Math 3

Lecture notes by Rangi

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Abstract

Abstract Here

1 Introduction

Previously: we had equations like

$$y^2 + 4y + 1 = 0 \tag{1}$$

where the solution is a number.

$$\int_a^b f(x)dt = \text{number} = F(b) - F(a) \tag{2}$$

New: $f'(x)$ is given, determine $f(x)$. The solution is a function.

$$\begin{aligned} \text{Velocity}(t) &= \text{Position}'(t) && \text{given} \\ &= \text{Position}(t) && \text{wanted} \end{aligned} \tag{3}$$

1.1 Differential equations

Example: Interest rate

$$\begin{aligned} y(t): &\quad \text{assets at time } t \\ \lambda < 0: &\quad \text{constant interest rate} \end{aligned} \tag{4}$$

[...]

2 First-order DE

For now: $m = 1, n = 1$

Consider: $y'(x) = f(x, y(x))$ explicit form of ODE

Function f is defined on

$$D = Dx \times Dy \in \mathbb{R}^2 \tag{5}$$

Ex. Strip: [...]

2.1 Geometric interpretation: Direction field

[...]

2.2 Observations

1. Through each point $(x_0, y_0) \in D$ there passes exactly one solution
2. Each solution curve is maximal (no blow up)
3. Solution curves don't intersect

2.3 Existence and Uniqueness of a solution to ODE

JVP: $y' = f(x, y), y(x_0) = y_0$, domain D

Theorem (Peano): Assume that f is continuous on D and $(x_0, y_0) \in D$. Then if JVP has at least one solution. This solution is maximal ... we can continue solv. until the boundary D.

Theorem (PicardLindelöt): Let f be continuous on D and let f be cont. diff. with respect y. ($\rightarrow$ Lipschitz cont.) Let $(x_0, y_0) \in D$. Then the IVP has a unique solution.

Ex.:

$$\begin{aligned} (x^2 - x)y' &= (2x - 1)y \qquad \text{implicit form} \\ y' &= \frac{2x - 1}{x^2 - x} * y \qquad \text{explicit form} \end{aligned} \tag{6}$$

[...]

1. $x_0 \notin \{0; 1\} \rightarrow$ unique sol. (due to Picard-Lindelöf Thm.)
2. $x_0 \in \{0; 1\} \rightarrow$ infinitely many solutions