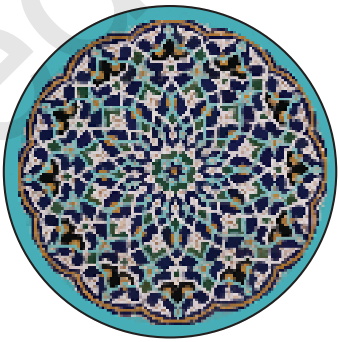
B. Xaydarov, E. Sariqov, A. Qo‘chqorov

GEOMETRIYA 9

Umumiy o‘rta ta’lim maktablarining 9- sinfi uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tomonidan tavsiya etilgan

To‘ldirilgan va qayta ishlangan to‘rtinchi nashr



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Taqrizchilar:

Shaniyazova M. – Sergeli tumanidagi 300- IDUMning oliy toifali matematika fani o‘qituvchisi;
Soibova I. – Yashnaobod tumanidagi 307- IDUMning oliy toifali matematika fani o‘qituvchisi;
Tajaddinova Sh. – Sergeli tumanidagi 104- maktabning oliy toifali matematika fani o‘qituvchisi.

O‘zbekiston Respublikasi Fanlar akademiyasining haqiqiy a'zosi, fizika-matematika fanlari doktori, professor A. Azamov tahriri ostida.

9- sinfda geometriyaning planimetriya qismini – yassi geometrik shakllarning xossalarini o‘rganish davom ettiriladi. Unda siz geometrik almashtirishlar, shakllarning o‘xshashligi, uchburchakning tomonlari va burchaklari orasidagi munosabatlar, aylana uzunligi va doira yuzi, uchburchak va aylanadagi metrik munosabatlar bilan tanishasiz.

Mazkur darslikning mazmuni qat'iy aksiomatik tizim asosiga qurilgan. Unda nazariy materiallar imkon boricha sodda va ravon tilda bayon etilgan. Barcha mavzu va tushunchalar turli-tuman hayotiy misollar orqali ochib berilgan. Har bir mavzudan so‘ng berilgan savollar, isbotlash, hisoblash, yasashga doir masala va misollar o‘quvchini ijodiy fikrlashga undaydi, unga o‘zlashtirilgan bilimlarni chuqurlashtirishga va mustahkamlab borishga yordam beradi. Darslik o‘zining o‘zgacha dizayni va dars materialining ko‘rgazmali qilib taqdim etilishi bilan ham ajralib turadi. Unda keltirilgan rasm va chizmalar dars materialini yaxshiroq o‘zlashtirishga xizmat qiladi.

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

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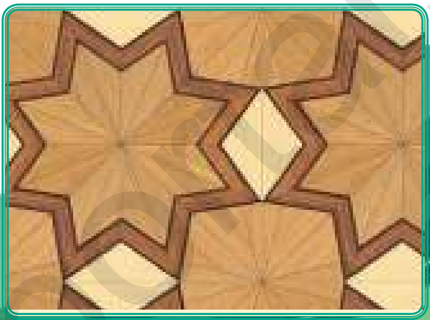
MUNDARIJA

5-8- SINFLARDA O‘TILGANLARNI TAKRORLASH



Mazkur bo‘limdagi masalalar 5-8- sinflarda o‘rganilgan geometrik shakllar va ularning xossalarini yodga olish uchun berilmoqda.

Bo‘limda PISA va TIMSS - o‘quvchilar bilimini baholashning xalqaro dasturlari masalalaridan ham keltirilmoqda.

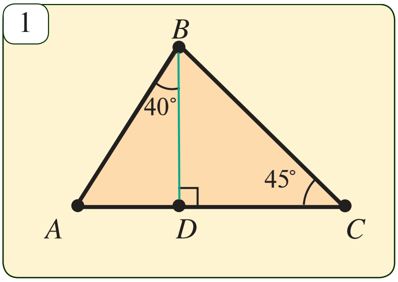
Bu bo‘limdagi materiallarni o‘rganish natijasida quyidagi bilim va ko‘nikmalarni yangilash imkoniyatiga ega bo‘lasiz:

√ 5-8- *sinflarda geometriyadan o‘tilgan mavzularni takrorlab, olgan bilimlaringizni esga olasiz va erishgan ko‘nikmalaringizni mustahkamlaysiz.*

√ *PISA va TIMSS - o‘quvchilar bilimini baholashning xalqaro dasturlari masalalari bilan tanishasiz;*

√ *Bu sizga 9- sinfda geometriyani o‘rganishni muvaffaqiyatli davom ettirishingizga zamin yaratadi.*

Mazkur bo‘limdagi masalalarni yechish uchun darslikning oxirida keltirilgan asosiy geometrik shakllarga oid ma’lumotlar hamda ularning xossalarini ifodalovchi formulalardan foydalanishingiz mumkin.



1.1. ABC uchburchakning BD balandligi o‘tkazilgan *(1-rasm)*. Agar $\angle ABD = 40°$, $\angle BCD = 45°$ bo‘lsa, uchburchakning A va B uchidagi burchaklarini toping.

Yechish. 1) To‘g‘ri burchakli ABD uchburchakda $\angle ABD = 40°$ va uchburchak ichki burchaklarining yig‘indisi 180° ga teng bo‘lgani uchun

$$\angle A = 180° - (90° + 40°) = 50°.$$

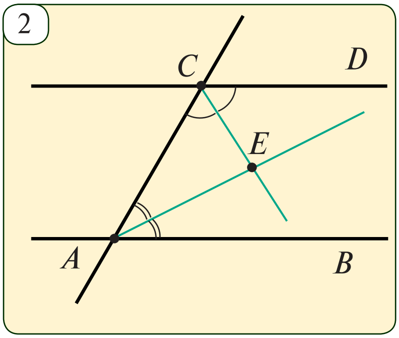
2) To‘g‘ri burchakli BCD uchburchakda $\angle BCD = 45°$ bo‘lgani uchun

$$\angle DBC = 180° - (90° + 45°) = 45°.$$

$\angle ABC = \angle ABD + \angle DBC$ bo‘lgani uchun

$$\angle B = 40° + 45° = 85°.$$

Javob: 50°, 85°.



1.2. Ikki parallel to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan ichki bir tomonli burchaklarning bissektrisalari orasidagi burchakni toping.

Yechish. AC to‘g‘ri chiziq AB va CD – parallel to‘g‘ri chiziqlarni 2-rasmda tasvirlangandek kesib o‘tgan bo‘lsin. Ichki bir tomonli BAC va ACD burchaklarning bissektrisalari E nuqtada kesishgan bo‘lib, $\angle EAC = x$, $\angle ECA = y$ bo‘lsin. Unda, burchak bissektrisasining ta’rifiga ko‘ra

$$\angle BAC = x + x = 2x, \ \angle ACD = y + y = 2y.$$

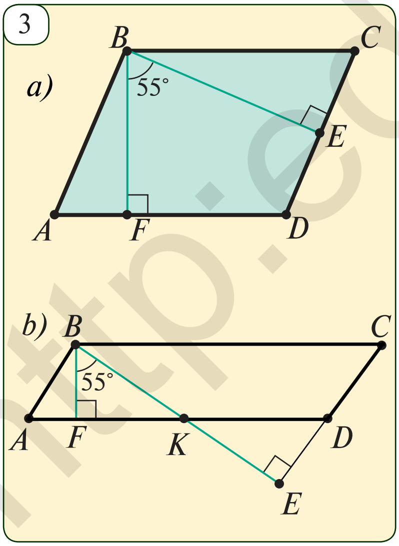
$AB \parallel CD$ bo‘lgani uchun ichki bir tomonli burchaklar xossasiga ko‘ra,

$$2x + 2y = 180°, \qquad x + y = 90°.$$

ACE uchburchak ichki burchaklari yig‘indisi 180° ga teng bo‘lgani uchun

$$\angle AEC = 180° - (x + y) = 180° - 90° = 90°.$$

Javob: 90°.



1.3. Agar parallelogrammning o‘tmas burchagi uchidan uning ikki tomoniga tushirilgan balandliklari orasidagi burchak 55° ga teng bo‘lsa, parallelogrammning burchaklarini toping.

Yechish. Parallelogrammning BF va BE balandliklari orasidagi burchak $55°$ bo'lsin *(3-rasm)*.

Rasmda tasvirlangan ikki hol: a) BE balandlik CD tomonga; b) BE balandlik CD tomon davomiga tushgan bo'lishi mumkin.

a) holda $BEDF$ to'rtburchak burchaklarining yig'indisi $360°$ bo'lgani uchun,

$$55° + 90° + \angle D + 90° = 360°. \qquad \text{Bundan, } \angle D = 125°.$$

b) holda BE balandlik AD tomon bilan kesishgan nuqta K bo'lsin. Unda,

$$\angle DKE = \angle BKF = 90° - 55° = 35°.$$

Uchburchak tashqi burchagining xossasiga ko'ra,

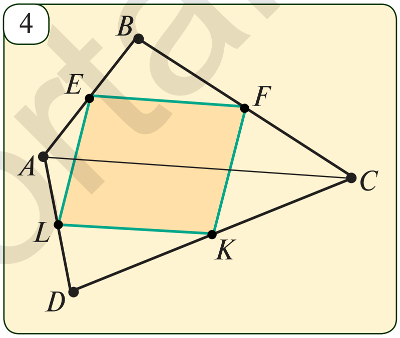
$$\angle ADC = \angle DKE + \angle KED = 35° + 90° = 125°.$$

Demak, har ikkala holda ham $\angle D = 125°$.

Unda, $\angle A = \angle C = 180° - \angle D = 55°$, $\angle B = \angle D = 125°$. *Javob:* $55°$, $125°$, $55°$, $125°$.

1.4. To'rtburchak tomonlarining o'rtalari parallelogramm uchlari bo'lishini isbotlang.

Yechish. $ABCD$ to'rtburchakning AB, BC, CD va DA tomonlari o'rtalari mos ravishda E, F, K va L nuqtalar bo'lsin. AC diagonalni o'tkazamiz *(4-rasm)*. $EFKL$ – parallelogramm ekanligini ko'rsatamiz.



EF kesma ABC uchburchakning, KL kesma esa ACD uchburchakning o'rta chizig'i bo'ladi. Unda, uchburchak o'rta chizig'ining xossalariga ko'ra,

$$EF \| AC,\ KL \| AC,\ EF = \frac{1}{2}AC,\ \ KL = \frac{1}{2}AC.$$

Bundan $EF \| KL$ va $EF = LK$. Shuning uchun, parallelogramm alomatiga ko'ra, $EFKL$ — parallelogramm.

1.5. ABC uchburchakda $\angle A = 47°$, $\angle C = 83°$ bo'lsa, uchburchakni uchinchi ichki burchagini va tashqi burchaklarini toping.

1.6. ABC uchburchakning AC tomoniga parallel to'g'ri chiziq AB va BC tomonlarni mos ravishda E va F nuqtalarda kesib o'tadi. Agar $\angle BEF = 65°$ va $\angle EFC = 135°$ bo'lsa, ABC uchburchak burchaklarini toping.

1.7. ABC uchburchak bissektrisalari I nuqtada kesishadi. Agar $\angle A = 80°$ va $\angle B = 70°$ bo'lsa, AIB, BIC va CIA burchaklarni toping.

1.8. Teng yonli uchburchakning bitta tashqi burchagi $70°$ ga teng. Uchburchak burchaklarini toping.

1.9. ABC uchburchakning AK bissektrisasi o'tkazilgan. Agar $\angle BAK = 47°$ va $\angle AKC = 103°$ bo'lsa, uchburchak burchaklarini toping.

1.10*. ABC uchburchak balandliklari H nuqtada kesishadi. Agar $\angle A = 50°$, $\angle B = 60°$ bo'lsa, AHB, BHC va CHA burchaklarni toping.

1.11. Uchburchakning o'rta chiziqlari uni to'rtta teng uchburchaklarga ajratishini isbotlang.

1.12*. ABC uchburchakda CD mediana davomiga bu medianaga teng DE kesma qo'yilgan. AF medrananing davomiga AF medianaga teng FH kesma qo'yilgan. B, H, E nuqtalar bitta to'g'ri chiziqda yotishini isbotlang.

1.13. ABC teng yonli uchburchakda ($AB=BC$) AN va CK bissektrisalar oʻtkazilgan.
a) KN kesma AC tomonga parallel ekanini koʻrsating.
b) $AK=KN=NC$ tenglik oʻrinli boʻlishini isbotlang.

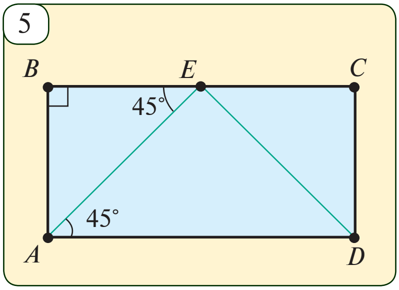
1.14. $ABCD$ toʻgʻri toʻrtburchak A va D burchaklarining bissektrisalari BC tomonda kesishadi. Agar $AB = 4$ cm boʻlsa, bu toʻgʻri toʻrtburchak yuzini toping.

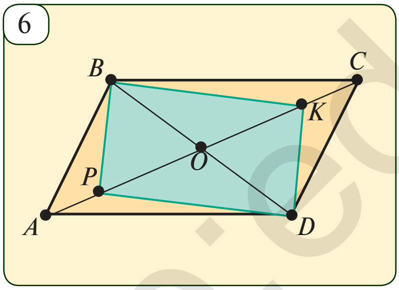
Yechish. Toʻgʻri toʻrtburchak A va D burchaklarining bissektrisalari kesishgan nuqta E boʻlsin *(5-rasm)*. $\angle B = 90°$, $\angle BAE = 45°$ boʻlgani uchun $\angle AEB = 180° - 90° - 45° = 45°$. Ya'ni, ABE — teng yonli uchburchak.

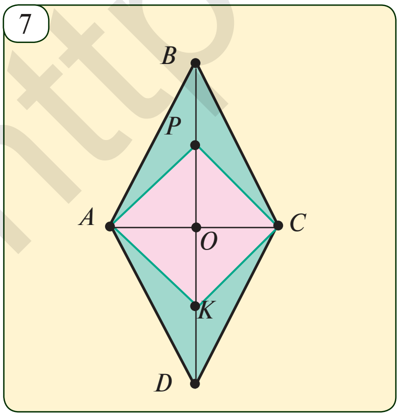
Unda, $AB = BE = 4$ (cm). Xuddi shunga oʻxshash $EC = CD = 4$ (cm) ekanligini koʻrsatish mumkin. Bundan $BC = BE + EC = 8\,(cm)$ va

$$S_{ABCD} = AB \cdot BC = 4 \cdot 8 = 32\ (cm^2).$$ *Javob:* 32 cm^2.

1.15. Toʻrtburchakning uchta burchagi 47°, 83° va 120° ga tengligi ma'lum. Uning toʻrtinchi burchagini toping.







1.16. Parallelogrammning ikki burchagi yigʻindisi 156° ga teng. Uning burchaklarini toping.

1.17. Toʻgʻri toʻrtburchak diagonallari orasidagi burchak 74°. Uning bir diagonali bilan tomonlari orasidagi burchaklarni toping.

1.18. Teng yonli trapetsiyaning ikkita burchagi ayirmasi 40° ga teng. Uning burchaklarini toping.

1.19. Romb burchaklaridan biri ikkinchisidan uch marta katta. Rombning burchaklarini toping.

1.20. $ABCD$ toʻgʻri toʻrtburchakning A burchagi bissektrisasi BC tomonini 2 cm va 6 cm ga teng kesmalarga ajratadi. Toʻgʻri toʻrtburchak perimetrini toping.

1.21. Tomonlari 3 cm va 6 cm, katta tomonlari orasidagi masofa esa 2 cm boʻlgan parallelogramm yasang.

1.22. $ABCD$ parallelogrammning AC diagonalida P va K nuqtalar tanlangan *(6-rasm)*. Agar $OP=OB=OK$ boʻlsa, $BKDP$ toʻgʻri toʻrtburchak boʻlishini isbotlang.

1.23*. $ABCD$ rombning BD katta diagonalida P va K nuqtalar tanlangan *(7-rasm)*. Agar $OA=OP=OK$ boʻlsa, $APCK$ toʻrtburchak kvadrat ekanligini isbotlang.

1.24*. $ABCD$ parallelogrammning BD diagonalida P va K nuqtalar tanlangan. Agar $BP=KD$ boʻlsa, $APCK$ toʻrtburchak parallelogramm ekanligini isbotlang.

2 PIFAGOR TEOREMASI VA UNING TATBIQLARI

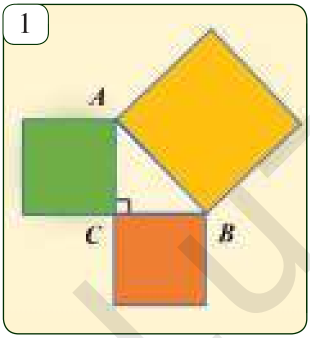
Bu mashhur teoremaning 3 xil ifodasini keltirib, uni esga olamiz.

a) matnli ifodasi: To‘g‘ri burchakli uchburchakning gipotenuzasi kvadrati katetlari kvadratlarining yig‘indisiga teng.

b) matematik ifodasi: ABC uchburchakda: $\angle C = 90°$, $AB = c$, $BC = a$, $AC = b$ bo‘lsa, $c^2 = a^2 + b^2$ bo‘ladi.

d) tasvirli ifodasi: (1-rasm).

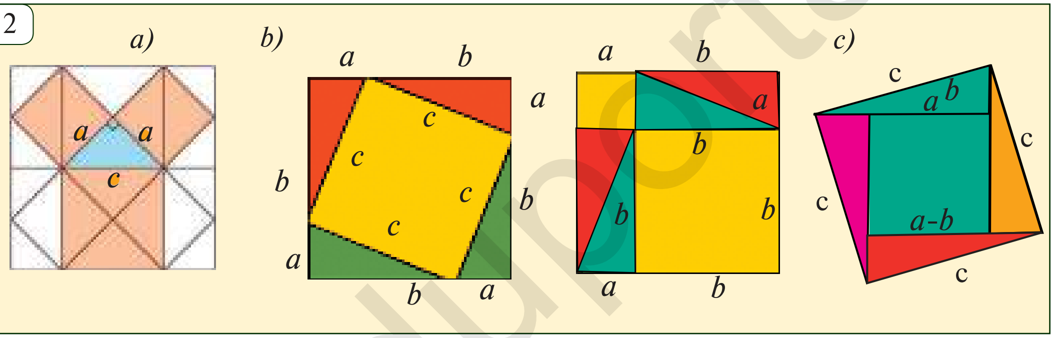
1



Masala va topshiriqlar

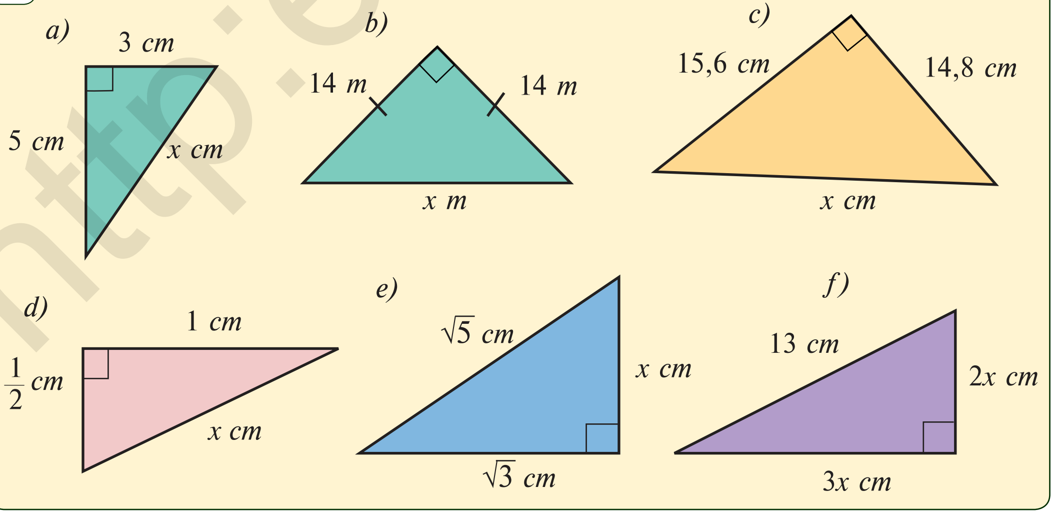
2.1. 2- rasmda keltirilgan rasmlar asosida Pifagor teoremasining bir nechta isbotini tiklang.

2

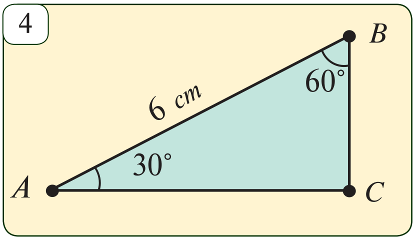


2.2. 3-rasmda berilganlarga ko‘ra noma’lumni toping.

3



2.3. *ABC* uchburchakning *AB* tomoni 6 *cm*, *A* va *B* burchaklari, mos ravishda, 30° va 60° bo'lsa, *ABC* uchburchak yuzini toping.



Yechish. Uchburchakning *C* burchagini topamiz:

$$\angle C = 180° - (\angle A + \angle B) = 180° - (60° + 30°) = 90°.$$

Demak, to'g'ri burchakli *ABC* uchburchakning *AB* gipotenuzasi 6 *cm* va *A* burchagi 30° ekan. To'g'ri burchakli uchburchakda 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng bo'lgani uchun, $BC = 3$ *cm* (*4-rasm*).

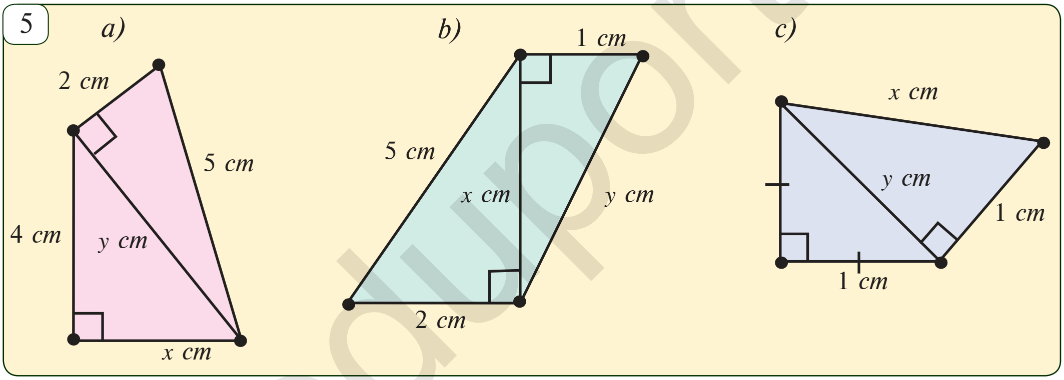
Pifagor teoremasidan foydalanib *AC* katetni topamiz:

$$AC^2 = AB^2 - BC^2 = 6^2 - 3^2 = 27\ (cm), \qquad AC = 3\sqrt{3}\ cm.$$

Endi uchburchak yuzini topamiz:

$$S_{ABC} = \frac{1}{2} AC \cdot BC = \frac{1}{2} \cdot 3\sqrt{3} \cdot 3 = \frac{9\sqrt{3}}{2}\ (cm^2).$$

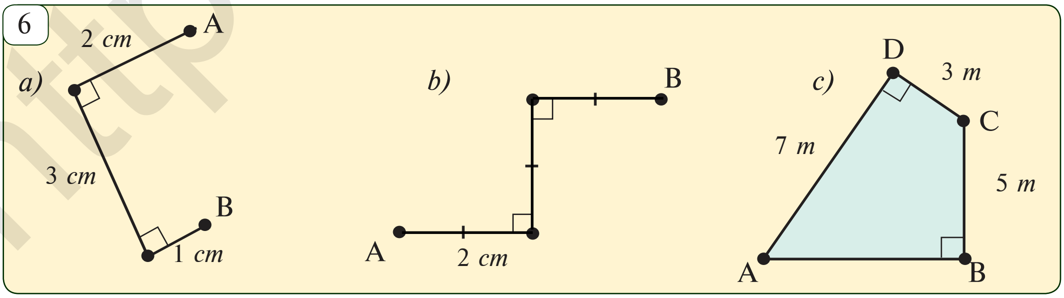
Javob: $\frac{9\sqrt{3}}{2}\ cm^2$.



2.4. 5-rasmda berilganlarga ko'ra noma'lumlarni toping.

2.5. Katetlari 15 *cm* va 20 *cm* bo'lgan to'g'ri burchakli uchburchak gipotenuzasiga tushirilgan balandligini toping.

2.6 6-rasmda tegishli kesma(lar)ni yasab, noma'lum AB kesmaning uzunligini toping.

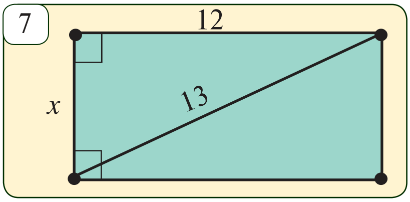


2.7. 7-rasmda berilganlardan foydalanib to'g'ri to'rtburchak yuzini toping.

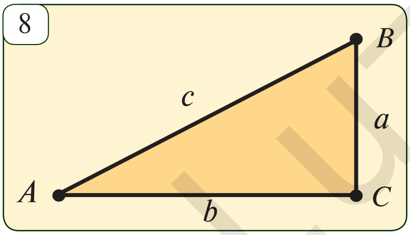
Yechish. To'g'ri to'rtburchakning kichik tomonini *x* bilan belgilasak, unda Pifagor teoremasiga ko'ra: $x^2 + 12^2 = 13^2$;

$x^2 + 144 = 169$; $x^2 = 169 - 144 = 25$; $x = \pm 5$. Uzunlik musbat kattalik bo'lgani uchun $x = 5$ *cm*. Unda to'g'ri to'rtburchak yuzi

$S = a \cdot b = 5 \cdot 12 = 60$ (cm²). *Javob:* 60 *cm²*.

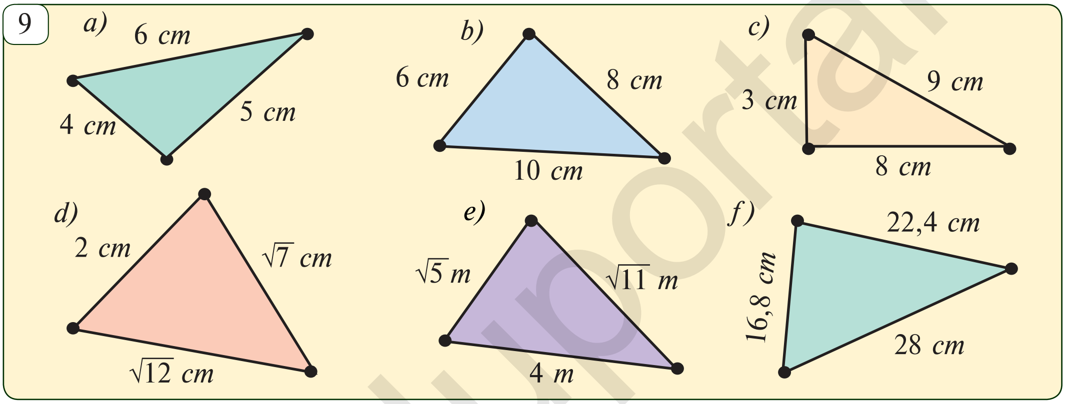


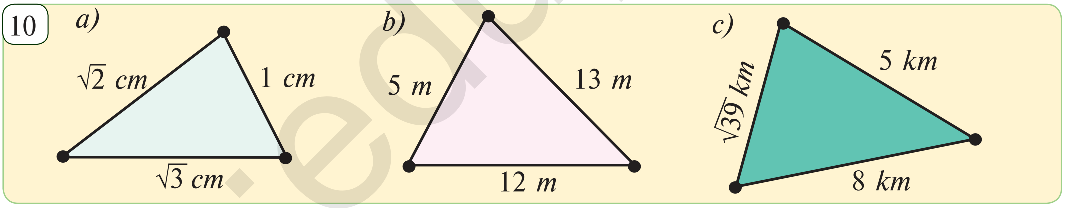
Teorema. ***Agar tomonlari a, b va c bo'lgan uchburchakda $c^2 = a^2 + b^2$ bo'lsa, bu uchburchak to'g'ri burchakli bo'ladi (8-rasm).***



2.8. 9-rasmdagi uchburchaklar noaniqroq tasvirlangan. Ularning qaysi biri to'g'ri burchakli?

2.9. 10-rasmdagi uchburchaklar noaniqroq tasvirlangan. Ularning qaysi biri to'g'ri burchakli?





2.10. 11- rasmda tasvirlangan noma'lum yuzani toping.

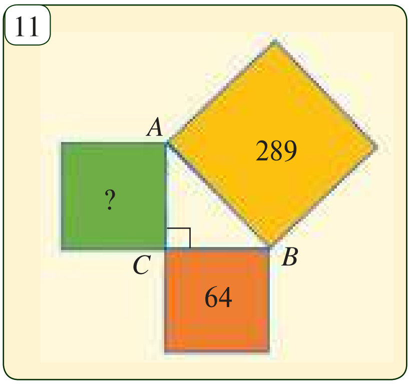
2.11. 12- rasmdagi rombning diagonallari 6 *cm* va 8 *cm* bo'lsa, uning tomonini toping.

2.12. 13- rasmdagi teng tomonli uchburchakning tomoni 6 *m* bo'lsa, uning balandligini toping.

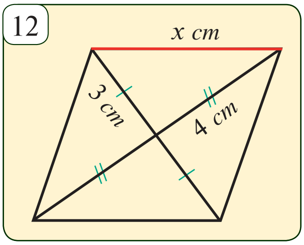
2.13. 14- rasmda tasvirlangan shaklning perimetrini toping.

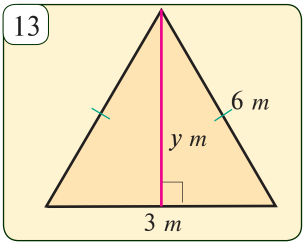
2.14. 15- rasmda berilganlardan foydalanib, noma'lum uzunlikni toping.

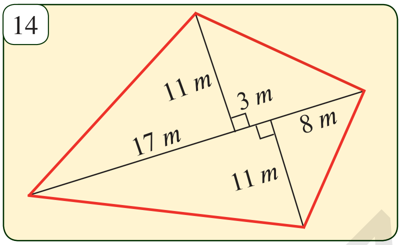
2.15. 16- rasmda berilganlardan foydalanib, *AB* kesma uzunligini toping.

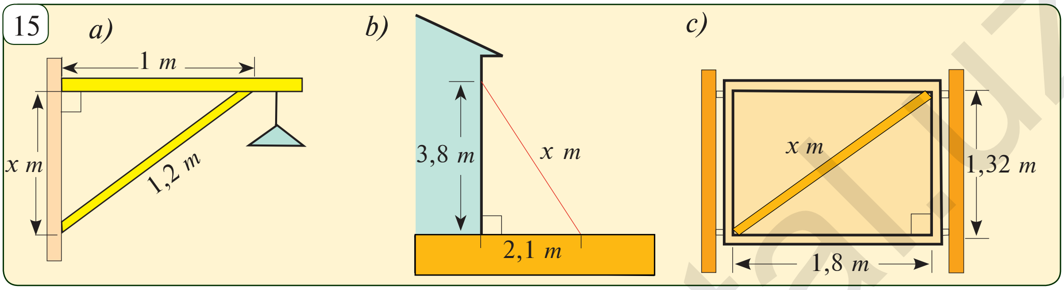


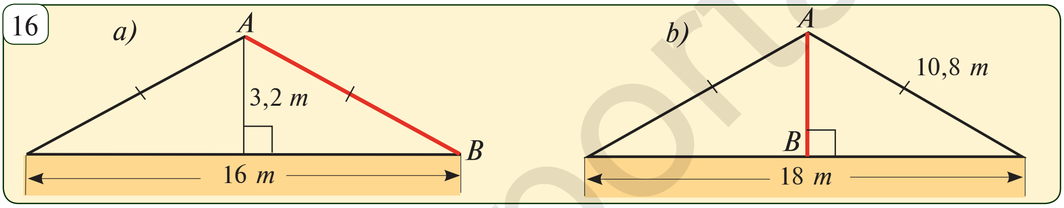
2.16. 17- rasmda tasvirlangan chodirning old tomoni teng tomonli uchburchak shaklida. Berilganlardan foydalanib chodir asosining yuzini toping.

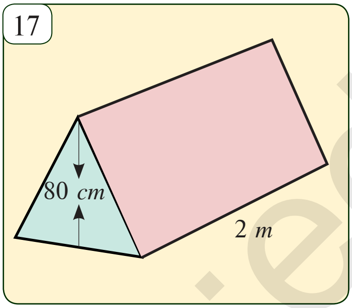


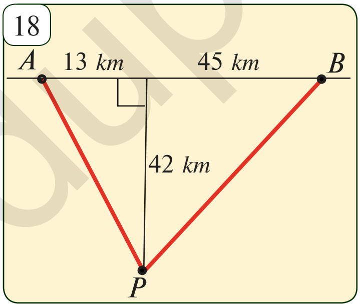


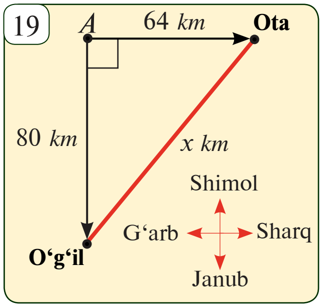












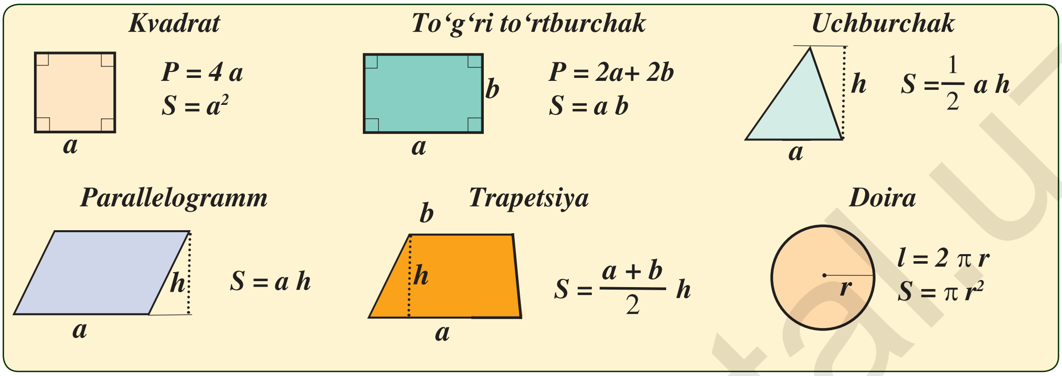
2.17. 18-rasmda P elektr stansiyadan A va B shaharchalarga sim tortishmoqchi. Buning uchun qancha sim kerak bo‘ladi?

2.18. A nuqtadan ota 16 *km/h* tezlik bilan sharqqa, o‘g‘il esa 20 *km/h* tezlik bilan velosipedda janubga qarab harakatlanmoqda (19-rasm). 4 soatdan keyin ular orasidagi masofa qancha bo‘ladi?

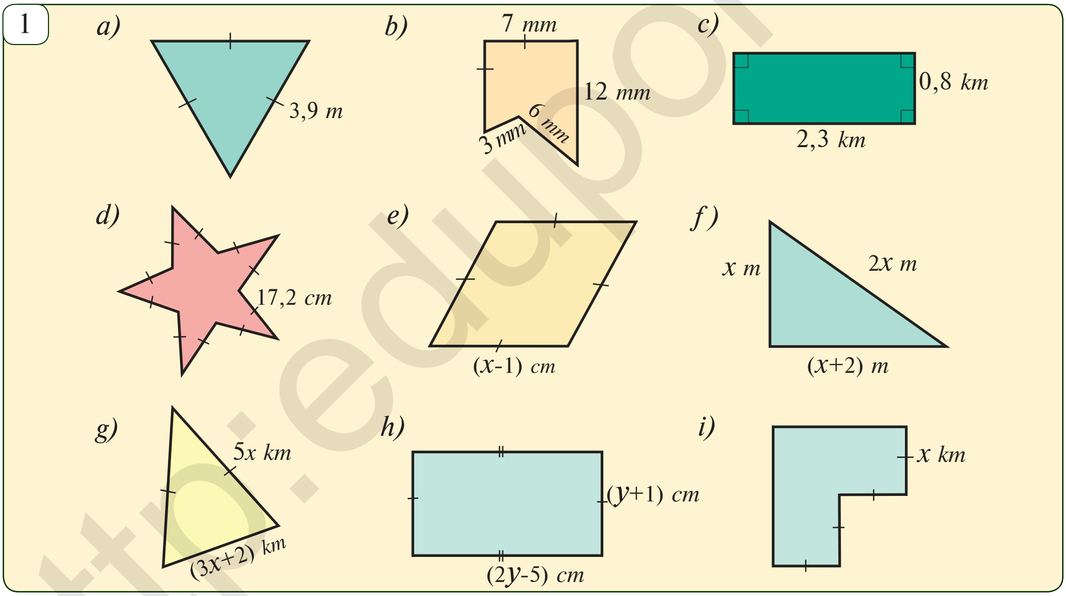
2.19. Ikki kapitan Jek va Huk Jumavoy orolidan o‘z kemalarida safarga chiqishdi (20-rasm). Birinchisi 15 *km/h* tezlik bilan shimolga, ikkinchisi esa 19 *km/h* tezlik bilan g‘arbga tomon suzib ketishdi. 2 soatdan keyin ular orasidagi masofa qancha bo‘ladi?

3 GEOMETRIK SHAKLLARNING PERIMETRI VA YUZINI HISOBLASHGA DOIR MASALALAR

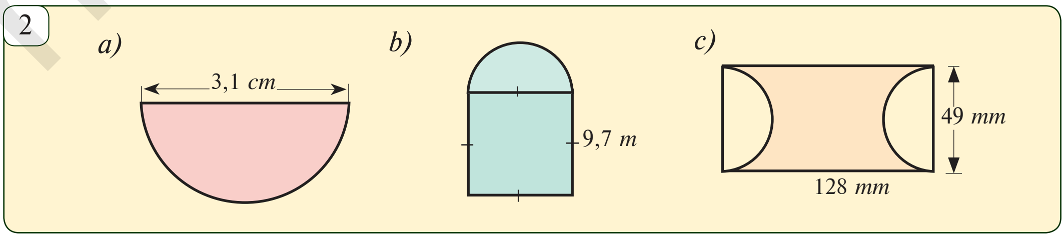
Quyida yassi geometrik shakllarning perimetri va yuzini hisoblashga doir turli masalalarni ko'rib chiqamiz.



3.1. 1-rasmda tasvirlangan ko'pburchaklar perimetrini hisoblang.



3.2. 2-rasmda tasvirlangan geometrik shakllarning perimetrini (chegarasining uzunligini) hisoblang.

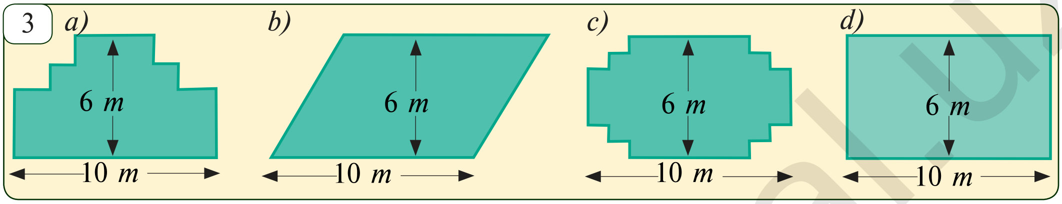


3.3. 3-rasmda tasvirlangan gulzorlarni 32 *m* sim bilan o‘rab bo‘ladimi?

3.4. 4-rasmda xona shifti tasvirlangan. Shift ichki qismini oq, tashqi qismini esa yashil rang bilan bo‘yash kerak. 1. Rasmda belgilangan noma’lum kesma uzunigini toping. 2. Yashil rangga bo‘yalgan shift bo‘lagining yuzini toping. 3. Oq rangga bo‘yalgan shift bo‘lagining yuzini toping.

3.5. Velosiped g‘ildiragining diametri 64 *cm*.(5-rasm) Ashraf velosipedda 100 *m* masofani bosib o‘tdi. Bunda velosipedning har bir g‘ildiragi necha marta to‘liq aylandi? (Eslatma: aylana uzunligi $C=2\pi r$ formula bilan hisoblanadi).

3



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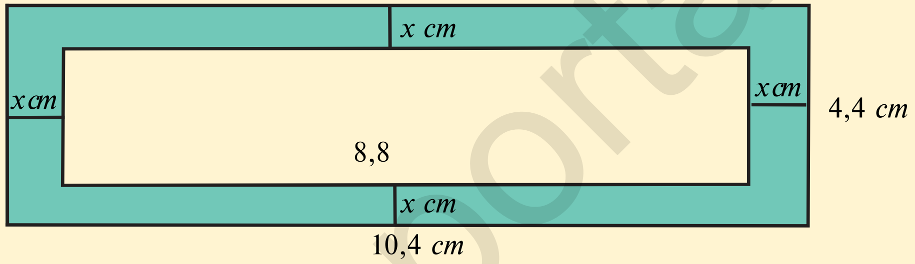


3.7. Avtomobil shinasi sirtidagi yozuv ma'lum o‘lchamlarni bildiradi (6.a-rasm). Masalan, 195/55 R16 yozuvda 195 soni shina kengligini mm larda anglatadi (6.b-rasm). Ikkinchi son 55 - shina profili balandligining shina kengligiga nisbatan foizini ko‘rsatadi. Bizning holda shina profili balandligi $195 \cdot 55\% = 107$ mm = 10,7 *cm*. R16 yozuv esa shina ichki diametrining dyumlardagi ifodasi. 1 dyum taxminan 2,54 *cm* ekanligini hisobga olsak, bizning shina ichki diametri $16 \cdot 2{,}54 = 40{,}64$ *cm* ga teng bo‘ladi.

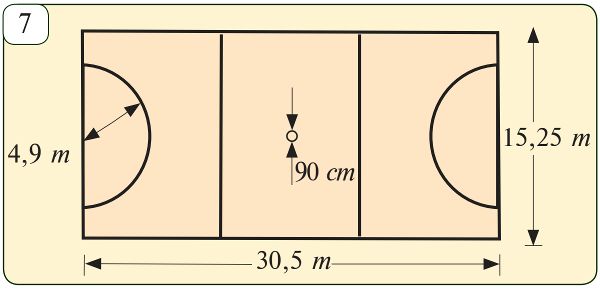
Ravon rusumidagi Neksiya avtomobili shinasida 175/60 R15 yozuv bor. Bu avtomobil shinasining kengligi, profili balandligi, ichki diametri va g‘ildirakning balandligi ya’ni tashqi diametrini santimetrlarda aniqlang.

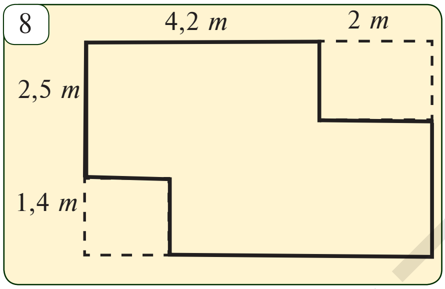
5

6

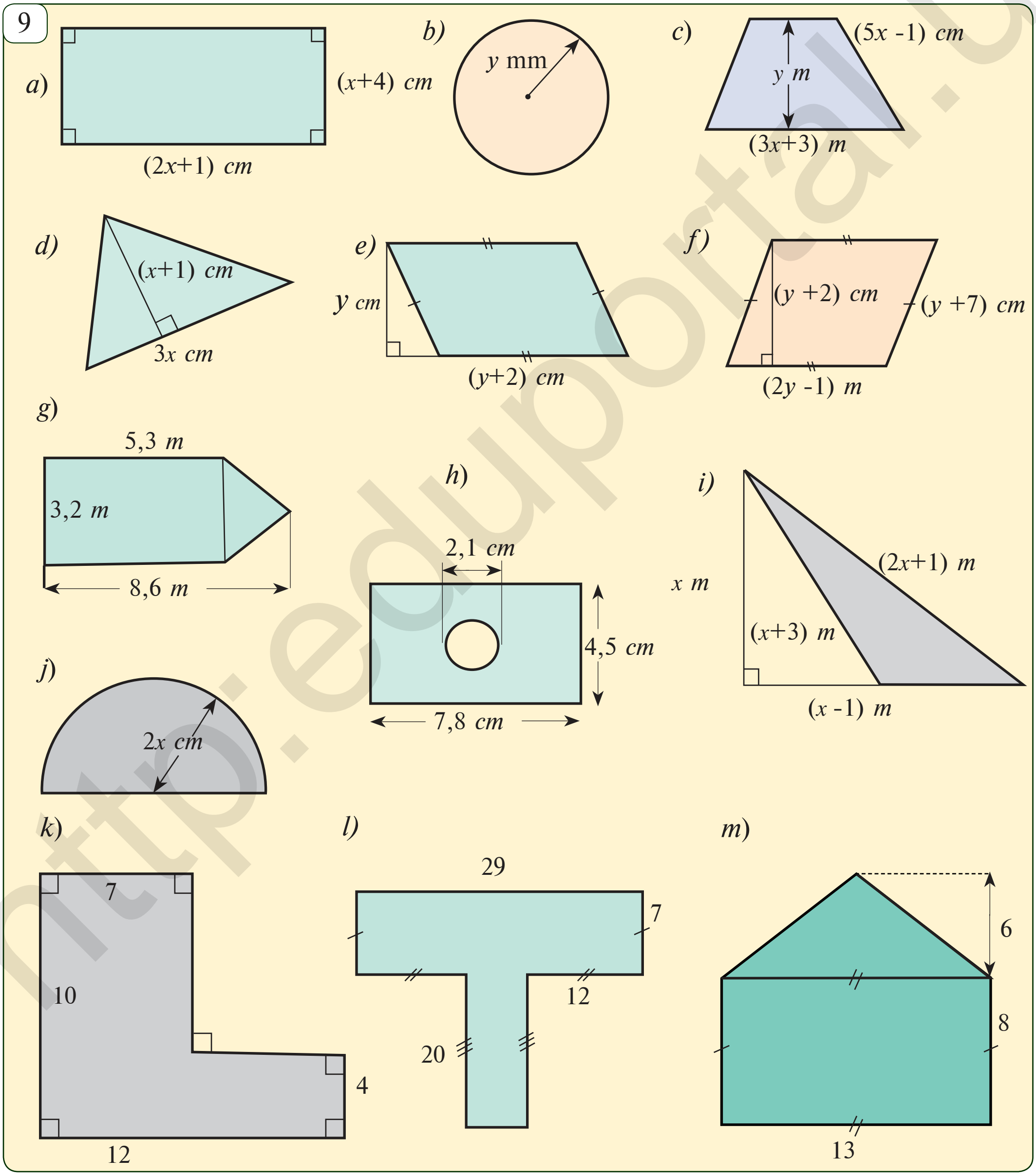
3.8. 7-rasmda berilgan amerikancha futbol stadionining perimetrini hisoblang. Bu stadion maydoni belgilash uchun chiziladigan chiziqlarning jami uzunligini toping.

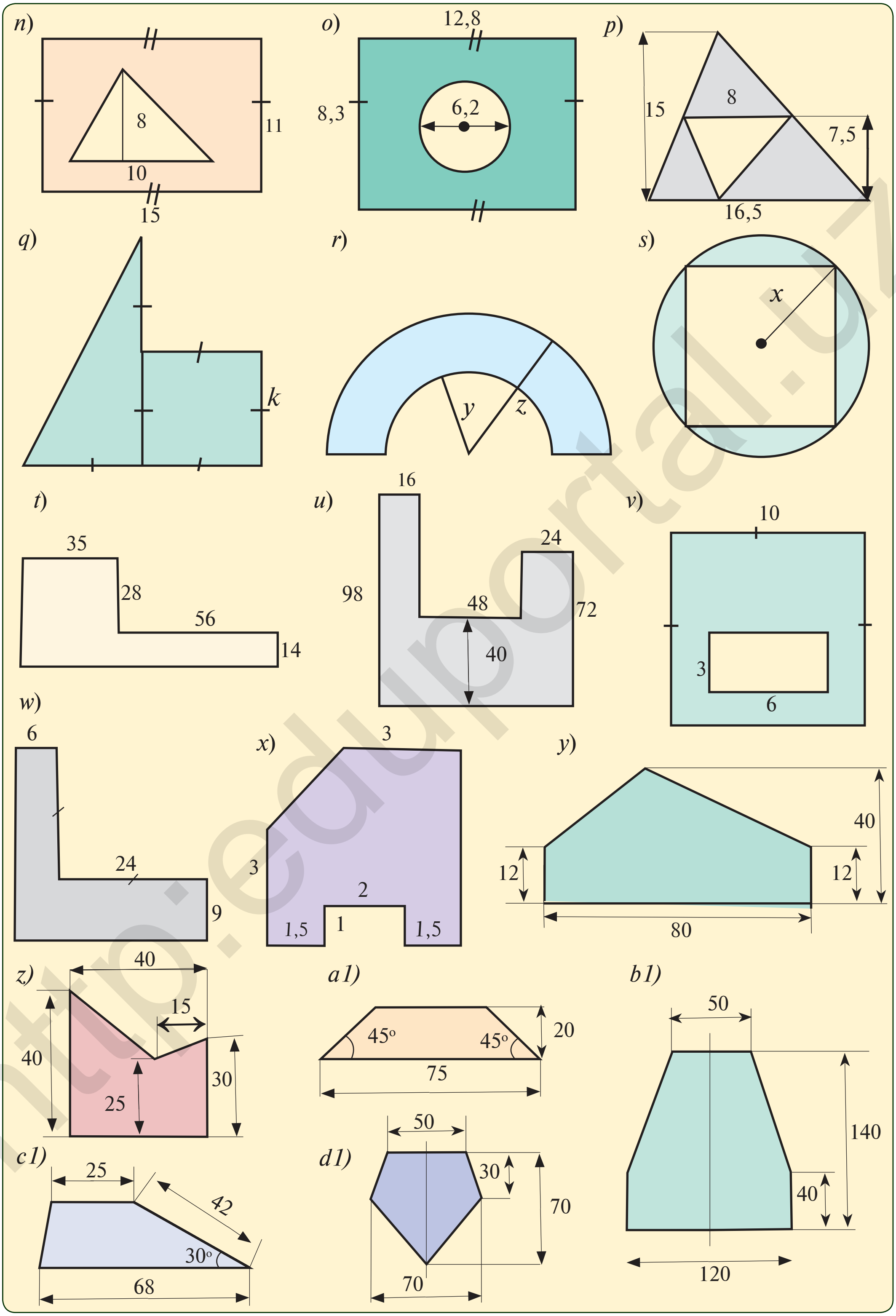
3.9. 8-rasmda tasvirlangan yer maydonining perimetrini toping.





3.10. 9- rasmda tasvirlangan turli shakldagi maydonlarning yuzini toping.

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b1)
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140
40
120
c1)
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42
30°
68
d1)
50
30
70
70

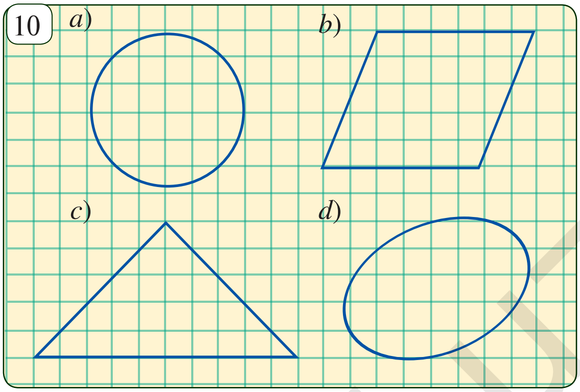
3.11. Amaliy topshiriq. 10-rasmda keltirilgan shakllarni katakli daftaringizga chizib oling. Ularning yuzini topish uchun qanday usullarini taklif qilasiz? Daftaringiz kataklaridan foydalangan holda ularning yuzini taxminan qanday aniqlash mumkin?

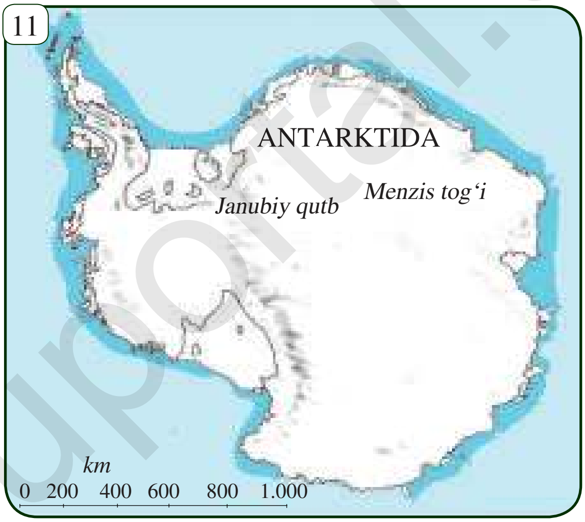
3.12. 11-rasmda Antraktida qit'asining xaritasi keltirilgan. Berilgan masshtabdan foydalanib va tegishli yordamchi yasashlarni bajarib, qit'aning yuzini taxminan aniqlang.

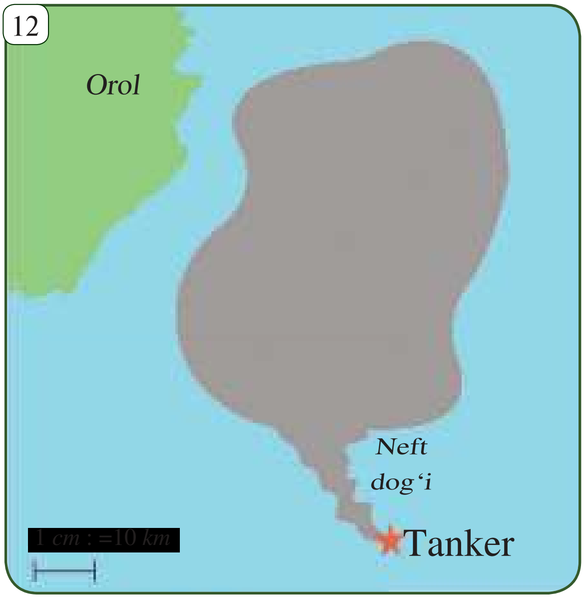
3.13. 12- rasmda neft tashib ketayotgan tanker halokatga uchrab, dengiz sirtida katta neft dog'i paydo bo'lgan. Berilgan masshtabdan va tegishli o'lchash ishlarini bajarib, neft dog'ining yuzini toping.

3.14. Tomorqa perimetri 48 *m* bo'lgan kvadrat shaklida. Uni 8 ta teng to'g'ri to'rtburchak shaklidagi uchastkalarga bo'lingan. Hosil bo'lgan to'g'ri to'rtburchakli uchastkalarning a) tomonlarini; b) yuzini aniqlang. Uchastkalarning yuzi tomorqa yuzidan necha foizga kichik?

3.15. Perimetri 20 *m*, bo'yi enidan 1,5 marta uzun bo'lgan to'g'ri to'rtburchak shaklidagi tomorqa kichik uchastkalarga bo'lingan. Agar uchastkalar a) kvadrat; b) to'g'ri to'rtburchak shaklida bo'lsa, ular orasida yuzi eng katta bo'lganining o'lchamlarini aniqlang.







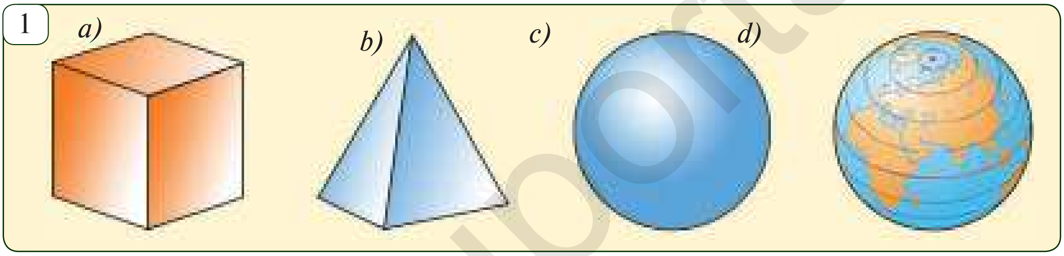
4 3D-GEOMETRIYA – FAZOVIY JISMLARDA PLANIMETRIYA MASALALARI

Ma’lumki, yassi shakllarni geometriyaning planimetriya bo‘limi, fazoviy jismlarni stereometriya bo‘limi o‘rganadi. Masalan, to‘g‘ri to‘rtburchak - yassi shakl bo‘lib, uning bo‘yi va eni, ya’ni ikkita o‘lchami bor. Parallelepiped esa fazoviy shakl bo‘lib, uning bo‘yi, eni va balandligi, ya’ni uchta o‘lchami bor.

Fazoviy jismlar haqida oldingi sinflarda tasavvurga ega bo‘lgansiz. Ularni 10-11- sinflarda stereometriya kursida atroflicha, tizimli ravishda o‘rganasiz. Shunday bo‘lsada, stereometriyaning qator masalalari borki, ularni faqat planimetriya yordamida ham yechish mumkin. Quyida planimetriyaga oid shunday 3D (3 demention - 3 o‘lchovli) geometrik masalalarni keltiramiz. Fazoviy jismlar haqidagi asosiy tushunchalarni qisqacha eslatib o‘tishni lozim topdik.

Fazoning chegaralangan qismi *fazoviy jism* deb ataladi. Fazoviy jismning chegarasiga (qobig‘iga) uning *sirti* deyiladi. Masalan, fazoviy shakl - kubning sirti 6 ta kvadratdan, sharning sirti sferadan iborat bo‘ladi.

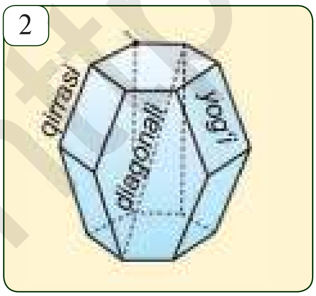
Ikki sirtning kesishmasidan chiziq hosil bo‘ladi. Masalan, 1- rasmdagi kub va

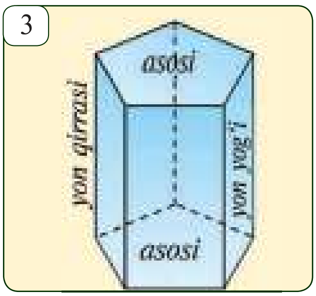


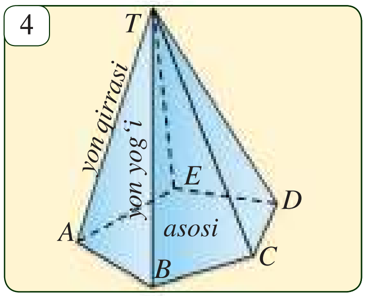
piramida qirralari shunday tekisliklarning kesishishidan hosil bo‘lgan. Sfera va tekislikning kesishishidan esa aylana hosil bo‘ladi.

Ikki chiziqning kesishishidan nuqta hosil bo‘ladi. Masalan, 1- rasmdagi kub va piramidaning qirralari keshishidan nuqtalar, ya’ni ularning uchlari hosil bo‘ladi.

Ko‘pyoq deb yassi ko‘pburchaklar bilan chegaralangan jismga aytiladi. Yassi ko‘pburchaklar bu *ko‘pyoqning yoqlari*, ko‘pburchaklarning uchlari *ko‘pyoqning uchlari*, tomonlari esa *ko‘pyoqning qirralari* deb ataladi. Bitta yoqqa tegishli bo‘lmagan uchlarni birlashtiruvchi kesma *ko‘pyoqning diagonali* deb ataladi (2-rasm).





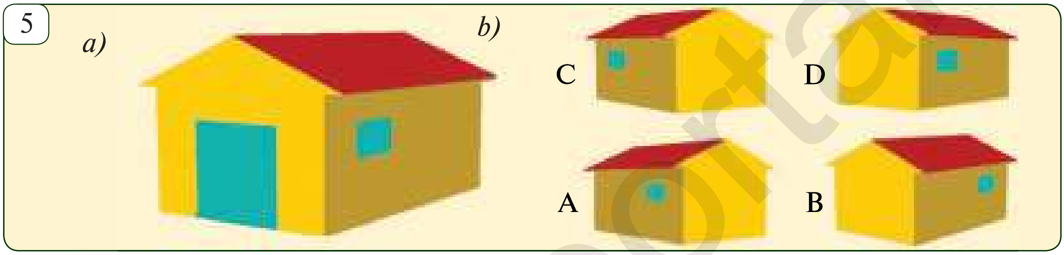


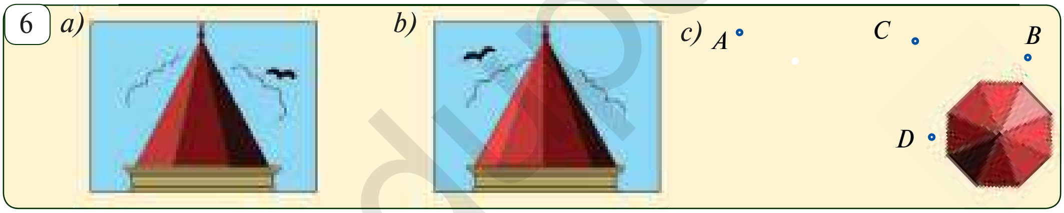
Prizma deb ikki yog‘i teng ko‘pburchakdan, qolgan yoqlari esa parallelogrammlardan iborat ko‘pyoqqa aytiladi (3- rasm). Teng yoqlar prizmaning *asoslari*, parallelogrammlar esa uning *yon yoqlari* deb ataladi. Asosining tomonlari soniga qarab prizmalar *uchburchakli, to'rtburchakli va hokazo n-burchakli prizmalar* deb yuritiladi.

Piramida deb bir yog'i ko'pburchakdan, qolgan yoqlari esa bitta uchga ega uchburchaklardan iborat ko'pyoqqa aytiladi. Ko'pburchak piramidaning *asosi*, uchburchaklar esa uning *yon yoqlari* deb ataladi. 4- rasmda *TABCDE* beshburchakli piramida tasvirlangan. *ABCDE* beshburchak piramidaning asosi, *ATB, BTC, CTD, DTE* va *ETA* uchburchaklar - uning yon yoqlari, T - esa uning uchi.

4.1 5.a- rasmda garaj tasvirlangan. 5.b- rasmda esa uning turli joydan ko'rinishlari berilgan. Ulardan faqat bittasi yuqoridagi garajga tegishli. Bu ko'rinish qaysi?

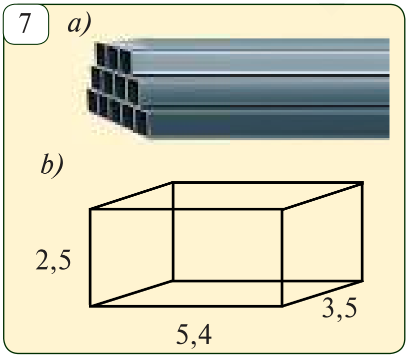
4.2. 6.a- va 6.b- rasmda binoning yon tomonidan qaraganda ko'ringan tasvirlar keltirilgan. 6.c- rasmda esa bino tepasidan ko'rinishi va unga qaralgan to'rtta nuqtalar o'rni belgilangan. Qaysi nuqtadan binoga qaraganda 1) 6.a- rasmdagi; 2) 6.b-rasmdagi tasvirlarni ko'rish mumkin?



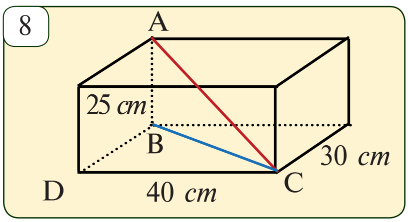


4.3. 12 dona 6 metrlik quvurlar bor (7.a-rasm). Ulardan eni 3,5 *m*, bo'yi 5,4 *m* va balandligi 2,5 *m* bo'lgan to'g'ri burchakli parallelepiped shaklidagi garaj karkasini tayyorlash kerak (7.b-rasm). Quvurlar kerakli uzunlikdagi bo'laklarga kesilib, so'ng payvandlanadi.

Eng tejamli variantda kesishda bu karkas uchun nechta quvur sarflanadi? Bunday kesishda qancha guvur chiqitga ketadi?



4.4. Ba'zi havo yo'llari kompaniyalari samolyotlariga olib chiqilayotgan yo'lovchilar jomadoni diagonalining uzunligi 56 *cm* dan katta bo'lmasligi kerak. 8-rasmda tasvirlangan to'g'ri burchakli parallelepiped shaklidagi, o'lchamlari 40 *cm* x 30 *cm* x 25 *cm* bo'lgan jomadonni samolyotga olib chiqish mumkinmi?



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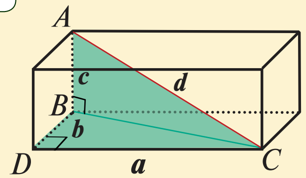

$$d^2 = a^2 + b^2 + c^2$$

Yechish: Avval jomadon asosidagi BC kesmaning uzunligini topamiz. Pifagor teoremasiga koʻra: $BC^2 = 40^2 + 30^2$.
ABC uchburchak toʻgʻri burchakli uchburchak. Yana Pifagor teoremasidan foydalanib, jomadonning diagonalini topamiz:
$AC^2 = AB^2 + BC^2$,
$AC^2 = 25^2 + 40^2 + 30^2 = 3125$. $AC = 55{,}9$ *cm*.

Javob: Mumkin, chunki $AC < 56$ *cm*.

Yuqoridagi masalaning yechimidan umumiy holda quyidagi ajoyib xossa kelib chiqadi. Uni Pifagor teoremasining fazodagi analogi (oʻxshashi) deb ham atashadi. Bu xossani mustaqil isbotlashga urinib koʻring (9 - rasm).

Teorema. Toʻgʻri burchakli parallelepiped diagonalining kvadrati uning uchta oʻlchamlari (boʻyi, eni va balandligi) kvadratlarining yigʻindisiga teng.

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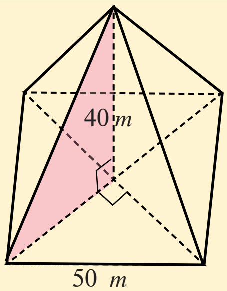

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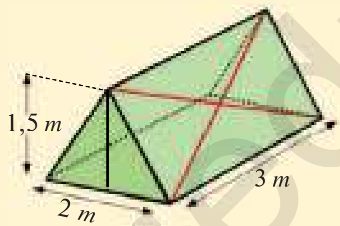

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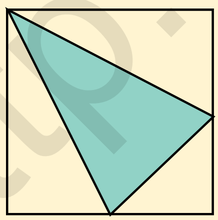

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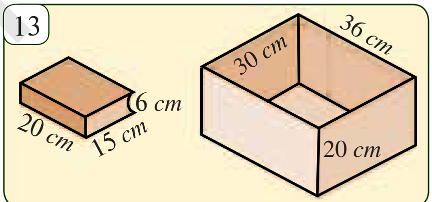


4.5. 10- rasmda tasvirlangan toʻgʻri piramidaning balandligi 40 *m* ga teng, asosi esa tomoni 50 *m* boʻlgan kvadratdan iborat. Piramidaning yon qirrasini toping.

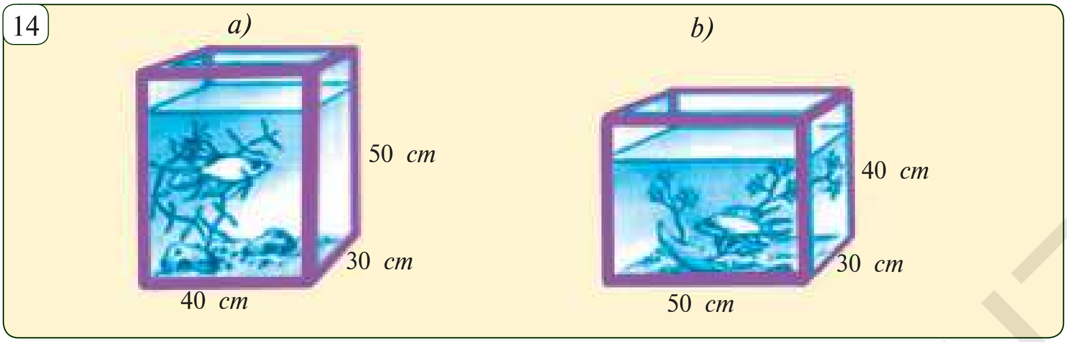
4.6. 11- rasmda tasvirlangan prizma shaklidagi chodirni tikish uchun qancha material kerak boʻladi?

4.7. Tomoni 8 *cm* ga teng boʻlgan kvadrat shaklidagi varaqni 12-rasmda koʻrsatilgandek qilib buklab piramida hosil qilindi. Piramidaning hajmini toping.

4.8. Toʻgʻri burchakli parallelepiped shaklidagi idishni hech qanday oʻlchov asboblaridan foydalanmasdan, hech qanday hisoblashlarni bajarmasdan qanday qilib yarmigacha suv bilan toʻldirish mumkin? Agar idishning boʻyi 4 *cm*, eni esa balandligidan 0,5 *cm* uzun, balandligi esa boʻyining 37,7 %ni tashkil qilsa, idishdagi suv hajmini hisoblang.

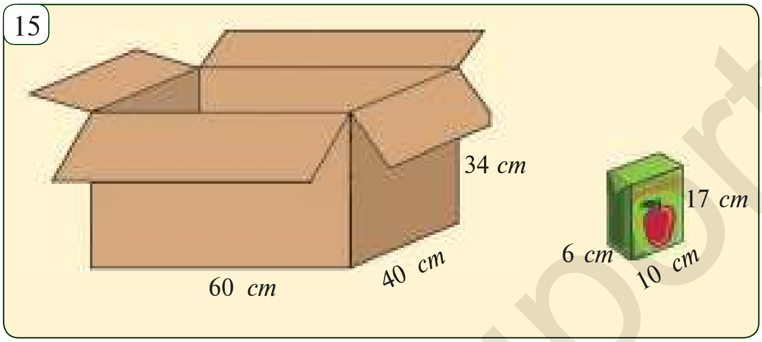
4.9. Bir xil oʻlchamdagi kitoblarni sandiqqa solish kerak (13-rasm). Bu sandiqqa nechta kitob sigʻadi?

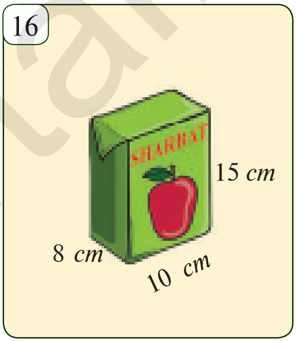
4.10. Ikkita akvariumga yuqori chetidan 10 *cm* past qilib suv quyildi (*14-rasm*). Qaysi akvariumda suv koʻp?



4.11. Qutiga necha paket meva sharbati sig'adi (*15-rasm*)?

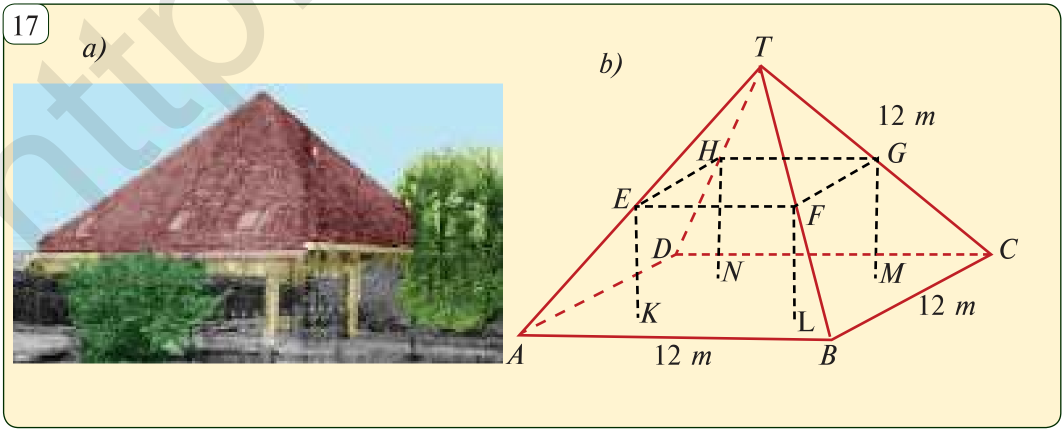
4.12. 1 litrli meva sharbati paketi to'g'ri to'rtburchakli parallelepiped shaklida (*16-rasm*). Bitta qadoq uchun qancha material kerak bo'ladi?



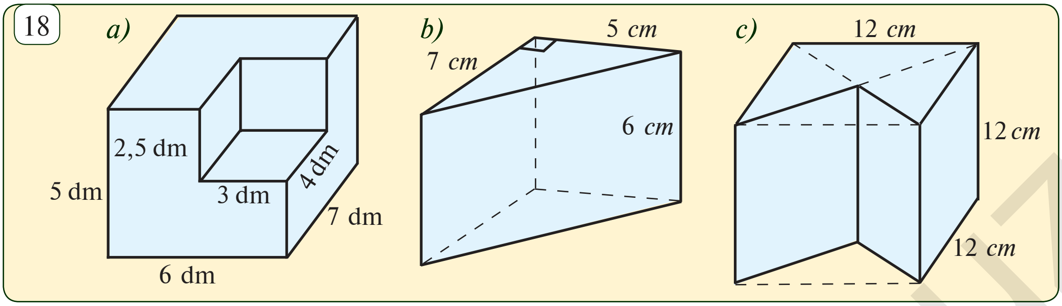


4.13. 17.a-rasmda tasvirlangan uyning tomi piramida shaklida. Quyida o'quvchilar tomonidan bu uyning chizmasi (matematik modeli) chizilgan (17.b-rasm) va ba'zi kesmalarning uzunligi ko'rsatilgan. Chizmaga ko'ra tom asosi *ABCD* kvadrat shaklida. Tom qirralari *EFGHKLMN* to'g'ri burchakli parallelepiped shaklidagi beton blokka tiralgan: *E* - *AT* qirraning, *F* - *BT* qirraning, *G* - *CT* qirraning va *H* - *DT* qirraning o'rtasi. Piramidaning hamma qirralari uzunligi 12 *m*.

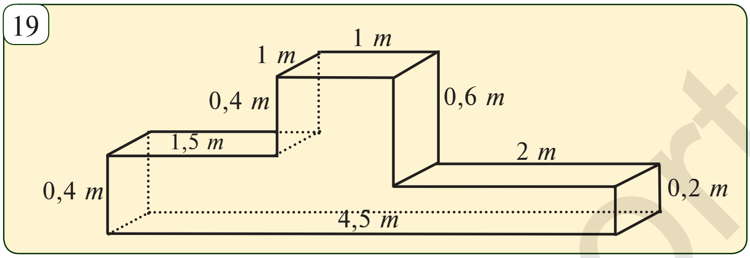
1. Tom asosi *ABCD* kvadratning yuzini toping.
2. Beton blokning tomoni - *EF* kesma uzunligini toping.

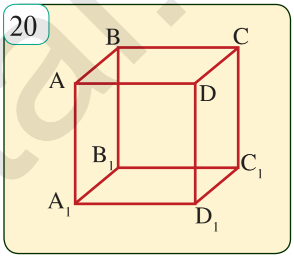


4.14*. 18-rasmda tasvirlangan yog‘och bo‘laklarining hajmini hisoblang.

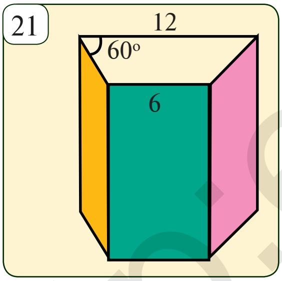


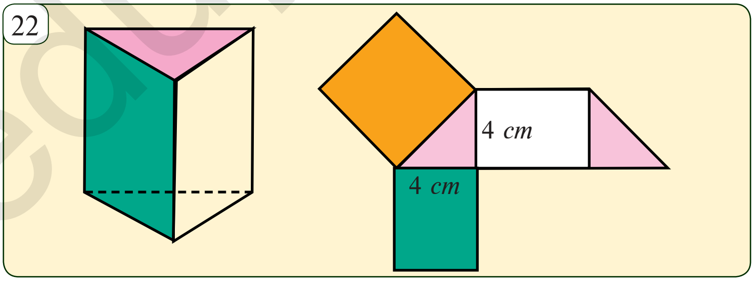
4.15. 19-rasmda sport arenasidagi g‘oliblik shohsupasi tasvirlangan. Berilganlardan foydalanib, uning hajmini toping (barcha ikki yoqli burchaklar to‘g‘ri.)

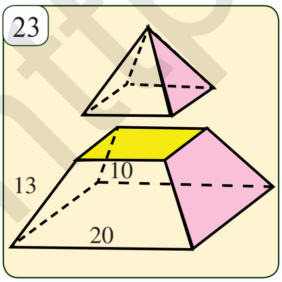




4.16. 20-rasmda to‘g‘ri burchakli parallelepipedning AA_1D_1D yog‘ining perimetri 20 *cm*, $ABCD$ yog‘i - perimetri 16 *cm* bo‘lgan kvadratdan iborat. a) $ABCC_1D_1A_1$ siniq chiziqning uzunligini; b) DD_1C_1C yoqning perimetri va yuzini; c) parallelepipedning hajmini toping.





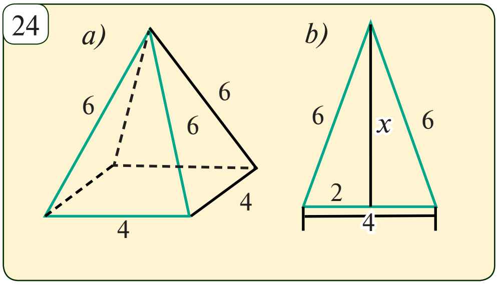


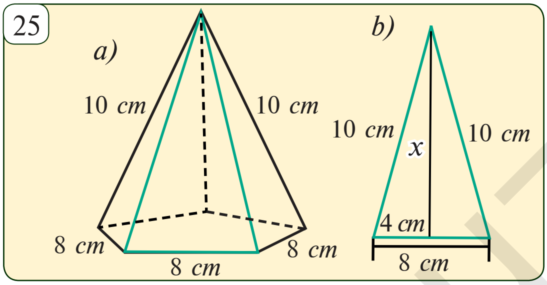
4.17. 21- rasmda tasvirlangan to‘g‘ri prizmaning asosi teng yonli trapetsiyadan iborat. Trapetsiyaning asoslari 12 *cm* va 6 *cm*, asosidagi o‘tkir burchaklaridan biri 60° ga teng. Agar prizma katta yog‘i kvadratdan iborat bo‘lsa, uning to‘la sirtining yuzini toping.

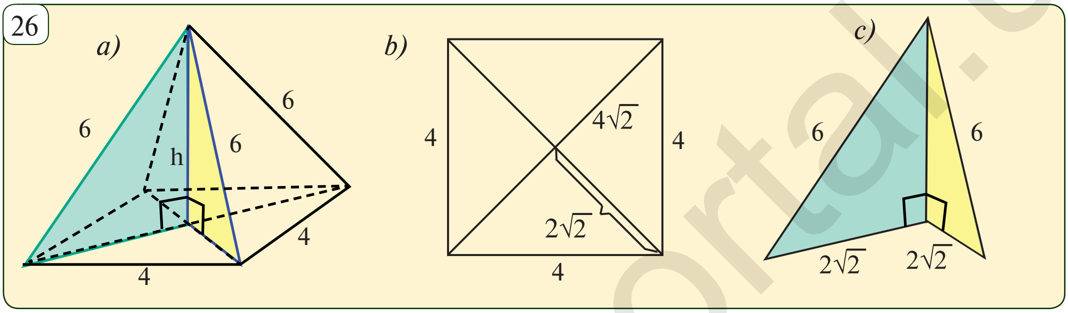
4.18. 22- rasmda prizma va uning yoyilmasi tasvirlangan. Agar prizma katta yog‘i kvadratdan iborat bo‘lsa, uning to‘la sirtining yuzini toping.

4.19. 23-rasmdagi muntazam to‘rtburchakli piramida asosiga parallel bo‘lgan tekislik bilan kesilganda, kesik piramida hosil bo‘ldi. Kesik piramida asoslarining tomoni 20 *cm* va 10 *cm*, yon qirrasi 13 *cm* bo‘lsa, uning to‘la sirtini toping.

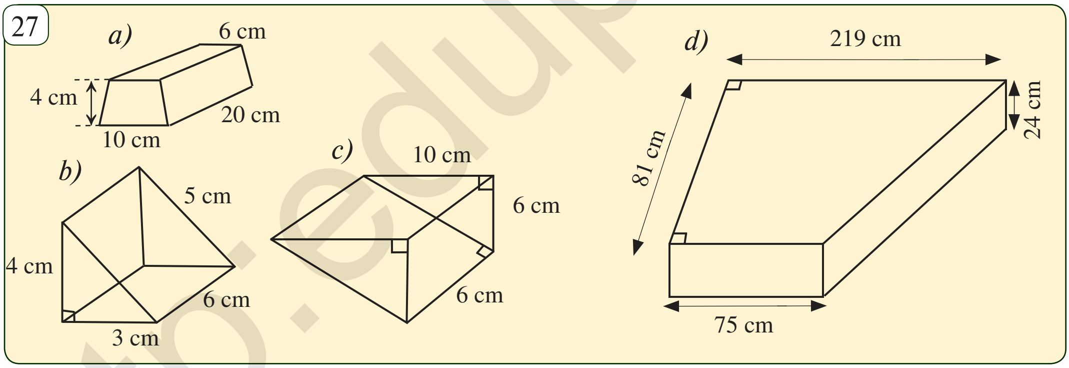
4.20. 24-26- rasmlarda berilgan ma’lumotlar va yordamchi chizmalar asosida noma’lum kattaliklarni toping.







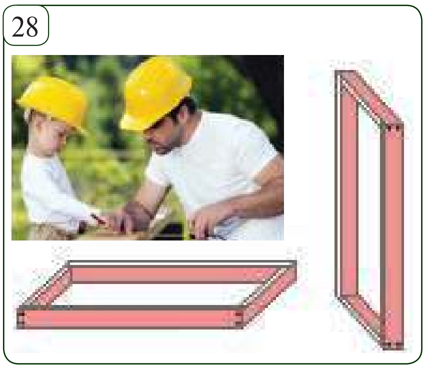
4.21. 27-rasmda berilgan ma’lumotlar asosida ko‘pyoqlarning to‘la sirti va hajmini toping.



Geometriya va duradgorlik

Uzunligi 2 *m* 20 *cm*, eni 12 *cm* va qalinligi 2 *cm* bo‘lgan reykalardan ota va bola eni 1 *m* bo‘yi 1 *m* 80 *cm* bo‘lgan rom yasamoqchi.

1. Bu romni yasash rejasini tuzing.
2. Yasalgan romning to‘g‘ri to‘rtburchak shaklida ekanligini a) burchakli chizg‘ich; b) ruletka yordamida qanday tekshirish mumkin.
3. 4 ta romni yasash uchun necha dona reyka talab qilinadi? (28- rasm).

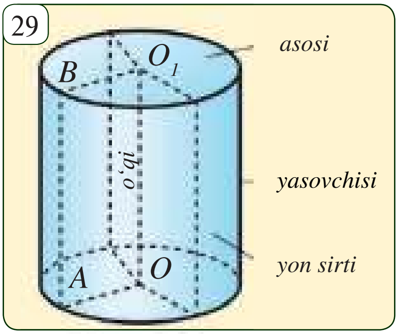


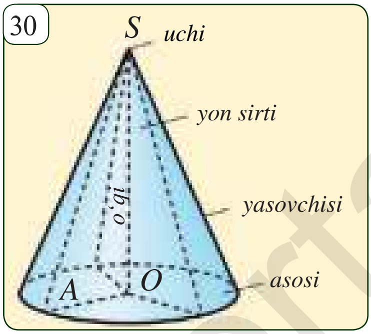
Fazoviy shakllarning yana muhim sinflaridan biri - bu aylanish jismlaridir. Ularga silindr, konus va shar kiradi.

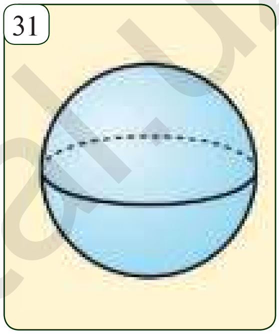
To‘g‘ri to‘rtburchakni bir tomoni atrofida aylantirishdan hosil bo‘lgan jismga *silindr* deb aytiladi 29-rasmda silindrning elementlari: asoslari, yasovchisi, o'qi, va yon sirti tasvirlangan.

To‘g‘ri burchakli uchburchakni bir kateti atrofida aylantirishdan hosil bo‘lgan jismga *konus* deb aytiladi 30-rasmda konusning uchi, yon sirti, yasovchisi va asosi tasvirlangan.

Doiraning o‘z diametri atrofida aylanishidan hosil bo‘lgan jismga *shar* deb

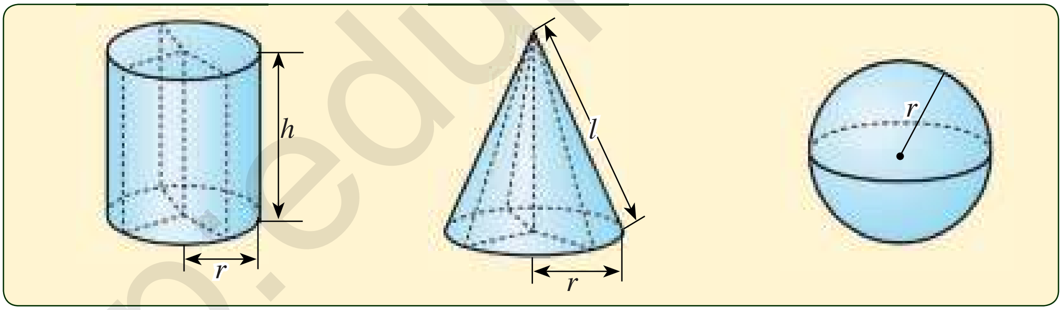






aytiladi (31- rasm). Bu aylantirishda aylana hosil qilgan sirt *sfera* deb ataladi. Ravshanki, sharning sirti sferadan iborat bo‘ladi. Sfera markazidan uning ixtiyoriy nuqtasigacha bo‘lgan masofa uning radiusini aniqlaydi.

Aylanish jismlarning yon va to‘la sirtining yuzi formulalari:



Silindr

$S_{yon} = 2\,\pi rh$

$S_{to‘la} = 2\,S_{asos} + S_{yon} =$
$= 2\,\pi r^2 + 2\,\pi rh$

Konus

$S_{yon} = \pi rl$

$S_{to‘la} = S_{asos} + S_{yon} =$
$= \pi r^2 + \pi rl$

Shar

$S = 4\,\pi r^2$

1- masala

$h = 5$ *cm*, $r = 6$ *cm* bo‘lsa,

$S_{yon} = 2 \cdot \pi \cdot r \cdot h \approx$

$\approx 2 \cdot 3,14 \cdot 5 \cdot 6 = 565(cm^2).$

2- masala

$r = 5$ *cm*, $l = 12$ *cm* bo‘lsa,

$S_{to'la} = \pi r^2 + \pi rl \approx$

$\approx 3,14 \cdot 5^2 + 3,14 \cdot 5 \cdot 12 =$

$= 267(cm^2).$

3- masala

$r = 8$ *cm* bo‘lsa,

$S = 4 \cdot \pi r^2 \approx$

$\approx 4 \cdot 3,14 \cdot 8^2 =$

$= 803,84(cm^2).$

Quyida planimetriya yordamida yechiladigan aylanish jismlarga doir masalalarni qarab chiqamiz.

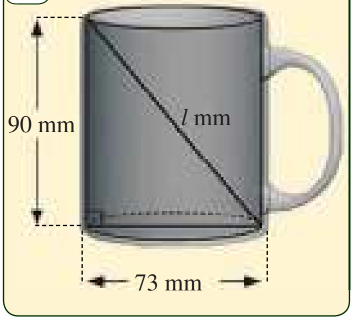
4.22. Shar markazidan uning sirtida yotgan 4 ta nuqtagacha bo‘lgan masofalar yig‘indisi 24 *cm* ga teng. Shar diametrini toping.

4.23. Ashrafning finjoni (qahva ichadigan idishi) balandligi 90 *mm*, asosining diametri 73 *mm* ga teng (32-rasm). Qahvaga solingan shakar yoki sutni aralashrirish vaqtida Ashrafning qo‘li kuymasligi uchun qoshiqchaning uzunligi kamida qancha bo‘lishi kerak?

Yechish: Qoshiqchaning uzunligini 32- rasmdagidek l deb olsak, unda Pifagor teoremasiga ko‘ra: $l^2 = 73^2 + 90^2 = 13429$ ga ega bo‘lamiz. Bundan $l = 115{,}9$ *mm*.

Javob: Qoshiqchaning uzunligi 116 *mm* dan kam bo‘lmasligi lozim.

32



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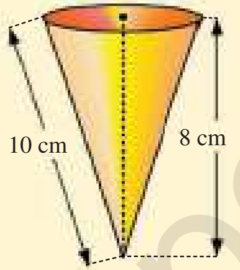

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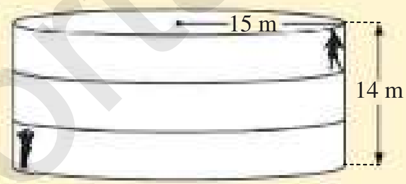


4.24. 33-rasmda berilganlardan foydalanib, konus shaklidagi muzqaymoq asosining radiusini toping. Uning sig‘imini toping.

4.25. 34-rasmda tasvirlangan London shahridagi Shekspir Globus teatri silindr shaklida. Rasmda berilganlardan foydalanib, teatrning pastki burchagidagi aktyor ovozi yuqorida turgan tomoshabinga yetib borishi uchun qancha masofani bosib o‘tishini aniqlang?

4.26. 35-rasmda tasvirlangan silindr shaklidagi idishning balandligi 12 *cm*, kengligi esa 8 *cm*. Tepa asosi qoq o‘rtasida teshik bor. Bu idishdan ichimlik ichish uchun mo‘ljallangan naycha uzunligi qancha bo‘lishi kerak? Naychaning ko‘rinib turgan qismining uzunligi 2 *cm*.

35

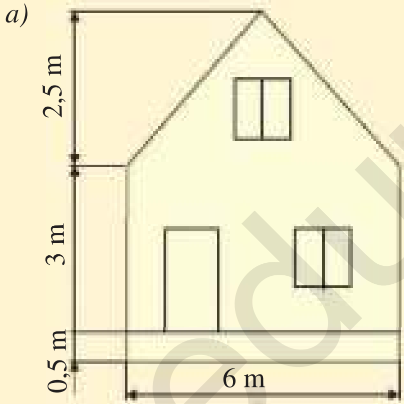
4.27. Misdan ishlangan balandligi 30 *cm* bo‘lgan konus eritilib, undan silindr yasaldi. Agar konus va silindr asoslari teng aylanalardan iborat bo‘lsa, hosil bo‘lgan silindr balandligini toping.

5 LOYIHA ISHINI BAJARISH BO‘YICHA KO‘RSATMALAR

Loyiha ishi mavzusi ustida o‘quvchilar alohida-alohida yoki 3-4 kishilik guruh bo‘lib ishlashlari mumkin. Loyiha ishi o‘quv yili oxirida o‘tkaziladigan himoya (kichik konferensiya) bilan tugaydi. Loyiha ishi ustida ish jarayoni quyidagi o‘quv faoliyatlarni o‘z ichiga olishi mumkin: izlanish faoliyatlarini rejalashtirish, vazifalarni o‘zaro taqsimlab olish, o‘quv maqsadlarini qo‘yish, kerakli ma’lumotlarni izlab topish, mavzuga doir muammoli vaziyat yechimlarini qidirish, ulardan eng maqbulini tanlash va uni asoslash, zarur hollarda so‘rovlar yoki tajribalar o‘tkazish, loyiha ishi natijalari bo‘yicha hisobot tayyorlash, o‘z faoliyatlarini tahlil qilish va baholash, loyiha ishi himoyasi uchun taqdimot tayyorlash va uni himoya qilish. O‘quvchilar loyiha ishi bo‘yicha izlanishlarini yil davomida odatda darsdan tashqari mustaqil mashg‘ulotlarda olib borishadi.

Loyiha ishi mavzulari amaliy, nazariy va tadqiqot xarakterida bo‘lishi mumkin. Amaliy ishda geometriyadan o‘zlashtirilgan bilim va ko‘nikmalar hayotiy vaziyatlardagi muammolarni (keyslarni) yechishga qo‘llaniladi. Nazariy loyiha ishlarida esa geometriyaning biror mavzusi chuqurroq o‘rganiladi. Tadqiqot

36



ishlarida esa biror nostandart geometrik masala yoki hayotiy muammoni yechish ustida kichik ilmiy izlanish olib boriladi.

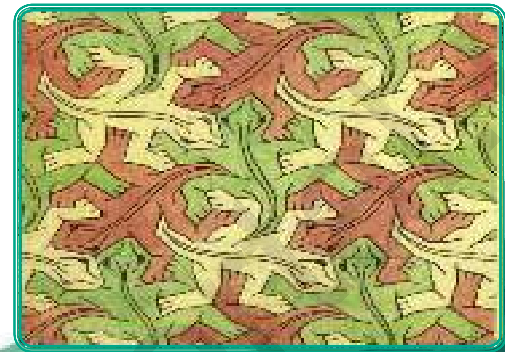
Amaliy loyiha ishi namunasi

Loyiha topshirig‘i. 36-rasmda tasvirlangan dala hovlidagi uy devorlarini bo‘yash kerak. Uyni qurish rejasi (topshiriqqa ilova qilinadi) asosida bu ishni bajarish uchun eng tejamli (arzon) loyihani ishlab chiqing.

Loyiha ishini bajarish jarayonida o‘quvchilar uy rejasini mustaqil o‘rganib chiqadilar. Vazifalarni aniqlab, reja tuzishadi va ishlarni o‘zaro taqsimlab olishadi. Dastlab, bo‘yaladigan yuzani aniqlab olishadi. Bo‘yash ishun qancha bo‘yoq kerakligini so‘rab surishtirishadi. Bir necha bo‘yoq turlari bo‘yicha hisob-kitob qiladilar. Qaysi bo‘yoq ishlatilsa, maqsadga muvofiq bo‘lishini aniqlashadi va asoslashadi. Tanlangan bo‘yoq bo‘yicha barcha hisob-kitob ishlarini bajarishadi va loyiha ishini hamda u bo‘yicha taqdimot tayyorlashadi. *Izoh: Rasmda uy rejasining hammasi keltirilmagan.*

I BOB

GEOMETRIK ALMASHTIRISHLAR VA O‘XSHASHLIK



Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilimlar:

√ *o‘xshash shakllarning ta‘rifini va belgilanishini bilish;*

√ *uchburchaklarning o‘xshashlik alomatlarini bilish;*

√ *gomotetiya tushunchasini bilish.*

Amaliy ko‘nikmalar:

√ *ikkita o‘xshash uchburchaklardan mos elementlarni topa olish;*

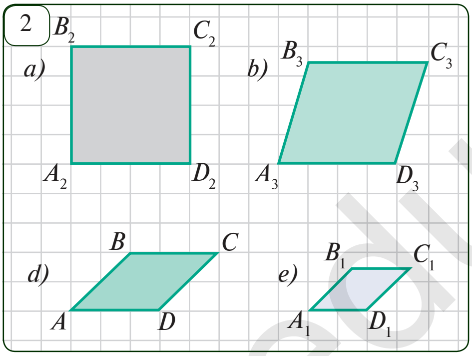
√ *uchburchaklarning o‘xshashlik alomatlarini isbotlashga va hisoblashga oid masalalarni yechishda qo‘llay olish;*

√ *gomotetiyadan foydalanib, o‘xshash ko‘pburchaklarni yasay olish.*

6 KO‘PBURCHAKLARNING O‘XSHASHLIGI

Kundalik turmushda teng shakllardan tashqari shakli (ko‘rinishi) bir xil, lekin o‘lchamlari turlicha bo‘lgan shakllarga ko‘p duch kelamiz. Tarix va geografiya fanlarida turli masshtabda ishlangan xaritalardan foydalangansiz. Sinf doskasiga ilinadigan va darsliklarda tasvirlangan respublikamizning xaritalari turli o‘lchamda, lekin ular bir xil shaklda (ko‘rinishda). Shuningdek, bitta fototasmadan turli o‘lchamdagi fotosuratlar tayyorlanadi. Bu suratlarning o‘lchamlari turlicha bo‘lsa-da, bir xil ko‘rinishda, ya’ni ular bir-biriga o‘xshaydi *(1-rasm)*.

Mashq. 2-rasmda to‘rtta romb tasvirlangan. Ulardan faqat *d*) va *e*) romblar bir xil ko‘rinishga ega. Bu romblar nimasi bilan boshqa romblardan ajralib turibdi?



Keling, buni birgalikda aniqlaylik.

1. Rasmdan ko‘rinib turibdiki, $AD=3$, $A_1D_1=2$. Rombning tomonlari teng bo‘lgani uchun,

$$\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CD}{C_1D_1}=\frac{AD}{A_1D_1}=\frac{3}{2}=1{,}5$$

tenglikni hosil qilamiz. Bu holatda romblarning mos tomonlari proporsional deb yuritiladi.

2. $ABCD$ va $A_1B_1C_1D_1$ romblarda $\angle A=\angle A_1=45°$, $\angle B=\angle B_1=135°$, $\angle C=\angle C_1=45°$, $\angle D=\angle D_1=135°$. Bu holatda romblarning mos burchaklari o‘zaro teng deb yuritiladi.

Shunday qilib, bu romblarning bir-biriga o‘xshashligining sababi — mos tomonlarining proporsionalligi va mos burchaklarining tengligi, deb ayta olamiz. Ixtiyoriy ko‘pburchaklarning o‘xshashligi tushunchasi ham shunga o‘xshash kiritiladi.

Ikkita ko‘pburchak (beshburchak) $ABCDE$ va $A_1B_1C_1D_1E_1$ tarzda belgilangan bo‘lib, mos ravishda $\angle A=\angle A_1$, $\angle B=\angle B_1$, $\angle C=\angle C_1$, $\angle D=\angle D_1$, $\angle E=\angle E_1$ ya’ni mos burchaklari o‘zaro teng bo‘lsin. Unda AB va A_1B_1, BC va B_1C_1, CD va C_1D_1, DE va D_1E_1, EA va E_1A_1 tomonlar ko‘pburchakning *mos tomonlari* deb yuritiladi.

Ta’rif. Ikki ko‘pburchakning burchaklari mos ravishda o‘zaro teng, barcha mos tomonlari esa o‘zaro proporsional bo‘lsa, bunday ko‘pburchaklar *o‘xshash ko‘pburchaklar* deb ataladi *(3-rasm)*.

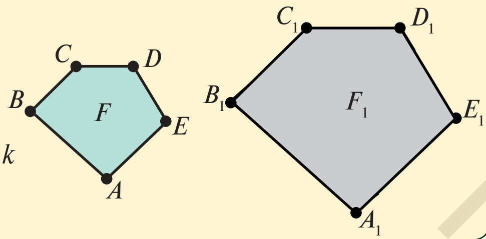
Ko‘pburchaklar o‘xshashligi ∾ belgisi bilan ko‘rsatiladi.

3

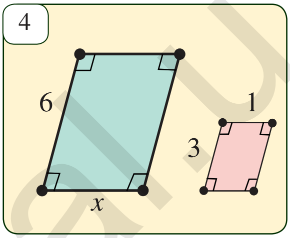
Mos burchaklar teng

$$F \backsim F_1 \quad \begin{cases} \angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1, \\ \angle D=\angle D_1,\ \angle E=\angle E_1 \\ \dfrac{A_1B_1}{AB}=\dfrac{B_1C_1}{BC}=\dfrac{C_1D_1}{CD}=\dfrac{D_1E_1}{DE}=\dfrac{E_1A_1}{EA}=k \end{cases}$$

Mos tomonlar proporsional



O‘xshash ko‘pburchaklar mos tomonlari nisbatiga teng bo‘lgan k songa bu ko‘pburchaklarning *o‘xshashlik koeffitsiyenti* deyiladi.

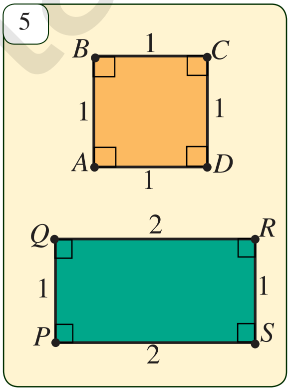


1- Masala. 4-rasmdagi ko‘pburchaklarning o‘xshashligi ma’lum bo‘lsa, noma’lum uzunlikni toping.

Yechish: Bu ko‘pburchaklar o‘xshashligidan ular mos tomonlarining proporsional ekanligi kelib chiqadi.

Demak, $\dfrac{x}{6}=\dfrac{1}{3}$. Bundan $x = 6 : 3 = 2$ ekanligini topamiz. *Javob.* $x = 2$.

2-masala. 5-rasmda tasvirlangan to‘rtburchaklar o‘xshashmi? Nega?



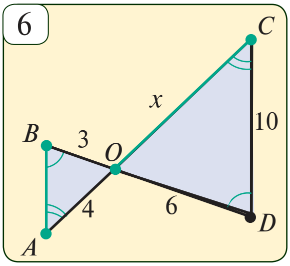
Yechish: Yo‘q. Chunki, ularning mos burchaklari teng (90°) bo‘lsada, mos tomonlari proporsional emas:

$$\frac{AB}{PQ}=1\neq\frac{BC}{QR}=\frac{1}{2}$$

Masala va topshiriqlar

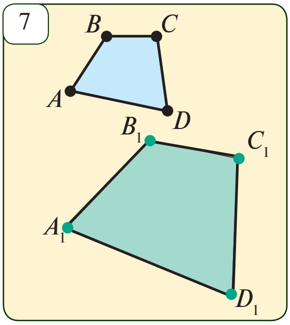
6.1. O‘xshashlik koeffitsiyenti nima va u qanday aniqlanadi?

6.2. Agar ABC va DEF uchburchaklarda $\angle A=105°$, $\angle B=35°$, $\angle E=105°$, $\angle F=40°$, $AC=4{,}4$ *cm*, $AB=5{,}2$ *cm*, $BC=7{,}6$ *cm*, $DE=15{,}6$ *cm*, $DF=22{,}8$ *cm*, $EF=13{,}2$ *cm* bo‘lsa, ular o‘xshash bo‘ladimi?



6.3. 2-rasmda tasvirlangan a) va b) romblar nima sababdan o‘xshash emas? b) va d) romblar-chi?

6.4. 6-rasmdagi ABO va CDO uchburchaklar o‘xshash bo‘lsa, AB, OC kesmalar uzunligini va o‘xshashlik koeffitsiyentini toping.



6.5. 7-rasmda $ABCD \backsim A_1B_1C_1D_1$. $AB = 24$, $BC = 18$, $CD = 30$, $AD = 54$, $B_1C_1 = 54$. A_1B_1, D_1A_1 va C_1D_1 kesmalarni toping.

6.6*. ABC uchburchak AB va AC tomonlarining o‘rtalari mos ravishda P va Q bo‘lsin. $\Delta ABC \backsim \Delta APQ$ ekanligini isbotlang.

7 O'XSHASH UCHBURCHAKLAR VA ULARNING XOSSALARI

Eng sodda ko'pburchak bo'lmish uchburchaklar o'xshashligini o'rganamiz.

Teorema. ***Ikkita o'xshash uchburchak perimetrlarining nisbati o'xshashlik koeffitsiyentiga teng.***

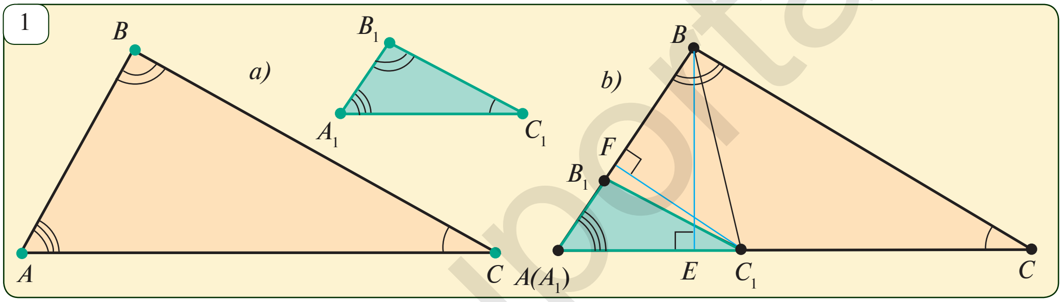
Bu teoremani mustaqil isbotlang.

Teorema. ***Ikkita o'xshash uchburchak yuzlari nisbati o'xshashlik koeffitsiyentining kvadratiga teng.***

$\Delta ABC \backsim \Delta A_1B_1C_1$ *(1-a rasm),*
k — o'xshashlik koeffitsiyenti
$\Rightarrow$
$S_{ABC} : S_{A_1B_1C_1} = k^2$

Isbot. Teorema shartiga ko'ra, $\Delta ABC \backsim \Delta A_1B_1C_1$. Demak, ko'pburchaklar o'xshashligi ta'rifiga ko'ra, $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = k$.

1



$\angle A = \angle A_1$ ekanligidan foydalanib, ularni 1-b rasmdagidek ustma-ust qo'yamiz va tegishli yasash hamda belgilashlarni amalga oshiramiz.

Quyidagi uchburchaklar yuzlarini topamiz va ularning nisbatlarini qaraymiz:

$$\left.\begin{aligned} S_{ABC} &= \frac{AC \cdot BE}{2}; \\ S_{ABC_1} &= \frac{A_1C_1 \cdot BE}{2}; \end{aligned}\right] \Rightarrow \frac{S_{ABC}}{S_{ABC_1}} = \frac{AC}{A_1C_1} \quad (1),$$

$$\left.\begin{aligned} S_{A_1B_1C_1} &= \frac{A_1B_1 \cdot C_1F}{2}; \\ S_{ABC_1} &= \frac{AB \cdot C_1F}{2}; \end{aligned}\right] \Rightarrow \frac{S_{A_1B_1C_1}}{S_{ABC_1}} = \frac{A_1B_1}{AB} \quad (2).$$

(1) tenglikni hadma-had (2) tenglikka bo'lsak, teng burchakka ega bo'lgan uchburchaklar yuzlarining nisbati uchun (3) tenglikni hosil qilamiz.

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AB \cdot AC}{A_1B_1 \cdot A_1C_1} \quad (3)$$

Bu yerda shartga ko'ra, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} = k$ ekanligini hisobga olsak,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AB \cdot AC}{A_1B_1 \cdot A_1C_1} = \frac{AB}{A_1B_1} \cdot \frac{AC}{A_1C_1} = k \cdot k = k^2$$

tenglik kelib chiqadi. *Teorema isbotlandi.*

1-masala. O‘xshash uchburchaklarning mos tomonlari nisbati shu tomonlarga tushirilgan balandliklar nisbatiga tengligini isbotlang *(2-rasm)*.

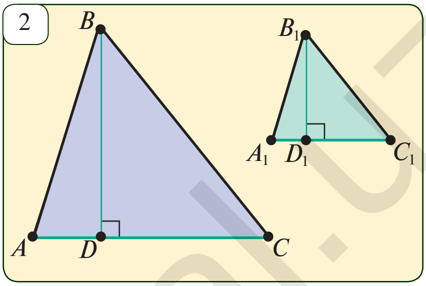
$\Delta ABC \backsim \Delta A_1B_1C_1,\ BD,\ B_1D_1$ — *balandliklar* $\Rightarrow$ $\dfrac{AC}{A_1C_1} = \dfrac{BD}{B_1D_1}$

Yechish. Berilgan uchburchaklarning o‘xshashlik koeffitsiyenti k bo‘lsin. Unda, $AC : A_1C_1 = k$; $\quad S_{ABC} : S_{A_1B_1C_1} = k^2 \quad (1)$ bo‘ladi. Ikkinchi tomondan,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{\frac{1}{2}AC \cdot BD}{\frac{1}{2}A_1C_1 \cdot B_1D_1} = \frac{AC}{A_1C_1} \cdot \frac{BD}{B_1D_1} = k \cdot \frac{BD}{B_1D_1}. \quad (2)$$

(1) va (2) tengliklardan $k \cdot \dfrac{BD}{B_1D_1} = k^2$ yoki $\dfrac{BD}{B_1D_1} = k$.

Shunday qilib, $\dfrac{BD}{B_1D_1}$ ham, $\dfrac{AC}{A_1C_1}$ nisbat ham k ga teng, ya’ni $\dfrac{AC}{A_1C_1} = \dfrac{BD}{B_1D_1}$.



Masala va topshiriqlar

7.1. O‘xshash uchburchaklar yuzlari nisbati haqidagi teoremani ayting va isbotlang.

7.2. Ikkita o‘xshash ABC va $A_1B_1C_1$ uchburchaklar berilgan. Agar $S_{ABC} = 25\ cm^2$ va $S_{A_1B_1C_1} = 81\ cm^2$ bo‘lsa, o‘xshashlik koeffitsiyentini toping.

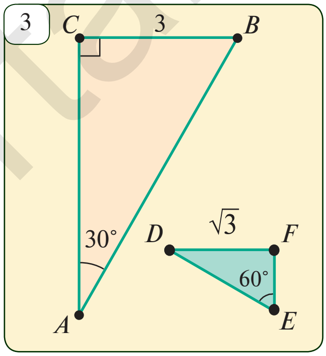
7.3. Ikkita o‘xshash uchburchak yuzlari 65 m^2 va 260 m^2. Birinchi uchburchakning bir tomoni 6 m bo‘lsa, ikkinchi uchburchakning unga mos tomonini toping.

7.4. Berilgan uchburchak tomonlari 15 cm, 25 cm va 30 cm. Agar perimetri 35 cm bo‘lgan uchburchak berilgan uchburchakka o‘xshash bo‘lsa, uning tomonlarini toping.

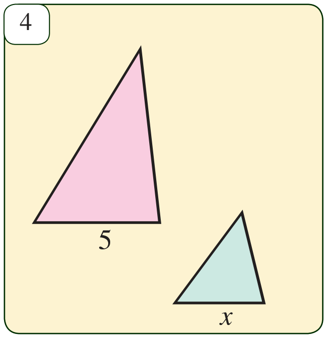
7.5. Tomonlari 12 cm, 20 cm va 13 cm bo‘lgan uchburchak berilgan. Agar kichik tomoni 9 cm bo‘lgan uchburchak berilgan uchburchakka o‘xshash bo‘lsa, uning qolgan tomonlarini toping.

7.6. $\Delta ABC \backsim \Delta A_1B_1C_1$ va bu uchburchaklarning mos tomonlari nisbati 7 : 5 ga teng. Agar ABC uchburchak yuzi $A_1B_1C_1$ uchburchak yuzidan 36 m^2 ga ortiq bo‘lsa, bu uchburchaklar yuzlarini toping.

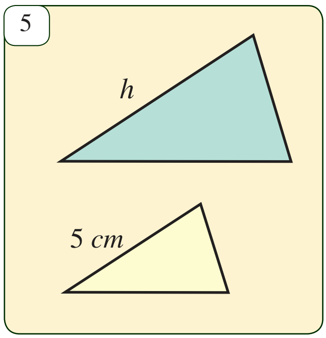
7.7. 3-rasmda berilganlardan foydalanib, uchburchaklarning o‘xshash yoki o‘xshash emasligini aniqlang.



7.8. 4-rasmdagi uchburchaklar o‘xshash va yuzlari nisbati 25 : 9 kabi bo‘lsa, noma’lum kesma uzunlini toping.



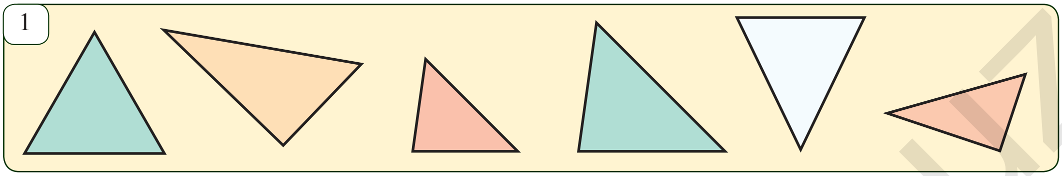
7.9. 5-rasmdagi uchburchaklar o‘xshash va $S_1 : S_2 = 49 : 25$ bo‘lsa, noma’lum tomonni toping.



UCHBURCHAKLAR O‘XSHASHLIGINING BIRINCHI ALOMATI

Faollashtiruvchi mashq

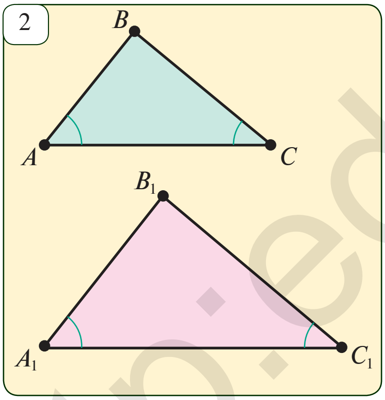
1-rasmda tasvirlangan uchburchaklar ichidan o‘xshashlarini aniqlang. Ularning o‘xshashligini qanday aniqladingiz?



Ta’rifga ko‘ra, ikkita uchburchakning o‘xshashligini aniqlash uchun ular burchaklarining tengligini va mos tomonlarining proporsional ekanligini tekshirish lozim bo‘ladi. Uchburchaklar uchun bu ish ancha osonlashar ekan. Quyida keltiriladigan teoremalar shu xususda bo‘lib, ular “uchburchaklar o‘xshashligining alomatlari” deb nomlanadi.

Teorema. (Uchburchaklar o‘xshashligining BB alomati). ***Agar bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi (2-rasm).***

$\Delta ABC,\ \Delta A_1B_1C_1,\ \angle A=\angle A_1,\ \angle C=\angle C_1 \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$



Isbot. 1. Uchburchak ichki burchaklari yig‘indisi haqidagi teoremaga ko‘ra,

$$\left.\begin{aligned}\angle B &= 180^\circ-(\angle A+\angle C),\\ \angle B_1 &= 180^\circ-(\angle A_1+\angle C_1)\end{aligned}\right\} \Rightarrow \quad \angle B=\angle B$$

Demak, ABC va $A_1B_1C_1$ uchburchaklarning burchaklari mos ravishda teng.

2. Shartga ko‘ra, $\angle A=\angle A_1$, $\angle C=\angle C_1$. Teng burchakka ega bo‘lgan uchburchaklar yuzlarining nisbati haqidagi teoremaga ko‘ra

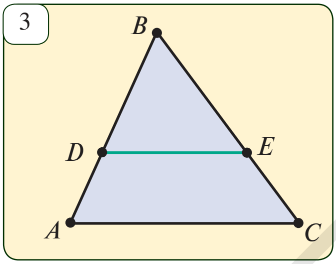
$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1} \text{ va } \frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{CA\cdot CB}{C_1A_1\cdot C_1B_1}.$$

Bu tengliklarning o‘ng qismlarini tenglab, bir xil hadlar qisqartirilsa, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ tenglik hosil bo‘ladi. Xuddi shu singari, $\angle A=\angle A_1$ va $\angle B=\angle B_1$ tengliklardan foydalanib, $\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}$ tenglikni olamiz. Shunday qilib, ABC va $A_1B_1C_1$ uchburchaklarning burchaklari teng va mos tomonlari proporsional, ya’ni bu uchburchaklar o‘xshash. *Teorema isbotlandi.*

Masala. ABC uchburchakning ikki tomonini kesib o‘tuvchi va uchinchi tomoniga parallel bo‘lgan DE to‘g‘ri chiziq uchburchakdan unga o‘xshash uchburchak ajratishini isbotlang *(3-rasm).*

Isbot. ABC va DBE uchburchaklarda $\angle B$ — umumiy, $\angle CAB = \angle EDB$ (AC va DE parallel to‘g‘ri chiziqlarni AB kesuvchi bilan kesganda hosil bo‘lgan mos burchaklar teng bo‘lgani uchun) *(3-rasm)*.

Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $ABC \backsim \Delta DBE$.

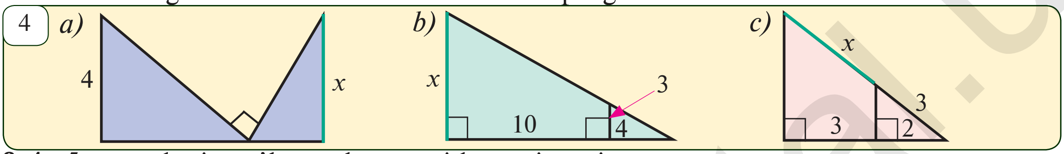


Masala va topshiriqlar

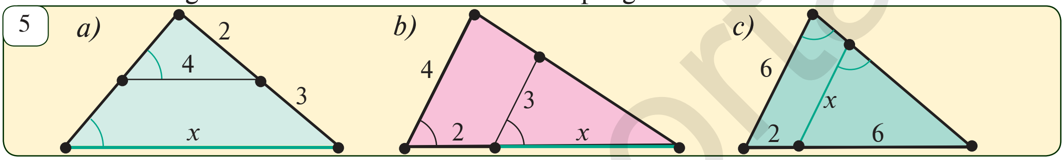
8.1. Uchburchaklar o‘xshashligining ta’rifi va BB alomatini o‘zaro solishtiring.

8.2. Uchburchaklar o‘xshashligining BB alomatini isbotlang.

8.3. Rasmdagi ma’lumotlar asosida x ni toping.



8.4. 5-rasmdagi ma’lumotlar asosida x ni toping.

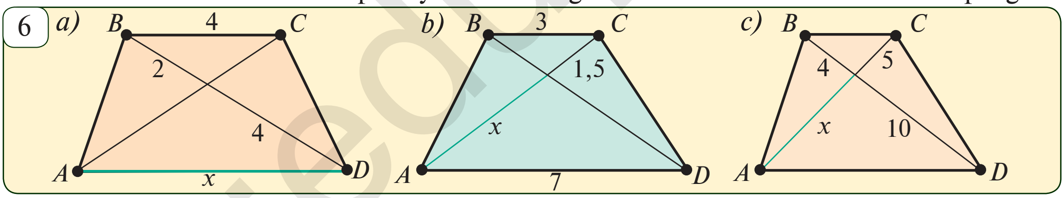


8.5. $ABCD$ parallelogrammning CD tomonida E nuqta olingan. AE va BC nurlar F nuqtada kesishadi.

a) Agar $DE = 8$ cm, $EC = 4$ cm, $BC = 7$ cm, $AE = 10$ cm bo‘lsa, EF va FC ni;

b) Agar $AB = 8$ cm, $AD = 5$ cm, $CF = 2$ cm bo‘lsa, DE va EC ni toping.

8.6. 6-rasmda $ABCD$ — trapetsiya. Rasmdagi ma’lumotlar asosida x ni toping.

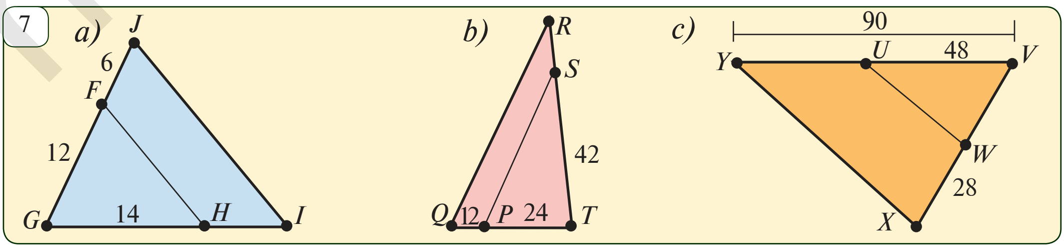


8.7*. Bittadan o‘tkir burchaklari teng bo‘lgan ikkita to‘g‘ri burchakli uchburchaklar o‘xshash ekanligini isbotlang.

8.8*. ABC uchburchakning AC tomonida D nuqta olingan. Agar $\angle ABC = \angle BDC$ bo‘lsa, ABC va BDC uchburchaklar o‘xshash ekanligini isbotlang. Shuningdek, $3AB = 4BD$ va $BC = 9$ cm bo‘lsa, AC kesmani toping.

8.9. 7-rasmda berilganlarga asosan noma’lum kesmani toping.

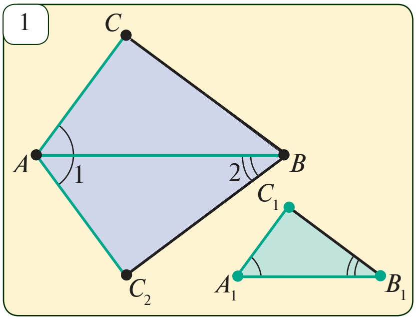
a) $IJ \parallel FH$, HI - ? b) $QR \parallel PS$, RS - ? d) $XY \parallel UW$, VX - ?



9 UCHBURCHAKLAR O‘XSHASHLIGINING IKKINCHI ALOMATI

Teorema. (Uchburchaklar o‘xshashligining TBT alomati). ***Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proporsional va bu tomonlar hosil qilgan burchaklar teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi (1-rasm).***

$\Delta ABC,\ \Delta A_1B_1C_1,\ \angle A=\angle A_1,\ \dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}$ $\Rightarrow$ $\Delta ABC \sim \Delta A_1B_1C_1$



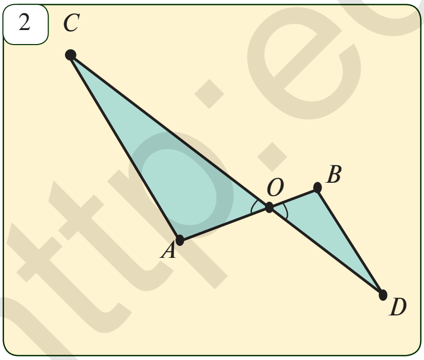
Isbot. $\angle 1=\angle A_1$, $\angle 2=\angle B_1$ bo‘ladigan qilib ABC_2 uchburchak yasaymiz *(1-rasm)*. U BB alomat bo‘yicha $A_1B_1C_1$ uchburchakka o‘xshash bo‘ladi.

$$\Delta A_1B_1C_1 \sim \Delta ABC_2: \quad \frac{AB}{A_1B_1}=\frac{AC_2}{A_1C_1}$$

Shartga ko‘ra: $\dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}$

Bu ikki tenglikdan, $AC_2=AC$ ekanligini aniqlaymiz. Unda, uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ABC=\Delta ABC_2$. Xususan, $\angle 2=\angle B$. Lekin yasashga ko‘ra, $\angle 2=\angle B_1$ edi. Demak, $\angle B=\angle B_1$. U holda, $\angle A=\angle A_1$ va $\angle B=\angle B_1$ bo‘lgani uchun, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\Delta ABC \sim \Delta A_1B_1C_1$. *Teorema isbotlandi.*

Masala. AB va CD kesmalar O nuqtada kesishadi, $AO=12$ *cm*, $BO=4$ *cm*, $CO=30$ *cm*, $DO=10$ *cm* bo‘lsa, AOC va BOD uchburchaklar yuzlari nisbatini toping.



Yechish: Shartga ko‘ra,

$$\left.\begin{array}{l}\dfrac{OA}{OB}=\dfrac{12}{4}=3\\[2mm] \dfrac{OC}{OD}=\dfrac{30}{10}=3\end{array}\right\} \Rightarrow \frac{OA}{OB}=\frac{OC}{OD}=3.$$

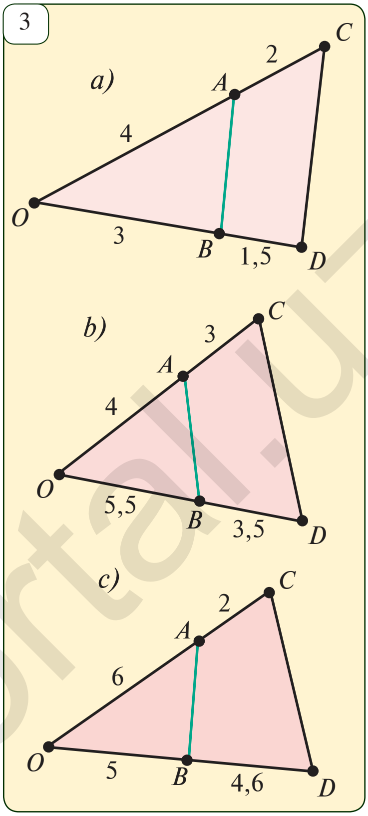
Demak, AOC uchburchakning ikki tomoni BOD uchburchakning ikki tomoniga proporsional va bu tomonlar orasidagi mos burchaklar vertikal burchaklar bo‘lgani uchun: $\angle AOC=\angle BOD$. Shuning uchun, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta AOC \sim \Delta BOD$ va o‘xshashlik koeffitsiyenti $k=\dfrac{OA}{OB}=3$. Endi o‘xshash uchburchaklar yuzlarining nisbati haqidagi teoremani qo‘llaymiz: $\dfrac{S_{AOC}}{S_{BOD}}=k^2=9$. *Javob:* 9.

Masala va topshiriqlar

9.1. Uchburchaklar o‘xshashligining ta’rifi va TBT alomatini o‘zaro solishtiring.

9.2. Uchidagi burchaklari teng bo‘lgan teng yonli uchburchaklarning o‘xshashligini a) BB; b) TBT alomatdan foydalanib isbotlang.

9.3. 3-rasmda tasvirlangan OAB va OCD uchburchaklar o‘xshash bo‘ladimi? Agar o‘xshash bo‘lsa, bu uchburchaklar perimetrining nisbatini toping.



9.4. AC va BD nurlar O nuqtada kesishadi. Agar $AO:CO=BO:DO=3$, $AB=7$ *cm* bo‘lsa, CD kesmani hamda AOB va COD uchburchaklar yuzlari nisbatini toping.

9.5. ABC va $A_1B_1C_1$ uchburchaklarda $\angle A=\angle A_1$, $AB:A_1B_1=AC:A_1C_1=4:3$.

a) Agar AB kesma A_1B_1 dan 5 *cm* ortiq bo‘lsa, AB va A_1B_1 tomonlarni toping.

b) Agar A_1B_1 kesma AB dan 6 *cm* kam bo‘lsa, AB va A_1B_1 tomonlarini toping.

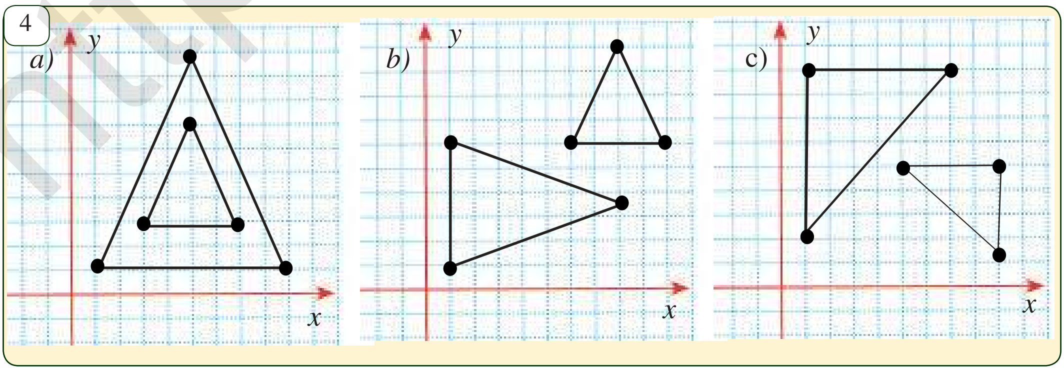
c) Agar berilgan uchburchaklarning yuzlari yig‘indisi 400 cm^2 bo‘lsa, har qaysi uchburchakning yuzini toping.

9.6. Agar bir to‘g‘ri burchakli uchburchakning katetlari ikkinchi to‘g‘ri burchakli uchburchakning mos katetlariga proporsional bo‘lsa, bu uchburchaklar o‘xshash bo‘lishini isbotlang.

9.7. Katetlari 3 *dm* va 4 *dm* bo‘lgan to‘g‘ri burchakli uchburchak bilan bir kateti 8 *dm* va gipotenuzasi 10 *dm* bo‘lgan to‘g‘ri burchakli uchburchak o‘xshash bo‘lishini isbotlang.

9.8*. AB kesma va l to‘g‘ri chiziq O nuqtada kesishadi. l to‘g‘ri chiziqqa AA_1 va BB_1 perpendikularlar tushirilgan. Agar $AA_1=2$ *cm*, $OA_1=4$ *cm* va $OB_1=3$ *cm* bo‘lsa, BB_1, OA va AB kesmalarni toping.

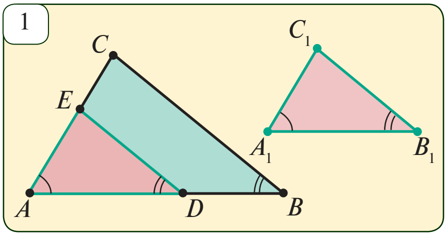
9.9*. 4-rasmda berilgan ma’lumotlar asosida uchburchaklarning o‘xshashligini asoslang.



10 UCHBURCHAKLAR O‘XSHASHLIGINING UCHINCHI ALOMATI

Teorema. (Uchburchaklar o‘xshashligining TTT alomati). ***Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda proporsional bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi.***

$$\Delta ABC, \Delta A_1B_1C_1, \ \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} \ (1\text{-rasm}) \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$$



Isbot. ABC uchburchakning AB tomonida $AD = A_1B_1$ bo‘ladigan qilib D nuqtani belgilaymiz. D nuqtadan BC tomonga parallel qilib o‘tkazilgan to‘g‘ri chiziq AC tomonni E nuqtada kessin. Unda uchburchaklar o‘xshashligining BB alomatiga ko‘ra, ΔADE va ΔABC o‘xshash bo‘ladi. U holda bu o‘xshashlik teorema shartiga ko‘ra quyidagi tengliklar juftiga ega bo‘lamiz:

$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} \text{ va } \frac{AB}{AD} = \frac{BC}{DE}, \quad (1) \qquad \frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \text{ va } \frac{AB}{AD} = \frac{AC}{AE}. \quad (2)$$

U holda $AD = A_1B_1$ ekanligini hisobga olsak, ularning birinchisidan $B_1C_1 = DE$, ikkinchisidan esa $A_1C_1 = AE$ ekanligi kelib chiqadi. Shunday qilib, uchburchaklar tengligining TTT alomatiga ko‘ra, $\Delta ADE = \Delta A_1B_1C_1$. Unda $\Delta ADE \sim \Delta ABC$.

Demak, $\Delta ABC \sim \Delta A_1B_1C_1$. *Teorema isbotlandi.*

Masala. Agar ikkita teng yonli uchburchakdan birining asosi va yon tomoni ikkinchisining asosi va yon tomoniga proporsional bo‘lsa, bu uchburchaklarning o‘xshash ekanligini isbotlang.

$$\left.\begin{array}{l} \Delta ABC, \ AB = BC, \\ \Delta A_1B_1C_1, \ A_1B_1 = B_1C_1, \end{array}\right| \ \frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$$

Isbot. Berilgan $AB = BC$, $A_1B_1 = B_1C_1$ tengliklar va $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}$ nisbatdan $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} = \frac{BC}{B_1C_1}$ tengliklarni hosil qilamiz. Demak, uchburchaklar o‘xshashligining TTT alomatiga ko‘ra, $\Delta ABC \sim \Delta A_1B_1C_1$.

Masala va topshiriqlar

10.1. Uchburchaklar o‘xshashligining TTT alomatini ayting va isbotini bayon qiling.

10.2. $AC = 14$ cm, $AB = 11$ cm, $BC = 13$ cm, $A_1C_1 = 28$ cm, $A_1B_1 = 22$ cm, $B_1C_1 = 26$ cm ekanligi ma’lum. ABC va $A_1B_1C_1$ uchburchaklar o‘xshash bo‘ladimi?

10.3. 2-rasmdagi o‘xshash uchburchaklar juftliklarini ko‘rsating.

10.4. $ABCD$ trapetsiyaning AB va CD yon tomonlari davom ettirilsa, E nuqtada kesishadi. Agar $AB = 5$ cm, $BC = 10$ cm, $CD = 6$ cm, $AD = 15$ cm bo‘lsa, AED uchburchak yuzini toping.

10.5. Trapetsiyaning asoslari 6 cm va 9 cm, balandligi 10 cm. Trapetsiyaning diagonallari kesishgan nuqtadan asoslarigacha bo‘lgan masofalarni toping.

10.6. Istalgan ikkita teng tomonli uchburchak o‘xshash bo‘lishini isbotlang.

10.7. Asosi 12 *cm*, balandligi 8 *cm* bo‘lgan teng yonli uchburchak ichiga kvadrat shunday ichki chizilganki, kvadratning ikkita uchi uchburchak asosida, qolgan ikki uchi esa yon tomonlarda yotadi. Kvadrat tomonini toping.

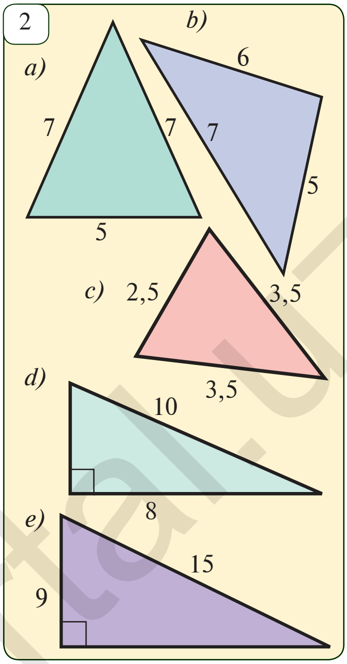
10.8*. O‘tkir burchakli ABC uchburchakning AA_1 va BB_1 balandliklari o‘tkazilgan. $\Delta ABC \backsim \Delta A_1B_1C$ ekanligini isbotlang.

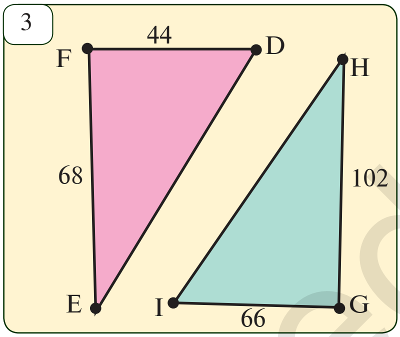
10.9. Ikkita o‘xshash uchburchak yuzlari 6 va 24 ga teng. Ulardan birining perimetri ikkinchisinikidan 6 ga ortiq. Katta uchburchakning perimetrini toping.

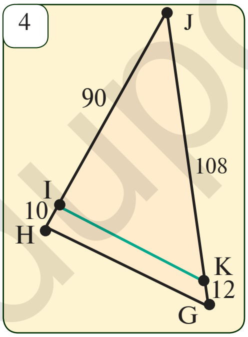
10.10. 3-rasmdagi uchburchaklar qaysi alomatga ko‘ra o‘xshash?

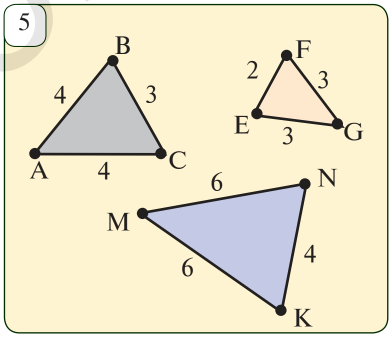
10.11. 4- rasmdagi JKI va JGH uchburchaklar qaysi alomatga ko‘ra o‘xshash?

10.12. 5- rasmdagi uchburchaklarning qaysilari bir-biriga o‘xshash?

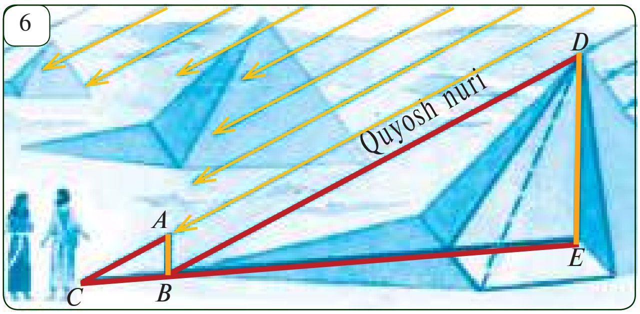








Tarixiy lavhalar. Bu voqea miloddan avvalgi VI asrda bo‘lgan. Bu vaqtda yunonlar geometriya bilan deyarli shug‘ullanishmas edi. Yunon faylasufi Fales misr fani bilan tanishish uchun tashrif buyurgan. Misrliklar unga qiyin masala beradi: ulkan piramidalardan birining balandligini qanday hisoblash mumkin? Fales bu masalaning sodda va jozibali yechimini topdi. U tayoqchani yerga qoqdi va shunday dedi: “Qachonki shu tayoqcha soyasining uzunligi tayoqchaning uzunligi bilan teng bo‘lsa, piramida soyasining uzunligi piramida balandligi bilan teng bo‘ladi”(6-rasm). Fales fikrini asoslashga harakat qiling!



11 TO‘G‘RI BURCHAKLI UCHBURCHAKLARNING O‘XSHASHLIK ALOMATLARI

Ma’lumki, to‘g‘ri burchakli uchburchaklarning bittadan burchaklari to‘g‘ri burchakdan iborat bo‘ladi. Shuning uchun bunday uchburchaklarning o‘xshashlik alomatlari ancha soddalashadi.

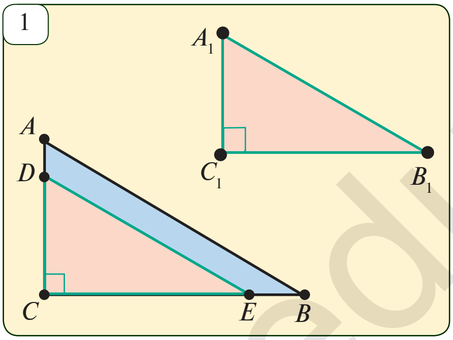
***1-teorema.** To‘g‘ri burchakli uchburchaklarning bittadan o‘tkir burchagi mos ravishda teng bo‘lsa, ular o‘xshash bo‘ladi.*

***2-teorema.** To‘g‘ri burchakli uchburchaklarning katetlari mos ravishda proporsional bo‘lsa, ular o‘xshash bo‘ladi.*

***3-Teorema.** To‘g‘ri burchakli uchburchaklardan birining gipotenuzasi va kateti ikkinchisining gipotenuzasi va katetiga mos ravishda proporsional bo‘lsa, ular o‘xshash bo‘ladi.*

Bu alomatlardan dastlabki ikkitasining to‘g‘riligi o‘z-o‘zidan ravshan. Keling, uchinchi alomatni isbotlaylik.

$\Delta ABC, \Delta A_1B_1C_1, \ \angle C = 90^\circ, \angle C_1 = 90^\circ, \quad \dfrac{AB}{A_1B_1} = \dfrac{BC}{B_1C_1}$ $\Rightarrow$ $\Delta ABC \sim \Delta A_1B_1C_1$



Isbot. ABC uchburchakning BC tomoniga $CE = C_1B_1$ bo‘ladigan qilib CE kesmani qo‘yamiz va $DE \parallel AB$ ni o‘tkazamiz *(1-rasm)*. Unda uchburchaklar o‘xshashligining BB alomatiga ko‘ra, ΔDEC va ΔABC o‘xshash bo‘ladi. O‘xshash uchburchaklar mos tomonlarining proporsionalligidan: $\dfrac{AB}{DE} = \dfrac{CB}{CE}$.

Yasashga ko‘ra, $CE = C_1B_1$. Demak,

$$\frac{AB}{DE} = \frac{CB}{C_1B_1} \qquad (1)$$

tenglik o‘rinli. Boshqa tomondan, teorema shartiga ko‘ra, $\dfrac{AB}{A_1B_1} = \dfrac{CB}{C_1B_1}$ (2)

(1) va (2) tengliklardan $DE = A_1B_1$ ekanligini aniqlaymiz.

$A_1B_1C_1$ va DEC uchburchaklarni qaraymiz: 1. $CE = C_1B_1$ (yasashga ko‘ra);
2. $DE = A_1B_1$ (isbotlangan tenglik).

To‘g‘ri burchakli uchburchaklarning bittadan kateti hamda gipotenuzasi bo‘yicha tenglik alomatiga ko‘ra, $\Delta A_1B_1C_1 = \Delta DEC$.

Ikkinchi tomondan esa $\Delta ABC \sim \Delta DEC$. U holda, $\Delta ABC \sim \Delta A_1B_1C_1$ bo‘ladi.

Teorema isbotlandi.

Masala. Agar ikkita teng yonli uchburchakdan birining yon tomoni va balandligi ikkinchisining yon tomoni va balandligiga proporsional bo‘lsa, bu uchburchaklarning o‘xshash ekanligini isbotlang *(2-rasm)*.

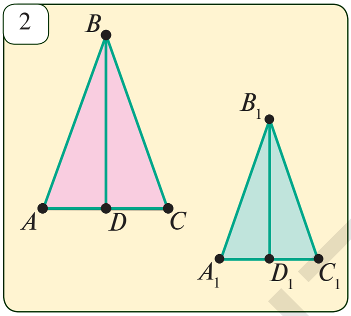
Isbot. To‘g‘ri burchakli ABD va $A_1B_1D_1$ uchburchaklarni qaraymiz. Shartga ko‘ra, ularning bittadan kateti va gipotenuzasi o‘zaro proporsional. Demak, 3-teoremaga asosan $\Delta ABD \backsim \Delta A_1B_1D_1$. Unda $\angle DBA = \angle D_1B_1A_1$. Teng yonli uchburchak asosiga tushirilgan balandlikning bissektrisa ham bo‘lishini hisobga olsak, $\angle B = 2\angle DBA = 2\angle D_1B_1A_1 = \angle B_1$ bo‘ladi.

Natijada, ABC va $A_1B_1C_1$ uchburchaklarda

$\angle B = \angle B_1$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ tengliklarga ega bo‘lamiz.

Demak, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta ABC \backsim \Delta A_1B_1C_1$. So‘ralgan tasdiq isbotlandi.

2



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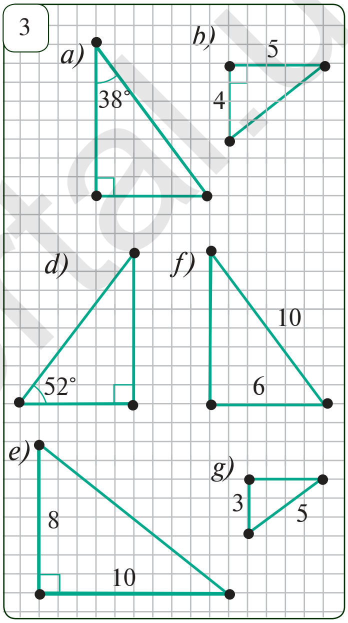

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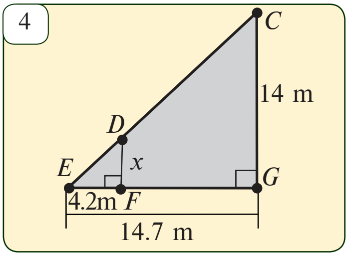

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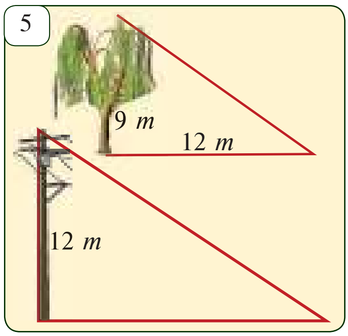


Masala va topshiriqlar

11.1. 3-rasmdan o‘xshash uchburchaklarni toping.

11.2. Katetlari 3 *m* va 4 *m* bo‘lgan to‘g‘ri burchakli uchburchakka o‘xshash uchburchakning bir kateti 27 *m* bo‘lsa, ikkinchi kateti necha *m* bo‘ladi?

11.3. Yuzlari 21 m^2 va 84 m^2 bo‘lgan ikkita to‘g‘ri burchakli uchburchaklar o‘xshash. Agar birinchi uchburchakning bir kateti 6 *m* bo‘lsa, ikkinchi uchburchak katetlarini toping.

11.4. Bir aylanaga ikkita o‘xshash to‘g‘ri burchakli uchburchak ichki chizilgan. Bu uchburchaklarning tengligini isbotlang.

11.5*. Katetlari 10 *cm* va 12 *cm* bo‘lgan to‘g‘ri burchakli uchburchakka bitta burchagi umumiy bo‘lgan kvadrat ichki chizilgan. Agar kvadratning bitta uchi gipotenuzada ekanligi ma’lum bo‘lsa, kvadratning tomonini toping.

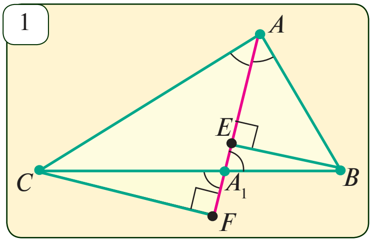
11.6*. ABC uchburchak berilgan. Unga $ADEF$ romb shunday ichki chizilganki, D, E va F nuqtalar mos ravishda uchburchakning AB, BC va CA tomonlarida yotadi. Agar $AB=c$, $AC=b$ bo‘lsa, romb tomonini toping.

11.7. 4-rasmda berilgan ma’lumotlar asosida noma’lum kesma uzunligini toping.

11.8. Tol daraxtining balandligi 9 *m*, simyog‘ochning balandligi esa 12 *m* (5-rasm). Agar tolning soyasi 12 *m* ni tashkil qilsa, simyog‘ochning soyasi uzunligini toping.

11.9. Anor daraxti balandligi 3 *m* bo‘lib uning soyasi kechga borib 6 *m* ni tashkil qildi. Balandligi 4,2 *m* bo‘lgan olma daraxtining soyasi bu paytda qanchani tashkil qiladi?

12 O‘XSHASHLIK ALOMATLARINING ISBOTLASHGA DOIR MASALALARGA TATBIQLARI



1-masala. Uchburchak bissektrisasi o‘zi tushgan tomonni qolgan ikki tomonga proporsional kesmalarga ajratishini isbotlang.

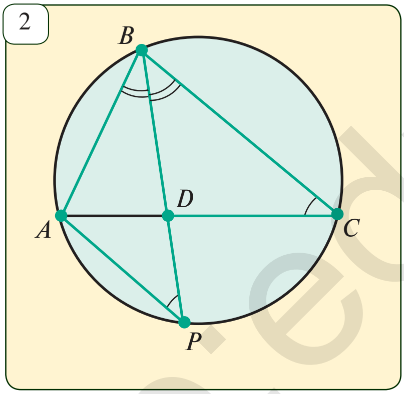
ΔABC, AA_1 — *bissektrisa (1-rasm)* ⇒ $\dfrac{AB}{A_1B}=\dfrac{AC}{A_1C}$

Yechish. AA_1 to‘g‘ri chiziqqa BE va CF perpendikularlar tushiramiz. Unda $\angle CAF=\angle BAE$ bo‘lgani uchun, to‘g‘ri burchakli CAF va BAE uchburchaklar o‘xshash bo‘ladi. O‘xshash uchburchaklarning mos tomonlari proporsionalligidan

$$\Delta CAF \sim \Delta BAE \Rightarrow \frac{AC}{AB}=\frac{CF}{BE}\ . \qquad (1)$$

Shunga o‘xshash
$$\Delta CA_1F \sim \Delta BA_1E \Rightarrow \frac{CA_1}{BA_1}=\frac{CF}{BE} \qquad (2)$$

(1) va (2) tengliklarni solishtirsak, $\dfrac{AC}{AB}=\dfrac{CA_1}{BA_1}$ yoki $\dfrac{AC}{AB}=\dfrac{CA_1}{BA_1}$ bo‘ladi. Bu A_1B va A_1C kesmalar AB va AC kesmalarga proporsional ekanligini anglatadi.



2-masala. ABC uchburchakning BD bissektrisasi uchburchakka tashqi chizilgan aylanani B va P nuqtalarda kesadi. $\Delta ABP \sim \Delta BDC$ ekanligini isbotlang *(2-rasm).*

Yechish. ΔABP va $\angle BDC$ da:

1. $\angle DBC=\angle ABP$ ⇐ shartga ko‘ra;
2. $\angle DCB=\angle APB$ ⇐ chunki ular bitta yoyga tiralgan.

Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\Delta ABP \sim \Delta BDC$.

Masala va topshiriqlar

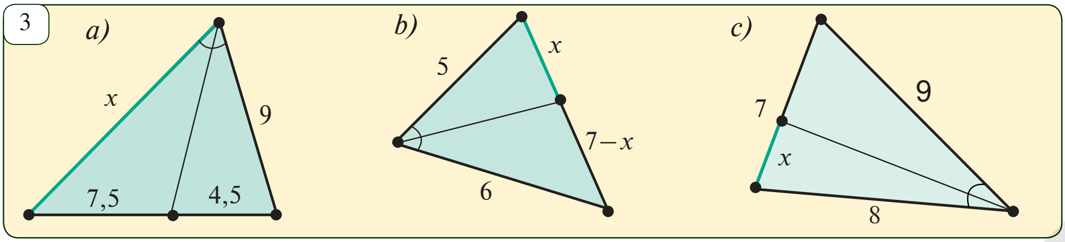
12.1. Uchburchak bissektrisasi o‘zi tushgan tomonda ajratgan kesmalari va uchburchakning qolgan tomonlari orasidagi proporsionallikni yozib ko‘rsating.

12.2. To‘g‘ri burchakli ABC uchburchakning C to‘g‘ri burchagidan CD balandlik o‘tkazilgan. $\angle ACD=\angle CBD$ bo‘lishini isbotlang. Hosil bo‘lgan shaklda nechta o‘zaro o‘xshash uchburchaklarni ko‘rsata olasiz?

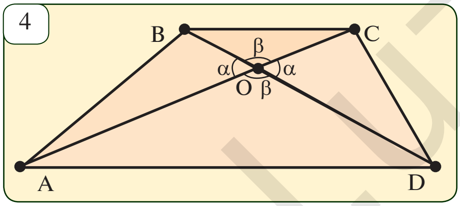
12.3. 3-rasmdagi ma’lumotlar asosida x ni toping.

12.4. ABC uchburchaklarning AD bissektrisasi o‘tkazilgan. Agar $CD=4{,}5$ *m*; $BD=13{,}5$ *m* va ABC uchburchak perimetri 42 *m* bo‘lsa, uning AB va AC tomonlarini toping.

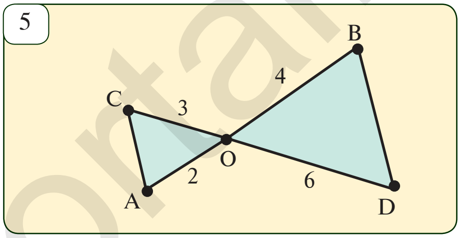
12.5. ABC uchburchak medianalari N nuqtada kesishadi. Agar ABC uchburchak yuzi 87 dm^2 bo‘lsa, ANB uchburchak yuzi nimaga teng?



12.6. ABC uchburchak medianalari kesishgan N nuqtadan AB va BC tomonlargacha bo‘lgan masofalar mos ravishda 3 *dm* va 4 *dm*. Agar $AB = 8$ *dm* bo‘lsa, BC tomonni hisoblang.



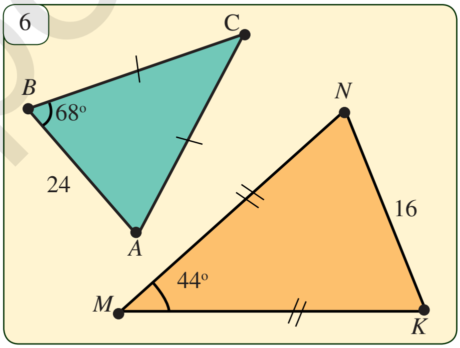
12.7*. Trapetsiyaning asosiga parallel to‘g‘ri chiziq yon tomonlaridan birini $m:n$ nisbatda bo‘lishi ma’lum. Bu to‘g‘ri chiziq uning ikkinchi yon tomonini qanday nisbatda bo‘ladi?



12.8. 4-rasmda trapetsiya tasvirlangan. AOD va COB uchburchaklarning o‘xshashligini isbotlang.

12.9. 5-rasmda AOC va DOB uchburchaklar o‘xshashligini ko‘rsating.

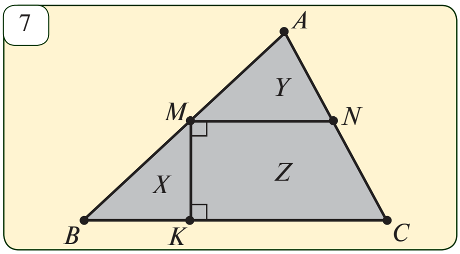
12.10. 6-rasmda tasvirlangan uchburchaklar o‘xshashmi?



Qiziqarli masalalar

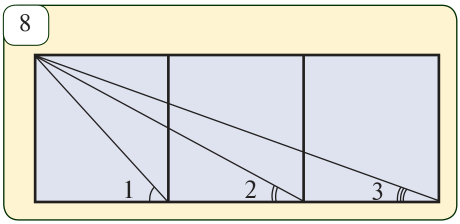
Geometriya va ingliz tili. Quyida ingliz tilida berilgan geometrik masalani yechib ko‘ring-chi! Bu bilan ham ingliz tilidan, ham geometriyadan nimaga qodirligingizni sinab olasiz.

1) Dissection Puzzle: Let M be the mid-point of the side AB of a given triangle ABC. The triangle has been dissected into parts X, Y, Z along the lines MN and MK passing through M such that MN is parallel while MK is perpendikular to the base BC (*picture 7*). Show how the three pieces can be fitted together to make a rectangle, respectively two different parallelograms.



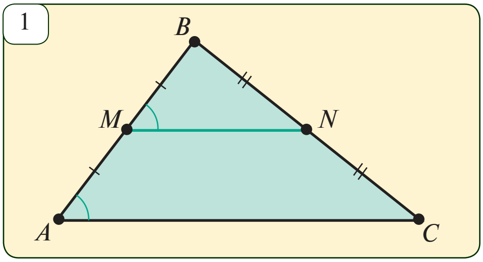
2) Look at the picture 8 and proof

$$\angle 1 + \angle 2 + \angle 3 = 90^\circ.$$



1-masala. Uchburchaklarning o‘xshashligidan foydalanib, uchburchak o‘rta chizig‘i uchburchakning bir tomoniga parallel va shu tomonning yarmiga teng ekanligini isbotlang.

$\Delta ABC, MN$ — *o‘rta chiziq (1-rasm):* $MA=MB, NC=NB$ ⇒ $MN \parallel AC,\ MN=\frac{1}{2}AC$



Yechish. ΔABC va ΔMBN uchun:

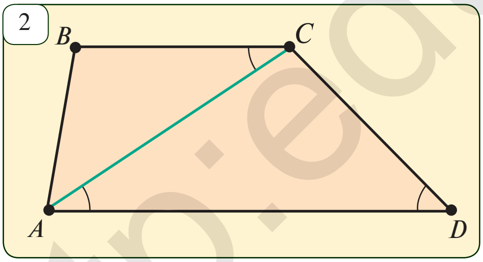
$\angle B$ — umumiy, $\frac{BM}{AB}=\frac{BN}{BC}=\frac{1}{2}$

Shuning uchun, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, bu ikki uchburchak o‘xshash. Endi mushohadani mana bunday davom ettiramiz:

$$\Delta MBN \backsim \Delta ABC \Rightarrow \begin{cases} \angle BMN=\angle A, \\ \frac{MN}{AC}=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} MN \parallel AC, \\ MN=\frac{1}{2}AC. \end{cases}$$

2-masala. Agar asoslari BC va AD bo‘lgan $ABCD$ trapetsiyaning AC diagonali uni ikkita o‘xshash uchburchakka ajratsa, $AC^2=BC\cdot AD$ bo‘lishini isbotlang.

$ABCD$ — trapetsiya, $BC \parallel AD$, $\Delta ABC \backsim \Delta DCA$ (2-rasm) ⇒ $AC^2=BC\cdot AD$



Yechish. **1-qadam.** ABC va ACD uchburchaklarning burchaklarini taqqoslaymiz. $\angle ACB=\angle CAD$, chunki bu burchaklar — ichki almashinuvchi burchaklar. $\angle B \neq \angle D$, chunki $ABCD$ — trapetsiya (aks holda,

$$\angle D+\angle A=\angle B+\angle A=180^\circ,$$

ya’ni $AB \parallel CD$ bo‘lib, $ABCD$ trapetsiya bo‘lmay qolar edi). U holda, $\angle D=\angle BAC$ va $\angle ACD=\angle B$.

2-qadam. Endi ABC va DCA o‘xshash uchburchaklarning mos tomonlari nisbatini yozamiz: , $\frac{AC}{BC}=\frac{AD}{AC}$ bundan $AC^2=BC\cdot AD$.

Masala va topshiriqlar

13.1. a) Bo‘yi 170 *cm* bo‘lgan odam soyasining uzunligi 1 *m* bo‘lsa, balandligi 5,4 *m* bo‘lgan simyog‘och soyasining uzunligini toping.

b) Ikkita teng yonli uchburchakning uchidagi burchaklari teng. Birinchi uchburchakning yon tomoni 17 *cm*, asosi 10 *cm* ga, ikkinchi uchburchakning asosi 8 *cm* ga teng. Ikkinchi uchburchakning yon tomonini toping.

13.2. 3-rasmdagi har bir chizmadan o‘xshash uchburchaklarni ko‘rsating.

13.3. *ABC* uchburchakning *AP* medianasi *BC* tomonga parallel va uchlari *AB* va *AC* tomonlarda yotgan istalgan kesmani teng ikkiga bo‘lishini isbotlang.

13.4. Uchburchakning uchlari uning o‘rta chizig‘ini o‘z ichiga olgan to‘g‘ri chiziqdan teng masofada yotishini isbotlang.

13.5. Aylanaga ichki chizilgan *ABCD* to‘rtburchak diagonallari *O* nuqtada kesishadi.
$\Delta AOB \backsim \Delta COD$ ekanligini isbotlang.

13.6. *ABC* uchburchak ichki sohasida *O* nuqta va *OA, OB, OC* nurlarda mos ravishda *E, F, K* nuqtalar olingan *(4-rasm)*. Agar $AB \| EF$ va $BC \| FK$ bo‘lsa, *ABC* va *EFK* uchburchaklar o‘xshash ekanligini isbotlang.

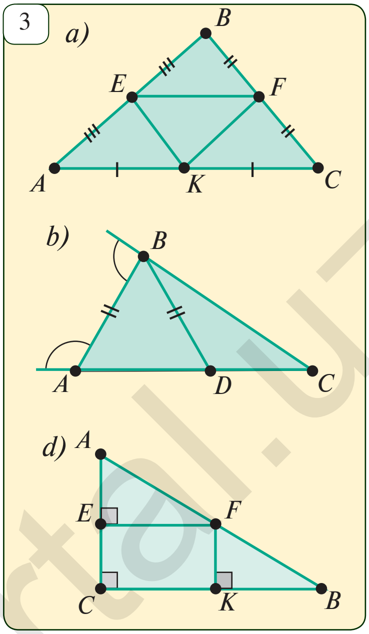
13.7*. Trapetsiyaning diagonallari kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziq trapetsiya asoslaridan birini $m:n$ nisbatda bo‘ladi. Bu to‘g‘ri chiziq ikkinchi asosni qanday nisbatda bo‘ladi?

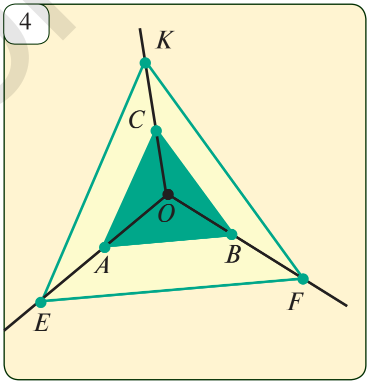
13.8. Agar *ABC* uchburchakning yuzi *S* ga teng bo‘lsa, 5-rasmda *x* bilan belgilangan soha yuzini toping.

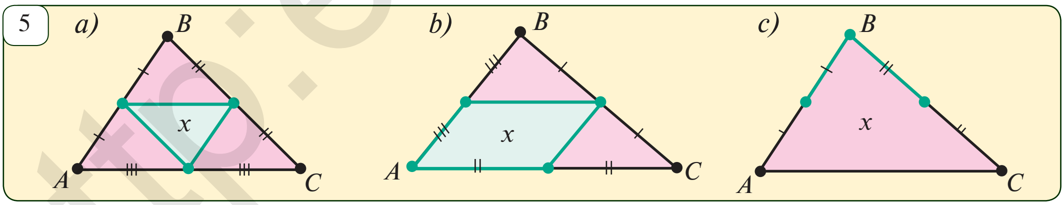
13.9. 6-rasmda $EQ \| BC$. *AQ* ni toping.

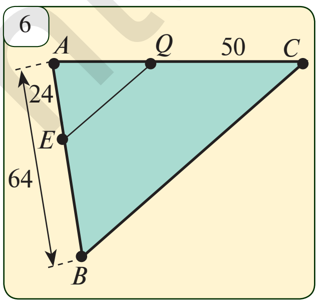
13.10. 7- rasmda $AB \| EC$. *QC* ni toping.

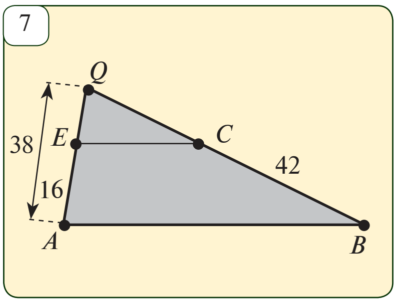
13.11. 8-rasmda *ABC* uchburchakning balandliklari o‘tkazilgan. Natijada nechta o‘xshash uchburchaklar hosil bo‘ldi?

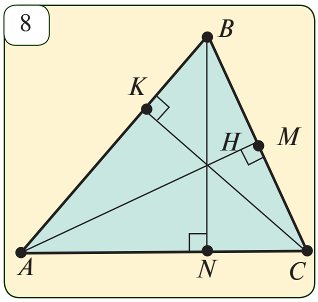












14 BILIMINGIZNI SINAB KO‘RING

I. Testlar

1. Quyidagi ta’riflardan qaysi biri to‘g‘ri?

A) Ikkita uchburchakning burchaklari mos ravishda teng bo‘lsa, ular o‘xshash deyiladi;

B) Ikkita uchburchakning tomonlari mos ravishda teng bo‘lsa, ular o‘xshash deyiladi;

D) Ikkita uchburchakning mos tomonlari proporsional va mos burchaklari teng bo‘lsa, ular o‘xshash deyiladi;

E) Ikkita uchburchakning mos tomonlari va mos burchaklari teng bo‘lsa, ular o‘xshash deyiladi.

2. Ikkita o‘xshash uchburchak yuzlarining nisbati nimaga teng?

A) O‘xshashlik koeffitsiyentiga;

B) Ularning mos tomonlari nisbatiga;

D) Ularning perimetrlari nisbatiga;

E) O‘xshashlik koeffitsiyentining kvadratiga.

3. Quyidagi tasdiqlardan qaysi biri to‘g‘ri?

A) Uchburchaklardan birining ikkita burchagi ikkinchisining ikkita burchagiga teng bo‘lsa, ular o‘xshash bo‘ladi;

B) Uchburchaklardan birining ikkita tomoni ikkinchisining ikki tomoniga teng bo‘lsa, ular o‘xshash bo‘ladi;

D) Ikkita uchburchakning bittadan burchaklari teng va ikkitadan tomonlari proporsional bo‘lsa, ular o‘xshash bo‘ladi;

E) Ikkita uchburchakning bittadan burchaklari teng va bittadan tomonlari proporsional bo‘lsa, ular o‘xshash bo‘ladi.

4. To‘g‘risini toping. Agar ikkita uchburchak o‘xshash bo‘lsa, ularning ...

A) Balandliklari teng bo‘ladi; B) Tomonlari proporsional bo‘ladi;

D) Tomonlari teng bo‘ladi; E) Yuzlari teng bo‘ladi.

5. O‘xshash uchburchaklarning perimetrlari nisbati nimaga teng?

A) Mos tomonlar nisbatining kvadratiga; B) O‘xshashlik koeffitsiyentiga;

D) O‘xshashlik koeffitsiyentining kvadratiga; E) Yuzlari nisbatiga.

6. Qaysi bandda 1-rasmda tasvirlangan romblar o‘xshashligi to‘g‘ri yozilgan?

A) $EHGF \sim PQRO$; B) $HGFE \sim PQRO$;

D) $GFEH \sim QROP$; E) $EHGF \sim QROP$.

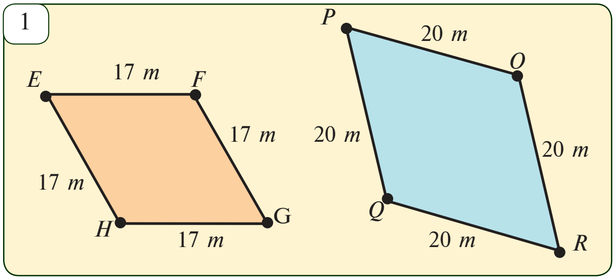
7. 2- rasmdagi ko‘pburchaklar o‘xshashmi? Nega?

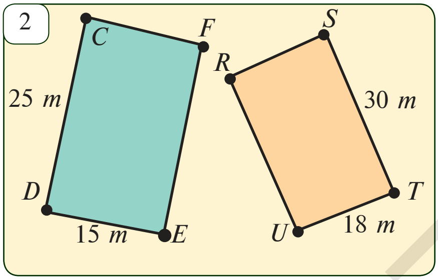
A) Ha, chunki bu ko‘pburchaklarning mos burchaklari teng va mos tomonlari proporsional;

B) Ha, chunki bu ko‘pburchaklarning mos burchaklari proporsional va mos tomonlari teng;

D) Ha, chunki bu ko‘pburchaklarning mos burchaklari teng;

E) Ha, chunki bu ko‘pburchaklarning mos tomonlari proporsional;





8. 3-rasmdagi SRQT va VWXU trapetsiyalar o‘xshashmi? Agar o‘xshash bo‘lsa, ularning o‘xshashlik koeffitsiyenti nimaga teng?

A. Ha, k= 0,4; B. Ha, k= 0,5;
D. Ha, k= 0,8; E. Yo‘q.

9. O‘xshash uchburchaklarning mos tomonlari 4 *cm* va 13 *cm*. Agar birinchi uchburchak yuzi 16 cm² ga teng bo‘lsa, ikkinchi uchburchak yuzini toping.

A. 169 cm²; B. 16 cm²;
D. 52 cm²; E. 189 cm²;

10. Ikkita o‘xshash uchburchak yuzlarining nisbati 144 ga teng. Ularning mos tomonlari nisbati nimaga teng?

A. 13 ga; B. 12 ga;
D. 14 ga; E. 16 ga;

11. 4-rasmdagi uchburchaklar o‘xshash. Rasmda berilgan kattaliklarga ko‘ra katta uchburchak yuzining kichik uchburchak yuziga nisbatini toping.

A. 9:4; B. 3:2;
D. 4:9; E. 2:3;

12. Ikkita o‘xshash uchburchak yuzlarining nisbati *a* ga teng bo‘lsa, bu uchburchaklar o‘xshashlik koeffitsiyenti nimaga teng bo‘ladi?

A. $1:a^2$; B. a^2 ; D. $\sqrt{a}$; E. $1:a$;

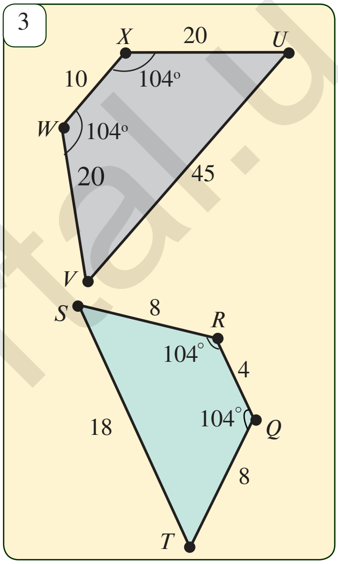
13. 5-rasmda keltirilgan teng yonli uchburchaklar o‘xshashmi? Nega?

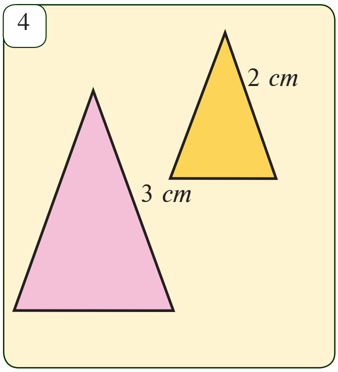
A. Ha, chunki ularning ikkitadan tomonlari propotsional va ular orasidagi burchagi teng;

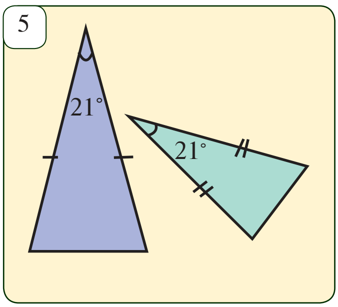
B. Yo‘q, chunki ularning ikkita burchagi o‘zaro teng emas;

D. Yo‘q, chunki ularning mos burchaklari teng emas;

E. Yo‘q, chunki ularning tomonlari proporsional emas;







II. Masalalar

1. ABC uchburchakning AB va AC tomonlari o‘rtalari mos ravishda E va F nuqtalar bo‘lsin. Agar AEF uchburchak yuzi 3 cm^2 bo‘lsa, ABC uchburchak yuzini toping.

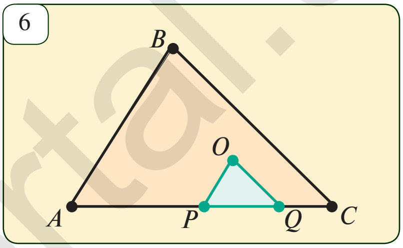
2. ABC uchburchakning AC tomoniga parallel to‘g‘ri chiziq AB va BC tomonlarni mos ravishda N va P nuqtalarda kesadi. Agar $AN = 4$, $NB = 3$, $BP = 3{,}6$ bo‘lsa, BC tomonni toping.

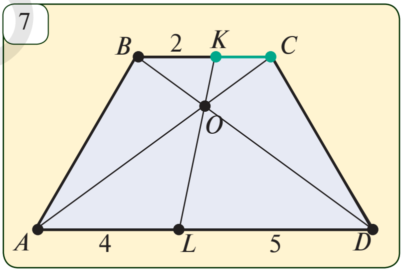
3. O‘tkir burchakli ABC uchburchakning AB tomonida K nuqta olingan. Agar $AK=3$, $BK=2$ va uchburchakning BD balandligi 4 ga teng bo‘lsa, K nuqtadan AC kesmagacha bo‘lgan masofani toping.

4. $ABCD$ parallelogrammning BC tomoni o‘rtasidagi K nuqtadan o‘tkazilgan DK nur bilan AB nur F nuqtada kesishadi. Agar $AD = 4$, $DK = 5$ va $DC = 5$ bo‘lsa, AFD uchburchak perimetrini hisoblang.

5. ABC uchburchak ichki sohasida olingan O nuqtadan AB va BC tomonlarga parallel to‘g‘ri chiziqlar o‘tkazilgan. Bu to‘g‘ri chiziqlar AC tomonni mos ravishda P va Q nuqtalarda kesadi. Agar $PQ = 2$, $AC = 7$ va ABC uchburchak yuzi 98 ga teng bo‘lsa, POQ uchburchak yuzini aniqlang (6-*rasm*).

6. $ABCD$ trapetsiyaning BC va AD asoslarida mos ravishda K va L nuqtalar olingan. KL kesma trapetsiyaning diagonallari kesishgan nuqtadan o‘tadi. Agar $AL=4$, $LD=5$ va $BK=2$ bo‘lsa, KC kesmani toping (7-*rasm*).

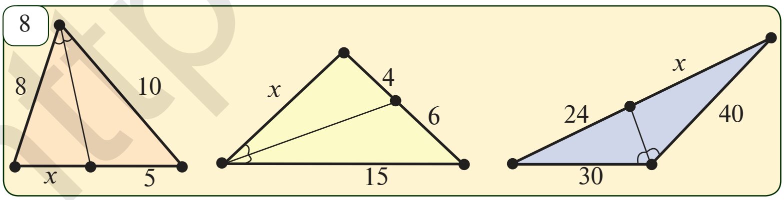


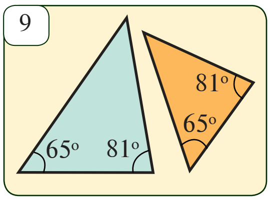


7. Ikkita o‘xshash uchburchaklardan birinchisining yuzi 15 mm², ikkinchisining yuzi esa 135 mm². Birinchi uchburchakning bitta tomoni 6 mm bo‘lsa, ikkinchi uchburchakning unga mos tomonini toping?

8. 8-rasmda berilganlarga ko‘ra noma’lum kesmani toping.

9. 9-rasmda keltirilgan uchburchak o‘xshashmi? Nega?



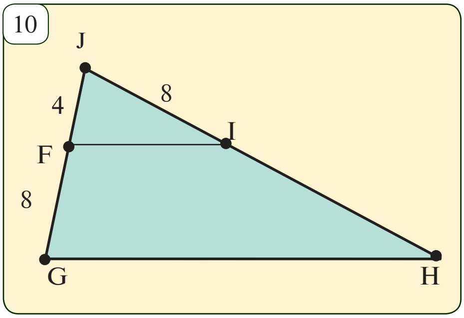


10. 10-rasmda $JIF \sim HJG$. IH kesma uzunligini toping

11. 11-rasmda tasvirlangan uchburchaklar o‘xshashmi? Agar o‘xshash bo‘lsa, ularning o‘xshashlik koeffitsiyentini toping.

12. Ikkita o‘xshash uchburchaklardan birinchisining yuzi 24 mm², ikkinchisining yuzi esa 216 mm². Birinchi uchburchakning balandliklaridan biri 8 mm bo‘lsa, ikkinchi uchburchakning mos balandligini toping.

10



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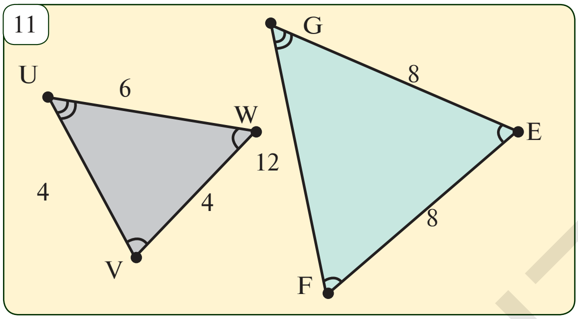

13. 12-rasmda tasvirlangan ko‘pburchaklar o‘xshash. Berilgan ma’lumotlardan foydalanib, noma’lum kattalikni toping

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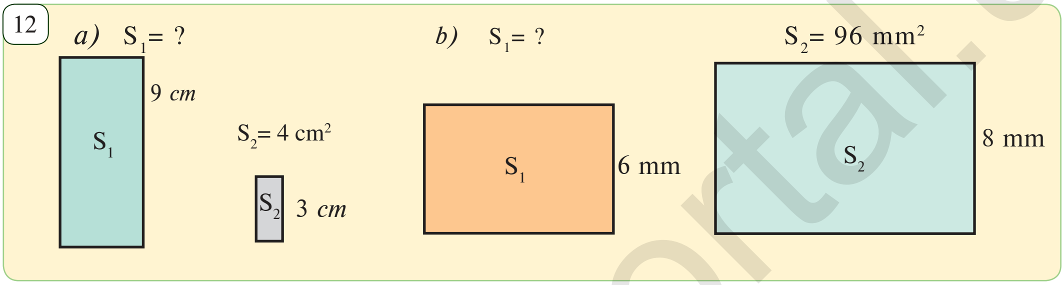
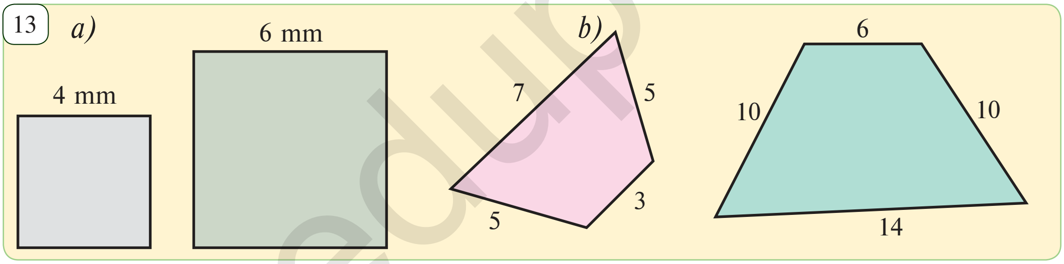


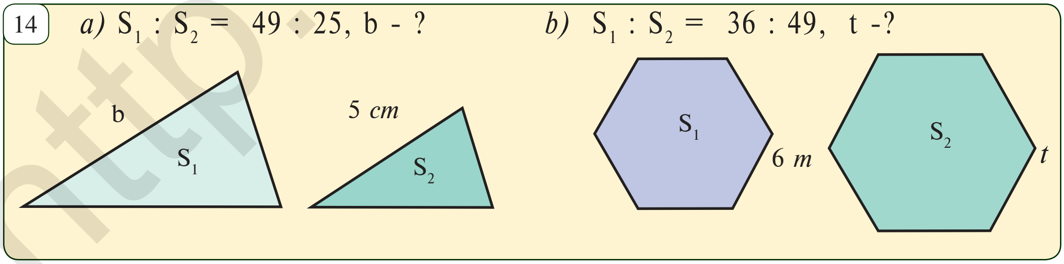
14. 13-rasmda tasvirlangan ko‘pburchaklar o‘xshash. Berilgan ma’lumotlardan foydalanib, ularning o‘xshashlik koeffitsiyentini toping

13



15. 14-rasmda tasvirlangan ko‘pburchaklar o‘xshash. Berilgan ma’lumotlar asosida noma’lumni toping.

14

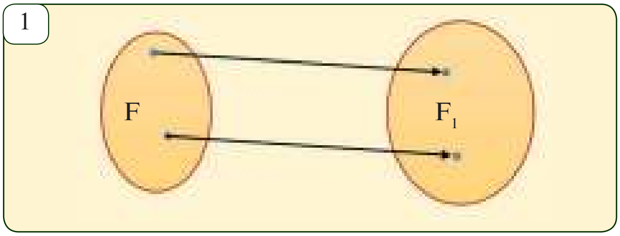


16. Chinor daraxtining soyasi 12 *m*. Uning yonidagi ko‘p qavatli uyning soyasi esa 6 *m*. Agar chinor daraxti uydan 16 *m* baland bo‘lsa, uyning balandligi qanchani tashkil qiladi?

17. Haykalning balandligi 9 *m* bo‘lib, uning soyasi 12 *m*. Haykal yonida o‘sayotgan terak daraxtining soyasi esa 16 *m*. Terakning balandligi qancha?

15 TEKISLIKDA GEOMETRIK ALMASHTIRISHLAR. HARAKAT VA PARALLEL KO‘CHIRISH

Tekislikda berilgan F shaklning har bir nuqtasi biror bir usulda ko‘chirilsa, yangi F_1 shakl hosil bo‘ladi (1- rasm). Agar bu ko‘chirishda (akslantirishda) birinchi shaklning har xil nuqtalari ikkinchi shaklning har xil nuqtalariga ko‘chsa (akslantirish o‘zaro bir qiymatli bo‘lsa), bu ko‘chirishga *geometrik shakl almashtirish* deb ataladi.

1


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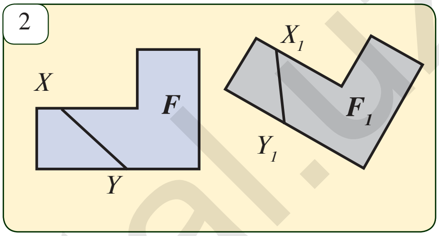


Agar shakl akslantirishda tekislikning barcha nuqtalari ko‘chsa, unda tekislikni o‘zini-o‘ziga akslantirish haqida ham gapirish mumkin. Quyida tekislikdagi ba‘zi bir geometrik almashtirishlar ustida to‘xtalamiz.

Nuqtalar orasidagi masofani saqlaydigan shakl almashtirish *harakat* deb ataladi.

Ta‘rifga ko‘ra, shakl almashtirshda F shaklning ixtiyoriy X va Y nuqtalari F_1 shaklning qandaydir X_1 va Y_1 nuqtalariga o‘tgan bo‘lib, $XY = X_1Y_1$ tenglik bajarilsa (ya‘ni masofa saqlansa), bunday shakl almashtirish harakat bo‘ladi (2-rasm).

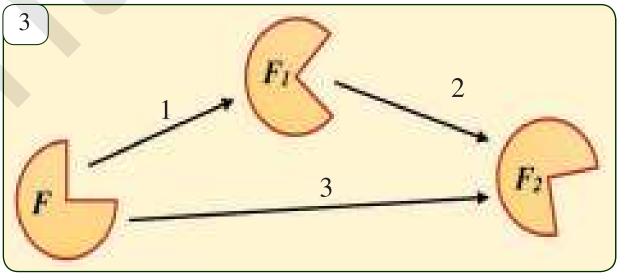
Harakatning quyidagi xossalarini keltirish mumkin.

Harakatda to‘g‘ri chiziq – to‘g‘ri chiziqqa, nur – nurga, kesma - unga teng kesmaga, burchak - unga teng burchakka, uchburchak - unga teng uchburchakka ko‘chadi (akslanadi).

Aytaylik, F shakl birinchi harakat natijasida F_1 shaklga, F_1 shakl esa ikkinchi harakat yordamida F_2 shaklga o‘tgan bo‘lsin. Natijada, F shakl bu ikki harakat yordamida F_2 shaklga ko‘chadi va bu ko‘chish o‘z navbatida yana harakat bo‘ladi (3-rasm).

Tekislikda biror harakat yordamida birini ikkinchisiga ko‘chirish mumkin bo‘lgan shakllar teng deyiladi.

Tekislikda biror $\overline{AB}$ vektor va ixtiyoriy X nuqta berilgan bo‘lsin. Agar X_1 nuqta uchun $XX_1 = AB$ shart bajarilsa, X nuqta X_1 nuqtaga AB vektor bo‘ylab *parallel ko‘chirilgan* deb ataladi.

3


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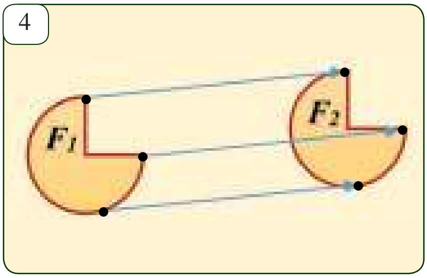
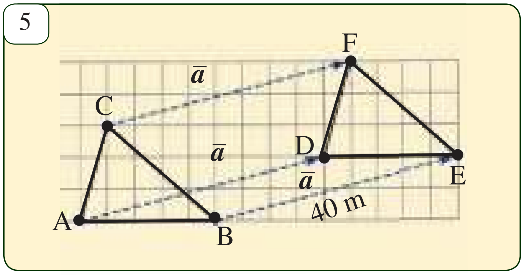
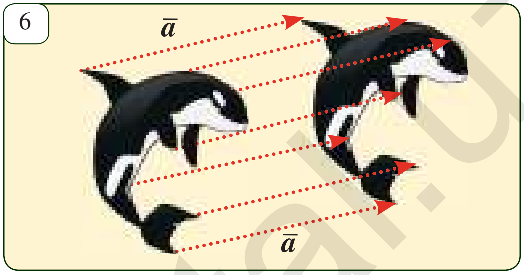


Agar tekislikda berilgan F shaklning har bir nuqtasi $\overline{AB}$ vektor bo‘ylab ko‘chirilsa (5- rasm), yangi F_1 shakl hosil bo‘ladi. Bu holda F shakl F_1 shaklga parallel ko‘chirilgan deyiladi. Parallel ko‘chirishda F shaklning har bir nuqtasi bir xil yo‘nalishda bir xil masofaga ko‘chirilgan bo‘ladi.

5- rasmda tasvirlangan uchburchakning har bir nuqtasi boshlang‘ich holatiga nisbatan 40 m ga parallel ko‘chgan. 6-rasmdagi delfin ham $\overline{a}$ vektor bo‘ylab parallel ko‘chirilgan.





Ravshanki, parallel ko‘chirish harakatdir. Shuning uchun, parallel ko‘chirishda to‘g‘ri chiziq - to‘g‘ri chiziqqa, nur – nurga, kesma - unga teng kesmaga ko‘chadi va hokazo.

Aytaylik, $\overline{AB} = (a; b)$ vektor bo‘ylab parallel ko‘chirishda F shaklning nuqtasi X $(x; y)$ va F_1 shaklning nuqtasi X_1 $(x_1; y_1)$ ga o‘tsin. Unda ta‘rifga ko‘ra quyidagilarga egamiz:

$x_1 - x = a$, $y_1 - y = b$ yoki $x_1 = x + a$, $y_1 = y + b$.

Bu tengliklar paralel ko‘chirish formulalari deb ataladi.

1-masala. $\overline{p} = (3; 2)$ vektor bo‘ylab parallel ko‘chirishda P (-2; 4) nuqta qaysi nuqtaga ko‘chadi?

Yechish. Yuqoridagi parallel ko‘chirish formulalardan foydalanamiz:

$x_1 = -2 + 3 = 1$, $y_1 = 4 + 2 = 6$. ***Javob:*** $P_1(1; 6)$.

? *Masala va topshiriqlar*

15.1. $\overline{p} = (-2; 1)$ vektor bo‘ylab parallel ko‘chirishda a) (3; -2); b) (0; 2); c) (2; -5) nuqta qaysi nuqtaga ko‘chadi?

15.2. Parallel ko‘chirishda $A(4; 2)$ nuqta $B(3; 7)$ nuqtaga ko‘chdi. Parallel ko‘chirish qaysi vektor bo‘ylab amalga oshirilgan?

15.3. Parallel ko‘chirishda a) to‘g‘ri chiziq – to‘g‘ri chiziqqa; b) nur – nurga; c) kesma – unga teng kesmaga ko‘chishini isbotlang.

15.4. Parallel ko‘chirishda (1; 2) nuqta (1; -1) nuqtaga o‘tadi. Koordinata boshi bu almashtirishda qaysi nuqtaga o‘tadi?

15.5. Parallel ko‘chirishda (3; 4) nuqta (2; -4) nuqtaga o‘tadi. Bu almashtirishda koordinata boshi qaysi nuqtaga o‘tadi?

15.6. $A(2; 1)$ nuqta $B(1; 0)$ nuqtaga, $C(3; -2)$ nuqta esa $D(2; -3)$ nuqtaga o‘tadigan parallel ko‘chirish mavjudmi?

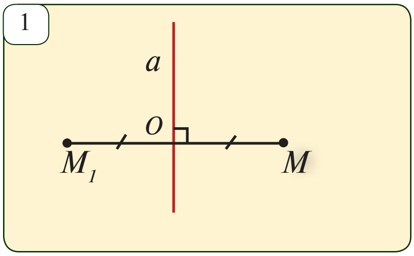
15.7. $A(-2; 3)$ nuqta $B(1; 2)$ nuqtaga, $C(4; -3)$ nuqta esa $D(7; -2)$ nuqtaga o‘tadigan parallel ko‘chirish mavjudmi?

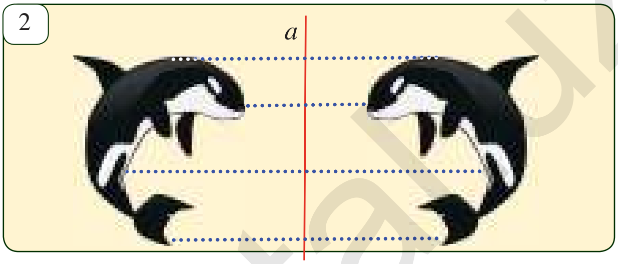
15.8. $ABCDA_1B_1C_1D_1$ kub berilgan. Parallel ko‘chirishda A_1D kesma B_1C kesmaga o‘tadi. Bu ko‘chirishda AA_1 kesma qaysi kesmaga o‘tadi?

O‘QQA NISBATAN SIMMETRIYA

Tekislikda biror *a* to‘g‘ri chiziq va unda yotmaydigan ixtiyoriy *M* nuqta berilgan bo‘lsin. *M* nuqtadan *a* to‘g‘ri chiziqqa perpendikular tushiramiz va uning asosini O bilan belgilaymiz (1 - rasm). Perpendikularda yotgan M_1 nuqta uchun

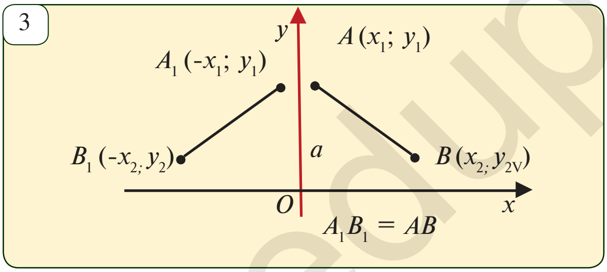
$MO = M_1O$ bo‘lsa, *M* va M_1 nuqtalarga *a* to‘g‘ri chiziq yoki *o‘qqa nisbatan simmetrik* nuqtalar deyiladi,

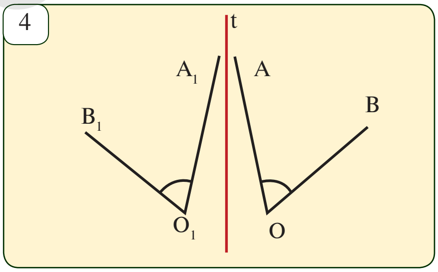




Tekislikning ixtiyoriy *M* nuqtasiga *a* to‘g‘ri chiziq (o‘qqa) nisbatan unga simmetrik bo‘lgan M_1 nuqtani mos qo‘yamiz. Tekislikni bunday o‘zini-o‘ziga akslantirishga *o‘qqa nisbatan simmetriya* deymiz. To‘g‘ri chiziqni esa *simmetriya o‘qi* deb yuritamiz.

2-rasmda tasvirlangan delfinlar o‘zaro *a* o‘qqa nisbatan simmetrik bo‘ladi.





O‘qqa nisbatan simmetriya harakatdir ya‘ni u nuqtalar orasidagi masofani saqlaydi.

Keling bu tasdiqni isbotlaylik. 3- rasmda ixtiyoriy $A\ (x_1;\ y_1)$ va $B\ (x_2;\ y_2)$ nuqtalar bo‘lib, $A_1\ (-x_1;\ y_1)$ va $B_1\ (-x_2;\ y_2)$ nuqtalar esa ularning *a* to‘g‘ri chiziqqa (Oy o‘qqa) nisbatan mos ravishda simmetrik akslari bo‘lsin. $AB = A_1B_1$ ekanligini ko‘rsatamiz.

Haqiqatan, ikki nuqta orasidagi masofani hisoblash formulasiga ko‘ra

$$AB = (x_2-x_1)^2+(y_2-y_1)^2,$$

$$A_1B_1 = (-x_2-(-x_1))^2+(y_2-y_1)^2 = (x_2-x_1)^2+(y_2-y_1)^2$$

ya‘ni bu masofalar o‘zaro teng. Bundan o‘qqa nisbatan simmetriyada har bir kesma o‘ziga teng kesmaga o‘tishi ham kelib chiqadi.

Xuddi shunga o‘xshash, o‘qqa nisbatan simmetriyada burchak – o‘ziga teng burchakka o‘tishini ham ko‘rsatish mumkin. Bunda faqat burchakning yo‘nalishi o‘zgarib qoladi (4- rasm).

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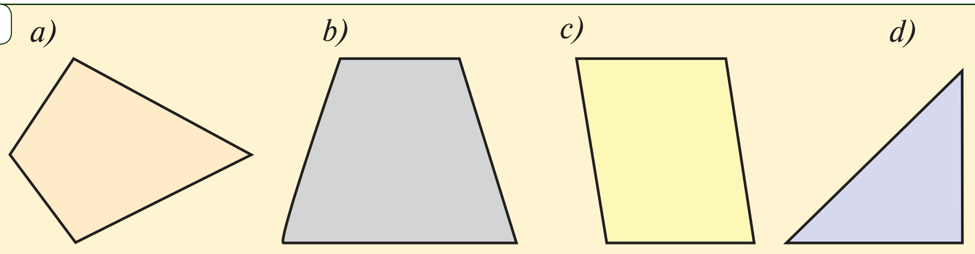

Koordinatalar tekisligida berilgan A $(x; y)$ nuqta Ox oʻqiga nisbatan simmetriyada A_1 $(x; -y)$ nuqtaga, Oy oʻqiga nisbatan simmetriyada esa A_2 $(-x; y)$ nuqtaga oʻtadi.

Masala va topshiriqlar

16.1. (1; 2), (0; 2), (2; 2) nuqtalar koordinata oʻqlariga nisbatan simmetriyalarda qaysi nuqtalarga oʻtadi? a) OX oʻqiga nisbatan; b) OY oʻqiga nisbatan.

16.2. (2; 4) nuqta koordinata oʻqiga nisbatan simmetrik akslantirishda (2; -4) nuqtaga oʻtdi. Akslantirish qaysi koordinata oʻqiga nisbatan amalga oshirilgan?

16.3. 5- rasmda tasvirlangan shakllarning qaysilari simmetriya oʻqiga ega? Bu shakllarni daftaringizga koʻchirib chizing va ularning simmetriya oʻqlarini yasang.

16.4. Toʻgʻri toʻrtburchak, kvadrat, romb, teng yonli trapetsiya va teng yonli uchburchakning nechta simmetriya oʻqi bor?

16.5. Ixtiyoriy ABC uchburchak chizing. Uning C uchidan oʻtuvchi toʻgʻri chiziqqa nisbatan unga simmetrik boʻlgan uchburchakni tasvirlang.

16.6. Koordinata tekisligida uchlari A (3; 2), B (2; 7), C (6; 7) va D (7; 2) nuqtalarda boʻlgan $ABCD$ parallelogrammga Oy oʻqiga nisbatan simmetrik boʻlgan $A_1B_1C_1D_1$ parallelogrammni tasvirlang.

16.7. Koordinata tekisligida $y = x + 4$ funksiya grafigini chizing. Bu grafikka Ox oʻqiga nisbatan simmetrik boʻlgan toʻgʻri chiziqni tasvirlang va u qaysi funksiya grafigi ekanligini aniqlang.

16.8. Chapdan oʻngga ham, oʻngdan chapga ham oʻqisa boʻladigan soʻzlarga polindromlar deyiladi. Quyidagi polindrom soʻzlarning qaysilarining simmetriya oʻqi bor?

KIYIK QOVOQ NON SOS KATAK BOB MUM RADAR

16.9. 6- rasmdagi koordinatalar tekisligida tasvirlangan shakllarni daftaringizga koʻchirib chizing. Shu koordinatalar tekisligida bu shakllarga Ox hamda Oy oʻqlariga nisbatan simmetrik boʻlgan shakllarni quring.

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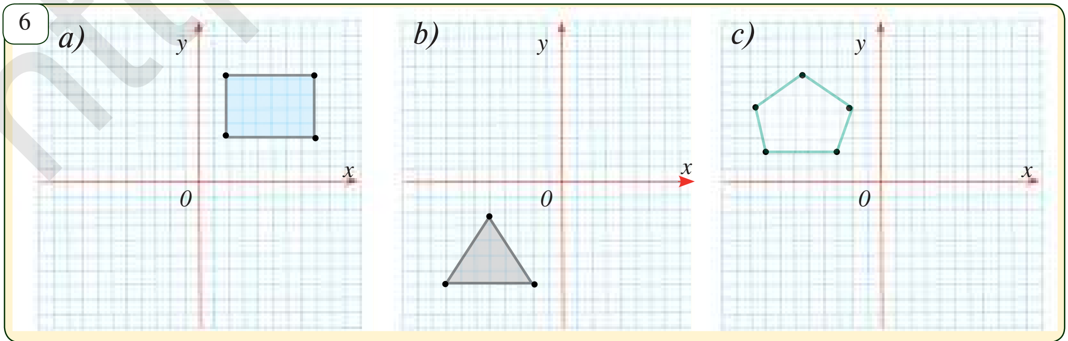


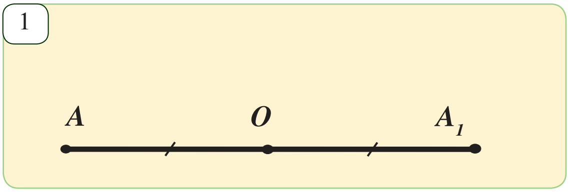
Tekislikda berilgan A va A_1 nuqtalar O nuqtaga nisbatan simmetrik deyiladi, agar $AO = OA_1$ ya'ni O nuqta AA_1 kesmaning o'rtasi bo'lsa (1- rasm).

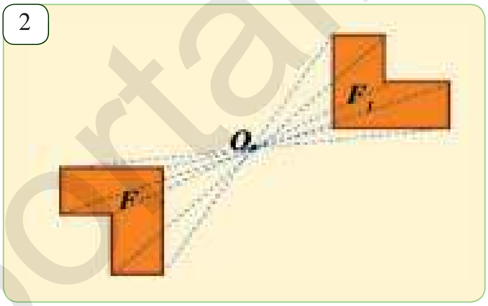
Agar tekislikda berilgan F shaklning har bir nuqtasi O nuqtaga nisbatan simmetrik nuqtaga ko'chsa (2- rasm), yangi F_1 shakl hosil bo'ladi. Bunday almashtirishda F va F_1 shakllar *O nuqtaga nisbatan simmetrik* deyiladi. 3- rasmlardagi delfinlar rasmi O nuqtaga nisbatan simmetrik shakllar bo'ladi.

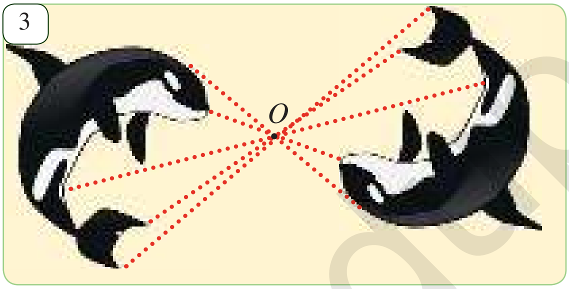
Nuqtaga nisbatan simmetriya – harakatdir.

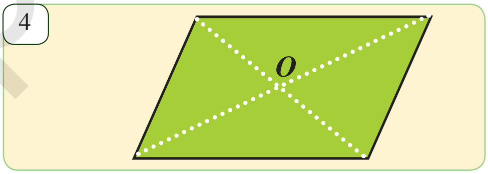
Agar F shakl O nuqtaga nisbatan simmetrik almashtirishda o'ziga ko'chsa, u *markaziy simmetrik shakl* deb ataladi.

Masalan, parallelogramm (4- rasm) diagonallari kesishish nuqtasi O ga nisbatan markaziy simmetrik shakl hisoblanadi.





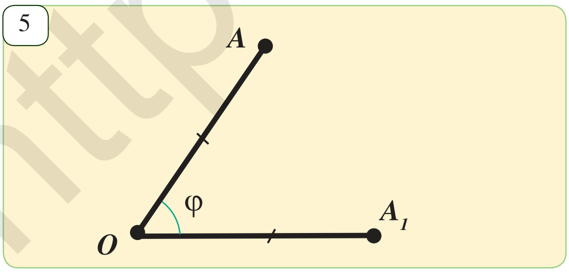


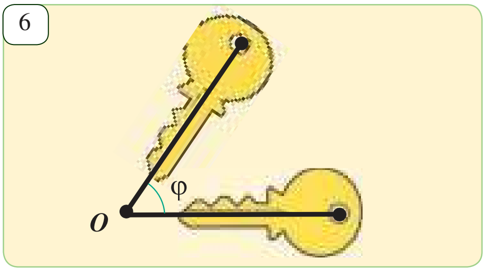


1-masala. O (2; 4) nuqtaga nisbatan simmetriyada A (1; 2) nuqta qaysi nuqtaga o'tadi?

Yechish. A_1 $(x; y)$ izlanayotgan nuqta bo'lsin. Ta'rifga ko'ra, O nuqta AA_1 kesmaning o'rtasi. Demak, $2 = (x+1)/2$, $4 = (y+2)/2$.

Bu tengliklardan $x = 4 - 1 = 3$, $y = 8 - 2 = 6$. *Javob:* A_1 (3; 6).





Aytaylik, tekislikda O nuqta va φ burchak berilgan bo'lib, shakl almashtirishda tekislikning ixtiyoriy A nuqtasi shunday A_1 nuqtaga ko'chsinki, $OA = OA_1$ va $\angle AOA_1 = \varphi$ bo'lsin. Bunday shakl almashtirish tekislikni O nuqta atrofida φ burchakka *burish* deb ataladi (5-rasm).

Agar tekislikda berilgan F shaklning har bir nuqtasini O nuqtaga nisbatan φ burchakka bursak, yangi F_1 shakl hosil bo'ladi. Bunda F shakl O nuqtaga nisbatan φ burchakka burishda F_1 shaklga o'tdi deyiladi. 6-rasmda kalit rasmi va uni biror burchakka burishda hosil bo'lgan shakl kerltirilgan.

Nuqtaga nisbatan burish ham harakat bo'ladi.

O nuqtaga nisbatan 180° burchakka burish O nuqtaga nisbatan markaziy simmetriyadan iborat bo'ladi.

Koordinatalari bilan berilgan $A\ (x;\ y)$ nuqta koordinata boshiga nisbatan simmetriyada $A_1(-x;\ -y)$ nuqtaga o'tadi: $A\ (x;\ y) \longrightarrow A_1(-x;\ -y)$.

Tabiatda simmetriyani har qadamda uchratish mumkin. Masalan, jonli mavjudodlarning ko'pchiligi, xususan, inson va hayvonlar gavdasi, o'simliklarning barglari va gullari simmetrik tuzilgan (7- rasm). Shuningdek, jonsiz tabiat unsurlari ham borki, masalan qor zarralari, tuz kristallari, moddalarning molekulyar tuzilishi ham ajoyib simmetrik shakllardan iboratdir. Tabiatdagi bu go'zallik va mukammallikdan andoza olgan quruvchi, injener va arxitektor kabi ijodkorlar yaratgan ko'plab inshoot va binolar, qurulma va mexanizmlar, texnika va transport vositalari ham simmetrik yaratilgan.

7

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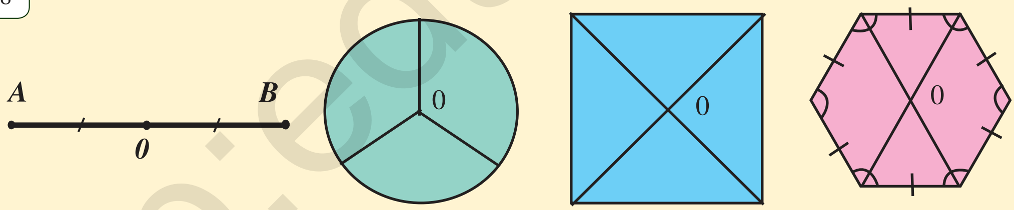


Masala va topshiriqlar

17.1. O (-2; 3) nuqtaga nisbatan markaziy simmetriyada A (4; 2) nuqta qaysi nuqtaga o'tadi?

17.2. 8– rasmda tasvirlangan shakllarda O nuqta simmetriya markazi ekanligini asoslang.

17.3. (-2; 5), (2; 2), (-6; 12) nuqtalar koordinata boshiga nisbatan markaziy simmetriyada qaysi nuqtalarga o'tadi?

17.4. Markaziy simmetriyaning harakat ekanligini isbotlang.

17.5. Parallelogrammning (4- rasm) diagonallari kesishish nuqtasi O ga nisbatan markaziy simmetrik shakl ekanligini isbotlang.

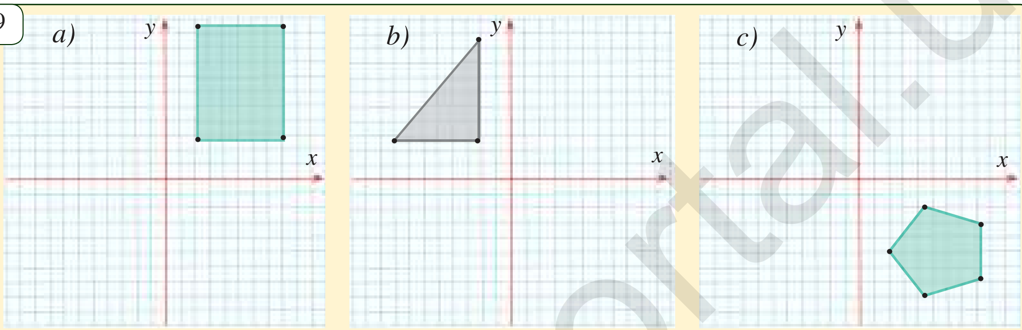
17.6. To‘g‘ri to‘rtburchak, kvadrat, parallelogramm, burchak, to‘g‘ri chiziq va teng yonli uchburchaklarning qaysi birlari markaziy simmetrik shakldan iborat bo‘ladi? Ularning simmetriya markazi qayerda joylashgan?

17.7. Ixtiyoriy AB kesma va unda yotmagan M nuqta chizing. AB kesmaga M nuqtaga nisbatan simmetrik bo‘lgan A_1B_1 kesmani tasvirlang.

17.8. Ixtiyoriy ABC uchburchak chizing. a) C uchiga nisbatan; b) medianalari kesishish nuqtasiga nisbatan simmetrik bo‘lgan uchburchakni tasvirlang.

17.9. Koordinata tekisligida uchlari A (3; 2), B (2; 7), C (6; 7) va D (6; 2) nuqtalarda bo‘lgan $ABCD$ parallelogrammga koordinata boshi O(0, 0) nuqtaga nisbatan simmetrik bo‘lgan $A_1B_1C_1D_1$ parallelogrammni tasvirlang.

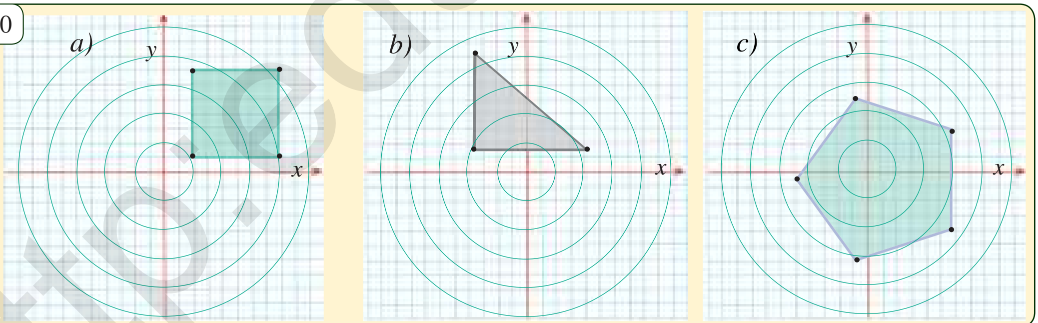
9



17.10. 9- rasmdagi koordinatalar tekisligida tasvirlangan shakllarni daftaringizga ko‘chirib chizing. Shu koordinatalar tekisligida bu shakllarga koordinata boshiga nisbatan simmetrik bo‘lgan shakllarni yasang.

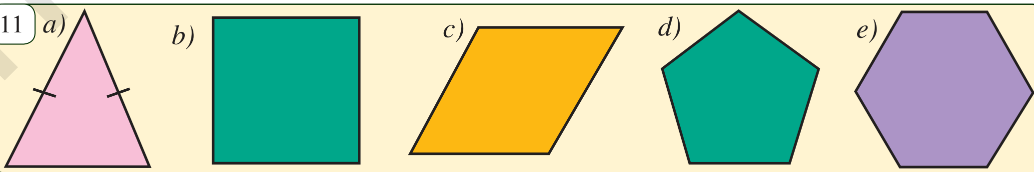
17.11. 10- rasmdagi koordinatalar tekisligida tasvirlangan shakllarni daftaringizga ko‘chirib chizing. Shu koordinatalar tekisligida kvadratni 90° ga, uchburchakni 180° ga va beshburchakni 120° ga buring.

10



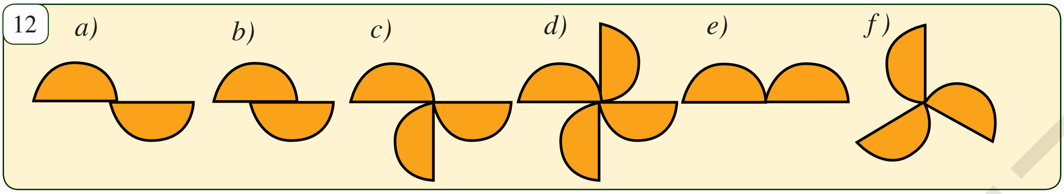
17.12. 11- rasmdagi ko‘pburchaklar qanday simmetriyaga ega ekanligini aniqlang. Ularning nechta simmetriya o‘qi bor?

11

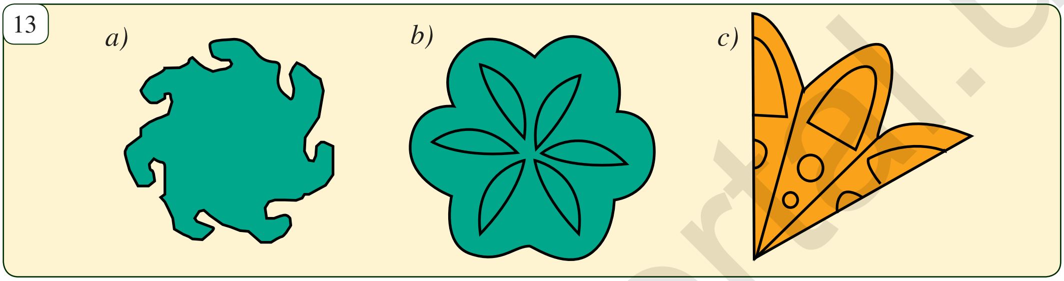


17.13. M, N, S, X, Z, V, T, Y, U, W, D, B, H, K, C, I, E, A harflari qanday simmetriyaga ega ekanligini aniqlang.

17.14. 12- rasmdagi shakllar bir nechta bir xil yarim doirachalardan tuzilgan. Bu shakllarni o‘zini-o‘ziga o‘tkazadigan burish bor yoki yo‘qligini aniqlang. Agar bor bo‘lsa, u qanday burish bo‘ladi?



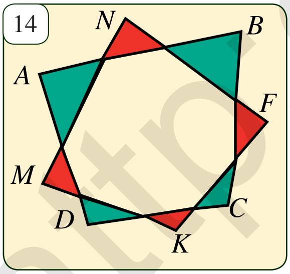
17.15. 13- rasmdagi shakllarning qaysilari simmetriya markaziga ega. Qanday burchakka burishda bu shakllar o‘ziga-o‘zi o‘tadi?

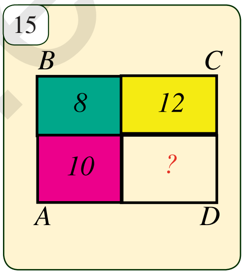


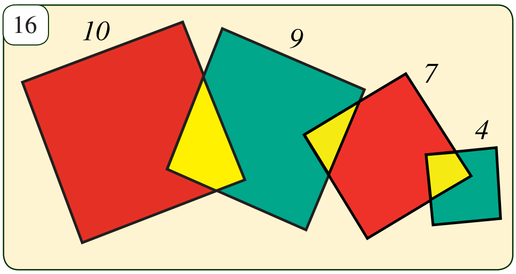
17.16. Ikkita *ABCD* va *MNPK* tengdosh ya’ni teng yuzga ega bo‘lgan to‘rtburchaklar bir-birining ustiga 14-rasmda ko‘rsatilgandek qilib qo‘yilgan. Qizil rangdagi uchburchak yuzlari yig‘indisi yashil rangga bo‘yalgan uchburchaklar yuzi yig‘indisiga tengligini ko‘rsating.

17.17. *ABCD* to‘g‘ri to‘rtburchak tomonlariga parallel to‘g‘ri chiziqlar bilan to‘rtta to‘g‘ri to‘rtburchakka bo‘lingan. 15-rasmda berilganlardan foydalanib, bo‘yalmagan to‘g‘ri to‘rtburchak yuzini toping.

17.18. 16-rasmdagi kvadratlarning tomonlari 10 *cm*, 9 *cm*, 7 *cm* va 4 *cm*. Qizil rangdagi kvadratlar yuzi yig‘indisi 112 cm^2 ga teng. Ko‘k rangdagi kvadratlar yuzi yig‘indisini toping.





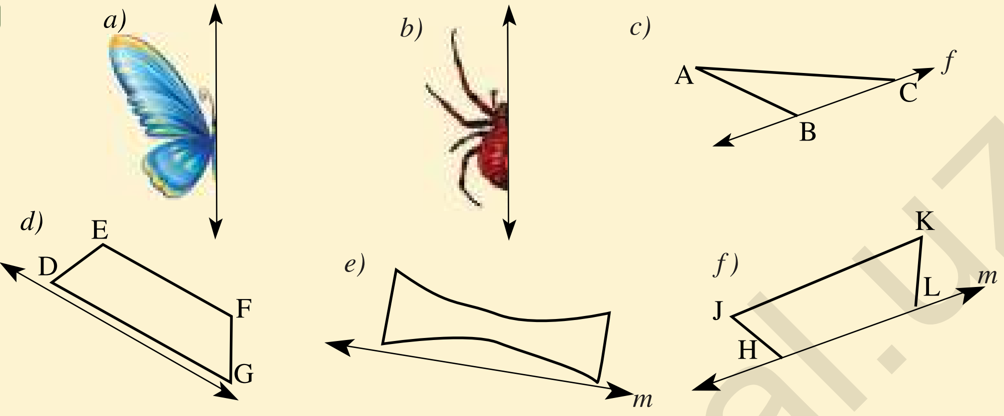


"Qor zarrachalari" loyiha ishi

Tabiatda barcha qor zarrachalari simmetrik shaklga ega bo‘ladi va bir-biriga o‘xshamaydi. Har bir qor zarrasi markaziga nisbatan 60° ga burishda o‘ziga-o‘zi o‘tadi. 60° ga burishda o‘ziga-o‘zi o‘tadigan shakllarni qog‘ozdan qanday qilib qirqib olish mumkin? Bir nechta turli shakldagi qor zarrachalarini qog‘ozdan qirqib oling.

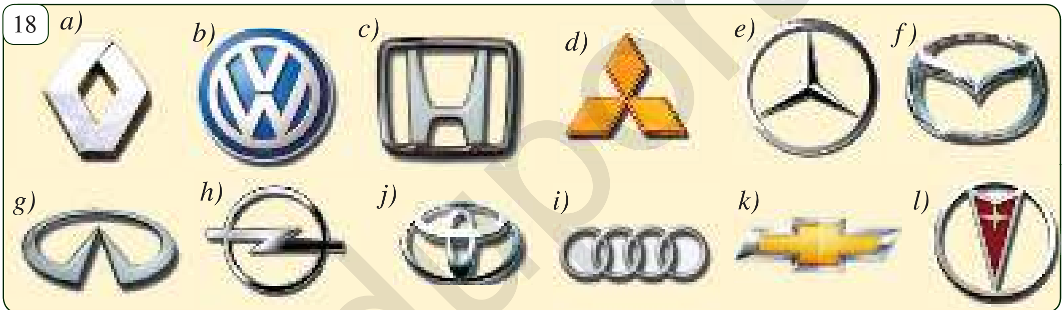
17.19. 17-rasmda tasvirlangan shakllarni daftaringizga chizib oling va berilgan o‘qqa nisbatan simmetrik aksini yasang.

17



17.20. 18-rasmda tasvirlangan avtomobil kompaniyalarining logotiplari qanday simmetriyaga ega ekanligini aniqlang.

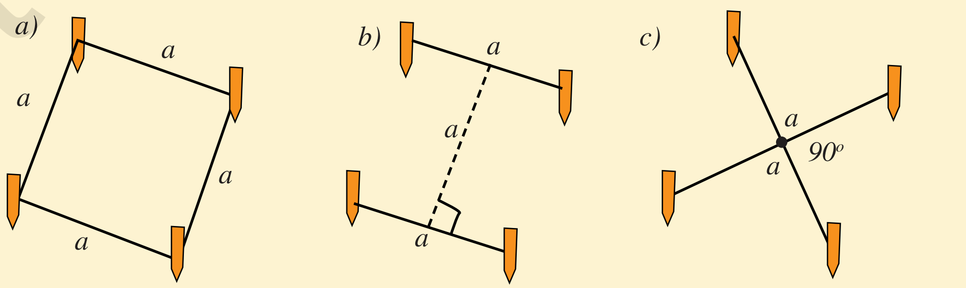
18



"Gulzorga geometriya" loyiha ishi.

Uch o‘rtoq Ali, Vali va Soli kvadrat shaklidagi gulzor yaratmoqchi. Ali kvadrat shaklidagi gulzorni 4 ta qoziqqa 4 ta bir xil uzunlikdagi iplarni tortib ajratmoqchi (19.a-rasm). Vali kvadrat shaklidagi gulzorni 2 ta bir xil uzunlikdagi iplarni qoziqlarga tortib, ularni parallel holda oralaridagi masofani ip uzunligiga teng qilib o‘rnatib ajratmoqchi (19.b-rasm). Soli esa 2 ta bir xil uzunlikdagi iplarning o‘rtalarini tugib, ularning o‘rtalari ustma-ust tushadigan va bir-biriga perpendikular qilib tortib qoziqlarga bog‘lab ajratmoqchi (19.c-rasm). Ayting-chi, ularning qaysi biri qo‘yilgan masalani to‘g‘ri hal qilgan? Nega?

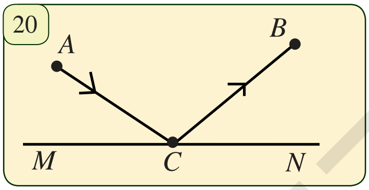
19



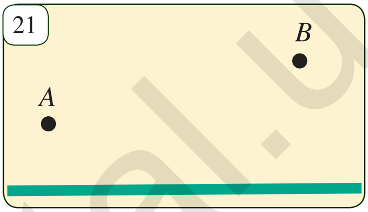
"Geometriya va optika" loyiha ishi.

XVII asrda buyuk fransuz matematik olimi Pyer Ferma quyidagi qonuniyatni kashf etdi: yorug‘lik nuri bir nuqtadan ikkinchi nuqtaga eng qisqa vaqt davomida yetib boradi.

1. Oynaning bir tomonidagi A va B nuqtalar berilgan. Yorug‘lik nuri A nuqtadan chiqib, oynaga urilib B nuqtadan o‘tdi (20-rasm). Ferma prinsipidan foydalanib, ACM (tushish burchagi) va BCN (qaytish burchagi) orasidagi munosabatni toping.

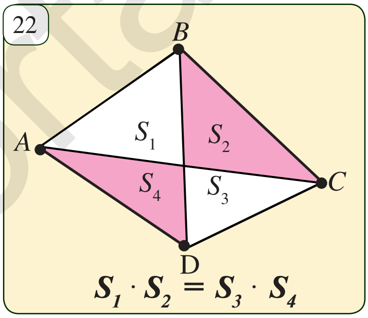


2. Daryoning qirg‘og‘idagi A nuqtada fermerning uyi va B nuqtada uning fermasi joylashgan (21-rasm). Fermer har kuni daryoga borib, idishlarga suv to‘ldirib fermasiga olib boradi. U bu ishni eng qisqa yo‘l bilan amalga oshirishi uchun qanday yo‘ldan yurgani ma’qul?



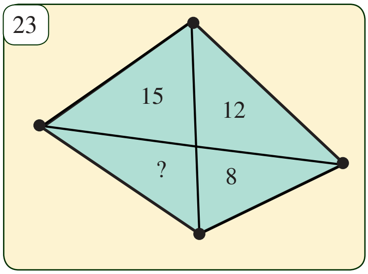
Qiziqarli geometriya

a) 22-rasmda ixtiyoriy qavariq to‘rtburchak tasvirlangan. To‘rtburchakning diagonallari uni to‘rtta uchburchakka ajratadi. Bu uchburchaklar yuzi uchun $S_1 \cdot S_2 = S_3 \cdot S_4$ bo‘lishini isbotlang.



Ko‘rsatma: o‘xshash shakllar xossalaridan foydalaning.

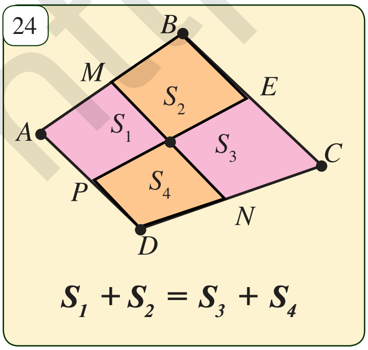
b) 23-rasmda berilganlardan foydalanib, noma’lum yuzani toping.

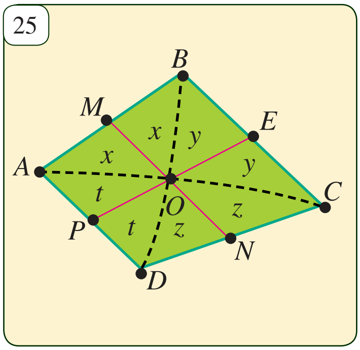


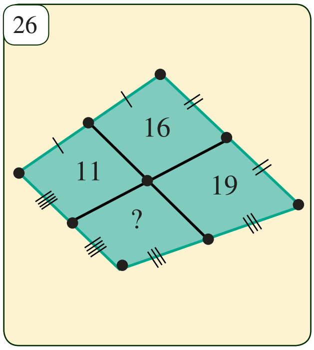
c) 24-rasmda ixtiyoriy qavariq to‘rtburchak tasvirlangan. To‘rtburchak qarama-qarshi tomonlarining o‘rtalari tutashtirilgan. Natijada to‘rtburchak to‘rtta to‘rtburchakka ajralgan. Bu to‘rtburchaklar yuzi uchun $S_1 + S_2 = S_3 + S_4$ bo‘lishini isbotlang.

Ko‘rsatma: isbotlash uchun 25-rasmdagi yordamchi shakldan foydalaning.

d) 26-rasmda berilganlardan foydalanib, noma’lum yuzani toping.

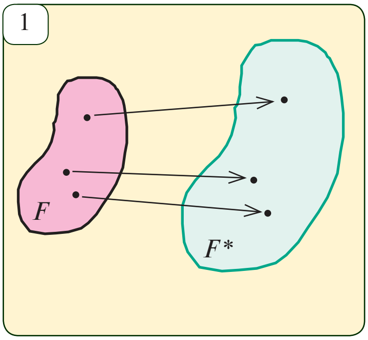






GEOMETRIK SHAKLLARNING O'XSHASHLIGI

1

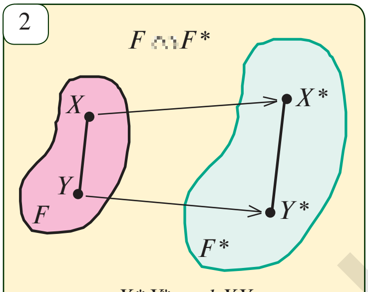


Oldingi darslarda ko'pburchaklarning o'xshashligi tushunchasi bilan tanishdik. Bu tushunchani faqat ko'pburchaklar uchun emas, balki istalgan geometrik shakllar uchun ham kiritish mumkin.

Agar F va F^* shakllar berilgan bo'lib, F shaklning har bir nuqtasiga F^* shaklning biror nuqtasi mos qo'yilgan bo'lsa va bunda F^* shaklning har bir nuqtasiga F shaklning faqat bitta nuqtasi mos kelsa, *(1-rasm)* *F shakl F* shaklga almashtirilgan* deyiladi.

Ta'rif. Agar F shaklni F^* shaklga almashtirishda nuqtalar orasidagi masofalar bir xil son marta o'zgarsa, bunday almashtirishga *o'xshashlik almashtirishi* deyiladi *(2-rasm)*.

2



$X^*Y^* = kXY$

k — o'xshashlik koeffitsiyenti

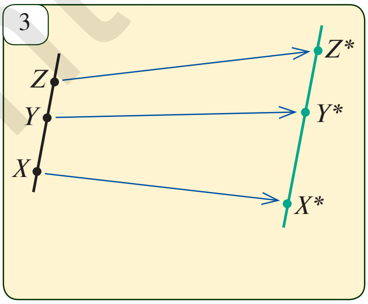
Bu ta'rifni quyidagicha talqin qilish mumkin: Aytaylik, biror almashtirish natijasida F shaklning ixtiyoriy X, Y nuqtalariga F^* shaklning X^*, Y^* nuqtalari mos qo'yilgan bo'lsin. Agar $\boldsymbol{X^*Y^* = k \cdot XY}$, $k>0$ bo'lsa, bunday almashtirishga *o'xshashlik almashtirishi* deyiladi. Bunda k — barcha X va Y nuqtalar uchun bir xil son bo'lib, u o'xshashlik koeffitsiyenti deb yuritiladi.

Agar F va F^* shakllar berilgan bo'lib, bu shakllardan birining ikkinchisiga o'tkazadigan o'xshashlik almashtirishi mavjud bo'lsa, F va F^* shakllar o'zaro **o'xshash** deyiladi. Shakllarning o'xshashligi $\boldsymbol{F \backsim F^*}$ kabi yoziladi. Agar o'xshashlik koeffitsiyenti k ni ham ko'rsatish lozim bo'lsa, $\boldsymbol{F \overset{k}{\backsim} F^*}$ tarzda ham belgilanadi.

Agar o'xshashlik almashtirishida X nuqtaga X^* nuqta mos qo'yilgan bo'lsa, *X nuqta X* nuqtaga almashdi* yoki *o'tdi* deyiladi.

Teorema. ***O'xshashlik almashtirishi a) to'g'ri chiziqni to'g'ri chiziqqa; b) nurni nurga; d) burchakni (uning kattaligini saqlagan holda) burchakka; e) kesmani (uzunligi bu kesmadan k marta uzun bo'lgan) kesmaga o'tkazadi.***

3



Isbot. a) O'xshashlik koeffitsiyenti k bo'lgan almashtirishda bir to'g'ri chiziqda yotgan turli X, Y va Z nuqtalar mos ravishda X^*, Y^* va Z^* nuqtalarga almashsin *(3-rasm)*.

X, Y, Z nuqtalardan biri, aytaylik, Y qolgan ikkitasining orasida yotsin. U holda $XZ = XY + YZ$. O'xshashlik almashtirishi ta'rifiga ko'ra:

$$X^*Z^* = k \cdot XZ = k \cdot (XY + YZ) = k \cdot XY + k \cdot YZ = X^*Y^* + Y^*Z^*.$$

Bu tenglikdan X^*, Y^* va Z^* nuqtalarning bir to‘g‘ri chiziqda yotishi kelib chiqadi.

Teoremaning isbotini faqat a) tasdiq uchun keltirdik. Qolgan tasdiqlarda isbotlashni sizga mashq tariqasida qoldiramiz.

Masala va topshiriqlar

18.1. O‘xshashlik almashtirishi nima?

18.2. Qanday shakllar o‘xshash deyiladi?

18.3. Eni 3 *cm*, bo‘yi 4 *cm* bo‘lgan to‘g‘ri to‘rtburchakka o‘xshash, o‘xshashlik koeffitsiyenti 2 ga teng bo‘lgan to‘rtburchak yasang.

18.4. 4-rasmda maktab hovlisining tarxi 1:1000 masshtabda tasvirlangan. O‘lchash ishlarini bajarib,
a) hovlining; b) maktab binosining;
d) gulzorlarning; e) sport maydonining;
f) bog‘ning haqiqiy o‘lchamlarini toping.

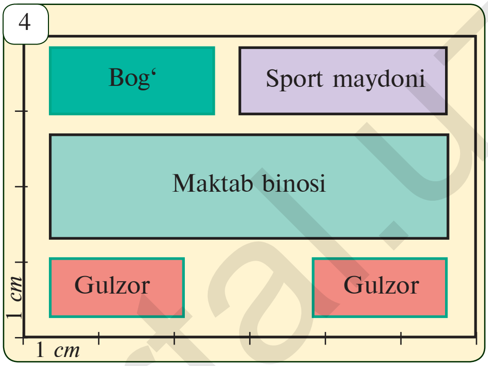
18.5. Agar xarita 1:50000 masshtabda tasvirlangan bo‘lsa *(5-rasm)*, Obod va Ozod qishloqlari markazlari orasidagi masofani toping.

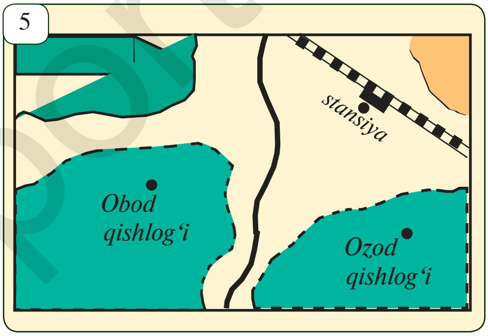
18.6. O‘xshashlik almashtirishida nurlar orasidagi burchak saqlanishini isbotlang.

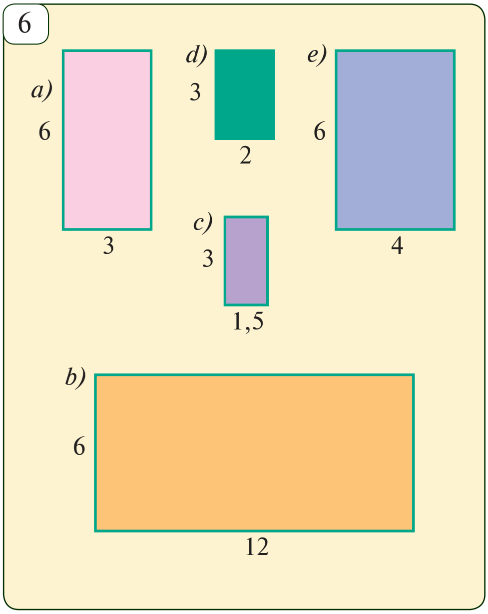
18.7*. O‘xshashlik almashtirishida a) parallelogramm parallelogrammga; b) kvadrat kvadratga; d) to‘g‘ri to‘rtburchak to‘g‘ri to‘rtburchakka; e) trapetsiya trapetsiyaga almashishini isbotlang.

18.8*. ABC uchburchak o‘xshashlik almashtirishida $A^*B^*C^*$ uchburchakka almashadi. Agar o‘xshashlik koeffitsiyenti 0,6 ga va ABC uchburchak perimetri 12 *cm* ga teng bo‘lsa, $A^*B^*C^*$ uchburchak perimetrini toping.

18.9. 6-rasmdan o‘xshash to‘g‘ri to‘rtburchaklar juftliklarini toping va o‘xshashlik koeffitsiyentlarini aniqlang.







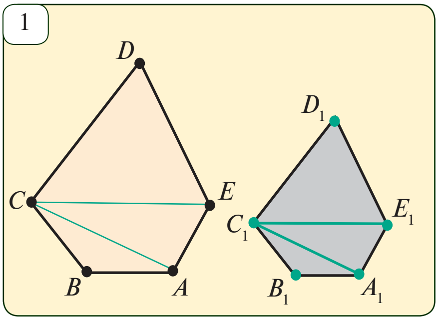
1-teorema. ***O‘xshash ko‘pburchaklar perimetrlarining nisbati o‘xshashlik koeffitsiyentiga teng.***

Isbot. Haqiqatan ham, $A_1A_2...A_n$ va $B_1B_2...B_n$ ko‘pburchaklar o‘xshash va o‘xshashlik koeffitsiyenti k bo‘lsa, $B_1B_2=k\cdot A_1A_2$, $B_2B_3=k\cdot A_2A_3$, ... , $B_nB_1=k\cdot A_nA_1$ bo‘ladi. Bundan

$$P=B_1B_2+B_2B_3+...+B_nB_1=k\cdot A_1A_2+k\cdot A_2A_3+...+k\cdot A_nA_1=k\cdot(A_1A_2+A_2A_3+...+A_nA_1)=k\cdot P_1$$

tenglikni hosil qilamiz. *Teorema isbotlandi.*

2-teorema. ***O‘xshash ko‘pburchaklarni bir xil sondagi o‘xshash uchburchaklarga ajratish mumkin.***



Isbot. Aytaylik, $ABCDE$ va $A_1B_1C_1D_1E_1$ ko‘pburchaklar o‘xshash bo‘lib, o‘xshashlik koeffitsiyenti k bo‘lsin.

O‘zaro mos C va C_1 uchlardan CA, CE va C_1A_1, C_1E_1 diagonallarni o‘tkazamiz (1-rasm). Natijada, ko‘pburchaklar bir xil sondagi uchburchaklarga ajraldi. Hosil bo‘lgan uch juft mos uchburchaklarning o‘xshashligini ko‘rsatamiz.

1. $\Delta ABC \sim \Delta A_1B_1C_1$. Chunki, bu uchburchaklarda, shartga ko‘ra, $\angle B=\angle B_1$, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\mathrm{k}$. Uchburchaklar o‘xshashligining TBT alomatiga ko‘ra,

$\Delta ABC \sim \Delta A_1B_1C_1$.

2. $\Delta CDE \sim \Delta C_1D_1E_1$. Bu o‘xshashlik 1-banddagi kabi isbotlanadi.

3. $\Delta ACE \sim \Delta A_1C_1E_1$. Haqiqatan, $\angle CAE$ va $\angle C_1A_1E_1$ burchaklarni qaraymiz:

$$\angle CAE=\angle BAE-\angle CAB,\ \angle C_1A_1E_1=\angle B_1A_1E_1-\angle C_1A_1B_1.$$

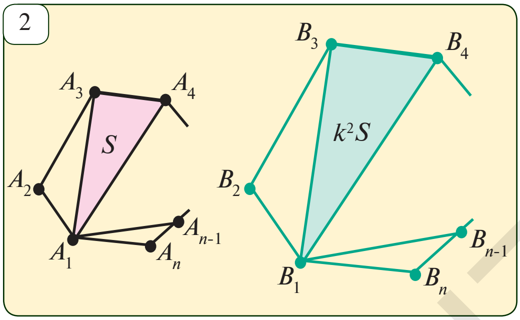
Bu yerda, $\angle BAE=\angle B_1A_1E_1$ (berilgan o‘xshash beshburchaklarning mos burchaklari). $\angle CAB=\angle C_1A_1B_1$ (o‘xshash ABC va $A_1B_1C_1$ uchburchaklarning mos burchaklari).

Demak, $\angle CAE=\angle C_1A_1E_1$.

AC va AE hamda A_1C_1 va A_1E_1 tomonlarni qaraymiz: $AC=kA_1C_1$, chunki ular o‘zaro o‘xshash ABC va $A_1B_1C_1$ uchburchaklarning mos tomonlari, $AE=kA_1E_1$, chunki ular ham berilgan o‘xshash beshburchaklarning mos tomonlari. Demak, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta ACE \sim \Delta A_1C_1E_1$. Ixtiyoriy o‘xshash ko‘pburchaklar uchun ham shu kabi mushohadalar o‘rinli bo‘lishi ravshan. *Teorema isbotlandi.*

3-Teorema. ***O‘xshash ko‘pburchaklar yuzlarining nisbati o‘xshashlik koeffitsiyentining kvadratiga teng.***

Isbot. Aytaylik, $A_1A_2...A_n$ va $B_1B_2...B_n$ koʻpburchaklar oʻxshash va k — oʻxshashlik koeffitsiyenti boʻlsin. U holda $A_1A_2A_3$, $A_1A_3A_4$, …, $A_1A_{n-1}A_n$ uchburchaklar mos ravishda, $B_1B_2B_3$, $B_1B_3B_4$, …, $B_1B_{n-1}B_n$ uchburchaklarga oʻxshash boʻlib, oʻxshash uchburchaklar yuzlarining nisbati k^2 ga teng boʻladi (*2-rasm*):



$S_{A_1A_2A_3} = k^2S_{B_1B_2B_3}$, $S_{A_1A_3A_4} = k^2S_{B_1B_3B_4}$, … , $S_{A_1A_{n-1}A_n} = k^2S_{B_1B_{n-1}B_n}$.

Bu tengliklarning mos qismlarini qoʻshsak,

$S_{A_1A_2...A_n} = k^2\,S_{B_1B_2...B_n}$ boʻladi. *Teorema isbotlandi.*

Masala. Perimetrlari 18 *cm* va 24 *cm* boʻlgan ikkita oʻxshash koʻpburchak yuzlarining nisbatini toping.

Yechish. 1) Oʻxshash koʻpburchaklar perimetrlarining nisbati oʻxshashlik koeffitsiyentiga teng ekanligidan foydalanib, $k = 24:18 = 4:3$ ekanligini topamiz.

2) Oʻxshash koʻpburchaklar yuzlarining nisbati oʻxshashlik koeffitsiyentining kvadratiga teng boʻlgani uchun izlangan nisbat $k^2 = \frac{16}{9}$ ga teng. *Javob:* $\frac{16}{9}$.

Masala va topshiriqlar

19.1. Oʻxshash koʻpburchaklar perimetrlarining nisbati nimaga teng?

19.2. Oʻxshash koʻpburchaklar yuzlarining nisbati haqidagi teoremani sharhlang.

19.3. Uchburchak bilan toʻrtburchak oʻxshash boʻlishi mumkinmi?

19.4. Yuzlari 6 m^2 va 24 m^2 boʻlgan ikkita toʻrtburchak oʻxshash. Oʻxshashlik koeffitsiyentini toping.

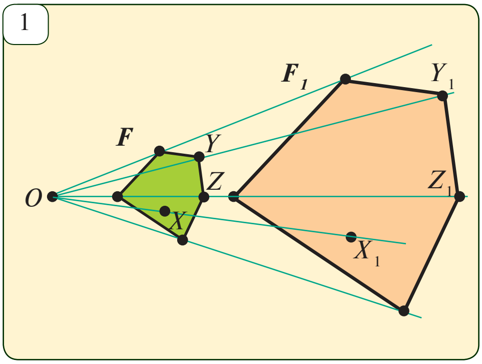
19.5. Ikkita koʻpburchakning perimetrlari 18 *cm* va 36 *cm* ga, yuzlarining yigʻindisi esa 30 cm^2 ga teng. Koʻpburchaklar yuzlarini toping.

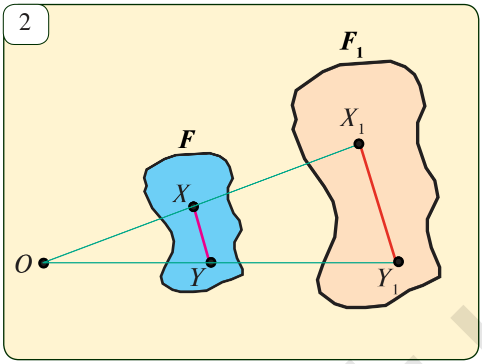
19.6. Perimetri 84 *cm* boʻlgan uchburchakning bir tomoniga parallel qilib oʻtkazilgan toʻgʻri chiziq undan perimetri 42 *cm* ga va yuzi 26 cm^2 ga teng uchburchak ajratdi. Berilgan uchburchak yuzini toping.

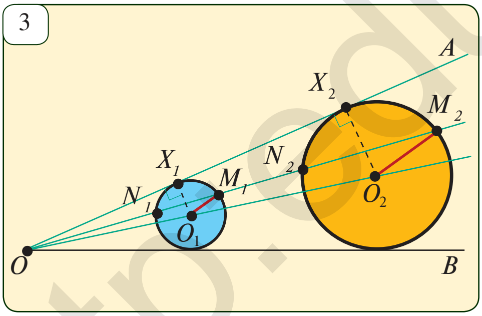
19.7. O nuqtaga nisbatan simmetrik shakllar oʻxshash boʻladimi? Oʻqqa nisbatan simmetrik shakllar-chi? Ularning oʻxshashlik koeffitsiyenti nimaga teng?

19.8. Toʻrtburchak shaklidagi paxta maydoni xaritada yuzi 12 cm^2 boʻlgan toʻrtburchak bilan tasvirlanadi. Agar xarita masshtabi 1:1000 boʻlsa, maydonning haqiqiy yuzini hisoblang.

19.9*. Yuzlari 8 cm^2 va 32 cm^2 boʻlgan ikkita oʻxshash uchburchak perimetrlarining yigʻindisi 48 *cm* ga teng. Uchburchaklarning perimetrlarini toping.







Eng sodda o‘xshash almashtirishlardan biri gomotetiyadir. Aytaylik, F — shakl, O — nuqta va k — musbat son berilgan bo‘lsin. F shaklning istalgan X nuqtasi orqali OX nur o‘tkazamiz va bu nurda uzunligi $k{\cdot}OX$ bo‘lgan OX_1 kesmani qo‘yamiz *(1-rasm)*. Shu usul bilan F shaklning har bir X nuqtasiga X_1 nuqtani mos qo‘yadigan almashtirish *gomotetiya* deyiladi. Bunda, O nuqta gomotetiya markazi, k soni gomotetiya koeffitsiyenti, F va gomotetiya natijasida F shakl almashadigan F_1 shakllar esa *gomotetik shakllar* deyiladi.

Teorema. Gomotetiya o‘xshashlik almashtirishi bo‘ladi.

Isbot. Ixtiyoriy O markazli, k koeffitsiyentli gomotetiyada F shaklning X va Y nuqtalari X_1 va Y_1 nuqtalarga o‘tsin *(2-rasm)*. U holda, gomotetiya ta’rifiga ko‘ra, XOY va X_1OY_1 uchburchaklarda $\angle O$ — umumiy va $\frac{OX_1}{OX}=\frac{OY_1}{OY}=k$ bo‘ladi.

Demak, XOY va X_1OY_1 uchburchaklar ikki tomoni va ular orasidagi burchagi bo‘yicha o‘xshash.

Shuning uchun $\frac{X_1Y_1}{XY}=\frac{OX_1}{OX}$, xususan, $X_1Y_1=k\cdot XY$

Teorema isbotlandi.

Masala. AOB burchak tomonlariga urinuvchi ixtiyoriy ikki aylana gomotetik bo‘lishini va O nuqta bu gomotetiya uchun markaz ekanligini isbotlang.

Isbot. Markazlari O_1 va O_2 bo‘lgan aylanalar AOB burchak tomonlariga urinsin *(3-rasm)*. Bu aylanalarning gomotetik ekanligini isbotlaymiz.

Aylanalar OA nurga mos ravishda X_1va X_2 nuqtalarda uringan bo‘lsin *(3-rasm)*.

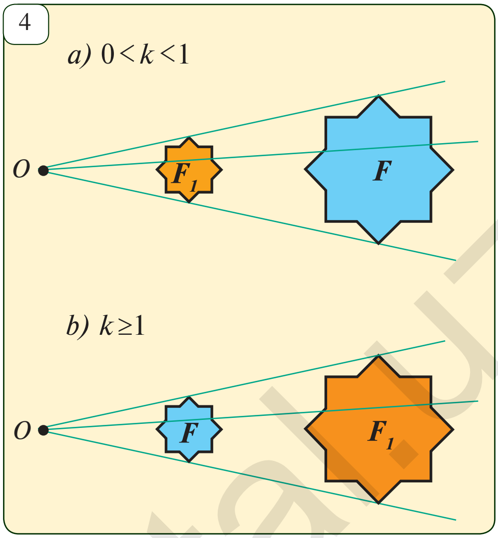
U holda, $\Delta OX_1O_1 \sim \Delta OX_2O_2$, chunki

$$\angle X_1OO_1=\angle X_2OO_2 \quad \text{va} \quad \angle OX_1O_1=\angle OX_2O_2=90^\circ.$$

Bundan, $\frac{O_2X_2}{O_1X_1}=\frac{OO_2}{OO_1}$.

O‘ng tomondagi nisbatni k bilan belgilaymiz va koeffitsiyenti $k = \dfrac{O_2X_2}{O_1X_1}$, markazi O bo‘lgan gomotetiyani qaraymiz. Aytaylik, bu gomotetiyada O_1 markazli aylananing istagan M_1 nuqtasi M_2 nuqtaga o‘tgan bo‘lsin. U holda, $O_2M_2 = k\ O_1M_1$ yoki $O_2M_2 = \dfrac{O_2X_2}{O_1X_1} \cdot O_1M_1$.

Bundan, $O_1X_1 = O_1M_1$ bo‘lgani uchun $O_2M_2 = O_2X_2$ tenglikni hosil qilamiz. Bu M_2 nuqta markazi O_2 nuqtada, radiusi O_2X_2 ga teng bo‘lgan aylanada yotishini bildiradi. Demak, qaralayotgan aylanalar o‘zaro gomotetik ekan.



Faollashtiruvchi mashq

4-rasmda gomotetiya koeffitsiyenti a) $0<k<1$; b) $k \geq 1$ bo‘lgan gomotetik shakllar tasvirlangan. Gomotetiya koeffitsiyentining qiymatiga qarab gomotetik shakllarning “siqilishi” yoki “cho‘zilishi” haqida qanday xulosa chiqarish mumkin?

Masala va topshiriqlar

20.1. Gomotetiya nima? Gomotetiya markazi, koeffitsiyenti-chi?

20.2. Gomotetiya o‘xshashlik almashtirishi ekanligini izohlang.

20.3. Uchburchak chizing. Uchburchak a) ichki sohasida; b) tashqi sohasida O nuqta belgilang va koeffitsiyenti 2 ga teng bo‘lgan O markazli gomotetiyani qarab, berilgan uchburchakka gomotetik uchburchak yasang.

20.4. Perimetrlari 18 *cm* va 27 *cm* bo‘lgan ikkita romb o‘zaro gomotetik. Bu romblar tomonlari va yuzlarining nisbatlarini toping.

20.5. Gomotetiyada X nuqta X_1 nuqtaga, Y nuqta Y_1 nuqtaga o‘tadi. Agar X, X_1, Y, Y_1 nuqtalar bir to‘g‘ri chiziqda yotmasa, shu gomotetiya markazini toping.

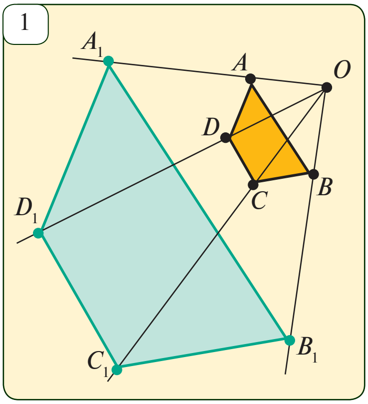
20.6. Koeffitsiyenti 2 ga teng bo‘lgan gomotetiyada X nuqta X_1 nuqtaga o‘tishi ma’lum. Shu gomotetiya markazini yasang.

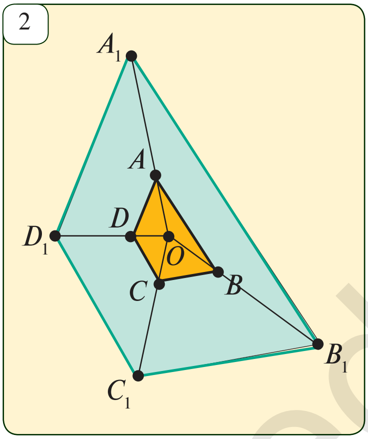
20.7. Aylanaga gomotetik shakl aylana bo‘lishini isbotlang.

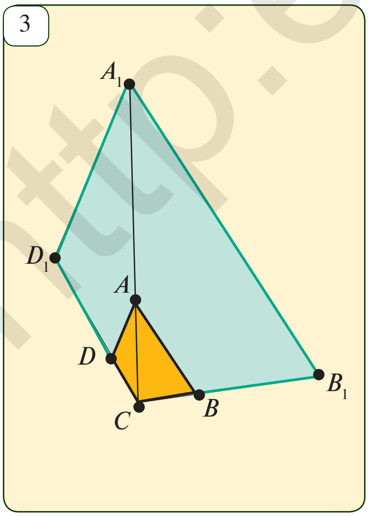
20.8. Aylana chizing. Markazi aylana markazida va koeffitsiyenti a) $\frac{1}{2}$; b) 2; d) 3; e) $\frac{1}{3}$ ga teng bo‘lgan gomotetiyada chizilgan aylanaga gomotetik bo‘lgan shakllarni quring.

20.9. Burchak va uning ichki sohasida A nuqta berilgan. Burchak tomonlariga urinib, A nuqtadan o‘tuvchi aylana yasang.

O‘XSHASH KO‘PBURCHAKLARNI YASASH







Shu paytgacha teoremalarni isbotlashda va masalalarni yechishda turli o‘xshash uchburchaklarni yasab keldik. O‘xshash ko‘pburchaklar qanday yasaladi? Quyida shu bilan tanishasiz.

Masala. Berilgan $ABCD$ to‘rtburchakka o‘xshash, o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan $A_1B_1C_1D_1$ to‘rtburchak yasang *(1-rasm)*.

Yasash. Tekislikda ixtiyoriy O nuqtani olamiz. Undan va to‘rtburchakning uchlaridan o‘tuvchi OA, OB, OC va OD nurlarni o‘tkazamiz. Bu nurlarda O nuqtadan $OA_1=3OA$, $OB_1=3OB$, $OC_1=3OC$ va $OD_1=3OD$ kesmalarni qo‘yamiz. Hosil bo‘lgan $A_1B_1C_1D_1$ to‘rtburchak izlangan to‘rtburchakdir.

Asoslash. $ABCD \sim A_1B_1C_1D_1$ ekanligini isbotlaymiz.

1. *Mos tomonlarning proporsionalligi.*

a) $\Delta AOD \sim \Delta A_1OD_1 \Rightarrow \dfrac{A_1D_1}{AD}=\dfrac{OD_1}{OD}=\dfrac{OA_1}{OA}=3;$ (1)

b) $\Delta DOC \sim \Delta D_1OC_1 \Rightarrow \dfrac{OD_1}{OD}=\dfrac{D_1C_1}{DC}=\dfrac{OC_1}{OC}=3.$ (2)

(1) va (2) tenglikdan $\dfrac{A_1D_1}{AD}=\dfrac{D_1C_1}{DC}$ ekanligini hosil qilamiz.

To‘rtburchaklarning boshqa mos tomonlari proporsionalligini xuddi shunga o‘xshash isbotlash mumkin.

2. *Mos burchaklarning tengligi.*

O‘xshash uchburchaklarning mos burchaklari teng bo‘lgani uchun, $\angle A_1D_1O=\angle ADO$, $\angle C_1D_1O=\angle CDO$. U holda,

$$\angle A_1D_1C_1=\angle A_1D_1O+\angle C_1D_1O=$$
$$=\angle ADO+\angle CDO=\angle ADC,$$

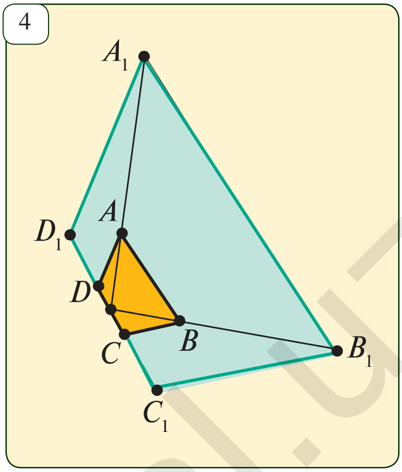
ya’ni to‘rtburchaklarning mos $A_1D_1C_1$ va ADC burchaklari teng.

Xuddi shunga o‘xshash to‘rtburchaklarning boshqa mos burchaklari tengligi isbotlanadi.

Demak, $ABCD$ va $A_1B_1C_1D_1$ to‘rtburchaklar o‘xshash.

Tomonlari ixtiyoriy sonda bo‘lgan ko‘pburchakka o‘xshash ko‘pburchak ham xuddi shu kabi yasaladi.

Gomotetiya markazini bu masalada to‘rtburchak tashqi sohasidan tanladik. Umuman olganda, gomotetiya markazini to‘rtburchakning ichki sohasida *(2-rasm)*, biror uchida *(3-rasm)* yoki biror tomonida *(4-rasm)* yotadigan qilib tanlashimiz ham mumkin edi. Gomotetiya markazini qayerda olmaylik, berilgan $ABCD$ to‘rtburchakka o‘xshash va o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan to‘rtburchaklar o‘zaro teng bo‘ladi.

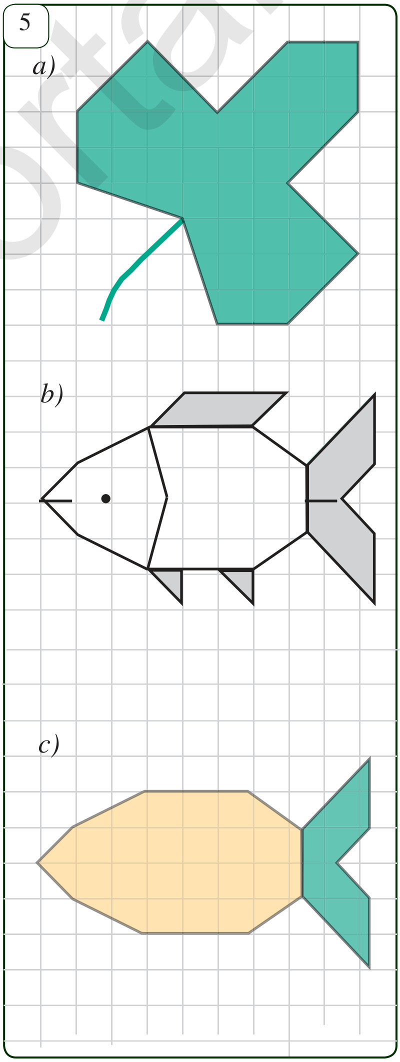


Masala va topshiriqlar

21.1. Berilgan ko‘pburchakka o‘xshash ko‘pburchakni yasash ketma-ketligini ayting.

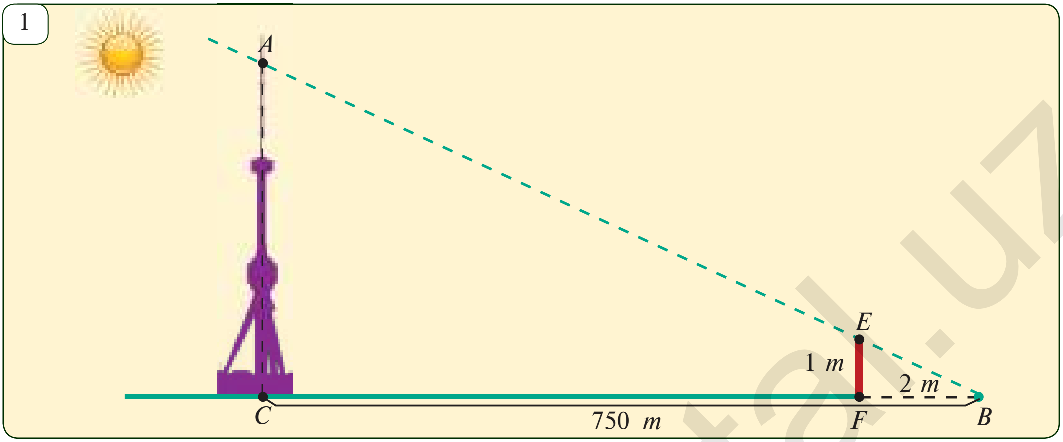
21.2. Daftaringizga biror $ABCDE$ beshburchak chizing. Gomotetiya yordamida bu beshburchakka o‘xshash, o‘xshashlik koeffitsiyenti 0,5 ga teng bo‘lgan beshburchak yasang. Gomotetiya markazi a) C nuqtada; b) beshburchak ichida; d) AB tomonda bo‘lgan hollarni alohida ko‘ring.

21.3. Kataklarni inobatga olgan holda, 5-rasmda berilgan shakllarni daftaringizga chizing: a) yaproqqa o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan yaproqni; b) baliqchaga o‘xshashlik koeffitsiyenti 0,8 ga teng bo‘lgan baliqchani c)sabziga o‘xshashlik koeffitsiyenti 1,8 ga teng bo‘lgan sabzini gomotetiya yordamida chizing.



21.4. F_1 ko‘pburchak F_2 ko‘pburchakka o‘xshash, k — o‘xshashlik koeffitsiyenti. P_1, P_2, S_1, S_2 harflar bilan mos ravishda bu ko‘pburchaklarning perimetrlari va yuzlari belgilangan. Quyidagi jadvalni daftaringizga ko‘chiring va uni to‘ldiring.

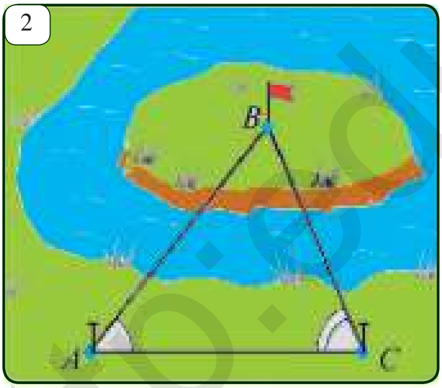
	P_1	P_2	S_1	S_2	k
a)	84		100	25	
b)	14	28		48	
d)		150	200	100	
e)		30	24		3

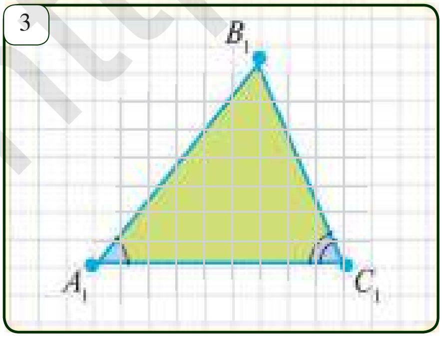


1. *Balandlikni aniqlash.*

Yerda turib, Toshkent teleminorasining balandligini topaylik. Minoraning uchi — A nuqtaning soyasi B nuqta bo'lsin. EF tayoqni vertikal tarzda shunday qoqamizki (*1-rasm*), tayoqning E uchi soyasi ham B nuqtada bo'lsin. Minoraning asosini C bilan belgilaymiz. Hosil bo'lgan, to'g'ri burchakli ABC va EBF uchburchaklar o'xshash bo'ladi. Shuning uchun,

$$\frac{AC}{EF}=\frac{BC}{BF} \quad \text{yoki} \quad AC=\frac{AC\cdot EF}{BF}$$





BC, BF masofalarni va EF tayoq uzunligini o'lchab, hosil bo'lgan formuladan teleminora balandligi — AC kesma uzunligini topamiz. Masalan, agar $EF=1$ *m*, $BC=750$ *m*, $FB=2$ *m* ekani ma'lum bo'lsa, u holda $AC=375$ *m* bo'ladi.

2. *Borib bo'lmaydigan joygacha bo'lgan masofani o'lchash.*

Aytaylik, A nuqtadan borish mumkin bo'lmagan B nuqtagacha bo'lgan masofani aniqlash lozim bo'lsin *(2-rasm)*. A nuqtadan borib bo'ladigan shunday C nuqtani belgilaymizki, undan qaraganda A va B nuqtalar ko'rinib tursin hamda AC masofani o'lchab bo'lsin.

Asboblar yordamida BAC va ACB burchaklarni o'lchaymiz. Aytaylik, $\angle BAC=\alpha$ va $\angle ACB=\beta$ bo'lsin.Qog'ozga $\angle A_1=\alpha$, $\angle C_1=\beta$ bo'lgan $A_1B_1C_1$ uchburchak yasaymiz. Unda

ABC va $A_1B_1C_1$ uchburchaklar ikki burchagi bo'yicha o'xshash bo'ladi (*2- va 3-rasmlar*). Bundan,

$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \quad \text{yoki} \quad AB = \frac{AC \cdot A_1B_1}{A_1C_1}.$$

AC masofa va A_1B_1, A_1C_1 kesmalarni o'lchab, natijada hosil bo'lgan formula yordamida AB kesma hisoblanadi. Hisoblash ishlarini osonlashtirish maqsadida $AC:A_1C_1$ nisbatni 100:1, 1000:1 kabi nisbatda olish mumkin. Masalan, $AC=130$ m, $\angle A=73^\circ$, $\angle C=58^\circ$ bo'lsa, qog'ozda $A_1B_1C_1$ uchburchakni $\angle A_1=73^\circ$, $\angle C_1=58^\circ$, $A_1C_1=130$ mm qilib chizamiz. A_1B_1 kesmani o'lchab, uning 153 mm ekanligini topamiz. Unda, izlangan masofa 153 m bo'ladi.

3. *Ko'l haqida amaliy ish.*

4-rasmda suv havzasining kosmik kemadan olingan surati tasvirlangan. U asosida tegishli o'lchash va hisoblash ishlarini bajarib, suv havzasi yuzining taqribiy qiymatini toping.

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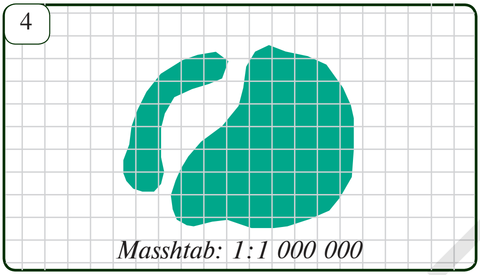

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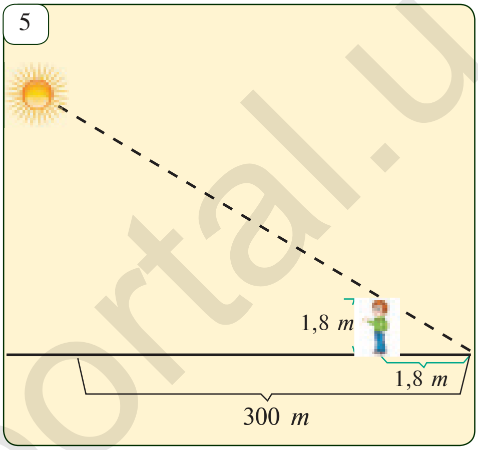

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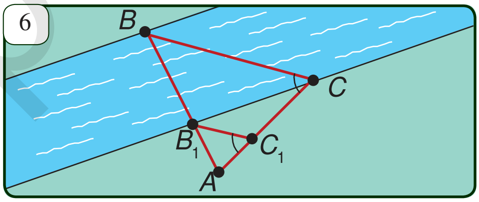

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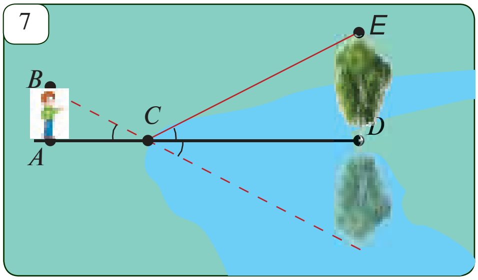


Masala va topshiriqlar

22.1. Agar bo'yi 1,7 m bo'lgan odam soyasining uzunligi 2,5 m bo'lsa, soyasining uzunligi 10,2 m bo'lgan daraxt balandligi qancha bo'ladi?

22.2. 5-rasmda tasvirlangan minora balandligini aniqlang.

22.3. 6-rasmdagi ikkita o'xshash AB_1C_1 va ABC uchburchaklar yordamida daryoning kengligini (enini) aniqlash zarur. Agar $AC=100$ m, $AC_1=32$ m va $AB_1=34$ m bo'lsa, daryoning eni (BB_1) ni toping.

22.4. Anhor qirg'og'idagi DE daraxtning suvdagi aksi A nuqtadagi odamga ko'rinayapti. Agar $AB=165$ cm, $AC=120$ cm, $CD=4,8$ m bo'lsa, daraxt balandligini toping *(7-rasm)*.

22.5. Hovlida biror daraxtni tanlang va uning balandligini aniqlang. Bu ishni qanday bajarganingiz haqida hisobot tayyorlang.

1-masala. $ABCD$ trapetsiyaning AB va CD yon tomonlarida M va N nuqtalar olingan. Bunda MN kesma trapetsiya asoslariga parallel va trapetsiya diagonallari kesishgan O nuqtadan o'tadi. Agar $BC = a$, $AD = b$ bo'lsa, a) MO; b) ON; d) MN kesmalarni toping (*1-rasm*).

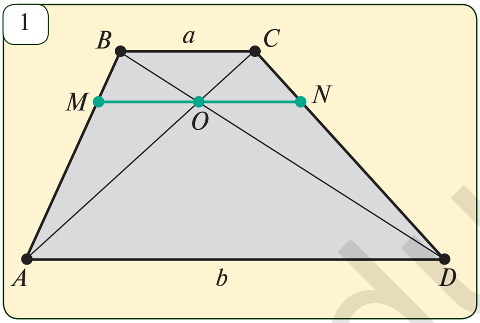
Yechish. 1) AOD va BOC uchburchaklar BB alomatga ko'ra o'xshash, chunki $\angle BOC = \angle AOD$, $\angle OBC = \angle ADO$. Bundan,

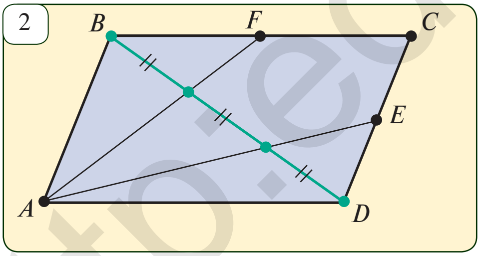
$$\frac{OC}{OA} = \frac{BC}{AD} \quad \text{yoki} \quad \frac{OC}{OA} = \frac{a}{b} \qquad (1)$$

2) ABC va AOM uchburchaklar ham BB alomatga ko'ra o'xshash, chunki $\angle AMO = \angle ABC$, $\angle ACB = \angle AOM$. Bundan,

$$\frac{AC}{OA} = \frac{BC}{MO} \quad \text{yoki} \quad \frac{OA+OC}{OA} = \frac{a}{MO} \Rightarrow 1+\frac{OC}{OA} = \frac{a}{MO}, \ \frac{OC}{OA} = \frac{a}{MO} - 1. \qquad (2)$$

3) (1) va (2) tengliklarning o'ng qismlarini tenglashtirib,





$$\frac{a}{MO} - 1 = \frac{a}{b}$$

tenglikni va undan

$$MO = \frac{ab}{a+b} \qquad (3)$$

ekanligini topamiz. Yuqoridagidek yo'l tutib

$$ON = \frac{ab}{a+b} \qquad (4)$$

tenglikni, keyin esa (3) va (4) tengliklarning mos tomonlarini qo'shib

$$MN = \frac{2ab}{a+b}$$

tenglikni hosil qilamiz.

Javob: a) $\frac{ab}{a+b}$; b) $\frac{ab}{a+b}$; d) $\frac{2ab}{a+b}$.

Eslatma. *Bu masala yechimidan $MO = ON$ ekanligi kelib chiqadi.*

Masala va topshiriqlar

23.1. ABC uchburchakning AB va BC yon tomonlarida D va E nuqtalar olingan. Agar $AC \| DE$, $AC = 6$, $DB = 3$ va $DE = 2$ bo'lsa, AB tomonni toping.

23.2. Ikkita o'xshash ko'pburchakning yuzlari 8 dm^2 va 72 dm^2 ga teng, ulardan birining perimetri ikkinchisinikidan 26 dm ga kam. Katta ko'pburchakning perimetrini toping.

23.3. Perimetri 1 m bo'lgan $A_1B_1C_1$ uchburchak $A_2B_2C_2$ uchburchakning tomonlari o'rtalarini, $A_2B_2C_2$ uchburchak $A_3B_3C_3$ uchburchak tomonlari o'rtalarini,

$A_3B_3C_3$ uchburchak esa $A_4B_4C_4$ uchburchak tomonlari oʻrtalarini tutashtirishdan hosil qilingan boʻlsa, $A_4B_4C_4$ uchburchakning perimetri qancha boʻladi?

3

23.4. Ikkita oʻxshash uchburchakning perimetrlari 18 *dm* va 36 *dm* ga, yuzlarining yigʻindisi 30 dm^2 ga teng. Katta uchburchakning yuzini toping.

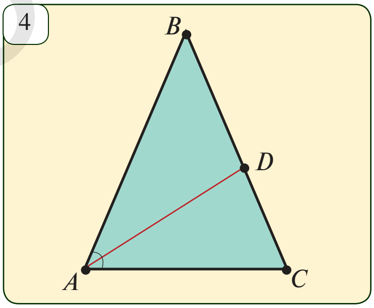
23.5. Romb tomonlarining oʻrtalari toʻgʻri toʻrtburchak uchlari boʻlishini isbotlang.

23.6. ABC uchburchak yasang. Bu uchburchakka oʻxshash va yuzi ABC uchburchak yuzidan 9 marta kichik boʻlgan $A_1B_1C_1$ uchburchakni yasang.

23.7*. E va F nuqtalar mos ravishda $ABCD$ parallelogramning CD va BC tomonlari oʻrtalari. AF va AE toʻgʻri chiziqlar BD diagonalni teng uch qismga boʻlishini isbotlang (*2-rasm*).

23.8. 3-rasmdaToshkent shahridagi Xalqlar doʻstligi saroyi oldida oʻrnatilgan eng katta Oʻzbekiston bayrogʻi tasvirlangan. Bayroqning oʻlchamlari 20 *m* x 30 *m* ekani ma’lum boʻlsa, chizmadan tegishli kesmalar uzunligini oʻlchab aniqlab, bayroq ustuning haqiqiy balandligini toping.

23.9. Teng yonli uchburchakning asosidagi burchak bissektrisasi bu uchburchakdan oʻziga oʻxshash uchburchak ajratadi. Uchburchak burchaklarini aniqlang (*4-rasm*, $AB=BC$, $\Delta ABC \sim \Delta CAD$).



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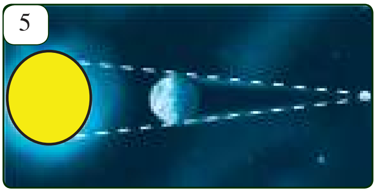
23.10. Aylana yasang va unda O nuqta belgilang. Markazi O nuqtada va koeffitsiyenti 2 ga teng boʻlgan gomotetiyada berilgan aylanaga gomotetik boʻlgan aylana yasang.

23.11. Ikkita oʻxshash koʻpburchak perimetrlarining nisbati 2:3 kabi. Katta koʻpburchakning yuzi 27 boʻlsa, kichik koʻpburchakning yuzini toping.

23.12. 5-rasmda Quyoshning toʻla tutilgan holati tasvirlangan. Agar Quyosh radiusi 686784 *km*, Oy radiusi 1760 *km* va Yerdan Oygacha boʻlgan masofa 384400 *km* boʻlsa, Yerdan Quyoshgacha boʻlgan masofani toping.



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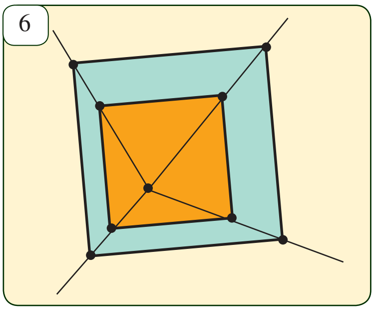
23.13. a) Bitta aylanaga ikkita oʻxshash koʻpburchak ichki chizilgan. Bu koʻpburchaklar teng boʻladimi?

b) Bitta aylanaga ikkita oʻxshash koʻpburchak tashqi chizilgan. Bu koʻpburchaklar teng boʻladimi?

23.14*. Bir kvadratning tomonlari ikkinchi kvadrat tomonlariga parallel. Agar kvadratlar bir-biriga teng boʻlmasa, ular gomotetik boʻlishini isbotlang (*6-rasm*).



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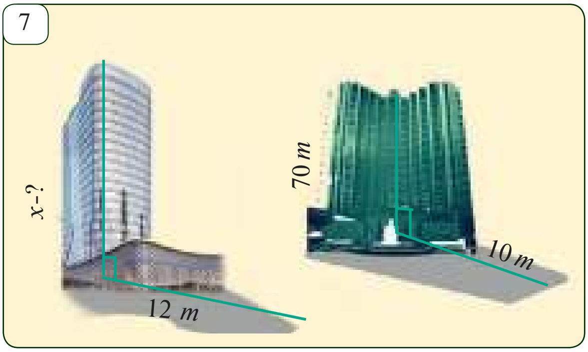
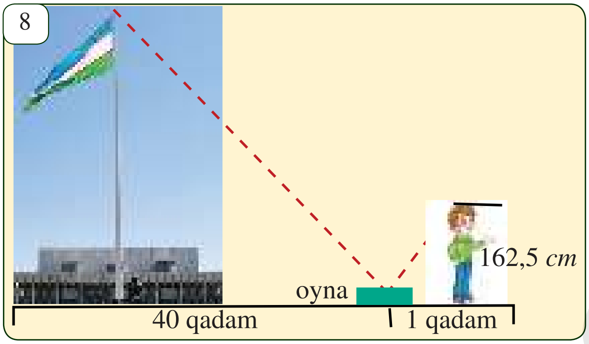
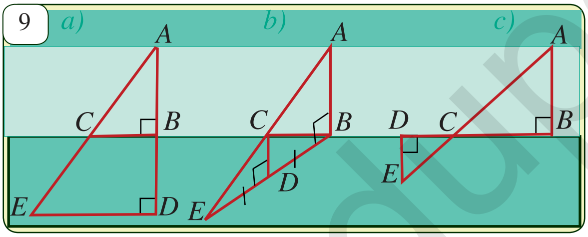






23.15. ABC uchburchakning AB va BC tomonlari to‘rtta teng kesmalarga bo‘lindi va bo‘linish nuqtalari AC tomonga parallel kesmalar bilan tutashtirildi. Agar $AC = 24$ *cm* bo‘lsa, hosil bo‘lgan kesmalar uzunliklarini toping.

23.16. Agar rasmlar ayni bir paytda suratga olingan bo‘lsa, berilgan ma’lumotlar asosida ikkinchi binoning balandligini toping (*7-rasm*).

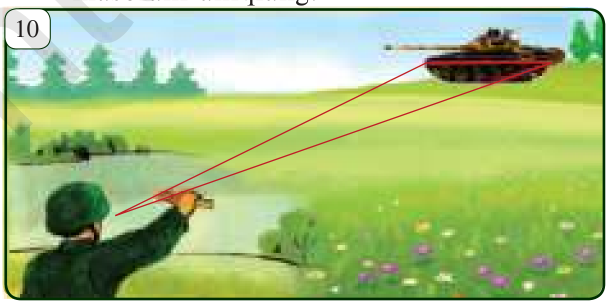
23.17. 8-rasmda berilganlardan foydalanib biror ob’ekt balandligini topish yo‘lini tushuntiring. "Xalqlar do‘stligi" saroyi oldida ko‘tarilgan vatanimiz bayrog‘i ustunining balandligini toping.

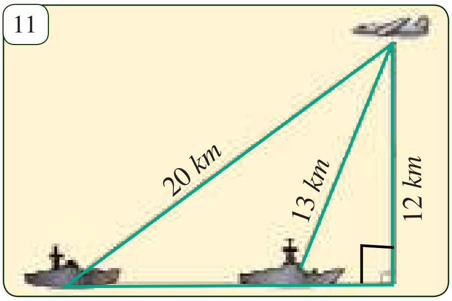
23.18. 9-rasmda berilganlardan foydalanib daryo kengligini aniqlashning 3 xil yo‘lini tushuntiring. Ularda geometriyaning qaysi teoremalaridan foydalanilayotganini aniqlang. O‘rgangan usullaringizni amalda boshqa vaziyatlarda qo‘llab ko‘ring.

Geometriya va harbiy ish

1. Harbiylar chizg‘ich va cho‘zilgan qo‘l yordamida nishongacha masofani aniqlay olishadi. Agar 10-rasmdagi chizg‘ichning tankni qoplaydigan uzunligi 5 *cm*, elkadan chizg‘ichgacha bo‘lgan masofa 50 *cm* va tankning uzunligi 6,86 *m* bo‘lsa, tankgacha bo‘lgan masofani toping.

2. 12 *km* balandlikda uchib borayotgan samolyot uchuvchisi undan 13 *km* uzoqlikda suzib borayotgan kemani va yana undan 20 *km* uzoqlikda birinchi kemani ta’qib qilib borayotgan boshqa kemani ko‘rdi (11-rasm). Bu kemalar orasidagi masofani aniqlang.





I. Testlar

1. **Ikkita o‘xshash uchburchak uchun noto‘g‘ri tasdiqni toping:**
   A. Yuzlari nisbati o‘xshashlik koeffitsiyentiga teng;
   B. Mos medianalari nisbati o‘xshashlik koeffitsiyentiga teng;
   D. Mos bissektrisalari nisbati o‘xshashlik koeffitsiyentiga teng;
   E. Mos balandliklari nisbati o‘xshashlik koeffitsiyentiga teng.
2. **Ikkita gomotetik ko‘pburchak uchun to‘g‘ri tasdiqni toping:**
   A. Ular teng; B. Ular o‘xshash;
   D. Ular tengdosh; E. To‘g‘ri javob yo‘q.
3. **Uchburchak medianalari uchun noto‘g‘ri tasdiqni ko‘rsating:**
   A. Bir nuqtada kesishadi; B. Kesishish nuqtasida 2:1 nisbatda bo‘linadi;
   D. Bir-biriga teng; E. Har biri uchburchakni ikkita tengdosh qismga ajratadi.
4. **Uchburchak bissektrisalari uchun noto‘g‘ri tasdiqni ko‘rsating:**
   A. Bir nuqtada kesishadi; B. Kesishish nuqtasida 2:1 nisbatda bo‘linadi;
   D. O‘zi tushgan tomonni qolgan ikki tomonga proporsional kesmalarga ajratadi;
   E. O‘zi chiqqan uchdagi burchakni teng ikkiga bo‘ladi.
5. **Ikkita o‘xshash ko‘pburchak uchun noto‘g‘ri tasdiqni toping:**
   A. Ularning tomonlari soni teng; B. Ularning burchaklari soni teng;
   D. Mos tomonlari proporsional; E. Yuzlarining nisbati o‘xshashlik koeffitsiyentiga teng.

II. Masalalar

24.1. Asoslari 6 *m* va 12 *m* bo‘lgan trapetsiya diagonallari kesishgan nuqtadan asoslarga parallel to‘g‘ri chiziq o‘tkazilgan. To‘g‘ri chiziqning trapetsiya ichidagi qismi uzunligini toping.

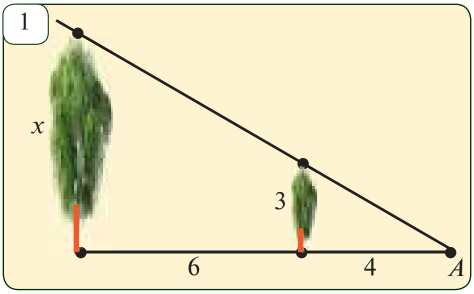
24.2. ABC uchburchakda $BC = BA = 10$, $AC = 8$. Agar AA_1 va CC_1 uchburchak bissektrisalari bo‘lsa, A_1C_1 kesmani toping.

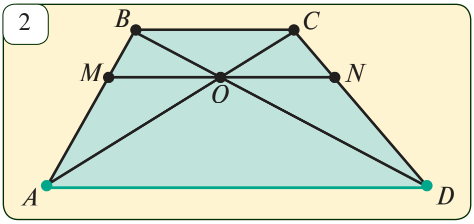
24.3. A nuqtadan borib bo‘lmaydigan B nuqtagacha bo‘lgan masofani aniqlash uchun tekis joyda C nuqta tanlandi. Keyin AC masofa, BAC va ACB burchaklar o‘lchandi va ABC uchburchakka o‘xshash $A_1B_1C_1$ uchburchak yasaldi. Agar $AC = 42$ *m*, $A_1C_1 = 6{,}3$ *cm*, $A_1B_1 = 7{,}2$ *cm* bo‘lsa, AB masofani toping.

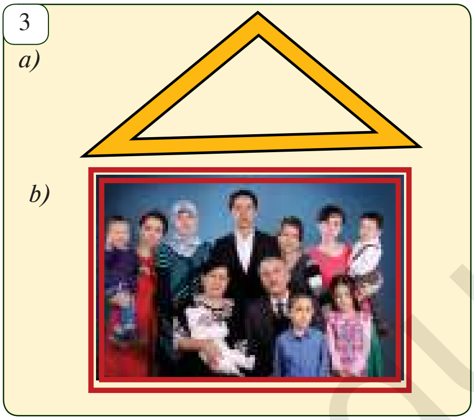
24.4. Koeffitsiyenti $k = 3$ bo‘lgan gomotetiyada F ko‘pburchak F_1 ko‘pburchakka almashadi. Agar F_1 ko‘pburchakning perimetri 12 *cm* va yuzi 4,5 cm^2 bo‘lsa, F ko‘pburchakning perimetri va yuzini toping.

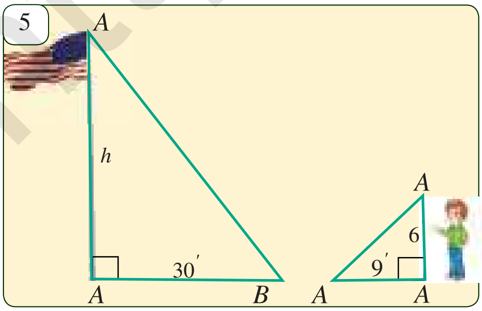
24.5. Bo‘yi 180 *cm* bo‘lgan odam soyasining uzunligi 2,4 *m* bo‘lgan paytda balandligi 4 *m* bo‘lgan simyog‘och soyasining uzunligi necha metr bo‘ladi?

24.6. Xaritada Toshkent va Urganch shaharlari orasidagi masofa 8,67 *cm*. Agar xarita masshtabi 1:10 000 000 bo‘lsa, Toshkent va Urganch shaharlari orasidagi masofani toping.









III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

24.7. 1-rasmda berilgan ma’lumotlar asosida daraxt balandligini toping.

24.8. ABC uchburchakning tomonlari $AB = 5$ cm, $AC = 6$ cm, $BC = 7$ cm. Bu uchburchakning AC tomoniga parallel to‘g‘ri chiziq AB tomonini P nuqtada, BC tomonini esa K nuqtada kesadi. Agar $PK = 2$ cm bo‘lsa, PBK uchburchak perimetrini toping.

24.9. 2-rasmda $AD||BC||MN$. Agar $BC = 6$ cm, $AD = 10$ cm bo‘lsa, MN kesmani toping.

24.10. *(Qo‘shimcha).* Romb tomonlarining o‘rtalari to‘g‘ri to‘rtburchakning uchlari bo‘lishini isbotlang.

Qiziqarli masalalar

1. 4 marta kattalashtirib ko‘rsatilgan ko‘z-gu-lupa bilan qaralganda 2° li burchak kattaligi qanchaga o‘zgaradi?

2. a) Uchburchakli chizg‘ich rasmida tasvir-langan ichki va tashqi uchburchaklar o‘xshashmi *(3-a rasm)*?

b) 3-b rasmdagi romning ichki va tashqi qirralarini tasvirlovchi to‘rtburchaklar o‘xshashmi?

3. Quyidagi chet tilida berilgan masalani yechib ko‘ring. Bu bilan ham rus va ingliz tilidan, ham geometriyadan nimaga qodirligingizni bilib olasiz.

а) На 4-рисунке ижображена русская игрушка “матрёшка”. Выпол-нив соответствующие измерения, найти коэффициент подобия игру-шек:

а) A и B; b) A и D; d) C и F; e) B и E.

b) Darnell is curious about the height of a flagpole that stands in front of his school. (pic.5) Darnell, who is 6 ft tall, casts a shadow that he paces off at 9 ft. He walks the length of the shadow of the flagpole, a distance of 30 ft. How tall is the flagpole?

c) The distance across a pond is to be measured indirectly by using similar triangles. (pic.6) If $XY = 160$ ft, $YW = 40$ ft, $TY = 120$ ft, and $WZ = 50$ ft, find XT.

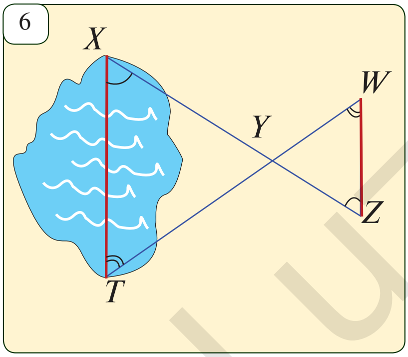
Geometrik modellashtirish

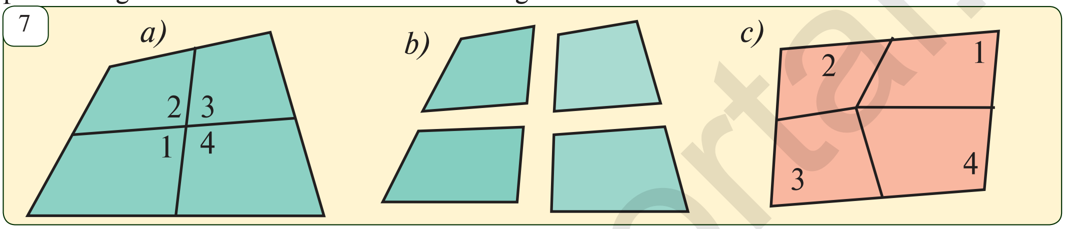
1. Ixtiyoriy to‘rtburchak chizing va qaychi bilan qirqib oling.

2. Uning qarama-qarshi tomonlari o‘rtalarini belgilang va kesmalar bilan tutashtiring (7.a- rasm) hamda shu kesmalar bo‘ylab to‘rtburchakni kesing (7.b- rasm).

3. Hosil bo‘lgan bo‘laklardan 7.c- rasmda ko‘rsatilgandek qilib parallelogramm tuzing.

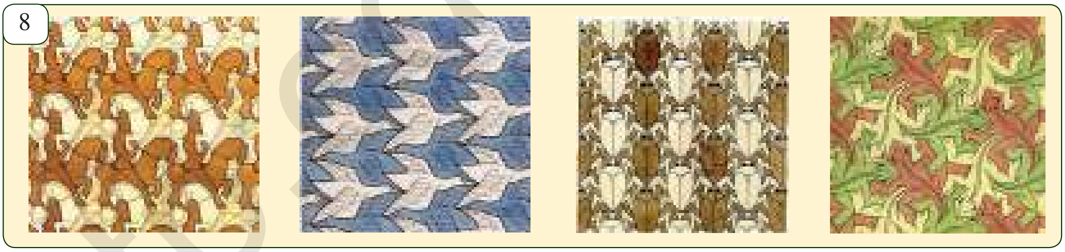
4. Bu ishni bajarishda haqiqatdan ham parallelogramm hosil bo‘lishini asoslang.



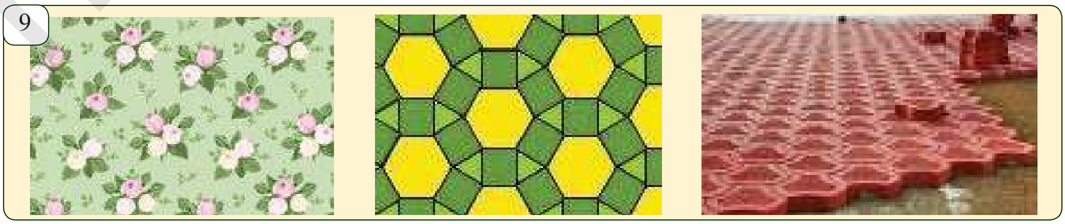


Naqshlar, panjaralar (bordyurlar) va parketlar

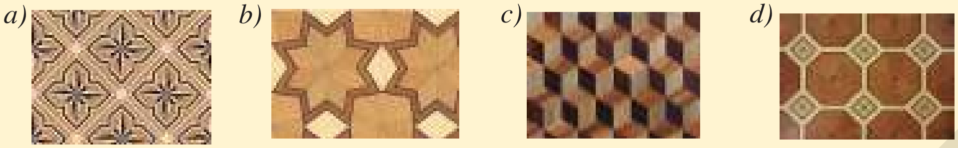
Uyimiz devorlaridagi gulqog‘ozlarga e’tibor berib qarasangiz, ularda bir xil shakl qayta-qayta takrorlanib butun devorni to‘ldirganini ko‘rish mumkin. Bitta shakl qayta-qayta takrorlanib butun tekislikni to‘ldirsa, bunday yig‘ma shakllarga naqsh deymiz. Mashhur golland rassomi Moris Esher qalamiga mansub mana bu ajabtovur rasmlar naqshlarga misol bo‘ladi (8-rasm). Bu naqshlarda aynan qaysi shakl qanday qaytarilayotganini aniqlang



Agar bitta shakl qayta-qayta takrorlanib ikki parallel to‘g‘ri chiziqlar orasidagi tasmani to‘ldirsa, bunday yig‘ma tasma shakllarga panjara yoki bordyur deymiz. Gulqog‘oz o‘rami, rasm solingan matolar va parklardagi panjaralar chekli uzunlikdagi bordyurlarga misol bo‘ladi (9-rasm).



Muntazam ko‘pburchaklar bilan qoplangan naqshlarni parket deb ataymiz. Parketlar bilan uyimiz pollari bezatiladi. Eng oddiy parketlar 10-rasmda keltirilgan. Ravshanki ular parallel ko‘chirishda o‘ziga-o‘zi o‘tadi.

10


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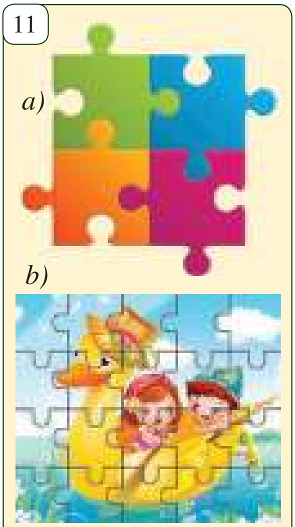
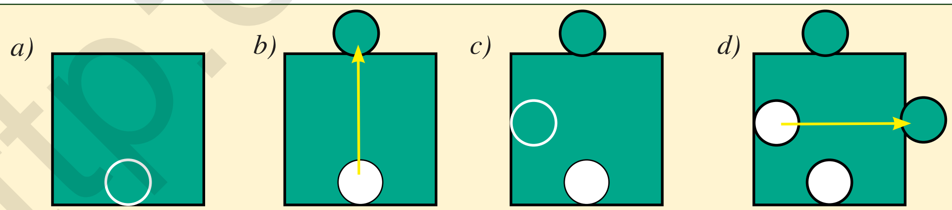


Geometrik modellashtirish.
Pazl shakllari qanday tuzilgan?

Pazl o‘yinchoqlarini yaxshi bilasiz? (11-rasm) Keling, ularni qanday yasash mumkinligini ko‘rib chiqaylik.

1. O‘lchamlari 5 *cm* x 5 *cm* bo‘lgan kvadrat chizing.
2. Uning pastki asosi o‘rtasidan doirasimon bo‘lakni kesib oling (12.a-rasm).
3. Kesib olingan bo‘lakni kvadratning yuqori asosi o‘rtasiga birlashtiring (12.b-rasm).
4. Endi kvadratning yon tomoni o‘rtasidan yana o‘shanday kattalikdagi doirasimon bo‘lakni kesib oling (12.c-rasm).
5. Kesib olingan bo‘lakni kvadratning ikkinchi yon tomoni o‘rtasiga birlashtiring (12.d-rasm).
6. Natijada pazl o‘yinchog‘ining bitta donasi tayyor bo‘ldi.
7. Bu pazl donalari bilan butun tekislikni qoplash mumkinligini asoslang.
8. Kvadrat tomonlaridan doirasimon emas boshqacha shakldagi bo‘laklarni qirqib, birlashtirish orqali boshqa ko‘rinishdagi pazl donalarini ham hosil qilish mumkin.
9. Marhamat, biror yangi pazl donasining chizmasini yarating. Bir nechta rangli pazl donalarini qirqib olib, ulardan turli naqshlar tuzing.

12


Geometrik tadqiqot.

73-betdagi "Geometrik modellashtirish" ruknida keltirilgan ma’lumotlar asosida ixtiyoriy qavariq to‘rtburchak bilan butun tekislikni qoplash mumkinligini isbotlang.

II BOB

UCHBURCHAK TOMONLARI VA BURCHAKLARI ORASIDAGI MUNOSABATLAR

Ushbu bobni o‘rganish natijasida siz quyidagi bilim, ko‘nikma va malakaga ega bo‘lasiz:

Bilimlar:

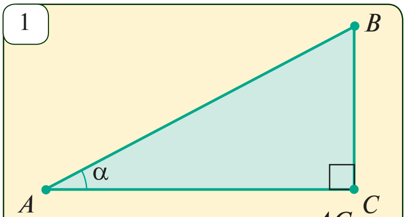
√ *ixtiyoriy burchakning sinusi, kosinusi, tangensi va kotangensi ta’riflarini bilish;*
√ *burchakning radian o‘lchovini bilish;*
√ *asosiy trigonometrik ayniyatlarni bilish;*
√ *uchburchakning yuzini burchak sinusi yordamida hisoblash formulasini bilish;*
√ *sinuslar va kosinuslar teoremasini bilish.*

Amaliy ko‘nikmalar:

√ *ba’zi burchaklarning sinusi, kosinusi, tangensi va kotangensini hisoblay olish;*
√ *asosiy trigonometrik ayniyatlarni misollar yechishda qo‘llay olish;*
√ *uchburchak yuzini uning ikki tomoni va ular orasidagi burchagi bo‘yicha hisoblay olish;*
√ *sinuslar, kosinuslar teoremasidan foydalanib hisoblashga va isbotlashga doir masalalarni yechish.*

0° DAN 180° GACHA BO‘LGAN BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI

1



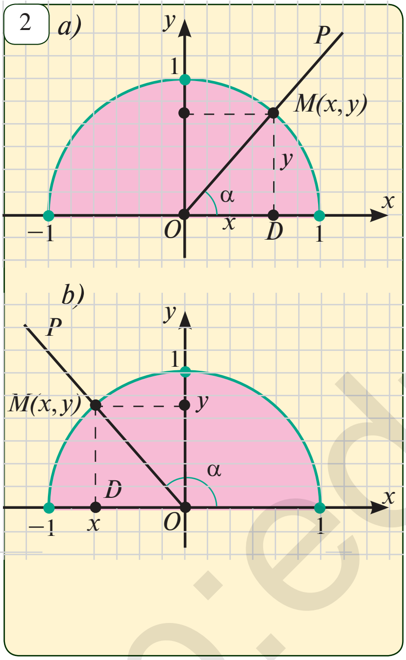
$\sin\alpha = \dfrac{BC}{AB}$; $\cos\alpha = \dfrac{AC}{AB}$;

$\mathrm{tg}\alpha = \dfrac{BC}{AC}$; $\mathrm{ctg}\alpha = \dfrac{AC}{BC}$;

To‘g‘ri burchakli ABC uchburchakda $\angle C=90°$ bo‘lsin. Ma’lumki, unda A o‘tkir burchakning sinusi, kosinusi, tangensi va kotangensi 1- rasmdagidek aniqlanar edi. Endi 0°dan 180° gacha bo‘lgan burchakning sinusi, kosinusi, tangensi va kotangensini aniqlaymiz.

Radiusi birlik kesmaga teng, markazi koordinatalar boshida bo‘lgan yarim aylanani qaraymiz (*2-rasm*). Aylanani M $(x;y)$ nuqtada kesuvchi OP nurni o‘tkazamiz. Bu nurning Ox nur bilan hosil qilgan burchagini α bilan belgilaymiz. OP nurning Ox nur bilan ustma-ust tushgan holdagi burchakni 0° li burchak sifatida qabul qilamiz.

2



Ma’lumki, α o‘tkir burchak bo‘lganda (*2-a rasm*), bu burchakning sinusi, kosinusi, tangensi va kotangensi to‘g‘ri burchakli ODM uchburchakdan

$$\sin\alpha=\frac{BC}{AB}\ ;\ \cos\alpha=\frac{OD}{MO}\ ;\ \mathrm{tg}\alpha=\frac{DM}{OD}\ ;\ \mathrm{ctg}\alpha=\frac{OD}{DM}\ .$$

tengliklar yordamida aniqlanadi. Agar $MO=1$, $DM=y$, $OD=x$ ekanligini hisobga olsak,

$$\boxed{\sin\alpha=y,\quad \cos\alpha=x,\quad \mathrm{tg}\alpha=\frac{y}{x}\ ,\quad \mathrm{ctg}\alpha=\frac{x}{y}} \qquad (1)$$

tengliklarga ega bo‘lamiz.

Umumiy holda, 0° dan 180° gacha bo‘lgan burchakning sinusi, kosinusi, tangensi va kotangensini ham (1) formula orqali aniqlaymiz (2.b-rasm):

OMD uchburchakda $OD^2+DM^2=MO^2$ yoki $x^2+y^2=1$. $\sin\alpha=y$ va $\cos\alpha=x$ ekanligini hisobga olsak, istalgan α $(0°\le\alpha\le180°)$ burchak uchun

$$\boxed{\sin^2\alpha+\cos^2\alpha=1} \qquad (2)$$

*asosiy trigonometrik ayniyat*ni hosil qilamiz.

Ta’rifga ko‘ra, $\mathrm{tg}\alpha=\dfrac{y}{x}$, $\mathrm{ctg}\alpha=\dfrac{x}{y}$, $x=\cos\alpha$, $y=\sin\alpha$ bo‘lgani uchun,

$$\boxed{\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}\ (\alpha\ne90°)} \qquad \boxed{\mathrm{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}\ (\alpha\ne0,\ \alpha\ne180°)}, \qquad \boxed{\mathrm{tg}\alpha\cdot\mathrm{ctg}\alpha=1\ (\alpha\ne0,\ \alpha\ne90°,\ \alpha\ne180°)}$$

ayniyatlar o‘rinlidir.

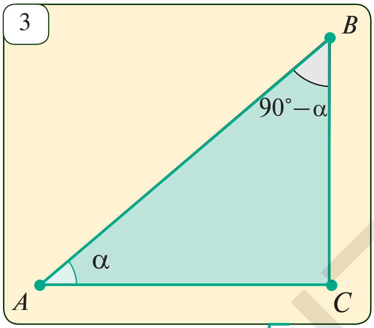
(2) tenglikning har ikki qismini oldin $\cos^2\alpha$ ga, keyin esa $\sin^2\alpha$ ga bo‘lib,

$$\boxed{1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha}\ (\alpha\ne90°)},\qquad \boxed{1+\mathrm{ctg}^2\alpha=\frac{1}{\sin^2\alpha},\ \ (\alpha\ne0,\ \alpha\ne180°)} \qquad (3)$$

ayniyatlarni hosil qilamiz.

Yuqoridagi (1) tengliklar asosida har bir α ($0^\circ \le \alpha \le 180^\circ$) burchakka bu burchak sinusining (kosinusi, tangensi va kotangensining) bitta qiymati mos qo‘yilayapti. Bu mosliklar burchakning "sinus", "kosinus", "tangens" va "kotangens" deb nomlanuvchi funksiyalarini aniqlaydi. Ular trigonometrik funksiyalar deb ataladi.

“Trigonometriya” so‘zi — yunoncha “uchburchaklarni yechish” degan ma’noni anglatadi.



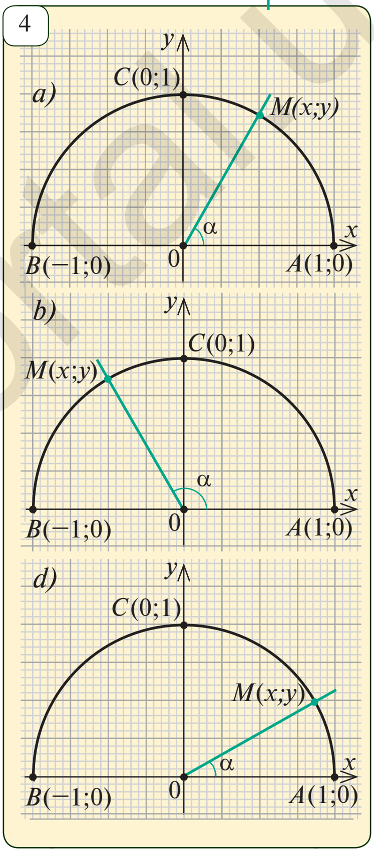
Har qanday o‘tkir α burchak uchun:

$$\sin(90^\circ-\alpha)=\cos\alpha, \quad \cos(90^\circ-\alpha)=\sin\alpha. \qquad (2)$$

Har qanday α ($0 \le \alpha \le 180^\circ$) burchak uchun:

$$\sin(180^\circ-\alpha)=\sin\alpha, \quad \cos(180^\circ-\alpha)=-\cos\alpha \qquad 3)$$

(2) va (3) formulalarga *keltirish formulalari* deyiladi. Ular algebra kursida isbotlanadi.



Masala va topshiriqlar

25.1. Agar $90^\circ<\alpha<180^\circ$ bo‘lsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ qiymatlarining ishorasini aniqlang.

25.2. 4-rasmdagi α burchakni o‘lchang va uning sinusi, kosinusi, tangensi va kotangensini tegishli o‘lchashlar yordamida aniqlang.

25.3. $\mathrm{tg}(90^\circ-\alpha)=\mathrm{ctg}\alpha$ ($\alpha\ne0^\circ$) va $\mathrm{ctg}(90^\circ-\alpha)=\mathrm{tg}\alpha$ ($\alpha\ne0^\circ$) ayniyatlarni isbotlang.

25.4. $\mathrm{tg}(180^\circ-\alpha)=-\mathrm{tg}\alpha$ ($\alpha\ne90^\circ$) va $\mathrm{ctg}(180^\circ-\alpha)=-\mathrm{ctg}\alpha$ ($\alpha\ne0^\circ$ va $\alpha\ne180^\circ$) ayniyatlarni isbotlang.

25.5. Soddalashtiring:

a) $\cos^2(180^\circ-\alpha)+\cos^2(90^\circ-\alpha)$;

b) $\sin^2(180^\circ-\alpha)+\sin^2(90^\circ-\alpha)$;

d) $\mathrm{tg}\alpha\cdot\mathrm{tg}(90^\circ-\alpha)$; e) $\mathrm{ctg}\alpha\cdot\mathrm{ctg}(90^\circ-\alpha)$.

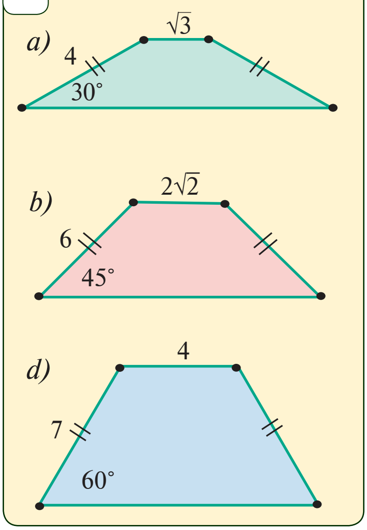
25.6. ABC uchburchakda $\angle A=150^\circ$ va $AC=7$ cm bo‘lsa, uchburchakning C uchidan tushirilgan balandligini toping.

25.7. Agar a) $\sin\alpha=\frac{\sqrt{3}}{2}$; b) $\sin\alpha=\frac{1}{4}$; d) $\sin\alpha=1$ bo‘lsa, $\cos\alpha$ ni toping.

25.8*. Agar a) $\sin\alpha=\frac{1}{2}$; b) $\mathrm{tg}\alpha=-1$; d) $\cos\alpha=-\frac{\sqrt{3}}{2}$ bo‘lsa, α ni toping.

25.9. Jadvalni to‘ldiring.

α	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\alpha$									
$\cos\alpha$									
$\mathrm{tg}\alpha$									
$\mathrm{ctg}\alpha$									

1


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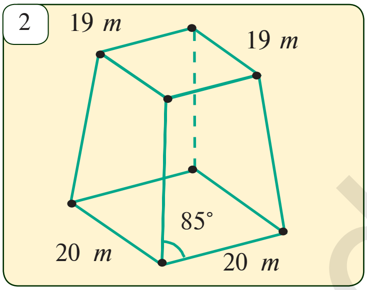

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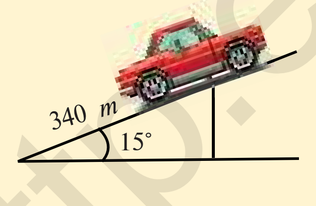


26.1. Balandligi 3 *cm* va o‘tkir burchagi 30° bo‘lgan rombning perimetri va yuzini hisoblang.

26.2. Teng yonli to‘g‘ri burchakli uchburchakning gipotenuzasi 12 *cm*. Uning yuzini hisoblang.

26.3. Balandligi $4\sqrt{3}$ *cm* bo‘lgan teng tomonli uchburchak perimetrini toping.

26.4. 1-rasmda berilganlarga ko‘ra teng yonli trapetsiyalar yuzini toping.

26.5. To‘g‘ri burchakli trapetsiyaning o‘tkir burchagi 30° ga, balandligi 4 *cm* ga va kichik asosi 6 *cm* ga teng. Trapetsiyaning perimetri va yuzini toping.

26.6. Aylana vatari 120 gradusli yoyni tortib turadi. Agar aylana radiusi 10 *cm* bo‘lsa, vatar uzunligini toping.

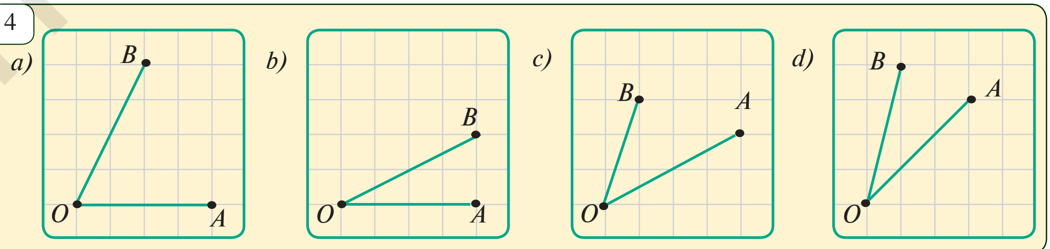
26.7*. Teng yonli uchburchakning uchidagi burchagi a) 120°; b) 90°; d) 60°. Uchburchak balandligining asosiga nisbatini hisoblang.

26.8*. 2-rasmda tasvirlangan paxta xirmonining yon yoqlari teng yonli trapetsiya, usti esa kvadrat shaklida. Rasmda berilganlardan foydalanib, xirmonni to‘liq yopish uchun qancha mato zarurligini aniqlang.

26.9. Yengil mashina dovonning yuqoriga ko‘tarilish qismida 340 *m* yo‘l bosdi. Agar yo‘lning gorizontga nisbatan ko‘tarilish burchagi 15° bo‘lsa, yengil mashina necha metr balandlikka ko‘tarilgan (3-rasm)?

26.10. Ashrafjon uyidan sharq tomonga qarab 800 *m*, so‘ng shimol tomonga qarab 600 *m* yo‘l yurdi. U uyidan necha metr uzoqlikka keldi? Endi u uyiga to‘g‘ri chiziq bo‘ylab yetib olishi uchun g‘arbga nisbatan qanday burchak ostida yurishi kerak?

26.11. 4-rasmda tasvirlangan burchaklarning sinusi, kosinusi, tangensi va kotangensini toping.

4


26.12. Poyezd har 30 *m* yo‘l yurganda 1 *m* tepaga ko‘tariladi. Temir yo‘lning gorizontga nisbatan ko‘tarilish burchagini toping.

26.13. Agar balandligi 30 *m* bo‘lgan bino soyasining uzunligi 45 *m* bo‘lsa, quyosh nurining shu bino joylashgan maydonga tushish burchagini toping.

26.14. To‘g‘ri burchakli uchburchakning bir burchagi 60° ga, katta kateti esa 6 ga teng. Uning kichik kateti va gipotenuzasini toping.

26.15. O markazli aylananing A nuqtasidan o‘tkazilgan urinmada B nuqta olingan. Agar $AB=9$ *cm*, $\angle ABO=30°$ bo‘lsa, aylana radiusini va BO kesma uzunligini toping.

26.16. m to‘g‘ri chiziq va uni kesib o‘tmaydigan AB kesma berilgan. Bunda $AB=10$, AB va m to‘g‘ri chiziqlar orasidagi burchak 60°. AB kesma uchlaridan m to‘g‘ri chiziqqa AC va BD perpendikularlar tushirilgan. CD kesmani toping.

26.17. Rombning o‘tkir burchagi 60° ga, balandligi esa 6 ga teng. Rombning katta diagonali uzunligini va yuzini toping.

26.18. Radiusi 5 *cm* bo‘lgan aylanaga teng yonli trapetsiya tashqi chizilgan. Agar trapetsiyaning o‘tkir burchagi 30° bo‘lsa, uning yon tomoni va yuzini toping.

26.19. Agar $ABCD$ to‘g‘ri to‘rtburchakda $AB = 4$, $\angle CAD = 30°$ bo‘lsa, unga tashqi chizilgan aylana radiusini va to‘g‘ri to‘rtburchak yuzini hisoblang.

26.20. To‘g‘ri to‘rtburchakning tomonlari 3 *cm* va $\sqrt{3}$ *cm*. Uning bir diagonali bilan tomonlari hosil qilgan burchaklarini toping.

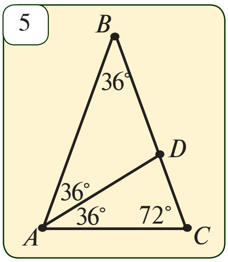
26,21. Agar a) $\sin A=\frac{4}{7}$; b) $\cos A=\frac{4}{7}$; d) $\cos A=-\frac{4}{7}$ bo‘lsa, A burchakni yasang.

26.22. To‘g‘ri burchakli uchburchakning bir burchagi 30°, gipotenuzasiga tushirilgan balandligi 6 *cm*. Uchburchak tomonlarini toping.

26.23. O‘tkir burchagi 30° ga, balandligi esa 4 *cm* ga teng bo‘lgan rombning yuzini hisoblang.

Tarixiy lavhalar. “Oltin” uchburchak

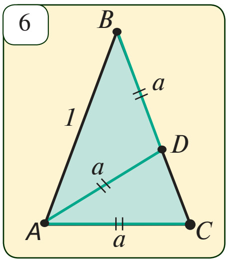
Yunonlar burchaklari 36°, 72° va 72° bo‘lgan teng yonli uchburchakni — *“oltin uchburchak”* deb atashgan. Sababi - u mana bunday ajoyib xossaga ega ekan: *asosidagi burchak bissektrisasi AD uni ikkita teng yonli uchburchakka bo‘ladi* (5-*rasm*).



Haqiqatan, AD bissektrisa bo‘lgani uchun, BAD va DAC burchaklar ham 36° dan. Demak, ABD uchburchak teng yonli. ADC uchburchakda ADC burchak $180°-36°-72°=72°$ bo‘lib, ACD burchakka teng. Demak, ADC uchburchak ham teng yonli.

Natija. ABC uchburchak ACD uchburchakka o‘xshash va

$$\frac{AC}{AB}=\frac{CD}{AC}. \qquad (1)$$



Agar ABC uchburchakning yon tomonlari $AB=BC=1$ deb olsak, uning asosi quyidagicha topiladi (6-*rasm*): $AC=a$ bo‘lsin. U holda, 1. $AD=a$ bo‘ladi, chunki ΔACD teng yonli.

2. $BD=a$ bo‘ladi, chunki ΔABD teng yonli.

3. $CD=BC-BD=1-a$.

(1) tenglikka ko‘ra: $\frac{a}{1}=\frac{1-a}{a}$

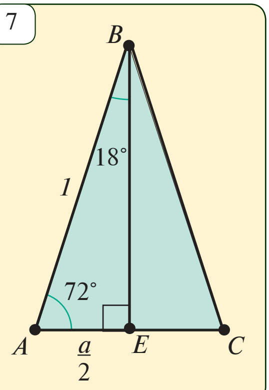
Bundan $a^2+a-1=0$. Bu kvadrat tenglamani yechib, $a=\frac{\sqrt{5}-1}{2}$ ekanligini topamiz.

Masala. $\sin 18^\circ$, $\cos 18^\circ$, $\sin 72^\circ$, $\cos 72^\circ$ qiymatlarni hisoblang.

Yechish: Yon tomoni $AB=BC=1$ va asosi $AC=a=\frac{\sqrt{5}-1}{2}$ ga teng bo‘lgan ABC "oltin uchburchak"ni qaraymiz (*7-rasm*). Uning BE balandligini o‘tkazamiz.

To‘g‘ri burchakli ABE uchburchakda: $\sin 18^\circ=\frac{AE}{AB}=\frac{a}{2}=\frac{\sqrt{5}-1}{4}$

7



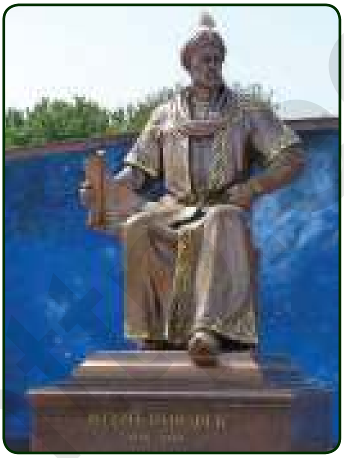
Bundan foydalanib, topilishi talab qilingan boshqa qiymatlarni hisoblaymiz:

$$\cos 18^\circ=\sqrt{1-\sin^2 18^\circ}=\frac{\sqrt{5}+1}{4};$$

$$\sin 72^\circ=\sin(90^\circ-18^\circ)=\cos 18^\circ=\frac{\sqrt{10+2\sqrt{5}}}{4};$$

$$\cos 72^\circ=\sin 18^\circ=\frac{\sqrt{5}-1}{2}.$$

Javob: $\sin 18^\circ=\cos 72^\circ=\frac{\sqrt{5}-1}{4}$; $\cos 18^\circ=\sin 72^\circ=\frac{\sqrt{10+2\sqrt{5}}}{4}$.



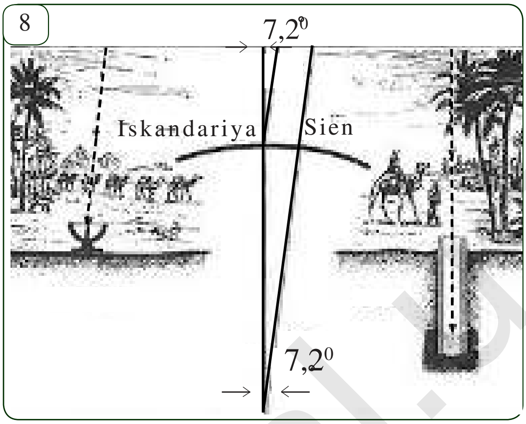
Tarixiy lavhalar

Mirzo Ulug‘bek (1394–1449) — buyuk o‘zbek olimi va davlat arbobi. Asl ismi Muhammad Tarag‘ay. U sohibqiron Amir Temurning nabirasi. Ulug‘bekning otasi Shohruh ham davlat arbobi bo‘lgan. Ulug‘bek taxminan 1425–1428-yillari Samarqand yaqinidagi Obi Rahmat tepaligida o‘zining mashhur rasadxonasini quradi. Rasadxonaning binosi uch qavatli bo‘lib, uning asosiy asbobi — kvadratning balandligi 50 metr edi. Ulug‘bekning eng mashhur asari "Ziji ko‘ragoniy" deb ataluvchi astronomik jadvaldir. U 1018 ta yulduzni o‘z ichiga olgan.

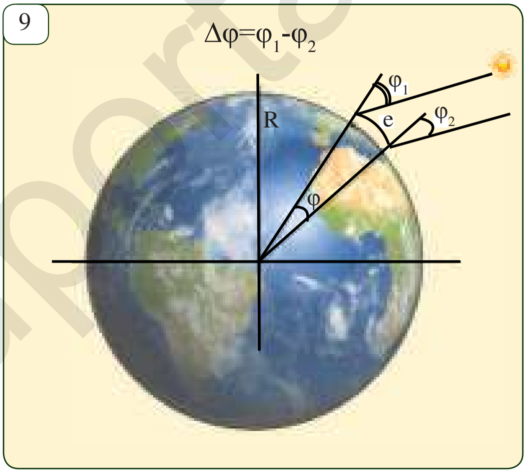
Shu bilan bir qatorda Ulug‘bekning trigonometrik jadvallari ham diqqatga sazovordir. Ulug‘bekning trigonometrik jadvallari 10 ta o‘nli xona aniqligida hisoblangan. Hisoblash vositalari deyarli bo‘lmagan bir davrda bu ishlarni bajarish uchun chuqur mushohadaga asoslangan nazariy salohiyat va aniq formulalar hamda anchagina hisobchilar talab qilingan bo‘lsa kerak. Zijda Ulug‘bek 1 gradusning sinusini hisoblash uchun alohida risola yozganligi qayd qilinadi.

Geometriya va astronomiyaga doir loyiha ishi

Qadimgi yunon olimi Eratosfen (miloddan avvalgi 276–194- yillar) Yer aylanasini birinchi bo‘lib o‘lchagan. U Sien (hozirgi Assuan) shahrida miloddan avvalgi 240-yil 19-iyun kuni, yozgi teng kunlikning tush paytida Quyosh qoq tikkada (zenitda) bo‘lishi va chuqur quduqning tagini ham yoritishini payqagan. Lekin, shu bilan birga, u yilning bu kuni va paytida Iskandariyada Quyosh qoq tikkadadan (zenitdan) aylana yoyining 1/50 qismi qadar og‘ishini ham aniqlagan.



Bundan Eratosfen qanday xulosaga kelgan? Uning fikrini davom ettiring va quyidagi 8-rasmda berilganlar asosida Yer radiusi uzunligini toping.



Zarur bo‘lishi mumkin bo‘lgan ba’zi ma’lumotlar va hisob-kitoblar:

Sien va Iskandariya shaharlari orasidagi masofa 787,5 *km*.

Aylana yoyining 1/50 qismi - $\alpha = 7{,}2^o$.

C - Yer aylanasining uzunligi.

$$\frac{\alpha^o}{360^o} = \frac{787{,}5}{C}$$

Bundan $C = 360 \cdot 787{,}5 : 7{,}2 = 39\ 375$ *km*.

Bugungi kungi hisob-kitoblarga ko‘ra, Yerning ekvator bo‘ylab aylanasining uzunligi 40 075,017 *km*, nolinchi metridan bo‘ylab aylanasining uzunligi esa- 40 007,86 *km* ni tashkil qiladi. Ko‘rib turganingizdek, qadimgi olim ozgina adashgan, xolos.

Topshiriq. 9 -rasmdan foydalanib, Yer aylanasi uzunligini topishning ixtiyoriy paytda qo‘llash mumkin bo‘lgan amaliy usulini ishlab chiqing va asoslang.

UCHBURCHAK YUZINI BURCHAK SINUSI YORDAMIDA HISOBLASH

1-teorema. ***Uchburchak yuzi uning ikki tomoni bilan shu ikki tomon orasidagi burchak sinusi ko'paytmasining yarmiga teng.***

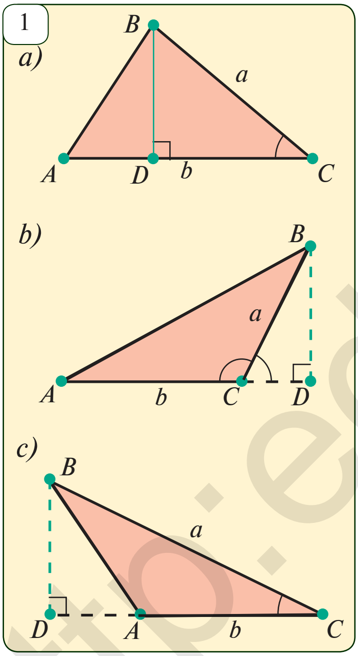
$\Delta ABC,\ BC=a,\ AC=b,\ \angle C$ (1-rasm) ⇨ $S_{ABC}=\frac{1}{2}ab\sin C$

Isbot. ABC uchburchakning BD balandligini tushiramiz. U holda 1-rasmda ko'rsatilgan uch hol bo'lishi mumkin.

Birinchi holni qaraymiz (1.a-rasm). BCD uchburchakda $\sin C=\frac{BD}{BC}$. Bundan $BD = BC \cdot \sin C = a \cdot \sin C$. Shunday qilib,

$$S_{ABC}=\frac{1}{2}\cdot AC\cdot BD=\frac{1}{2}\cdot b\cdot a\cdot \sin C=\frac{1}{2}\ ab\sin C.$$

Ikkinchi va uchunchi hollarning isbotini mustaqil bajaring. *Teorema isbotlandi.*



1-teoremaga ko'ra, uchburchak yuzi uchun

$$S_{ABC}=\frac{1}{2}bc\sin A \quad \text{va} \quad S_{ABC}=\frac{1}{2}ac\sin B$$

formulalar ham o'rinli bo'ladi.

1-masala. ABC uchburchakning yuzi 24 cm^2. Agar $AC=8$ cm va $\angle A=30^\circ$ bo'lsa, AB tomonni toping.

Yechish. Uchburchak yuzini burchak sinusi orqali topish formulasiga ko'ra,

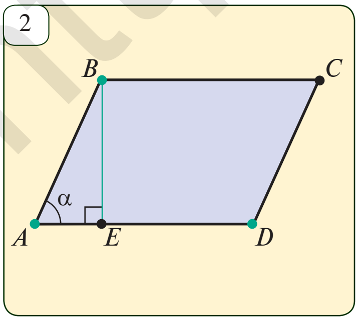
$$S_{ABC}=\frac{1}{2}AB\cdot AC\cdot \sin A$$

Bundan,

$$AB=\frac{2S_{ABC}}{AC\cdot \sin A}=\frac{2\cdot 24}{8\cdot \sin 30^\circ}=\frac{2\cdot 24}{8\cdot 0,5}=12\ (cm).$$

Javob: 12 *cm*.

2- masala Parallelogramm yuzi uning ikkita qo'shni tomoni va shu tomonlar orasidagi burchagi sinusining ko'paytmasiga teng ekanligini isbotlang.



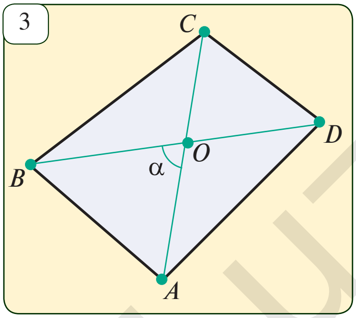
ABCD parallelogramm, $AB=a,\ AD=b,\ \angle A=\alpha$ (2-rasm) ⇨ $S_{ABCD}=ab\sin\alpha$

Yechish. BE balandlik tushiramiz. ABE uchburchakda $\sin A = \frac{BE}{AB}$ yoki $BE= AB\sin A =a\sin\alpha$.

U holda, $S_{ABCD} = AD\cdot BE =ab\sin\alpha$.

2-teorema. ***To'rtburchak yuzi uning diagonallari bilan diagonallar orasidagi burchak sinusi ko'paytmasining yarmiga teng.***

Isbot. Diagonallar kesishishidan hosil bo'lgan burchaklarni qaraymiz (*3-rasm*):



shartga ko'ra $\angle AOB = \alpha$

$\angle AOB$ ga vertikal bo'lgani uchun $\angle COD = \alpha$,

$\angle AOB$ ga qo'shni bo'lgani uchun $\angle BOC = 180^\circ - \alpha$,

$\angle BOC$ ga vertikal bo'lgani uchun $\angle DOA = 180^\circ - \alpha$

Uchburchak yuzini burchak sinusi yordamida hisoblash formulasiga ko'ra:

$$S_{AOB} = \frac{1}{2} AO \cdot OB \sin\alpha;$$

$$S_{BOC} = \frac{1}{2} BO \cdot OC \sin(180^\circ - \alpha) = \frac{1}{2} BO \cdot OC \sin\alpha;$$

$$S_{COD} = \frac{1}{2} CO \cdot OD \sin\alpha; \qquad S_{DOA} = \frac{1}{2} DO \cdot OA \sin(180^\circ - \alpha) = \frac{1}{2} DO \cdot OA \sin\alpha.$$

Yuzning xossasiga ko'ra:

$$S_{ABCD} = S_{AOB} + S_{BOC} + S_{COD} + S_{DOA} =$$

$$= \frac{1}{2} AO \cdot OB \sin\alpha + \frac{1}{2} BO \cdot OC \sin\alpha + \frac{1}{2} CO \cdot OD \sin\alpha + \frac{1}{2} DO \cdot OA \sin\alpha =$$

$$= \frac{1}{2}(AO \cdot OB + BO \cdot OC + CO \cdot OD + DO \cdot OA)\sin\alpha = \frac{1}{2}\{(OB \cdot (AO + OC) +$$

$$+ OD \cdot (CO + OA)\}\sin\alpha = \frac{1}{2}(OB \cdot AC + OD \cdot AC)\sin\alpha = \frac{1}{2} AC \cdot BD \sin\alpha.$$

Teorema isbotlandi.

Masala va topshiriqlar

27.1. 1-teoremani 1-b va 1-c rasmda tasvirlangan hollar uchun isbotlang.

27.2. Agar a) $AB = 6$ *cm*, $AC = 4$ *cm*, $\angle A = 30^\circ$; b) $AC = 14$ *cm*, $BC = 7\sqrt{3}$ *cm*, $\angle C = 60^\circ$;
d) $BC = 3$ *cm*, $AB = 4\sqrt{2}$ *cm*, $\angle B = 45^\circ$ bo'lsa, ABC uchburchak yuzini toping.

27.3. Diagonali 12 *cm* va diagonallari orasidagi burchagi 30° bo'lgan to'g'ri to'rtburchak yuzini toping.

27.4. Tomoni $7\sqrt{2}$ *cm* va o'tmas burchagi 135° bo'lgan romb yuzini toping.

27.5. Rombning katta diagonali 18 *cm* va bir burchagi 120°. Romb yuzini toping.

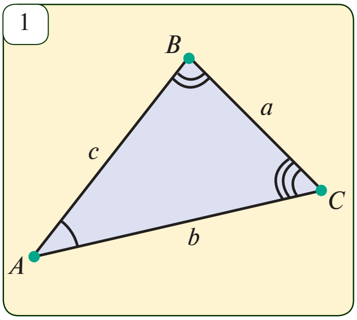
27.6. Yuzi $6\sqrt{2}$ cm^2 ga teng bo'lgan ABC uchburchakda $AB = 9$ *cm*, $\angle A = 45^\circ$. Uchburchakning AC tomonini va shu tomonga tushirilgan balandligini toping.

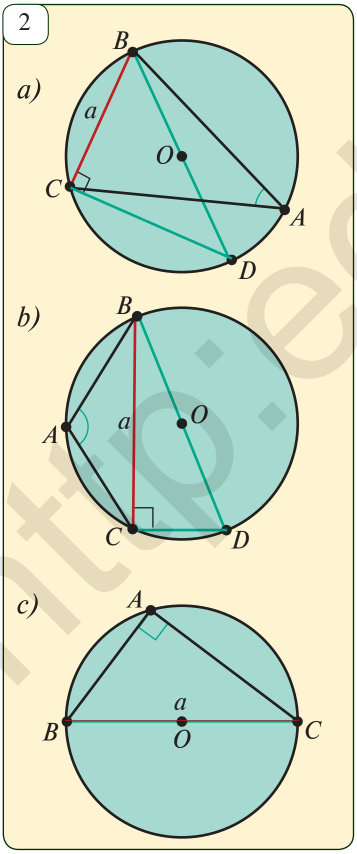
27.7*. ABC uchburchakda $\angle A = \alpha$, uning B va C uchlaridan tushirilgan balandliklari esa mos ravishda h_b va h_c bo'lsa, uchburchak yuzini toping.

27.8*. ABC uchburchakda $AB = 8$ *cm*, $AC = 12$ *cm* va $\angle A = 60^\circ$ bo'lsa, uning AD bissektrisasini toping (*ko'rsatma:* $S_{ABC} = S_{ABD} + S_{ADC}$).

Teorema. (Sinuslar teoremasi). ***Uchburchakning tomonlari qarshisidagi burchaklarning sinuslariga proporsional.***

$\Delta ABC,\ AB=c,\ BC=a,\ CA=b$ (*1-rasm*) $\Rightarrow$ $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$





Isbot. Uchburchak yuzini burchak sinusi orqali topish formulasiga ko‘ra,

$$S=\frac{1}{2}ab\sin C,\quad S=\frac{1}{2}bc\sin A,\quad S=\frac{1}{2}ac\sin B.\quad (❖)$$

Bu tengliklarning dastlabki ikkitasiga ko‘ra,

$$\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A, \text{ demak } \frac{a}{\sin A}=\frac{c}{\sin C}$$

Shuningdek, (❖) tengliklarning birinchi va uchinchidan $\dfrac{c}{\sin C}=\dfrac{b}{\sin B}$ tenglikni hosil qilamiz.

Shunday qilib, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$.

Teorema isbotlandi.

1-masala. ABC uchburchakda $AB=14\ dm$, $\angle A=30^\circ$, $\angle C=65^\circ$ (*1-rasm*). BC tomonni toping.

Yechish: Sinuslar teoremasiga ko‘ra,

$$\frac{AB}{\sin C}=\frac{BC}{\sin A} \qquad \text{Undan,}$$

$$BC=\frac{AB\cdot\sin A}{\sin C}=\frac{14\cdot\sin 30^\circ}{\sin 65^\circ}\approx\frac{14\cdot 0{,}5}{0{,}9}\approx 7{,}78\ (dm).$$

Eslatma: Trigonometrik funksiyalarning qiymatlari maxsus kalkulator yoki jadvallar yordamida topiladi. Bu yerda $\sin 65^\circ\approx 0{,}9$ ekanligini darslikning 153-betidagi jadvaldan aniqladik.

Javob: 7,78 dm.

2-masala. Uchburchak tomonining shu tomon qarshisidagi burchagi sinusiga nisbati uchburchakka tashqi chizilgan aylana diametriga teng, ya’ni

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

ekanligini isbotlang *(2-rasm)*.

Isbot. Ravshanki, sinuslar teoremasiga ko'ra, $\frac{a}{\sin A} = 2R$ tenglikni isbotlash kifoya. Uch hol bo'lishi mumkin:

1-hol: $\angle A$ — o'tkir burchak (*2-a rasm*); 2-hol: $\angle A$ — o'tmas burchak (*2-b rasm*);
3-hol: $\angle A$ — to'g'ri burchak (*2-c rasm*).

1-holni qaraymiz: C va D nuqtalarni tutashtiramiz. BCD — to'g'ri burchakli uchburchak, chunki $\angle BCD$ burchak BD diametrga tiralgan.

ΔBCD da: $BC = BD \cdot \sin D = 2R\sin D$. Lekin, $\angle D = \angle A$, chunki ular bitta BC yoyga tiralgan ichki chizilgan burchaklar. Unda,

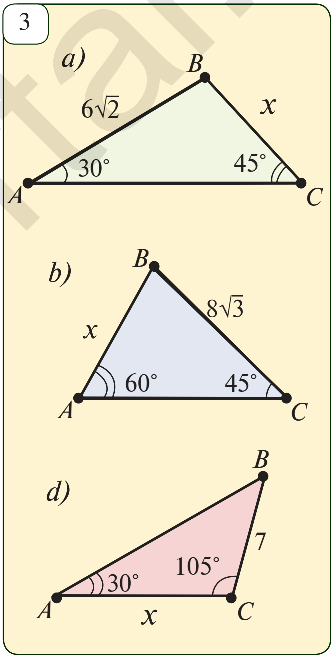
$$BC = 2R\sin A \qquad \text{yoki} \quad \frac{a}{\sin A} = 2R.$$

Qolgan hollarni mustaqil isbotlang (*ko'rsatma:* 2-holda $\angle D = 180° - \angle A$ ekanligidan, 3-holda $a = 2R$ ekanligidan foydalaning).

Masala va topshiriqlar

28.1. Uchburchak istalgan tomonining shu tomon qarshisidagi burchak sinusiga nisbati uchburchakka tashqi chizilgan aylana diametriga teng ekanligini 2-masalada keltirilgan 2- va 3-hollar uchun isbotlang.

28.2. 3-rasmda berilganlarga ko'ra, so'ralgan kesmalarni toping.



28.3. Agar ABC uchburchakda:

a) $\sin A = 0,4$; $BC = 6$ *cm* va $AB = 5$ *cm* bo'lsa, $\sin C$ ni;

b) $\sin B = \frac{1}{2}$; $AC = 8$ *dm* va $BC = 7$ *dm* bo'lsa, $\sin A$ ni;

d) $\sin C = \frac{1}{2}$; $AB = 6$ *m* va $AC = 8$ *m* bo'lsa, $\sin B$ ni toping.

28.4. Uchburchakning bir burchagi 30° ga teng. Uning qarshisidagi tomon 4,8 *dm*. Uchburchakka tashqi chizilgan aylana radiusini hisoblang.

28.5. Uchburchakning bir tomoni uchburchakka tashqi chizilgan aylana radiusiga teng. Uchburchakning shu tomoni qarshisidagi burchagini toping. Bunda, ikki holni qarashga to'g'ri kelishiga e'tibor qiling.

28.6. ABC uchburchak uchun $AB:BC:CA = \sin C : \sin A : \sin B$ tenglik o'rinli bo'lishini asoslang. $\sin A : \sin B : \sin C = 3:5:7$ tenglik to'g'ri bo'lishi mumkinmi?

28.7. Agar ABC uchburchakda $BC = 20$ *m*, $AC = 13$ *m* va $\angle A = 67°$ bo'lsa, uchburchakning AB tomonini, B va C burchaklarini toping.

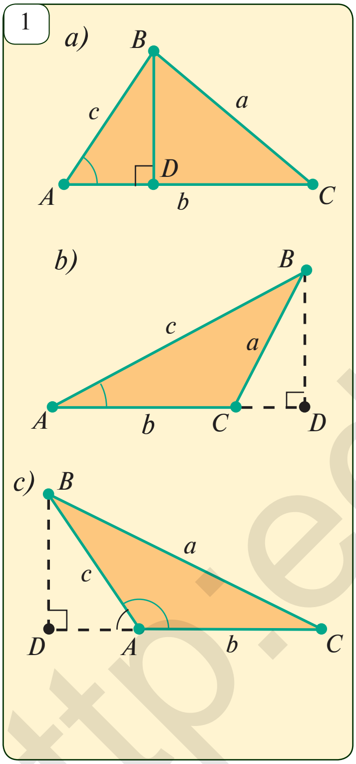
28.8*. Agar ABC uchburchakda $BC = 18$ *dm*, $\angle A = 42°$, $\angle B = 62°$ bo'lsa, uchburchakning C burchagini, AB va AC tomonlarini toping.

To‘g‘ri burchakli uchburchakda to‘g‘ri burchak qarshisidagi tomon (gipotenuza) kvadrati qolgan tomonlar (katetlar) kvadratlari yig‘indisiga teng.

Xo‘sh, to‘g‘ri bo‘lmagan burchak uchun-chi? Quyidagi teorema shu xususda.

Teorema. ***(Kosinuslar teoremasi). Uchburchak istalgan tomonining kvadrati qolgan ikki tomoni kvadratlari yig‘indisi shu ikki tomon bilan ular orasidagi burchak kosinusi ko‘paytmasining ikkilangani ayirmasiga teng.***

$\Delta ABC,\ AB=c,\ BC=a,\ CA=b$ *(1-rasm)* $\Rightarrow$ $a^2=b^2+c^2-2bc\cos A$

1


Isbot. ABC uchburchakning BD balandligini o‘tkazamiz. D nuqta AC tomonda *(1-a rasm)* yoki uning davomida *(1-b va 1-d rasmlar)* bo‘lishi mumkin. Birinchi holni qaraymiz. To‘g‘ri burchakli BCD uchburchakda Pifagor teoremasiga ko‘ra,

$$BC^2=BD^2+DC^2.$$

$DC=AC-AD$ bo‘lgani uchun:

$$BC^2=BD^2+(AC-AD)^2=BD^2+AC^2-2\cdot AC\cdot AD+AD^2.$$

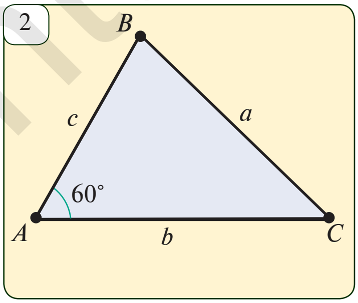
To‘g‘ri burchakli ABD uchburchakda $BD^2+AD^2=AB^2$ va $AD=AB\cos A$ ekanligini hisobga olib, oxirgi tenglikdan

$$BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos A,$$

ya’ni $a^2=b^2+c^2-2bc\cos A$ tenglikka ega bo‘lamiz.

Teorema isbotlandi.

1-b rasmda tasvirlangan holda $DC=AD-AC$, 1-d rasmda tasvirlangan holda $DC=AD+AC$ va $\cos(180^\circ-A)=-\cos A$ tengliklardan foydalanib, kosinuslar teoremasini mustaqil isbotlang.

2


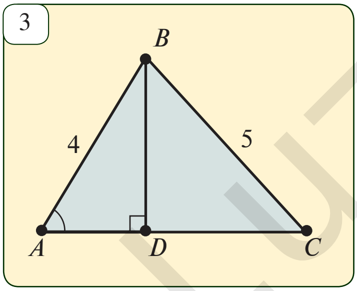
Eslatma. Kosinuslar teoremasi Pifagor teoremasining umumlashganidir. $\angle A=90^\circ$ bo‘lganda ($\cos 90^\circ=0$ bo‘lgani uchun) kosinuslar teoremasidan Pifagor teoremasi kelib chiqadi.

1-masala. ABC uchburchakda $AB=6$ *cm*, $AC=7$ *cm*, $\angle A=60^\circ$ (*2-rasm*). BC tomonni toping.

Yechish. Kosinuslar teoremasiga ko‘ra, $a^2=b^2+c^2-2bc\cos A$ yoki $BC^2=AC^2+AB^2-2AC\cdot AB\cdot\cos A$ bo‘lgani uchun

$$BC^2=7^2+6^2-2\cdot7\cdot6\cdot\cos60^\circ=49+36-84\cdot\frac{1}{2}=43,$$

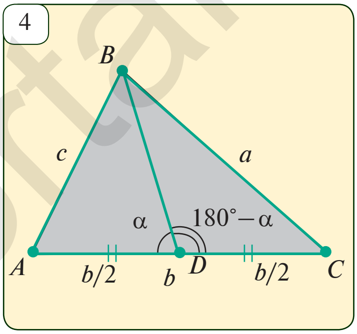
ya’ni $BC=\sqrt{43}$ *cm*. *Javob:* $\sqrt{43}$ *cm*.



Kosinuslar teoremasidan foydalanib, tomonlari ma’lum bo‘lgan uchburchakning burchaklarini topish mumkin:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}. \qquad (1)$$

2-masala. ABC uchburchakning tomonlari $a=5$ *m*, $b=6$ *m* va $c=4$ *m*. Kichik tomonning katta tomondagi proyeksiyasini toping (*3-rasm*).



Yechish. (1) formula asosida $\cos A$ ni topamiz:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{6^2+4^2-5^2}{2\cdot6\cdot4}=\frac{9}{16}.$$

To‘g‘ri burchakli ABD uchburchakda $AD=AB\cdot\cos A$ bo‘lgani uchun $AD=4\cdot\frac{9}{16}=2{,}25$ (*m*).

Javob: 2,25 *m*.

Masala va topshiriqlar

29.1. Kosinuslar teoremasini 1-b va 1-d rasmda tasvirlangan hollarda isbotlang.

29.2. ABC uchburchakda

a) $AC=3$ *cm*, $BC=4$ *cm* va $\angle C=60^\circ$ bo‘lsa, AB ni;

b) $AB=4$ *m*, $BC=4\sqrt{2}$ *m* va $\angle B=45^\circ$ bo‘lsa, AC ni;

d) $AB=7$ *dm*, $AC=6\sqrt{3}$ *dm* va $\angle A=150^\circ$ bo‘lsa, BC ni toping.

29.3. Tomonlari 5 *cm*, 6 *cm*, 7 *cm* bo‘lgan uchburchak burchaklari kosinuslarini toping.

29.4. ABC uchburchakda $AB=10$ *cm*, $BC=12$ *m* va $\sin B=0{,}6$ bo‘lsa, AC tomonni toping.

29.5. Parallelogrammning diagonallari 10 *cm* va 12 *cm*, ular orasidagi burchagi 60° ga teng. Parallelogramm tomonlarini toping.

29.6. Tomonlari 5 *cm* va 7 *cm* bo‘lgan parallelogrammning bir burchagi 120° ga teng. Uning diagonallarini toping.

29.7*. Tomonlari a, b, c bo‘lgan ABC uchburchakning BD medianasi $BD=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}$ formula bilan hisoblanishini isbotlang (*4-rasm*).

29.8*. Tomonlari 6 *m*, 7 *m* va 8 *m* bo‘lgan uchburchak medianalarini toping.

29.9. Tomonlari 5 *cm*, 6 *cm*, 7 *cm* bo‘lgan uchburchak bissektrisalarini toping.

29.10. Tomonlari 5 *cm*, 6 *cm*, 7 *cm* bo‘lgan uchburchak balandliklarini toping.

SINUSLAR VA KOSINUSLAR TEOREMALARINING BA’ZI TATBIQLARI

Oldingi darslarda isbotlangan sinuslar va kosinuslar teoremalaridan uchburchaklarga oid turli-tuman masalalarni yechishda samarali foydalanish mumkin. Bu darsda bu teoremalarning ba’zi bir tatbiqlariga to‘xtalamiz.

1. Kosinuslar teoremasi uchburchak burchaklarini topmasdan, uning burchaklar bo‘yicha turini (o‘tkir, o‘tmas yoki to‘g‘ri burchakli ekanligini) aniqlashga imkon beradi. Haqiqatan,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

formulada

1) agar $b^2+c^2>a^2$ bo‘lsa, $\cos A>0$. Demak, A — o‘tkir burchak;
2) agar $b^2+c^2=a^2$ bo‘lsa, $\cos A=0$. Demak, A — to‘g‘ri burchak;
3) agar $b^2+c^2<a^2$ bo‘lsa, $\cos A<0$. Demak, A — o‘tmas burchak.

$b^2+c^2=a^2$ tenglik yoki $b^2+c^2<a^2$ tengsizlik a — uchburchakning eng katta tomoni bo‘lgan holdagina bajariladi. Demak, uchburchakning to‘g‘ri yoki o‘tmas burchagi uning eng katta tomoni qarshisida yotadi.

Uchburchakning eng katta tomoni qarshisidagi burchakning kattaligiga qarab, bu uchburchakning qanday (o‘tkir, o‘tmas, to‘g‘ri burchakli) uchburchak ekanligi haqida xulosaga kelish mumkin.

1-masala. Tomonlari 5 *m*, 6 *m* va 7 *m* bo‘lgan uchburchak burchaklarini topmasdan uning turini aniqlang.

Yechish. Eng katta burchak qarshisida eng katta tomon yotadi. Shuning uchun, agar $a=7$, $b=6$, $c=5$ bo‘lsa, $\angle A$ eng katta burchak bo‘ladi.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 25 - 49}{2 \cdot 6 \cdot 5} = \frac{12}{60} = \frac{1}{5} > 0.$$

Demak, A — o‘tkir burchak, berilgan uchburchak esa o‘tkir burchakli.

2. Uchburchak yuzini uning ikki tomoni va ular orasidagi burchagi orqali hisoblash formulasi

$$S = \frac{1}{2} bc \sin A$$

va $\sin A = \frac{a}{2R}$ formulalardan uchburchak yuzini hisoblash uchun

$$S = \frac{abc}{4R}$$

formulani va uchburchakka tashqi chizilgan aylana radiusini hisoblash uchun

$$R = \frac{abc}{4S}$$

formulani hosil qilamiz.

2-masala. Tomonlari $a=5$, $b=6$, $c=10$ bo‘lgan uchburchakka tashqi chizilgan aylana radiusini toping.

Yechish. Geron formulasidan foydalanib, uchburchak yuzini topamiz:

$$p=\frac{a+b+c}{2}=\frac{5+7+10}{2}=11,$$

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{11(11-5)(11-7)(11-10)}=\sqrt{11\cdot6\cdot4}=\sqrt{264}\approx16{,}3.$$

Unda, $R=\dfrac{abc}{4S}\approx\dfrac{5\cdot7\cdot10}{4\cdot16{,}3}\approx5{,}4.$ *Javob:* $\approx5{,}4.$

Masala va topshiriqlar

30.1. Agar $AB=7$ *cm*, $BC=8$ *cm*, $CA=9$ *cm* bo‘lsa, ABC uchburchakning eng katta va eng kichik burchagini toping.

30.2. Agar ABC uchburchakda $\angle A=47^\circ$, $\angle B=58^\circ$ bo‘lsa, uchburchakning eng katta va eng kichik tomonlarini aniqlang.

30.3. Uchburchakning uchta tomoni berilgan:
a) $a=5$, $b=4$, $c=4$; b) $a=17$, $b=8$, $c=15$; d) $a=9$, $b=5$, $c=6$.
Uchburchak o‘tkir burchakli, to‘g‘ri burchakli yoki o‘tmas burchakli ekanligini aniqlang.

30.4. Tomonlari a) 13, 14, 15; b) 15, 13, 4; d) 35, 29, 8; e) 4, 5, 7 bo‘lgan uchburchakka tashqi chizilgan aylana radiusini toping.

30.5. ABC uchburchakning AB tomonida D nuqta belgilangan. CD kesma AC va BC kesmalarning kamida bittasidan kichik ekanligini isbotlang.

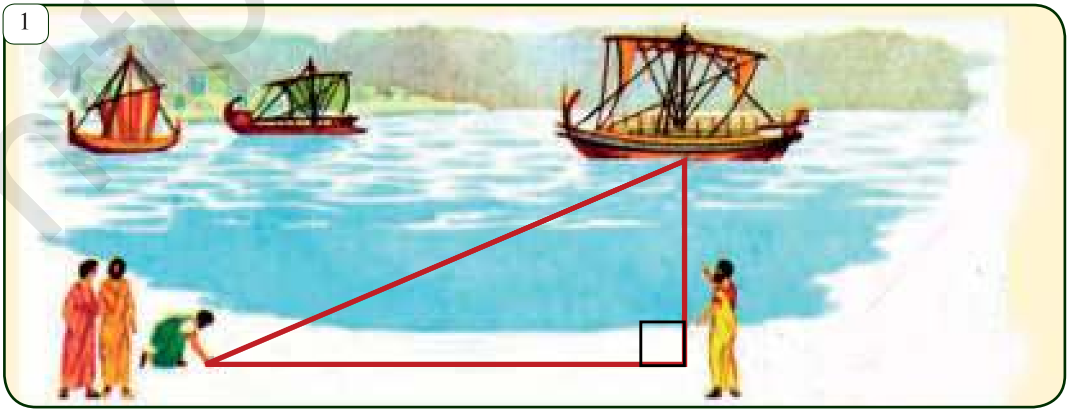
30.6. Uchburchakning katta burchagi qarshisida katta tomoni yotishini isbotlang.

30.7. Uchburchakning katta tomoni qarshisida katta burchagi yotishini isbotlang.

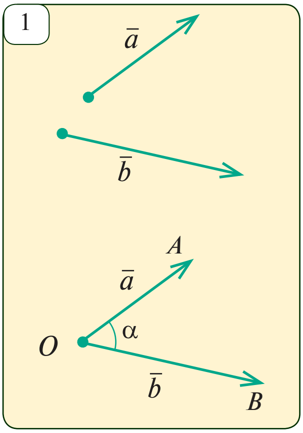
30.8*. ABC uchburchakning CD medianasi o‘tkazilgan. Agar $AC>BC$ bo‘lsa, ACD burchak BCD burchakdan kichik bo‘lishini isbotlang.

30.9*. 1-rasmda berilganlarga asoslanib, qadimda yunonlar qirg‘oqdan kemagacha bo‘lgan masofani qanday o‘lchaganlarini aniqlang.

1



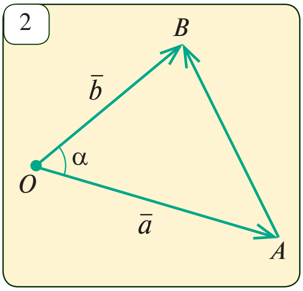
31 IKKI VEKTOR ORASIDAGI BURCHAK VA ULARNING SKALAR KO‘PAYTMASI



Nol vektordan farqli $\bar{a}$ vaa $\bar{b}$ *vektorlar orasidagi burchak* deb O nuqtadan chiquvchi $\overline{OA}=\bar{a}$ va $\overline{OB}=\bar{b}$ vektorlarning yo‘naltiruvchi kesmalari orasidagi AOB burchakka aytiladi (1- rasm).

Bir xil yo‘nalgan vektorlar orasidagi burchak 0° ga teng deb hisoblanadi. Agar ikkita vektor orasidagi burchak 90° ga teng bo‘lsa, ular *perpendikular* deyiladi.

$\bar{a}$ vaa $\bar{b}$ vektorlarning *skalar ko‘paytmasi* deb, bu vektorlar uzunliklarining ular orasidagi burchak kosinusi ko‘paytmasiga aytiladi.



Agar vektorlarning biri nol vektor bo‘lsa, ularning skalar ko‘paytmasi nolga teng bo‘ladi.

Skalar ko‘paytma $\bar{a}\cdot\bar{b}$ yoki $(\bar{a},\bar{b})$ tarzda belgilanadi. Ta’rifga ko‘ra

$$(\bar{a},\bar{b}) = |\bar{a}|\cdot|\bar{b}|\cos\varphi. \qquad (1)$$

Ta’rifdan ko‘rinadiki, $\bar{a}$ va $\bar{b}$ vektorlarning skalar ko‘paytmasi nolga teng bo‘lsa, ular *perpendikular* bo‘ladi va aksincha.

Fizikada jismni $\bar{F}$ kuch ta’siri ostida $\bar{s}$ masofaga siljitishda bajarilgan A ish $\bar{F}$ va $\bar{s}$ vektorlarning skalar ko‘paytmsiga teng bo‘ladi:

$$A = (\bar{F},\bar{s}) = |\bar{F}|\cdot|\bar{s}|\cos\varphi.$$

Xossa. $\bar{a}\,(a_1;\ a_2)$ va $\bar{b}(b_1;\ b_2)$ vektorlar uchun $(\bar{a};\ \bar{b}) = a_1b_1+a_2b_2$.

Isbot. $\bar{a}$ va $\bar{b}$ vektorlarni koordinata boshi O nuqtaga qo‘yamiz (2- rasm). Unda $\overline{OA} = (a_1;\ a_2)$ va $\overline{OB} = (b_1;\ b_2)$ bo‘ladi. Agar berilgan vektorlar kollinear bo‘lmasa, ABO uchburchakdan iborat bo‘ladi va uning uchun kosinuslar teoremasi o‘rinli bo‘ladi: $AB^2 = OA^2 + OB^2 - 2OA\cdot OB\cdot\cos\varphi$.

Unda $OA\cdot OB\cdot\cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2)$ bo‘ladi.

Lekin, $OA^2 = a_1^2 + a_2^2$, $OB^2 = b_1^2 + b_2^2$ va $AB^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2$.

Demak, $(\bar{a},\bar{b}) = |\bar{a}|\cdot|\bar{b}|\cos\varphi = OA.OB.\cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2) =$

$$= \frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 + (b_1 - a_1)^2 + (b_2 - a_2)^2\right) = a_1b_1 + a_2b_2.$$

Berilgan vektorlar kollinear bo‘lgan ($\varphi = 0°$, $\varphi = 180°$) holda ham bu tenglik o‘rinli bo‘lishini mustaqil ko‘rsating. $\square$

Vektorlar skalar ko'paytmasining xossalari

1. $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{a}$ o'rin almashtirish xossasi.

2. $(\bar{a}+\bar{b})\ \bar{c} = \bar{a}\ \ \bar{c} + \bar{b}\ \ \bar{c}$ taqsimot xossasi.

3. $\lambda\cdot(\bar{a}\cdot\bar{b})=(\lambda\cdot\bar{a})\cdot\bar{b}=\bar{a}\cdot(\lambda\cdot\bar{b})$ guruhlash xossasi.

4. Agar a va b vektorlar bir xil yo'nalishdagi kollinear vektorlar bo'lsa, $\bar{a}\ \bar{b} = |\bar{a}|\ \ |\bar{b}|$ bo'ladi, chunki $\cos 0° = 1$.

5. Agar qarama-qarshi yo'nalgan bo'lsa, $\bar{a}\ \bar{b} = -|\bar{a}||\bar{b}|$, chunki $\cos180° = -1$.

6. $\bar{a}\ \ \bar{a} = |\bar{a}||\bar{a}|\cos0° = |\bar{a}|^2 \Rightarrow \bar{a}^{\,2} = |\bar{a}|^2$.

7. $\bar{a}$ va $\bar{b}$ vektorlar o'zaro perpendikular bo'lsa, $\bar{a}\ \bar{b} = 0$ bo'ladi.

Natijalar:

a) $\bar{a}=(a_1;a_2)$ vektorning uzunligi: $|\bar{a}|=\sqrt{a_1^{\,2}+a_2^{\,2}}$; (1)

b) $\bar{a}=(a_1;a_2)$ va $\bar{b}=(b_1;b_2)$ vektorlar orasidagi burchak kosinusi:

$$\cos\varphi=\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} \quad \text{yoki} \quad \cos\varphi=\frac{a_1b_1+a_2b_2}{\sqrt{a_1^{\,2}+a_2^{\,2}}\cdot\sqrt{b_1^{\,2}+b_2^{\,2}}}$$

Masala. $\bar{a}(1;2)$ va $\bar{b}(4;-2)$ vektorlar orasidagi burchakni toping.

Yechish. Berilgan vektorlar orasidagi burchakni α deb belgilasak, formulaga ko'ra, $\cos\alpha=\dfrac{1\cdot4+2\cdot(-2)}{\sqrt{1^2+2^2}\cdot\sqrt{4^2+(-2)^2}}=\dfrac{4-4}{\sqrt{5}\cdot\sqrt{20}}=0.$ Demak, $\alpha=90°$. *Javob:* 90°.

Masala va topshiriqlar

31.1. Agar $\bar{a}$ va $\bar{b}$ vektorlar uchun *a)* $|\bar{a}|=4$, $|\bar{b}|=5$, $\alpha=30°$; *b)* $|\bar{a}|=8$, $|\bar{b}|=7$, $\alpha=45°$; *d)* $|\bar{a}|=2.4$, $|\bar{b}|=10$, $\alpha=60°$; *e)* $|\bar{a}|=0{,}8$, $|\bar{b}|=\frac{1}{2}$, $\alpha=40°$ bo'lsa, bu vektorlarning skalar ko'paytmasini toping (bu yerda — $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak).

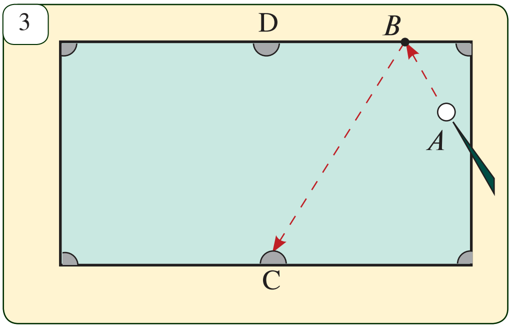
31.2. *a)* $\bar{a}(\frac{1}{2};-1)$ va $\bar{b}(2;3)$; b) $\bar{a}(-5;6)$ va $\bar{b}(6;5)$; d) $\bar{a}(1{,}5;2)$ va $\bar{b}(4;-2)$ vektorlarning skalar ko'paytmasini hisoblang va ular orasidagi burchakni toping.

31.3. *ABCD* rombning diagonallari *O* nuqtada kesishadi va bunda $\overline{BD}=\overline{AB}=4$ *cm*.a) $\overline{AB}$ va $\overline{AD}$; b) $\overline{AB}$ va $\overline{AC}$; d) $\overline{AD}$ va $\overline{DC}$; e) $\overline{OC}$ va $\overline{OD}$ vektorlarning skalar ko'paytmasini va bu vektorlar orasidagi burchakni toping.

31.4. Nol vektordan farqli $\bar{a}$ va $\bar{b}$ vektorlar berilgan bo'lsin. $\bar{a}\cdot\bar{b}=0$ bo'lganda bu vektorlar perpendikular bo'lishini va aksincha, $\bar{a}$ va $\bar{b}$ vektorlar perpendikular bo'lsa, $\bar{a}\cdot\bar{b}=0$ bo'lishini isbotlang.

31.5*. x ning qanday qiymatida a) $\bar{a}(4;5)$ va $b(x;6)$; b) $\bar{a}(x;1)$ va $\bar{b}(3;2)$; d) $\bar{a}(0;-3)$ va $\bar{b}(5;x)$ vektorlar o'zaro perpendikular bo'ladi?

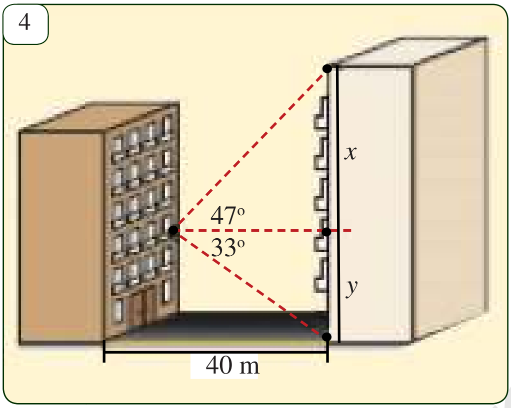
31.6. $\bar{a}(3;3)$, $\bar{b}(2;-2)$, $\bar{c}(-1;-4)$ va $\bar{d}(-4;1)$ vektorlar orasidan o'zaro perpendikular juftlarini toping.



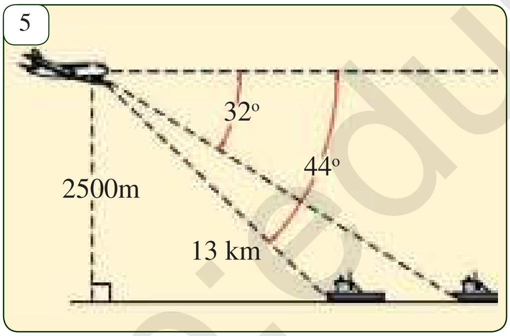
31.7*. Bilyard o‘yinida A nuqtada turgan shar zarbadan keyin bilyard stoli tomoniga B nuqtada urildi va yo‘nalishini o‘zgartirib C nuqtadagi savatchaga tushdi (*3-rasm*).
Agar $AB=40$ *cm*, $BC=150$ *cm* va $ABD=120°$ bo‘lsa, $AB \cdot BC$ skalar ko‘paytmani toping.

31.8. $F(-3, 4)$ kuch ta’siri ostida nuqta $A(5,-1)$holatdan $B(2, 1)$ holatga o‘tdi. Bu jarayonda qanday ish bajarildi?

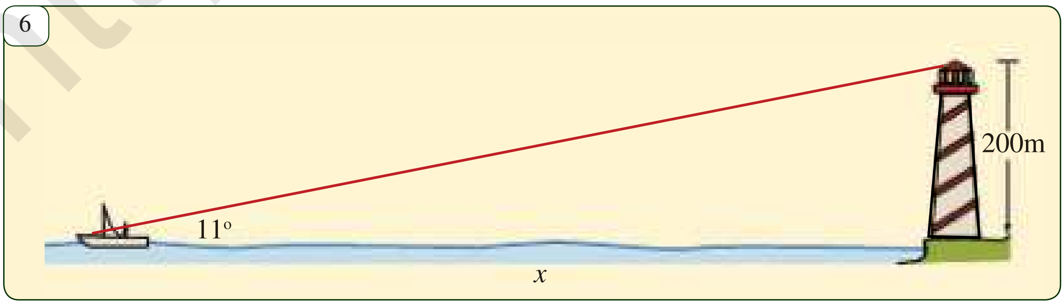
31.9. Lola ko‘p qavatli uyning 3-qavatida yashaydi. Uning oynasidan uyidan 40 *m* masofada turgan boshqa bir uy ko‘rinib turadi (4-rasm). Agar ro‘paradagi uyning tomi Lolaga 47° burchak ostida, partki asosi esa 33° burchak ostida ko‘rinsa, ro‘paradagi uyning balandligini toping.



31.10. 2500 *m* balandlikda uchib borayotgan samolyotdan birinchi kema ufqqa nisbatan 44° burchak ostida, ikkinchi kema esa 32° burchak ostida ko‘rinadi (5-rasm). Kemalar orasidagi masofani toping.



31.11. Baliqchilar qayig‘idan balandligi 200 *m* bo‘lgan mayoq 11° burchak ostida ko‘rinadi (6-rasm). Qayiqdan qirg‘oqqacha bo‘lgan masofani toping.



Geometriya va geografiyadan loyiha ishi

Geografiya fanidan ma'lumki, Yer shari sirtidagi joylar geografik koordinatalar yordamida aniqlanadi. 7-rasmda bu koordinatalar keltirilgan. Unda

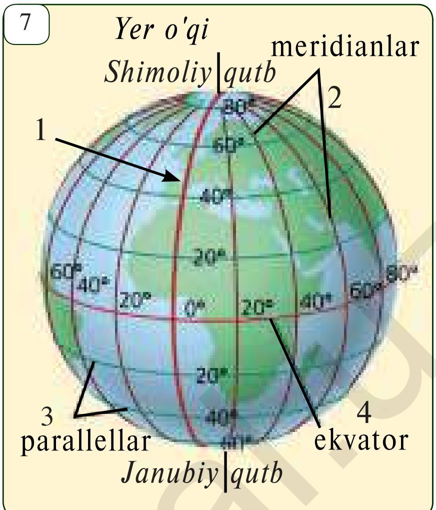
1 - nolinchi (Grinvich) meridiani;

2- nolinchi meridiandan o'ngda (sharqda) joylashgan meridianlar;

3- ekvatordan pastda (janubda) joylashgan parallellar;

4 - ekvator.

7

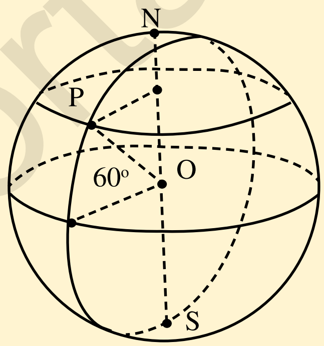


Nolinchi (Grinvich) meridiani (1) ning ekvator (4) bilan kesishish nuqtasi geografik koordinatalarning sanoq boshi hisoblanadi.

Ekvatordan shimolga tomon meridian bo'ylab chorak aylana yoyi 90° shimoliy kenglikni, ekvatordan janubga tomon ham 90° janubiy kenglikni o'z ichiga oladi.

Nolinchi meridiandan sharqqa tomon ekvator bo'ylab yarim aylana yoyi 180° sharqiy uzoqlikni, nolinchi meridiandan garbga tomon ham 180° g'arbiy uzoqlikni o'z ichiga oladi.

8



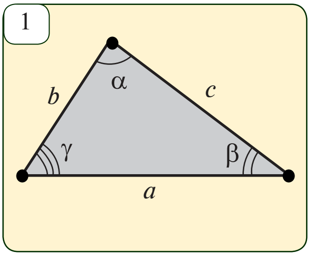
1. Toshkent shahrining geografik koordinatalarini toping.
2. Vatanimiz poytaxti bilan yana qaysi katta shaharlar taxminan bir xil meridianda joylashgan.
3. Toshkent shahridan Tokio, Pekin, Seul, Vashington va Nyu-York shaharlarigacha (meridian bo'ylab) bo'lgan masofalarni aniqlang (yetishmayotgan ma'lumotlarni o'zingiz qidirib toping).
4. Shahar 60° shimoliy kenglikda joylashgan. Agar Yerning radiusi 6400 *km* bo'lsa, bu shahar joylashgan parallel radiusini toping.

Qiziqarli geometriya

Ovchi ovga chiqdi. Dastlab u janubga tomon 1 *km* yurdi. So'ng sharqqa tomon 1 *km*, so'ng esa shimolga tomon 1 *km* yo'l yurdi va boshlang'ich holatiga kelib qoldi. Qarasa, ayiq turibdi. Uni otdi.

1. Ovlangan ayiq rangi qanaqa?

2. Yer sharining yana qaysi joylaridan yo'lga chiqib, yuqorida tasvirlanganidek 3 tomonga yurib yana boshlang'ich nuqtaga kelib qolish mumkin? U joylarda ayiq yashaydimi?



Uchburchakning tomonlarini *a, b, c* bilan, bu tomonlar qarshisidagi burchaklarni mos ravishda α, β, γ bilan belgilaymiz *(1-rasm)*. Uchburchakning tomonlari va burchaklarini bitta nom bilan — uning *elementlari* deb atashadi.

Uchburchakni aniqlovchi berilgan elementlariga ko'ra, uning qolgan elementlarini topish *uchburchakni yechish deb yuritiladi.*

1-masala. *(Uchburchakni berilgan bir tomoni va unga yopishgan burchaklari bo'yicha yechish).* Agar uchburchakda $a=6$, $\beta=60^\circ$ va $\gamma=45^\circ$ bo'lsa, uning uchinchi burchagi va qolgan ikki tomonini toping.

Yechish. 1. Uchburchak burchaklari yig'indisi 180° bo'lgani uchun

$$\alpha=180^\circ-\beta-\gamma=180^\circ-60^\circ-45^\circ=75^\circ.$$

Sinuslar teoremasidan foydalanib, qolgan ikki tomonni topamiz:

2. $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}$ tenglikdan $b=a\cdot\dfrac{\sin\beta}{\sin\alpha}=6\cdot\dfrac{\sin 60^\circ}{\sin 75^\circ}\approx 6\cdot\dfrac{0{,}8660}{0{,}9659}\approx 5{,}3794\approx 5{,}4.$

(sin60° va sin75° qiymatlari mikrokalkulatorda topib qo'yildi, ularni darslikning 153-betidagi jadvaldan ham topishingiz mumkin).

3. $\dfrac{a}{\sin\alpha}=\dfrac{c}{\sin\gamma}$ tenglikdan $c=a\cdot\dfrac{\sin\gamma}{\sin\alpha}=6\cdot\dfrac{\sin 45^\circ}{\sin 75^\circ}\approx 6\cdot\dfrac{0{,}7071}{0{,}9659}\approx 4{,}3924\approx 4{,}4.$

Javob: $\alpha=75^\circ$; $\beta\approx 5{,}4$; $c\approx 4{,}4$.

2-masala. *(Uchburchakni berilgan ikki tomoni va ular orasidagi burchagi bo'yicha yechish).* Agar uchburchakda $a=6$, $b=4$ va $\gamma=120^\circ$ bo'lsa, uning uchinchi tomoni va qolgan burchaklarini toping.

Yechish. 1. Kosinuslar teoremasidan foydalanib, uchburchakning uchinchi c tomonini topamiz.

$$c=\sqrt{a^2+b^2-2ab\cos\gamma}=\sqrt{36+16-2\cdot 6\cdot 4\cdot(-0{,}5)}=\sqrt{76}\approx 8{,}7.$$

2. Endi, uchburchakning uchta tomonini bilgan holda, kosinuslar teoremasidan foydalanib, uchburchakning qolgan burchaklarini topamiz:

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}=\frac{4^2+76-6^2}{2\cdot 4\cdot\sqrt{76}}\approx 0{,}8046.$$

$\cos\alpha\approx 0{,}8046$ tenglik asosida α burchakning qiymatini 153-betdagi jadvaldan aniqlaymiz (α — o'tkir burchak): $\alpha\approx 36^\circ$.

3. $\beta=180^\circ-\alpha-\gamma\approx 180^\circ-(36^\circ+120^\circ)=24^\circ$. *Javob:* $c\approx 8{,}7$; $\alpha\approx 36^\circ$, $\beta\approx 24^\circ$.

3-masala. *(Uchburchakni berilgan uch tomoni bo'yicha yechish).* Agar uchburchakda $a=10$, $b=6$ va $c=13$ bo'lsa, uning burchaklarini toping.

Yechish: **1.** Uchburchak o'tmas burchakli bo'lishi yoki bo'lmasligini katta tomon qarshisidagi burchak kosinusining ishorasiga qarab aniqlaymiz:

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{100+36-169}{2\cdot10\cdot6}=-\frac{33}{120}\approx-0{,}275<0.$$

Demak, C — o'tmas burchak ekan. Buni 153-betdagi jadvaldan C burchakning kattaligini aniqlashda hisobga olamiz. Jadvaldan kosinusi 0,275 ga teng burchak $\angle C_1=74^\circ$ ekanligini topamiz. Unda $\cos(180^\circ-\alpha)=-\cos\alpha$ formulaga ko'ra,

$$\angle C=180^\circ-\angle C_1=180^\circ-74^\circ=106^\circ.$$

2. Sinuslar teoremasiga ko'ra,

$\frac{a}{\sin A}=\frac{c}{\sin C}$. Bundan, $\sin A=\frac{a\cdot\sin C}{c}=\frac{10\cdot\sin106^\circ}{13}=\frac{10\cdot\sin74^\circ}{13}\approx\frac{10\cdot0{,}9615}{13}\approx0{,}7396.$

A — o'tkir burchak bo'lgani uchun 153-betdagi jadvaldan $\angle A\approx47^\circ$ ekanligini aniqlaymiz.

3. $\angle B\approx180^\circ-(106^\circ+47^\circ)=26^\circ$. *Javob:* $\angle A\approx47^\circ$, $\angle B\approx26^\circ$, $\angle C\approx106^\circ$.

Masala va topshiriqlar

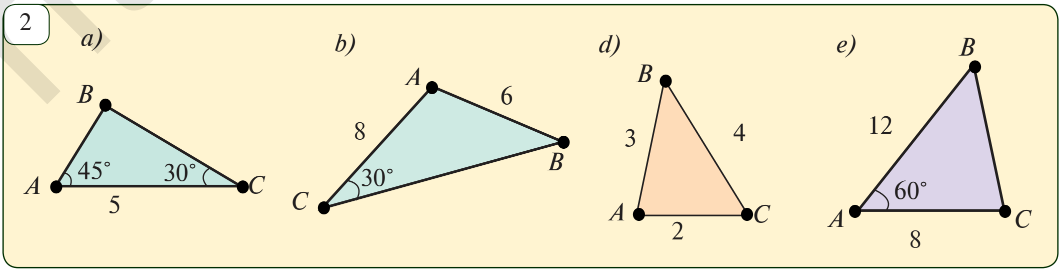
32.1. Uchburchakning bir tomoni va unga yopishgan ikkita burchagi berilgan:
a) $a=5$ *cm*, $\beta=45^\circ$, $\gamma=45^\circ$; b) $c=20$ *cm*, $\alpha=75^\circ$, $\beta=60^\circ$;
d) $a=35$ *cm*, $\beta=40^\circ$, $\gamma=120^\circ$; e) $c=12$ *cm*, $\alpha=36^\circ$, $\beta=25^\circ$.
Uchburchakning uchinchi burchagi va qolgan ikki tomonini toping.

32.2. Uchburchakning ikki tomoni va ular orasidagi burchagi berilgan:
a) $a=6$, $b=4$, $\gamma=60^\circ$; b) $a=14$, $b=43$, $\gamma=130^\circ$;
d) $b=17$, $c=9$, $\alpha=85^\circ$; e) $b=14$, $c=10$, $\alpha=145^\circ$.
Uchburchakning qolgan burchaklarini va uchinchi tomonini toping.

32.3. Uchburchakning uchta tomoni berilgan:
a) $a=2$, $b=3$, $c=4$; b) $a=7$, $b=2$, $c=8$;
d) $a=4$, $b=5$, $c=7$; e) $a=15$, $b=24$, $c=18$.
Uchburchakning burchaklarini toping.

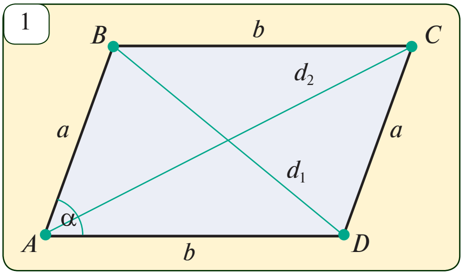
32.4. Uchburchakning ikki tomoni va bu tomonlardan birining qarshisidagi burchagi berilgan. Uchburchakning qolgan tomoni va burchaklarini toping:
a) $a=12$, $b=5$, $\alpha=120^\circ$; b) $a=27$, $b=9$, $\alpha=138^\circ$;
d) $b=2$, $c=2$, $\alpha=60^\circ$; e) $b=6$, $c=8$, $\alpha=30^\circ$.

32.5. 2-rasmda berilgan ma'lumotlar asosida uchburchakni yeching.



1-masala. Parallelogramm diagonallari kvadratlarining yig'indisi tomonlari kvadratlari yig'indisining ikkilanganiga teng ekanligini isbotlang.

ABCD — parallelogramm, $AB=a$, $AD=b$, $BD=d_1$, $AC=d_2$ *(1-rasm).* ⇒ $d_1^2+d_2^2=2(a^2+b^2)$



Yechish. $ABCD$ parallelogrammning A burchagi α ga teng bo'lsin. Unda $\angle B=180^\circ-\alpha$. ABD va ABC uchburchaklarga kosinuslar teoremasini qo'llaymiz *(1-rasm)*:

$$d_1^2=a^2+b^2-2ab\cos\alpha, \qquad (1)$$

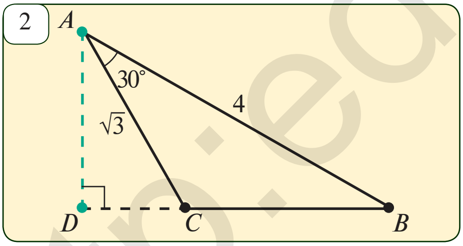
$$d_2^2=a^2+b^2-2ab\cos(180^\circ-\alpha).$$

$\cos(180^\circ-\alpha)=-\cos\alpha$ tenglikni hisobga olsak,

$$d_2^2=a^2+b^2+2ab\cos\alpha. \qquad (2)$$

(1) va (2) tengliklarning mos qismlarini qo'shib, $d_1^2+d_2^2=2(a^2+b^2)$ tenglikni hosil qilamiz.

2-masala. ABC uchburchakda $\angle A=30^\circ$, $AB=4$, $AC=\sqrt{3}$ bo'lsa, uchburchakning A uchidan tushirilgan AD balandligini toping (*2-rasm*).



Yechish. 1) Kosinuslar teoremasidan foydalanib, uchburchakning BC tomonini topamiz:

$$BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos A=$$

$$=4^2+(\sqrt{3})^2-2\cdot4\cdot\sqrt{3}\cdot\cos30^\circ=7, \qquad BC=\sqrt{7}.$$

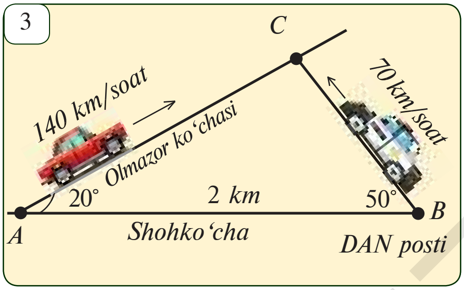
2) Endi uchburchakning yuzini topamiz:

$$S=\frac{1}{2}\cdot AB\cdot AC\cdot\sin A=\frac{1}{2}\sqrt{3}\cdot4\cdot\sin30^\circ=\sqrt{3}.$$

3) Topilganlardan foydalanib, uchburchakning AD balandligini topamiz:

$S=\frac{1}{2}\cdot BC\cdot AD$ formuladan $AD=\frac{2S}{BC}=\frac{2\sqrt{3}}{\sqrt{7}}=\frac{2\sqrt{21}}{7}$. *Javob:* $\frac{2\sqrt{21}}{7}$.

3-masala. Haydovchi yo'l harakati qoidasini buzib, soat 12^{00} da shohko'chaning A nuqtasidan Olmazor ko'chasi tomon burildi va 140 *km/soat* tezlikda harakatini davom ettirdi (*3-rasm*). Soat 12^{00} da DAN xodimi B nuqtadan toshloq yo'l bo'ylab 70 *km/soat* tezlikda qoidabuzar haydovchi yo'lini kesib chiqish uchun yo'lga chiqdi. DAN xodimi chorrahada, ya'ni C nuqtada qoidabuzar haydovchini to'xtatib qola oladimi?



Yechish: ABC uchburchakda

$\angle C=180^\circ-(\angle A+\angle B)=180^\circ-(20^\circ+50^\circ)=$
$=180^\circ-70^\circ=110^\circ$.

1. Olmazor ko'chasidagi yo'lning AC qismi uzunligini topamiz: sinuslar teoremasiga ko'ra,

$$\frac{AC}{\sin B}=\frac{AB}{\sin C}.\ \text{Bu tenglikdan}\ AC=\frac{AB\cdot\sin B}{\sin C}=\frac{2\cdot\sin50^\circ}{\sin110^\circ}=\frac{2\cdot\sin50^\circ}{\sin(90^\circ+20^\circ)}=\frac{2\cdot\sin50^\circ}{\cos20^\circ}\approx$$

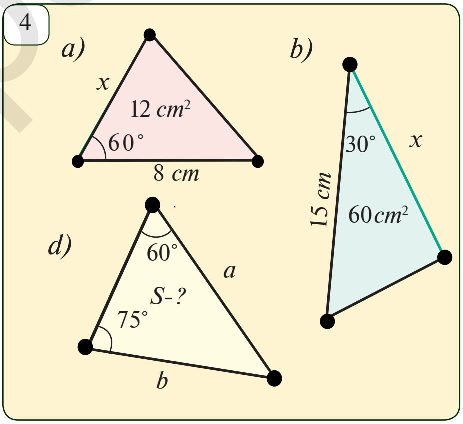
$\approx\frac{2\cdot0{,}766}{0{,}940}=\frac{1{,}532}{0{,}94}\approx$ 1,630 (km). Bu yo'lni qoidabuzar haydovchi $\frac{1{,}630\ km}{140\ km/h}\approx$ $\approx$ 0,0116 *soat*=0,012·3600 *sekund*≈42 *sekund*da bosib o'tadi.

2. Endi toshloq yo'lning BC qismi uzunligini topamiz: sinuslar teoremasiga ko'ra, $\frac{BC}{\sin A}=\frac{AC}{\sin B}$. Bu tenglikdan $BC=\frac{AC\cdot\sin A}{\sin B}=\frac{2\cdot\sin20^\circ}{\sin50^\circ}=\frac{2\cdot0{,}342}{0{,}766}\approx0{,}893$ (km).

Bu yo'lni DAN xodimi $\frac{0{,}893\ km}{70\ km/h}$ ≈0,0128 *soat*=0,0128·3600 *sekund*≈ ≈46 *sekund*da bosib o'tadi. Demak, C chorrahaga DAN xodimi haydovchidan kechroq yetib kelar ekan. *Javob:* Yo'q.

Masala va topshiriqlar

33.1. 4-rasmdagi ma'lumotlar bo'yicha x ning qiymatini toping.



33.2. ABC uchburchakning CD balandligi 4 m. Agar $\angle A=45^\circ$, $\angle B=30^\circ$ bo'lsa, uchburchak tomonlarini toping.

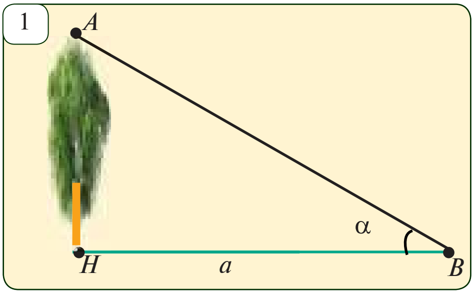
33.3. Bir nuqtaga kattaligi bir xil bo'lgan ikkita kuch qo'yilgan. Agar bu kuchlar yo'nalishlari orasidagi burchak 60° va kuchlarning teng ta'sir etuvchisi 150 kg bo'lsa, bu kuchlar kattaligini toping.

33.4. Uchburchakning ikki tomoni 7 dm va 11 dm, uchinchi tomoniga tushirilgan medianasi esa 6 dm. Uchburchakning uchinchi tomonini toping.

33.5. Tomonlari 6 cm va 8 cm bo'lgan parallelogrammning bir diagonali 12 cm bo'lsa, uning ikkinchi diagonalini toping.

33.6. Uchburchakning 18 cm ga teng tomoni qarshisidagi burchagi 60° ga teng. Uchburchakka tashqi chizilgan aylana radiusini toping.

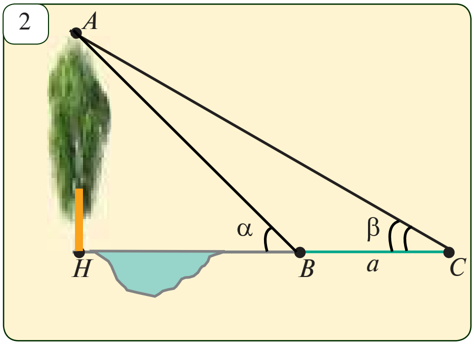
33.7. Teng yonli trapetsiyaning kichik asosi yon tomoniga teng, katta asosi esa 20 cm. Agar trapetsiyaning bir burchagi 120° bo'lsa, uning perimetrini toping.



1. *Balandlikni o'lchash.* Aytaylik, daraxtning AH balandligini o'lchash zarur bo'lsin (*1-rasm*).

a) Buning uchun B nuqtani belgilaymiz va BH masofa a ni va HBA burchak α ni o'lchaymiz. Unda, to'g'ri burchakli ABH uchburchakda

$$AH=BH\,\mathrm{tg}\alpha=a\,\mathrm{tg}\alpha.$$



b) Agar balandlikning asosi H nuqta borib bo'lmaydigan nuqta bo'lsa *(2-rasm)*, yuqoridagi usul bilan AH balandlikni aniqlay olmaymiz. Unda quyidagicha yo'l tutamiz:

1) H nuqta bilan bir to'g'ri chiziqda yotgan B va C nuqtalarni belgilaymiz;

2) BC masofani o'lchab a ni topamiz;

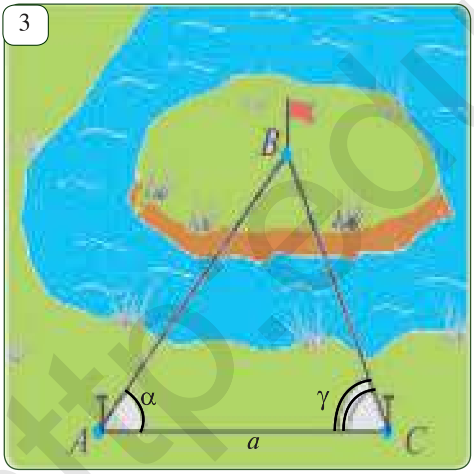
3) ABH va ACH burchaklarni o'lchab $\angle ABH=\alpha$ va $\angle ACH=\beta$ larni topamiz;

4) ABC uchburchakka sinuslar teoremasini qo'llasak ($\angle BAC=\alpha-\beta$)

$$\frac{AB}{\sin\beta}=\frac{a}{\sin(\alpha-\beta)}, \quad \text{ya'ni } AB=\frac{a\sin\beta}{\sin(\alpha-\beta)}.$$

5) to'g'ri burchakli ABH uchburchakda AH balandlikni topamiz:

$$AH=AB\sin\alpha=\frac{a\sin\alpha\cdot\sin\beta}{\sin(\alpha-\beta)}.$$



2. *Borib bo'lmaydigan nuqtagacha bo'lgan masofani hisoblash.* Aytaylik, A nuqtadan borib bo'lmaydigan B nuqtagacha bo'lgan masofani hisoblash kerak *(3-rasm)*. Bu masalani uchburchaklarning o'xshashlik alomatlaridan foydalanib javobini topganimizni eslatib o'tamiz.

Endi bu masalani sinuslar teoremasidan foydalanib yechamiz.
1) A va B nuqtalardan ko'rinib turgan tekis joyda C nuqtani belgilaymiz.
2) AC masofani o'lchaymiz: $AC=a$.
3) Asboblar yordamida ACB va BCA burchaklarni o'lchaymiz: $\angle BAC=\alpha$, $\angle BCA=\gamma$.
4) ABC uchburchakda $\angle B=180°-\alpha-\gamma$ bo'lgani uchun,

$$\sin B=\sin(180°-\alpha-\gamma)=\sin(\alpha+\gamma).$$

Sinuslar teoremasiga ko'ra, $\dfrac{AB}{\sin C}=\dfrac{AC}{\sin B}$ yoki $AB=\dfrac{a\sin\gamma}{\sin(\alpha+\gamma)}$.

Masala va topshiriqlar

34.1. 1-rasmda $a = 12$ *m*, $\alpha = 42°$ bo'lsa, daraxt balandligini hisoblang.

34.2. 2-rasmda $a = 8$ *m*, $\alpha = 43°$, $\beta = 32°$ bo'lsa, daraxt balandligini hisoblang.

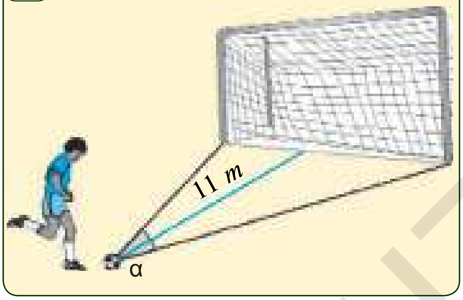
34.3. 3-rasmda $a = 60$ *m*, $\alpha = 62°$, $\gamma = 44°$ bo'lsa, AB masofani toping.

34.4. Futbol o'yinida 11 metrlik jarima to'pini darvozaga yo'naltirish burchagi α ni toping (*4-rasm*). Darvozaning kengligi 7 *m*.

34.5. 5-rasmda Xiva shahridagi Kaltaminor tasvirlangan. Agar $\beta = 30{,}7°$, $\gamma = 45°$, $BC = 50$ *m* bo'lsa, Kaltaminor balandligini toping.

34.6. Sayohatchi Shahrisabz shahridagi Oqsaroyni undan 47 *m* masofada tomosha qilyapti (*6-rasm*). Agar u Oqsaroy asosini gorizontga nisbatan 2,5° ga teng burchak ostida, tepa qismini esa 45° ga teng burchak ostida ko'rayotgan bo'lsa, Oqsaroy balandligini toping.

34.7. Uchta yo'l ABC uchburchakni tashkil qiladi. Bu uchburchakda $\angle A = 20°$, $\angle B = 150°$. A nuqtadan yo'lga chiqqan haydovchi C nuqtaga imkon boricha tezroq yetib bormoqchi. AC va CB yo'llar toshloq, AB asfalt yo'l bo'lib, asfalt yo'lda toshloq yo'lga qaraganda 2 baravar tezroq harakatlanish mumkin. Haydovchiga qaysi yo'ldan yurishni maslahat berasiz?

4


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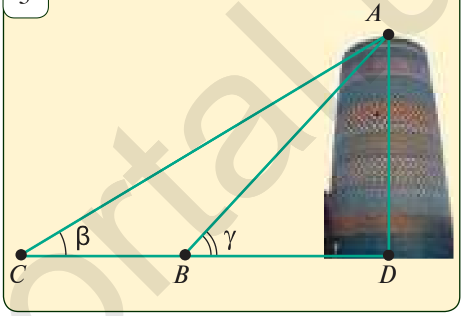

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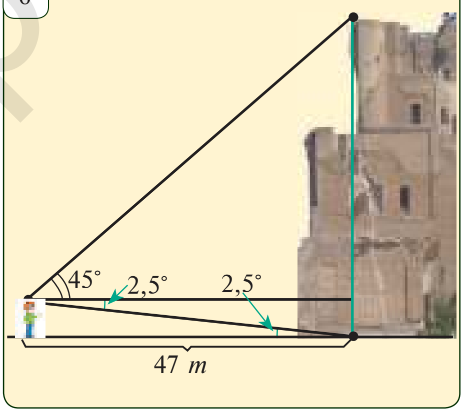


Qiziqarli masala

Pifagor teoremasining yana bir "isboti"

To'g'ri burchakli ABC uchburchakda $a = c\sin\alpha$, $b = c\cos\alpha$. Bu ikki tenglikni kvadratga oshirib, hadma-had qo'shsak va $\sin^2\alpha + \cos^2\alpha = 1$ ekanligini hisobga olsak,

$$a^2 + b^2 = c^2\sin^2\alpha + c^2\cos^2\alpha = c^2(\sin^2\alpha + \cos^2\alpha) = c^2.$$

Demak, $a^2 + b^2 = c^2$. Bu "isbot" mantiqan noto'g'ri ekanligini asoslang.

I. Testlar

1. Tomonlari a, b, c, mos burchaklari α, β, γ, yuzi S bo‘lgan uchburchak uchun qaysi tenglik noto‘g‘ri?

A. $a^2 = b^2 + c^2 - 2bc\cos\alpha$; B. $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$;

D. $S = \frac{1}{2}ab\sin\gamma$; E. $S = \frac{1}{2}ab\sin\alpha$.

2. Noto‘g‘ri tenglikni toping:

A. $\sin^2\alpha + \cos^2\alpha = 1$; B. $\sin(180° - \alpha) = \sin\alpha$;

D. $\cos(180° - \alpha) = \cos\alpha$; E. $\sin(90° - \alpha) = \cos\alpha$.

3. Uchburchakning uchta tomoni ma’lum bo‘lsa, qaysi teoremadan foydalanib uning burchaklarini topish mumkin?

A. Sinuslar teoremasi; B. Kosinuslar teoremasi;

D. Fales teoremasi; E. Geron formulasi.

4. Uchburchakning bir burchagi 137° ga, ikkinchi burchagi 15° ga teng. Agar bu uchburchakning katta tomoni 22 ga teng bo‘lsa, uning kichik tomonini toping.

A. 8,3; B. 9,3; D. 3,8; E. 6,5.

5. Uchburchakning 14 va 19 ga teng bo‘lgan tomonlari orasidagi burchagi 26°. Shu uchburchakning uchinchi tomonini toping.

A. 1,2; B. 5,4; D. 6,9; E. 19,7.

6. Agar ikki vektorning uzunliklari $|\bar{a}|=2$, $|\bar{b}|=5$ va ular orasidagi burchak 45° bo‘lsa, $\bar{a}$ va $\bar{b}$ vektorlarning skalar ko‘paytmasini toping.

A. 52; B. 32 D. 102; E. 2.

7. $\bar{a}(4; -1)$ va $\bar{b}(2; 3)$ vektorlarning skalar ko‘paytmasini toping.

A. 5; B. 3; D. 4; E. 9.

8. $\bar{a}\left(-\frac{1}{2}; \frac{\sqrt{3}}{2}\right)$ va $\bar{b}(\sqrt{3}; 1)$ vektorlar orasidagi burchakni toping.

A. 30°; B. 60°; D. 90°; E. 45°.

9. Uchburchak burchaklarining nisbati 3:2:1 kabi bo‘lsa, uning tomonlari nisbatini toping.

A. 3:2:1; B. 1:2:3; D. $2:\sqrt{3}:1$; E. $\sqrt{3}:\sqrt{2}:1$.

10. Tomoni 3 cm bo‘lgan muntazam uchburchakka tashqi chizilgan aylana radiusini toping.

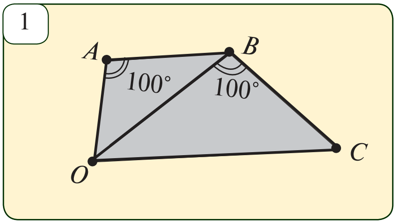
A. $\sqrt{3}$; B. $\frac{\sqrt{3}}{3}$ D. $2\sqrt{3}$; E. $\frac{\sqrt{3}}{2}$

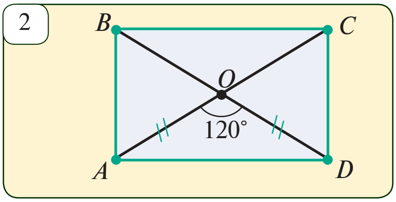
II. Masalalar

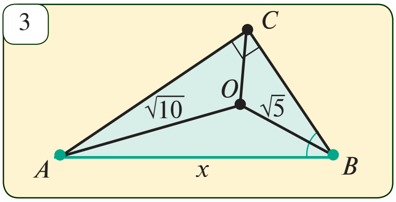
1. *ABC* uchburchakka $AB = 6$ *cm*, $\angle A = 60°$, $\angle B = 75°$. *BC* tomonni hamda *ABC* uchburchakka tashqi chizilgan aylananing radiusini toping.

2. Tomonlari 5 *cm*, 6 *cm* va 10 *cm* bo‘lgan uchburchak burchaklarining kosinuslarini toping.

3. ABC uchburchakda $\angle B=60^\circ$, $AB=6$ *cm*, $BC=4$ *cm*. AC tomonni hamda ABC uchburchakka tashqi chizilgan aylananing radiusini toping.
4. Tomonlari 51 *cm*, 52 *cm* va 53 *cm* bo‘lgan uchburchakka tashqi chizilgan aylananing radiusini toping.
5. Uchburchakning ikkita tomoni 14 *cm* va 22 *cm*, uchinchi tomoniga o‘tkazilgan medianasi esa 12 *cm*. Uchburchakning uchinchi tomonini toping.
6. Parallelogrammning diagonallari 4 *cm*, $4\sqrt{2}$ *cm* va ular orasidagi burchak 45°. Parallelogrammning a) yuzini; b) perimetrini; d) balandliklarini toping.
7. Tomonlari 3 va 5 bo‘lgan parallelogrammning bir diagonali 4 ga teng. Uning ikkinchi diagonalini toping.
8. Tomonlari a) 2, 2 va 2,5; b) 24, 7 va 25; d) 9, 5 va 6 bo‘lgan uchburchak turini aniqlang.
9. Parallelogrammning tomonlari $7\sqrt{3}$ va 6 *cm*. Agar uning o‘tmas burchagi 120° bo‘lsa, uning yuzini toping.
10. ABC uchburchakning AB, BC tomonlarida N, K nuqtalar olingan. Unda $BN=2AN$, $3BK=2KC$. Agar $AB=3$, $BC=5$, $CA=6$ bo‘lsa, NK kesmani toping.
11. ABC uchburchakda $\angle A=30^\circ$, $BC=7$ *cm*. Uchburchakka tashqi chizilgan aylana radiusini toping.
12. ABC uchburchakning BE bissektrisasi o‘tkazilgan. E nuqtadan BC tomonga EF perpendikular tushirilgan. Agar $EF=3$, $\angle A=30^\circ$ bo‘lsa, AE ni toping.
13. $ABCD$ to‘g‘ri to‘rtburchak AD tomonining o‘rtasi N nuqtada. Agar $AB=3$, $BC=6$ bo‘lsa, $\overline{NB}\cdot\overline{NC}$ skalar ko‘paytmani toping.
14. $\bar{a}(2;x)$, $\bar{b}(-4;1)$ bo‘lib, $\bar{a}+\bar{b}$ va $\bar{b}$ vektorlar perpendikular. x ni toping.
15. $\bar{m}(7;3)$ va $\bar{n}(-2;-5)$ vektorlar orasidagi burchakni toping.
16. 1- rasmda berilganlardan foydalanib, rasmdagi eng katta kesmani aniqlang.
17. $ABCD$ to‘g‘ri to‘rtburchakning diagonallari O nuqtada kesishadi(2-rasm). Agar $AO=12$ *cm*, $\angle AOD=120^\circ$ bo‘lsa, to‘rtburchak perimetrini toping.
18. To‘g‘ri burchakli ABC uchburchak bissektrisalari O nuqtada kesishadi ($\angle C=90^\circ$). Agar $OA=\sqrt{10}$, $OB=\sqrt{5}$ bo‘lsa, AB gipotenuzani toping (3-rasm).







III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Tomonlari $a=45$, $b=70$, $c=95$ bo‘lgan uchburchakning eng katta burchagini toping.
2. Uchburchakda $b=5$, $\alpha=30^\circ$, $\beta=50^\circ$ bo‘lsa, uchburchakni yeching.
3. PKH uchburchakda $PK=6$, $KH=5$, $\angle PKH=100^\circ$. HF mediana uzunligini va PFH uchburchak yuzini toping.
4. *(Qo‘shimcha)*. Uchburchakda $a=\sqrt{3}$, $b=1$, $\alpha=135^\circ$ bo‘lsa, β burchakni toping.

Tarixiy lavhalar. Sinus haqida

Sinus haqidagi ma’lumot dastlab IV–V asrlardagi hind astronomlarining asarlarida uchraydi.

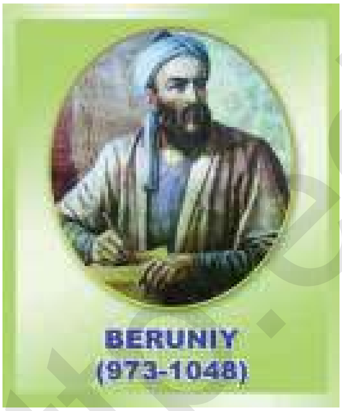
O‘rta Osiyolik olimlar al-Xorazmiy, Beruniy, Ibn Sino, Abdurahmon al-Haziniy (XII asr) sinus uchun *«al-jayb»* atamasini ishlatishgan.

Hozirgi sinus belgisini Simpson, Eyler, D’alamber, Lagranj (XVII asr) va boshqalar qo‘llagan.

«Kosinus» atamasi lotincha «komplimenti sinus» atamasining qisqartirilgani, u «qo‘shimcha sinus», aniqrog‘i «qo‘shimcha yoyning sinusi» demakdir.

Kosinuslar teoremasini yunonlar ham bilgan, uning isboti Yevklidning “Negizlar” asarida keltirilgan. Sinuslar teoremasining o‘ziga xos isbotini Abu Rayhon Beruniy bayon qilgan.

Tarixiy lavhalar.



Beruniy (to‘liq ismi – Abu Rayhon Muhammad ibn Ahmad) (973 — 1048) – o‘rta asrning buyuk qomusiy olimi. U Xorazm o‘lkasining Qiyot shahrida tug‘ilgan. Qiyot Amudaryoning o‘ng qirg‘og‘i – hozirgi Beruniy shahrining o‘rnida bo‘lgan, u yaqin vaqtlargacha Shabboz deb atalgan. Beruniyning matematika va fanning boshqa sohalariga qo‘shgan hissasini yozib qoldirgan 150 dan ortiq asaridan ham ko‘rish mumkin. Ulardan eng yiriklari – “Hindiston”, “Yodgorliklar”, “Mas’ud qonunlari”, “Geodeziya”, “Mineralogiya” va “Astronomiya”.

Beruniyning shoh asari “Mas’ud qonunlari”, asosan, astronomiyaga oid bo‘lsa ham, uning matematikaga oid anchagina kashfiyotlari shu asarda bayon etilgan.

Bu asarda Beruniy ikki burchak yig‘indisi va ayirmasining sinuslari, ikkilangan va yarim burchakning sinuslari haqidagi teoremalar bilan teng kuchli bo‘lgan vatarlar haqidagi teoremalarni isbotlagan, ikki gradusli yoyning vatarlarini hisoblab topgan, sinuslar va tangenslar jadvallarini tuzgan, sinuslar teoremasini o‘ziga xos usulda isbotlagan.

III BOB

AYLANA UZUNLIGI VA DOIRA YUZI

Ushbu bobni o'rganish natijasida siz quyidagi bilim va amaliy ko'nikmalarga ega bo'lasiz:

***Bilimlar**:*

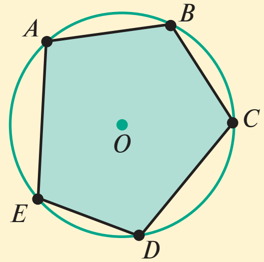
√ ko'pburchakka tashqi va ichki chizilgan aylanalarning xossalarini bilish;
√ muntazam ko'pburchaklarning xossalarini bilish;
√ muntazam ko'pburchakning yuzini hisoblash formulalarini bilish;
√ aylana va uning yoyi uzunligini hisoblash formulalarini bilish;
√ doira va uning bo'laklari yuzini topish formulalarini bilish;
√ burchakning radian o'lchovini bilish.

Amaliy ko'nikmalar:

√ muntazam ko'pburchaklarni tasvirlay olish;
√ muntazam ko'pburchakka tashqi va ichki chizilgan aylanalarning radiuslarini topa olish;
√ aylana va yoy uzunligini hisoblay olish;
√ doira va uning bo'laklari yuzini hisoblay olish.

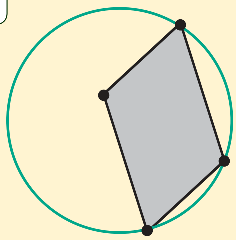
36 AYLANAGA ICHKI CHIZILGAN KO‘PBURCHAK

1



Aylanaga ichki chizilgan beshburchak yoki beshburchakka tashqi chizilgan aylana.

2



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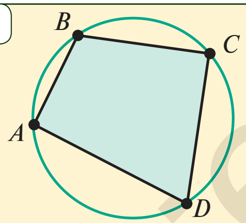
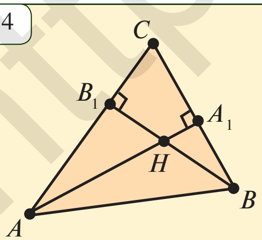


$\angle A+\angle C=180^\circ$
$\angle B+\angle D=180^\circ$

4



Ta'rif. Agar ko‘pburchakning barcha uchlari aylanada yotsa, bu ko‘pburchak aylanaga *ichki chizilgan*, aylana esa ko‘pburchakka *tashqi chizilgan* deyiladi *(1-rasm)*.

Istalgan uchburchakka tashqi aylana chizish mumkinligi va bu aylana markazi uchburchak tomonlarining o‘rta perpendikularlari kesishgan nuqtada yotishini 8-sinfda o‘rgangansiz.

Agar ko‘pburchak burchaklari soni uchtadan ortiq bo‘lsa, ko‘pburchakka har doim ham tashqi aylana chizib bo‘lavermaydi. Masalan, to‘g‘ri to‘rtburchakdan farqli parallelogramm uchun tashqi chizilgan aylana mavjud emas *(2-rasm)*.

8-sinf geometriya kursidan ma’lumki, to‘rtburchakka qarama-qarshi burchaklari yig‘indisi 180° ga teng bo‘lganda va faqat shu holda unga tashqi aylana chizish mumkin *(3-rasm)*.

1-masala. O‘tkir burchakli ABC uchburchakning AA_1 va BB_1 balandliklari H nuqtada kesishadi. A_1HB_1C to‘rtburchakka tashqi aylana chizish mumkinligini isbotlang.

Yechish. $AA_1 \perp BC$ va $BB_1 \perp AC$ bo‘lgani uchun (*4-rasm*)

$$\angle HB_1C=\angle HA_1C=90^\circ.$$

Unda $\angle HB_1C+\angle HA_1C=180^\circ$. To‘rtburchak ichki burchaklari yig‘indisi 360° bo‘lgani uchun:

$$\angle B_1CA_1+\angle B_1HA_1=180^\circ.$$

Demak, A_1HB_1C to‘rtburchakka tashqi aylana chizish mumkin.

Aylanaga ichki chizilgan ko‘pburchak uchlari aylana markazidan teng uzoqlikda yotgani uchun aylana markazi ko‘pburchak tomonlarining o‘rta perpendikularlarida yotadi *(5-rasm)*. Demak, aylanaga ichki chizilgan ko‘pburchak tomonlarining o‘rta perpendikularlari bir nuqtada kesishishi shart.

2-masala. Asosiga tushirilgan balandligi 16 *cm* bo‘lgan teng yonli uchburchak radiusi 10 *cm* bo‘lgan aylanaga ichki chizilgan. Uchburchak tomonlarini toping.

Yechish. ABC uchburchakka tashqi chizilgan aylana markazi O nuqta AC tomonning o'rta perpendikulari bo'lgan BD balandlikda yotadi (*6-rasm*). Unda,

$$OD=BD-OB=16-10=6\ (cm)$$

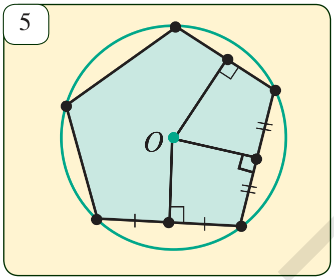
bo'ladi va Pifagor teoremasiga ko'ra,

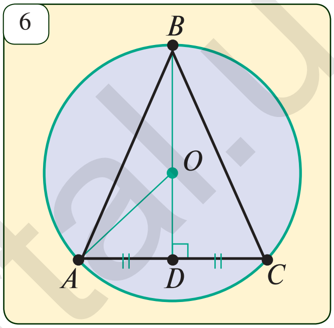
$$AD=\sqrt{OA^2-OD^2}=\sqrt{10^2-6^2}=8\ (cm),\ AC=2AD=16(cm).$$

Shuningdek, to'g'ri burchakli ABD uchburchakda

$$AB=\sqrt{AD^2+BD^2}=\sqrt{8^2+16^2}=8\sqrt{5}\ (cm).$$

Javob: $8\sqrt{5}$ *cm*, $8\sqrt{5}$ *cm*, 16 *cm*.





Masala va topshiriqlar

36.1. Agar ko'pburchak aylanaga ichki chizilgan bo'lsa, uning tomonlari o'rta perpendikulari bir nuqtada kesishishini isbotlang.

36.2. Qanday uchburchak aylanaga ichki chizilgan bo'lishi mumkin? To'rtburchak-chi?

36.3. $ABCDE$ beshburchak aylanaga ichki chizilgan bo'lsa, $\angle ACB=\angle AEB$ bo'lishini isbotlang.

36.4. Katetlari 16 *cm* va 12 *cm* bo'lgan to'g'ri burchakli uchburchakka tashqi chizilgan aylana radiusini toping.

36.5. Radiusi 25 *cm* bo'lgan aylanaga bir tomoni 14 *cm* bo'lgan to'g'ri to'rtburchak ichki chizilgan. To'g'ri to'rtburchak yuzini toping.

36.6. Radiusi 10 *cm* bo'lgan aylanaga ichki chizilgan a) teng tomonli uchburchak; b) kvadrat; d) teng yonli to'g'ri burchakli uchburchak tomonlarini toping.

36.7. Tomonlari 16 *cm*, 10 *cm* va 10 *cm* bo'lgan uchburchakka tashqi chizilgan aylana radiusini toping.

36.8. Aylanaga ichki chizilgan $ABCDEF$ oltiburchakda $\angle BAF+\angle AFB=90^\circ$ bo'lsa, aylana markazi AF tomonda yotishini isbotlang.

36.9. Istalgan teng yonli trapetsiya aylanaga ichki chizilishi mumkinligini isbotlang.

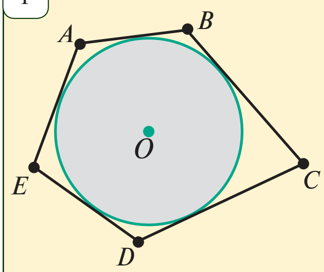
36.10. Teng yonli trapetsiya chizing. Unga tashqi chizilgan aylana yasang.

Qiziqarli masala

O'n olti yoshli Galua (E.Galua — fransuz matematigi, 1811—1832) kollejda o'qib yurgan chog'larida, unga o'qituvchisi bir soat ichida uchta masalani yechib berishni so'ragan. Galua yechimi uncha oson bo'lmagan bu masalalarni 15 daqiqada yechib, hammani hayron qoldirgan. Mana, shu masalalardan biri. Uni siz ham yechib ko'ring-chi!

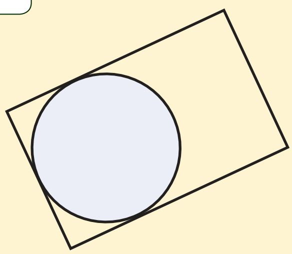
Masala. Aylanaga ichki chizilgan to'rtburchakning to'rtta tomoni a, b, c va d ga teng. Uning diagonallarini toping.

1



Aylanaga tashqi chizilgan ABCDE beshburchak yoki ABCDE beshburchakka ichki chizilgan aylana.

2



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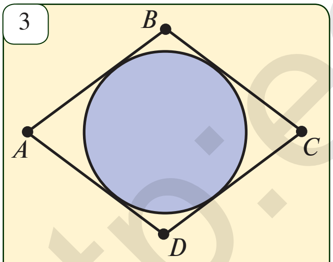
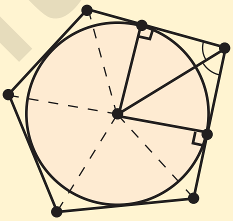


$AB + CD = AD + BC$

4



Ta’rif. Agar ko‘pburchakning barcha tomonlari aylanaga urinsa, u holda ko‘pburchak aylanaga *tashqi chizilgan,* aylana esa ko‘pburchakka *ichki chizilgan* deyiladi *(1-rasm).*

Istalgan uchburchakka ichki aylana chizish mumkinligi va bu aylana markazi uchburchak bissektrisalari kesishgan nuqtada ekanligi bilan 8-sinfda tanishgansiz.

Agar ko‘pburchak burchaklari soni uchtadan ortiq bo‘lsa, bu ko‘pburchakka har doim ham ichki aylana chizib bo‘lavermaydi. Masalan, kvadratdan farqli to‘g‘ri to‘rtburchakka ichki aylana chizib bo‘lmaydi *(2-rasm).*

Yana 8-sinf geometriya kursidan ma’lumki, to‘rtburchakka faqat va faqat qarama-qarshi tomonlari yig‘indisi teng bo‘lganda ichki aylana chizish mumkin *(3-rasm).*

Aylanaga tashqi chizilgan ko‘pburchak tomonlari aylanaga uringani uchun aylana markazi shu ko‘pburchak burchaklari bissektrisasida yotadi *(4-rasm).* Demak, aylanaga tashqi chizilgan ko‘pburchak burchaklarining bissektrisalari bir nuqtada kesishadi.

Teorema. ***Agar r radiusli aylanaga tashqi chizilgan ko‘pburchakning yuzi S, yarim perimetri p bo‘lsa, S = pr bo‘ladi.***

Isbot. Teorema isbotini aylanaga tashqi chizilgan *ABCDEF* oltiburchak uchun keltiramiz. Aylana markazi *O* nuqtani ko‘pburchak uchlari bilan tutashtirib, ko‘pburchakni uchburchaklarga ajratamiz. Bu uchburchaklarning balandliklari *r* ga teng *(5-rasm).* Unda,

$$S = S_{AOB} + S_{BOC} + \ldots + S_{FOA} =$$
$$= \frac{1}{2} AB \cdot r + \frac{1}{2} BC \cdot r + \ldots + \frac{1}{2} FA \cdot r =$$
$$= \frac{AB + BC + \ldots + FA}{2} \cdot r = pr.$$

Teorema isbotlandi.

Masala. Aylanaga tashqi chizilgan to‘rtburchakning yuzi 21 cm^2 ga, perimetri esa 7 cm ga teng. Aylana radiusini toping.

Yechish. $S=pr$ formulaga ko‘ra,

$$r=\frac{S}{p}=\frac{21}{3,5}=6\ (cm).$$

Javob: 6 cm.

Masala va topshiriqlar

37.1. Tomoni 6 cm bo‘lgan a) teng tomonli uchburchakka; b) kvadratga ichki chizilgan aylana radiusini toping.

37.2. Radiusi 5 cm bo‘lgan aylanaga tashqi chizilgan ko‘pburchak yuzi 18 cm^2. Ko‘pburchak perimetrini toping.

37.3. 6-rasmdagi to‘rtburchaklarning perimetrini toping.

37.4. 7-rasmdagi ma’lumotlar asosida so‘ralgan kesmani toping.

37.5. Aylanaga tashqi chizilgan parallelogramm romb bo‘lishini isbotlang.

37.6. To‘g‘ri burchakli uchburchakka ichki chizilgan aylana radiusi katetlar yig‘indisi bilan gipotenuza ayirmasining yarmiga tengligini isbotlang.

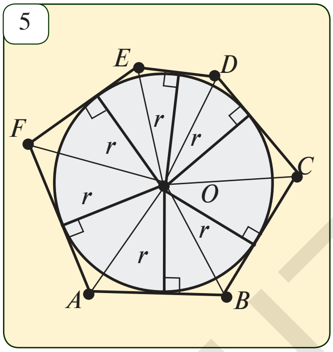
37.7. Aylanaga tashqi chizilgan teng yonli trapetsiyaning o‘rta chizig‘i uning yon tomoniga teng ekanligini isbotlang.

37.8. Asoslari 9 cm va 16 cm bo‘lgan teng yonli trapetsiya aylanaga tashqi chizilgan. Aylana radiusini toping.

37.9*. $ABCD$ to‘rtburchak O markazli aylanaga tashqi chizilgan. AOB va COD uchburchaklar yuzlarining yig‘indisi to‘rtburchak yuzining yarmiga tengligini isbotlang.

37.10*. Aylanaga tashqi chizilgan teng yonli trapetsiyaning asoslari a va b bo‘lsa, uning balandligi $\frac{\sqrt{ab}}{2}$ ga teng ekanligini isbotlang.

37.11*. Uchlari $ABCD$ to‘rtburchak bissektrisalarining kesishgan nuqtalarda bo‘lgan $EFPQ$ to‘rtburchakka tashqi aylana chizish mumkinligini isbotlang.

5


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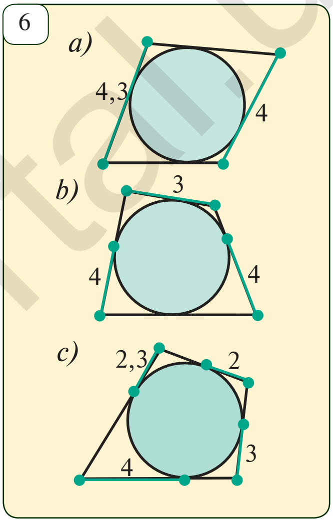

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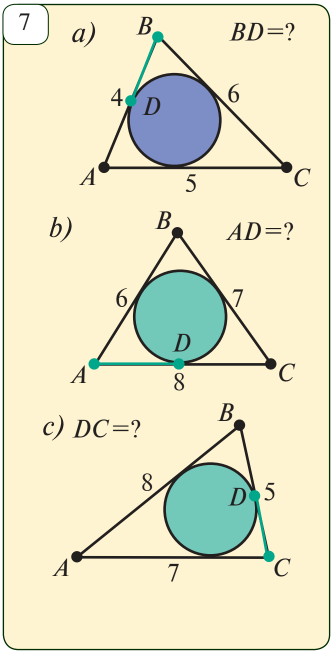


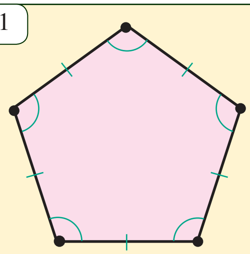
MUNTAZAM KO‘PBURCHAKLAR

Faollashtiruvchi mashq

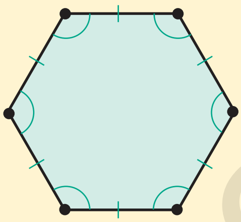
1. Qanday shakllar ko‘pburchak deyiladi?
2. Ko‘pburchak burchaklari, qo‘shni tomonlari, diagonallari deb nimaga aytiladi?
3. Qavariq ko‘pburchak deb qanday ko‘pburchakka aytiladi?
4. Qavariq ko‘pburchak ichki burchaklari yig‘indisi haqidagi teoremani ayting.

Ta’rif. Hamma tomonlari teng va hamma burchaklari teng bo‘lgan qavariq ko‘pburchakka *muntazam ko‘pburchak* deyiladi.

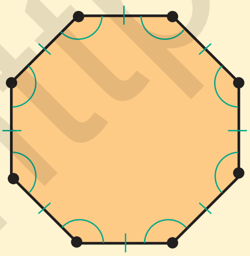
1



muntazam beshburchak



muntazam oltiburchak



muntazam sakkizburchak

Teng tomonli uchburchak, kvadrat muntazam ko‘pburchakka misol bo‘ladi. 1-rasmda muntazam beshburchak, oltiburchak va sakkizburchaklar tasvirlangan.

Teorema. Muntazam n burchakning har bir burchagi $\frac{n-2}{n}\cdot 180^\circ$ ga teng.

Isbot. Muntazam n burchakning burchaklari yig‘indisi $(n-2)\cdot 180^\circ$ ga teng (8-sinf). Demak, uning har bir burchagi $\frac{n-2}{n}\cdot 180^\circ$ga teng. *Teorema isbotlandi.*

Masala. Muntazam $A_1A_2A_3A_4A_5$ beshburchakda A_1A_3 va A_1A_4 diagonallar teng ekanligini ko‘rsating (*2-rasm*).

$A_1A_2A_3A_4A_5$ — *muntazam beshburchak* ⇒ $A_1A_3 = A_1A_4$

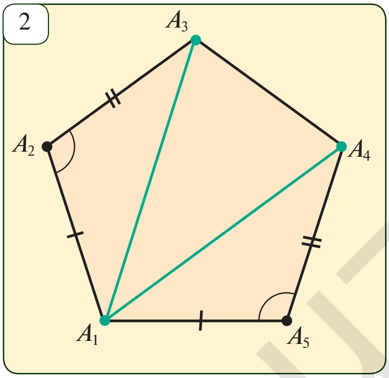
Yechish. Uchburchaklar tengligining *TBT* alomatiga ko‘ra, $A_1A_2A_3$ va $A_1A_5A_4$ uchburchaklar o‘zaro teng. Haqiqatan ham, muntazam ko‘pburchakning tomonlari teng va burchaklari teng bo‘lgani uchun,

$$A_1A_2 = A_1A_5,\ A_2A_3 = A_5A_4 \text{ va } \angle A_1A_2A_3 = \angle A_1A_5A_4.$$

Demak, $\Delta A_1A_2A_3 = \Delta A_1A_5A_4$. Bundan

$A_1A_3 = A_1A_4$ ekanligi kelib chiqadi.

Natija. ***Muntazam beshburchakning barcha diagonallari o‘zaro teng.***



Masala va topshiriqlar

38.1. Muntazam bo‘lmagan ko‘pburchaklarga misollar ayting va nima uchun muntazam emasligini tushuntiring.

38.2. Quyidagi tasdiqlardan to‘g‘rilarini toping:

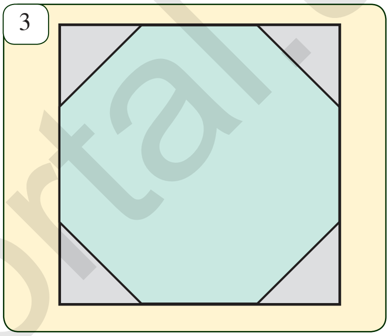
a) barcha tomonlari teng bo‘lgan uchburchak muntazam bo‘ladi;

b) barcha tomonlari teng to‘rtburchak muntazam bo‘ladi;

d) barcha burchaklari teng to‘rtburchak muntazam bo‘ladi;

e) barcha burchaklari teng romb muntazam bo‘ladi;

f) barcha tomonlari teng to‘g‘ri to‘rtburchak muntazam bo‘ladi.



38.3. Agar a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=18$ bo‘lsa, muntazam n burchak burchaklarini toping.

38.4. Muntazam n burchakning tashqi burchagi nimaga teng bo‘ladi? Agar a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=12$ bo‘lsa, muntazam n burchakning tashqi burchagini toping.

38.5. Muntazam n burchakning har uchidan bittadan olingan tashqi burchaklari yig‘indisi $360°$ ga teng ekanligini isbotlang.

38.6. Agar muntazam ko‘pburchakning har bir burchagi a) $60°$; b) $90°$; d) $135°$; e) $150°$ bo‘lsa, bu ko‘pburchak tomonlari sonini toping.

38.7. Muntazam $ABCDEF$ oltiburchak berilgan.

a) AC va BD diagonallar tengligini isbotlang.

b) ACE — muntazam uchburchak bo‘lishini isbotlang.

d) AD, BE va CF diagonallar o‘zaro tengligini isbotlang.

38.8. Tomoni 10 cm bo‘lgan muntazam a) beshburchakning; b) oltiburchakning; d) sakkizburchakning; e) o‘n ikkiburchakning; f) o‘n sakkizburchakning kichik diagonalini hisoblang.

38.9. Muntazam to‘rtburchakning kvadrat bo‘lishini isbotlang.

38.10*. Kvadratning tomoni a ga teng. Uning tomonlariga har bir uchidan boshlab diagonalining yarmiga teng kesmalar qo‘yildi. Natijada, 3-rasmda tasvirlangan sakkizburchak hosil bo‘ldi. Uning turini aniqlang va yuzini toping.

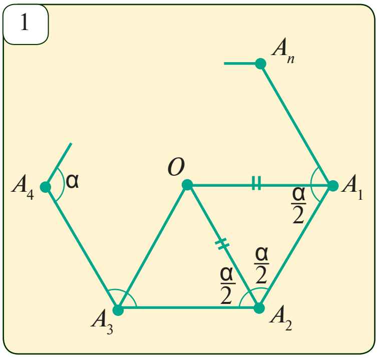
MUNTAZAM KO‘PBURCHAKKA ICHKI VA TASHQI CHIZILGAN AYLANALAR

Faollashtiruvchi mashq

1. Qanday ko‘pburchak aylanaga ichki chizilgan ko‘pburchak deyiladi?
2. Qanday ko‘pburchak aylanaga tashqi chizilgan ko‘pburchak deyiladi?
3. Istalgan ko‘pburchak aylanaga ichki (tashqi) chizilgan bo‘lishi mumkinmi?

Teorema. ***Har qanday muntazam ko‘pburchakka ichki aylana ham, tashqi aylana ham chizish mumkin.***

1



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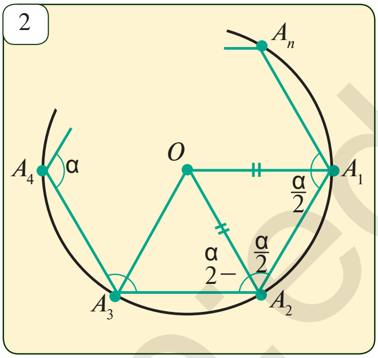

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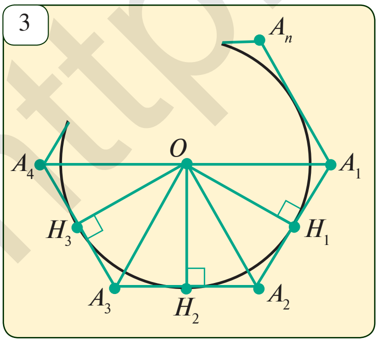


Isbot. Aytaylik, $A_1A_2 \ldots A_n$ — muntazam ko‘pburchak, O — A_1 va A_2 burchaklari bissektrisalarining kesishish nuqtasi bo‘lsin. Bu muntazam ko‘pburchakning burchagini α bilan belgilaylik.

1. $OA_1 = OA_2 = \ldots = OA_n$ ekanligini isbotlaymiz (*1-rasm*). Burchak bissektrisasining ta’rifiga ko‘ra,

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Demak, A_1OA_2 — teng yonli uchburchak. Bundan, $OA_1 = OA_2$ kelib chiqadi. ΔA_1A_2O va ΔA_3A_2O uchburchaklar tengligining TBT alomatiga ko‘ra teng, chunki $A_1A_2 = A_3A_2$, A_2O — tomon umumiy hamda

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Shuning uchun $OA_3 = OA_1$. Xuddi shunday yo‘l tutib, $OA_4 = OA_2$, $OA_5 = OA_3$ va hokazo tengliklar o‘rinli bo‘lishi ko‘rsatiladi.

Shunday qilib, $OA_1 = OA_2 = \ldots = OA_n$, ya’ni markazi O va radiusi OA_1 bo‘lgan aylana ko‘pburchakka tashqi chizilgan aylanadan iborat bo‘ladi (*2-rasm*).

2. Yuqorida aytilganlarga ko‘ra, teng yonli A_1OA_2, A_2OA_3, ... A_nOA_1 uchburchaklar teng. Shuning uchun bu uchburchaklarning O uchidan tushirilgan balandliklari ham teng bo‘ladi (*3-rasm*):

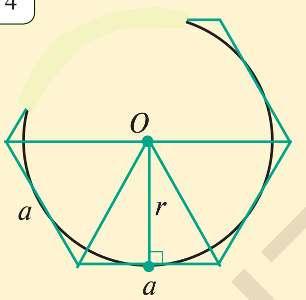
$$OH_1 = OH_2 = \ldots = OH_n.$$

Demak, O markazli va radiusi OH_1 kesmaga teng bo‘lgan aylana ko‘pburchakning barcha tomonlariga urinadi. Ya’ni, bu aylana ko‘pburchakka ichki chizilgan aylana bo‘ladi. *Teorema isbotlandi.*

Natija. ***Muntazam ko‘pburchakka ichki chizilgan va tashqi chizilgan aylanalarning markazlari bitta nuqtada bo‘ladi.***

Bu nuqta muntazam ko‘pburchakning *markazi* deyiladi. Muntazam ko‘pburchak markazini uning ikki qo‘shni uchlari bilan tutashtiruvchi nurlardan iborat burchak (1-rasmdagi A_1OA_2, A_2OA_3 ... burchaklar) uning *markaziy burchagi* deyiladi. Muntazam ko‘pburchakning markazidan tomonlariga tushirilgan perpendikularlar (3-rasmdagi OH_1, OH_2, ... kesmalar) uning *apofemasi* deyiladi.

4



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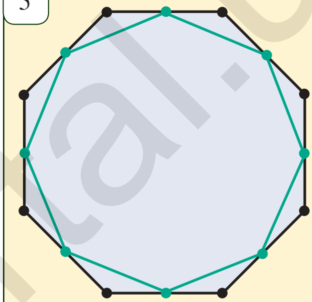


Masala. Agar muntazam n burchakning tomoni a, unga ichki chizilgan aylananing radiusi r bo‘lsa, uning yuzi $S = \frac{1}{2}nar$ formula bilan hisoblashini isbotlang. (4-rasm).

Isbot. Ko‘pburchakning yarim perimetri $p = \frac{1}{2}na$ bo‘lgani uchun, aylanaga tashqi chizilgan ko‘pburchak yuzini topish formulasi $S = pr$ ga ko‘ra, $S = \frac{1}{2}nar$ bo‘ladi.

? Masala va topshiriqlar

39.1. Yuzi 36 cm^2 bo‘lgan kvadratga ichki va tashqi chizilgan aylanalar radiuslarini toping.

39.2. Perimetri 18 cm bo‘lgan muntazam uchburchakka ichki va tashqi chizilgan aylanalar radiuslarini hisoblang.

39.3. Muntazam oltiburchakka tashqi chizilgan aylana radiusi uning tomoniga teng bo‘lishini isbotlang.

39.4. Muntazam ko‘pburchak tomonlarining o‘rtalari boshqa bir muntazam ko‘pburchak uchlari bo‘lishini isbotlang (*5-rasm*).

39.5. Muntazam uchburchakka ichki chizilgan aylana radiusi tashqi chizilgan aylana radiusidan ikki marta kichik ekanligini isbotlang.

39.6*. Muntazam ko‘pburchakning istalgan ikkita tomonining o‘rta perpendikularlari bir nuqtada kesishishi yoki bir to‘g‘ri chiziqda yotishini isbotlang.

39.7. Aylanaga ichki chizilgan muntazam ko‘pburchakning bir tomoni aylanadan a) 60°; b) 30°; d) 36°; e) 18°; f) 72° ga teng yoy ajratadi. Ko‘pburchakning nechta tomoni bor?

39.8. Qog‘ozdan oltita teng muntazam uchburchak qirqib oling. Ulardan foydalanib, muntazam oltiburchak yasang. Tomonlari teng bo‘lgan muntazam oltiburchak va uchburchak yuzlari nisbatini toping.

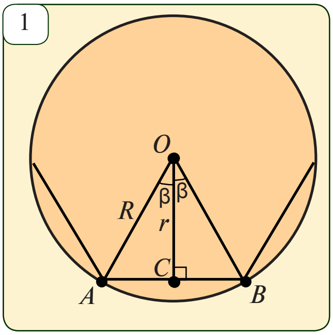
40 MUNTAZAM KO‘PBURCHAKNING TOMONI BILAN TASHQI VA ICHKI CHIZILGAN AYLANALAR RADIUSLARI ORASIDAGI BOG‘LANISH

Faollashtiruvchi mashq

To‘g‘ri burchakli uchburchak o‘tkir burchagining a) sinusi; b) kosinusi; d) tangensi deb nimaga aytiladi?

Tomoni a_n ga teng bo‘lgan muntazam n burchakka tashqi chizilgan aylananing R radiusi va ichki chizilgan aylananing r radiusini hisoblash uchun formulalar topamiz. Buning uchun to‘g‘ri burchakli ACO uchburchakdan foydalanamiz. Bu yerda O — ko‘pburchakning markazi, C — ko‘pburchakning AB tomoni o‘rtasi (*1-rasm*). Unda,

1



$$\beta=\angle AOC=\frac{1}{2}\angle AOB=\frac{1}{2}\cdot\frac{360^\circ}{n}=\frac{180^\circ}{n};$$

$$R=OA=\frac{AC}{\sin\beta}=\frac{a_n}{2\sin\frac{180^\circ}{n}}; \qquad r=OC=\frac{AC}{tg\beta}=\frac{a_n}{2tg\frac{180^\circ}{n}};$$

$$r=OC=OA\cdot\cos\beta=R\cos\frac{180^\circ}{n}.$$

Bu formulalardan foydalanib, ayrim muntazam ko‘pburchaklar tomoni, ichki va tashqi chizilgan aylanalar radiuslari orasidagi bog‘lanishlarni topamiz.

1. *Muntazam uchburchak uchun (n=3):*

$$\beta=\frac{180^\circ}{3}=60^\circ; \qquad R=\frac{a_3}{2\sin 60^\circ}=\frac{a_3}{\sqrt{3}}; \qquad r=\frac{a_3}{2tg\,60^\circ}=\frac{a_3}{2\sqrt{3}}; \qquad R=2r.$$

2. *Kvadrat uchun (n=4):*

$$\beta=\frac{180^\circ}{4}=45^\circ; \qquad R=\frac{a_4}{2\sin 45^\circ}=\frac{a_4}{\sqrt{2}}; \qquad r=\frac{a_4}{2tg\,45^\circ}=\frac{a_4}{2}; \qquad R=r\sqrt{2}.$$

3. *Muntazam oltiburchak uchun (n=6):*

$$\beta=\frac{180^\circ}{6}=30^\circ; \qquad R=\frac{a_6}{2\sin 30^\circ}=a_6; \qquad r=\frac{a_6}{2tg\,30^\circ}=\frac{a_6\sqrt{3}}{2}; \qquad R=\frac{2r}{\sqrt{3}}.$$

Masala. Muntazam n burchakning a_n tomonini shu ko‘pburchakka tashqi chizilgan aylananing R radiusi va ichki chizilgan aylananing r radiusi orqali ifodalang.

Yechish. $R=\dfrac{a_n}{2\sin\frac{180^\circ}{n}}$ va $r=\dfrac{a_n}{2tg\frac{180^\circ}{n}}$ formulalardan $a_n=2R\sin\dfrac{180^\circ}{n}$ va $a_n=2r\,tg\dfrac{180^\circ}{n}$ formulalarni hosil qilamiz. Xususan, $n=3$ bo‘lsa, $a_3=R\sqrt{3}=2r\sqrt{3}$.

Masala va topshiriqlar

40.1. Tomoni 15 *cm* bo‘lgan a) muntazam uchburchakka; b) muntazam to‘rtburchakka; d) muntazam oltiburchakka ichki va tashqi chizilgan aylanalar radiuslarini hisoblang.

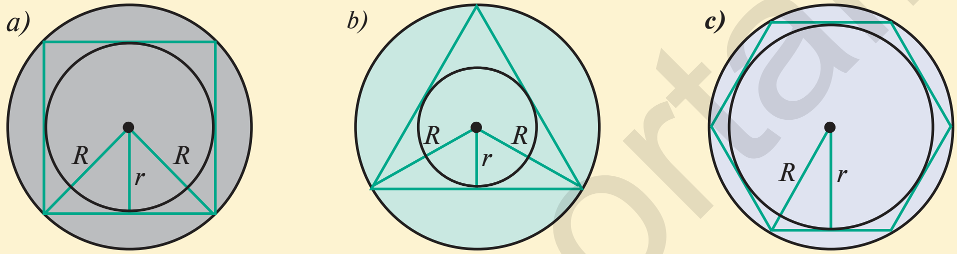
40.2. 2-rasmda R radiusli aylanaga ichki chizilgan kvadrat, muntazam uchburchak va muntazam oltiburchak tasvirlangan. Daftaringizga berilgan jadvallarni ko‘chirib, uning bo‘sh kataklarini to‘ldiring (a_n — ko‘pburchak tomoni, P — ko‘pburchak perimetri, S — uning yuzi, r — unga ichki chizilgan aylana radiusi).

40.3. Radiusi 8 *cm* bo‘lgan aylanaga ichki chizilgan muntazam o‘n ikkiburchakning bir uchidan chiqqan diagonallarini toping.

40.4. Aylanaga ichki chizilgan muntazam uchburchak perimetri 24 *cm*. Shu aylanaga ichki chizilgan kvadrat tomonini toping.

40.5. Silindr shaklidagi yog‘ochdan asosining tomoni 20 *cm* bo‘lgan: a) kvadrat; b) muntazam oltiburchak bo‘lgan prizma shaklidagi ustun tayyorlash kerak.

2



	R	r	a_4	P	S
1.			6		
2.		2			
3.	4				
4.				28	
5.					16

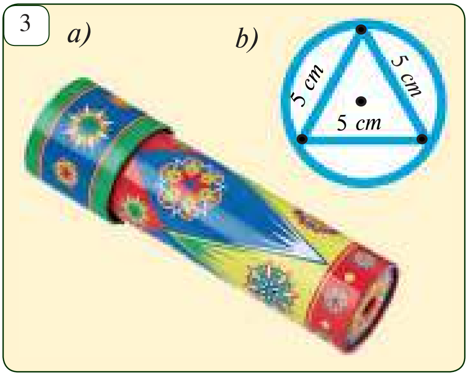
	R	r	a_3	P	S
1.	3				
2.					10
3.		2			
4.			5		
5.				6	

	R	r	a_6	P	S
1.	4				
2.		5			
3.			6		
4.				42	
5.					$24\sqrt{3}$

Yog‘och ko‘ndalang kesimining diametri kamida qancha bo‘lishi zarur?

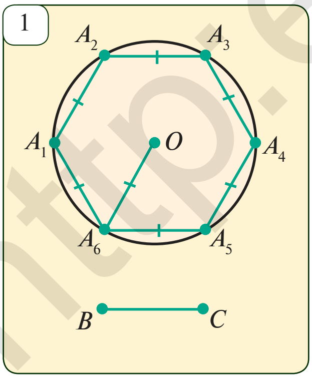
40.6. 3-*a* rasmda tasvirlangan, rang-barang naqshlarni tomosha qilsa bo‘ladigan “Kaleydoskop” deb nomlangan o‘yinchoq sizga tanish bo‘lsa kerak. O‘yinchoq quvur va 3 ta oyna bo‘laklaridan iborat. 3-*b* rasmda uning ko‘ndalang kesimi tasvirlangan va o‘lchamlari berilgan. Kaleydoskop ko‘ndalang kesimining radiusini toping.

3



I. Testlar

1. **Quyidagi ko‘pburchaklarning qaysi biriga ichki chizilgan aylana mavjud emas?**
   A) Uchburchakka; D) Kvadratdan farqli rombga;
   B) Kvadratga; E) Rombdan farqli to‘g‘ri to‘rtburchakka.
2. **Quyidagi ko‘pburchaklarning qaysi birida tashqi chizilgan aylana mavjud emas?**
   A) Uchburchakda; D) Kvadratdan farqli rombda;
   B) Kvadratda; E) Rombdan farqli to‘g‘ri to‘rtburchakda.
3. **Aylanaga ichki chizilgan barcha *ABCD* to‘rtburchaklar uchun noto‘g‘ri tenglikni** toping.
   A) $\angle A+\angle B+\angle C+\angle D=360^\circ$; D) $AB+CD=BC+AD$;
   B) $\angle A+\angle C=180^\circ$; E) $\angle B+\angle D=180^\circ$.
4. **Aylanaga tashqi chizilgan barcha ABCD to‘rtburchaklar uchun noto‘g‘ri tenglikni toping.**
   A) $\angle A+\angle B+\angle C+\angle D=360^\circ$; D) $AB+CD=BC+AD$;
   B) $\angle A+\angle C=180^\circ$; E) $AB-BC=AD-CD$.
5. **Tomonlari 5 *cm* va 12 *cm* bo‘lgan to‘g‘ri to‘rtburchakka tashqi chizilgan aylana radiusini toping.**
   A) 6 *cm*; B) 6,5 *cm*; D) 7 *cm*; E) 7,5 *cm*.
6. **Muntazam 24 burchakning ichki burchagini toping.**
   A) 120°; B) 135°; D) 150°; E) 165°.
7. **Har bir tashqi burchagi 60° bo‘lgan muntazam ko‘pburchakning ichki burchaklari yig‘indisini toping.**
   A) 540°; B) 360°; D) 90°; E) 720°.



II. Yasashga doir masalalar.

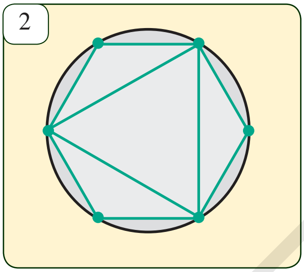
1. Tomoni berilgan kesmaga teng muntazam oltiburchak yasang. Bunda muntazam oltiburchakka tashqi chizilgan aylananing radiusi oltiburchakning tomoniga teng ekanligidan va 1-rasmdan foydalaning.

2. 2–4-rasmlardagi ma’lumotlardan foydalanib, berilgan aylanaga ichki chizilgan a) muntazam uchburchak; b) kvadrat; d) muntazam sakkizburchak yasang.

3. 5-rasmdan foydalanib, berilgan aylanaga tashqi chizilgan muntazam oltiburchak yasang (5-rasmda tasvirlangan aylanaga tashqi chizilgan oltiburchak tomonlari shu aylanaga ichki chizilgan muntazam oltiburchak uchlaridan aylanaga o‘tkazilgan urinmalarda yotadi).

III. Hisoblashga doir masalalar.

1. Muntazam uchburchak, kvadrat va muntazam oltiburchaklarning tomonlari bir-biriga teng. Ularning yuzlari nisbatini toping.
2. Bitta aylanaga ichki chizilgan muntazam oltiburchak va tashqi chizilgan oltiburchak yuzlari nisbatini toping.
3. Muntazam a) oltiburchak; b) sakkizburchak; d) o‘n ikkiburchakning parallel tomonlari orasidagi masofa 10 *cm* ga teng. Ko‘pburchak tomonini toping.
4. Radiusi R bo‘lgan aylanaga $A_1A_2 \ldots A_8$ muntazam sakkizburchak ichki chizilgan. $A_3A_4A_7A_8$ to‘rtburchakning to‘g‘ri to‘rtburchak ekanini isbotlang va uning yuzini toping.
5. Aylanaga tashqi chizilgan to‘g‘ri burchakli uchburchakning gipotenuzasi shu aylanaga urinish nuqtasida 4 *cm* va 6 *cm* uzunlikdagi kesmalarga bo‘linadi. Uchburchak yuzini toping.
6. Muntazam o‘nburchakning bir uchidan chiqqan eng katta va eng kichik diagonallari orasidagi burchakni toping.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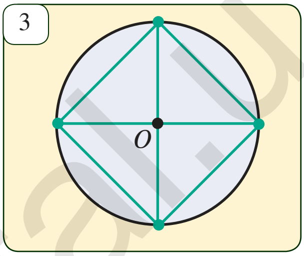

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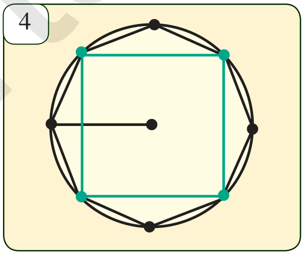
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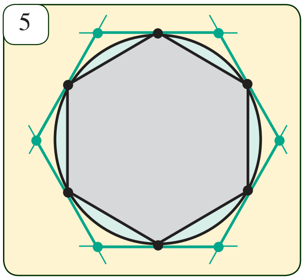
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IV. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi).

1. Katetlari 10 *cm* va 24 *cm* bo‘lgan to‘g‘ri burchakli uchburchakka ichki chizilgan va tashqi chizilgan aylanalarning radiuslarini toping.
2. Radiusi 5 *cm* bo‘lgan aylanaga tashqi chizilgan rombning bir burchagi 150° ga teng. Rombning a) perimetrini; b) diagonallarini; d) yuzini toping.
3. Tomoni 4 *cm* bo‘lgan muntazam oltiburchakning bir uchidan chiqqan diagonallarini toping.
4. *(Qo‘shimcha).* Radiusi 3 *cm* bo‘lgan aylanaga ichki chizilgan muntazam oltiburchak va muntazam uchburchaklar yuzlarining ayirmasini toping.

Tarixiy lavhalar. Istalgan muntazam ko‘pburchakni ham sirkul va chizg‘ich yordamida yasab bo‘lavermaydi. Buni 1801-yilda nemis matematigi Karl Gauss (1777−1855) algebraik usulda isbotlagan. U agar n sonning $2^m p_1 p_2 \ldots p_n$ yoyilmasidagi p_1, p_2, … p_n turli tub sonlar $2^{2^k}+1$ ko‘rinishida bo‘lsagina muntazam n burchakni sirkul va chizg‘ich yordamida yasash mumkinligini isbotladi. Bu yerda m va k manfiy bo‘lmagan butun sonlar.

1

Faollashtiruvchi mashq

1. Odatda quvur bo'lagining ko'ndalang kesimi aylanadan iborat bo'ladi. Ingichka ipni bir uchidan boshlab, quvurga bir marta o'rang. Bir marta o'rashga ketgan ip bo'lagi quvur ko'ndalang kesimi, ya'ni aylananing uzunligi bo'ladi. Uni 1-rasmda ko'rsatilgandek qilib chizg'ich yordamida o'lchang.
2. Yuqoridagi usul bilan quvur ko'ndalang kesimi diametrini aniqlang.
3. Aniqlangan aylana uzunligini uning diametriga nisbatini hisoblang.
4. Yuqorida keltirilgan o'lchash va hisoblash ishlarini yana bir nechta turli o'lchamdagi quvur bo'laklari uchun ham bajarib, aylana uzunligini uning diametriga nisbatini toping.
5. Mashq natijasiga ko'ra, aylana uzunligining uning diametriga nisbati haqida qanday xulosa chiqarish mumkin?

Teorema. ***Aylana uzunligining aylana diametriga nisbati aylana radiusiga bog'liq emas, ya'ni har qanday aylana uchun bu nisbat o'zgarmas sondir.***

Isbot. Ikkita ixtiyoriy aylana olamiz. Ularning radiuslari R_1 va R_2, uzunliklari esa mos ravishda C_1 va C_2 bo'lsin. $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ tenglikni isbotlashimiz kerak. Har ikki aylanaga ichki muntazam n-burchakni chizamiz. Ularning perimetrlarini mos ravishda P_1 va P_2 deb belgilaylik. Unda,

$P_1 = n \cdot 2R_1 \sin\frac{180^\circ}{n}$, $P_2 = n \cdot 2R_2 \sin\frac{180^\circ}{n}$ bo'lgani uchun $\frac{P_1}{P_2} = \frac{2R_1}{2R_2}$ (*) bo'ladi.

Bu tenglik istalgan n uchun to'g'ri. n soni kattalashib borsa, berilgan aylanaga ichki chizilgan n-burchak perimetri P_1 shu aylana uzunligi C_1 ga yaqinlashib boradi. Shu singari P_2 ham C_2 ga yaqinlashib boradi.

Shuning uchun $\frac{P_1}{P_2}$ nisbat $\frac{C_1}{C_2}$ nisbatga teng bo'ladi (buning to'liq isboti matematikaning yuqori bosqichlarida o'rganiladi). Shunday qilib, (*) tenglikdan $\frac{C_1}{C_2} = \frac{2R_1}{2R_2}$, bundan esa $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ tenglik kelib chiqadi. *Teorema isbotlandi.*

Aylana uzunligini uning diametriga nisbatini yunon alifbosining π harfi bilan belgilash qabul qilingan *("pi" deb o'qiladi)*. Aylana uzunligining uning diametriga nisbatini "π" harfi bilan belgilashni buyuk matematik Leonard Eyler (1707—1783) fanga kiritgan. Yunonchada "aylana" so'zi shu harf bilan boshlanadi. π irratsional son bo'lib, amaliyotda uning 3,1416 ga teng bo'lgan taqribiy qiymatidan foydalaniladi.

Shunday qilib, $\frac{C}{2R}=\pi$. Bu tenglikdan radiusi R ga teng aylana uzunligi uchun $C=2\pi R$ formulani hosil qilamiz.

Masala. Tomoni 6 *cm* bo'lgan muntazam uchburchakka tashqi chizilgan aylana uzunligini toping.

Yechish. Muntazam uchburchakka tashqi chizilgan aylana radiusini topish formulasi $R=\frac{a_3}{\sqrt{3}}$ ga ko'ra, $R=\frac{6}{\sqrt{3}}=2\sqrt{3}$ (*cm*). Endi, aylana uzunligini topish formulasidan

$$C=2\pi R=2\pi\cdot 2\sqrt{3}=4\pi\sqrt{3}\ (cm).$$ *Javob:* $4\pi\sqrt{3}$ *cm*.

Masala va topshiriqlar

42.1. Qanday son π bilan belgilanadi? Radiusi R ga teng aylana uzunligini topish formulasidan foydalanib, jadvalni to'ldiring ($\pi\approx3,14$ deb hisoblang).

C			82	18π		6,28	
R	4	3			0,7		101,5

42.2. Agar aylana radiusi a) 3 marta oshsa; b) 3 *cm* ga oshsa; d) 3 marta kamaysa; e) 3 *cm* ga kamaysa, aylana uzunligi qanchaga o'zgaradi?

42.3. Agar Yer shari ekvatorining 40 milliondan bir qismi 1 *m* ga teng bo'lsa, Yer sharining radiusini toping.

42.4. a) Tomoni *a* ga teng bo'lgan muntazam uchburchakka; b) katetlari *a* va *b* bo'lgan to'g'ri burchakli uchburchakka; d) asosi *a* va yon tomoni *b* bo'lgan teng yonli uchburchakka tashqi chizilgan aylana uzunligini toping.

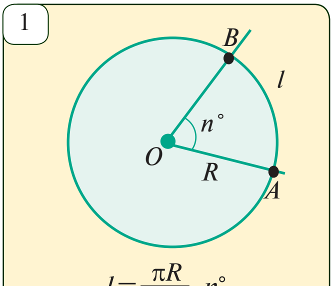
42.5. a) Tomoni *a* ga teng kvadratga; b) gipotenuzasi *c* ga teng bo'lgan teng yonli to'g'ri burchakli uchburchakka; d) gipotenuzasi *c*, o'tkir burchagi α bo'lgan to'g'ri burchakli uchburchakka ichki chizilgan aylana uzunligini toping.

2

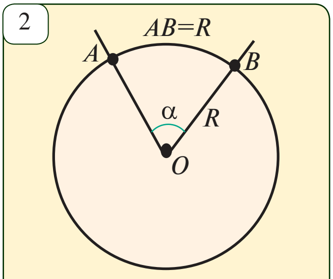
42.6. Teplovoz 1413 *m* yo'l yurdi. Bunda uning g'ildiragi 300 marta aylandi. Teplovoz g'ildiragining diametrini toping.

42.7. Yengil avtomobil g'ildiragi aylanasining radiusi 24 *cm* ga teng. Avtomobil 100 *km* yo'l yursa, uning g'ildiragi necha marta aylanadi (*2-rasm*)?

43 AYLANA YOYI UZUNLIGI. BURCHAKNING RADIAN O‘LCHOVI



$l=\frac{\pi R}{180^\circ}\cdot n^\circ$



$\alpha=1$ *radian* $\approx 57^\circ17'45''$

1. n° li markaziy burchak tiralgan yoy uzunligi.

Aytaylik, radiusi R ga teng bo‘lgan aylanada n° li AOB markaziy burchak berilgan bo‘lsin (*1-rasm*). Bunda aylananing AOB markaziy burchakka tiralgan AB yoyining gradus o‘lchovini n° yoki n° li yoy deb yuritilishini eslatib o‘tamiz.

Radiusi R ga teng butun aylana, ya’ni o‘lchovi 360° bo‘lgan yoy uzunligi $2\pi R$ ga teng bo‘lgani uchun,

1° li yoy uzunligi $\frac{2\pi R}{360^\circ} = \frac{\pi R}{180^\circ}$ ga teng bo‘ladi.

U holda, n° li yoy uzunligi $l = \frac{\pi R}{180^\circ} \cdot n^\circ$ formula bilan aniqlanadi *(1-rasm)*.

2. Burchakning radian o‘lchovi.

Burchakning gradus o‘lchovi bilan bir qatorda uning radian o‘lchovi ham ishlatiladi.

Aylana yoyi uzunligining radiusga nisbatini yuqoridagi formulaga asosan: $\frac{l}{R}=\frac{\pi}{180^\circ}\cdot n^\circ$ ga teng.

Demak, aylana yoyi uzunligining radiusiga nisbati faqat shu yoyga tiralgan markaziy burchak kattaligiga bog‘liq ekan. Bu xossadan foydalanib, burchakning radian o‘lchovi sifatida xuddi shu nisbatni olamiz:

$$\alpha = \frac{l}{R} = \frac{\pi}{180^\circ}\cdot n^\circ.$$

Odatda, radian so‘zi yozilmaydi. Masalan: 5 radian o‘rniga 5 deb yoziladi. Bir radian $\frac{180^\circ}{\pi}$ gradusga teng: $\quad$ 1 radian $=\frac{180^\circ}{\pi} \approx 57^\circ17'45''$.

Burchakning gradus o‘lchovidan radian o‘lchoviga o‘tish uchun

$$\alpha=\frac{\pi}{180^\circ}\cdot n^\circ$$

formuladan foydalaniladi.

Shunday qilib, n° li burchakning radian o‘lchovini topish uchun uning gradus o‘lchovini $\frac{\pi}{180^\circ}$ ga ko‘paytirish kifoya ekan. Xususiy holda, 180° li burchakning radian o‘lchovi π ga teng, 90° li, ya’ni to‘g‘ri burchakning radian o‘lchovi $\frac{\pi}{2}$ ga teng bo‘ladi.

α radianga teng markaziy burchakka mos yoyining uzunligi $l = \alpha R$ formula bilan hisoblanadi.

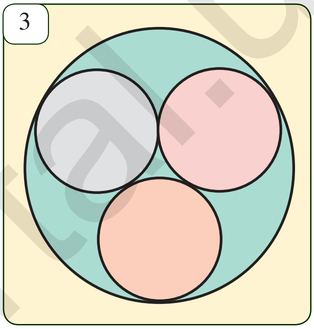
Masala. Ikkita burchagi mos ravishda 30° va 45° bo'lgan uchburchak burchaklarining radian o'lchovlarini toping.

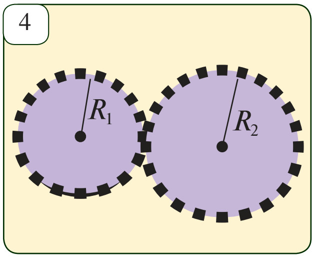
Yechish. Uchburchakning 30° li burchagi radian o'lchovi $30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$, 45°li burchagi radian o'lchovi $45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$. Uchburchak ichki burchaklari yig'indisi 180° ga, ya'ni π ga tengligi haqidagi teoremaga asosan uchburchakning uchinchi burchagining radian o'lchovini topamiz

$$\pi - \frac{\pi}{6} - \frac{\pi}{4} = \frac{7\pi}{12}.$$

Javob: $\frac{\pi}{6}$; $\frac{\pi}{4}$ va $\frac{7\pi}{12}$.

Masala va topshiriqlar





43.1. Radiusi 6 *cm* bo'lgan aylananing gradus o'lchovi a) 30°; b) 45°; c) 90°; d) 120° bo'lgan yoyi uzunligini toping.

43.2. a) 40°; b) 60°; d) 75° ga teng burchakning radian o'lchovini toping.

43.3. a) 1,2; b) $\frac{2\pi}{3}$; c) $\frac{5\pi}{6}$ radianga teng burchakning gradus o'lchovini toping.

43.4. Agar aylana radiusi 5 *cm* bo'lsa, uning a) $\frac{\pi}{8}$; b) $\frac{2\pi}{5}$; c) $\frac{3\pi}{4}$ radianga teng markaziy burchagi tiralgan yoy uzunligini toping.

43.5. Radiusi 12 *cm* bo'lgan aylanaga *ABC* uchburchak ichki chizilgan. Agar a) $\angle A=30°$; b) $\angle A=120°$ bo'lsa, *A* nuqtani o'z ichiga olmagan *BC* yoy uzunligini toping.

43.6. Aylananing teng vatarlari aylanadan teng yoylar ajratishini isbotlang.

43.7*. Ikkita aylana bir-birining markazidan o'tadi. Bu aylanalarning umumiy vatari har ikki aylanadan ajratgan yoylar uzunliklari nisbatini toping.

43.8*. Radiuslari teng bo'lgan uchta aylanalar bir-biriga tashqaridan va radiusi *R* ga teng bo'lgan aylanaga ichkaridan urinadi (*3-rasm*): a) aylanalar radiusini toping; b) bo'yalgan shaklni chegaralovchi yoylar uzunliklari yig'indisini toping.

Qiziqarli masala

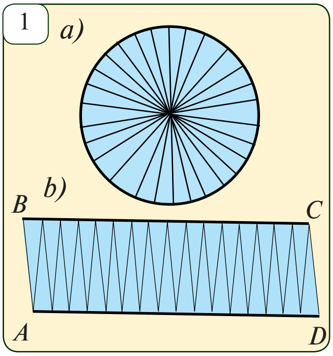
4-rasmda tasvirlangan ikkita tishli g'ildiraklar bir-biriga "tishlatilgan". G'ildiraklar radiusi R_1 va R_2. Birinchi g'ildirak *n* marta aylanganda ikkinchi g'ildirak necha marta aylanadi?

DOIRA YUZI

Ta'rif. Tekislikning berilgan O nuqtasidan berilgan R masofadan katta bo'lmagan masofada yotuvchi barcha nuqtalaridan tashkil topgan shaklga *doira* deb ataladi.

Bunda O nuqta doiraning markazi, R esa doiraning radiusi deb ataladi. Mazkur doiraning chegarasi markazi O nuqtada, radiusi esa R ga teng bo'lgan aylanadan iborat bo'ladi.

1



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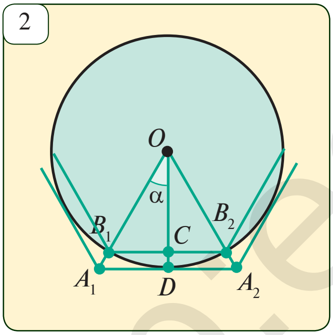


Faollashtiruvchi mashq

Bir varaq qog'ozga yo'g'on chiziq bilan aylana chizing va 1-*a* rasmda ko'rsatilgandek, uning bir nechta diametrlarini o'tkazib, doirani teng bo'laklarga bo'ling. So'ng bu bo'laklarni qiyib oling va 1-b rasmda ko'rsatilgandek terib, F shaklni hosil qiling. Agar doira istalgancha ko'p teng bo'laklarga bo'linib, bu bo'laklar rasmda ko'rsatilgan tartibda terilsa, natijada to'g'ri to'rtburchakka juda yaqin F shakl paydo bo'ladi.

a) F shaklni to'g'ri to'rtburchak shakliga juda yaqinligini hisobga olib, uning AB tomoni taxminan nimaga teng bo'lishini toping (ko'rsatma: AB tomonni doira radiusi bilan taqqoslang).

b) F shaklning BC "tomoni" taxminan nimaga teng bo'ladi? (Ko'rsatma: BC va AD tomonlar yo'g'on chiziq bilan chizilganiga, ya'ni aylana yoychalaridan iborat ekanligiga e'tibor bering).

d) F shaklning $ABCD$ to'g'ri to'rtburchak shakliga juda yaqin ekanligini hisobga olib, uning yuzini taqriban hisoblang. F shakl yuzi doira yuziga juda yaqin ekanligini nazarda tutib, doira yuzi haqida xulosa chiqaring.

Teorema. ***Radiusi R ga teng bo'lgan doiraning yuzi πR^2 ga teng.***

Isbot. Radiusi R va markazi O nuqtada bo'lgan aylanani qaraymiz.

Aylanaga tashqi chizilgan $A_1A_2 \dots A_n$ va ichki chizilgan $B_1B_2 \dots B_n$ muntazam n burchaklarning yuzlari mos ravishda S_n' va S_n'' bo'lsin *(2-rasm)*.

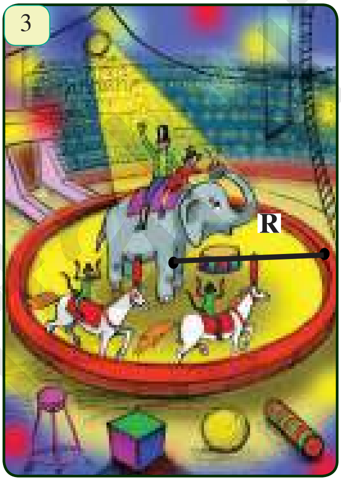
A_1OA_2 va B_1OB_2 uchburchaklar yuzlarini topamiz:

$$S'_{A_1OA_2}=\frac{1}{2}A_1A_2\cdot OD=\frac{1}{2}A_1A_2\cdot R;\quad S''_{B_1OB_2}=\frac{1}{2}B_1B_2\cdot OC=\frac{1}{2}B_1B_2\cdot OB_1\cos\alpha=\frac{1}{2}B_1B_2\cdot R\cos\alpha.$$

Unda, $S_n''=n\cdot\frac{1}{2}AA_1\cdot R=\frac{1}{2}P_nR,\quad S_n'=n\cdot\frac{1}{2}B_1B_2\cdot R\cos\alpha=\frac{1}{2}P_nR''\cos\alpha$ (1)

Bu yerda P'_n va P''_n mos ravishda $A_1A_2...A_n$ va $B_1B_2...B_n$ ko'pburchaklarning perimetrlari. $\alpha = \frac{180^\circ}{n}$ bo'lgani uchun n ning yetarlicha katta qiymatlarida $\cos\alpha$ ning qiymati birdan, P'_n va P''_n larning qiymatlari aylana uzunligi, ya'ni $2\pi R$ dan istalgancha kam farq qiladi. Unda, (1) tengliklarga ko'ra, n ning yetarlicha katta qiymatlarida ko'pburchaklarning yuzi πR^2 ga yaqinlashib boradi. Bundan, doira ning yuzi uchun $S=\pi R^2$ formula kelib chiqadi.

Teorema isbotlandi.



Masala. Sirk arenasi aylanasining uzunligi 41 *m*. Arena radiusi va yuzini toping.

Yechish. 1) Aylana uzunligini topish formulasidan radiusni topamiz *(3-rasm)*:

$$R = \frac{C}{2\pi} \approx \frac{41}{2\cdot 3{,}14} \approx 6{,}53 \ (m).$$

2) Doira yuzini hisoblash formulasidan arenaning yuzini topamiz: $S=\pi R^2 \approx 3{,}14\cdot 6{,}53^2 \approx 133{,}84\ (m^2)$.

Javob: $R\approx 6{,}53\ m$; $S\approx 133{,}84\ m^2$.

Masala va topshiriqlar

44.1. Doira yuzini hisoblash formulasini asoslang.

44.2. Radiusi R ga teng bo'lgan doiraning S yuzini topish formulasidan foydalanib jadvalni to'ldiring ($\pi=3{,}14$ deb oling).

R	2	5		$\frac{2}{7}$		54,3		6,25
S			9		49π		$\sqrt{3}$	

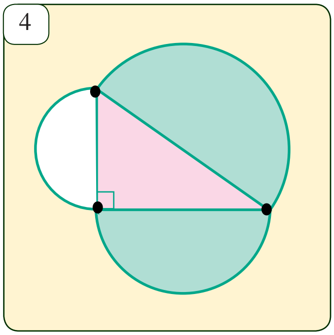
44.3. Agar doira radiusi a) k marta oshsa; b) k marta kamaysa, doira yuzi qanday o'zgaradi?

44.4. Tomoni 5 *cm* bo'lgan kvadratga ichki chizilgan va tashqi chizilgan doiralarning yuzini toping.

44.5. Tomoni $3\sqrt{3}$ *cm* bo'lgan muntazam uchburchakka ichki va tashqi chizilgan doiralarning yuzini toping.

44.6. Radiusi R bo'lgan doiradan eng katta kvadrat qirqib olindi. Doiraning qolgan qismi yuzini toping.

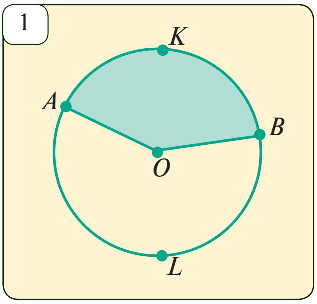
44.7. Tomonlari 6 *cm* va 7 *cm* bo'lgan to'g'ri to'rtburchakka tashqi chizilgan doira yuzini toping.



44.8. Tomoni 10 *cm* va o'tkir burchagi 60° bo'lgan rombga ichki chizilgan doira yuzini toping.

44.9*. To'g'ri burchakli uchburchak tomonlarini diametr qilib yarim doiralar chizilgan. Gipotenuzaga chizilgan yarim doira yuzi katetlarga chizilgan yarim doiralar yuzlari yig'indisiga teng bo'lishini ko'rsating *(4-rasm)*.

DOIRA BO‘LAKLARI YUZI



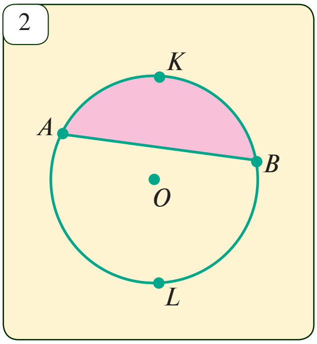
Ta’rif. Doiraning yoyi va bu yoy oxirlarini doira markazi bilan tutashtiruvchi ikkita radiusi bilan chegaralangan qismi *sektor* deyiladi. Sektorning chegarasi bo‘lgan yoy *sektor yoyi* deyiladi.

1-rasmda AKB va BLA yoyli ikkita sektor tasvirlangan (ulardan birinchisi bo‘yalgan).

Radiusi R ga va yoyining gradus o‘lchovi $n°$ ga teng bo‘lgan sektorning S yuzini topish uchun formula keltirib chiqaramiz. Yoyi 1° ga teng sektorning yuzi doira (ya’ni yoyi 360° ga teng sektor) yuzining $\frac{1}{360}$ qismiga teng bo‘lgani uchun, yoyi n gradus bo‘lgan sektorning yuzi

$$S = \frac{\pi R^2}{360} \cdot n \qquad \text{yoki} \qquad S = \frac{1}{2} R \cdot l$$

formula orqali topiladi. Bu yerda l – $n°$ li sektor yoyining uzunligi.

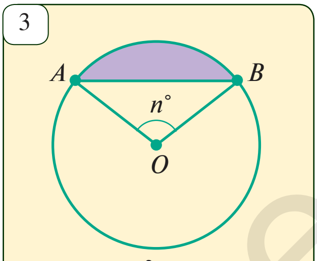


Ta’rif. Doiraning yoyi va bu yoy oxirlarini tutashtiruvchi vatari bilan chegaralangan qismi **segment** deyiladi.

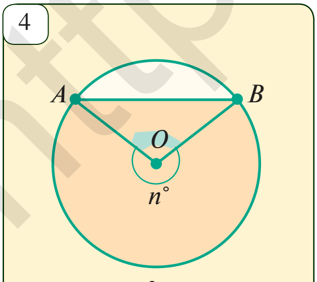
2-rasmda AKB va BLA yoyli ikkita segment tasvirlangan (ulardan birinchisi bo‘yalgan). Yarim doiradan farqli segmentning S yuzi

$$S = S_{sektor} \pm S_{\Delta} = \frac{\pi R^2}{360} \cdot n \pm S_{AOB}$$

formula bo‘yicha hisoblanadi *(3- va 4-rasmlarga qarang)*.



$S = \frac{\pi R^2}{360} \cdot n - S_{AOB}$



$S = \frac{\pi R^2}{360} \cdot n + S_{AOB}$

Masala. Yoyning gradus o‘lchovi 72° bo‘lgan sektorning yuzi 45π ga teng. Sektor radiusini toping.

Yechish. Sektor yuzini topish formulasiga ko‘ra,

$$\frac{\pi R^2}{360} \cdot 72 = 45\pi.$$

Bundan, $R^2 = \frac{45\pi \cdot 360}{72\pi} = 225$, demak, $R = 15$.

Javob: 15.

Masala va topshiriqlar

45.1. Sektor yuzini topish formulasini keltirib chiqaring.

45.2. Segment yuzini topish formulasini keltirib chiqaring.

45.3. Yoyining gradus o'lchovi a) $30°$; b) $45°$; d) $120°$; e) $90°$ va radiusi 7 *cm* bo'lgan sektor va segment yuzlarini toping.

45.4. 5-rasmda tomoni *a* ga teng bo'lgan muntazam uchburchak, kvadrat va muntazam oltiburchak tasvirlangan. Bo'yalgan shakllar yuzini toping. Bunda sektorlarning radiuslari ko'pburchak tomonining yarmiga teng.

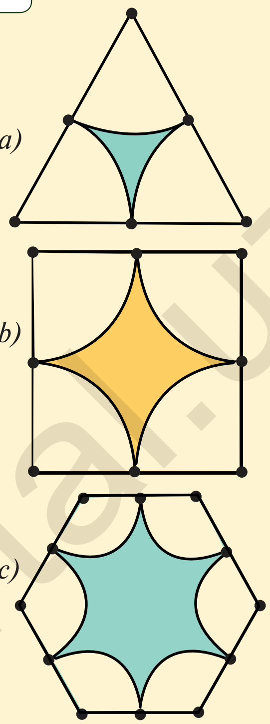
45.5. Nishonda radiuslari 1, 2, 3, 4 ga teng bo'lgan to'rtta aylana bor. Eng kichik doira yuzini va har bir halqa yuzini toping *(6-rasm)*.

45.6. Radiusi 10 *cm* ga teng bo'lgan doirada radiusga teng vatar o'tkazilgan. Hosil bo'lgan segmentlar yuzini hisoblang.

45.7. Radiuslari 15 *cm* dan bo'lgan ikkita doira markazlari orasidagi masofa 15 *cm*. Doiralar umumiy qismining yuzini toping.

45.8. Radiusi 10 *cm* bo'lgan doiraga ichki va tashqi chizilgan muntazam o'n ikkiburchaklar yuzini hisoblang. Natijalarni doira yuzi bilan solishtiring.

5



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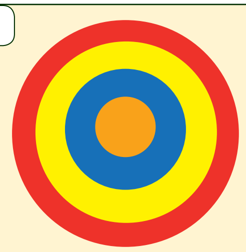


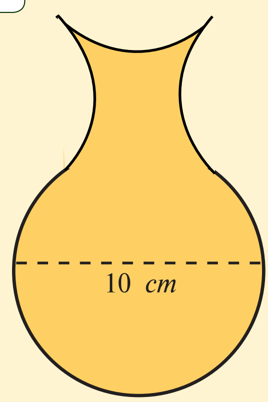
Qiziqarli masala

7-rasmda tasvirlangan guldon rasmini:

a) uchta to'g'ri chiziq bilan shunday to'rt bo'lakka bo'lingki, ulardan to'g'ri to'rtburchak yig'ish mumkin bo'lsin;

b) ikkita to'g'ri chiziq bilan shunday uch qismga bo'lingki, ulardan kvadrat yig'ish mumkin bo'lsin.

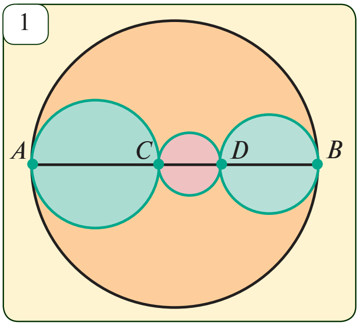
7



Tarixiy lavhalar. Uzoq vaqtlar mobaynida dunyoning ko'plab matematiklari "doira kvadraturasi" deb nom olgan quyidagi masalani yechishga harakat qilganlar: sirkul va chizg'ich yordamida yuzi berilgan doira yuziga teng bo'lgan kvadrat yasash. Faqat XIX asrning oxirida bu masala yechimga ega emasligi isbotlangan.

1-masala. C va D nuqtalar aylananing AB diametrini uchta AC, CD va DB kesmalarga ajratadi. AC, CD va DB diametrli aylanalar uzunliklarining yig'indisi AB diametrli aylana uzunligiga teng ekanligini isbotlang *(1-rasm)*.

1



Yechish. Aylana uzunligini topish formulasidan foydalanib, AC, CD va DB diametrli aylanalarning C_1, C_2, C_3 uzunliklari yig'indisini topamiz:

$C_1+C_2+C_3=AC\cdot\pi+CD\cdot\pi+DB\cdot\pi=\pi(AC+CD+DB)$.

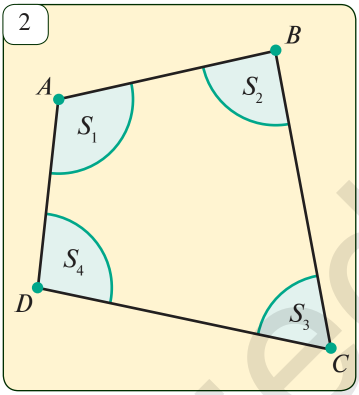
$AC+CD+DB=AB$ va AB diametrli aylananing C uzunligi $AB\cdot\pi$ ga teng bo'lgani uchun

$$C_1+C_2+C_3=C.$$

Shu tenglikni isbotlash talab qilingan edi.

2-masala. $ABCD$ to'rtburchakning uchlarini markaz qilib bir xil radiusli sektorlar yasalgan *(2-rasm)*. Bu sektorlardan ixtiyoriy ikkitasi umumiy nuqtaga ega emas hamda barchasining radiusi 1 *cm*. Sektorlar yuzlarining yig'indisini toping.

2



Yechish. 1) To'rtburchakning A, B, C, D burchaklari mos ravishda α_1, α_2, α_3, α_4 bo'lsin. Unda, ko'pburchak ichki burchaklarining yig'indisi haqidagi teoremaga ko'ra,

$$\alpha_1+\alpha_2+\alpha_3+\alpha_4=360°.$$

2) Sektor yuzini topish formulasiga ko'ra ($R=1$ *cm*),

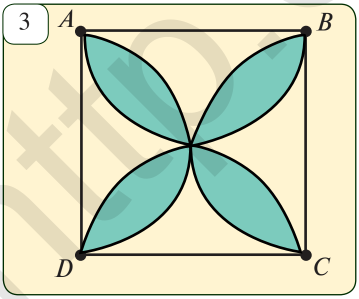
$$S_1=\frac{\pi}{360°}\cdot\alpha_1,\quad S_2=\frac{\pi}{360°}\cdot\alpha_2,\quad S_3=\frac{\pi}{360°}\cdot\alpha_3,\quad S_4=\frac{\pi}{360°}\cdot\alpha_4.\quad (1)$$

3) (1) tengliklarning mos tomonlarini qo'shamiz. Unda,

$$S_1+S_2+S_3+S_4=\frac{\pi}{360°}(\alpha_1+\alpha_2+\alpha_3+\alpha_4)=\frac{\pi}{360°}\cdot 360°=$$
$$=\pi\ (cm^2).$$

Javob: π cm^2.

3



Masala va topshiriqlar

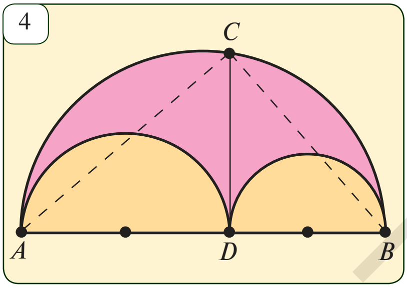
46.1. Perimetri 1 *m* bo'lgan kvadrat va uzunligi 1 *m* bo'lgan aylana berilgan. Bu aylana bilan chegaralangan doira yuzi bilan kvadrat yuzini taqqoslang.

46.2. Radiusi 8 *cm* bo'lgan doiradan 60° li sektor qirqib olingan. Doiraning qolgan qismi yuzini toping.

46.3. Diagonallari 6 *cm* va 8 *cm* bo'lgan rombga ichki chizilgan doira yuzini hisoblang.

46.4. 3-rasmda bo‘yab ko‘rsatilgan shakl yuzini toping. Unda $ABCD$ — kvadrat, $AB = 4$ cm.

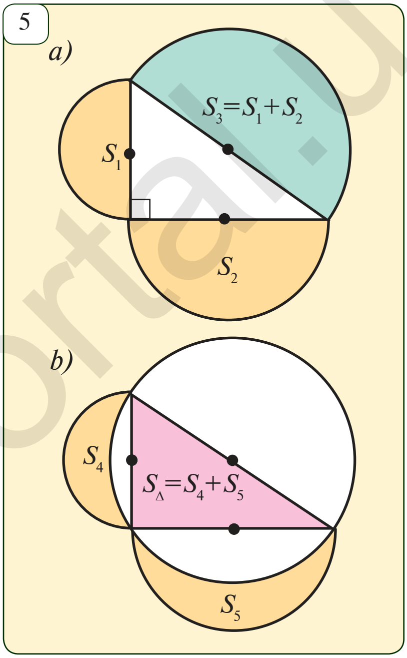
46.5*. 4-rasmda "Arximed pichog‘i" deb ataluvchi shakl bo‘yab ko‘rsatilgan. Uning yuzi $\frac{\pi \cdot CD^2}{4}$ formula bilan hisoblanishini isbotlang (bunda, $\angle ACB = 90°$ va $CD^2 = AD \cdot DB$ ekanligidan foydalaning).



46.6. Agar $AD = 6$ cm, $BD = 4$ cm bo‘lsa, 4-rasmda bo‘yab ko‘rsatilgan shaklning yuzi va perimetrini (uni o‘rab turgan yoylar uzunligi yig‘indisini) toping.

Tarixiy lavhalar. Gippokrat oychalari.

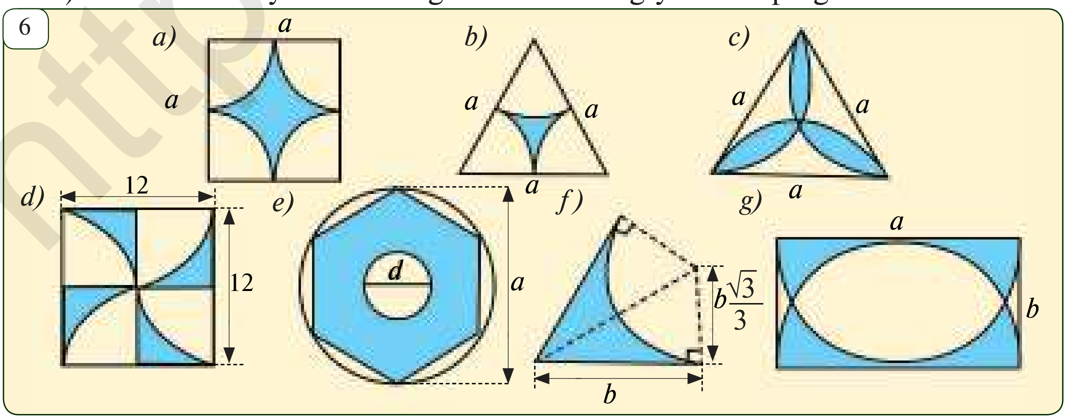
a) Gippokrat oychasi – ikki aylana yoylari bilan chegaralangan va quyidagi xossaga ega bo‘lgan shakldir: agar aylanalar radiuslari va oycha yoylari tiralgan vatar berilgan bo‘lsa, oychaga tengdosh kvadrat yasash mumkin.



Pifagor teoremasi qo‘llanilsa, 5-a rasmda tasvirlangan gipotenuzaga qurilgan yarim doira yuzi katetlarga qurilgan yarim doiralar yuzlari yig‘indisiga teng bo‘ladi (121-betdagi 44.9- masalaga qarang). Shuning uchun 5-b rasmdagi oychalar yuzlarining yig‘indisi uchburchak yuziga teng (mushohada qilib ko‘ring!). Agar rasmdagi uchburchak o‘rniga teng yonli to‘g‘ri burchakli uchburchak olsak, hosil bo‘lgan ikki oychadan har birining yuzi uchburchak yuzining yarmiga teng bo‘ladi. Doira kvadraturasi haqidagi masalani yechishga urinib, yunon matematigi Gippokrat (miloddan avvalgi V asr) ko‘pburchak bilan tengdosh bir necha xil oychalarni ixtiro qilgan.

Gippokrat oychalarining to‘la jadvali faqat XIX–XX asrlarda tuzilgan.

b) 6-rasmda bo‘yab ko‘rsatilgan shakllarning yuzini toping.



BILIMINGIZNI SINAB KO‘RING.

I. Testlar

1. 45 gradusli burchakning radian o‘lchovi nimaga teng?
A. 1 ga teng; B. $\frac{\pi}{2}$ ga teng; D. $\frac{\pi}{4}$ ga teng; E. $\sqrt{2}$ ga teng.
2. Radiusi 3 *cm* bo‘lgan aylananing gradus o‘lchovi 150° bo‘lgan markaziy burchagi tiralgan yoy uzunligini toping.
A. $\frac{5\pi}{2}$ *cm*; B. $\frac{5\pi}{3}$ *cm*; D. $\frac{10\pi}{3}$ *cm*; E. $\frac{5\pi}{4}$ *cm*.
3. Radiusi 6 *cm* bo‘lgan aylanada $\frac{5\pi}{4}$ radianga teng markaziy burchak tiralgan yoy uzunligini toping.
A. $\frac{15\pi}{2}$ *cm*; B. $\frac{5\pi}{6}$ *cm*; D. $\frac{4\pi}{3}$ *cm*; E. $\frac{5\pi}{2}$ *cm*.
4. Tomoni 5 *cm* ga teng bo‘lgan kvadratga tashqi chizilgan aylana uzunligini toping.
A. $5\sqrt{2}\pi$; B. $\sqrt{2}\pi$; D. $3\sqrt{2}\pi$; E. 5π.
5. Diametri 6 ga teng doira yuzini toping.
A. 9π; B. 6π; D. $3\sqrt{2}\pi$; E. 12π.
6. Yoyining gradus o‘lchovi 150°, radiusi 6 cm bo‘lgan doiraviy sektorning yuzini toping.
A. 15π cm²; B. 6π cm²; D. $30\sqrt{2}\pi$ cm²; E. 24π cm².
7. Yoyining uzunligi 12 cm va radiusi 6 cm bo‘lgan doiraviy sektorning yuzini toping.
A. 15π cm²; B. 6π cm²; D. $30\sqrt{2}\pi$ cm²; E. 24π cm².
8. Yoyining gradus o‘lchovi 120°, radiusi 3 ga teng bo‘lgan doiraviy segmentning yuzini toping.
A. $6\pi - 4\sqrt{3}$; B. $6\pi + 4\sqrt{3}$; D. $3\pi - 4\sqrt{3}$; E. $3\pi + 4\sqrt{3}$.

II. Masalalar

1. *ABCDEFKL* muntazam sakkizburchakning tomoni 6 *cm*. Uning *AC* diagonalini toping.

2. Kvadrat radiusi 4 *dm* bo‘lgan aylanaga ichki chizilgan. Kvadrat qo‘shni tomonlarining o‘rtalaridan o‘tuvchi vatarni aylanadan ajratgan yoylarning uzunligini toping.

3. Aylananing 90° li yoyi uzunligi 15π *cm*. Aylana radiusini toping.

4. Radiusi 20 ga teng aylanadan uzunligi 10π ga teng yoy ajratildi. Bu yoyga mos markaziy burchakni toping.

5. Ikkita doiraning umumiy vatari bu doiralarni chegaralovchi aylanalardan 60° li va 120° li yoylarni ajratadi. Doiralar yuzlarining nisbatini toping.

6. Tomonlari 3, 4, 5 bo‘lgan uchburchakka ichki va tashqi chizilgan doiralar yuzlarini toping.

7. Doira vatari 60° li yoyni tortib turadi. Bu vatar ajratgan segmentlar yuzlari nisbatini toping.

8. Muntazam oltiburchak yuzining unga ichki chizilgan doira yuziga nisbatini toping.

9. Tomoni a ga teng bo‘lgan $ABCDEF$ muntazam oltiburchak berilgan. Markazi A nuqtada va radiusi a bo‘lgan aylana bu oltiburchakni ikki qismga ajratadi. Har bir qism yuzini toping.

10. To‘g‘ri burchakli ABC uchburchakda $\angle A=72°$, $\angle C=90°$, $BC=15\ cm$. BC diametrli aylananing ABC uchburchak ichida yotgan yoyi uzunligini toping.

11. Doiraga ichki chizilgan muntazam sakkizburchak berilgan. Uning ikki qo‘shni uchlariga o‘tkazilgan radiuslar doirani ikkita sektorga ajratadi. Bu sektorlar yuzlarining nisbatini toping.

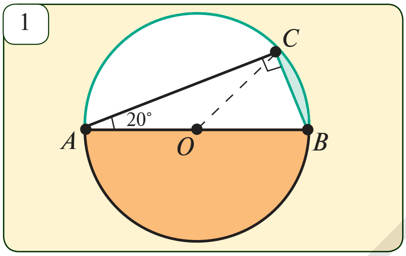
12. To‘g‘ri burchakli ABC uchburchakda $\angle A=20°$, $\angle C=90°$, $AB=18\ cm$. BC kesma uchburchakka tashqi chizilgan doirani ikki segmentga ajratadi. Bo‘yab ko‘rsatilgan segment yuzini toping (*1-rasm*).

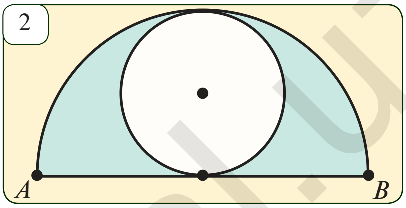
13. Kichik aylana katta aylanaga hamda uning AB diametriga urinadi. Agar diametrga urinish nuqtasi aylana markazi va $AB=4$ bo‘lsa, rasmda bo‘yalgan shakl yuzini toping (*2-rasm*).

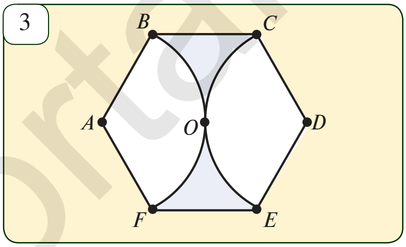
14. Muntazam $ABCDEF$ oltiburchakning tomoni 6 ga teng va markazi O nuqtada. Markazlari A va D nuqtada va radiuslari teng bo‘lgan aylanalar O nuqtada urinadi. Bo‘yalgan soha yuzini toping (*3-rasm*).

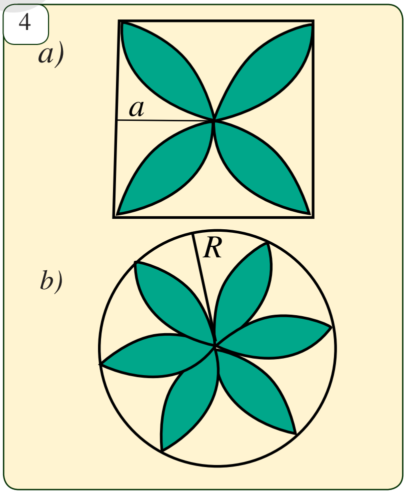
15. To‘g‘ri burchakli ABC uchburchakda $\angle C=90°$, $AC=4$, $CB=2$. Markazi gipotenuzada bo‘lgan aylana uchburchak katetlariga urinadi. Bu aylana uzunligini toping.

16. 4-rasmda bo‘yab ko‘rsatilgan shakllarning yuzini toping. Ular qanday chizilganligini aniqlang.









III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Tomoni 6 cm bo‘lgan kvadratga tashqi chizilgan aylana uzunligini va ichki chizilgan doira yuzini toping.

2. Tomoni 24 cm bo‘lgan muntazam ko‘pburchakka ichki chizilgan aylana radiusi $4\sqrt{3}$ cm ga teng bo‘lsa, unga tashqi chizilgan aylana radiusini toping.

3. 240° li aylana yoyining uzunligi 24 cm bo‘lsa,
a) aylana radiusini; b) yoyi 240° bo‘lgan sektor yuzini;
d) yoyi 240° bo‘lgan segmentning yuzini toping.

Qiziqarli masala

In va Yan

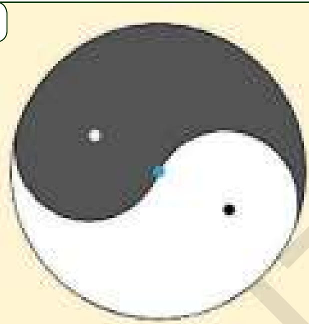
5-rasmda tabiatdagi qarama-qarshiliklarni ifodalovchi “In va Yan” deb nomlangan xitoy ramzi tasvirlangan.

a) In va Yan ramzlari yuzlari tengligini ko‘rsating;

b) bitta to‘g‘ri chiziq bilan bu ramzlarning har birini yuzlari teng bo‘lgan ikki bo‘lakka bo‘ling.

d) In va Yan ramzlar perimetrlarini (ularni o‘rab turgan yoylar uzunliklari yig‘indisini) toping.

1



Tarixiy lavhalar. Aylana uzunligini hisoblash juda qadimdan dolzarb muammo bo‘lgan. Aylana uzunligini unga ichki chizilgan ko‘pburchak perimetriga almashtirish usuli keng tarqalgan.

O‘rta Osiyolik matematiklar ham doiraga ichki chizilgan muntazam ko‘pburchaklarni yasash, ularning tomonlarini doiraning radiusi orqali ifodalash masalalari bilan shug‘ullanganlar. Abu Rayhon Beruniy “Qonuni Mas’udiy” asarida doiraga ichki chizilgan ko‘pburchaklarning tomonini aniqlash bilan shug‘ullanib, ichki chizilgan beshburchak, oltiburchak, yettiburchak,..., o‘nburchak tomonlarini aniqlash usulini ko‘rsatadi. Bu hisoblash natijasida u $\pi \approx 3{,}14$ qiymatga ega ekanligini aniqlaydi.

Qadimgi Bobil va Misr qo‘lyozmalari va mixxatlarida π uchga teng deb olingan. Bu o‘sha davr aniqlik talabi uchun yetarli bo‘lgan. Keyinchalik rimliklar π uchun 3,12 ni ishlatishgan. π soni uchun Arximed bergan qiymat 3,14 bo‘lib, bu amaliy masalalarni hal qilishda juda ma’qul edi.

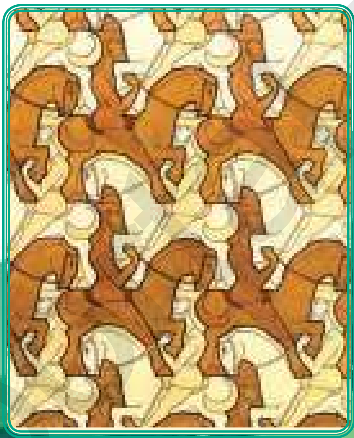
Xitoy matematiklarida $\pi \approx 3{,}155$ ... va 22/7. Hindlarning “Sulva Sutra” (“Arqon qoidasi”) asarida π uchun 3,008 va 3,1416 ... va $\sqrt{10} \approx 3{,}162$ ... qiymatlar uchraydi.

Mirzo Ulug‘bekning “Astronomiya maktabi” namoyandalaridan biri Jamshid G‘iyosiddin al-Koshiy 1424-yilda yozgan “Aylana uzunligi haqida kitob” nomli risolasida aylanaga ichki va tashqi chizilgan muntazam ko‘pburchak tomonlari sonini ikkilantirish yo‘li bilan $3 \cdot 2^{28} = 800\ 335\ 168$ tomonli muntazam ko‘pburchaklar perimetrini hisoblab, π uchun $\pi = 3{,}1\ 415\ 826\ 535\ 897\ 932$ qiymatni hosil qilgan. Bu 16 ta o‘nli raqamgacha aniqdir.

Ammo al-Koshiyning asari uzoq vaqtgacha Yevropada noma’lum bo‘lgan. Yevropaliklardan belgiyalik Van Romen 1597- yilda 2^{30} tomonli muntazam ko‘pburchakka Arximed usulini tatbiq etib, π uchun 17 ta o‘nli raqamlari aniq bo‘lgan qiymat topgan. Gollandiyalik Rudolf van Seylon (1540−1610) bu aniqlikni 35 ta o‘nli raqamlargacha olib borgan. Hozirgi davrda elektron hisoblash mashinalari yordamida π uchun milliondan ortiq o‘nli raqamlari aniq bo‘lgan qiymatlar topilgan. Kundalik hisoblashlar uchun 3,14 qiymat, matematik hisoblashlar uchun 3,1416 qiymat, hatto astronomiya va kocmonavtika uchun 3,1415826 qiymat kifoyadir.

IV BOB

UCHBURCHAK VA AYLANADAGI METRIK MUNOSABATLAR



Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilimlar:

√ *proporsional kesmalarning xossalarini bilish;*

√ *to‘g‘ri burchakli uchburchakda gipotenuzaga tushirilgan balandlikning xossalarini bilish;*

√ *o‘zaro kesishuvchi vatarlar kesmalari to‘g‘risidagi hamda aylanani kesuvchi to‘g‘ri chiziq kesmalari to‘g‘risidagi xossalarni bilish.*

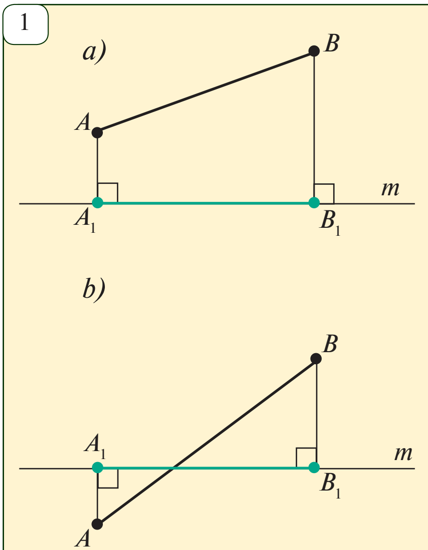
Ko‘nikmalar:

√ *kesmalarning nisbati va proporsional kesmalarga doir masalalarni yecha olish;*

√ *to‘g‘ri burchakli uchburchakda gipotenuzaga tushirilgan balandlikning xossalaridan foydalanib, masalalar yecha olish;*

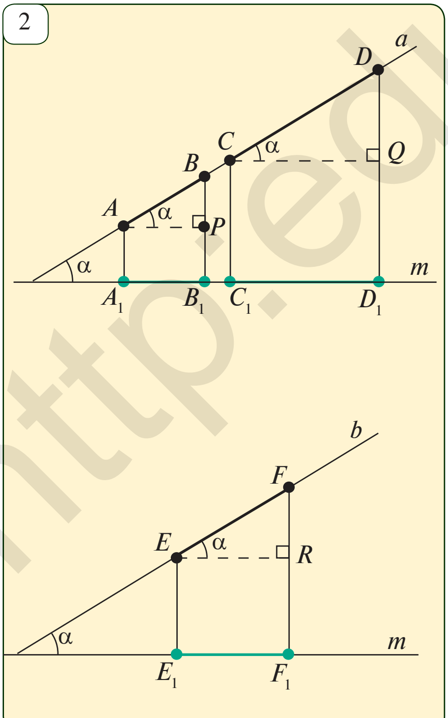
√ *kesuvchi vatarlar kesmalarining va kesuvchi to‘g‘ri chiziq kesmalarining xossalaridan foydalanib, masalalar yechish.*

1



A_1 — A nuqtaning,
B_1 — B nuqtaning,
A_1B_1 — AB kesmaning ***m* to'g'ri chiziqdagi proyeksiyasi**

2



Faollashtiruvchi mashq

1. Kesmalar nisbati nimani anglatadi?
2. Qanday kesmalar proporsional deyiladi?
3. Fales teoremasini ayting.

Tekislikda m to'g'ri chiziq va AB kesma berilgan bo'lsin. A va B nuqtalardan m to'g'ri chiziqqa AA_1 va BB_1 perpendikularlar tushiramiz (*1-rasm*). A_1B_1 kesma AB kesmaning m to'g'ri chiziqdagi *proyeksiyasi (soyasi)* deyiladi.

AB kesmaning m to'g'ri chiziqdagi A_1B_1 proyeksiyasini qurish amali AB kesmani m to'g'ri chiziqqa *proyeksiyalash* deyiladi.

Teorema. ***Bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotadigan kesmalar berilgan bo'lsin. Ularning ayni bir to'g'ri chiziqqa proyeksiyalari berilgan kesmalarga proporsional bo'ladi.***

$a||b$,
A_1B_1 — AB ning,
C_1D_1 — CD ning,
E_1F_1 — EF ning
m to'g'ri chiziqdagi proyeksiyalari (2-rasm)

$$\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF} \quad (1)$$

Isbot. a) Agar a va b to'g'ri chiziqlar m to'g'ri chiziqqa parallel bo'lsa, $AB=A_1B_1$, $CD=C_1D_1$, $EF=E_1F$ bo'lishi hamda (1) tengliklar o'rinli ekanligi ravshan.

b) Bordi-yu a va b to'g'ri chiziqlar m to'g'ri chiziqqa perpendikular bo'lsa, A_1 va B_1, C_1 va D_1, E_1 va F_1 nuqtalar ustma-ust tushadi. Shuning uchun A_1B_1, C_1D_1, E_1F_1 kesmalarning uzunligi nolga teng bo'ladi va (1) tengliklar bajariladi.

d) Endi boshqa holni qaraymiz. 2-rasmda tasvirlanganidek, to'g'ri burchakli ABP, CDQ, EFR uchburchaklarni quramiz. Unda $a||b$ bo'lgani uchun, $\angle BAP=\angle DCQ=\angle FER$.

Demak, *ABP, CDQ* va *EFR* to‘g‘ri burchakli uchburchaklar o‘xshash.

Bundan $\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF}$ tengliklarni hosil qilamiz.

Teorema isbotlandi.

Masala. *AB* va *CD* kesmalar parallel to‘g‘ri chiziqlarda yotadi. Agar $AB=12$ *cm*, $CD=15$ *cm* va *AB* kesmaning biror *m* to‘g‘ri chiziqdagi proyeksiyasi 8 *cm* bo‘lsa, *CD* kesmaning shu *m* to‘g‘ri chiziqdagi proyeksiyasini toping.

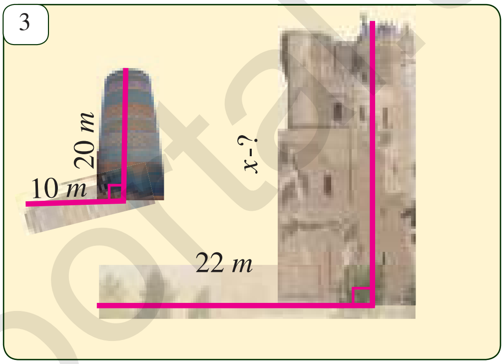
Yechish. *CD* kesmaning *m* to‘g‘ri chiziqdagi proyeksiyasi *x* bo‘lsin. Unda, isbotlangan teorema va masala shartidan foydalanib, proporsiya tuzamiz:

$$\frac{x}{15}=\frac{8}{12}.$$

Bu tenglikdan $x=10$ bo‘lishini topamiz.

Javob: 10 *cm*.

3



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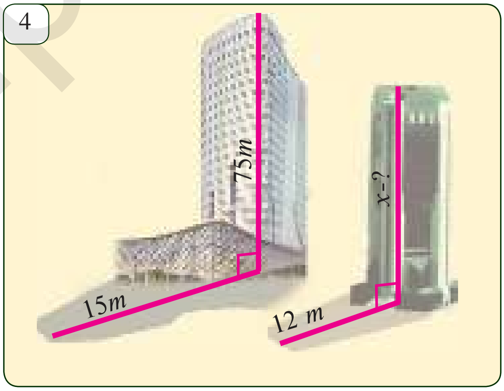


Masala va topshiriqlar

48.1. Kesmaning berilgan to‘g‘ri chiziqdagi proyeksiyasi nima?

48.2. Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotgan kesmalarning ayni boshqa bir to‘g‘ri chiziqqa proyeksiyalari berilgan kesmalarga proporsional ekanligini isbotlang.

48.3. *a* va *b* to‘g‘ri chiziqlar orasidagi burchak 45° ga teng. *a* to‘g‘ri chiziqda uzunligi 10 *cm* bo‘lgan *AB* kesma olingan. *AB* kesmaning *b* to‘g‘ri chiziqdagi proyeksiyasini toping.

48.4. *AB* kesmaning uchlari *l* to‘g‘ri chiziqdan 9 *cm* va 14 *cm* uzoqlikda yotadi. Agar *AB* kesma *l* to‘g‘ri chiziqni kesib o‘tmasa va $AB=13$ *cm* bo‘lsa, *AB* kesmaning *l* to‘g‘ri chiziqdagi proyeksiyasini toping.

48.5. 3- va 4-rasmlardagi ma’lumotlar asosida binolarning balandliklarini toping.

48.6. To‘g‘ri chiziq va unga parallel bo‘lmagan kesma chizing. Kesmaning to‘g‘ri chiziqdagi proyeksiyasini yasang.

48.7. Koordinatalar tekisligida *A*(2;3) va *B*(3;-4) nuqtalar belgilangan. *AB* kesmaning koordinata o‘qlaridagi proyeksiyalarining uzunliklarini toping.

48.8. *a* va *b* to‘g‘ri chiziqlar orasidagi burchak α ekanligi ma’lum. *a* to‘g‘ri chiziqda *AB* kesma olingan. *AB* kesmaning *b* to‘g‘ri chiziqdagi proyeksiyasini toping.

48.9*. *AB* va *CD* kesmalarning *l* to‘g‘ri chiziqdagi proyeksiyalari o‘zaro teng. *AB* va *CD* kesmalarning uzunliklari haqida nima deyish mumkin? Misollar keltiring.

49 PROPORSIONAL KESMALARNING XOSSALARI

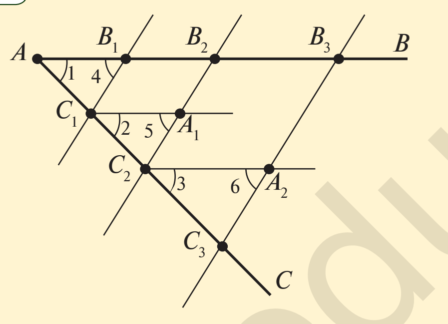
Fales teoremasining umumlashmasi bo‘lgan muhim xossani isbotlaymiz.

Teorema. ***Burchakning har ikkala tomonini kesib o‘tgan parallel to‘g‘ri chiziqlar uning tomonlaridan proporsional kesmalar ajratadi.***

$\angle BAC,\ B_1C_1 || B_2C_2 || B_3C_3$ (*1-rasm*) $\Rightarrow$ $\dfrac{AB_1}{AC_1}=\dfrac{B_1B_2}{C_1C_2}=\dfrac{B_2B_3}{C_2C_3}$

Isbot. C_1 va C_2 nuqtalardan AB ga parallel C_1A_1 va C_2A_2 to‘g‘ri chiziqlarni o‘tkazamiz. U holda, birinchidan, $\angle 1=\angle 2=\angle 3$ bo‘ladi, chunki ular o‘zaro parallel bo‘lgan AB, C_1A_1 va C_2A_2 to‘g‘ri chiziqlarni AC to‘g‘ri chiziq kesganda hosil bo‘lgan mos burchaklardir. Ikkinchidan, $\angle 4=\angle 5=\angle 6$, chunki ular tomonlari parallel bo‘lgan burchaklardir.

1



Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\Delta AB_1C_1 \backsim \Delta C_1A_1C_2 \backsim \Delta C_2A_2C_3$ bo‘ladi.

U holda, $\dfrac{AB_1}{AC_1}=\dfrac{C_1A_1}{C_1C_2}=\dfrac{C_2A_2}{C_2C_3}$ (1)

tengliklarni hosil qilamiz.

Bundan tashqari, $B_1C_1A_1B_2$ va $B_2C_2A_2B_3$ to‘rtburchaklar parallelogramm, chunki

$B_1C_1||B_2C_2||B_3C_3$ — shartga ko‘ra;

$AB||C_1A_1||C_2A_2$ — yasashga ko‘ra.

Shuning uchun, bu parallelogrammlarning qarama-qarshi tomonlari o‘zaro teng bo‘ladi:

$$C_1A_1=B_1B_2 \quad \text{va} \quad C_2A_2=B_2B_3. \qquad (2)$$

(1) va (2) tengliklardan $\dfrac{AB_1}{AC_1}=\dfrac{B_1B_2}{C_1C_2}=\dfrac{B_2B_3}{C_2C_3}$ bo‘lishi kelib chiqadi.

Teorema isbotlandi.

Amaliy mashq. ***Kesmani berilgan nisbatda bo‘lish.***

Berilgan a kesmani to‘rt bo‘lakka shunday bo‘lingki, bo‘laklarning o‘zaro nisbati $m{:}n{:}l{:}k$ kabi bo‘lsin.

Buning uchun quyidagilarni qadam-baqadam bajaramiz:

1-qadam. Ixtiyoriy o‘tkir burchak chizib, uning bir tomoniga uzunliklari $OA=m$, $AB=n$, $BC=l$ va $CD=k$ ga teng bo‘lgan kesmalarni 2-rasmda ko‘rsatilgandek qilib, ketma-ket qo‘yib chiqamiz.

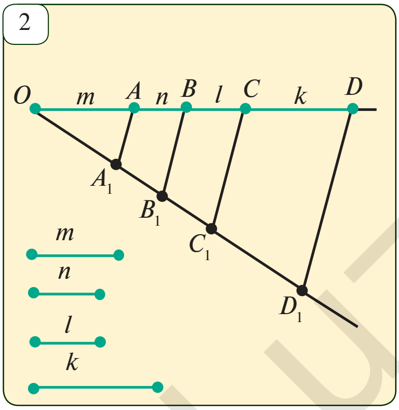
2-qadam. Burchakning ikkinchi tomoniga berilgan a kesmaga teng OD_1 kesmani qo‘yamiz.

3-qadam. D va D_1 nuqtalarni tutashtiramiz.

4-qadam. A, B, C nuqtalar orqali DD_1 ga parallel AA_1, BB_1 va CC_1 kesmalarni o‘tkazamiz.

Yuqoridagi teoremaga ko‘ra, berilgan $a=OD_1$ kesma A_1, B_1, C_1 va D_1 nuqtalar bilan $m:n:l:k$ nisbatda bo‘lingan bo‘ladi.

Topshiriq: Bu tasdiqni mustaqil ravishda asoslang.



Amaliy topshiriq. To‘rtinchi proporsional kesmani yasash.

a, b va *c* kesmalar berilgan. *a* va *b* kesmalar *c* va *d* kesmalarga proporsional, ya’ni *a:b=c:d* ekanligi ma’lum. *d* kesmani yasang *(3-rasm)*.

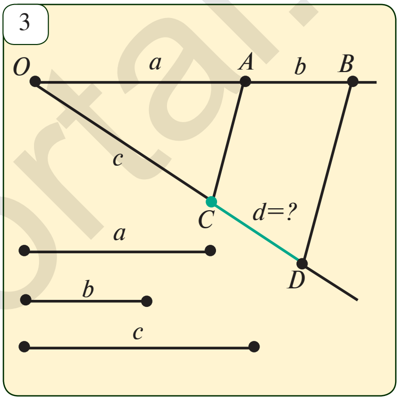
1-qadam. Ixtiyoriy o‘tkir burchak chizib, uning bir tomoniga $OA=a$ va $AB=b$ kesmalarni 3-rasmda ko‘rsatilgandek qo‘yamiz.

2-qadam. Ikkinchi tomoniga esa $OC=c$ kesmani qo‘yamiz.

3-qadam. A va C nuqtalarni tutashtiramiz.

4-qadam. B nuqtadan AC ga parallel BD to‘g‘ri chiziq o‘tkazamiz.

Topshiriq: CD izlanayotgan d kesma bo‘lishini asoslang.



Masala va topshiriqlar

49.1. Uzunligi 42 *cm* bo‘lgan kesma berilgan. Uni a) 5:2; b) 3:4:7; d) 1:5:1:7 nisbatdagi bo‘lakchalarga bo‘ling.

49.2. Rasmda har bir bo‘lak birlik kesmadan iborat bo‘lsa, AB va CD, EF va MN, AC va DF, AN va CE, EN va BM kesmalarning nisbatlarini toping.

49.3. m, n kesmalar l va k kesmalarga proporsional. Agar a) $m=4$ *cm*, $n=3$ *cm* va $l=8$ *cm*; b) $m=2$ *cm*, $n=3$ *cm* va $l=7$ *cm* bo‘lsa, k — to‘rtinchi kesmani quring va uzunligini toping.

49.4. To‘rtburchakning perimetri 54 *cm* va tomonlari 3:4:5:6 kabi nisbatda bolsa, uning har bir tomonini aniqlang.

49.5. To‘rtburchakning burchaklari o‘zaro 3:4:5:6 kabi nisbatda bo‘lsa, uning kichik burchagi nimaga tengligini toping.

49.6. Uzunligi 4, 5 va 6 bo‘lgan kesmalar berilgan. Uzunligi 4,8 ga teng kesma yasang.

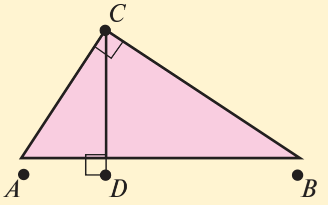
49.7*. Perimetri 60 *cm* bo‘lgan to‘rtburchakning bir tomoni 15 *cm*, qolgan tomonlari esa 2:3:4 nisbatda ekanligi ma’lum. Uning katta tomonini toping.

50 TO'G'RI BURCHAKLI UCHBURCHAKDAGI PROPORSIONAL KESMALAR

Xossa. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandligi uni o'ziga o'xshash ikkita uchburchakka ajratadi.

$\Delta ABC, \angle C=90^\circ$, CD — *balandlik* (*1-rasm*) ⇒ $\Delta ABC \sim \Delta ACD$, $\Delta ABC \sim \Delta CBD$

1



Isbot. ABC va ACD uchburchaklar to'g'ri burchakli bo'lib, A burchak esa ular uchun umumiy. Demak, $\Delta ABC \sim \Delta ACD$. Shu singari, ΔABC va ΔCBD to'g'ri burchakli bo'lib, ular uchun $\angle B$ umumiy. Demak, $\Delta ABC \sim \Delta CBD$.

1-rasmda tasvirlangan AD va DC kesmalar mos ravishda AC va BC katetlarning gipotenuzadagi proyeksiyalari deb yuritiladi.

Ta'rif. Agar a, b va c kesmalar uchun $a:b=b:c$ bo'lsa, b kesma a va c kesmalar orasidagi *o'rta proporsional kesma* deb ataladi.

O'rta proporsionallik shartini $b^2=ac$ yoki $b=\sqrt{ac}$ ko'rinishda yozish mumkin.

Yuqorida isbotlangan xossaga asoslanadigan bo'lsak, o'rta proporsional kesmalar haqidagi quyidagi teoremalar osonlikcha isbotlanadi:

1-teorema. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandlik katetlarning gipotenuzadagi proyeksiyalari orasida o'rta proporsional bo'ladi.

Haqiqatan ham, isbotlangan xossaga ko'ra, $\Delta ACD \sim \Delta CBD$. Bundan,

$$\frac{AD}{CD}=\frac{CD}{BD} \Rightarrow CD^2=AD\cdot BD \Rightarrow CD=\sqrt{AD\cdot BD}.$$

2-teorema. To'g'ri burchakli uchburchakning kateti gipotenuza bilan shu katetning gipotenuzadagi proyeksiyasi orasida o'rta proporsionaldir (*1-rasm*).

Haqiqatan ham, isbotlangan xossaga ko'ra, $\Delta ABC \sim \Delta ACD$. Bundan,

$$\frac{AB}{AC}=\frac{AC}{AD} \Rightarrow AC^2=AB\cdot AD \Rightarrow AC=\sqrt{AB\cdot AD}.$$

Xuddi shunga o'xshash $BC=\sqrt{BD\cdot AB}$ ekanligini isbotlash mumkin.

Masala. Katetlari 15 *cm* va 20 *cm* bo'lgan to'g'ri burchakli uchburchak kichik katetining gipotenuzadagi proyeksiyasini toping.

$\Delta ABC, \angle C=90^\circ$, CD — *balandlik*, $AC=15$ *cm*, $BC=20$ *cm* (*1-rasm*) ⇒ $AD=?$

Yechish. 1) Pifagor teoremasidan foydalanib, uchburchak gipotenuzasini topamiz: $AB^2=AC^2+BC^2=15^2+20^2=625$, ya'ni $AB=25$ *cm*.

2) Ikkinchi teoremadan foydalanib, AD ni topamiz:

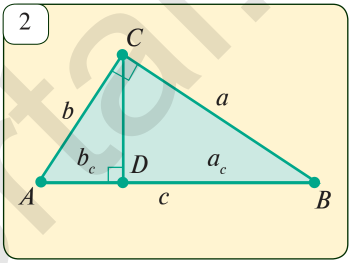
$$AC^2=AB\cdot AD \Rightarrow AD=\frac{AC^2}{AB}=\frac{15^2}{25}=9\ (cm).$$ *Javob:* 9 *cm*.

Ikkinchi teoremadan natija sifatida Pifagor teoremasining **Pifagorning o'zi yozib qoldirgan isboti** kelib chiqadi *(1-rasm)*. 2- teoremaga ko'ra,

$$\left.\begin{aligned} AC^2=AD\cdot AB \\ BC^2=BD\cdot AB \end{aligned}\right\} \Rightarrow AC^2+BC^2=AD\cdot AB+BD\cdot AB=AB\cdot(\underbrace{AD+BD}_{AB})=AB\cdot AB=AB^2.$$

Shunday qilib, $AC^2+BC^2=AB^2$.

Masala va topshiriqlar



50.1. Isbotlang *(2-rasm)*:

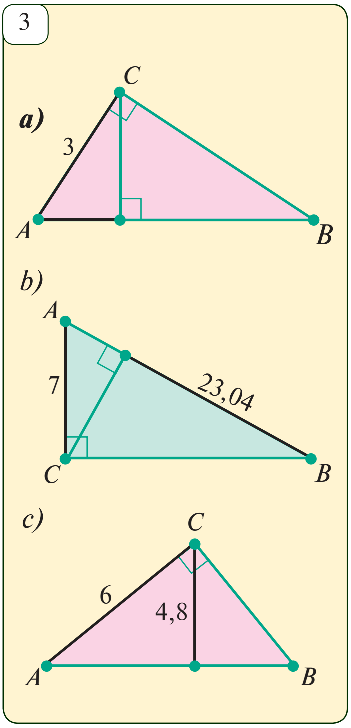
a) $\Delta ACD \sim \Delta CBD \sim \Delta ABC$;

b) $b^2=b_c\cdot c$, $a^2=a_c\cdot c$; d) $h_c^2=a_c\cdot b_c$.

50.2. To'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi gipotenuzani 9 *cm* va 16 *cm* ga teng kesmalarga bo'ladi. Uchburchak tomonlarini toping.

50.3. To'g'ri burchakli uchburchakning gipotenuzasi 15 *cm* ga, bir kateti esa 9 *cm* ga teng. Ikkinchi katetning gipotenuzadagi proyeksiyasini toping.

50.4. 3-rasmdagi ma'lumotlar asosida ABC uchburchakning tomonlarini toping.



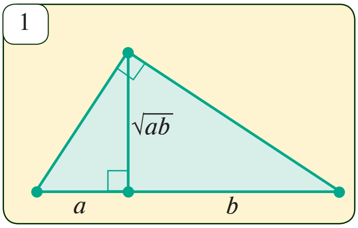
50.5*. Katetlarining nisbati 4:5 kabi bo'lgan to'g'ri burchakli uchburchak katetlarining gipotenuzadagi proyeksiyalari nisbatini toping.

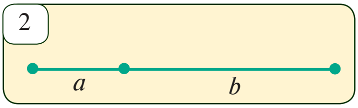
50.6*. Katetlarining nisbati 3:2 kabi bo'lgan to'g'ri burchakli uchburchak berilgan. Katetlarning gipotenuzasidagi proyeksiyalaridan biri ikkinchisidan 6 *cm* ga uzun. Uchburchak yuzini toping.

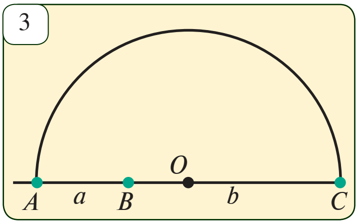
50.7. Katetlarining gipotenuzasidagi proyeksiyalari 2 *cm* va 18 *cm* bo'lgan to'g'ri burchakli uchburchak yuzini toping.

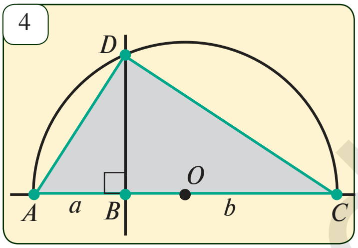
50.8*. ABC uchburchakda $\angle C=90^\circ$, CD — balandlik, CE — bissektrisa va $AE:EB=2:3$. a) $AC:BC$; b) $S_{ACE}:S_{BCE}$; d) $AD:BD$ nisbatlarni toping.

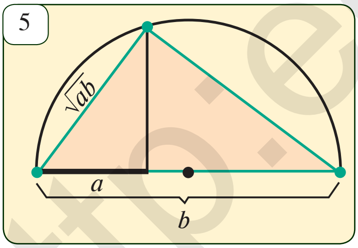
51 BERILGAN IKKITA KESMAGA O‘RTA PROPORSIONAL KESMANI YASASH











To‘g‘ri burchakli uchburchakning to‘g‘ri burchagidan tushirilgan balandligi gipotenuzani a va b kesmalarga bo‘lsa, balandlik $\sqrt{ab}$ ga teng bo‘lishini ko‘rgan edik (*1-rasm*).

Demak, berilgan ikki kesmaga o‘rta proporsional kesmani yasash uchun:

1) gipotenuzasining uzunligi $a+b$ ga teng (*2-rasm*);

2) to‘g‘ri burchagidan tushirilgan balandligi shu gipotenuzani a va b bo‘laklarga bo‘ladigan to‘g‘ri burchakli uchburchak yasash kifoya.

Buning uchun to‘g‘ri burchakli uchburchakka tashqi chizilgan aylana markazi gipotenuzaning o‘rtasida joylashganidan foydalanamiz (*3-rasm*).

Yasash:

1) To‘g‘ri chiziq chizamiz va unda $AB=a$ va $BC=b$ bo‘ladigan qilib A, B va C nuqtalarni belgilaymiz (*3-rasm*).

2) AC kesmaning o‘rtasi — O nuqtani topamiz. Markazi O nuqtada bo‘lgan AC diametrli yarim aylana yasaymiz *(3-rasm)*.

3) B nuqtadan AC to‘g‘ri chiziqqa perpendikular to‘g‘ri chiziq o‘tkazamiz *(4-rasm)*. Bu to‘g‘ri chiziq yarim aylanani D nuqtada kesib o‘tgan bo‘lsin. Unda ΔADC — to‘g‘ri burchakli uchburchak, $BD=\sqrt{ab}$ — biz yasashimiz zarur bo‘lgan kesma bo‘ladi.

Yasash bajarildi.

O‘rta proporsional kesmani yasashda to‘g‘ri burchakli uchburchakning kateti gipotenuza bilan shu katetning gipotenuzadagi proyeksiyasi orasida o‘rta proporsional ekanligidan foydalanish ham mumkin *(5-rasm)*.

Masala va topshiriqlar

51.1. Uzunliklari a va b bo‘lgan kesmalar berilgan. Uzunligi $\sqrt{ab}$ bo‘lgan kesmani yasang.

51.2. Uzunligi a va b ga teng kesmalar berilgan. Pifagor teoremasidan foydalanib, uzunligi a) $\sqrt{a^2+b^2}$; b) $\sqrt{a^2-b^2}$ bo‘lgan kesmalarni yasang.

51.3. Uzunligi 1 ga teng kesma berilgan. Uzunligi a) $\sqrt{2}$; b) $\sqrt{3}$; d) $\sqrt{5}$; e) $\sqrt{6}$; f) $\sqrt{18}$; g) $\sqrt{30}$ bo‘lgan kesmalarni yasang.

51.4. 6-rasmdagi ma’lumotlar asosida ABC uchburchakning yuzini toping.

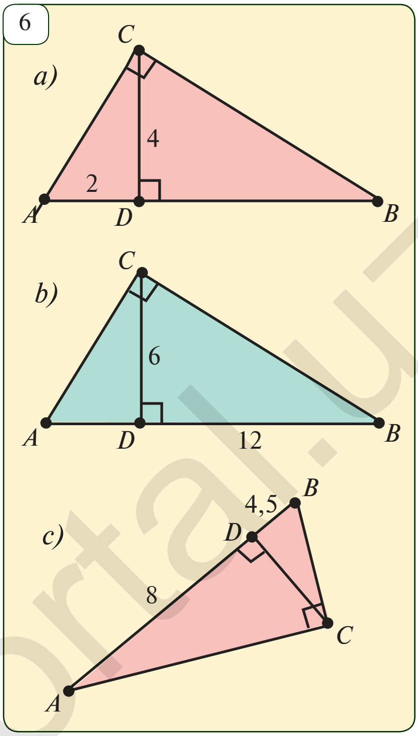
51.5. Aylanadagi C nuqtadan AB diametrga CD perpendikular tushirilgan. Agar $CD=12$ cm, $AD=24$ cm bo'lsa, doira yuzini toping.

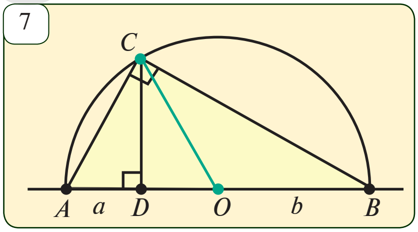
51.6. Oldingi masaladagi ABC uchburchak yuzini toping.

51.7. To'g'ri burchakli uchburchak to'g'ri burchagining bissektrisasi gipotenuzani 5:3 kabi nisbatda bo'ladi. To'g'ri burchak uchidan tushirilgan balandlikning gipotenuzadan ajratgan kesmalari nisbatini toping.

51.8. Radiusi 8 cm ga teng doiraga bir burchagi 30° bo'lgan to'g'ri burchakli uchburchak ichki chizilgan. Doiraning uchburchakdan tashqaridagi qismi 3 ta segmentdan iborat. Ana shu segmentlar yuzlarini toping.

51.9*. 7-rasmda $AD=a$, $DB=b$, demak, $OC=\frac{a+b}{2}=$ (O— aylana markazi). Rasmdan foydalanib, $\frac{a+b}{2} \geq \sqrt{ab}$ tengsizlikni isbotlang.

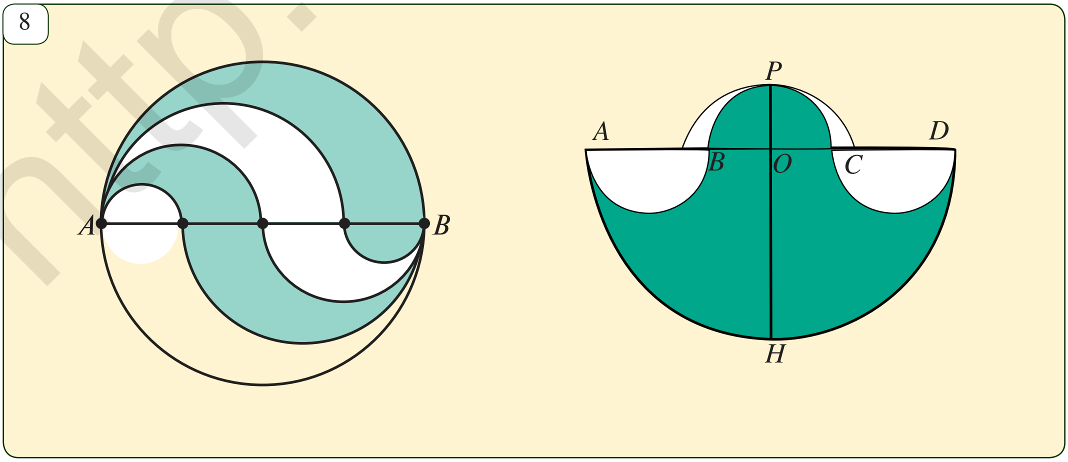




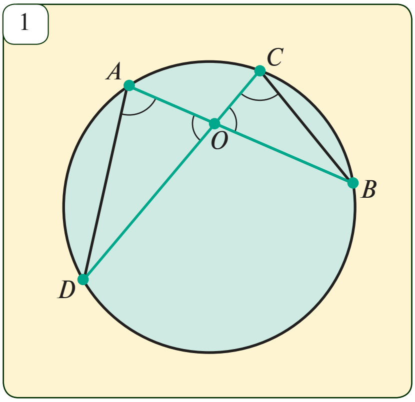
Qiziqarli masalalar

1. Aylananing AB diametri to'rtta teng bo'lakka bo'lindi va 8-rasmda ko'rsatilgandek yarim aylanalar yasaldi. Agar $AB=d$ bo'lsa, rasmda bo'yab ko'rsatilgan har bir shakl yuzini hisoblang.

2. 9-rasmda AB va CD kesmalar teng. O nuqta AD kesmaning o'rtasi. AB, CD, AD va BC kesmalar yarimdoiralarning diametri. Bu yarimdoiralar bilan chegaralangan shaklning yuzi diametri PH ga teng doira yuziga tengligini isbotlang. Bu yerda PH kesma AD kesmaning o'rtasi O nuqtaga o'rkazilgan perpendikular.



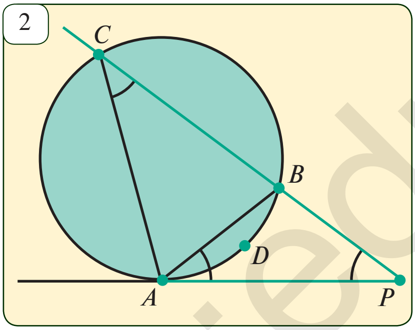
***1-teorema.** Aylananing AB va CD vatarlari O nuqtada kesishsa, $AO \cdot OB = CO \cdot OD$ tenglik o‘rinli bo‘ladi.*



Isbot. AB va CD vatarlar (*1-rasm*) ko‘rsatilgan tartibda joylashgan bo‘lsin. Uchlarini AD va BC vatarlar bilan tutashtiramiz. Shunda BAD va BCD burchaklar bitta yoyga tiraladi, demak, $\angle BAD = \angle BCD$. Yana ravshanki, $\angle AOD = \angle BOC$. Bu ikki tenglikdan, BB alomatga ko‘ra, AOD va COB uchburchaklarning o‘xshashligi kelib chiqadi. O‘xshash uchburchaklar mos tomonlari esa proporsional: $\frac{OD}{OB} = \frac{AO}{CO}$ yoki $AO \cdot OB = CO \cdot OD$.

Teorema isbotlandi.

***2-teorema.** Aylana tashqi sohasidagi P nuqtadan aylanaga PA urinma (A – urinish nuqtasi) va aylanani B va C nuqtalarda kesib o‘tuvchi to‘g‘ri chiziq o‘tkazilgan bo‘lsa, $PA^2 = PB \cdot PC$ bo‘ladi.*



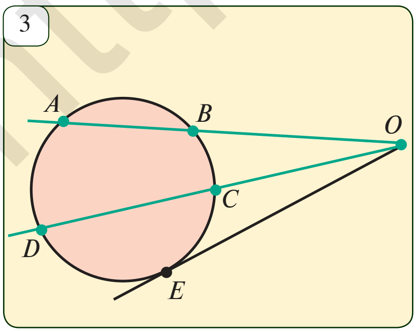
Isbot. ABP va CPA uchburchaklarni qaraymiz (*2-rasm*). Unda,

$\angle C = \frac{\smile ADB}{2} = \angle BAP$ hamda $\angle P$ — bu uchburchaklar uchun umumiy burchak. Demak, ABP va CPA uchburchaklar ikki burchagi bo‘yicha o‘xshash.

Bundan, $\frac{PA}{PC} = \frac{PB}{PA}$ yoki $PA^2 = PB \cdot PC$.

Teorema isbotlandi.

Masala. A, B, C va D nuqtalar aylanani AB, BC, CD va AD yoylarga ajratadi. Agar AB va DC nurlar O nuqtada kesishsa, u holda $OA \cdot OB = OC \cdot OD$ tenglik o‘rinli bo‘lishini isbotlang.



Yechish. Masala shartiga mos chizma chizamiz (*3-rasm*) va O nuqtadan OE urinma o‘tkazamiz. Unda, 2-teoremaga ko‘ra,

$$\left.\begin{aligned} OB \cdot OA &= OE^2 \\ OC \cdot OD &= OE^2 \end{aligned}\right\} \Rightarrow OA \cdot OB = OC \cdot OD.$$

Masala va topshiriqlar

52.1. 4-rasmda x bilan belgilangan noma'lum kesmani toping.

52.2. A nuqtadan aylanaga AB urinma (B — urinish nuqtasi) va aylanani C va D nuqtalarida kesadigan kesuvchi o'tkazilgan. Agar
a) $AB=4$ cm, $AC=2$ cm bo'lsa, AD kesmani;
b) $AB=5$ cm, $AD=10$ cm bo'lsa, AC kesmani;
d) $AC=3$ cm, $AD=2{,}7$ cm bo'lsa, AB kesmani toping.

52.3. Aylanaga $ABCD$ to'rtburchak ichki chizilgan. AB va DC nurlar O nuqtada kesishadi. Agar
a) $AO=10$ dm, $BO=6$ dm, $DO=15$ dm bo'lsa, OC kesmani;
b) $CD=10$ dm, $OD=8$ dm, $AB=4$ dm bo'lsa, OB kesmani toping.

52.4. Aylananing AB diametri va bu diametrga perpendikular CD vatari E nuqtada kesishadi. Agar $AE=2$ cm, $EB=8$ cm bo'lsa, CD vatarni toping.

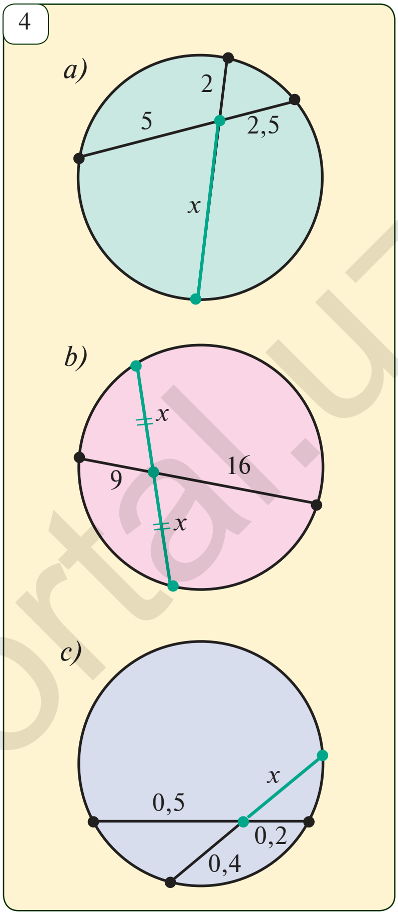
52.5. AB va CD kesmalar O nuqtada kesishadi. Agar $AO \cdot OB=BO \cdot OD$ bo'lsa, A, B, C va D nuqtalarning bir aylanada yotishini isbotlang.

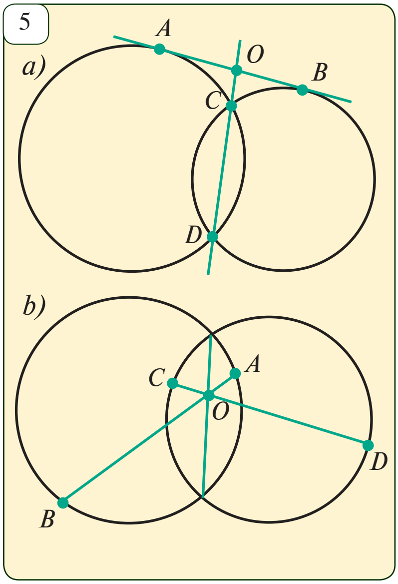
52.6. Radiusi 13 dm bo'lgan aylana markazidan 5 dm uzoqlikda P nuqta olingan. P nuqtadan uzunligi 25 dm bo'lgan AB vatar o'tkazilgan. AP va PB kesmalarni toping.

52.7. 3-rasmda $AO \cdot OB=CO \cdot OD$ tenglikni AOD va BOC uchburchaklarning o'xshash ekanligidan foydalanib isbotlang.

52.8*. 5-rasmlardagi ma'lumotlar asosida $AO \cdot OB= =CO \cdot OD$ tenglikni isbotlang.

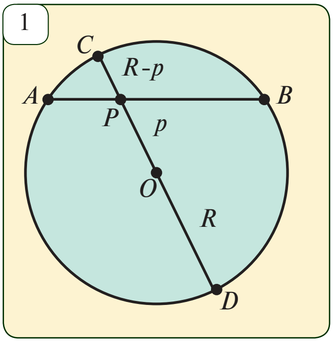
52.9*. Ikki aylana C nuqtada urinadi. AB to'g'ri chiziq birinchi aylanaga A nuqtada, ikkinchi aylanaga esa B nuqtada urinadi. $\angle ACB=90°$ ekanligini isbotlang.





Oldingi darsda aylana kesuvchilari va vatarlarining xossalarini isbotlagan edik. Endi shu xossalarning ayrim xususiy hollari bilan tanishamiz.

1



1-masala. R radiusli aylananing ichki sohasidagi P nuqta aylana markazidan p masofada joylashgan bo‘lsin. Unda P nuqtadan o‘tuvchi ixtiyoriy AB vatar uchun

$$AP \cdot PB = R^2 - p^2 \qquad (1)$$

tenglik o‘rinli bo‘lishini isbotlang.

Yechish. P nuqta orqali aylananing CD diametrini o‘tkazamiz. Unda, $PC = R - p$, $PD = R + p$ (*1-rasm*). Kesuvchi vatarlar haqidagi teoremaga ko‘ra,

$$AP \cdot PB = CP \cdot PD = (R-p)(R+p) = R^2 - p^2.$$

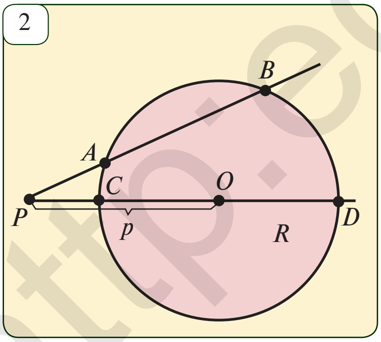
(1) tenglik isbotlandi.

2-masala. Radiusi 6 *cm* bo‘lgan aylananing O markazidan 4 *cm* uzoqlikda P nuqta olindi. P nuqta orqali AB vatar o‘tkazildi. Agar $AP = 2$ *cm* bo‘lsa, PB kesmani toping.

Yechish. Masala shartiga ko‘ra, $R = 6$ *cm*, $d = 4$ *cm*, $AP = 2$ *cm*. U holda (1) tenglikka ko‘ra, $2 \cdot PB = 6^2 - 4^2 = 36 - 16 = 20$. Bundan, $PB = 10$ *cm*.

Javob: $PB = 10$ *cm*.

2



3-masala. R radiusli aylananing tashqi sohasidagi P nuqta aylana markazidan p masofada joylashgan bo‘lsin. Unda P nuqta orqali o‘tuvchi va aylanani A va B nuqtalarda kesuvchi ixtiyoriy to‘g‘ri chiziq uchun

$$PA \cdot PB = p^2 - R^2 \qquad (2)$$

tenglik o‘rinli bo‘lishini isbotlang.

Yechishi. Aylananing O markazi orqali o‘tuvchi PO to‘g‘ri chiziq aylana bilan C va D nuqtalarda kesishsin (*2-rasm*). Unda, shartga ko‘ra, $PC = p - R$, $PD = p + R$. Aylana tashqi sohasidagi nuqtadan o‘tkazilgan kesuvchilar haqidagi teoremaga ko‘ra,

$$PA \cdot PB = PC \cdot PD = (p-R)(p+R) = p^2 - R^2.$$

Shunday qilib (2) tenglik isbotlandi.

4-masala. Radiusi 7 *cm* bo‘lgan aylananing markazidan 13 *cm* uzoqlikdagi P nuqtadan o‘tuvchi to‘g‘ri chiziq aylanani A va B nuqtalarda kesadi. Agar $PA=10$ *cm* bo‘lsa, AB vatarni toping.

Yechish. Shartga ko‘ra, $R=7$ *cm*, $p=13$ *cm*. U holda (2) formulaga ko‘ra,

$$PA \cdot PB = p^2 - R^2 = 13^2 - 7^2 = 169 - 49 = 120.$$

Bundan, $PB = \dfrac{120}{PA} = \dfrac{120}{10} = 12\ (cm)$. Demak,

$$AB = PB - PA = 12 - 10 = 2\ (cm).$$ *Javob:* 2 *cm*.

Masala va topshiriqlar

53.1. Radiusi 5 *cm* bo‘lgan aylana markazidan 3 *cm* uzoqlikda P nuqta olingan. AB vatar P nuqta orqali o‘tadi. Agar $PA=2$ *cm* bo‘lsa, AB vatar uzunligini toping.

53.2. Radiusi 5 *m* bo‘lgan aylana markazidan 7 *m* uzoqlikda P nuqta olingan. P nuqta orqali o‘tuvchi to‘g‘ri chiziq aylanani A va B nuqtada kesadi. Agar $PA=4\ m$ bo‘lsa, AB vatar uzunligini toping.

53.3. 3-rasmdagi ma’lumotlar asosida x bilan belgilangan kesmani toping (O — aylana markazi).

53.4. 4-rasmdan foydalanib, masalani yeching. Unda,

a) $PC=5\ dm$, $OD=7\ dm$, $AB=2\ dm$, $PA=?$

b) $PA=5\ dm$, $AB=4\ dm$, $PC=3\ dm$, $OD=?$

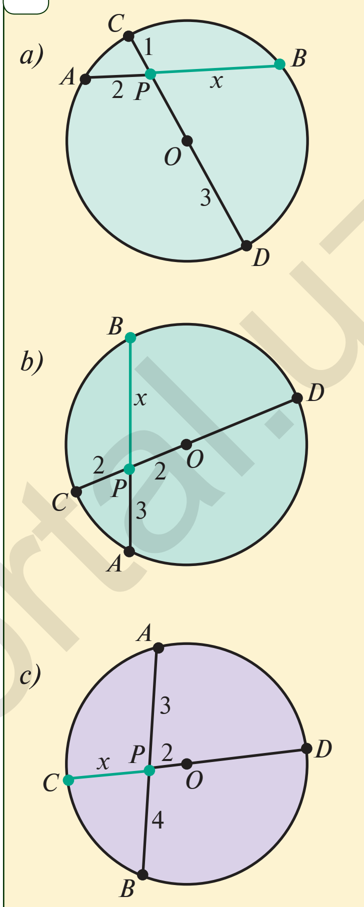
53.5. Aylananing $AB=7$ *cm* va $CD=5$ *cm* vatarlari P nuqtada kesishadi. Agar $CP:PD=2:3$ bo‘lsa, P nuqta AB vatarni qanday nisbatda bo‘ladi?

53.6. Aylananing C nuqtasidan AB diametrga CD perpendikular tushirilgan. Agar $AD=2$ *cm*, $DB=18$ *cm* bo‘lsa, CD kesmani toping.

53.7*. Aylanaga ichki chizilgan $ABCD$ to‘rtburchakning diagonallari K nuqtada kesishadi. Agar $AB=2$, $BC=1$, $CD=3$ va $CK:KA=1:2$ bo‘lsa, AD kesmani toping.

53.8*. Aylanaga ichki chizilgan $ABCD$ to‘rtburchakda $AB:DC=1:2$ va $BD:AC=2:3$ bo‘lsa, $DA:BC$ nisbatni toping.

3



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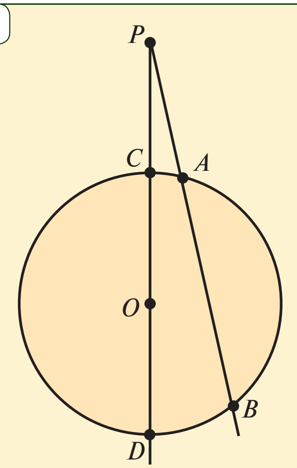


BILIMINGIZNI SINAB KO‘RING

I. Testlar

1. **To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi haqida noto‘g‘ri tasdiqni ko‘rsating:**
   A. Katetlaridan kichik;
   B. Uchburchakni ikkita o‘xshash uchburchaklarga ajratadi;
   D. Katetlarining gipotenuzadagi proyeksiyalari orasida o‘rta proporsional;
   E. Gipotenuzaning yarmiga teng.
2. ***AB* va *CD* vatarlar *O* nuqtada kesishadi. Noto‘g‘ri tasdiqni toping:**
   A. $\angle DAB = \angle DCB$; B. AOD va COB uchburchaklar o‘xshash;
   D. $AO \cdot OB = CO \cdot OD$; E. $AO = CO$.
3. **To‘g‘ri tasdiqni toping:**
   A. Teng kesmalarning proyeksiyalari ham teng bo‘ladi;
   B. Katta kesmaning proyeksiyasi katta bo‘ladi;
   D. Bir to‘g‘ri chiziqdagi teng kesmalarning proyeksiyalari teng bo‘ladi;
   E. Proyeksiya uzunligi proyeksiyalanuvchi kesma uzunligiga teng bo‘ladi.
4. **To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik uni ikkita uchburchakka ajratadi. Bu uchburchaklar:**
   A. Teng; B. Tengdosh; D. O‘xshash; E. Teng yonli.
5. **Uzunligi *a* va *b* bo‘lgan kesmalarning o‘rta proporsionali nimaga teng?**
   A. $a+b$; B. $\sqrt{ab}$; D. $\frac{a+b}{2}$; E. $a:b$.
6. ***ABCD* to‘rtburchak *O* markazli aylanaga ichki chizilgan. Noto‘g‘ri tasdiqni ko‘rsating:**
   A. $\Delta AOB \sim \Delta COD$; B. $\angle A + \angle C = \angle B + \angle D$;
   D. $AO \cdot OB = CO \cdot OD$; E. $AB \cdot CD = BC \cdot AD$.

II. Masalalar.

1. To‘g‘ri burchakli uchburchak katetlarining nisbati 3:4 ga teng. Bu uchburchakning gipotenuzasi 50 *cm*. Uchburchakning to‘g‘ri burchagi uchidan tushirilgan balandligi gipotenuzadan qanday uzunlikdagi kesmalar ajratadi?
2. Aylananing AB va CD vatarlari E nuqtada kesishadi. Agar $AE = 5$ *cm*, $BE = 2$ *cm* va $EC = 2{,}5$ *cm* bo‘lsa, ED ni toping.
3. Radiusi 6 *m* bo‘lgan aylananing markazidan 10 *m* uzoqlikda K nuqta olindi va K nuqtadan aylanaga urinma o‘tkazildi. Urinmaning urinish nuqtasi P bilan K nuqta orasidagi masofani toping.
4. ABC uchburchakda $\angle C = 90^\circ$ va CD balandlik 4,8 *dm*. Agar $AD = 3{,}6$ *dm* bo‘lsa, AB tomonni toping.
5. Aylananing AB va CD vatarlari O nuqtada kesishadi. Agar $AO = 6$, $OB = 4$ va $CO = 3$ bo‘lsa, OD kesmani toping.

6. Aylanada A, B, C, D nuqtalar belgilangan, BA va CD nurlar O nuqtada kesishadi. Agar $OA=5$, $AB=4$, $OD=6$ bo‘lsa, DC vatarni toping.

7. Aylanaga B nuqtada urinuvchi to‘g‘ri chiziq ustida A nuqta olindi. Agar $AB=12$ va A nuqtadan aylanagacha bo‘lgan eng qisqa masofa 8 bo‘lsa, aylana radiusini toping.

8. Yarim aylanadagi C nuqtadan AB diametrga tushirilgan CD perpendikular AB kesmada 4 va 9 ga teng kesmalar ajratadi. CD kesmani toping.

9. To‘g‘ri burchakli uchburchakning balandligi gipotenuzani 3 *dm* va 12 *dm* ga teng kesmalarga bo‘ladi. Uchburchak yuzini toping.

10. Radiusi 5 *cm* bo‘lgan O markazli aylananing AB vatarida D nuqta olingan. Agar $AD=2$ *cm*, $DB=4{,}5$ *cm* bo‘lsa, OD kesmani toping.

11. Radiusi 5 *m* bo‘lgan O markazli aylanani A va B nuqtalarda kesuvchi to‘g‘ri chiziqda P nuqta olindi. Agar $PA=5$ *m*, $AB=2{,}8$ *m* bo‘lsa, OP masofani toping.

12. To‘rtta parallel to‘g‘ri chiziq berilgan. Ular burchak tomonlarini A va A_1, B va B_1, C va C_1 hamda D va D_1 nuqtalarda kesadi. Bunda A, B, C, D nuqtalar burchakning bitta tomonida yotadi. Agar $AB=8$, $CD=12$ va $C_1D_1=9$ bo‘lsa, A_1B_1 kesmani toping.

13. Aylana burchakka ichki chizilgan. Agar burchak uchidan aylanagacha bo‘lgan masofa radiusga teng bo‘lsa, burchak kattaligini toping.

14. Aylanaga AB diametrning B uchidan BC urinma va AC kesuvchi o‘tkazilgan. AC aylana bilan D nuqtada kesishadi. Agar $AD=DC$ bo‘lsa, CBD burchakni toping.

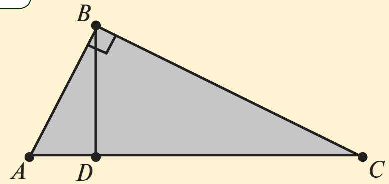
15. To‘g‘ri burchakli uchburchakning katetlari nisbati 2:3 kabi. Uchburchakning gipotenuzasiga tushirilgan balandlik uni ikkita uchburchakka bo‘ladi. Ular yuzlarining nisbatini toping.

III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Aylana tashqarisidagi nuqtadan aylanaga urinma o‘tkazilgan. Bu nuqtadan aylanagacha bo‘lgan eng qisqa masofa 2 *cm* ga, urinish nuqtasigacha bo‘lgan masofa esa 6 *cm* ga teng. Aylananing radiusini toping.

2. ΔABC to‘g‘ri burchakli, $AD=9$ *dm*, $DC=16$ *dm* bo‘lsa, shu uchburchakka ichki chizilgan aylana radiusini hisoblang.(1-rasm)

1



3. Nuqtadan to‘g‘ri chiziqqa ikkita og‘ma o‘tkazilgan. Agar og‘malar 1:2 nisbatda bo‘lib, ularning proyeksiyalari 1 *m* va 7 *m* bo‘lsa, og‘malarning uzunliklarini toping.

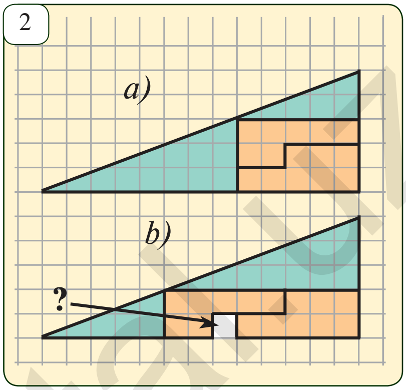
4.* (Qo‘shimcha masala) PQ va undan uzun ET kesmalar berilgan. Shunday $ABCD$ to‘rtburchak yasangki, $AB=BC=PQ$; $BD=ET$

bo‘lib, diagonallari kesishadigan O nuqta uchun $AO \cdot OC = BO \cdot OD$ tenglik o‘rinli bo‘lsin.

Qiziqarli masala

Uchburchak 2-a rasmda ko‘rsatilganidek qilib to‘rtta bo‘lakka bo‘lingan va 2-b rasmda ko‘rsatilganidek qilib qayta yig‘ilgan. Ayting-chi, ortiqcha kvadrat qayerdan paydo bo‘lib qoldi?

2

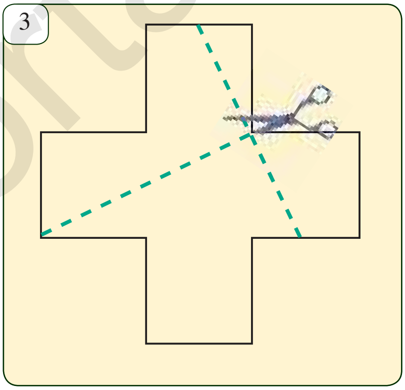


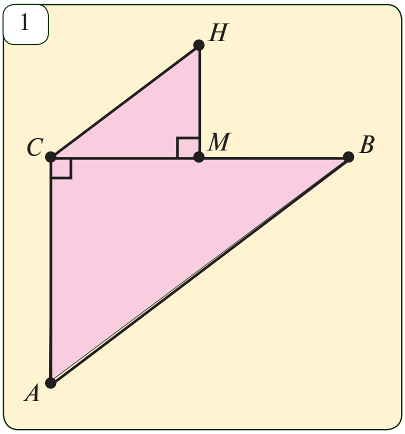
Yunon xochi

Eramizdan avvalgi 500-yillarda paydo bo‘lgan bu shaklni (3-rasm) hayotning ramzi sifatida non ustiga chizganlar.

Bu shaklni qalin qog‘ozga chizib olib, uni rasmda ko‘rsatilgan chiziqlar bo‘ylab qirqing. Hosil bo‘lgan bo‘laklardan kvadrat yasash mumkinligiga ishonch hosil qiling.

3





I. Namunaviy nazorat ishi

1. $ABCD$ parallelogrammda $\angle A=45°$, $AD=4$. Parallelogramm AB tomonining davomiga $\angle PDA=90°$ ga teng bo‘ladigan BP kesma qo‘yildi. BC va PD kesmalar T nuqtada kesishadi, bunda $PT:TD=3:1$.

a) $\Delta BPT \sim \Delta CDT$ ekanligini isbotlang, bu uchburchaklar yuzlari nisbatini toping.

b) $ABCD$ parallelogramm yuzini toping.

d) AB va TD kesmalarning o‘rtalarini tutashtiruvchi kesmaning uzunligini toping.

e) $\overline{AB}$ vektorni $\overline{CA}$ va $\overline{TB}$ vektorlar orqali ifodalang.

f) CAD burchakning sinusini toping.

2. *(Qo‘shimcha)* 1-rasmda $BC\perp AC$, $MH\perp BC$, $2MC=BC$, $MH=0,5AC$ bo‘lsa, $AB||CH$ ekanligini isbotlang.

II. Nazorat ishi uchun namunaviy testlar

1. Agar to‘g‘ri burchakli uchburchakning balandligi gipotenuzasini 6 *cm* va 54 *cm* kesmalarga ajratsa, bu uchburchakning yuzini toping:
A) 648 cm^2; B) 324 cm^2; D) 1080 cm^2; E) 540 cm^2.

2. C nuqtadan o‘tkazilgan bir kesuvchi aylanani A va B, ikkinchisi esa D va E nuqtalarda kesadi. Agar $CA=18$ *cm*, $CB=8$ *cm*, $CD=8$ *cm* bo‘lsa, DE kesma uzunligini toping:
A) 17 *cm*; B) 1 *cm*; D) 9 *cm*; E) to‘g‘ri javob ko‘rsatilmagan.

3. Agar $A(-5;2\sqrt{3})$, $B(-4;2)$, $C(-2;\sqrt{3})$, $D(0;2)$ bo‘lsa, $ABCD$ to‘rtburchakning diagonallari orasidagi burchakni toping:
A) 30°; B) 60°; D) 90°; E) to‘g‘ri javob ko‘rsatilmagan.

4. Agar parallelogrammning diagonallari 10 *cm* va $8\sqrt{2}$ *cm* ga teng va ular orasidagi burchak 45° bo‘lsa, parallelogrammning tomonlarini toping:
A) $\sqrt{17}$ *cm* va $\sqrt{97}$ *cm*; B) 5 *cm* va 6 *cm*;
D) $\sqrt{34}$ *cm* va $\sqrt{63}$ *cm*; E) to‘g‘ri javob ko‘rsatilmagan.

5. Radiusi 8 *cm* bo‘lgan aylanaga ichki chizilgan muntazam oltiburchakning yuzini toping:
A) $48\sqrt{3}$ cm^2; B) $192\sqrt{3}$ cm^2; D) $96\sqrt{2}$; E) to‘g‘ri javob yo‘q.

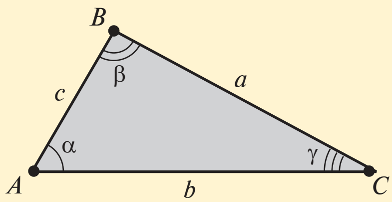
6. Markaziy burchagi 140°, yuzi $31,5\pi$ cm^2 bo‘lgan doiraviy sektorning radiusini aniqlang:

A) 9 *cm*; B) 18 *cm*; D) 9π *cm*; E) to‘g‘ri javob ko‘rsatilmagan.

7. Asosining uzunligi 15 *cm* bo‘lgan uchburchak asosiga parallel kesmao‘tkazilgan. Agar hosil bo‘lgan trapetsiyaning yuzi uchburchak yuzining $\frac{3}{4}$ qismini tashkil qilishi ma’lum bo‘lsa, kesmaning uzunligini toping:
   A) 6,5; B) 7; D) 7,5; E) 5.
8. Yon tomoni $2\sqrt{39}$ *cm* bo‘lgan teng yonli uchburchak balandligining asosiga nisbati 3:4 ga teng bo‘lsa, uchburchakning yuzini toping:
   A) 260; B) 245; D) 310; E) 72.
9. $\bar{a}(4;4\sqrt{3})$ va $\bar{b}(8\sqrt{3};8)$ vektorlar orasidagi burchakni toping:
   A) 45°; B) 90°; D) 30°; E) 60°.
10. Teng yonli trapetsiyaning asoslari 10 *cm* va 16 *cm*, yon tomoni esa 5 *cm*. Trapetsiyaning yuzini toping:
    A) 45; B) 50; D) 48; E) 52.
11. To‘g‘ri burchakli uchburchakning gipotenuzasi 13 *cm* bo‘lib, katetlaridan biri ikkinchisidan 7 *cm* katta. Uchburchakning yuzini toping:
    A) 30 cm^2; B) 25 cm^2; D) 45 cm^2; E) 40 cm^2.
12. Tomoni 5 *cm* bo‘lgan rombning bitta diagonali 6 *cm* ga teng. Rombning yuzini toping:
    A) 24 cm^2; B) 30 cm^2; D) 29 cm^2; E) 40 cm^2.
13. Diagonali $6\sqrt{2}$ bo‘lgan kvadratga ichki chizilgan aylana uzunligini toping:
    A) 10π; B) 8π; D) 9π; E) 6π.
14. Tomoni $6\sqrt{2}$ *cm* bo‘lgan kvadratga tashqi chizilgan doira yuzini toping:
    A) 9π; B) 12π; D) 15π; E) 18π.
15. Balandliklari 4 *cm* va 6 *cm* bo‘lgan parallelogramm yuzi 36 cm^2 ga teng. Uning perimetrini toping:
    A) 26 *cm*; B) 30 *cm*; D) 29 *cm*; E) 36 *cm*.
16. Perimetri 30 *cm* bo‘lgan parallelogrammning tomonlari 2:3 nisbatda. Agar parallelogrammning o‘tkir burchagi 30° bo‘lsa, uning yuzini toping:
    A) 26 cm^2; B) 27 cm^2; D) 29 cm^2; E) 30 cm^2.
17. Agar ABC uchburchakda $AB=6\sqrt{3}$ *cm*, $BC=12$ *cm* va $\angle C=60°$ bo‘lsa, uchburchakning A burchagini toping:
    A) 45°; B) 90°; D) 30°; E) 60°.

PLANIMETRIYAGA OID ASOSIY TUSHUNCHA VA MA'LUMOTLAR

1 **UCHBURCHAKLAR**



1°. Asosiy tushunchalar

Tekislikda bir to'g'ri chiziqda yotmagan uchta nuqta berilgan bo'lsin. Shu nuqtalarning har ikkitasini kesmalar bilan tutashtiramiz. Hosil bo'lgan shakl *uchburchak* deyiladi. Nuqtalar uchburchakning *uchlari*, kesmalar esa *tomonlari* deyiladi. Belgilanishi: A, B, C – uchlar, a, b, c – tomonlar (*1-rasm*).

Uchburchak uchta ichki burchakka ega: $\angle BAC$, $\angle CBA$, $\angle ACB$. Belgilanishi: α, β, γ.

Mediana — uchburchak uchini uning qarshisidagi tomon o'rtasi bilan tutashtiruvchi kesma. Uchburchakda 3 ta mediana bo'lib, ular m_a, m_b, m_c kabi belgilanadi.

Bissektrisa — uchburchak uchini uning qarshisidagi tomon bilan tutashtiruvchi va shu uchdagi burchak bissektrisasida yotuvchi kesma. Uchburchakda uchta bissektrisa bo'lib, ular l_a, l_b, l_c kabi belgilanadi.

Balandlik — uchburchak uchidan uning qarshisidagi tomon yotgan to'g'ri chiziqqa tushirilgan perpendikular.

Uchburchakda uchta balandlik bo'lib, ular h_a, h_b, h_c kabi belgilanadi.

O'rta chiziq — ikki tomon o'rtalarini tutashtiruvchi kesma.

O'rta chiziqlar soni ham 3 ta.

Perimetr — uchala tomon uzunliklari yig'indisi. Belgilanishi: P.

Uchburchaklar tomonlariga qarab uch turga ajratiladi:

a) teng tomonli ($a=b=c$); b) teng yonli (a, b, c larning qaysidir ikkisi teng);

d) turli tomonli (a, b, c larning hech qaysi ikkisi teng emas).

Uchburchakning uchala tomoniga urinib o'tuvchi aylana unga ichki chizilgan *aylana* deyiladi (bunday aylana mavjud va yagona). Ichki chizilgan aylana radiusi r orqali belgilanadi.

Uchburchakning uchala uchidan o'tuvchi aylana unga *tashqi chizilgan aylana* deyiladi va uning radiusi R orqali belgilanadi (bunday aylana mavjud va yagona).

2°. Asosiy munosabatlar

1) $\alpha+\beta+\gamma=180°$. Uchburchak ichki burchaklari yig'indisi 180° ga teng.

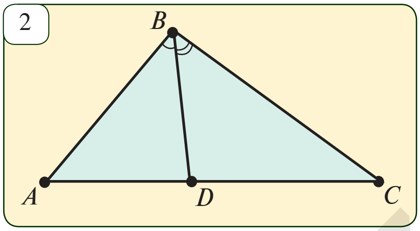
2) Uchala mediana bir nuqtada kesishadi. Bu nuqta medianani 2:1 nisbatda bo'ladi. Mediana uchburchakni ikkita yuzlari teng uchburchaklarga ajratadi. Medianalar uzunliklari $m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$; $m_b=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}$; $m_c=\frac{1}{2}\sqrt{2a^2+2b^2-c^2}$ formulalardan topiladi.

3) Uchala bissektrisa bir nuqtada kesishadi. Bu nuqta ichki chizilgan aylana markazi bo'ladi. Bissektrisa o'zi tushirilgan tomonni qolgan tomonlarga proporsional bo'laklarga ajratadi (*2-rasm*).

BD bissektrisa bo'lsa, $\frac{AB}{AD}=\frac{BC}{DC}$.

$$l_a=\frac{2\sqrt{bc}}{b+c}\sqrt{p(p-a)};\quad l_b=\frac{2\sqrt{ac}}{a+c}\sqrt{p(p-b)};$$

$$l_c=\frac{2\sqrt{ab}}{a+b}\sqrt{p(p-c)},\qquad p=(a+b+c)$$

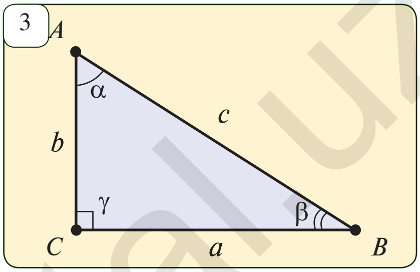


Bissektrisa uzunliklarini ushbu formulalardan topish mumkin.

4) Uchburchak balandliklari yoki ularning davomlari bir nuqtada kesishadi.

Balandlik uzunliklarini

$$h_a=\frac{2S}{a};\qquad h_b=\frac{2S}{b};\qquad h_c=\frac{2S}{c}$$



formulalardan topish mumkin. Bu yerda S — uchburchak yuzi.

5) Uchburchak tomonlarining o‘rta perpendikulari bir nuqtada kesishadi. Bu nuqta uchburchakka *tashqi chizilgan aylana markazi* bo‘ladi.

6) Uchburchakning *o‘rta chizig‘i* uchinchi tomonga parallel va uning yarmiga teng.

7) Sinuslar teoremasi:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}=2R.$$

8) Kosinuslar teoremasi:

$$a^2=b^2+c^2-2bc\cos\alpha,\quad b^2=a^2+c^2-2ac\cos\beta,\quad c^2=a^2+b^2-2ab\cos\gamma$$

9. Uchburchak yuzini hisoblash formulalari:

$$S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c;\qquad S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ac\sin\beta;$$

10. Geron formulasi:

$$S=\sqrt{p(p-a)(p-b)(p-c)},\qquad p=\frac{a+b+c}{2};\qquad S=\frac{abc}{4R},\qquad S=pr.$$

3°. Muhim xususiy hollar

a) *To‘g‘ri burchakli uchburchak* (*3-rasm*).

$\angle\gamma=90^\circ$, $\alpha+\beta=90^\circ$, AC va BC — katetlar, AB — gipotenuza. Pifagor teoremasi: $a^2+b^2=c^2$.

$$S=\frac{1}{2}ab;\qquad R=\frac{c}{2};\qquad r=\frac{a+b+c}{2};$$

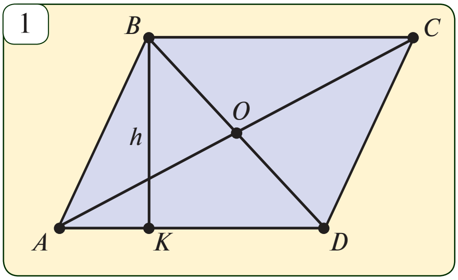
$$\frac{a}{c}=\sin\alpha;\qquad \frac{a}{c}=\cos\beta;\qquad \frac{b}{c}=\sin\beta;\qquad \frac{b}{c}=\cos\alpha.$$

$$\frac{a}{b}=\mathrm{tg}\alpha;\qquad \frac{a}{b}=\mathrm{ctg}\beta;\qquad \frac{b}{a}=\mathrm{ctg}\alpha;\qquad \frac{b}{a}=\mathrm{tg}\beta.$$

b) *Teng tomonli uchburchak*

$$\alpha=\beta=\gamma=60^\circ,\qquad S=\frac{a^2\sqrt{3}}{4},\qquad r=\frac{a\sqrt{3}}{6},\qquad R=\frac{a\sqrt{3}}{3}.$$

TO‘RTBURCHAKLAR



1°. Parallelogramm

Qarama-qarshi tomonlari parallel bo‘lgan to‘rtburchak *parallelogramm* deyiladi (*1-rasm*).

Qo‘shni bo‘lmagan uchlarni tutashtiruvchi kesma *diagonal* deyiladi.

AB va CD; AD va BC parallel tomonlar;

BD va AC diagonallar.

Asosiy xossalar va munosabatlar

1) Diagonallar kesishish nuqtasi parallelogrammning simmetriya markazi bo‘ladi.

2) Qarama-qarshi tomonlarning uzunliklari o‘zaro teng:

$$AB=CD \quad \text{va} \quad AD=BC.$$

3) Parallelogrammning qarama-qarshi burchaklari o‘zaro teng:

$$\angle BAD=\angle BCD \quad \text{va} \quad \angle ABC=\angle ADC.$$

4) Qo‘shni burchaklar yig‘indisi 180° ga teng.

5) Diagonallar kesishish nuqtasida teng ikkiga bo‘linadi: $BO=OD$ va $AO=OC$

6) Tomonlari kvadratlarining yig‘indisi diagonallari kvadratlarining yig‘indisiga teng:

$$AB^2+BC^2+CD^2+DA^2=AC^2+BD^2 \quad \text{yoki} \quad 2(AB^2+BC^2)=AC^2+BD^2.$$

7) Parallelogramm yuzi: a) $S=ah_a$, bu yerda: $a=AD$ tomon, $h_a=BK$ — balandlik; b) $S=ab\sin\alpha$, bu yerda: $b=AB$ — tomon, $\alpha=\angle BAD$ — AB va AD tomonlar orasidagi burchak.

2°. Romb

Barcha tomonlari o‘zaro teng bo‘lgan parallelogramm *romb* deyiladi.

Parallelogramm uchun o‘rinli bo‘lgan barcha xossalar romb uchun ham o‘rinli.

Rombning qo‘shimcha xossalari.

1) Romb diagonallari o‘zaro perpendikular.

2) Romb diagonallari ichki burchaklarning bissektrisalari bo‘ladi.

3) Romb yuzi $S=\frac{1}{2}d_1d_2$, bu yerda: d_1, d_2 — romb diagonallari.

3°. To‘g‘ri to‘rtburchak

Barcha burchaklari 90° ga teng bo‘lgan parallelogramm *to‘g‘ri to‘rtburchak* deyiladi.

1) To‘g‘ri to‘rtburchak diagonallari o‘zaro teng.

2) To‘g‘ri to‘rtburchak yuzi $S=ab$, bu yerda: a va b — to‘g‘ri to‘rtburchakning qo‘shni tomonlari.

4°. Kvadrat

Barcha tomonlari o‘zaro teng bo‘lgan to‘g‘ri to‘rtburchak *kvadrat* deyiladi.

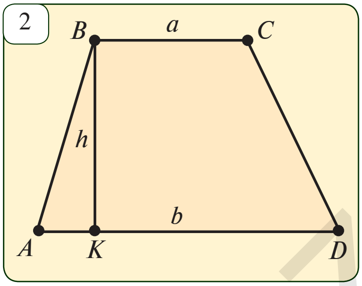
Romb va to‘g‘ri to‘rtburchaklar uchun o‘rinli bo‘lgan barcha xossalar kvadrat uchun ham o‘rinli.

Agar a — kvadrat tomoni, d esa diagonali bo‘lsa: $S=a^2$; $\quad S=\frac{d^2}{2}$; $\quad d=a\sqrt{2}$.

5°. Trapetsiya

Asoslar deb ataluvchi ikki tomoni o‘zaro parallel va yon tomonlar deb ataluvchi, qolgan ikki tomoni esa parallel bo‘lmagan to‘rtburchak *trapetsiya* deyiladi.

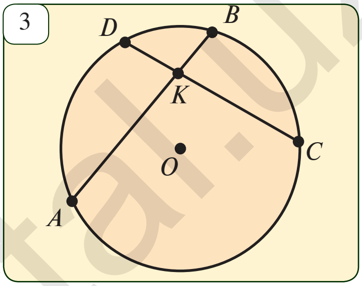
Yon tomonlar o‘rtalarini tutashtiruvchi kesma trapetsiyaning *o‘rta chizig‘i* deyiladi.



Asosiy xossalar

1) Trapetsiya o‘rta chizig‘i asoslarga parallel va asoslar yig‘indisining yarmiga teng bo‘ladi.

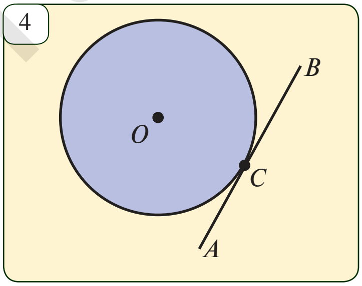
2) Trapetsiya yuzi $S=\frac{a+b}{2}h$, bu yerda a va b — asoslar, h esa balandlik (*2-rasm*).



AYLANA, DOIRA

1°. Musbat son R va tekislikda O nuqta berilgan bo‘lsin. O nuqtadan R masofada joylashgan nuqtalardan tashkil topgan shakl *aylana* deyiladi. O nuqta *aylana markazi*, markaz bilan aylanadagi nuqtani tutashtiruvchi kesma *radius*, R son esa *radius uzunligi* deyiladi. Aylanadagi ikki nuqtani tutashtiruvchi kesma *vatar*, markazdan o‘tuvchi vatar esa *diametr* deyiladi.

Tekislikning aylana bilan chegaralangan chekli qismi *doira* deb ataladi.



Asosiy munosabatlar

1) $D=2R$, bu yerda: D — diametr uzunligi.
2) $l=2\pi R$ — aylana uzunligi.
3) $S=\pi R^2$ — doira yuzi.
4) AB va CD vatarlar K nuqtada kesishsa (*3-rasm*), $AK\cdot KB=CK\cdot KD$ munosabat bajariladi.
5) Vatarni teng ikkiga bo‘luvchi diametr shu vatarga perpendikulardir.
6) Teng vatarlar markazdan teng masofalarda joylashgan va aksincha, markazdan teng masofada joylashgan vatarlar o‘zaro teng.

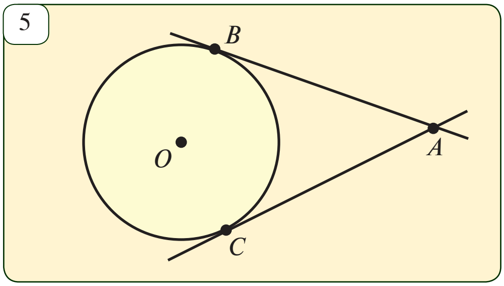
2°. Urinma

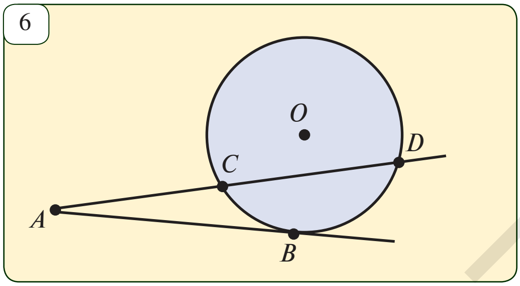
Aylana (yoki doira) bilan yagona umumiy nuqtaga ega bo‘lgan to‘g‘ri chiziq *urinma* deyiladi. Nuqta esa *urinish nuqtasi* deyiladi (*4-rasm*).

Aylana bilan 2 ta umumiy nuqtaga ega bo‘lgan to‘g‘ri chiziq *kesuvchi* deb ataladi.

Urinmaning xossalari

1) Urinish nuqtasiga o‘tkazilgan radius urinmaga perpendikulardir.
2) Doira tashqarisidagi nuqtadan shu doiraga ikkita urinma o‘tkazish mumkin. Bu urinmalarning kesmalari o‘zaro teng (*5-rasm*): $AB=AC$.
3) Agar AC kesuvchi bo‘lib, aylanani C va D nuqtalarda kesib o‘tsa, AB esa urinma bo‘lsa, $AB^2=AD\cdot AC$ tenglik o‘rinli (*6-rasm*).





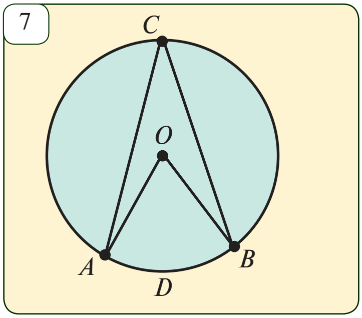
3°. Markaziy va ichki chizilgan burchaklar

Aylanadagi ikki nuqta yordamida aylana ikki bo‘lakka ajraladi. Bu bo‘laklar *yoylar* deb ataladi. Belgilanishi: ADB; ACB.

AOB burchak ADB yoyga tiralgan *markaziy burchak* (*7-rasm*), ACB burchak esa ADB yoyga tiralgan va aylanaga *ichki chizilgan burchak* deyiladi. Bu burchaklar orasida

$$\angle ACB = \frac{1}{2}\angle AOB$$

munosabat o‘rinli.



Xususan, yarim aylanaga tiralgan ichki burchak to‘g‘ri burchak bo‘ladi (*8-rasm*). Bitta yoyga tiralgan aylanaga ichki chizilgan burchaklar o‘zaro teng bo‘ladi.

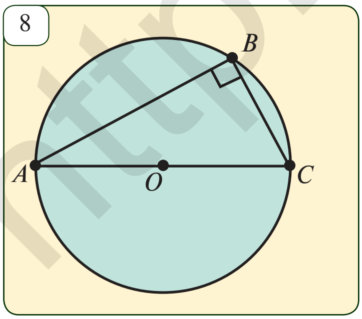
4°. Sektor va segment

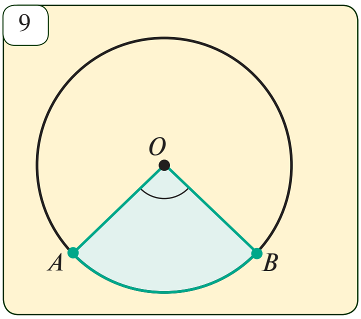
Doiraning ikki radius bilan chegaralangan bo‘lagi *sektor* deyiladi (*9-rasm*). Sektor yoyining uzunligi:

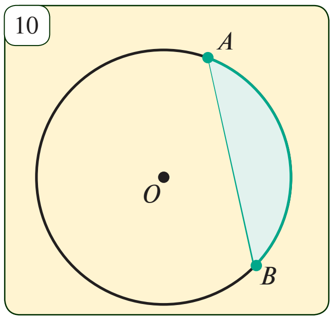
$l = \frac{\pi R \alpha}{180^\circ}$, bu yerda α — markaziy burchakning gradus o‘lchovi.

Sektor yuzi: $S = \frac{\pi R^2 \alpha}{360^\circ}$; $S = \frac{1}{2} Rl$.

Segment — doiraning vatar va shu vatar tiralgan yoy bilan chegaralangan bo‘lagi (*10-rasm*).







Segment yuzi: $S = S_{sektor} \pm S_{\Delta} = \frac{\pi R^2}{360^\circ} \cdot \alpha \pm \frac{1}{2} R^2 \sin \alpha$

MUNTAZAM KO‘PBURCHAKLAR

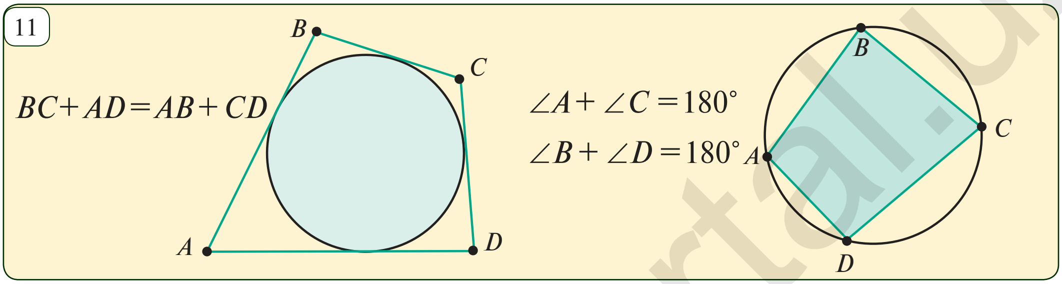
Muntazam n burchakning tomoni a_n, perimetri P_n, yuzi S_n, ichki chizilgan aylana radiusi r_n, tashqi chizilgan aylana radiusi R_n, ichki burchagi α_n bo‘lsa,

$$P_n=na_n,\qquad S_n=\frac{1}{2}P_nr_n=\frac{1}{2}na_nr_n,\qquad \alpha_n=\frac{(n-2)\cdot 180^\circ}{n}$$

$$R_n=\frac{a_n}{2\sin\frac{180^\circ}{n}},\qquad r_n=\frac{a_n}{2\operatorname{tg}\frac{180^\circ}{n}}$$

Aylanaga tashqi va ichki chizilgan to‘rtburchaklar (11- rasm).

11



10 dan 99 gacha bo‘lgan natural sonlar kvadratlarining jadvali

birlik \ o‘nlik	1	2	3	4	5	6	7	8	9
0	100	400	900	1600	2500	3600	4900	6400	8100
1	121	441	961	1681	2601	3721	5041	6561	8281
2	144	484	1024	1764	2704	3844	5184	6724	8464
3	169	529	1089	1849	2809	3969	5329	6889	8649
4	196	576	1156	1936	2916	4036	5476	7056	8836
5	225	625	1225	2025	3025	4225	5625	7225	9025
6	256	676	1296	2116	3136	4356	5776	7396	9216
7	289	729	1369	2209	3249	4489	5929	7569	9409
8	324	784	1444	2304	3364	4624	6084	7744	9604
9	361	841	1521	2401	3481	4761	6241	7921	9801

Ayrim kattaliklar jadvali

$\pi\cong 3{,}1416$	$\sqrt{8}\cong 2{,}8284$
$\sqrt{2}\cong 1{,}4142$	$\sqrt{10}\cong 3{,}1623$
$\sqrt{3}\cong 1{,}7320$	$\frac{1}{\sqrt{2}}\cong 0{,}7071$
$\sqrt{5}\cong 2{,}2360$	$\frac{1}{\sqrt{3}}\cong 0{,}5774$
$\sqrt{6}\cong 2{,}4495$	$\frac{1}{\sqrt{\pi}}\cong 0{,}3183$
$\sqrt{7}\cong 2{,}6457$	

Trigonometrik funksiyalar qiymatlarining jadvali

α°	sinα	cosα	tgα	ctgα	α°	sinα	cosα	tgα	ctgα
1	0,0175	1,000	0,0175	57,3	**46**	0,719	0,695	1,036	0,966
2	0,0349	0,999	0,0349	28,6	**47**	0,731	0,682	1,072	0,933
3	0,0523	0,999	0,0524	19,1	**48**	0,743	0,669	1,111	0,900
4	0,0698	0,998	0,0699	14,3	**49**	0,755	0,656	1,150	0,869
5	0,0872	0,996	0,0875	11,4	**50**	0,766	0,643	1,192	0,839
6	0,1045	0,995	0,1051	9,51	**51**	0,777	0,629	1,235	0,810
7	0,1219	0,993	0,1228	8,14	**52**	0,788	0,616	1,280	0,781
8	0,139	0,990	0,141	7,11	**53**	0,799	0,602	1,327	0,754
9	0,156	0,988	0,158	6,31	**54**	0,809	0,588	1,376	0,727
10	0,174	0,985	0,176	5,67	**55**	0,819	0,574	1,428	0,700
11	0,191	0,982	0,194	5,145	**56**	0,829	0,559	1,483	0,675
12	0,208	0,978	0,213	4,507	**57**	0,839	0,545	1,540	0,649
13	0,225	0,974	0,231	4,331	**58**	0,848	0,530	1,600	0,625
14	0,242	0,970	0,249	4,011	**59**	0,857	0,515	1,664	0,601
15	0,259	0,966	0,268	3,732	**60**	0,866	0,500	1,732	0,577
16	0,276	0,961	0,287	3,487	**61**	0,875	0,485	1,804	0,554
17	0,292	0,956	0,306	3,271	**62**	0,883	0,469	1,881	0,532
18	0,309	0,951	0,325	3,078	**63**	0,891	0,454	1,963	0,510
19	0326	0,946	0,344	2,904	**64**	0,899	0,438	2,050	0,488
20	0,342	0,940	0,364	2,747	**65**	0,906	0,423	2,145	0,466
21	0,358	0,934	0,384	2,605	**66**	0,914	0,405	2,246	0,445
22	0,375	0,927	0,404	2,475	**67**	0,921	0,391	2,356	0,424
23	0,391	0,921	0,424	2,356	**68**	0,927	0,375	2,475	0,404
24	0,405	0,914	0,445	2,246	**69**	0,934	0,358	2,605	0,384
25	0,423	0,906	0,466	2,145	**70**	0,940	0,342	2,747	0,364
26	0,438	0,899	0,488	2,050	**71**	0,946	0,326	2,904	0,344
27	0,454	0,891	0,510	1,963	**72**	0,951	0,309	3,078	0,325
28	0,469	0,883	0,532	1,881	**73**	0,956	0,292	3,271	0,306
29	0,485	0,875	0,554	1,804	**74**	0,961	0,276	3,487	0,287
30	0,500	0,866	0,577	1,732	**75**	0,966	0,259	3,732	0,268
31	0,515	0,857	0,601	1,664	**76**	0,970	0,242	4,011	0,249
32	0,530	0,848	0,625	1,600	**77**	0,974	0,225	4,331	0,231
33	0,545	0,839	0,649	1,540	**78**	0,978	0,208	4,507	0,213
34	0,559	0,829	0,675	1,483	**79**	0,982	0,191	5,145	0,194
35	0,574	0,819	0,700	1,428	**80**	0,985	0,174	5,67	0,176
36	0,588	0,809	0,727	1,376	**81**	0,988	0,156	6,31	0,158
37	0,602	0,799	0,754	1,327	**82**	0,990	0,139	7,11	0,141
38	0,616	0,788	0,781	1,280	**83**	0,993	0,1219	8,14	0,1228
39	0,629	0,777	0,810	1,235	**84**	0,995	0,1045	9,51	0,1051
40	0,643	0,766	0,839	1,192	**85**	0,996	0,0872	11,4	0,0875
41	0,656	0,755	0,869	1,150	**86**	0,998	0,0698	14,3	0,0699
42	0,669	0,743	0,900	1,111	**87**	0,999	0,0523	19,1	0,0524
43	0,682	0,731	0,933	1,072	**88**	0,999	0,0349	28,6	0,0349
44	0,695	0,719	0,966	1,036	**89**	1,000	0,0175	57,3	0,0175
45	0,707	0,707	1,000	1,000	**90**	1,000	0,0000	-	0,0000

1-mavzu. **5.** 50°; 130°; 133°; 97°. **6.** 65°; 70°; 45°. **7.** 105°; 130°; 125°. **8.** 35°; 35°; 110°. **9.** 94°; 56°; 30°. **10.** 110°; 130°; 120°. **11.** *Ko'rsatma:* to'rtta uchburchakning har birining tomonlari dastlabki uchburchakning mos tomonlarining yarmiga teng. **12.** *Ko'rsatma: DF* kesma *ABH* uchburchakning ham, *CEB* uchburchakning ham o'rta chizig'i bo'ladi. **13.** *Ko'rsatma: ANC* va *CKA* uchburchaklarning hamda ichki almashinuchi burchaklarning tengligidan foydalaning.

2-mavzu. **2.** a) $\sqrt{34}$ yoki $\approx 5{,}8$ *cm*; b)$14\sqrt{2}$ *m*; c) $\approx 21{,}5$ *cm; d)*$\frac{\sqrt{5}}{2}$*cm; e)* $\sqrt{2}$ *cm; f)* $\sqrt{13}$ *cm*. **4.** a) $\sqrt{21}$ *cm,* $\sqrt{5}$ *cm;* b) $\sqrt{21}$ *cm,*$\sqrt{22}$ *cm;* c) $\sqrt{2}$ *cm,* $\sqrt{3}$ *cm*. **5.** 12 *cm*. **6.** a) $\sqrt{10}$ *cm;* b) $2\sqrt{5}$ *cm*; c) $\sqrt{33}$ *m*. **8.** b), *e) va.* f) **9.** hammasi. **10.** 225. **11.** 5 *cm*. **12.** $\sqrt{27}$ *m*. **14.** b) $\approx 4{,}3$ *m;* c). $\approx 2{,}23$. **15.** a) 8,62 *m*; b) $\approx 5{,}97$ *m*. **16.** $\approx 1{,}84$ m^2. **17.** $\approx 105{,}6$ *m*. **18.** $\approx 102{,}5$ km. **19.** $\approx 48{,}4$ km.

3-mavzu. **1.** a) 11,7 *m*; b) 35 *mm*; c) 6,2 *km*; d) 172 *cm*; e) $4(x\text{-}1)$ *cm*; f) $(4x+2)$ *m*; g) $(13x+2)$ *km*; h) $(6y\text{-}8)$ *cm*; *i)* $8x$ *km* . **2.** a).$\approx 7{,}967$ *cm*; b) $\approx 44{,}329$ *m*; c) $\approx 409{,}86$ *mm*. **3.** a) Ha; b) Yo'q; c) Ha; d) Ha. **4.** 0,8 *m*; 24,64 m^2; 21,12 m^2. **5.** ≈ 50 marta. **7.** 17,5 *cm*; 10,5 *cm*; 38,1 *cm*; 59,1 *cm*. **8.** 91,5 *m*.

4-mavzu. **1.** *c*. **2.** a)*C*; b) *A* ; **3.** 8 ta, 2,4*m*. **5.** $\approx 53{,}4$ *m*. **6.** $\approx 19{,}25$ m^2. **9.** 12 ta **10.** Birinchisida. **11.** 80 ta. **12.** 7 dm^2. **14.** a)180 dm^3; b)105 cm^3 c)1364 cm^3. **15.** 1,8 m^3. **16.** a) 22 *cm;* b) 20 *cm va 24* cm^2*;* c) 96 cm^3. **20.** a) $4\sqrt{2}$; b) $2\sqrt{21}$; c) h= $2\sqrt{7}$. **21.** a) $(384+80\sqrt{5})$ cm^2, 640 cm^3; b) 84 cm^2, 36 cm^3. c) $(12\sqrt{34}+156)m^2$, *180* m^3. d) $36564+306\sqrt{97}$ cm^2, 404838 cm^3.

6-mavzu. **2.** Uchburchaklar o'xshash. **4.** 5; 8; $\frac{1}{2}$. **5.** 72; 162; 90.

7-mavzu. **3.** 12 *m*. **4.** 7,5 *cm*; 12,5 *cm*; 15 *cm*. **6.** 73,5 m^2; 37,5 m^2. **7.** Uchburchaklar o'xshash.

8-mavzu. **3.** a) 4,5; b) 10,5; c) 4,5. **4.** a) 10; b) 6; c) 4,5. **5.** a) 5 *cm*, 3,5 *cm*; b) $5\frac{5}{7}$*cm*, $2\frac{2}{7}$*cm*. **6.** a) 8; b) 3,5; c) 12,5. **8.** 12 *cm*.

9-mavzu. **3.** a) ha; b) yo'q; c) yo'q. **4.** $2\frac{1}{3}$*cm*, 9. **5.** a) 15 *cm*; 20 *cm*; b) 24 *cm*; 18 *cm*; c) 144 cm^2; 256 cm^2.

10-mavzu. **2.** ha. **3.** a) va c); d) va e). **4.** 108 cm^2. **5.** 4 *cm*; 6 *cm*. **7.** 4,8 *cm*. **9.** 12.

11-mavzu. **1.** a) va d); b) va e); g) va f). **2.** 36 *m* yoki 20,25 *m*. **3.** 12 *cm*; 14 *cm*. **5.** $5\frac{5}{11}$*cm*. **7.** 4 *m* . **8.**16 *m* **9.** 8,4 *m*.

12-mavzu. **3.** a) 15; b) $3\frac{2}{11}$; c) $3\frac{5}{17}$ **4.** 18 *cm*; 6 *cm*. **5.** 29 dm^2. **6.** 6 *dm*. **7.** *m*:*n*. **10.** Ha .

13-mavzu. **1.** $3\frac{3}{17}$*m*; 13,6 *cm*. **7.** *n*:*m*. **8.** a) *S*:4; b) *S*:2; c) *S*:4.**9.** 30. **10.** 57,75.

14-mavzu. **II.** **1.** 12 cm^2. **2.** 8,4. **3.** 2,4. **4.** 24. **5.** 8. **6.** 1,6. **7.** 18 *mm*. **8.** a)4; b)10; c)32. **9.** Ha.Uchburchaklar o'xshashligining 2-alomatigako'ra.**10.** 16.**11.** Ha, *k*=2. **12.** 24 *mm*. **13.** a) 36cm^2; b) *54* mm^2. **14.** a) ; b) . **15.** a) 7; b) 7. **16.** 6*m*. **17.** 12 *m*.

15-mavzu. **1.** a)(1;-1); b)(-2;3); c)(0;-4). **2.** (-1;5). **4.**(0;-3). **5.** (-1;-8). **6.** Ha. **7.** Yo‘q. **8.** BB_1 ga.

16-mavzu. **1.** a) *Ox* o‘qiga nisbatan simmetriyada: (1;-2), (0;-2), (2;-2). b) *Oy* o‘qiga nisbatan simmetriyada: (-1;2), (0;2), (-2;2). **2.** *Ox* o‘qiga nisbatan. **4.** Mos ravishda: 2ta, 4ta, 2ta, 1ta, 1ta. **8.** BOB, MUM

17-mavzu. **1.** (8;3). **3.** (2;-5), (-2;-2), (6;-12). **6.** To‘g‘ri to‘rtburchak, kvadrat, parallelogrammning simmetriya markazi - diagnallari kesishish nuqtasida, to‘g‘ri chiziqning simmetriya markazi - uning ixtiyoriy nuqtasida.

12. a) o‘qqa nisbatan simmetriya (bitta).
b) markaziy simmetriya, o‘qqa nisbatan simmetriya (4ta).
c) markaziy simmetriya.
d) o‘qqa nisbatan simmetriya (5ta).
e) markaziy simmetriya, o‘qqa nisbatan simmetriya (6ta)

13. a) o‘qqa nisbatan simmetriya:
M, X, V, T, Y, V, W, D, B, H, K, C, I, E, A. b) markaziy simmetriya: N, S, Z, X, H, I.

14. a) 180°ga; b) Yo‘q; c) Yo‘q; d) 90°ga; e) Yo‘q; f) 120°ga.

15. a) $\frac{360^\circ}{7}$; b) 60°; c) 360°. **17.** 15

18-mavzu. **5.** 1 *km* 750 *m*. **8.** 7,2 *cm*. **9.** $k=\frac{1}{2}$ yoki $k=2$.

19-mavzu. **4.** $k=2$. **5.** 6 cm^2; 24 cm^2. **6.** 104 cm^2. **7.** Har ikki holda $k=1$. **8.** 1,2 m^2. **9.** 16 *cm*, 32 *cm*.

20-mavzu. **4.** $\frac{2}{3}$; $\frac{4}{9}$. **5.** X_1X va Y_1Y nurlarning kesishish nuqtasi gomotetiya markazidir. **6.** $OX_1=2\cdot OX$. **7.** *Ko‘rsatma:* Mavzuda yechilgan masaladan foydalaning.

21-mavzu. **4.** a) $P_2=42$; $k=\frac{1}{2}$; b) $S_1=12$, $k=2$; d) $P_1=150\sqrt{2}$, $k=\sqrt{2}$; e) $P_1=10$, $S_2=216$.

22-mavzu. **1.** ≈6,94 *m*. **2.** 300 *m*.**3.** ≈72 *m*. **4.** 6,6 *m*.

23-mavzu. **1.** 9. **2.** $P_2=39$ *dm*. **3.** 8 *m*. **4.** 24 dm^2. **6.** *Ko‘rsatma: ABC* uchburchak chizing, ko‘pburchaklar yasash mavzusidagi 1-masaladan foydalanib, chizilgan uchburchak tomonlaridan uch marta kichik uchburchak yasang.
9. 72°; 72°; 36°. **11.** 12 cm^2. **12.** 150 000 000 *km*. **13.** a) Ha; b) Ha. **15.** 6 *cm*, 12 *cm*, 18 *cm*. **16.** 84 *m*.

24-mavzu. **II.** **1.** 8 *cm*. **2.** $4\frac{4}{9}$*cm*. **3.** 48 *m*. **4.** 4 *cm*; 0,5 cm^2. **5.** $5\frac{1}{3}$*m*. **6.** 867 *km*. **III.** **7.** 7,5 *m*. **8.** 6 *cm*. **9.** a) 7,5 *cm*; b) 6 *cm*; d) 16,2 *cm*. *Qiziqarli masalalar:* **1.** O‘zgarmaydi. **2.** a) Ha; b) Yo‘q. **3.** *Ko‘rsatma:* Chizg‘ich bilan har bir qo‘g‘irchoqning bo‘yini o‘lchang va ularning nisbatini toping.

25-mavzu. **1.** $\sin\alpha>0$, $\cos\alpha<0$, $\mathrm{tg}\alpha<0$, $\mathrm{ctg}\alpha<0$. **5.** a) 1; b) 1; d) 1. **6.** 3,5 *cm*. . **7.** a) $\frac{1}{2}$; $-\frac{1}{2}$; b) $\pm\frac{\sqrt{15}}{4}$; d) 0. **8*.** a) 30°; b) 135°; d) 150°.

26-mavzu. **2.** 36 cm^2. **3.** 24 *cm*. **4.** a) $6\sqrt{3}$; b) 30; d) $\frac{105\sqrt{3}}{4}$. **5.** $(24+4\sqrt{3})$ *cm*; $(24+8\sqrt{3})$ cm^2.

6. $10\sqrt{3}$ *cm*. **7.** a) $\frac{\sqrt{3}}{6}$; b) $\frac{1}{2}$; d) $\frac{\sqrt{3}}{2}$. **8.** ≈ 807 m^2. **9.** ≈ 88 *m*.
10. 1000, 37°. **12.** 2°. **13.** 34°. **14.** $2\sqrt{3}$; $4\sqrt{3}$. **15.** R = $3\sqrt{3}$ *cm; BO* = $6\sqrt{3}$. **16.** 5 *cm*. **17.** 12, $24\sqrt{3}$. **18.** 20 *cm*, 200 cm^2. **19.** 4, $16\sqrt{3}$. **20.** 30°; 60°. **22.** 12 *cm*; $4\sqrt{3}$ *cm*; $8\sqrt{3}$ *cm*. **23.** 32 cm^2.

27-mavzu **2.** a) 6 cm^2; b) 73,5 cm^2; d) 6 cm^2. **3.** 36 cm^2. **4.** $49\sqrt{2}$ cm^2. **5.** $54\sqrt{3}$ cm^2. **6.** $2\frac{2}{3}$ *cm*; $4{,}5\sqrt{2}$ *cm*. **7.** $S = \frac{h_b \cdot h_c}{2\sin\alpha}$ **8.** $4{,}8\sqrt{3}$ *cm*.

28-mavzu. **2.** a) $BC=6$; b) $AB=8\sqrt{2}$; d) $AC=7\sqrt{2}$. **3.** a) $\sin C=\frac{1}{3}$; b) $\sin A=\frac{7}{16}$; d) $\sin B=\frac{2}{3}$. **4.** 4,8 *dm*. **5.** 30° yoki 150°. **6.** Mumkin. **7.** $AB\approx 21{,}1$ *m*; $\angle B\approx 37°$, $\angle C\approx 76°$. **8.** 76°; 26,1 *cm*; 23,8 *cm*.

29-mavzu. **2.** a) $\sqrt{13}$ *cm*; b) 4 *m*; d) $\sqrt{283}$ *dm*. **3.** $\frac{1}{5}$; $\frac{19}{35}$; $\frac{5}{7}$. **4.** $2\sqrt{13}$ *cm* yoki $2\sqrt{109}$ *cm*. **5.** $\sqrt{31}$ *cm*, $\sqrt{91}$ *cm*. **6.** $\sqrt{109}$ *cm*, $\sqrt{39}$ *cm*.
7. *Ko'rsatma: ADB* va *BDC* uchburchaklarga kosinuslar teoremasini qo'llab, a^2 va c^2 ni toping, so'ngra bu tengliklarni hadma-had qo'shing. **8.** $\frac{\sqrt{106}}{2}$ *cm*; $\frac{\sqrt{151}}{2}$ *cm*; $\frac{\sqrt{190}}{2}$ *cm*.

30-mavzu. **1.** $\angle B$ va $\angle C$. **2.** *AB* va *BC*. **3.** a) o'tkir burchakli; b) to'g'ri burchakli; d) o'tmas burchakli. **4.** a) $8\frac{1}{8}$; b) $8\frac{1}{8}$; d) $24\frac{1}{6}$; e) $\frac{35\sqrt{6}}{24}$. **6.** *Ko'rsatma:* Sinuslar teoremasidan foydalaning. **7.** *Ko'rsatma:* 6-masalaga o'xshash yechiladi. **8.** *Ko'rsatma:* Sinuslar teoremasidan foydalaning.

31-mavzu. **1.** a) $10\sqrt{3}$; b) $28\sqrt{2}$; d) 12; e) $\approx 0{,}3064$. **2.** a) —2; b) 0; d) 2. **3.** a) 8; b) 24; d) 8; e) 0. **5.** a) —7,5; d) 0. **6.** $a\perp b$, $c\perp d$.

32-mavzu. **1.** a) $\alpha=90°$, $c=\frac{5\sqrt{2}}{2}$. b) $\gamma\approx 45°$; $a\approx 27{,}3$, $b\approx 24{,}5$; d) $\alpha=20°$; $b\approx 65.8$; $c\approx 88{,}6$; e) $\gamma=119°$; $a\approx 8{,}1$; $b\approx 5{,}8$. **2.** a) $c\approx 5{,}29$; $\alpha\approx 79°6'$; $\beta\approx 138°21'$; b) $c\approx 53{,}09$; $\alpha\approx 11°39'$; $\beta\approx 38°21'$; d) $a\approx 19{,}9$; $\beta\approx 58°19'$; $\gamma\approx 936°41'$; e) $a\approx 22{,}9$; $\beta\approx 21°$; $\gamma\approx 15°$. **3.** a) $\alpha\approx 29°$; $\beta\approx 47°$; $\gamma\approx 104°$; b) $\alpha\approx 54°$; $\beta\approx 13°$; $\gamma\approx 113°$; d) $\alpha\approx 34°$; $\beta\approx 44°$; $\gamma\approx 102°$; e) $\alpha\approx 39°$; $\beta\approx 93°$; $\gamma\approx 48°$.

33-mavzu. **1.** a) $2\sqrt{3}$ *cm*; b) 16 *cm*; d) $\frac{ab\sqrt{2}}{4}$. **2.** $4\sqrt{2}$ *m*; 8 *m* va $4+4\sqrt{3}$ *m*. **3.** $50\sqrt{3}$ *kg*. **4.** 14 *cm*. **5.** $2\sqrt{14}$ *cm*. **6.** $6\sqrt{3}$ *cm*. **7.** 50 *cm*.

34-mavzu. **1.** $\approx 10{,}8$ *m*. **2.** ≈ 15 *m*. **3.** $\approx 43{,}4$ *m*. **4.** $\approx 35°$. **5.** $\approx 73{,}2$ *m*. **6.** ≈ 49 *m*. **7.** Asfalt yo'ldan.

35-mavzu. **II.** **1.** $3\sqrt{6}$, $3\sqrt{2}$. **2.** $\frac{111}{120}$; 0,89; —0,65. **3.** $2\sqrt{7}$ *cm*; $\frac{2\sqrt{21}}{3}$ *cm*. **4.** $30\frac{1}{30}$ *cm*. **5.** 28 *cm*. **6.** 8 cm^2; $(4+4\sqrt{5})$ *cm*; $h_a=4$ *cm*, $h_b=0{,}8\sqrt{5}$ *cm*. **7.** $2\sqrt{13}$. **8.** a) o'tkir burchakli; b) to'g'ri burchakli, d) o'tmas burchakli. **9.** 63 cm^2. **10.** $\approx 3{,}7$ *cm*. **11.** 7 *cm*. **12.** 6. **13.** 0. **14.** —9. **15.** 135°. **16.** $OC\approx 9{,}6$. **17.** $(24+24\sqrt{3})$ *cm*. **18.** 5. **III.** **1.** $\approx 109°$. **2.** $\gamma=100°$, $a\approx 3{,}25$; $c\approx 6{,}43$. **3.** 6,25; 14,76.

36-mavzu. **2.** a) Har qanday uchburchak aylanaga ichki chizilishi mumkin; b) Qarama-qarshi burchaklari yig'indisi 180° bo'lgan to'rtburchaklar. **3.** Bitta yoyga tiralgan burchaklar teng. **4.** 10 *cm*. **5.** 672 cm^2. **6.** a) $10\sqrt{3}$ *cm*; b) $10\sqrt{2}$ *cm*; d) $10\sqrt{2}$ *cm*; $10\sqrt{2}$ *cm*; 20 *cm*. **7.** $8\frac{1}{3}$ *cm*. **8.** ΔABF da, $\angle BAF+\angle AFB=90°$, $\angle ABF=90°$. Demak, *AF* – diametr. **9.** Qarama-qarshi burchaklari yig'indisi

180°, ya'ni aylanaga ichki chizish mumkin. **10.** *Ko'rsatma:* Bitta asos va bir yon tomonning o'rta perpendikulari kesishgan nuqta aylana markazi bo'ladi.

37-mavzu. **2.** 7,2 *cm*. **3.** a) 16,6; b) 22; d) 22,6. **4.** a) 2,5; b) 3,5; d) 2. **8.** 6 *cm*.

38-mavzu. **3.** a) 60°; b) 108°; d) 120°; e) 144°; f) 160°. **4.** a) 120°; b) 72°; d) 120°; e) 36°; f) 30°. **5.** a) 3; b) 4; d) 8; e) 12.

39-mavzu. **1.** 3 *cm* va $3\sqrt{2}$ *cm*. **2.** $\sqrt{3}$ va $2\sqrt{3}$. **7.** a) 6; b) 12; d) 10; e) 20; f) 5.

40-mavzu. **3.** 8 *cm*; $8\sqrt{2}$ *cm*; $8\sqrt{3}$ *cm*; $8\sqrt{2+\sqrt{3}}$ *cm*; 16 *cm*.
4. $\frac{8\sqrt{6}}{3}$ *cm*; **5.** a) $20\sqrt{2}$ *cm*; b) 40 *cm*. **6.** $\frac{5\sqrt{3}}{3}$ *cm*.

41-mavzu. **I.** **1.** E; **2.** D; **3.** D; **4.** B; **5.** B; **6.** E; **7.** E. **III.** **1.** $\sqrt{3{:}4}$: $6\sqrt{3}$. **2.** 3:4. **3.** a) ≈5,780 *cm*; b) ≈4,142 *cm*; d) ≈2,679 *cm*. **4.** $S=\sqrt{2}R^2$. **5.** 24 cm^2. **IV.** **1.** 4 *cm*; 13 *cm*. **2.** a) 80 *cm*; b) $20\sqrt{2}-\sqrt{3}$ *cm*; $40\sqrt{2}-\sqrt{3}$ *cm*; d) 200 cm^2. **3.** $4\sqrt{3}$ *cm*; 8 *cm*. 4. $\frac{27\sqrt{3}}{4}$ cm^2.

42-mavzu. **2.** a) 3 marta ortadi; b) 6π *cm* ga ortadi; d) 3 marta kamayadi; e) 6π *cm* ga kamayadi. **3.** 6369 *km*. **4.** a) $\frac{2\pi\sqrt{3}a}{3}$; b) $\pi\sqrt{a^2+b^2}$; d) $\frac{2\pi b^2}{\sqrt{4b^2-a^2}}$. **5.** a) πa; b) $\pi c(\sqrt{2}-1)$; d) $\pi c(\sin\alpha+\cos\alpha-1)$. **6.** 1,5 *m*. **7.** 66348 marta.

43-mavzu. **1.** a) π *cm*; b) $1{,}5\pi$ *cm*; d) 3π *cm*; e) 4π *cm*. **2.** a) $\frac{2\pi}{9}$; b) $\frac{\pi}{3}$; d) $\frac{5\pi}{12}$. **3.** a) ≈69°; b) 120°; d) 150°. **4.** a) $\frac{5\pi}{8}$ *cm*; b) 2π *cm*; d) $\frac{15\pi}{4}$ *cm*; **5.** a) 4π; b) 16π. **7.** 2.

44-mavzu. **3.** k^2 marta ortdi; b) k^2 marta kamayadi. **4.** $6{,}25\pi$ cm^2; $12{,}5\pi$ cm^2. **5.** $2{,}25\pi$ cm^2; 9π cm^2. **6.** $(\pi-2)R^2$. **7.** $21{,}25\,\pi$ cm^2. **8.** 18,75 cm^2.

45-mavzu. **3.** a) $\frac{49}{12}\pi$ cm^2; $\frac{49(\pi-3)}{12}$ cm^2; b) $6{,}125\pi$ cm^2; $\frac{49(\pi-2\sqrt{2})}{8}$ cm^2; d) $\frac{49\pi}{3}$ cm^2; $\frac{49(4\pi-3\sqrt{3})}{2}$ cm^2; e) $\frac{49\pi}{4}$ cm^2; $\frac{49(\pi-2)}{4}$ cm^2. **4.** a) $a^2\left(\frac{\sqrt{3}}{4}-\frac{\pi}{8}\right)$; b) $a^2\left(1-\frac{\pi}{4}\right)$ d) $\frac{3\sqrt{3}-\pi}{2}\,a^2$; **5.** π cm^2; 3π cm^2; 5π cm^2; 7π cm^2. **6.** $\frac{25(2\pi-3\sqrt{3})}{3}$ cm^2; $\frac{25(10\pi-3\sqrt{3})}{3}$ cm^2; **7.** $\frac{75(4\pi-3\sqrt{3})}{2}$ cm^2. **8.** $S_1<S<S_2$; 300 cm^2 < 314 cm^2 < 321,48 cm^2.

46-mavzu. **1.** Doiraniki katta. **2.** $\frac{160}{3}\pi$ cm^2. **3.** $5{,}76\pi$ cm^2. **4.** $8.(\pi-2)$ cm^2. **6.** 6π cm^2; 10π *cm*.

47-mavzu. **II.** **1.** $6\sqrt{2+\sqrt{2}}$. **2.** $\frac{8\pi}{3}$ *dm*. **3.** 30 *cm*. **4.** 90°. **5.** 3. **6.** π va $6{,}25\pi$. **7.** $\frac{10\pi+3\sqrt{3}}{2\pi-3\sqrt{3}}$. **8.** $\frac{2\sqrt{3}}{\pi}$ **9.** $\frac{9\sqrt{3}-2\pi}{6}\,a^2$. **10.** $1{,}5\pi$. **11.** 7. **12.** $\approx 9\pi-26{,}04$. **13.** π. **14.** $54\sqrt{3}-24\pi$. **15.** $\frac{3\pi}{8}$. **III.** **2.** $8\sqrt{3}$ *cm*. **3.** a) $\frac{18}{\pi}$ *cm*; b) $\frac{216}{\pi}$ cm^2; d) $\frac{216\pi+81\sqrt{3})}{\pi^2}$ cm^2.

48-mavzu. **3.** $5\sqrt{2}$ *cm*. **4.** 12 *cm*. **5.** 44 *m*, 60 *m*. **7.** 1:7. **8.** $AB\cos\alpha$.

49-mavzu. **1.** a) 30 *cm*, 12 *cm*; b) 9 *cm*, 12 *cm*, 21 *cm*; d) 3 *cm*, 15 *cm*, 3 *cm*, 21 *cm*. **3.** 6 *cm*; 10,5 *cm*. **4.** 9 *cm*, 12 *cm*, 15 *cm*, 18 *cm*. **5.** 60°. **6.** 21 *cm*. **7.** 20 *cm*.

50-mavzu. **1.** *Ko'rsatma:* $\triangle ACD \sim \triangle CBD \sim \triangle ABC$. **2.** 25 *cm*, 15 *cm*, 20 *cm*. **3.** $9\frac{3}{5}$ *cm*. **4.** a) 5, 4; b) 24, 25; d) 8,10. **5.** 16:25. **6.** 56,16 cm^2. **7.** 60 cm^2. **8.** $\frac{2}{3}$; $\frac{4}{9}$; $\frac{4}{9}$.

51-mavzu. **2.** *Ko'rsatma:* a) katetlari *a* va *b* bo'lgan to'g'ri burchakli uchburchak yasang; b) gipotenuzasi *a*, bir kateti *b* bo'lgan to'g'ri burchakli uchburchak yasang. **3.** *Ko'rsatma:* Katetlari $AB=BC=1$ bo'lgan $\triangle ABC$ yasang. So'ng kateti

$CC_1=1$ va $\angle C_1=90°$ bo‘lgan ΔBCC_1 yasang va hokazo. **4.** a) 20; b) 45; d) 37,5. **5.** 225π *cm*². **6.** 180 *cm*². **7.** 25:9. **9.** $OC \geq OD$ bo‘lgani uchun tengsizlik har doim to‘g‘ri.

52-mavzu. **1.** a) 6,25; b) 12; d) 0,25. **2.** a) 8 *cm*; b) 2,5 *cm;* d) 0,9 *cm*. **3.** a) 4 *dm*; b) 4 *dm*. **4.** 8 *cm*. **6.** 9 *dm*; 16 *dm*.

53-mavzu. **1.** 10 *cm*. **2.** 2 *cm*. **3.** a) 2,5; b) 4; d) 2. **4.** a) $4\sqrt{6}-1$ *cm*; b) 6 *cm*. **5.** 1:6. **6.** 6 *cm*. **7.** 3. **8.** 1:4.

54-mavzu. **II.** **1.** 18 *cm*; 32 *cm*. **2.** 4 *cm*; **3.** 8 *cm*; **4.** 6,4 *dm*. **5.** 8 *cm*. **6.** 1,5. **7.** 5. **8.** 6. **9.** 45 *dm*². **10.** 4 *cm*. **11.** 8 *cm*. **12.** 6. **13.** 60°. **14.** 45°. **15.** 4:9.

III. **1.** 8 *cm*. **2.** 5 *dm*. **3.** 4 *cm*; 8 *cm*.

55-mavzu. **1.** a) 9; b) 4 *cm*²; d) 3,5 *cm*; e) $\frac{1}{2}TB-CA$; f) 0,2. **2.** $\Delta CMH \sim \Delta BCA$.

Darslikni tuzishda foydalanilgan va qo‘shimcha o‘rganishga tavsiya etilayotgan o‘quv adabiyotlari va electron resurslar

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Boxodir Qayumovich Xaydarov,
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GEOMETRIYA

9- sinf uchun darslik

To'ldirilgan va qayta ishlangan to'rtinchi nashr
(O'zbek tilida)

"Huquq va Jamiyat" nashriyoti, 2019
Toshkent, Yunusobod 6, Jumamasjid ko'chasi.

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Badiiy muharrir	*A.Umarova*
Bosh dizayner	*H&J dizayn jamoasi*
Sahifalovchi	*D.Iskandarbekov*

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Ijaraga berilgan darslik holatini ko'rsatuvchi jadval

T/r	O'quvchining ismi va familiyasi	O'quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshiril-gandagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o'quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to'ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko'chmagan, betlarida yozuv va chiziqlar yo'q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta'mirlangan. Ko'chgan varaqlari qayta ta'mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqovaga chizilgan, yirtilgan, asosiy qismdan ajralgan yoki butunlay yo'q, qoniqarsiz ta'mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo'yab tashlangan. Darslikni tiklab bo'lmaydi.