# DISCONTINUITIES OF FUNCTIONS AND MATTER FIELDS

ZULFIKAR MOINUDDIN AHMED

## 1. Final Answer To Matter Fields In Physics

I will today write about some insights that are valuable for all human beings that I have slowly over many years appreciated. Yesterday I had proposed that all of Science, and of course theoretical physics specifically consider *measurable spinor fields* on a scaled four-sphere as the fundamental model of matter in our actual universe.

We do not tell you technical details of what are Killing spinor fields on a four-sphere. I will refer to the beautiful article of Christian Bär for this [3].

I ask you to take on faith that there are sixteen Killing spinors $\sigma_1, \dots, \sigma_{16}$ that have the property that $(\sigma_1(x), \dots, \sigma_{16}(x))$ for $x \in S^4$ form a basis for the spinor bundle $\Sigma S^4$. Once you take this on faith as true, it is quite easy to see that an arbitrary *measurable spinor field* $s$ will have the representation

$$s = f_1\sigma_1 + \cdots + f_{16}\sigma_{16} \tag{1}$$

where each of the $f_j$ for $1 \leq j \leq 16$ are Lebesgue measurable real-valued functions on $S^4$.

This is simple, but this is also one of the most profound advances of the history of Science. Why? Well it is because mathematicians from 1890-1910 including Henri Lebesgue and many others were finicky about measurability and so measurable real-valued functions $f_j : S^4 \to \mathbf{R}$ have a depth of mathematical understanding that is quite strong. It is part of the ordinary lexicon of mathematicians, especially analysts. On the other hand, physicists have been struggling with chaos and anarchy on precision of models for matter ever since the rise of quantum field theory in the 1920s.

Here the simplicity is the achievement, for mathematical precision for the *full scope* of measurable real-valued functions is worked out in detail by mathematicians. Physics theory gains immediately fruits of this vast effort in Mathematics. *All other theories of matter fields in physics* are going to be more confused than this. Four-Sphere Theory allows this because we do not have (a) special relativity and deformation of time, and (b) expansion and deformation of space-time. Because of these, the models of matter fields in four-sphere theory is direct and precise.

Time and experience will show all theoretical physicists that *measurable spinor fields* are the absolutely best model for matter fields in history of the world.

## 2. What Is An "Arbitrary Function"?

I want to tie this with another insight regarding notion of an *arbitrary function* that is quite basic to all of Analysis. First let me tell you the insight?

*Date*: January 24, 2022.

After a century of confusion in the nineteenth century of notion of an *arbitrary function* on the interval $[0,1] \subset \mathbf{R}$ which led in the end to Lebesgue Integration Theory in 1902-1905 which clarified the mathematical issues here with *Lebesgue measurable functions* $f : [0,1] \to \mathbf{R}$ *are arbitrary.* Further work by Paul Lévy had clarified something much more central to Analysis than is still understood.

As I have said, it was my experience in Finance, and specifically at Lehman Brothers 1995-1996, that was the start of a long journey seeking truth and wisdom. And it is now clear to me.

*Sample paths of Levy processes* $X_t$*, with* $0 \le t \le 1$ *is an arbitrary function of Analysis* $f : [0,1] \to \mathbf{R}$ In other words these belong in Analysis and not in any associated field of probability theory. This is the *resolution* of a deep confusion of nineteenth century mathematics, i.e. what is an arbitrary function. All other solutions to this problem are ad hoc and without as clear concrete insight.

Only when you have a precise notion of an arbitrary function can you begin to appreciate how special are $C^k([0,1])$ functions. These are very far from arbitrary, and this is the problem of human intuition that took an entire century to understand deeply.

You need to appreciate the whole history of Analysis to see these insights that apply on fundamental questions. Sample paths of Lévy Processes are not esoteric special isses. They are the canonical answer to what an arbitrary function ought to be with clear precision, and this represents basic parts of Analysis.

## 3. Historical Perspective

I will tell you about my observation of mathematical literature. Mathematicians do not have a forest of lemmas, propositions and theorems that are obscure, arcane, and difficult to understand when *they know what they are doing.* Vast array of propositions, lemmas, and esoteric and arcane theorems are a sign that mathematicians do **not** know what they are doing yet.

When mathematicians know what they are doing, the results are crisp and propositions and lemmas and forest of confusion is limited.

In this example of the question of *arbitrary function on* $[0,1]$ the confusion is long and begins before 1811 when Jean Baptiste Joseph Fourier first raised this issue of their representation by trigonometric polynomials when the domain is $[-\pi, \pi]$ and the functions are periodic. But I will take the strong view that it was Paul Lévy, another brilliant Frenchman, who had truly explored arbitrary functions as *sample paths of Lévy Processes.* It took me a lifetime to truly understand this.

I therefore recommend that the first mathematical analysis courses introduce L/'evy Processes immediately and without much technicalities so that discontinuity is rationalised. Even the great Peter Gustave Lejeune Dirichlet was only willing to consider functions with minor set of discontinuities because it actually is important to understand measure spaces $(X, \mathcal{F}_t, m)$, probability spaces with a filtration of sigma-algebras and stochastic processes with jumps to appreciate the sorts of discontinuities that are *natural.* And here although Paul L/'evy was a pure mathematician, he has explored Nature in my view, and this is part of what young people taking their first real analysis course ought to learn immediately so there is some deeper and clearer intuition about Nature's Mathematical gifts to humanity.

## 4. Mathematical Knowledge Is Part Of Human Understanding Of Nature

I am 49 now, and over the years I have strengthened my *taste* in Mathematics and in Science and in Humanities and Art. What becomes clearer to me is that Mathematics contains deep insights about Nature. It is nothing arbitrary, and these insights are not gained easily. Generations of mathematicians struggle before some insight is clear enough that young people in high schools and colleges can simply absorb mathematical discoveries easily.

You see, L/'evy Processes are much more basic part of human understanding of Nature than their technical history would allow us to appreciate. I know this because I have a lot of experience with *actual data* from price changes in capital markets in real-time. Once you actually see the data, there is no doubt that these are Nature's data, and that L/'evy Processes tell us about them. The weak view of statistical theory won't give you as much power to understand the data as the Mathematical Analysis view. And so these are things that tell us about *arbitrary real-valued functions* $f : [0, 1] \to \mathbf{R}$ and technicalities of measurability are not as pedantic and useless *when you deal with actual examples daily.* Human Understanding of Mathematics here is the way in which we can have any insight about these Natural Phenomena. And then you will suddenly find this everywhere in Nature, as Benoit Mandelbrot did. Then you will begin to marvel at the naturality of Lebesgue measurability to be even able to talk about things ubiquitous in Nature. Then, after all this is done, you can revisit the definition of real numbers and marvel at this because you suddenly realise that rational numbers would not have given you understanding of unruly Nature that defies all human concepts.

## 5. My Attitude Toward Benoit Mandelbrot's Work

Benoit Mandelbrot was a student of Paul L/'evy. He was fascinated by Self-Similarity, and he broke the Gaussian barrier in distributions in Nature and focused on Self-Similar processes. He wanted to challenge Riemann's Geometry.

I took a different path, roughly verus Mandelbrot. I did spend many years absorbing his papers. He was deeply wrong about Riemann's Geometry. Four-Sphere Theory vindicates Riemann's 1854 view of Geometry of Space, for it is beautiful and smooth homogeneous geometry that underlies all of existence. He did not successfully challenge Riemann here as my success with Four-Sphere Theory shows.

He was also wrong about Self-Similarity being the the main concept of stochastic features of Nature. Nature's distributions are much better described by Generalised Hyperbolic Distributions of Ole Barndorff-Nielsen than Benoit Mandelbrot. Mandelbrot was right about Paul L/'evy's insights in L/'evy Processes but wrong about the actual class that matter for Nature.

Now let me tell you how I see things. My S4 Electromagnetic Law, or Ahmed-d'Alembert Law, is the fundamental law of macroscopic Nature. There is no stochasticity at bottom in Nature, and Quantum Mechanics is *fooled by randomness.* At bottom Nature is serene and deterministic.

But stochasticity occurs with *distribution of Matter* in the universe. This is my own insight. There is no substance to Heisenberg Uncertainty Principle. That's not important. What is important is that complexity of distribution of matter in the universe is the sole source of stochasticity. And that is all there is to Nature's stochasticity.

Mandelbrot was right to defy Gaussian distribution assumption and choose L/'evy Process transition densities generally but he picked the *wrong L/'evy Processes* granting that once we do not seek stochasticity in fundamental law but in distribution of matter, *then* we do want to see what stochastic processes are able to tell us about the mysteries of Nature. Generalised Hyperbolic Distributions are better fit to Nature's distributions than Mandelbrot's Self-Similar Distributions. These are my discoveries, and they are based on measurements. I once examined fits of 3000+ time series noise and determined better fit by Generalised Hyperbolic Distributions than Self-Similar.

This should tell you there is deeper substance to my efforts about 'arbitrary functions' in Analysis.

## 6. Attempts To Reshuffle Order Of Technicalities

The usual order of learning about mathematical analysis is based on various principles, but perhaps logical development is the primary driver. We consider rigorous construction of real numbers from rational numbers $\mathbf{Q}$. The one that I remember is that real numbers are in one-to-one correspondence with *Cauchy sequences* of rational numbers. Then on $[0,1] \subset \mathbf{R}$ we can study real-valued functions. Continuity is a central constraint to mathematical understanding. For any $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that $|f(x) - f(y)| < \epsilon$ whenever $|x - y| < \delta(\epsilon)$. Bernard Bolzano had a definition from 1819 but Augustin Louis Cauchy gets credit for it in 1821-3 because he was in Ecole Polytechnique in Paris the center of mathematical world.

A century passes by and in 1922 we have Henri Lebesgue's Integration theory and we have measurable functions that allow discontinuities in functions.

But Paul L/'evy's work follows this period. He is thought to have worked in probability theory. And that is what we would like to change.

We'd like to see sample paths of L/'evy Processes be part of elementary real analysis as *concrete examples* of arbitrary real-valued functions.

We'd like to see this being introduced right at the beginning with a lower technical barrier so younger students immediately gain concrete intuition about the sorts of things that real-valued functions do and the sorts of discontinuities that arise naturally and then put continuous and $C^k([0,1])$ functions in their proper context.

The time has come when we can assess what we have learned about arbitrary real-valued functions from Nature, and adapt Mathematics to Nature rather than remain in a cocoon. We are in an age now where Mathematics needs to be able to serve deeper understanding and cannot remain locked away behind enormous technical barriers that require doctoral work to understand.

You see, arbitrary real-valued functions on $[0,1]$ are natural things about which today there is very little elementary insight. This has to change, because young people are quite capable of absorbing the special nature of $C^k([0,1])$ from the sample paths of L/'evy Processes with controlled jumps.

In early nineteenth century, without computers, this would be invitation to anarchy and chaos, but today, with the help of computers, we can gain some depth of understanding for arbitrary functions and be able to get a fuller typology of functions without getting caught into the wrong mindset regarding what is good and what is pathological among functions.

I implore mathematicians to give deeper sense of what elementary real-valued function theory on $[0,1]$ that will allow them flexibility in their future travails.

L/'evy Processes are defined thus: Let $(\Omega, \mathcal{F}_t, m)$ be a probability space with a filtration of sigma-algebras and $0 \leq t \leq 1$. A one parameter set of functions $X_t : \Omega \to \mathbf{R}$ is a L/'evy Process if

- $X_0 = 0$ almost surely
- For an arbitrary partition $t_n$ various $X_{t_{n+1}} - X_{t_n}$ are independent
- $X_{t+s} - X_s$ does not depend on $s$

Then we have a large amount of concrete examples of processes with jump discontinuities in their sample paths from particular families of L/'evy Processes $(X_t)_{0 \leq t \leq 1}$.

You see these examples are not 'pathological' at all even with discontinuities. They ought to be comfortable for students of real analysis but they are not yet. Discomfort comes from just not habituating to concrete examples from early education.

Today, they still seem exotic for real analysis students. That is not good. Analysis students ought to feel very very comfortable with these because they are central examples in what allows Mathematics to tell us something deep about how Nature works.

## 7. The Primordial Discontinuous Poisson Process

On $\mathbf{N}$ the Poisson distribution with mean $c$ is the probability distribution

$$\mu(k) = e^{-c} \frac{c^k}{k!}$$

A Lévy Process $(X_t, t \geq 0)$ is a Poisson Process if $X_t$ for $t > 0$ has Poisson distribution with mean $ct$.

I think that sample paths of the Poisson process ought to be well-understood by elementary real analysis students because in various ways, they give a full spectrum of understanding for Lévy process with jumps. These rationalise the discontinuities in a reasonable way and give sense of the richness of arbitrary real-valued functions.

Without context for jump discontinuities, it is hard for students to appreciate the substance of discontinuities. Years of experience with price data gives one a sense for jump discontinuities in natural measurements of stochastic processes but the novice student of analysis ought to be able to gain intuition about what is arbitrary about $f : [0,1] \to \mathbf{R}$ without having to be subjected to the data. For me personally, these discontinuous processes had always been a headache because I was not educated in real analysis to worry about discontinuities at all; I suspect that the same will be the case for most students.

## 8. Real Analysis Is Idiosyncratic Today

Analysis is thought to be a vast field and there are too many different directions, we are told in our classes. Mathematicians teaching real analysis are mostly specialists in partial differential equations. Their training is often in abstract aspects maybe of functional analysis, and a great deal of a priori estimates. This is not very surprising in a way, since nineteenth century analysis was concerned with analytic and smooth functions. Discontinuities were considered 'badly behaved'. A great mathematician such as Marcel Riesz was discouraging Laurent Schwartz regarding the distributions $\mathcal{D}'([0,1])$. But you see, in Nature, discontinuities are common in sample paths of stochastic processes with jumps, in various limiting operations, smoothness is broken.

In an ideal world an elementary real analysis course should prepare the young student to face the full spectrum of real valued functions. But they are not so prepared, having absorbed the prejudices of various mathematicians teaching these courses. The result is that we are flung into the world where our pure mathematical training is not adequate. But the problem is that the mathematical literature for these issues is not clear and elementary but abstruse and impenetrable and too costly for people who just wanted a decent broad education.

This is a serious concern for future of elementary real analysis. They ought to be refactored so they are able to produce graduates who are flexible and feel comfortable with the spectrum of situations where real-valued functions arise naturally.

No one ought to need *encyclopaedic knowledge* to have clear understanding of real-valued functions. Their elementary real analysis courses ought to organise all the things that are valuable, and remove all the esoteric rubbish and give them a good preparation to engage with the world out there they will face the challenges themselves without any oracle to guide them. They will curse their real analysis courses when they suddenly find that they have deficiencies in their understanding of the mathematical situations.

## 9. Let Me Illustrate Better What I Mean

For mathematicians, this is a good book to know about Quantitative Finance [1]. This tells you about relatively standard modeling of jump processes in Finance. You can read also Peter Tankov's notes [2]. These are good people, just so that you know. I have one of the best long memory stochastic volatility option pricing models in the world, and I know what I am doing in this arena. If I say these are good people, they are good ok?

So you can look through their work, but the substantial question is whether the generic elementary real analysis courses around the world prepare all students to appreciate arbitrary real-valued functions? And I think the answer is that it's not good enough. There is no standard for systematic treatment of discontinuities that arise either in partial differential equations or in finance and elsewhere (such as engineering). The result is that valuable time is wasted on rubbish abstract issues while there is no finer intuition developed for the issues where real analysis could have given firmer pure mathematical confidence with dealing with discontinuous functions. And that is frankly, bad education. We need to change that.

## 10. A New Philosophy For World's Mathematics Education

I am strongly a Republic Soul and thoroughly Romantic in the 1780-1820 sense of English Romantic Era. I strongly disagree with the philosophy that Mathematics courses exist in universities to weed out the weak ones so that those who survive have proven their worth to do esoteric mathematical research and the rest are sent to the shredder and cast out.

Instead, I believe that it is the professors and instructors who are on trial, for the failure of the students is their fault. So my philosophy is that all students ought to have deeper confidence in their abilities and understanding of mathematics depending on the talents of the teachers. This seems like an over-idealistic viewpoint only to bad teachers and those who are interested in their own research and just give some shoddy effort in cultivating young people. This should affect their salaries because if they want that, they should take a pay cut. Great universities are for cultivation of young people, not weeding out people for various monastic research work.

You see, as the world grows in population, we are less interested in cults of personality and more interested in uplifting the world's Mathematics to higher levels, and so the profession of teaching can no longer be a spot for those who are involved in research to give some shoddy effort any more. This is the Achilles' Heel of Mathematics and it has to end now.

## 11. What I Think That I Can Deliver

I have had a wide variety of experiences in matters of intellect, in matters of industry, in matters of life generally. I had an opportunity to think about what I can deliver *for the human race* instead of to particular institutions. I think I can deliver some very strong results in Quantitative Positive Psychology, which is a field that does not exist yet. I want to create several institutions in San Francisco in Technology and Quantitative Positive Psychology which I can produce some nontrivial progress; this could improve the human condition. I can deliver some things in Global Individual Debt and Finance. I believe that it is a question of Primitive Age of Man that makes money such a difficult issue for all people's lives and I think I can change that for eight billion people eventually. Then I can produce original progress in psychology, mathematics, physics. I will not go into specific issues because these are issues of evolution and history. But I do want full professorship and a good life financially in San Francisco to deliver on these things. They are specifically interesting to me and I am confident that I will have world class accomplishments if various pragmatic issues are handled by Stanford University.

I am not money oriented but I am not seeking a life of penury for saintliness. I expect there will be some financial success but this is a secondary concern for me. What is primary is to move the Human Civilisation in some positive directions. I am quite senior in my mindset, and I am confident that I am prepared to move things in healthy positive directions.

One of the reasons I do not worry about the details is because I am experienced in nontrivial projects and I know that the details work themselves out when the right issues are understood and the right people are happy with the strategic direction. Experience of ten thousand year of Civilisation shows that it is the strategic direction that really drives the enthusiasm and energy of engaged talented people and not small details; they are variable.

## 12. A Return to Foundations Of Analysis

Moritz Epple has a nice essay "End Of Science Of Quantity: Foundations of Analysis 1860-1910" in /em A History Of Analysis. I like to think about these issues a great deal recently. "Today, analysis is usually thought of as founded on an axiomatic definition of real numbers, within general framework of set theoretic foundations for mathematics."

I like to think about this not because I have a deep suspicion about set-theoretic foundations of mathematics, but *continuous question of whether the knowledge of the external world, Nature, is fully and precisely available by this approach.*

It is not exactly valuable to attempt to haphazardly tinker with foundations. Axiomatic foundations of real numbers has been valuable to give precision to our mathematical knowledge, and certainly helped us think more clearly about mathematical objects. But there is, in Science, always the wonder about whether our axiomatic foundations are endorsed by or is compatible with some *fundamental objective reality* or whether, at least in relation to Nature, these are parlour games that will amuse us but will not give us the deepest secrets of Nature.

So let me formulate this question in the following way: How do we gain certainty of knowledge that our methods and results in mathematics is guaranteed to tell us some truths about Nature? How do we know that our axiomatised objects, such as real numbers, are exactly able to represent the Natural world or even *adequately* able to represent the Natural world?

This is in fact the most important question in all of Science, that is not seen yet to be so important yet.

## 13. Starcraft II Very Hard Defeat Jan 24 2022

I decided to play a 15 minute Very Hard AI game and lost quite handily and got dropped back into Harder territory [4]. I won't comment much on the game. I am not able yet to compete with Very Hard AI in production and in other ways. What occurs to me as important is that there is here a microcosm of any activity of human beings, that habituation leads to advancement in skills. For many years, my concerns about mathematics has not been advanced technical skills or purely mathematical issues at all but rather the philosophical issues of whether the Mathematical objects that exist and the problems that have been solved are adequately able to provide us with certain knowledge about existence and Nature.

You see, as I get closer to death, I don't really care as much about smaller issues. They don't really give me any satisfaction. I will die at some point, and so smaller issues are not worthy of my time at all. I have nothing really to prove to anyone about anything either. Perhaps *others want me to show themselves something or other* but I personally don't give a damn what they want. I am more concerned with how I will be spending the last days of my life on Earth and I just don't give a damn what other people want from me all that much.

Then you will say, "Ah, that Zulf, what a bad attitude he has." That's your problem, honestly. I am more interested in things that are important to me, personally, and don't give a damn about what is important to you.

## 14. Priority For Distributions S. L. Sobolev and Laurent Schwartz

I am unhappy about the credit for distribution theory going only to Laurent Schwartz and not given to Sergei Lvovich Sobolev (1909-1989) with sufficient enthusiasm. These sorts of things irritate me a great deal.

I am the sole originator of Four-Sphere Theory and no one else. I do not want the world to give credit for my great advances to human understanding of Nature to other people. These sorts of things when Laurent Schwartz gets the whole credit for originating distribution theory which was really initiated by Sergei L. Sobolev, is extremely disturbing to me, because it bespokes the sort of world where criminals like Bill Gates who is fully committed to denying me credit for my work, predominates. I am the immortal genius behind four-sphere theory, and no one else. I better put in enormous efforts to ensure that there is no confusion about this matter. It's annoying that the world works the way it does, but what can one do?

I better ensure that people understand clearly that I respect all my teachers such as Peter Sarnak and Daniel Stroock, regardless of my particular agreements or disagreements with them, because I have great self-respect and do believe that in order to have great self-respect one ought to respect one's teachers and elders despite intellectual disagreements or agreements. On the other hand I think it a great duty to *disrespect and continuously denigrate Bill Gates.* This is because I do not think he has had *any literacy* or *any achievement* that is not menial and low, and because he is a pathological charlatan and liar, and he is a menial intellectual midget who is an unimportant amoeba in the world who deserves 50 lashes and torture to death for his insolence. He is a pure scumbag, and his intellect is significantly lower than a parakeet. He is malevolent and conniving and parasitic, but that is not intellect; that is his evil nature.

## 15. A Priori Science Has No Interest In Pure Mathematics

After graduating with magna cum laude from Princeton in 1995, with a pure Mathematics thesis that won a prize on Several Complex Variables, I was young and found that I could get an extension of my F-1 Student Visa for *Practical Training.* I heard about Finance jobs in New York where they wanted some mathematicians for something called *Quantitative Finance.* I was quite naive and had gotten out of various Science requirements from Princeton with AP scores of 5 in Chemistry and Physics C. To be honest, I did not think highly of spending time with manual labour taking measurements all my life. I thought I was made for higher loftier things, and I did not like ruining my manicured nails with hard labour. It's metaphorical; I did not manicure my nails. I took up employment at Lehman Brothers Fixed Income Derivative Research in Andrew Morton's group. That was the beginning of a slow transformation of my life and return to Science. At first I was really not clear that I was dealing with a totally open area where nothing much was really known. I thought that there were some well-developed *mathematical theories* that worked and I would apply them and implement them and that would be that. The world, in my mind, was mapped out by exact mathematics, and all I had to do was just mathematical things, implement, get paid a good salary, and focus on my romantic love life and so on. Well life is never how you expect it to be. In due time, a decade passed and I slowly became aware that Science looks good on glossy Nature and Science articles but it is in total anarchy, disarray, and no one knows what they are doing at all. It's not because they are not bright people. It's because that's just how Nature is. Nature is cruel, enigmatic, and I did go through a phase where I felt that the natural laws were just to sabotage my happiness. Then I got out of it because I did know *Notes From The Underground* and refused to become a Dostoevskian figure for the rest of my life. For those not in the know, the Underground Man did feel that laws of Nature were against him. I have a better attitude these days.

## 16. Can David Applebaum's Presentation Merge With Elementary Real Analysis

The intuition for arbitrary real-valued functions in elementary analysis is a concern of mine. Let us think of ways that completely elementary real analysis course could have realistic presentation of sample paths of Levy Processes.

I think this does require introduction of the Brownian Motion sample paths first and use of Lévy-Khinchine formula to parametrise jumps. This is therefore an issue of introducing Brownian Motion and jump processes informally.

I have done this myself to people who are not mathematicians, and it is easy as pie. This is how you do it. You introduce standard random walk with Bernoulli random variables. Then you take the intuitive limit and first replace the values keeping times the same and then cut up times and invoke consistency. Rigour does not matter because people can intuitively get some feel for these and introduce the third term with the Lévy measure in Lévy-Khinchine formula and begin tweaking it. Elementary real analysis students can get the hang of this quite easily.

A lot of real analysis courses are concerned about rigour too much and not enough about mathematical substance. This is a bad decision, because mathematical substance is more important than rigour and mathematical substance requires its own time to appreciate even without rigorous proofs. You do not need a presentation of refined proof of the Lévy-Khinchine formula to understand the mathematical substance here at all. It will be infinitely superior to elementary real analysis where there is no clue that besides a finite set of points that we are *encouraged to ignore* that functions are continuous. In fact, this is what often I do with Lusin's Theorem. This point of view is valuable, but there is no control of all the trillions of situations where *only jumps have any serious meaning or content* such as every single application of actual Poisson Processes and then suddenly we feel that our analysis education did not prepare us for them.

## References

[1] R. Cont and Peter Tankov,Financial Modelling with Jump Processes,Chapman & Hall CRC Press, 2004

[2] Peter Tankov, Financial modeling with Lévy processes, `http://public.econ.duke.edu/~get/browse/courses/883/Spr15/COURSE-MATERIALS/Z_Papers/tankov.pdf`

[3] Christian Bär, The Dirac Operator On Space Forms Of Positive Curvature, J. Math Soc. Japan, 48(1), 1996, pp. 69–83

[4] `https://drive.google.com/file/d/1Wo3-NVf0k7nhrrDsES0-en_L6RmZEWGY/view?usp=sharing`