Hashing

Data Structures & Algorithms

CONTENT

- Introduction
- Static hashing
- Dynamic hashing

CONTENT

- **Introduction**
- Static hashing
- Dynamic hashing

- A *table* has several *fields* (types of information)
  - A telephone book may have fields **name**, **address**, **phone number**
  - A user account table may have fields **user id**, **password**, **home folder**

⇨ To find an *entry* in the table, you only need know the contents of <u>one</u> of the fields (not <u>all</u> of them).

- The *key* field
  - In a telephone book, the key is usually **name**
  - In a user account table, the key is usually **user id**

⇨ Key *uniquely identifies* an entry

- If the key is **name** and no two entries in the telephone book have the same name, the key uniquely identifies the entries

- **How should we implement a table?**
  - How often are entries inserted and removed?
  - How many of the possible key values are likely to be used?
  - What is the likely pattern of searching for keys?
    - e.g. Will most of the accesses be to just one or two key values?
  - Is the table small enough to fit into memory?
  - How long will the table exist?

- **Element: a key and its entry**

For searching purposes, it is best to store the key and the entry separately (even though the key's value may be inside the entry)

Element

key	entry
"Smith"	"Smith", "124 Hawkers Lane", "9675846"
"Yeo"	"Yeo", "1 Apple Crescent", "0044 1970 622455"

…Introduction

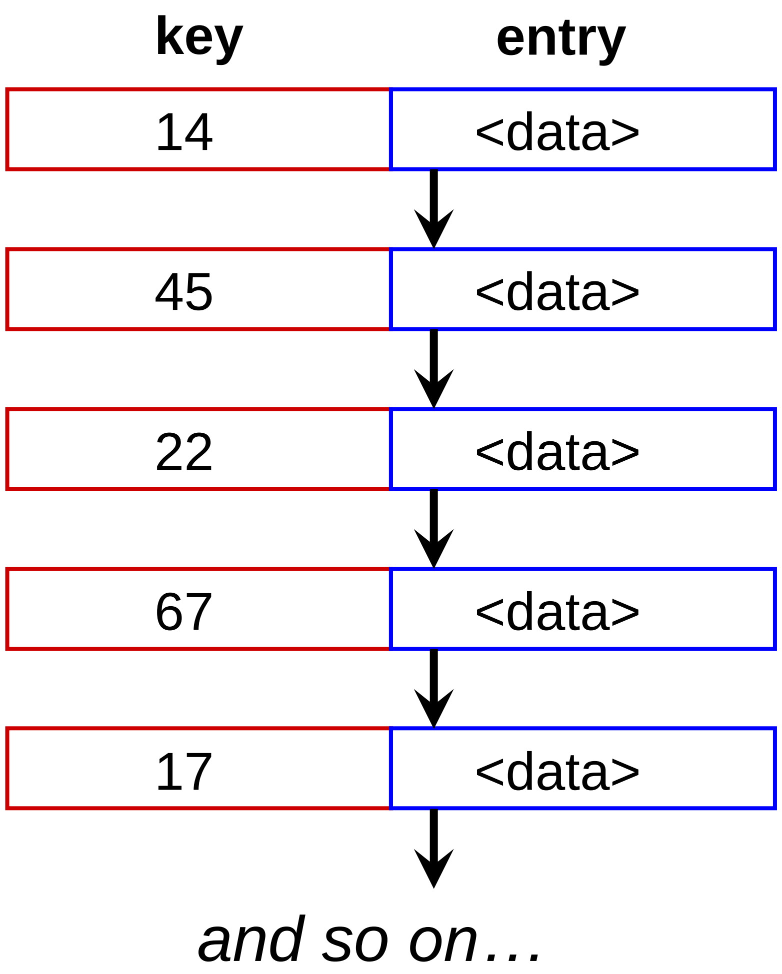
- **Implementation 1- Unsorted Sequential Array**
  - An array in which elements are stored consecutively in *any* order
  - **insert**: add to back of array; O(1)
  - **find**: search through the keys one at a time, potentially all of the keys; O(n)
  - **remove**: find + replace removed node with last node; O(n)

	key	entry
0	14	<data>
1	45	<data>
2	22	<data>
3	67	<data>
4	17	<data>
⋮	*and so on…*	

- **Implementation 2 - Sorted Sequential Array**
  - An array in which elements are stored consecutively, *sorted* by key
  - **insert**: add in sorted order; O(*n*)
  - **find**: binary search; O(log *n*)
  - **remove**: find, remove element; O(log *n*)

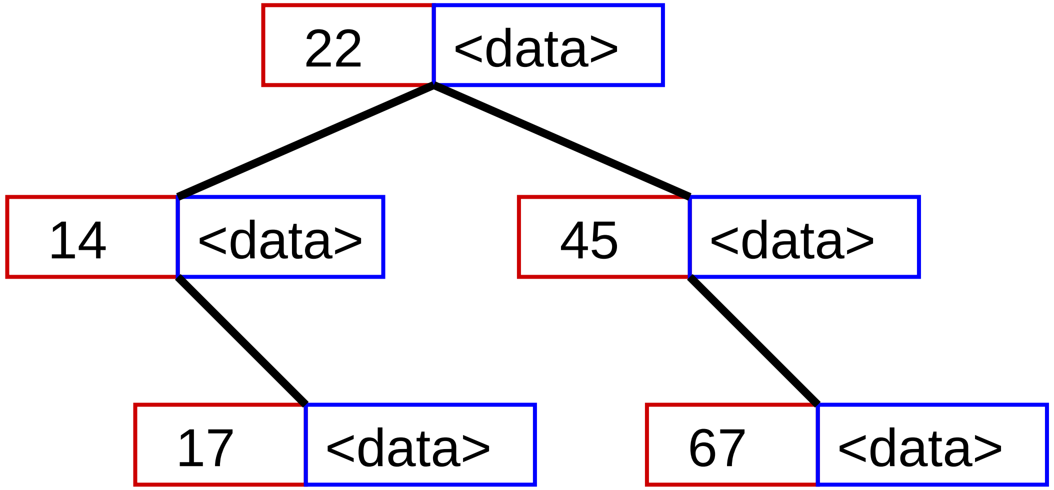
	key	entry
0	15	<data>
1	17	<data>
2	22	<data>
3	45	<data>
4	67	<data>
⋮	*and so on…*	

- **Implementation 3 - Linked List (Unsorted or Sorted)**
  - Elements are again stored consecutively
  - **insert**: add to front; O(1) or *O(n)* for a sorted list
  - **find**: search through potentially all the keys, one at a time; O(*n*) still *O(n)* for a sorted list
  - **remove**: find, remove using pointer alterations; O(*n*)



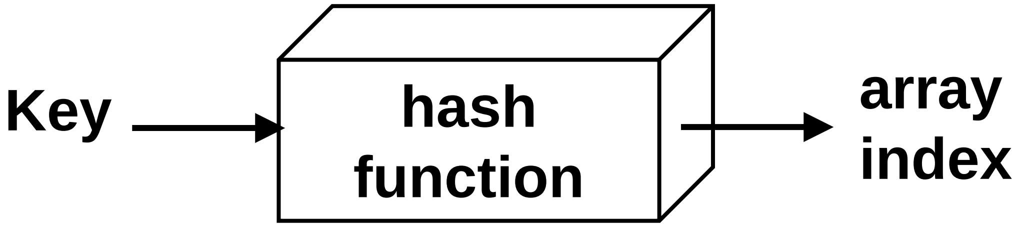
…Introduction

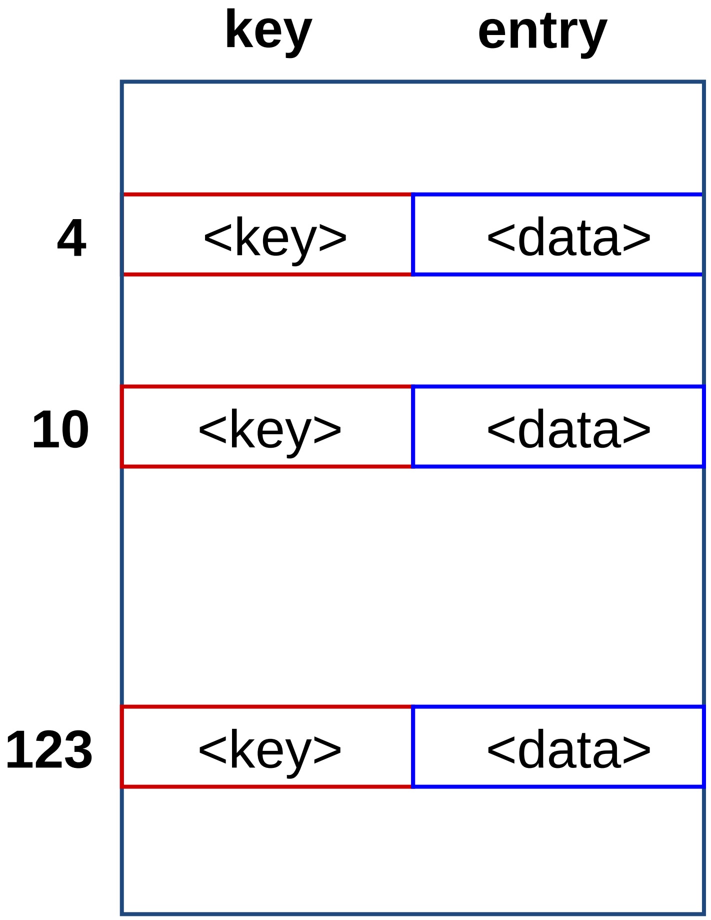
- **Implementation 4 - BST**
  - A BST, ordered by key
  - **insert**: a standard insert; O(log *n*)
  - **find**: a standard find (without removing, of course); O(log *n*)
  - **remove**: a standard remove; O(log *n*)



and so on…

- **Implementation 5 - Hash Table**
  - An array in which elements are ***not*** stored consecutively - their place of storage is calculated using the key and a *hash function*

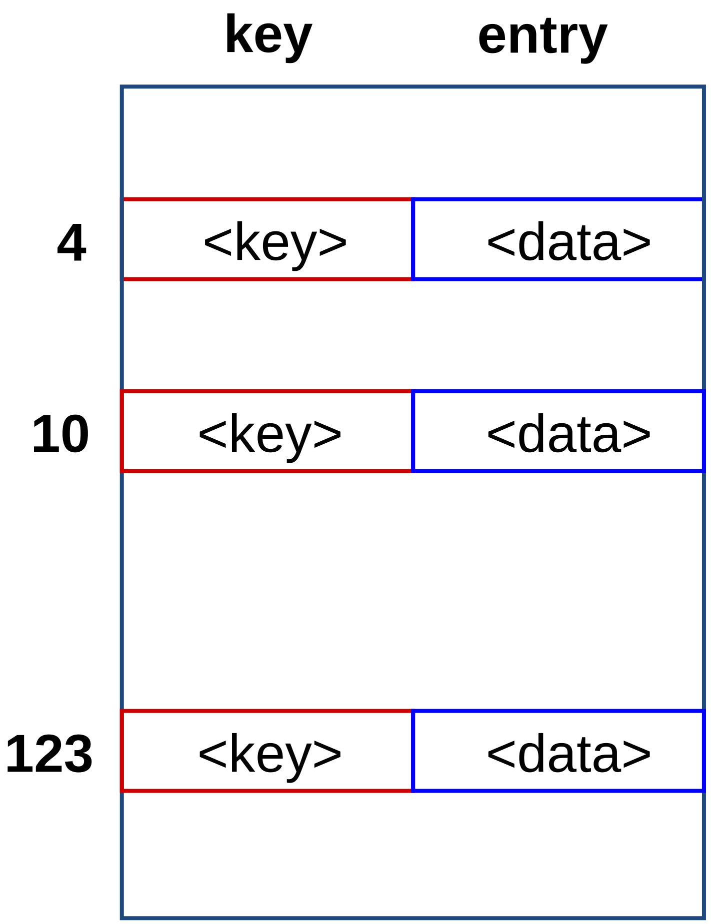




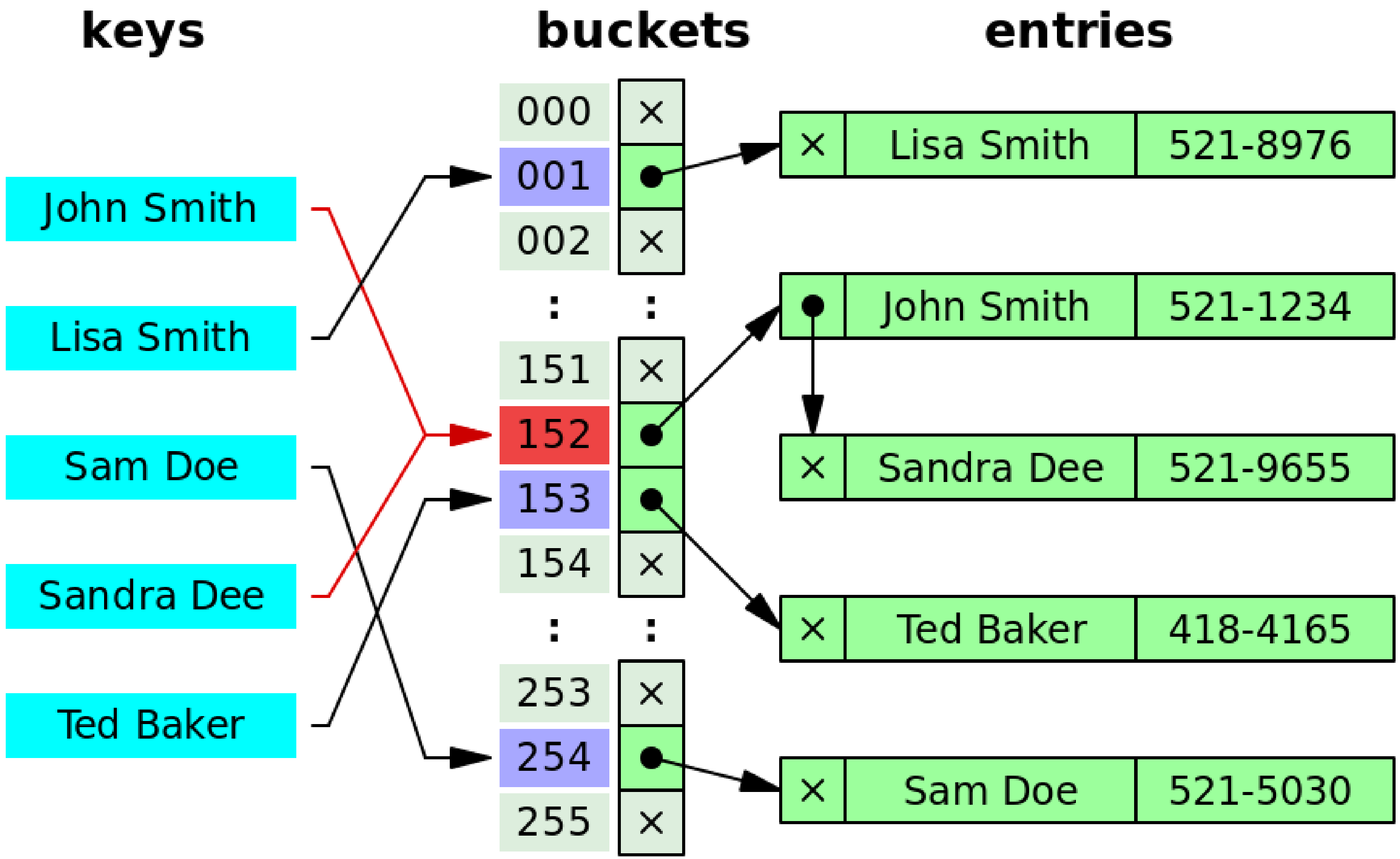
Hash values → mappings of the keys as the indexes in the hash table

- **Implementation 5 - Hash Table**
  - **Hashed key**: result of applying a hash function to a key
  - Keys and entries are scattered throughout the array
  - Array elements are not stored consecutively, their place of storage is calculated using the key & a hash function
  - **insert**: calculate place of storage, insert TableNode; O(1)
  - **find**: calculate place of storage, retrieve entry; O(1)
  - **remove**: calculate place of storage, set it to null; O(1)

All are O(1) !



- **Implementation 5 - Hash Table**



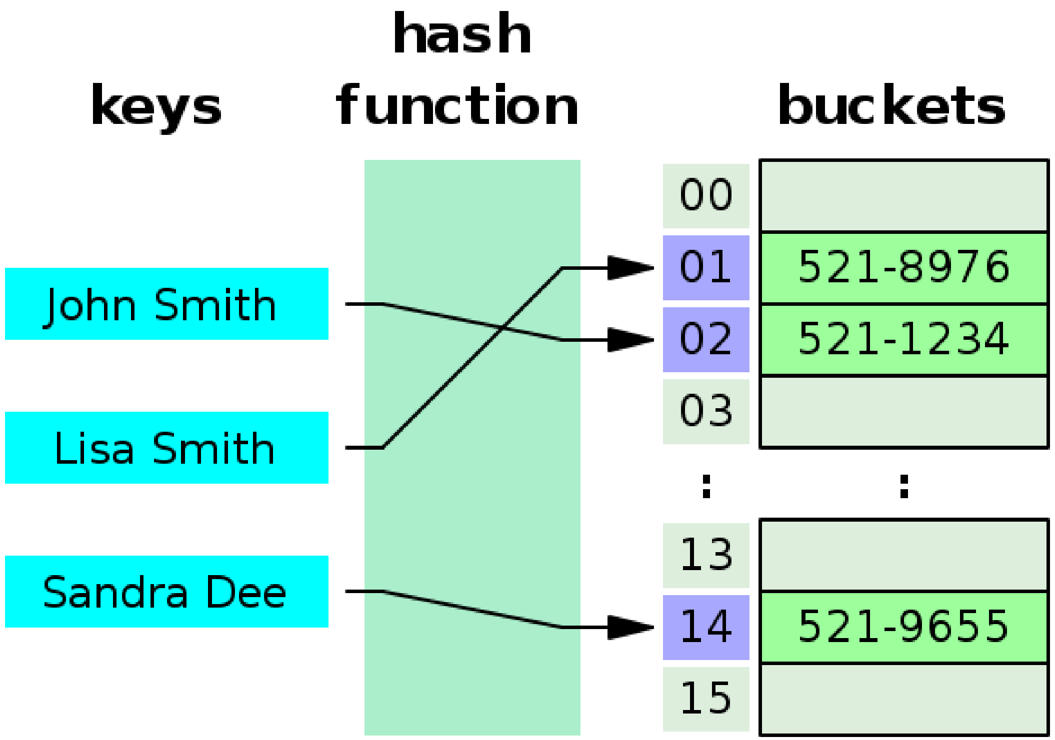
- **Implementation 5 - Hash Table**
  - **Độ phức tạp**
    - Nếu các kết quả của hàm hash được phân bố đều, các bucket sẽ có kích thước xấp xỉ nhau => tìm kiếm rất hiệu quả.
    - Gọi **n** là số phần tử cần lưu trong Hash table

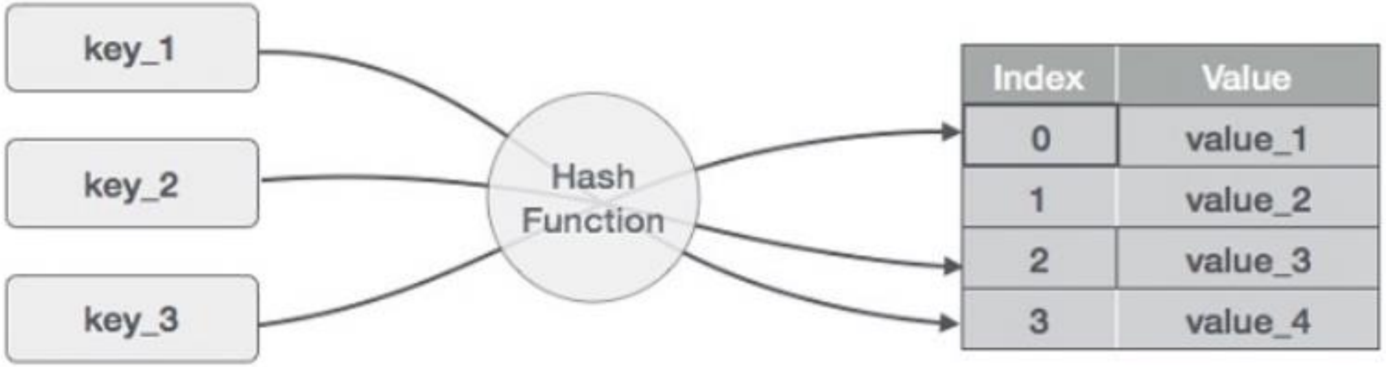
      k là số bucket

      Giá trị **n/k** được gọi là load factor.

      Khi load factor nhỏ (xấp xỉ 1),

và giá trị của hàm Hash phân bố đều, độ phức tạp của các thao tác trên Hash table là O(1)





- **Applications of Hashing**
  - Compilers use hash tables to keep track of declared variables
  - A hash table can be used for on-line spelling checkers — if misspelling detection (rather than correction) is important, an entire dictionary can be hashed and words checked in constant time
  - Game playing programs use hash tables to store seen positions, thereby saving computation time if the position is encountered again
  - Hash functions can be used to quickly check for inequality — if two elements hash to different values they must be different
  - Storing sparse data

- **When are other representations more suitable than hashing?**
  - Hash tables are very good if there is a need for many searches in a reasonably stable table
  - Hash tables are not so good if there are many insertions and deletions, or if table traversals are needed

- **Types of hashing**
  - Static hashing
    - Tables with a fixed size
  - Dynamic hashing
    - Table sizes may vary

CONTENT

- Introduction
- **Static hashing**
  - Hash table
  - Hash methods
  - Collision resolution
- Dynamic hashing

Static Hashing - Hash table

- **Key-value pairs are stored in a fixed size table called a *hash table***
  - A hash table is partitioned into many ***buckets***
  - Each bucket has many ***slots***
  - Each slot holds one record
  - A hash function *f*(*x*) transforms the identifier (i.e. key) into an address in the hash table

- **Uses an array hash_table[0..b-1].**
  - Each position of this array is a **bucket**
  - A bucket can normally hold only one dictionary pair
- **Uses a hash function *f***
  - that converts each key *k* into an index in the range [0, *b*-1].

⇨**Every dictionary pair (key, element) is stored in its home bucket hash_table[f(key)]**

- **Data Structure for Hash Table**

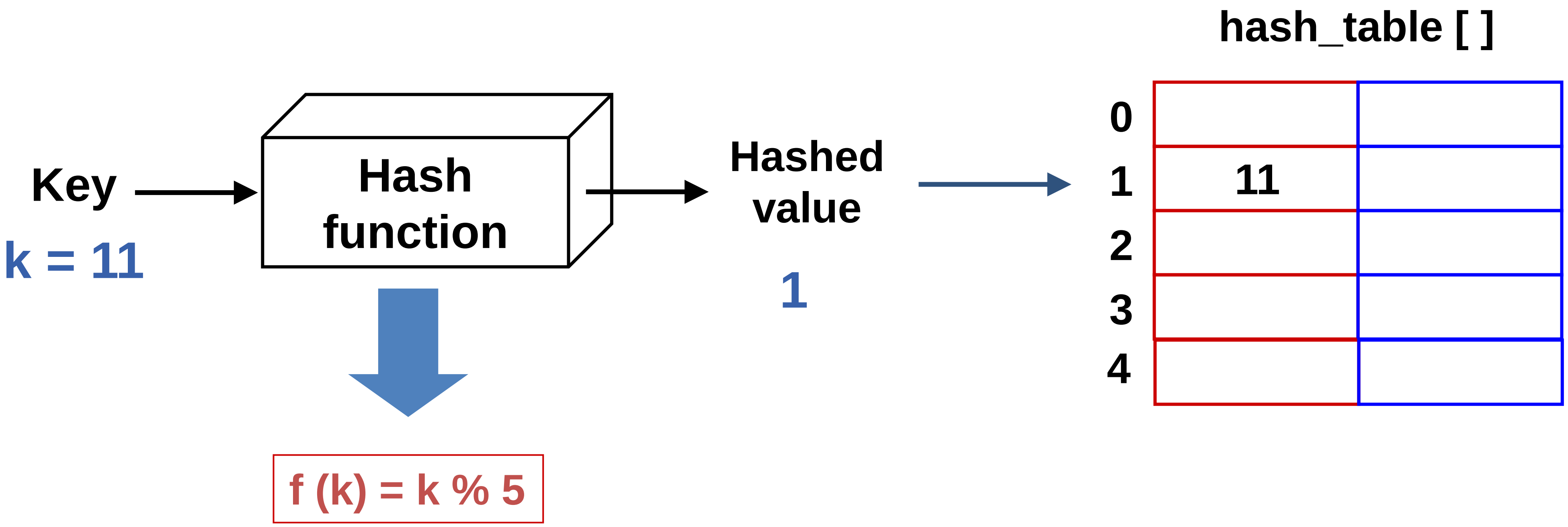
```
#define MAX_CHAR  10
#define TABLE_SIZE  13
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

- **Hash Function - Example**

```
void init_table(element ht[]){
        int i;
        for (i=0; i<TABLE_SIZE; i++)
        ht[i].key[0]=NULL;
}

int hash( char *key, int TABLE_SIZE ) {
        unsigned int hash_val = 0;
        while( *key != '\0' )
            hash_val += *key++;
        return( hash_val % H_SIZE );
}
```

- **Example**



hash_table [ ]
Key
k = 11
Hash function
f (k) = k % 5
Hashed value
1
0
1
11
2
3
4

- **Collision**
  - In a hash table with a single array table (single slot bucket), two different keys may be hashed to **the same hash value**
    - Two different k1 and k2
    - Hash (k1) = Hash (k2)
    - Keys that have the same home bucket are synonyms
  - This is called **collision.** Example: k1 = 11, k2 = 21

    Hash (11) = 11 % 5 = 1

    Hash (21) = 21 % 5 = 1
- **Choice of hash method**
  - To avoid collision (two different pairs are in the same the same bucket)
  - Size (number of buckets) of hash table
- **Overflow handling method**
  - Overflow: there is no space in the bucket for the new pair

- **Overflow Example**

	Slot 0	Slot 1
0	acos	atan
1		
2	char	ceil
3	define	
4	exp	
5	float	floor
6		
...		
25		

synonyms (row 0)

synonyms (row 2)

synonyms:
char, ceil,
clock, ctime

overflow

- **Choice of Hash Method**
  - Requirements
    - easy to compute
    - minimal number of collisions
  - If a hashing function groups key values together, this is called **clustering** of the keys
    - The larger the cluster, the longer the search
  - A good hashing function **distributes the key values uniformly** throughout the range
    - For a random variable X, $P(X = i) = 1/b$ (b is the number of bucket)

- **Choice of Hash Method**
  - The **worst hash function** maps all keys to the same bucket
  - The **best hash function** maps all keys to distinct addresses
  - **Ideally, distribution of keys to addresses is uniform and random**

- **Many hashing methods**
  - Truncation
  - Division
  - Mid-square
  - Folding
  - Digit analysis
  - and so on

- **Truncation method**
  - Ignore part of the key and use the rest as the array index (converting non-numeric parts)
  - Example
    - If students have an 9-digit identification number, take the last **3** digits as the table position
    - e.g. 925371**622** becomes 622

- **Division method**
  - Hash function ***f*(*k*) = *k* % *b***
    - Requires only a single division operation (quite fast)
  - Certain values of *b* should be avoided
    - if $b=2^p$, then *f*(k) is just the *p* lowest-order bits of *k*; the hash function does not depend on all the bits
  - It's a good practice to set the table size *b* to be a **prime number**

- **Mid-square method**
  - Middle of square method
  - This method squares the key value, and then takes out the number of bits from the middle of the square
  - The number of bits to be used to obtain the bucket address depends on the table size
    - If *r* bits are used, the range of values is 0 to 2^r-1
  - This works well because most or all bits of the key value contribute to the result

- **Mid-square method**
  - Example
    - consider records whose keys are 4-digit numbers in base 10
    - The goal is to hash these key values to a table of size 100
    - This range is equivalent to two digits in base 10, that is, r = 2
    - If the input is the number 4567, squaring yields an 8-digit number, 20857489
    - The middle two digits of this result are 57

- **Folding method**
  - Partition the key into several parts of the same length except for the last
  - These parts are then added together to obtain the hash address for the key
  - Two ways of carrying out this addition
    - shift folding
    - folding and reverse

Shift-folding

123 203 241 112 020

$$\begin{array}{r} 123 \\ 203 \\ 241 \\ 112 \\ 020 \\ \hline 699 \end{array}$$

Folding and reverse

123 203 241 112 020

$$\begin{array}{r} 123 \\ 302 \\ 241 \\ 211 \\ 020 \\ \hline 897 \end{array}$$

Example

Digit analysis method

- All the identifiers/keys in the table are known in advance
- The index is formed by extracting, and then manipulating specific digits from the key
- For example, the key is 925371622, we select the digits from 5 through 7 resulting 537
- The manipulation can then take many forms
  - Reversing the digits (735)
  - Performing a circular shift to the right (753)
  - Performing a circular shift to the left (375)
  - Swapping each pair of digits (357)

- **Hash Function Implementations**
  - A generic hashing function does not exist
  - However, there are several forms of a hash function
  - Let’s discuss some specific hash function implementations

```
typedef unsigned        HASH_VALUE
typedef unsigned short  KEY_TYPE

HASH_VALUE  ModulusHashFunc (KEY_TYPE key){
        return (key % HT_SIZE);
}
```

- **Folding hash function for integers**

```
typedef unsigned long int  KEY_TYPE

HASH_VALUE Fold_Integer_Hash (KEY_TYPE key){
        return ( (key / 10000 + key % 10000) % HT_SIZE);
}
```

High-order digits

Low-order digits

New integer

Source code above is for a 8-digit integer key

- **Folding hash function for integers**

```
typedef unsigned long int  KEY_TYPE

HASH_VALUE Fold_Integer_Hash (KEY_TYPE key){
        return ( (key / 10000 + key % 10000) % HT_SIZE);
}
```

Example: key = 87629426, HT_SIZE = 251

hash value = (87629426 / 10000 + 87629426 % 10000) % 251

hash value = (8762 + 9426) % 251 = 116

key = 87629426

High-order digits Low-order digits

- **Folding hash function for pointer-based character strings**

  Useful for applications involving symbols and names.

  Folding hash → adds ASCII values of each character and takes the modulus with respect to the hash table size.

```
typedef char  *KEY_TYPE

HASH_VALUE Fold_String_Hash (KEY_TYPE key){
        unsigned sum_ascii_value = 0;

        while (*key != '\0')
                sum_ascii_values += *key++;

        return (sum_ascii_values % HT_SIZE);
}
```

- **Folding hash function for pointer-based character strings**

Example:

key = “PRATIVA”,   HT_SIZE = 31

hash value = (P + R + A + T + I + V + A) % 31

hash value = (80 + 82 + 65 + 84 + 73 + 86 + 65) % 31

hash value = 8

- **Digit Analysis-Based Folding**

```
static unsigned DigitFoldStringHash (char *key){
        unsigned long hash;

        hash = ( (key[0] ^ key[3]) ^ (key[1] ^ key[2]) ) % HT_SIZE;

        return (hash);
}
```

- **Collision Resolution/Overflow Handling**
  - An overflow occurs when the home bucket for a new pair **(key, element)** is full
  - Methods of solving collisions/overflows
    - **Open Addressing**
      - Insert the element into the next free position in the table
    - **Separate Chaining**
      - Each table position is a linked list

- **Open addressing**
  - relocate the key *k* to be inserted if it collides with an existing key. That is, we store *k* at an entry different from *hash_table*[*f*(*k*)].

- **Two issues arise**
  - what is the relocation scheme?
  - how to search for k later?

- **Common methods for resolving a collision in open addressing**
  - Linear probing
  - Quadratic probing
  - Double hashing
  - Rehashing

- **Open Addressing**
  - To insert a key k, compute $f_0(k)$. If *hash_table*[$f_0(k)$] is empty, insert it there.
  - If collision occurs, probe alternative cell $f_1(k)$, $f_2(k)$, ... until an empty cell is found
    - $f_i(k) = (f(k) + g(i)) \% b$, with $g(0) = 0$
    - g: **collision resolution strategy**

- **Linear Probing**
  - **g(i) =i**
    - cells are probed **sequentially** (with wraparound)
    - $f_i(k) = (f(k) + i) \% b$
  - **Insertion**
    - Let k be the new key to be inserted. We compute f(k)
    - For i = 0 to b-1
      - compute L = (f(k) + i) % b
      - hash_table[L] is empty, then we put k there and stop
    - If we cannot find an empty entry to put *k*, it means that the table is full and we should report an error

- **Linear Probing – Insert**

  - divisor = b (number of buckets) = 17
  - Home bucket = key % 17

0				4				8				12				16
34	**0**	**45**				**6**	**23**	**7**			**28**	**12**	**29**	**11**	**30**	**33**

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

- **Linear Probing – Insert**

```
void linear_insert(element item, element ht[]){
    int i, hash_value;
    i = hash_value = hash(item.key);
    while(strlen(ht[i].key)) {
        if (!strcmp(ht[i].key, item.key)) {
            fprintf(stderr, “Duplicate entry\n”);
            exit(1);
        }
        i = (i+1)%TABLE_SIZE;
        if (i == hash_value)
            fprintf(stderr, “The table is full\n”); exit(1);
    }
    ht[i] = item;
}
```

- **Data Structure for Hash Table**

```
#define MAX_CHAR  10
#define TABLE_SIZE  13
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

- **Linear Probing – Delete**

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- **Delete(0)**

0				4				8				12				16
34		45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
34	45					6	23	7			28	12	29	11	30	33

- **Linear Probing – Delete**
  - **Delete(34)**

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
	0	45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
0		45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
0	45					6	23	7			28	12	29	11	30	33

- **Linear Probing – Delete**
  - **Delete(29)**

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
34	0	45				6	23	7			28	12		11	30	33

- Search cluster for pair (if any) to fill vacated bucket

0				4				8				12				16
34	0	45				6	23	7			28	12	11		30	33

0				4				8				12				16
34	0	45				6	23	7			28	12	11	30		33

0				4				8				12				16
34	0					6	23	7			28	12	11	30	45	33

- **Performance Of Linear Probing**

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Worst-case find/insert/erase time is **Θ(n)**, where n is the number of pairs in the table
- This happens when all pairs are in the same cluster (the same index/bucket)

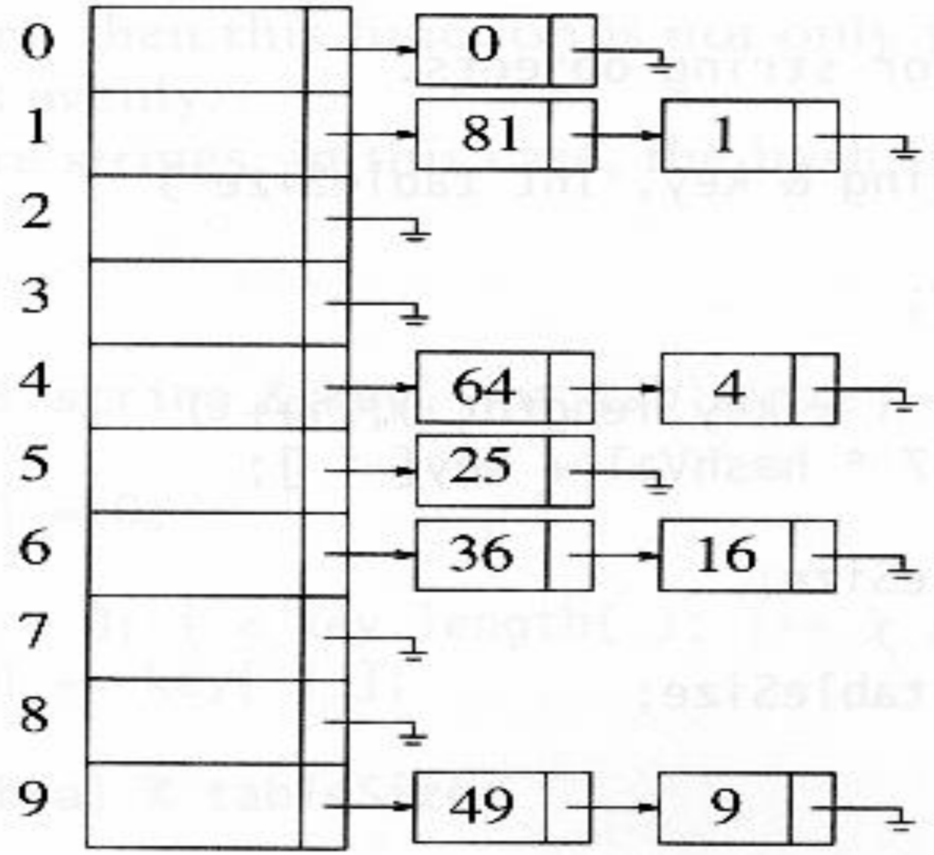
- **Quadratic Probing**
  - Linear probing searches buckets (f(x)+i)%b
  - Quadratic probing uses a quadratic function of *i* as the increment
  - Examine buckets f(x), $(f(x)+i^2)\%b$, $(f(x)-i^2)\%b$, for 1<=i<=(b-1)/2
  - b is a prime number of the form 4j+3, j is an integer

- **Random Probing**
  - Random Probing works incorporating with random numbers
    - f(x) = (f'(x) + S[i]) % b
      - S[i] is a table with size b-1
      - S[i] is a random permuation of integers [1,b-1]

- **Double hashing**
  - Double hashing is one of the best method for dealing with collisions
  - If the slot is full, then a second hash function (which is different from the first one) is calculated and combined with the first hash function
    - $f(k, i) = (f_1(k) + i\, f_2(k)) \,\%\, b$

- **Rehashing**
  - Enlarging the Table
  - To rehashing
    - Create a new table of double the size (adjusting until it is again prime)
    - Transfer the entries in the old table to the new table, by recomputing their positions (using the hash function)
  - Rehashing when the table is completely full

- **Separate Chaining**
  - Instead of a hash table, we use a table of linked list
  - keep a linked list of keys that hash to the same value

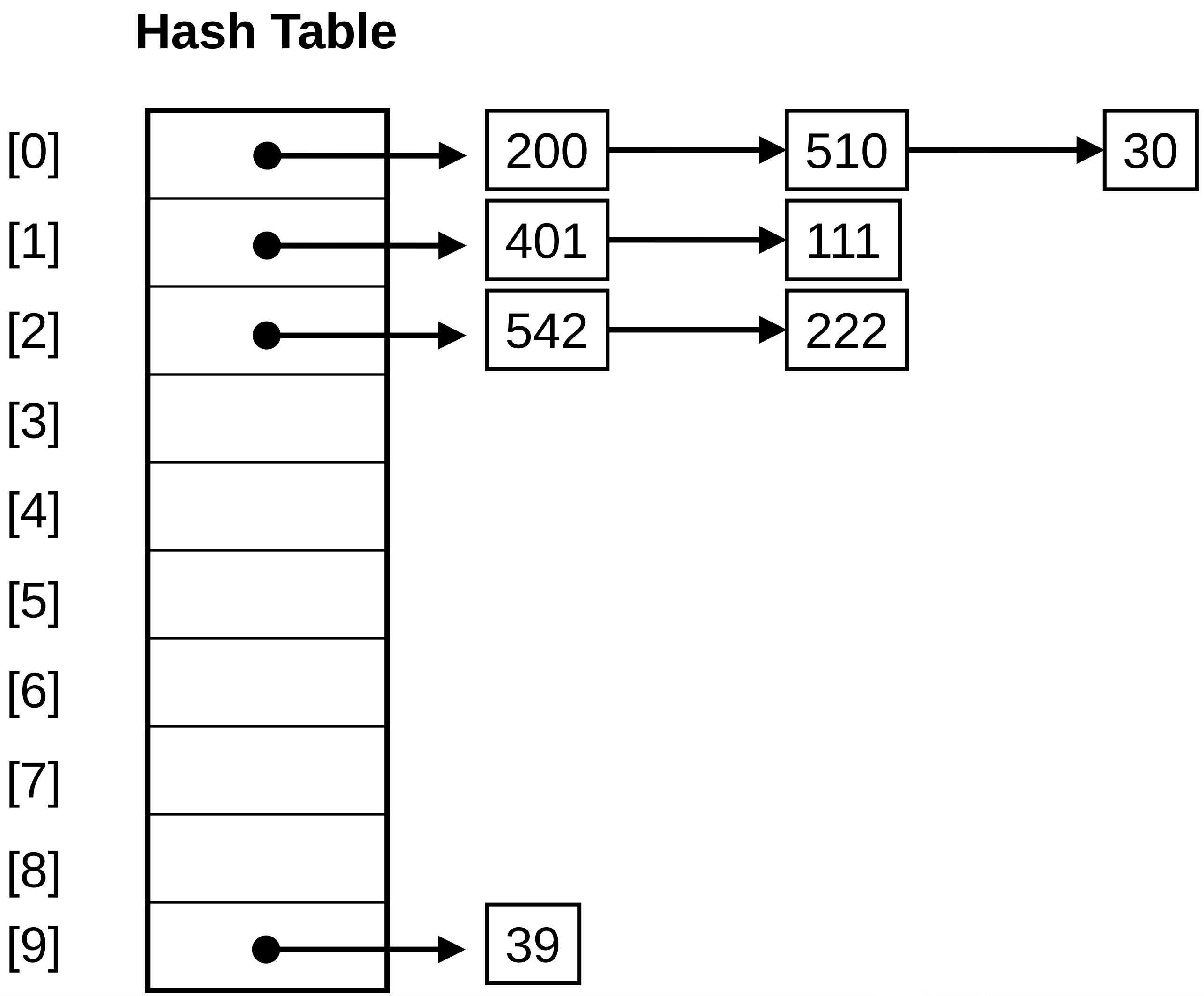


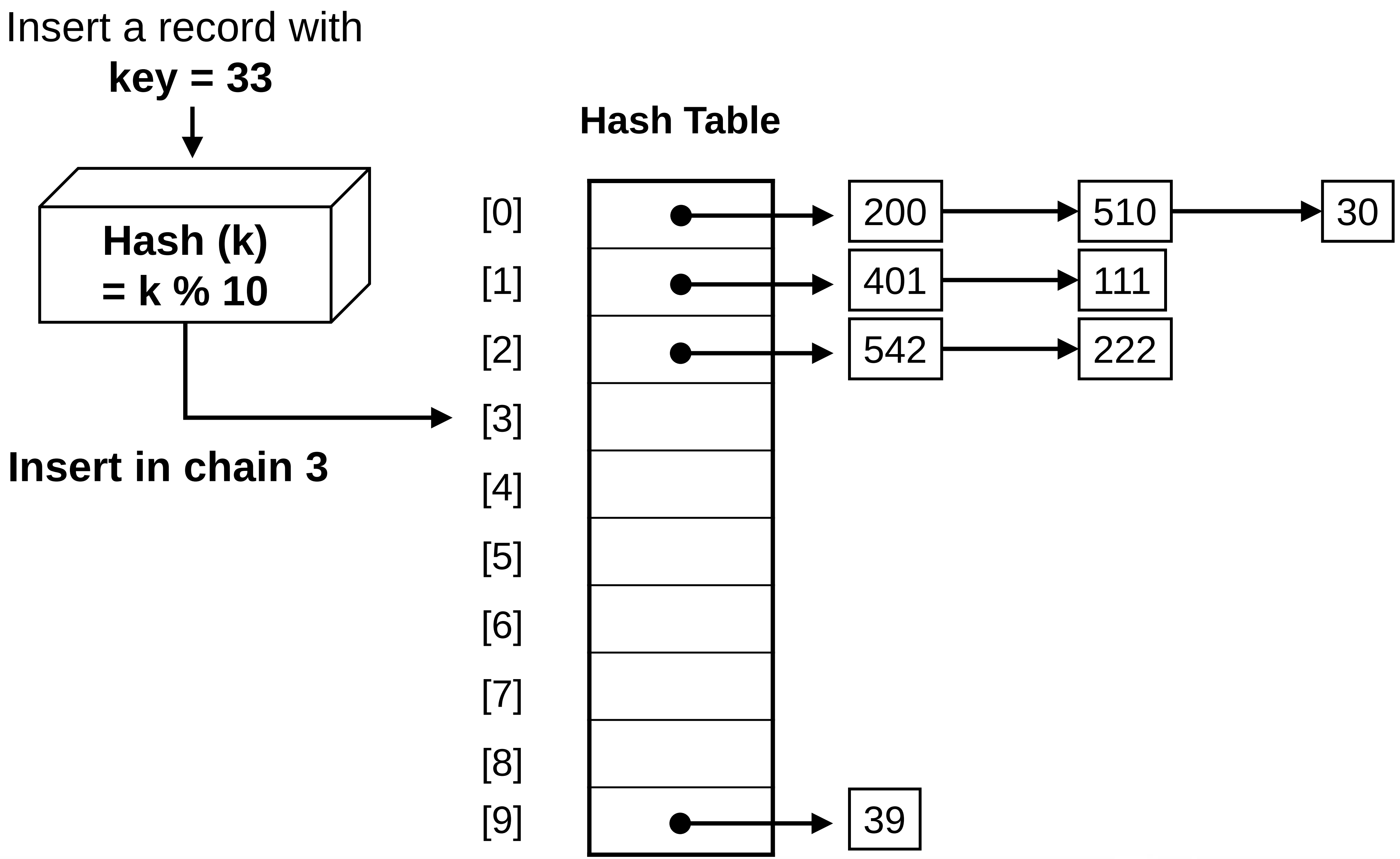
$f(k) = k \bmod 10$

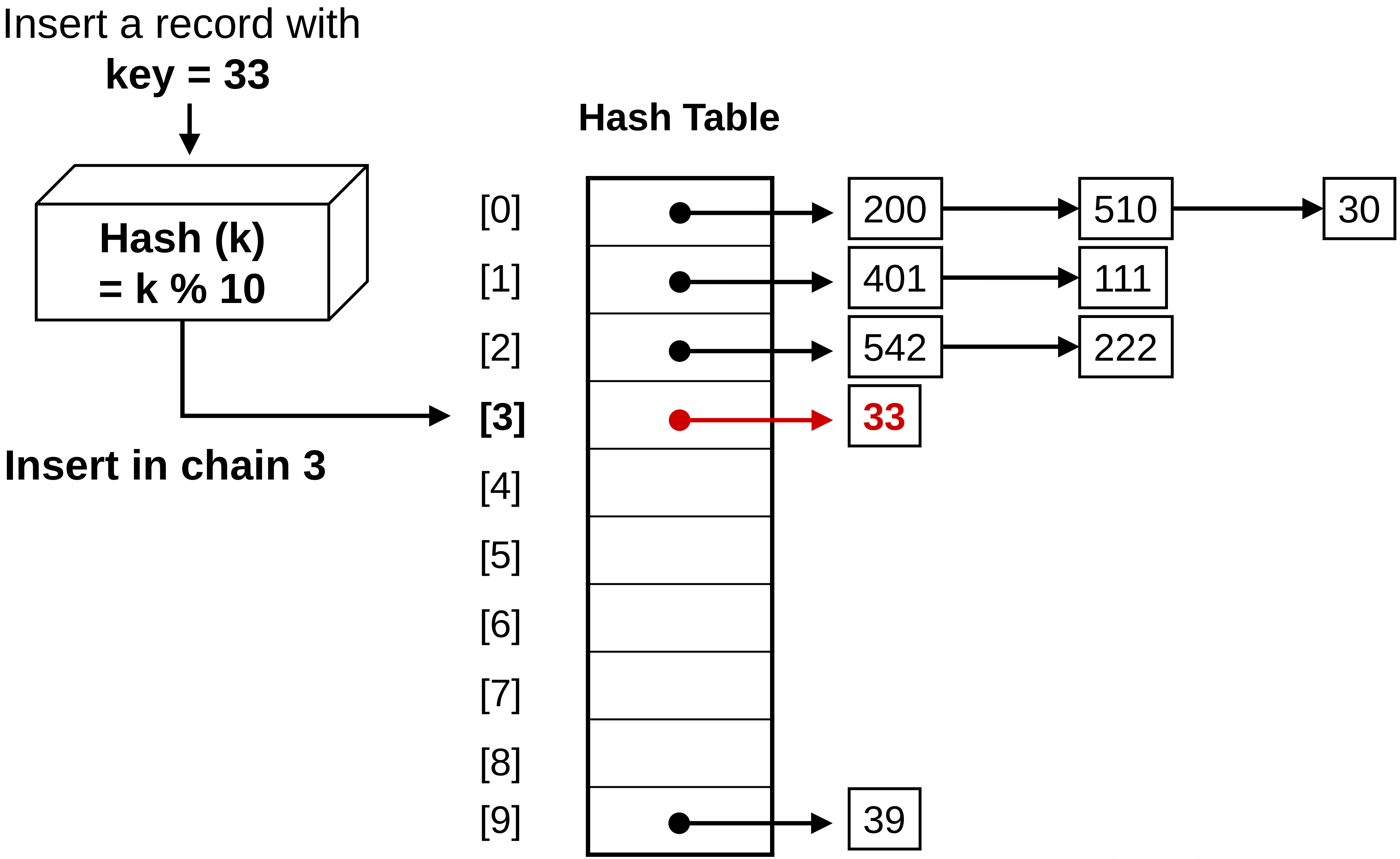
- **Separate Chaining**
  - To insert a key k
    - Compute f(k) to determine which list to traverse
    - If hash_table[f(k)] contains a null pointer, initiatize this entry to point to a linked list that contains k alone
    - If hash_table[f(k)] is a non-empty list, we add k at the beginning of this list
  - To delete a key k
    - compute f(k), then search for k within the list at hash_table[f(k)]. Delete k if it is found.

- **Separate Chaining**
  - If the hash function works well, the number of keys in each linked list will be a small constant
  - Therefore, we expect that each search, insertion, and deletion can be done in constant time
  - Disadvantage
    - Memory allocation in linked list manipulation will slow down the program
  - Advantage
    - Deletion is easy
    - Array size is not a limitation

- **Example**




Insert a record with
key = 33
Hash (k)
= k % 10
Insert in chain 3
Hash Table
[0]
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
200
510
30
401
111
542
222
39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[0]
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[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
200
510
30
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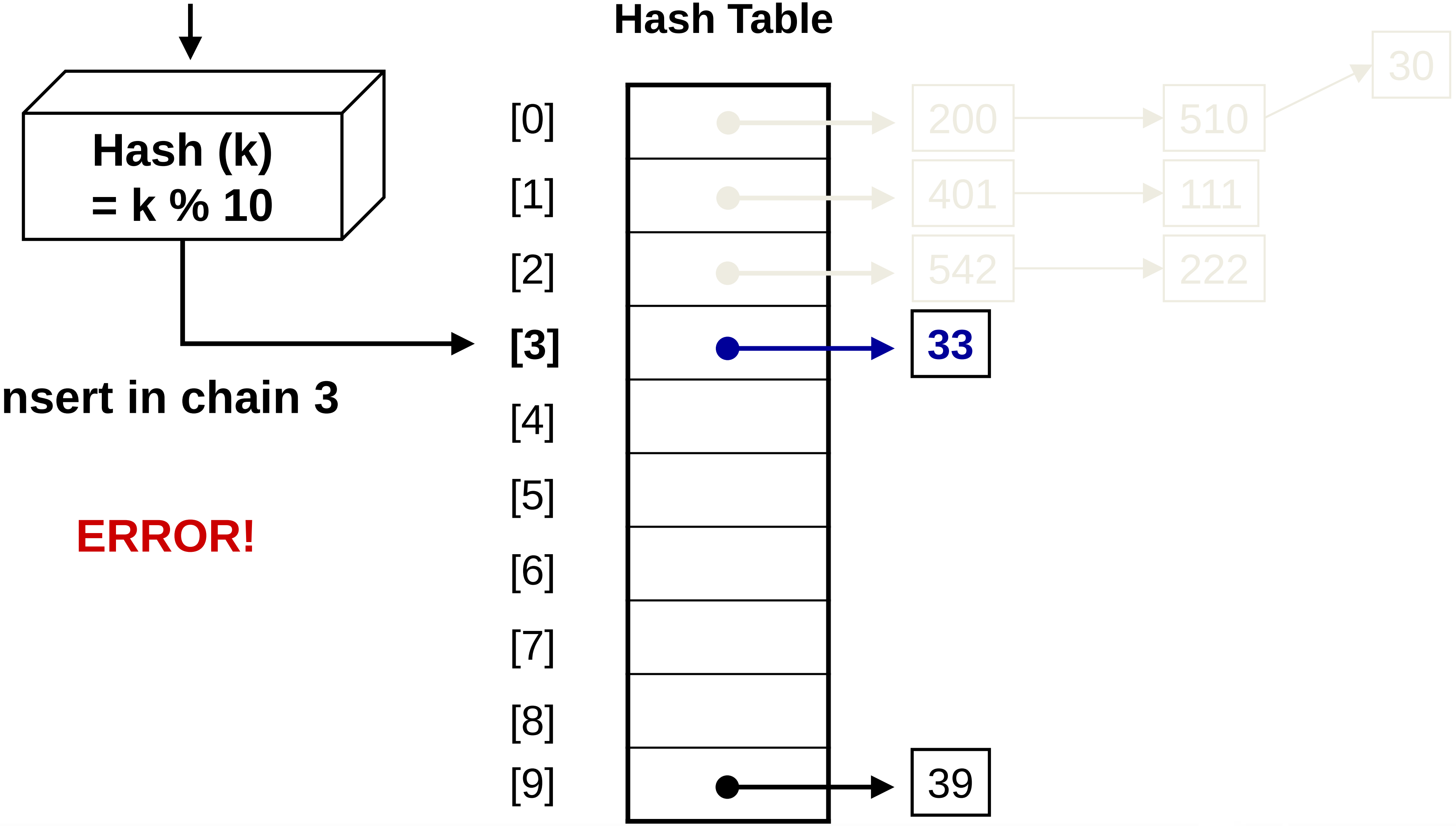
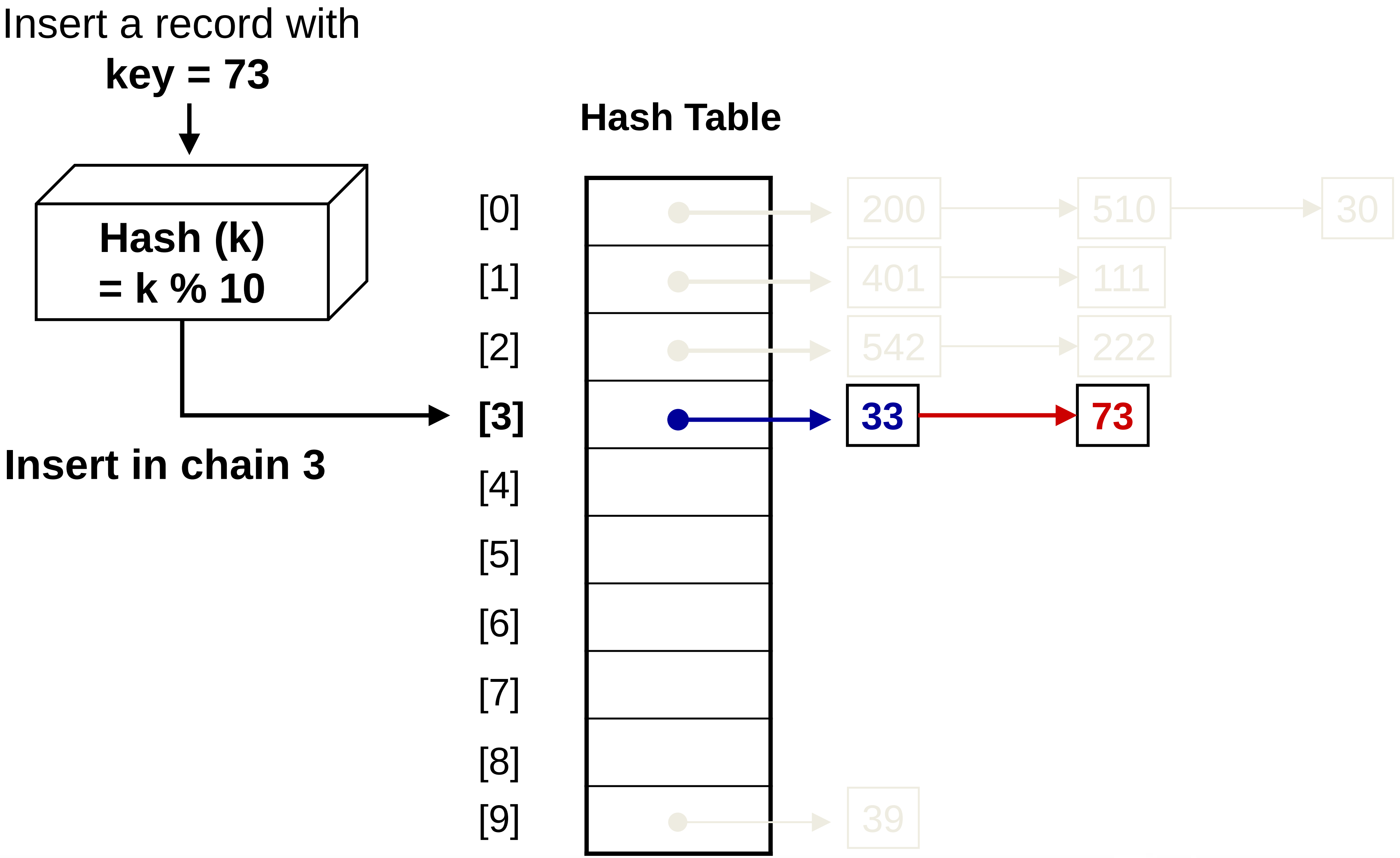
Insert a record with
key = 33

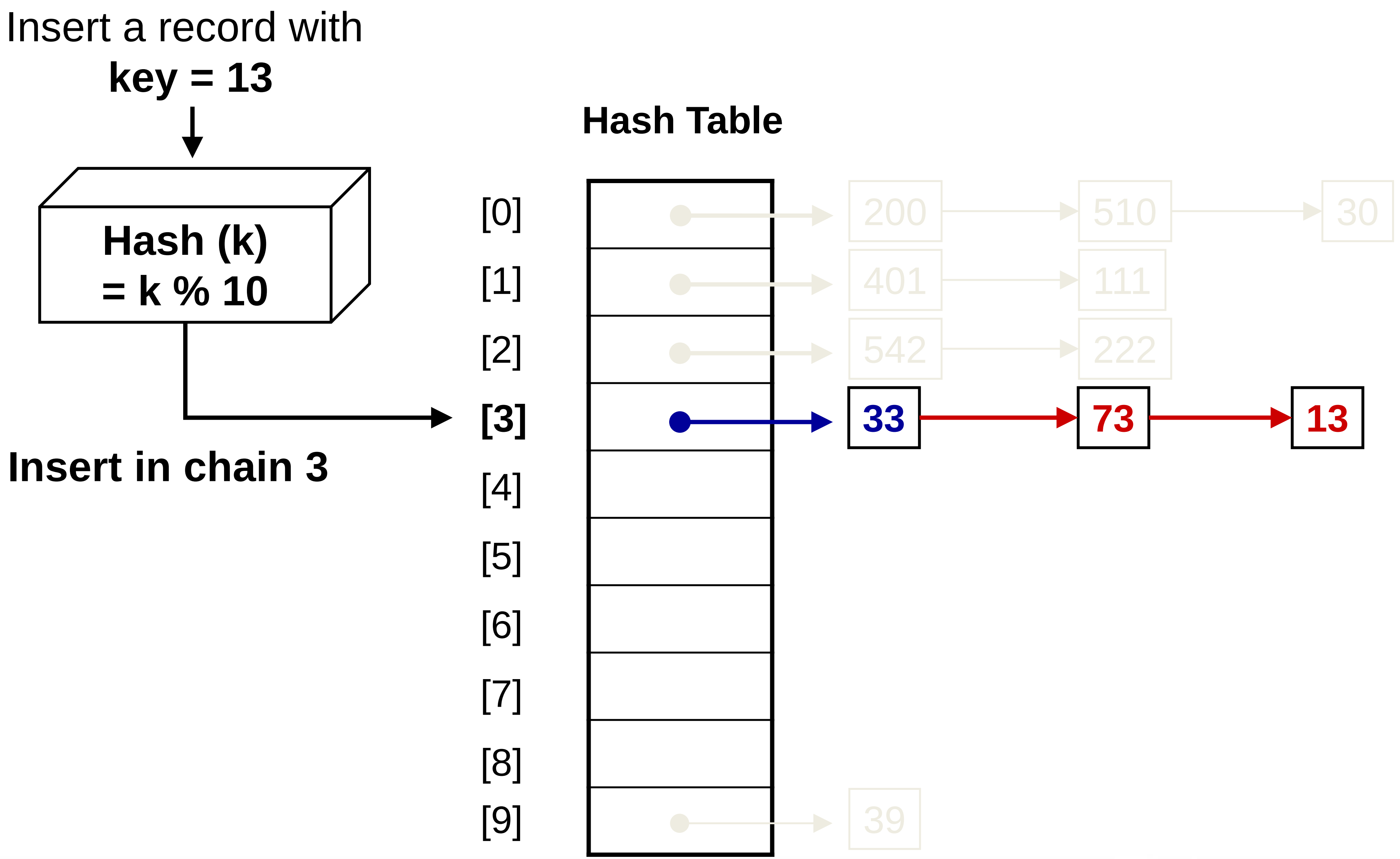


Insert in chain 3

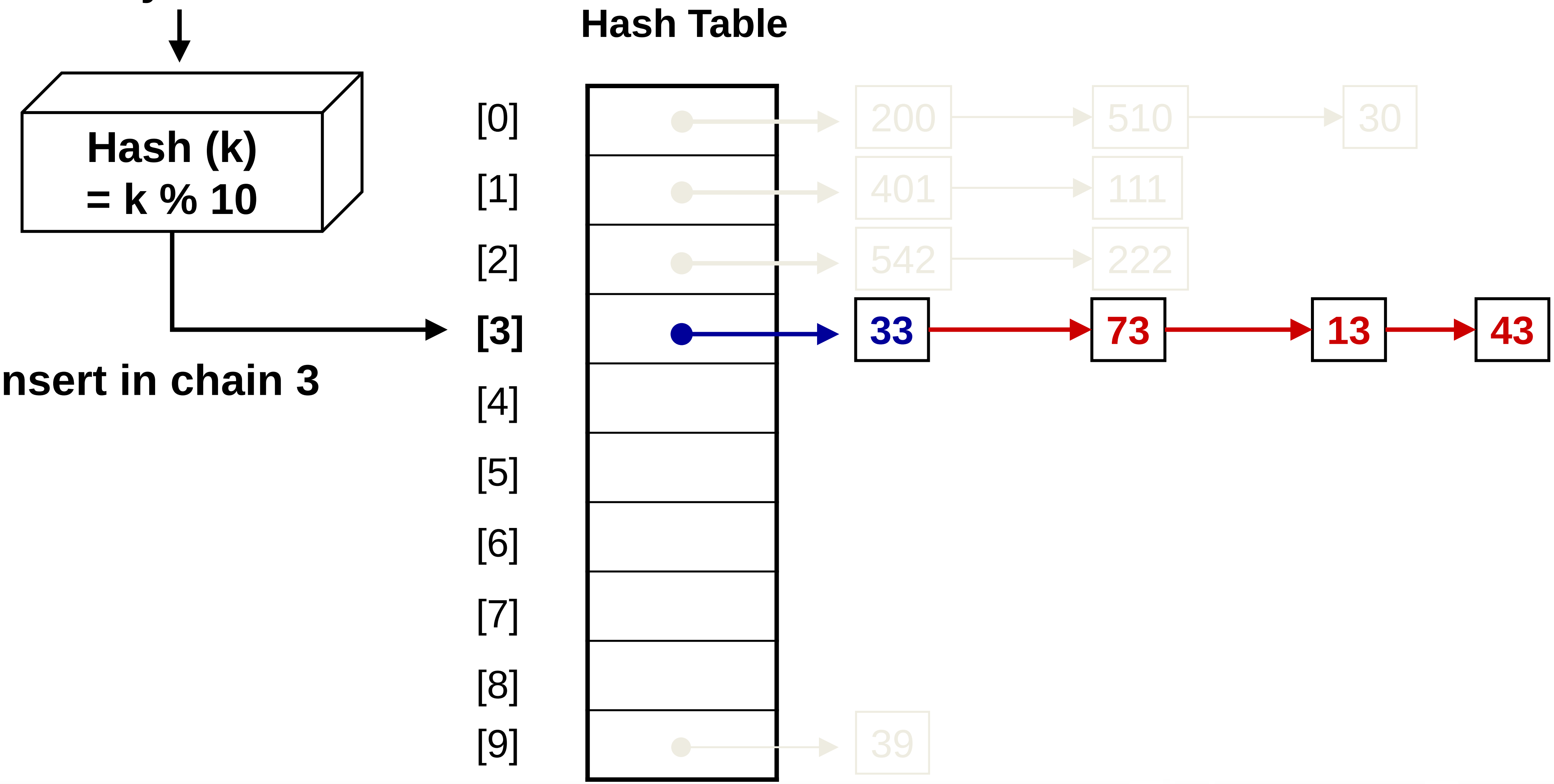
ERROR!



Insert a record with
key = 73
Hash (k)
= k % 10
Insert in chain 3
Hash Table
[0]
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
200
510
30
401
111
542
222
33
73
39


Insert a record with
key = 13
Hash (k)
= k % 10
Insert in chain 3
Hash Table
[0]
[1]
[2]
[3]
[4]
[5]
[6]
[7]
[8]
[9]
200
510
30
401
111
542
222
33
73
13
39

Insert a record with
key = 43



Insert in chain 3

- **Data Structure for Chaining**

```
#define MAX_CHAR 10
#define TABLE_SIZE 13
#define IS_FULL(ptr) (!(ptr))
typedef struct {
        char key[MAX_CHAR];
        /* other fields */
} element;
typedef struct list *list_pointer;
typedef struct list {
        element item;
        list_pointer link;
};
list_pointer hash_table[TABLE_SIZE];
```

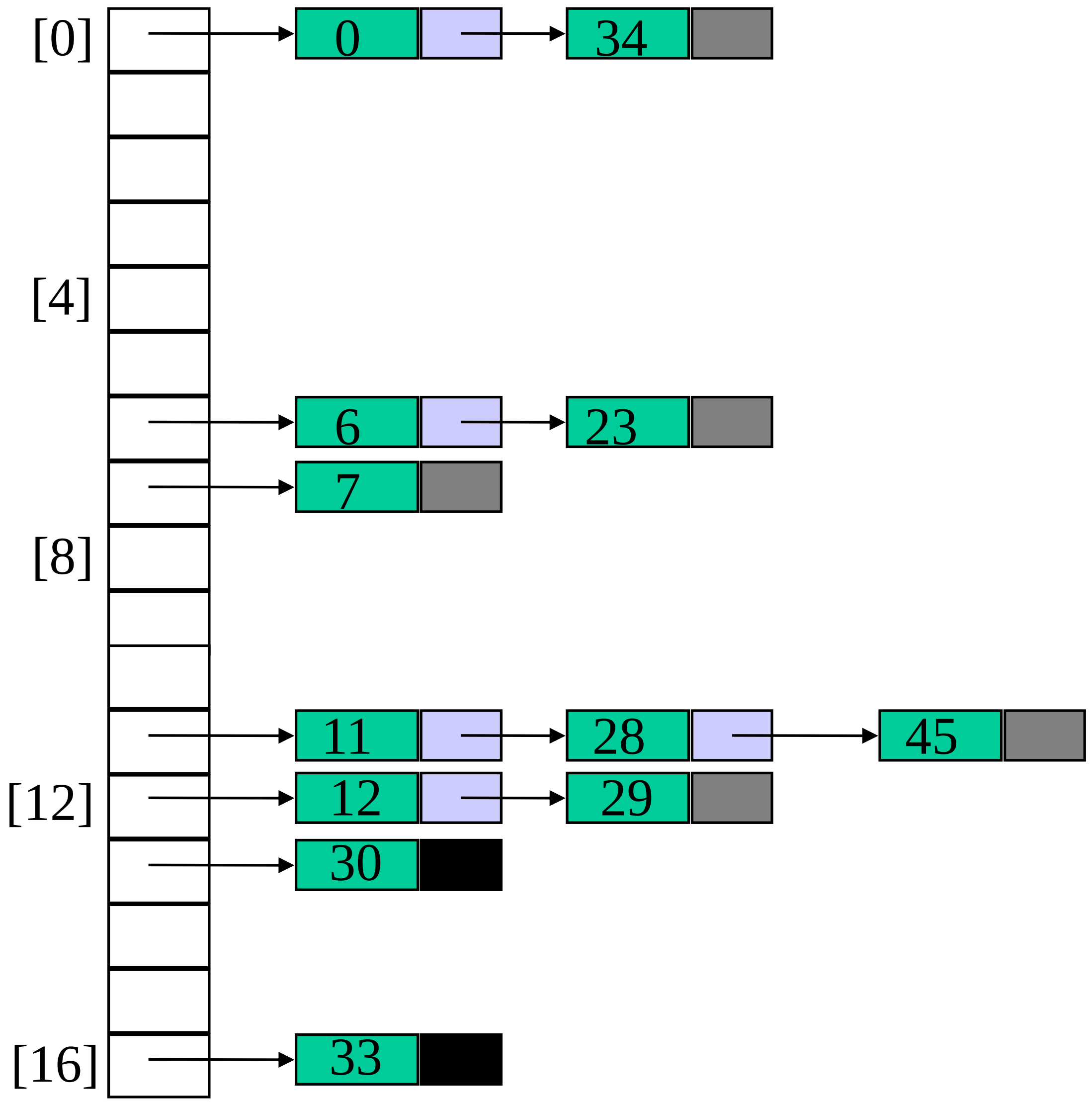
- **Separate Chaining – Insert**

```
void insert( element_type key, HASH_TABLE H ){
    position pos, new_cell; LIST L;
    pos = find( key, H );
    if( pos == NULL ) {
        new_cell = (position) malloc(sizeof(struct list_node));
        if( new_cell == NULL )   fatal_error("Out of space!!!");
        else {
            L = H->the_lists[ hash( key, H->table size ) ];
            new_cell->next = L->next;
            new_cell->element = key; /* Probably need strcpy!! */
            L->next = new_cell;
        }
    }
}
```

- **Other Implementation**

Sorted chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Bucket = key % 17



CONTENT

- Introduction
- Static hashing
- **Dynamic hashing**

- **Dynamic hashing**
  - The number of identifiers in a hash table may vary
  - Use a small table initially; when a lot of identifiers are inserted into the table, we may increase the table size
  - When a lot of identifiers are deleted from the table, we may reduce the table size
  - This is called **dynamic hashing** or **extendible hashing**
  - Dynamic hashing usually involves databases and buckets may also be called **pages**

- We assume each page contains *p* records
- Each record is identified by a key (i.e., the identifiers in static hashing)
- Space utilization = *n/mp*
  - where *n* is the number of actual records
  - *m* is the number of pages reserved
  - *p* is the number of records per page

$$\frac{NumberOfRecord}{NumberOfPages * PageCapacity}$$

- **Objective: Find an extendible hashing function such that**
  - it minimizes the number of pages accessed
  - space utilization is as high as possible

- **There are two approaches**
  - Dynamic Hashing with Using Directories
  - Dynamic Hashing without Using Directories

SUMMARY

- Introduction
- Static hashing
- Dynamic hashing

Nhân bản – Phụng sự – Khai phóng

Enjoy the Course…!