LX331: Assignment 3

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1 The syntax and semantics of Propositional Logic

A. B.

(1) $\sim s \& r$: Stuart is not in the kitchen and Fred left.

r	s	$\sim s$ & $\sim s \& r$	
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

(2) $\sim (p \vee q)$: It is not true that Mary or Sue is at home.

p	q	$p \vee q$ & $\sim (p \vee q)$	
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(3) $\sim (p \& s) \rightarrow q$: If it is not true that Marry is at home and Stuart is in the kitchen, then Sue is at home.

p	q	s	$p \& s$	$\sim (p \& s)$	$\sim (p \& s) \rightarrow q$
T	T	T	T	F	T
T	T	F	F	T	T
T	F	T	T	F	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	F
F	F	F	F	T	F

(4) $\sim q \& ((s \vee \sim r) \rightarrow \sim s)$: Sue isn't at home and if Stuart is in the kitchen or Fred didn't left then Stuart is not in the kitchen.

p	r	s	$s \vee \sim r$	$(s \vee \sim r) \rightarrow \sim s$	$\sim q \& ((s \vee \sim r) \rightarrow \sim s)$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	F
F	T	F	F	T	T
F	F	T	T	F	F
F	F	F	T	T	T

C. D.

(5) Stuart is not in the kitchen and Fred didn't leave.
$\sim s \& \sim r$
We can rewrite this logic formula as $\sim (s \vee r)$ due to DeMorgan's law.

s	r	$\sim s \& \sim r \equiv \sim (s \vee r)$
T	T	F
T	F	F
F	T	F
F	F	T

(6) If Sue is at home or Fred didn't leave, then Stuart is not in the kitchen.
$(q \vee \sim r) \rightarrow \sim s$

p	r	s	$(q \vee \sim r)$	$(q \vee \sim r) \rightarrow \sim s$
T	T	T	T	F
T	T	F	F	T
T	F	T	T	F
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	F
F	F	F	F	T

(7) It's not the case that Mary and Sue are both at home.
$\sim (p \& q)$

p	q	$\sim (p \& q)$
T	T	F
T	F	T
F	T	T
F	F	T

(8) Neither Mary nor Sue is at home.
$\sim (p \vee q)$

p	q	$\sim (p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

2 Representing semantic ambiguity in Propositional Logic

(9) I didn't talk to Fred and Barney
We can interpret two understanding from (9).
(a) I did not talk to Fred and Barney at the same time.
$\sim (p\&q)$
(b) I did not talk to Fred, and I did not talked to Barney
$\sim p\& \sim q$

If we look into the truth table, we can see that the two interpretations of the English sentence is not equivalent.

p	q	$\sim (p\&q)$	$\sim p\& \sim q$
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

3 A new logical connective

I can't type the connective arrow so $->$ is a substitution.

A	B	$A -> B$	Comment
T	T	F	It's the last week so there can't be homework
T	F	T	It's not the last week so there have to be homework
F	T	T	It's the last week, so it is true that there isn't homework
F	F	F	It's not the last week, so if there is no homework it is false.

4 Logical relations between sentences

A.

So we have that $p \to q \equiv\sim p \vee q \equiv\sim (p\& \sim q)$. So $p \to q$ and $\sim (p\& \sim q)$ should be equivalent. Let's check with the truth table.

p	q	$p \to q$	$\sim (p\& \sim q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

B.

Logical compatible means that there are rows in that both formulae is both true or both false.

r	s	$\sim r \& \sim s$	$\sim (r \vee \sim s)$
T	**T**	**F**	**F**
T	**F**	**F**	**F**
F	T	F	T
F	F	T	F

As we can see, the first two row are show that these two formulae are compatible.

C.

One formula entails the other when the truth of the first "forces" the truth of the second. The table for two formulae is as follow:

p	s	$\sim (p \& s)$	$\sim (p \vee s)$
T	T	F	F
T	F	T	F
F	T	T	F
F	**F**	**T**	**T**

From the truth table, we can see that $\sim (p \& s)$ does not entails $\sim (p \vee s)$. However, the last line show us that $\sim (p \vee s)$ entails $\sim (p \& s)$.