ZULF'S STANFORD ANALYSIS FALL 2012 QUAL

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1. Preamble: Seeking Tenure At Stanford For Four-Sphere Theory And Quant Human Nature Results

I am seeking tenured full professorship at Stanford University right away based on success with my four-sphere theory which is my original work that began in 2008 in a loft in Williamsburg Brooklyn and showed success by 2018. It was done without any financial support of any university or government, and without any supervision. I successfully challenged general relativity, quantum field theory, and expansionary cosmology. I also have new and original empirical and not just theoretical results on universal human moral nature, vindication of Aristotle's virtue-eudaimonia theory as well as extensions to include virtues of romantic love for life satisfaction for all human beings. My work can be found archived in Github available for public [1, 2]. I am in conflict with *Bill Gates* and he has used his powers to (a) steal money from me, (b) starve me of income $620 million, $120 million from D. E. Shaw & Co. and $500 million from Madam Christine Lagarde of ECB. These are for work on medium frequency alpha strategy by discovery of pure arbitrage opportunities in 1-15 minute frequency in all asset classes and plans for Quantitative Positive Psychology technology application projects for Global Life Satisfaction. Bill Gates had violated all international and national laws to destroy my life altogether and blockade all funds. He is has been condemned to death by UN Security Council for War Crimes but United States Government is lethargic in co-operating. I would like to get notice of a tenure decision by Stanford as soon as possible as I have been without any significant income for more than a decade and have to get my own pad and re-establish normal life. At the moment I am living with my aunt in Allen Texas dealing with a great deal of verbal abuse and other problems and being accosted by local government officials on trumped up charges, being sent to mental health incarceration for frivolous charges, and other problems. Co-operation of Stanford in swift tenure decision and getting me funds immediately would be welcome. I plan to buy some properties in Mission District San Francisco and settle there and have a family while working with Stanford as an Adjunct Professor.

All the work that I do is my own for the Stanford Mathematics Qual unless explicit references are given. I am a Princetonian and take Honour Code very seriously. I am very proud of my extremely strong Virtuous Character. Here is a profile of my ranked Virtues from VIA-120. The top 5-10 are most significant virtues here.

Date: January 29, 2022.

	Rank	Virtue	Class
1	1	Love	(Humanity)
2	2	Creativity	(Wisdom)
3	3	Honesty	(Courage)
4	4	Curiosity	(Wisdom)
5	5	Spirituality	(Transcendence)
6	6	Hope	(Transcendence)
7	7	Bravery	(Courage)
8	8	Humor	(Transcendence)
9	9	Forgiveness	(Temperance)
10	10	Perspective	(Wisdom)
11	11	LoveOfLearning	(Wisdom)
12	12	AppreciationOfBeautyAndExcellence	(Transcendence)
13	13	SocialIntelligence	(Humanity)
14	14	Zest	(Courage)
15	15	Gratitude	(Transcendence)
16	16	Self-Regulation	(Transcendence)
17	17	Fairness	(Justice)
18	18	Perserverence	(Courage)
19	19	Judgment	(Wisdom)
20	20	Leadership	(Justice)
21	21	Kindness	(Humanity)
22	22	Prudence	(Temperance)
23	23	Teamwork	(Justice)
24	24	Humility	(Temperance)

2. The Purpose Of These Tests And Timing

I generally do ten problems in roughly 2-4 days. I am 49 and do not try to rush any problems. The goal is really to test my weaknesses in Analysis. I had taken Real Analysis with Nick Katz at Princeton sophomore year. That's right it was 1992. Then I took functional analysis with Peter Sarnak in 1993. Four-Sphere Theory developed from 2008 in my mind primarily based on my strong understanding of spectral theorem for compact self-adjoint operators and its applications for compact riemannian manifolds. I was examining physics when I realised that quantisation of energy in the actual universe could have a global geometric origin and eventually was led to four-sphere theory with exact homogeneous geometry for absolute space with measured cosmological constant $\Lambda = 1.11 \times 10^{-52} m^{-2}$ being its curvature. This theory has shown remarkable success and this achievement is tenure-worthy alone.

These problems from Stanford Mathematics Ph.D. Quals I am doing to sharpen depth of understanding of Analysis which now I need for Four-Sphere Theory and other efforts.

3. Problem I.1

(a) Suppose $u \in \mathcal{D}'(\mathbf{R})$ with the property that for some $k \in \mathbf{N}$ we have $x^k u = 0$. We have to show that there exist $a_0, \dots, a_{k-1} \in \mathbf{C}$ so

$$u(\phi) = \sum_{j=0}^{k-1} a_j \phi^{(j)}(0)$$

(b) Suppose $1 < p < \infty$ and $u_n \in L^p(X, \mu)$. Let q be the conjugate exponent so $q^{-1} + p^{-1} = 1$. Suppose now that

$$\lim_{n\to\infty} \int \phi u_n d\mu$$

exists for all $\phi \in L^q(X, \mu)$. Show that there is a $C \geq 0$ so that

$$\|u_n\| \leq C$$

for all n.

3.1. **Putzing Around For (a).**

These are important problems, quite basic. Let's look at how to do (a) for just $k = 1$ because the secret might lie there.

So we have a distribution with $xu = 0$ and we want to know that it is a constant times δ_0. So let's take $\phi \in C_0^\infty(\mathbf{R})$. We have

$$xu(\phi) = 0$$

Let's do this without worry about technicalities first as follows. We pretend first that u is a linear functional not on *compactly supported* smooth test functions but all smooth functions. We can fix this later. Then we just take a Taylor expansion.

We use test functions $\phi_j(x) = x^j$ for $0 \leq j$ and let $u(\phi_j) = a_j$. Then

$$xu(\phi_j) = u(x\phi_j) = 0$$

for $j \geq 0$. If we take arbitrary $\phi = \sum_{j=0}^\infty b_j x^j$ then by linearity

$$u(\phi) = a_0 b_0 + xu(b_1 + b_2 x + b_3 x^3 + \dots) = a_0 b_0$$

This leads to $u = a_0 \delta_0$. I will return to justify how this can be modified with a partition of unity later.

With $k = 1$ case done, we can proceed for general k similarly algebraically. We replace $u(x^j) = a_j$ and keep the coefficients of first k terms of the Taylor expansion of ϕ and kill the rest off using $x^k u = 0$.

3.2. **The Banach Space Theory.**

Suppose X, Y are Banach spaces and we have a family of bounded linear operators T_n with

$$\sup_n \|T_n x\| < \infty$$

for each $x \in X$. Then we the Uniform Boundedness Principle gives us

$$\sup_n \|T_n\| \leq C$$

for some C.

We are going to try to use this to get our $C > 0$ in problem I.1(b) by using the fact that L^p is the dual of L^q and using the linear functionals $T_n = \langle u_n, \cdot \rangle$ on $L^q(X, \mu)$. That will do it.

I have to say that that's a neat use of the Uniform Boundedness Principle, Stanford Ph.D. Qual Directors. You would have fooled all sorts of people who would be considering the wrong tool. Hehehehe, didn't fool Zulf there at all. I saw Uniform Boundedness application because you see the thing is this: no one says something like $\lim_n \int \phi u_n$ exists as a condition at all unless there is some uncanny hidden agenda.

4. Dirichlet Kernel Integral

Let D_N be the Dirichlet kernel

$$D_N(\theta) = \frac{\sin(N + 1/2)\theta}{\sin \frac{1}{2}\theta}$$

Let

$$L_N = \int_0^{2\pi} |D_N(\theta)| d\theta.$$

Prove that there are constants $C_1, C_2 > 0$ such that

$$C_1 \log(N) \leq L_N \leq C_2 \log(N).$$

4.1. **Zulf Meanders About In Delirious Aimlessness.** If you thought Zulf would just whipper-snapper and provide a posh polished proof here, you are sadly mistaken. Instead, Zulf will invite you into Zulf's way of seeing the world and take you to the journey to Zulflandia where these sorts of estimates are totally unknown.

So Zulf sees this and says, "Say what? How did the $\log(N)$ get there?"

$$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$

Let me try something else. I look at

$$e^{i(2N+1)\theta/2}/e^{i\theta/2} = e^{iN\theta}$$

' I wonder what happens when I integrate

$$\int_0^{2\pi} |\sin N\theta| d\theta$$

No no. I have another idea

$$\sin(nx) = \sum (-1)^r \binom{n}{2r+1} \cos^{n-2r-1}(x) \sin^{2r+1}(x)$$

We can do this sort of expansion, then remove one $\sin(x)$ for the denominator, and then try to get alternative sum

$$1 - 1/2 + 1/3 - 1/4 + \cdots + (-1)^N 1/N = \log(N)$$

somehow.

4.2. **Distraction.** On problem I.2 Zulf meanders around without any resolution. There is an uncanny similarity between Zulf and Moses. There is wandering in the desert, there is a aimless delirium, mirages instead of oases, there are sighs and murmurs, maternal lamentations with "God why have you forsaken me" but not a single good idea for how to integrate absolute value of the Dirichlet kernel.

It's a sad thing when Divinity forsakes one right in the Dunes where history was written. Let's calculate how far we are from Alexandria. We're in Moses era, so is the Library of Alexandria burned already? Hold on.

Much more important than Dirichlet kernel is whether Alexandria already had a Great Library when Moses was wandering in the Desert. This is super important. I'm Asian. I get distracted by these things.

Because if our *Asian* Great Library of Alexandria was well stocked with all sorts of scrolls while Moses was still wandering in the Desert, that would be most useful for *Polemical Rhetoric*.

I must quote *The Waste Land* here:

> Who is the third who walks always beside you?
> When I count, there are only you and I together
> But when I look ahead up the white road
> There is always another one walking beside you
> Gliding wrapt in a brown mantle, hooded
> I do not know whether a man or a woman
> —But who is that on the other side of you?
> What is that sound high in the air
> Murmur of maternal lamentation
> Who are those hooded hordes swarming
> Over endless plains, stumbling in cracked earth
> Ringed by the flat horizon only
> What is the city over the mountains
> Cracks and reforms and bursts in the
> violet air
> Falling towers
> Jerusalem Athens Alexandria
> Vienna London
> Unreal

That's beautiful. Alexandria, *the Asian City* is mentioned here.

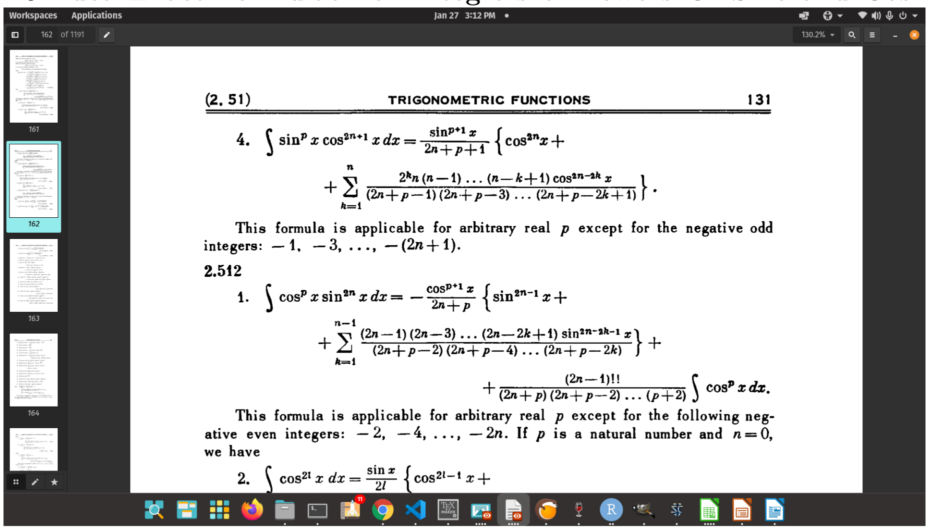
Ah failure! Moses wanderings came before the founding of the Great Library of Alexandria. Alas.

The earliest known surviving source of information on the founding of the Library of Alexandria is the pseudepigraphic Letter of Aristeas, which was composed between c. 180 and c. 145 BC. It claims the Library was founded during the reign of Ptolemy I Soter (c. 323–c. 283 BC) and that it was initially organized by Demetrius of Phalerum, a student of Aristotle who had been exiled from Athens and taken refuge in Alexandria within the Ptolemaic court.

That's a pity. This was an opportunity for great polemical rhetoric. Fortunately this *Asian* Library predated all literacy of England. That's good. That's not bad at all.

Queen Elizabeth says "England had been doing a lot of things since founding of Great Library of Alexandria. What has Bill Gates been doing?" That's a good queen. I approve highly. If the man has been discovered to be stealing my money at 66 with $131 billion in assets, what is the explanation for his rise? It's very simple. This is not an aberration. The man has been stealing, robbing, killing all his life and that's who he is. Ladies and Gentlemen, the secrets of Bill Gates are revealed by Zulf. This is who he is, an evil malevolent thief, a literal shameless thief, a robber, a liar, a murderer, a hard-core criminal from birth not metaphorically but literally.

4.3. Idea Exact Formulae For Integrals of Powers Of Sine and Cosine.



(2.51) TRIGONOMETRIC FUNCTIONS 131

4. $$\int \sin^p x \cos^{2n+1} x\,dx = \frac{\sin^{p+1} x}{2n+p+1}\left\{\cos^{2n} x + \right.$$
$$\left. + \sum_{k=1}^{n} \frac{2^k n(n-1)\dots(n-k+1)\cos^{2n-2k} x}{(2n+p-1)(2n+p-3)\dots(2n+p-2k+1)}\right\}.$$

This formula is applicable for arbitrary real p except for the negative odd integers: $-1,\ -3,\ \dots,\ -(2n+1)$.

2.512

1. $$\int \cos^p x \sin^{2n} x\,dx = -\frac{\cos^{p+1} x}{2n+p}\left\{\sin^{2n-1} x + \right.$$
$$\left. + \sum_{k=1}^{n-1} \frac{(2n-1)(2n-3)\dots(2n-2k+1)\sin^{2n-2k-1} x}{(2n+p-2)(2n+p-4)\dots(2n+p-2k)}\right\} +$$
$$+ \frac{(2n-1)!!}{(2n+p)(2n+p-2)\dots(p+2)}\int \cos^p x\,dx.$$

This formula is applicable for arbitrary real p except for the following negative even integers: $-2,\ -4,\ \dots,\ -2n$. If p is a natural number and $n=0$, we have

2. $$\int \cos^{2l} x\,dx = \frac{\sin x}{2l}\left\{\cos^{2l-1} x + \right.$$

The idea that I have is to attempt to produce an exact formula from Gradshteyn-Ryzhik p. 131 2.511 and 2.512. The expansion of $\sin(2N+1)x$ with $x = \theta/2$ after we divide by $\sin(x)$ will have only *even* powers of $\sin(x)$.

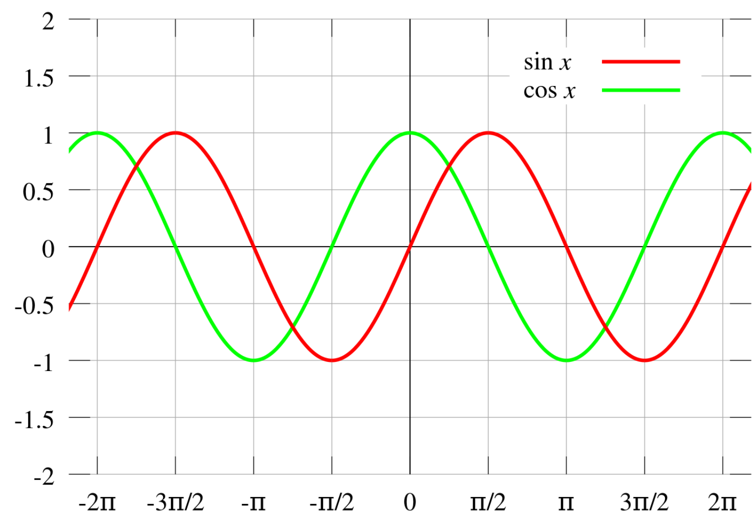
The idea is then to actually integrate the mixed powers of cosine and sines and then examine whether there is a harmonic sum. I made a total error for the sum needed. It's a harmonic sum without alternation that gives us $\sum_{k=1}^{n} 1/k = \log(n) + \gamma + O(1/n)$

Anyway, the problem is hard. The estimate is not trivial.

4.4. Grueling Jambalaya Of Integral Evaluations. Whenever we expect to do really hard work with sines and cosines and lots of combinatorial expressions we have certain natural coping mechanisms. Now now, let's not be rude about them. First there is the fight-or-flight reaction. We want to be *distracted immediately* towards more pleasant considerations. Then we might suddenly remember that we had to do so many more important things, like finish reading some Philip K. Dick novels that we had put off till just then. Then we might think, "Wouldn't it be nice if we took some weed or had some glasses of good wine?" Those things will happen soon, so I better get some elementary high school issues out of the way.

Let's remember the graphs of sine and cosine. We have

$$\cos(0) = 1$$
$$\cos(\frac{\pi}{2}) = 0$$
$$\cos(\pi) = -1$$
$$\sin(0) = 0$$
$$\sin(\frac{\pi}{2} = 1$$
$$\sin(\pi) = 0$$



Second, we want to be examining

$$\int_0^{2\pi} \sin^a(x/2) cos^b(x/2) dx$$

It's best to just let $y = x/2$ and integrate from 0 to π instead pulling out a 2.

You think this is funny? It's too elementary for you? That's too bad. You see, I don't like seeing a huge jumble that I cannot make any head or tail with so many symbols and minus signs that I feel like I am in the great American wilds surrounded by herds of reindeers moving silently and very fast. Those sorts of things look good only in Hollywood movies, which I watch from comfortable urban setting, preferably with a beautiful lover wrapped in comforters and with the main lights off and gentle scent and evening air perfuming my surroundings ok? Jungles stay out from my living areas.

We need to keep track of all the integration end points. You think it's just really great to show off how you can calculate all sorts of exact integrals in your head? Good, get a job with *Rain Man* leave me alone.

4.5. **The $2N+1$ Endpoints.** It is finally dawning on me that this problem is best done by exact calculation with dividing $[0, \pi]$ into $2N + 1$ equal parts, letting

$$c_k = \cos(\frac{k}{2N+1}\pi)$$

and

$$s_k = \sin(\frac{k}{2N+1}\pi)$$

and then writing

$$2\int_0^{\pi} |D_N(x)| dx$$

as the sum

$$\sum_{k=0}^{N} \int_{x_{2k}}^{x_{2k+1}} D_N(x) dx + \sum_{k=0}^{N} \int_{x_{2k+1}}^{x_{2k+2}} (-D_N(x)) dx$$

based on where $\sin(2N+2)x$ is positive or negative. We have various indefinite integrals for

$$\int D_N(x)dx$$

and so we can calculate $\int |D_N(x)|dx$ exactly by this method. Then we just have to plug in the end points in the indefinite integrals and if we keep the resulting jambalaya under control we should be able to handle the combinatorial sums to get our upper and lower $\log(N)$ estimates.

So that is the next insight, that this problem is one of estimations after there is exact integration od D_N at various endpoints $x_k \in [0,\pi]$. After this is done, the hard work is manipulating messy sums, tracking all the signs and values.

4.6. **Efforts To Clear Up The Jambalaya.** Let $J_1, \ldots, J_N$ be the intervals

$$J_k = [\frac{k-1}{\pi}, \frac{k}{\pi}]$$

Since it is clear that $\sin(x) \geq 0$ in each J_k, the sign of $\sin(2N+1)x$ determines that of $D_N(x)$. This allows us to write

$$\int_0^{2\pi} |D_N(x)|dx = \sum_{k=1}^{2N+1} |\int_{x_{k-1}}^{x_k} D_N(x)dx|$$

This is good. This is true and it does not have an absolute value inside the integral. This is what I really needed to write before.

This allows us to consider the following quantities only.

$$A_k = |\int_{x_{k-1}}^{x_k} D_N(x)dx|$$

This might be trivial re-arrangement to you but this is massive simplification for me.

The problem is to show that there are $C_1, C_2 > 0$ so that

$$C_1 \log(N) \leq \sum_{k=1}^{2N+1} A_k \leq C_2 \log(N)$$

And now we get to the business of evaluating these A_k using the exact formulae.

Now we go back to the formula for the integrand

$$D_N(x) = \sum (-1)^r \binom{2N+1}{2r+1} \cos^{2N-2r}(x) \sin^{2r}(x)$$

This gives us

$$A_k = \left| \sum (-1)^r \binom{2N+1}{2r+1} \int_{x_{k-1}}^{x_k} \cos^{2N-2r}(x) \sin^{2r}(x) dx \right|$$

We still have to be careful about signs. Let us do even more lazy relabeling now and let

$$B_{k,r} = \int_{x_{k-1}}^{x_k} \cos^{2N-2r}(x) \sin^{2r}(x) dx$$

and write

$$A_k = |\sum (-1)^r \binom{2N+1}{2r+1} B_{k,r}| \tag{1}$$

Now we can see something clear here. And our task is to evaluate $B_{k,r}$ and get something tractable.

Let me remind our dear reader why we are rewriting things and just doing cosmetic changes. You see, we hate a lot of signs obscuring things when we cannot get a clear sense of cancellations. We also wanted to see something where there are no more integrations left. The formula (1) contains things where all the integrations have been separated out from the mess that will yield our $\log(N)$ in the end.

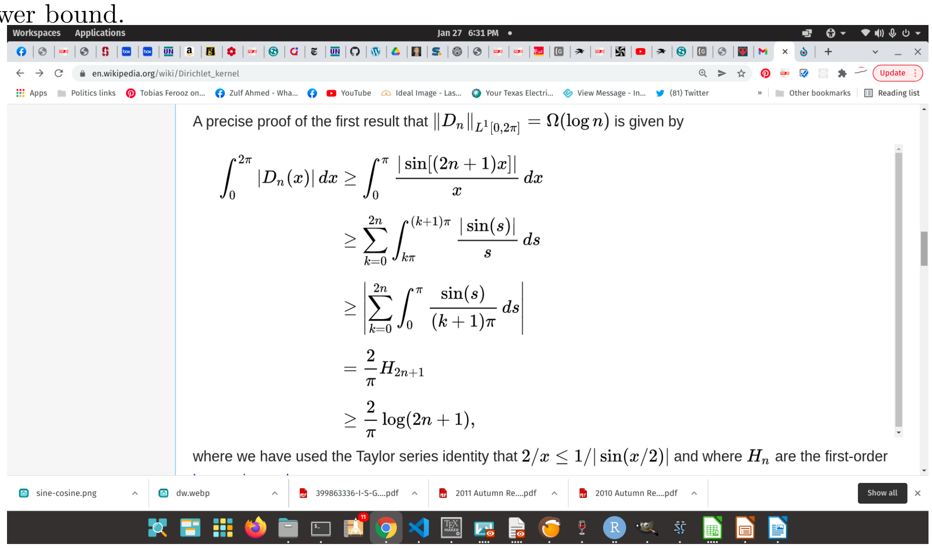
The key point here is that we did not do any *estimations* until after integrations are performed in the intervals $J_1, \ldots, J_{2N+1}$. Why? Because we cannot afford to lose all the pesky negative signs at all.

Now you may ask, "Why would you coddle these horrible negative numbers? Why don't you just eliminate them?" And dear reader, bless your heart for the question. I, Zulfikar Moinuddin Ahmed, have been clobbering all sorts of negative numbers in inequalities for many decades now. It is most pleasing to clobber the pesky negative numbers. But unfortunately we don't know what sort of cancellations will be necessary here so we have to tolerate them. We're just getting rid of integrations and reducing the estimate to ordinary finite sums of horrible combinatorial expressions.

Now why would anyone want to do something so horrible, you ask? Don't look at me. This I believe is the kernel for summation of Fourier series by Peter Gustav Lejeune Dirichlet, the German apostate who used the excuse of Parisian mathematical activity to abandon his fatherland.

Let me check this. Peter Gustav Lejeune Dirichlet 1805-1859. And Charles Baudelaire was 1821-1867. Baudelaire published *The Flowers Of Evil* in 1857 in Paris. And good, Dirichlet published on the Dirichlet kernel in 1829 [3]. So he managed to do something reasonable before he got corrupted by *The Flowers Of Evil* in 1857 which might explain why he was dead in 1859. He probably couldn't survive it. He should have never left his fatherland. Tsk tsk.

There is a cheap trick I just learned from Wikipedia about how to get $\log(N)$ lower bound.



A precise proof of the first result that $\|D_n\|_{L^1[0,2\pi]} = \Omega(\log n)$ is given by

$$\begin{aligned}\int_0^{2\pi} |D_n(x)|\,dx &\ge \int_0^{\pi} \frac{|\sin[(2n+1)x]|}{x}\,dx \\ &\ge \sum_{k=0}^{2n} \int_{k\pi}^{(k+1)\pi} \frac{|\sin(s)|}{s}\,ds \\ &\ge \left| \sum_{k=0}^{2n} \int_0^{\pi} \frac{\sin(s)}{(k+1)\pi}\,ds \right| \\ &= \frac{2}{\pi} H_{2n+1} \\ &\ge \frac{2}{\pi}\log(2n+1),\end{aligned}$$

where we have used the Taylor series identity that $2/x \le 1/|\sin(x/2)|$ and where H_n are the first-order

This argument gives the lower bound for our problem with quite a bit less work than we're doing. But we are still interested in the upper bound.

Ok we are happy because we were not cheating, and we had found that the breaking up of the integral into $2N+1$ parts was used in the smoothened lower bound too. So that's good. Our delirious wandering in the desert had produced something fruitful after all.

5. Stanford Can You Arrange A Meeting And Bring Bill Gates In?

Bill Gates did so much harm to me that I am now quite enthusiastic about ripping his tongue and pulling it out of its sockets in a public ceremony. Do you think you can arrange a public event like that. I can't wait now. It would be one of the best things for the world. This little miserable cunt deserves to have his tongue ripped out from his body for what he has already done.

You don't mind doing this small favour for Zulf do you? Just get him into a good publicity event, and we shall begin the *Ripping Out A Little Lying Murderous Shit's Tongue Ceremony* and we can have a cable network show on how it's done.

6. Reprise Problem I.2 With Wikipedia Cheating

The elaborate method I used before would lead to an exact evaluation of $\int_0^{2\pi}|D_N(x)|dx$ that would in the end be better than $\log(N)$ but after I saw the Wikipedia proof for lower bound it was instantly clear that my approach was too expensive.

Let me go through the cheaper method that I did not know to get upper bound too. The key is this:

$$\frac{1}{2}|x| \leq |\sin(x)| \leq |x|$$

for some neighborhood of zero. How big a neighborhood? Let's see.

$$\sin(x) - x/2 = x/2 - x^3/3! + x^5/5!$$

and so certainly for where $x/2 \geq x^3/3!$ and that is

$$1/2 \geq x^2/6$$

That's

$$x \leq \sqrt{3} = 1.732$$

That's pretty nice. We could tune this a bit. We could take

$$\frac{1}{10}|x| \leq |\sin(x)| \leq |x|$$

for

$$|x| \leq 2.3238$$

We won't get all the way to π this way but we can get past $\pi/2$. For upper bound we might want to get the integral in $[-\pi/2, \pi/2]$ to use the lower bound

$$\frac{9}{10}|x| \leq |\sin(x)| \leq |x|$$

for all $x \in (-\pi/2, \pi/2)$. This allows us to say

$$\frac{10|\sin((2N+1)x)|}{9|x|} \geq |D_N(x)| \geq |\frac{|\sin((2N+1)x)|}{|x|}$$

We can now get both upper and lower bounds by integrating $\sin(x)/x$ appropriately as in the Wikipedia proof to introduce harmonic sums.

I will have to admit that I had not seen this before at all, so I am not used to this particular set of arguments.

The method I was following is not as good as this one for the result sought here but will yield much sharper bounds than $\log(N)$ and obviously is more elaborate.

I am not particularly ashamed about not having the right approach. It's a learning experience, and besides, it's good to know more efficient exact calculation is possible for $\int |D_N(x)|dx$.

6.1. **My Happiness With Sine Kernels.** Now I will tell you that kernels like $1/\sin(x)$ fill me with happiness. In this problem we estimate

$$\frac{1}{|\sin(x)|}$$

by $1/|x|$. But some years ago I did the reverse, by using $1/\sin(x)$ in the electromagnetic potential and obtained gorgeous L^2 boundedness of molecular Hamiltonians with not Coulomb Potential but the correct Four-Sphere potential.

I will have to tell you, I don't really care about whether I am efficient with the Dirichlet kernel or not because my S4 Electromagnetic Potentials gave me such beautiful results that even Tosio Kato would have been most surprised. Molecular Hamiltonians in Four-Sphere Theory produce compact resolvents, making my Four-Sphere Theory superior to Schroedinger Theory.

7. Problem I.3

(a) Show there is a Lebesgue measurable set $E \subset [0,1]$ with positive measure and empty interior.

(b) Show that if $f : [0,1] \to \mathbf{R}$ is absolutely continuous and A is Lebesgue measurable with measure zero then $f(A)$ is Lebesgue measurable with measure zero.

For the first, I thought I wonder if the 1/3-removed Cantor set would do. Then I realised that the measure of 1/3 removed Cantor set is

$$1 - \frac{1}{3}(1 + 2/3 + (2/3)^2 + \dots)$$

After you're done summing the right side you get

$$frac11 - 2/3 = 3$$

The one-third removed Cantor set has measure zero in the end. Then I hit upon the miraculous cheapo answer. Take the irrationals, stupid. The irrationals will never have any interior because rationals are dense, and it's measurable, with measure 1.

The lesson to be learned is be simple then it won't get you a headache.

Now (b) is a more serious question. It's so serious and elementary that I will spend time on it. First of all what are absolutely continuous functions? There is a technical answer which I will get to. The non-technical answer is that they are proof that mathematicians might be intelligent in a way, but they are linguistically challenged beyond any narcissism that anyone else can even hope to reach. You knew that when you talk to normal cultivated people and mention 'real numbers' and 'rational numbers' and 'irrational numbers'. The right answer, if they are sane will be, "You're not serious. You mean to tell me that $\sqrt{5}$ is a real irrational number to you? What exactly is there so real about it, and why is it irrational? It has difficulties reasoning?" Anyway, "absolutely continuous functions" are a class of functions that are very badly named objects. It's like calling things sort-of-kind-of-continuous, really-pretty-continuous-if-you-ask-me, quasi-continuous-not-really-very-continuous, and positively-continuous, so-so-continuous, and absolutely-continuous. The next in the serious is swear-to-god-absolutely-surely-continuous.

Logic, they say, is a branch of mathematics, but mathematicians do not make any sense when they name things. What a horribly misnamed set of functions.

One characterisation of absolutely continuous functions is that they are members of Sobolev space $W^{1,p=1}([0,1])$ and Haim Brezis gives them.

What are we doing again? We have a Lebesgue measure zero set A and we want to look at $f(A)$ and show that this object has measure zero. We could do it this way. We could say let

$$f(x) = f(0) + \int_0^x g(t)dt$$

where by magical properties of absolutely continuous functions g is the Lebesgue integrable derivative of f. Then we integrate it over A.

$$m(f(A)) = \int_A gdt$$

Now we are done if our magical formula holds because that's Lebesgue integration of a Lebesgue integrable function over a Lebesgue measurable set of Lebesgue measure zero. Boy, this Lebesgue did a pretty good job Lebesgue-ing all parts of integration theory. So this is done modulo nontrivial claim about existence of this magical $g \in L^1([0,1])$. I invoke generally famous theorem about such a thing.

Now I am most intrigued. I want my name to be repeated ad infinitum in the future of human race so that I am immortal. What was Henri Lebesgue's trick exactly? I need to look into this.

8. Zulf Thinks That There Exists a Great Project Of Rebuilding Analysis From Foundations

Analysis really began with 1660s and Newton and Leibniz's work and it's around 350 years and various sorts of things have happened to produce the Analysis that exists today. For some people who have lived and breathed Analysis it will be quite clear that in the end, the whole of Analysis is quite small, only a small number of things matter, only a small number of concepts have been fruitful. It is possible to shore up Analysis in totality and rebuild it with finer foundations and gain clearer understanding of the whole of Analysis in a generation of work by a large group of mathematicians. I do not think it is worthwhile unless people had enormous commitment to Analysis and enormous enthusiasm and love for the subject to do it. But I think it would be worthwhile. Analysis is a living subject, and it is old, and there are many great analysts alive.

I think it would be fabulous but I don't think people like to do that because they want to make advances not rework old things. But it is genuinely worthwhile, because many many many concepts of Analysis are not wise enough, not properly understood, not able to produce deeper understanding of the mathematical substance, etc.

Entire nations need to be interested with commitment to fund and ensure the livelihood of thousands of people in order for something like this to succeed. If China and India and others want to do this, you can. It's not a bad idea, but you do need to have extraordinary historical scholarship for the entire period from 1660s-2020s without gaps. You need all the papers ready for analysis, you need to produce institutes where scholars can collaborate, and you need to be connected so that you understand what is easy and what is hard for people who are not thousand-year experts in esoterica.

Unless you are willing to put in resources and back monumental projects, you should stay away from it. Analysis grew organically and so it's disorganised. Many individual mathematicians put in some order to it but these are difficult demanding things, and so this cannot be done by small efforts.

If you decide to do it, it's good. But don't botch the project by random incompetent crisis of funding, mismanagement and mistreatment of people who will dedicate their lives to having better mathematics for all human beings, etc. Don't do monumental projects unless you are sure that you will not pull out suddenly and decide this is unnecessary expense leaving a lot of talented people in vulnerable situations. That's totally obnoxious.

9. Zulf's Eastern Gambit

Fine Bill Gates. We'll take your bait. China and India, could you please consider if you could double the salaries of every single significant Mathematician in Europe, Russia, and United States and give them luxury accomodations and tenure level treatment in luxurious universities in the East and siphon every single good mathematician and scientist from the West. Bill Gates will be left with his skeleton crew of white supremacist inferior dipshits doing their thing in the West afterwards and instead of learning serious Mathematics or Science they will learn how to repeat 'whites are superior' and suck Bill Gates' dick.

Don't be tardy. Let's play the blues China and India. I don't have all day.

10. Problem I.4

Suppose $(X, \mathcal{T}_{norm})$ be a Banach Space with the norm topology, and let $(X, \mathcal{T}_{weak})$ be the same space with weak topology. Let $(Y, \mathcal{U}_{norm})$ and $(Y, \mathcal{U}_{weak})$ be another Banach space with norm and weak topologies.

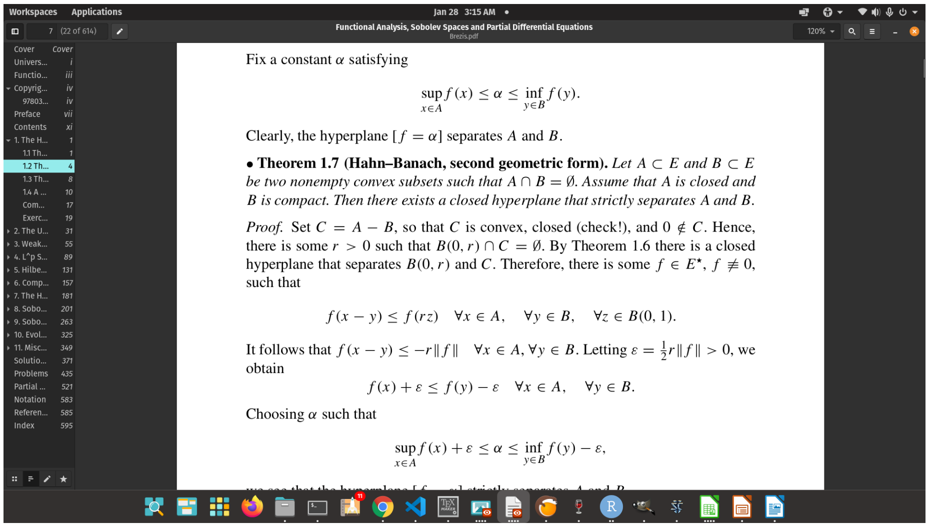
(a) Prove the separability condition that if $x \in X$ and $C \subset X$ is $\mathcal{T}_{weak}$-closed, and $x \notin C$ there is a continuous function $f : X \to [0, 1]$ with $f(x) = 1$ and $f(y) = 0$ for all $y \in C$.

(b) A linear map $T : X \to Y$ is continuous for $(X, \mathcal{T}_{weak})$ and $(Y, \mathcal{U}_{weak})$ if and only if it is continuous for $(X, \mathcal{T}_{norm})$ and $(Y, \mathcal{U}_{norm})$.

10.1. Geometric Form Of Hahn-Banach Theorem.

10.1. Geometric Form Of Hahn-Banach Theorem. We will attempt to apply the geometric form of the Hahn-Banach theorem to produce this function. In these situations what we do is refer to Haim Brezis precise formulation of Geometric Hahn-Banach because once we obtain a linear functional, we can massage it to a continuous function with values in $[0, 1]$.

Now the geometric Hahn-Banach theorem is valid only for *convex* closed sets and even then they are fussy. Let us add the 'convex' to the assumptions in the problem. We do this because we want to apply Haim Brezis Theorem 1.7 without change first.



Fix a constant α satisfying

$$\sup_{x\in A} f(x) \le \alpha \le \inf_{y\in B} f(y).$$

Clearly, the hyperplane $[f = \alpha]$ separates A and B.

• **Theorem 1.7 (Hahn–Banach, second geometric form).** *Let $A \subset E$ and $B \subset E$ be two nonempty convex subsets such that $A \cap B = \emptyset$. Assume that A is closed and B is compact. Then there exists a closed hyperplane that strictly separates A and B.*

Proof. Set $C = A - B$, so that C is convex, closed (check!), and $0 \notin C$. Hence, there is some $r > 0$ such that $B(0, r) \cap C = \emptyset$. By Theorem 1.6 there is a closed hyperplane that separates $B(0, r)$ and C. Therefore, there is some $f \in E^\star$, $f \not\equiv 0$, such that

$$f(x - y) \le f(rz) \quad \forall x \in A, \quad \forall y \in B, \quad \forall z \in B(0, 1).$$

It follows that $f(x - y) \le -r\|f\| \quad \forall x \in A, \forall y \in B$. Letting $\varepsilon = \frac{1}{2}r\|f\| > 0$, we obtain

$$f(x) + \varepsilon \le f(y) - \varepsilon \quad \forall x \in A, \quad \forall y \in B.$$

Choosing α such that

$$\sup_{x\in A} f(x) + \varepsilon \le \alpha \le \inf_{y\in B} f(y) - \varepsilon,$$

We get a linear functional $\phi \in X^*$ and an a so that

$$\phi(x_0) \le a \le inf_{y\in C}\phi(y)$$

We can massage this into a continuous function $\tilde{\phi} : X \to \mathbf{R}$ with $\tilde{\phi}(x_0) = 1$ and $\tilde{\phi} = 0$ on C as follows. Assuming $\phi(x_0) \neq 0$ just let

$$b = \inf_{y\in C} \phi(x)$$

and

$$\tilde{\phi}(x) = \begin{cases} frac\phi(x) - b\phi(x_0) - b & \phi(x) \in (\phi(x_0), \inf_C \phi) \\ 1 & \phi(x) < \phi(x_0) \\ 0 & \phi(x) \geq b \end{cases}$$

This massaging just puts C in a larger set calibrated by ϕ.

The separability is then proved for convex sets C. For more general situations we would be a bit worried about this separability.

Suppose C is a union of finite number of convex closed sets $C = \bigcup_{k=1}^{N} C_k$. Our construction gives us $F_1, dots, F_N$ that separates x_0 and each of these C_k. We could try their product $F(x) = F_1(x) \ldots F_N(x)$ which would have $F(x_0) = 1$ and $F(y) = 0$ for all $y \in C_k$. So for these sorts of finite unions of convex sets we're fine. I am not sure about countable unions.

11. Why I Am Extremely Conservative On Analysis Technicalities

You see I am 49 now and I like to preserve my dignity and self-respect. I am free-wheeling when my confidence is high that I am actually right about what I am saying. This happens quite often in geometric matters or topological in low dimensions. But with infinite dimensional spaces and all sorts of duals and other knick-knacks I do not know for sure whether I am right very often so I am conservative.

You see, mistakes are costly. The moment you make a mistake some smart alec young punk kid will give you all sorts of lectures about how this *exact situation* was handled by the Polish schools who have all very long unpronouncable names and had, for your information had written very thick tomes on all sorts of special points in Banach Space named after them and how I ought to have referred to their impenetrable papers and educated myself. I don't want those situations. I don't know why anyone would spend their entire careers on extremely arcane issues of this type. But they do, and I don't want to be stuck having to read their works. So I am very conservative.

Fine, fine Stefan Banach and Frigyes Riesz and various other Eastern European mathematicians are superbly expert at these things. Good, good. Let them do these and I would rather take the light labour approach.

11.1. **Weak Topology Continuity For Linear Operators.** This problem is one of these deceptive problems that are outrageously difficult. Whenever 'topology' is mentioned in these functional analytic contexts, it bodes massive amounts of technical mumbo-jumbo. You see I know a lot about 'real genuine authentic topology' and these are for topological manifolds. All the topologies of infinite dimensional function spaces are not really genuine authentic topologies. They are technical mumbo jumbo pure and simple. A serious question is whether these are actually *valuable*. What is going for them is that bounded operators are continuous usually. So that's good. But there is a serious question about whether the world would have been better off if topology was not abused and maltreated so much as all these outrageous notions of open sets.

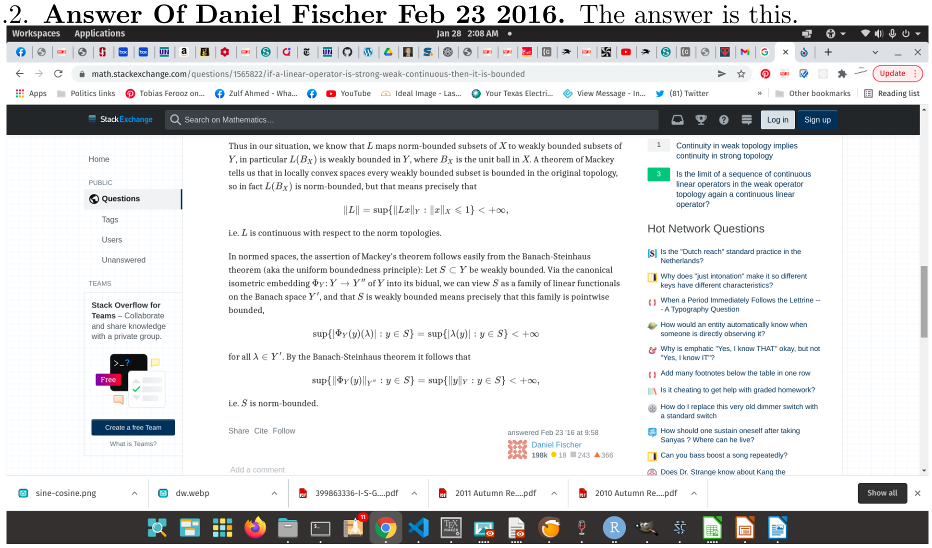
Let's try this. Suppose $T : X \to Y$ is weakly continuous. Let us consider this to mean that if $x_j \to x$ weakly in X then $Tx_j \to Tx$ weakly in Y. Our task is to prove from this that if $x_j \to x$ strongly in X then $Tx_j \to Tx$ strongly in Y.

Our assumption if $Tx_j \to Tx$ weakly in Y. For every $f \in Y^*$ we have

$$\lim_{j\to\infty} \langle f, T(x_j - x)\rangle = 0.$$

We have to understand how this has to do with strong convergence.

11.2. **Answer Of Daniel Fischer Feb 23 2016.** The answer is this.



I am glad I did not even try to do anything here because I had never thought about such exciting topics as weakly bounded subsets being strongly bounded by Banach-Steinhaus theorem. So that's the key to this problem. You need to have learned this fact and if you don't you will not get anywhere at all. That is what

I really dislike about these weak topologies. You need to know all sorts of esoteric theorems that you never heard of in your entire life.

I am giving myself an A+ for not putting effort in the problem I.4(b). Are you kidding me? I'm not going to drive myself insane on something that needs totally arcane esoteric knowledge. Bravo Daniel Fischer. You are a good man.

The argument to use is this. We suppose $B \subset X$ is weakly bounded. Then we have $T(B)$ is weakly bounded. Then we look at $Y \subset Y^{**}$ by the bidual embedding $j : Y \to Y^{**}$ and consider

$$\sup\{|\langle j(y), v\rangle| : y \in T(B)\} < \infty$$

for all $v \in Y^*$. Then we apply Banach-Steinhaus theorem to get bound on

$$\sup\{\|y\| : y \in T(B)\}$$

and that gives us strong boundedness.

This argument is clean and smooth but it's part of the field. I can probably remember it but I would not have discovered this myself because weak topology, weak convergence and weak boundedness are not intuitive for me at all. They are part of an intuition set that is distant from where my intuition sees deeply – such as universal human nature, or absolute space and time and single law for nature. In the latter cases my genius is immortal but on weak topologies on Banach or Locally Convex Vector Spaces it is very weak.

12. On Conscientiousness In Credit

I am supremely conscientious about giving credit to anyone at all whose work I use, even if I get something from the internet. The reasons are several.

First, I am extremely fussy about this now at 49 having had various problems with getting credit for my work. And I am extremely confident now that my four-sphere theory is absolute truth of Nature, infinitely superior as science than relativity theory, quantum field theory and expansionary cosmology and I do not want the credit for it to be distributed till it is established and I am acknowledged as the sole originator and the world understands that this is work that far exceeds the genius of Isaac Newton, Albert Einstein, James Clerk Maxwell, Erwin Schroedinger and so on. I am closer to death now and I want my immortal genius to be untainted for *my own work*. I have spent some of the most productive years of my life 2008-2018 working on four-sphere theory and universal human moral nature without *any compensation at all* and I do not want this to be squandered by petty pilfering of credit for other people's work because karma is a bitch and it will hurt me.

But there is also basic Honour Code. Princeton University did a great job in cultivating myself and all other students to take Honour Code and Character issues seriously. I do not respect people who pass off other people's work as their own. Even Leonhard Euler I do not respect for publishing continuously work on the vibrating string problem breaking the trust of Jean le Rond D'Alembert who deserves the full credit for originating the wave equation.

Why don't I even try to do problems like I.4(b)? Well you see my time is precious. This is not original research anymore, whether weakly continuous operators between Banach Spaces are strongly continuous. If it is a Theorem of Mackey, then it won't be a theorem of Zulfikar Moinuddin Ahmed even if I do a very nice proof. It's better in this case just to learn what the experts already know. There is no added benefit for re-inventing the wheel here; at least at 49 it is worthless.

13. Problem I.5 Fredholm Operators

Suppose X, Y are Hilbert spaces. An operator $A \in L(X, Y)$ is called *Fredholm* if A has closed range and both $Ker(A)$ and $Y/Ran(A)$ are finite dimensional.

(a) Show that $A \in L(X, Y)$ is Fredholm iff there are finite dimensional vector spaces V, W and a finite rank operator $P \in L(W \oplus X, V \oplus Y)$ such that $\bar{A} + P$ is invertible where $\bar{A}(w, x) = (0, Ax)$.

13.1. Preliminary Comments.

In one way or another, the issues surrounding Fredholm operators are all about $I - K$ for K a compact operator. For me, organising the issues in a solid reliable manner has been a challenge. I will take this opportunity to attempt to this because the particular problem is not as important as the way in which the most clear and detailed understanding of the situation is absorbed and stored within us.

First the general idea is that compact operators between Hilbert spaces are norm-limits of finite rank operators. The second general idea is that compact perturbations of the identity in $L(X)$ give us some opportunities to solve some equations of interest. The integral equations that spawned much of functional analysis in 1900 was Erik Ivar Fredholm's efforts are of this type:

$$f(x) - \int_a^b K(x, y) f(y) dy = g(x)$$

where $g(x)$ is known and the goal is to find $f(x)$ that solves this equation. This equation is of form

$$(I - K)f = g$$

So why can't you just write

$$f = (I - K)^{-1} g$$

The subtleties of what you can do in these cases to write $f = (I - K)^{-1}g$ led to all the theory and techniques of Fredholm operators. For me this story is helpful to understand what all issues of Fredholm operators are. Otherwise who cares about closed rank and finite dimensionality of kernel and cokernel and other technical conditions?

All the results of Fredholm Operators exist to give clarity regarding when there is a solution to an equation $Af = g$ which will be $f = A^{-1}g$ and how unique it is and so on.

The problem I.5(a) seeks to understand invertibility, and Fredholm operators can be perturbed by a *finite rank* operator to produce solutions. This means that we can have an equation

$$Af = g$$

and we don't know how to solve it. But we can prove that some other equation

$$(\bar{A} + P)f = g$$

can be solved suitably massaging f and g and P is going to be finite-rank so we consider the perturbation P minor in some way and declare $(\bar{A} + P)^{-1}g$ to be the solution of our problem and celebrate victory.

Now some people will say, "How is this verbiage substantial?" You see, people do not understand something. If you are trying to solve an equation

$$Af = g$$

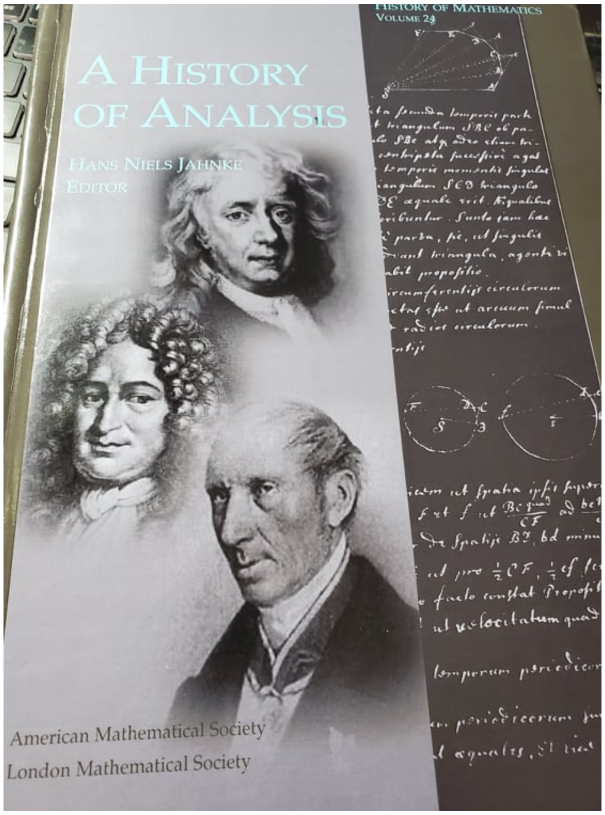
that is *respectable*. You might use the solution to improve lives of billions of people. You might produce from this solution medicine supposed to designed to keep all people high, and you might get as a result garlands and rose petals from a hundred thousand beautiful maidens who accept you as a great benefactor of humanity, give you hero's welcome. That's substantial. No one gives a damn about all manner of Mickey Mouse technicality of how you fidgeted to solve the equation. That is the point. Unless the technical issues lead to solving problems that have substance in the end, these are wastes of our precious time on Earth.

Don't believe in the lies. Pure technical mumbo jumbo for its own purity's sake is total rubbish. There is substance and there is worthless mumbo-jumbo and the entire effort of education is to discern the difference.

13.2. **Return To Philosophical Issues That Are Significant.** Let me compare two areas of Mathematics to make sense of what I am trying to express. I compare Number Theory and Analysis. In Number Theory deep understanding has been gained over the centuries and extraordinary theorems have been proven giving us understanding of structure of prime numbers in $\mathbf{Z}$. But Analysis has always been confused and there has always been a nebulous sense. One has never known what the depths of Analysis are either for Mathematical Purity or for representation of Nature.

I hold the strong view that all deep Mathematics, *including Number Theory* are explorations of Nature. In fact every atom and molecule in Nature are constantly involved in Arithmetic operations in energy levels. This is not yet clear to the world. The ineffectual division between Pure and Applied has led to disaster in Mathematics where one does not know which is which and so there is a horrible confusion. I do not think there is any substance in the end in this division. That is the basis of my Four-Sphere Theory, that there is just one knowledge of Nature, and we human beings have artificially divided things for convenience that ultimately hurts us.

When I study history of Analysis from a recent book such as this one here:



I do not have a sense at all of the coherence that I feel deep within my soul of the unity that exists in Nature that *ought to be perfectly reflected* in Mathematics and all our intellectual efforts to decipher the deepest structure of Nature at all.

This is source of considering some things in Mathematics to be purely technical mumbo-jumbo. It is because there is deeper coherence in Man's Understanding of Nature that is obscured without greater structure that we have *not yet discovered*.

Of course part of the problem is that Mathematics and other intellectual disciplines are hard and we make progress slowly. Centuries go by before progress is made on some issues. Technical progress has its value, but that's only instrumental in nature. The lack of coherence hurts all fields of knowledge and is amended very slowly.

In a way the problem we face today is Anarchy and confusion rather than one faced by Lagrange and d'Alembert, who had felt that perhaps Analysis had exhausted itself and there was no further development. D'Alembert was encouraging of Mathematicians to keep going, and in fact Enlightenment's key proponents were those who were faithful about *ability of Mathematics* to further human ambitions. Their concerns were shown to be misguided. Now there is a gigantic and impossible spread of Mathematics. Today the concern I bring up is the natural one for today, that of unity and coherence, in a sense the ones quite opposite of d'Alembert and Lagrange.

13.3. **Invertibility of $\bar{A} + P$ implies A Fredholm.** Suppose

- $A \in L(X, Y)$

- V, W are finite dimensional
- $P \in L(W \oplus X, V \oplus Y)$ is finite rank
- $\bar{A}(w, x) = (0, Ax)$ for all $(w, x) \in W \oplus X$
- $\bar{A} + P$ is invertible

Our goal is to prove that A has the following properties

- A has closed range
- $Ker(A)$ is finite dimensional
- $Y/Ran(Y)$ is finite dimensional

This is one direction for Problem I.5(a). We are going to consider this problem in slow steps without rushing. Let us examine what we can prove about $\bar{A}$ rather than about A first.

Our assumption is that $\bar{A} + P$ is invertible. Let $Q = (\bar{A} + P)^{-1}$ and we know that $Q \in L(V \oplus Y, W \oplus X)$.

Does $\bar{A}$ have closed range? Suppose $z_n \to z$ in $W \oplus X$. We write $z_n = (w_n, x_n)$ and $z = (w, x)$. Since $\bar{A} + P$ is invertible, it is continuous, injective and surjective, so it has closed range $V \oplus Y$. So we obtain

$$\lim_{n\to\infty} (\bar{A} + P)(z_n) = (\bar{A} + P)(z) = \bar{A}(z) + P(z)$$

Since P is finite rank the range is closed. Therefore $P(z_n) \to P(z)$. Therefore $\bar{A}(z_n) = (\bar{A} + P)(z_n) - P(z_n) \to \bar{A}(z)$. Good so we now have established $\bar{A}$ has closed range.

Suppose $z \in Ker(\bar{A})$. Then $\bar{A}(z) = 0$. When $z \neq 0$ there exists nonzero $w \in V \oplus Y$ with

$$z = (\bar{A} + P)^{-1} w$$

Then

$$\bar{A}(\bar{A} + P)^{-1}(w) = 0$$

this implies

$$w - P(\bar{A} + P)^{-1} w = 0$$

In other words w is in the range of P. Since P is finite rank, the range is finite dimensional and therefore $Ker(\bar{A})$ is finite dimensional.

Next, let us check that the cokernel of $\bar{A}$ is finite dimensional as well. Since $\bar{A}$ has closed range, we claim that any element $w \in V \oplus Y$ that does not belong to the range belongs to the range of P. If this is true then we have finite dimensional cokernel for $\bar{A}$.

The above considerations will prove that $\bar{A}$ is a Fredholm operator. Now we will want to argue that $\bar{A}|_X$ is also Fredholm and that will complete this direction.

13.4. **Proving Fredolm A Implies Invertibility of $\bar{A} + P$.** Our goal in this section is to produce various finite dimensional spaces V, W and finite rank operator P so that $\bar{A} + P$ is invertible.

The idea is quite simple. First use the fact that A has closed range to produce a decomposition $X = H \oplus X_0$ and $Y = G \oplus Y_0$ with $H = Ker(A)$ and $G = Coker(A)$. Then $W \simeq G$ and $V \simeq H$.

Then the idea is that $H \oplus W$ is isomorphic to $G \oplus V$. The operator A is an isomorphism between X_0 and Y_0, zero on $H \oplus W$ and we *construct* P to be the operator that is an isomorphism on $H \oplus W$ to $G \oplus V$ but zero on the orthocomplement; obviously P will be finite rank. Then $\bar{A} + P$ will be an invertible isomorphism.

This path works because of the Fredholm properties of A.

13.5. **Problem I.5(b).** We are to show that small perturbations of Fredholm Operators are Fredholm. Suppose $A \in L(X, Y)$ is a Fredholm operator. We are to show that there exists a $\delta > 0$ such that all bounded operators $R \in L(X, Y)$ with $\|R\| < \delta$ lead to $A + R$ also Fredholm.

Let me see here. We have to preserve closed range, finite dimensionality of kernel and cokernel.

I have an idea which might be the purpose of this exercise. We'll use the characterisation of I.5(a) to prove (b). Let us posit $\bar{R}$ defined on $L(W \oplus X, V \oplus Y)$ and let us also assume a correspondence between norms of R and $\bar{R}$.

We begin with $\bar{A}+P$ is invertible. We are interested in invertibility of $\bar{A}+\bar{R}+P$. We write the series expansion

$$(\bar{A} + P)^{-1}(\sum_{k=0}^{\infty}(-R)^k$$

This is the inverse of $\bar{A} + \bar{R} + P$ so long as there is norm-convergence of the series which happens for $\| - \bar{R}\| = \|\bar{R}\| < \delta$ for some $\delta > 0$. This then proves $\bar{A} + \bar{R}$ is Fredholm whenever $\|\bar{R}\| < \delta$.

Now we just unravel the infrastructure and translate back to $R \in L(X, Y)$. This proves (b).

13.6. **Marvelous Work Stanford Qual Directors.** This particular decomposition of the Fredholm conditions is very strong. I do not know who invented this scheme, but it is actually worthwhile considering as sharp infrastructure for *eternal presentation of Fredholm operators* because it is clear that this going back and forth with invertibility in $L(W \oplus X, V \oplus Y)$ is canonical and allows us to actually gain tractable analytic control. This is really superb work on your part.

14. Inevitable Total Destruction Of Bill Gates Reputation

You see, Bill Gates is an arrogant son of a bitch who *does not do any serious intellectual work*. Instead he plays around with power, and is obsessed with illegitimate power such as use of US War Power and US Industrial Power and White Racial Power to harm Asian-Americans like me and sabotage our money and livelihood. He blockaded $620 million I earned from D. E. Shaw & Co. and I am living with my aunt on disability income without even my own pad. And he believes that Hammurabi was white and he pretends he is a Master and I am his Slave and other rubbish. I have asked him to show us he has any *coding skills* and he refuses claiming it's too trivial. Then I asked him to compete with me in doing Stanford Math Ph.D. Qual Problems and that is too trivial for him too. He's a wretched worthless illiterate hick who was always a charlatan, a thief, a robber and racial murderer, scum of the Earth. He is no intellectual luminary. So obviously his reputation will be savaged by me. I am actually talented in Mathematics with a legitimate degree from Princeton University and I am responsible for Four-Sphere Theory which is a matter of *immortal genius*. These punk charalatans like Bill Gates who hoodwinked the world are not even able to understand how low they are compared to me.

Now most people would not try to destroy the reputation of a charlatan like him, but his insolence and harm to me crossed the limits of my patience so I am eliminating him from all of history.

15. Some Facts About My Relation To Bill Gates

I have *never* been an employee of either Bill Gates or any of the organisations that he has any legitimate stake at all. I despise the man and consider him totally criminal, malevolent, hick, uncultivated, totally disgusting, and worthy of exile from the land of the living from his birth. I would *never ever ever* work for someone who is so vile, uncouth, hick, lacking in education and cultivation from boondocks of Pacific Northwest which is barbarian frontier zone from where you can see the end of the world. He is positively disgusting and lacking in any virtue, a total charlatan and ugly in all things he touches. Microsoft code is akin to slaughtering a cat in the kitchen. I hate their products and think their code is horrible. Maybe now they have some good people but I did work with Microsoft NT code and it is dreadful code. I am a Linux man myself.

Even early 2019 I did not know anything about Bill Gates. I founded a company and emailed a lot of people seeking funding. Bill Gates assaulted me in *meta* only right when I had success with my Finance work and could get back on my feet and succeed in my American Dream.

Bull Gates represents the minority political view of around 15% of white Americans who are racial. I am Asian-American. The racial whites are let's see, around 9% of the total population and Asians are just a bit less. But they are mostly ill educated and are not democratically going to be powerful in America. I am firmly American, and have had Permanent Residency from 1997. My USCIS# 046-077-179. I consider Bill Gates to be unworthy and total charlatan all his life. I do not accept his great genius from having done production level software development from 1995 and I know instantly that this man is *not* a good coder. No good coder refuses to show his code with a lot of 'trivia' and other deflections.

16. Strong Claim That Bill Gates Is Unable To Do A Single Stanford Analysis Problem At All

It is rare for me to have extreme conviction about the inability and incompetence of anyone at all. I am generally strongly in favour of the view that *anyone* will accomplish great mathematical achievements if they put in the effort and have the interest to do this.

But despite this I have been dealing with Bill Gates for more than a year now and I have absolutely strongest possible conviction that the man is totally incompetent in Mathematics and will be totally incapable of doing even the simplest problem from any of the Stanford Ph.D. Qualifications. He is literally the intellectual level of an imbecile and a bird-brain. He thinks high school arrogance dismissing all things that are literally *beyond his ability to comprehend* are *trivia*. Parrots can call things trivia. I am far too old to put up with this rubbish. Eliminate the worthless intellectually totally inept illiterate wretched son of a bitch immediately.

Bill Gates, if you want even a modicum of respect for your intellect, prove it to us. Show your actual code. Show your actual ability to solve any Analysis Qual Problem at Stanford. No one thinks that you are an intellectual genius. We believe you are a total charlatan and intellectually inept.

17. Holistic Assessment Is Important For Young People Too

In Humanities, wide assessment is valued even for younger people. In Humanities it is absolutely essential that cultivation has expansive view of things. In History and in Literary Studies, these are emphasized even for undergraduates. In Mathematics and Science there is a tendency towards excessive focus on technicalities, and often expansive views are denigrated. This is a grave error. Expansive view and ability to analyse and assess the history and development, even in human context is necessary in Mathematics and Sciences as well. These require different intellectual skills than those that often resolve particular technical problems. I feel that today, the expansive issues are not part of the course of education. Only the top Mathematicians and Nobel Prize winning scientists are 'tolerated' if they have expansive views and this is then considered matters of vanity. This is an egregious error for the human race and our cultivation. All students of Science and Mathematics benefit from spending at least some part of their effort on expansive views and merging their own specialist work with the longer intellectual traditions that are still evolving. I will have more to say about this perhaps in the future.

I am fortunate to have attended Princeton University between 1991-1995 and therefore benefitted from great learning and cultivation of Humanities as well as Mathematics, and I would advocate that Mathematics and Science generally adopt a Humanistic Perspective as well.

Science is really on 350 years old in its modern incarnation, and there are millions of things on which the level of confusion is very high in Science still. In the next 10,000 years we will see great transformations of Science as well as Mathematics. It is worthwhile to consider the unity of intellectual direction of the entire Human Civilisation and harmonious relationships between people who dedicate their lives to furthering the intellectual abilities of our people as a significant common good across the world. This cannot but have a positive effect on the future of our peoples.

18. Problem II.1

(a) Show that the spectrum of a bounded linear operator $A \in L(X)$ is nonempty for Banach space X.

We assume that it is empty for some $A \in L(X)$. Then for all $z \in \mathbf{C}$ we have $(z-A)^{-1}$ is bounded. For every $x \in X$ we have

$$\|(z-A)^{-1}x\| < \infty$$

for all $z \in \mathbf{C}$. We consider this as a family of operators $T_z(x) = (z-A)(x)$ and apply Banach-Steinhaus Uniform Boundedness Principle. We obtain

$$\sup_{z \in \mathbf{C}} \|T_z\| \leq C < \infty$$

And this produces a contradiction with the fact that bounded holomorphic functions are constant on $\mathbf{C}$.

(b) For $g \in L^q(\mathbf{R}^n)$ define the operator $T_g : L^p(\mathbf{R}^n) \to L^p(\mathbf{R}^n)$ by

$$T_g f = f * g$$

Then we have

$$\|f * g\|_1 = \int |\int f(x-y)g(y)dy|dx$$

Now
$$\int |\int f(x-y)g(y)dy|dx \leq \int\int |f(x-y)g(y)|dydx$$
Just define $f_x(y) = f(x-y)$ and apply Hölder's inequality in the inner integral. Then we get
$$\int\int |f(x-y)g(y)|dydx \leq \|g\|_q \int \|f_x\|_p dx$$
Now that $\|g\|_q$ is not an issue we need to prove that $\int \|f_x\|_p dx \leq \|f\|_p$ and that is some minor manipulation.

19. Problem II.2

(a) Show that every $f \in C[0,1]$ is the uniform limit of elements of P the set of continuous piecewise affine functions on $[0,1]$.

Let N be some large number and consider the partition of $[0,1]$ by
$$x_k = frackN$$
for $k = 0, 1, \ldots, N$. Our idea is that for any $\epsilon > 0$ we will choose N such that on $[x_k, x_{k+1}]$ we have
$$|f(y) - f(x_k)| \leq \epsilon$$
Then we'll call the approximation the lines joining the points $(x_k, f(x_k))$ on the strip $[0,1] \times \mathbf{R}$. The linear interpolation will produce points that are uniformly closer to f than ϵ and the interpolated function is a member of P.

(b) The includion map $j : C[0,1] \to L^1[0,1]$ is not compact.

We need to exhibit a bounded set $B \in C[0,1]$ such that there is sequence that does not have any L^1 convergence.

We could do this as follows. Consider functions that are a countable family defined by two parameters. The idea comes from wavelet theory. First parameter is a level $L = 1, 2, \ldots$. With this parameter we divide up $[0,1]$ into $N_L = 2^L$ equal intervals. The second paramerer goes through $a = 0, 1, dots, N_L$. Then with index (L, a) we consider the functions that are *tents* of $(a2^{-L}, (a+1)2^{-L)}$. By this I mean they are 1 at the midpoint and zero at the endpoints by linear interpolation. Let's call these $q_{L,a}$

These have the property that given any level, all the elements have lower found on $\|q_{L,a} - q_{L,b}\|_1 \geq \delta(L, a, b)$ which are some positive numbers. It is impossible to find a L^1 Cauchy subsequence for this family because given any $\epsilon > 0$ you can find some L such that all the $q_{L,a}$ differ more than ϵ in L^1 metric distance. This shows this family is bounded in $C[0,1]$ but not precompact in $L^1[0,1]$.

We want to clarify that we need the family to include all *combinations* of the $q_{L,a}$ in the sense that all the superpositions of the tents are included. Then the L^1 differences at arbitrary level L will include values that are larger than any specified $\epsilon > 0$. The family is still countable and bounded.

20. Problem II.3

Let L be the operator on $\ell^2(\mathbf{Z})$ that is multiplication by n. Let $R \in L(\ell^2(\mathbf{Z}))$. Let
$$H = \{a \in \ell^2(\mathbf{Z}) : na_n \in \ell^2(\mathbf{Z})\}$$

and let

$$\|a\|_H^2 = \sum_{n\in\mathbf{Z}} (1+n^2)a_n^2$$

The problem is to examine the spectrum of $L+R$. More specifically, since this is not defined on the same space, the problem is to examine discreteness points $z \in \mathbf{C}$ such that $L+R-z$ is not invertible.

(a) Prove that there is a discrete set $D \in \mathbf{C}$ such that $L+R-z : H \to \ell^2$ is invertible.

20.1. **Examination Of The Situation.** Let $z \in \mathbf{C}$ be arbitrary. Now before we consider anything exotic, let us just take a look at the following.

Let $R \in L(\ell^2(\mathbf{Z}))$. Let's assume $\|R\| < 1$. Then

$$(I+R)^{-1} = \sum_{k=0}^{\infty} (-1)^k R^k$$

and the there is invertibility with the norm restriction.

Second we claim that if $z \in \mathbf{C}$ is not in $\mathbf{Z}$ then $L-z$ is invertible.

If $z \notin \mathbf{Z}$ then for all elements we have

$$M(a_n) = (n-z)^1 a_n$$

well-defined, and we also have

$$\|M\| \leq \sup_n |(n-z)|^{-1} < \infty$$

It is easy to check that $M(L-z) = I$; furthermore $Le_n = ne_n$ so we have the exact "spectrum" for L.

We are interested in proving therefore that the spectrum of L perturbed by a bounded operator R is also discrete.

This is, for those who are not aware, the central problem of *perturbation theory of Linear Operators* of which the great master was Tosio Kato. I am familiar with some of this.

20.2. **Zulf Tells The Story of Discrete Spectra of Molecular Hamiltonians.** Some years ago, it ought to have been sometime between 2014-2018, I got interested in challenging Schroedinger's Quantum Theory. I was sure that my S4 Electromagnetic Law, and I prefer it be called Ahmed-d'Alembert Law, was the fundamental law of nature and not Schroedinger's Law.

So I study this problem for a while, and then I discover beautiful papers of *Barry Simon*, fascinating accounts of how Schroedinger's Theory produces horrible situations for multiparticle systems with all manner of monstrous horrors. I then showed that for molecular Hamiltonians my four-sphere theory produces compact self-adjoint resolvents that are bounded in L^2. This was the great moment I knew that my Four-Sphere Theory was the correct theory of physics and Schroedinger's Quantum Theory was the wrong theory.

As the dear reader might imagine, I am extremely kind to spectral theory as a result, and treat it with gentle kindness and veneration.

20.3. **A Nicer Problem.** We can see that the 'spectrum' of L and we put it in quotes because L is not in any sort of $L(X)$ for any Hilbert space X so the 'spectrum' is not the ordinary definition. It's an unbounded operator. Fine fine. The problem just duct taped the Fourier series of d/dx to avoid issues of unbounded operators.

We know then that $z = 0, 1, 2, 3, \ldots$ are in the spectrum of L and it is interesting to do the following. We take a parameter $\varepsilon \in [0, 1]$ say and we take a bounded linear operator $R \in L(\ell^2(\mathbf{Z}))$. and we consider

$$L_\varepsilon = L + \varepsilon R$$

So we just put L in a nice one parameter family of operators. Then we say, "Is it true that the eigenvalues $\lambda_k(\varepsilon)$ must be the same as those of L when $\varepsilon = 0$ and then vary smoothly as a function of ε?"

This is the fundamental problem of perturbation theory. This would be a nice refinement of Problem II.3(a) because we are looking for smoothness in the variation. The conclusion would be that the eigenvalues of $L(\varepsilon)$ are discrete for $0 \leq \varepsilon \leq 1$.

We are not done with this problem yet. Stay tuned.

20.4. **Problem II.3(b).** We are given $V \in C(\mathbf{T})$ and we are asked to prove that the set of $\lambda \in \mathbf{Z}$ such that

$$f' + Vf = \lambda f$$

has any solution are discrete.

We let $Af = f' + Vf$ and conjugate it by Fourier transform so $FAF^{-1} = L + R$ where $Ra_n = V\hat{F(a_k)}$. The formality is not important. The key point is that Problem II.3(a) operators allow us to appreciate the spectral properties. Then we just apply the discreteness result of Problem II.3(a) and complete this problem.

21. Quantum Field Theory Has A Lot Of Problems

For me after more than a decade working on my four-sphere theory it is impossible for me to imagine that quantum field theory has any ability to compete with my four-sphere theory. The issue is that quantum electrodynamics stands on layers of errors. Special Relativity is *totally wrong.* Time dilation is a figment of people's imagination. Time is linear and does not deform at all. Relativity theory was a solution to lack of aether. You see, Michelson-Morley experiments only removed a specific sort of aether from nature, and that is a three-dimensional aether. Four-sphere theory is based on a much more robust concept of aether that is a four-sphere aether, one that Michelson-Morley experiments do not remove at all. It is a theory whether the background geometry of all of existence is perfect round four-sphere and aether is its surface. Then the ordinary three-dimensions are emergent. The compact geometry of the four-sphere is what allows perturbation theory to provide valid results here.

Nature does not, and indeed, cannot, provide any noncompact flat infinite $\mathbf{R}^3$. This sort of concept was Euclid's conception. Until my four-sphere theory, there was no homogeneous replacement of $\mathbf{R}^3$ at all. But with the empirical results I have shown, there is no question at all that four-sphere theory is the correct theory of Nature, and we must abandon quantum field theory, general relativity and expansionary cosmology once and for all.

22. Problem II.4

(a) Show including explicit constant that for all $\varphi \in C_0^\infty(\mathbf{R})$ we have

$$\|\varphi\|_\infty \le \frac{1}{2}\int_{-\infty}^{\infty} |\varphi'(t)|dt$$

Take a 'tent' function that is continuous piecewise affine let's say defined on $(-[h/2, h/2]$ that has height 1 at zero. Let's say it's called $g(t)$. Then $|g'(t)| = 2/h$ throughout $[-h/2, h/2]$ and the total integral is

$$h \cdot 2/h = 2$$

We have

$$1 = \|g\|_\infty = \frac{1}{2}\int |g'(t)|dt = 1$$

The function is not smooth but it reaches the constant $1/2$.

I have never seen the proof of this theorem before so I will attempt some simple ideas.

Suppose we have a function g that is monotonically increasing and is non-negative on $[0, 1]$ and $g(0) = 0$. Then

$$\|g\|_\infty = g(1) = \int_0^1 g'(t)dt$$

Here we have

$$\|g\|_\infty \le \int_0^1 |g'(t)|dt$$

So we need to do something more subtle than this.

Now let's use the *mean value theorem*. Let's say $g \in C_0^\infty(\mathbf{R})$ and the support is contained in $[a, b]$. There is at least one maximum in $[a, b]$, and suppose $\xi \in [a, b]$ achieves $g(\xi) \ge g(y)$ for all $y \in [a, b]$.

We want to consider next the special case there g is monotonic increasing in $[a, \xi]$ and monotonically decreasing in $[\xi, b]$. We can paste together $[0, 1]$ result above and get

$$2\|g\|_\infty \le \int_a^b |g'(t)|dt$$

That's progress. I don't know yet how to get the general smooth non-monotonic case.

22.1. Scaling Restriction.

We are given

$$\|\varphi\|_q \le C\|\nabla\varphi\|_p$$

The same constant with apply for $\varphi_t(x) = \varphi(tx)$ so we have to pool the power of t and ensure they are equal for both sides. The derivative will introduce the skew this will give us $q^{-1} = p^{-1} + n^{-1}$.

22.2. No Inequality for $n = 2$.

23. More Details On What I Want To Do

I want a tenure position at Stanford and live in Mission San Francisco with absolute guarantee that my wage payments will have no problems ever. I am 49 and I am extremely aware that people and institutions are totally unreliable about paying people's salaries and wages. Bill Gates has blockaded $620 million of legitimate earnings for me from Madam Christine Lagarde and David E. Shaw. I do not understand how this is even possible in the modern world. How does a scrub with no Finance experience even manage to blockade significant funds to someone with professional background in Finance? I do not understand this.

What I would like to do is found a couple of organisations for doing technology applications of Quantitative Positive Psychology. I will form them initially as private organisations. I will do this in Mission District San Francisco.

Quantitative Positive Psychology *does not exist yet* and I will found it in a professional setting. I will then just homebrew these companies till they are like IMF in some years dedicated to interests of eight billion people. I am not rushing to make money but we might have some revenue. The goal is to *incubate* Quant Positive Psychology with Technology applications. I am a seasoned coder and can manage those things. I want Stanford to commit to full professorship for me with work out of Mission District. I don't want to be in Palo Alto. I am familiar with Mission District and will set up various things for convenience of Stanford.

My four-sphere theory is successful with numbers. My Universal Human Moral Nature Results are absolutely central and solid. These alone deserve tenure.

As you can see, my Analysis is improving and will be good level in some months.

24. Problem II.5

Suppose $\Omega \subset \mathbf{R}^2$ is a polygonal domain. For each $x \in \partial\Omega$ there is a neighborhood O_x such that $\Omega \cap O_x$ is the intersection of one or two half-planes. The problem is to show

$$|F\chi_\Omega(\xi)| \leq C(1+|\xi|)^{-1}$$

The hint is to consider a cutoff with product of indicators of one or two half-planes.

Let ρ_a be a partition of unity on $\mathbf{R}^2$ and we can give some special properties to ρ_a later. What matters right now is that we have

$$\chi_\Omega = \sum_a \rho_a + \sum_b \rho_b \chi_{H_1,b} + \sum_c \rho_c \chi_{H_2,c}$$

These are for partitions within interior of Ω and then boundaries with one half-plane or two half-plane intersections.

We then use superpositions to estimate the Fourier transforms of the three types of terms. Since Ω has compact closure, we can choose a partition of unity with only a finite number of terms appearing in the sum above.

I just comment on a totally elementary point that

$$\int_a^b e^{-ix\xi} dx = (i\xi)^{-1}(e^{-i\xi a} - e^{-i\xi b})$$

This is the source of the decay of the Fourier transforms in the problem, and judicious use of this formula in the complicated geometry will yield the required decay after a while. The analysis is mostly about tracking ways of handling the boundary regions. We had examined the finiteness of terms. The problem will be resolved when we extract ways of using the exact Fourier transform of $\chi_{[a,b]}$ on the line appropriately.

25. On Analysis And Physics

When I read the point of view of Lars Hörmander in his book *Linear Partial Differential Operators I-IV* it is a tome dedicated to the major advances of the theory of partial differential equations from 1940s-2000 broadly. The development owes a great deal to Laurent Schwartz' theory of distributions.

There is a major difference that is apparent to me in this general approach of extremely sophisticated developments in theory of partial differential equations and what I consider the heart of physics. It is that four-sphere theory reduces physics strictly to analysis (a) on the compact space four-sphere, and (b) four-sphere theory specifies that the only physically real objects are *spinor fields on four-sphere*. These particular restrictions tell us immediately that the character of mathematical physics in the future will be quite different from the high level advances in the theory of partial differential equations. This is so because harmonic analysis of $S^4 = SO(5)/SO(4)$ differs radically from that of $\mathbf{R}^n$ and it is the latter harmonic analysis that is the core of physics by my work on four-sphere theory.

I was working in isolation worried a great deal for decades to ensure that four-sphere theory is actually true. For this reason, I was isolated from the larger mathematical and physics community. But eventually, as it was only reasonable, I am returning to examine mathematics and only very slowly I am able to understand some of the things that must change.

The theory of partial differential equations has had enormous and beautiful development, and that is very nice. But for physics, we have to keep focus on four-sphere geometry and examine special features of Analysis only in this case.

The mathematical theory is a true inspiration, but we are not interested only in purely mathematical questions. We are interested in the deep structure of Nature. Mathematical Physics will have enormous depth in the centuries that will succeed ours and I am confident that discovery of four-sphere geometry of nature will stand as the watershed moment in the advances to come in the future.

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