Individual work	Description here
9	Independent learning ability	Description here

Appendix B

Addenda

B.1 Ethics Forms

Appendix C

GitHub Repository

[Click here to access the GitHub repository.](#)

GitHub Structure