HISTORY OF ANALYSIS NOTES DECEMBER 30 2021

ZULFIKAR MOINUDDIN AHMED

In 1992 I took real analysis at Princeton University with Nick Katz, the great number theorist. I am reviewing the book we had used there, H. L. Royden's *Real Analysis*. The book delves into Lebesgue integration theory in the second chapter. In the intervening 29 years, I have not pursued a career as a mathematician and am turning to mathematics again because my Four-Sphere Theory necessitated a serious return to mathematics. It was really the overwhelming surprise for me the spectacular success of Four-Sphere Theory as the prediction of the width of an electron from its mass, several simple constants like the cosmological constant and light speed was so strong that I am quite driven to return to study Mathematics to understand this phenomenon.

I only began to gain interest in the history that led to the Lebesgue integration, something that is discussed without sufficient clarity in mathematical courses for the main focus is of course mathematical development.

Royden briefly motivates the Lebesgue integral with how when decomposition of the domain into subinterval is replaced by a richer class of Lebesgue measurable sets then upper and lower approximations of functions are more better and the properties of the integral as well.

I use from Thomas Hawkins *Lebesgue's Theory Of Integration: It's History And Development* some material. Newton and Leibniz were working in 1660s. Leonard Euler introduced the notion of *arbitrary functions* in a pioneering work in partial differential equations in 1734. And two centuries later Laurent Schwartz would introduce distributions. Regardless of what the various issues had arisen, this is truly fascinating to me, that these notions had evolved in a particular way, that the real line, the Euclidean plane, these mathematical objects all ended with the sorts of things that allowed me to calculate the *radius of existence itself* between 2008 and 2018 and be absolutely certain that it is *finite* and that it is roughly $R = 3075.69Mpc$, a concrete value. It is *this deep mystery* that propels me forward into these interests and not issues that are more specific and bounded in historical or civilisational extent. The issue is not to celebrate Man's great ascent and so on. Rather the issue is pure wonder at what these objects really are, and how they relate to actual Nature. This success of Mathematics is not an invitation at all to relegate profound and important work in Humanities and Arts and other aspects of life of Human Civilisation of course. But this is a matter of genuine wonder; after all Mathematics in its modern incarnation is not even 350 years old while great Civilisations had grown and died many times before in the past 20 thousand years. Celestial Mechanics was the concern of many scientists of the past ages but Four-Sphere Theory is a profound breakthrough for many other reasons because its reach is all of existence. The fundamental law governing all things in the universe

Date: January 2, 2022.

above scale $\delta = 10^{-13}$ centimeters will find Mathematical precision with four-sphere theory, and that is a great moment for the history of our race, the human race.

It was Leonard Euler in 1732 who in a pioneering book on partial differential equations began consideration of 'arbitrary functions', i.e. assignments $f : X \to Y$ that had become standard. I was a mathematics student at Princeton between 1991-1995 and even today, the later developments such as Laurent Schwartz distributions, i.e. linear functionals on spaces like $\mathcal{S}(\mathbf{R})$ are not as intuitive to me. In fact I am familiarising myself with these issues at 49 now because I am just absolutely fascinated by the larger history of these concepts that I never was even when I was a good student in John Adams High School and tenth grade obtained a 5 on my Calculus BC Advanced Placement examination. What fascinates me about functions (including generalised functions) is not their formal properties but their *ability to exactly model Nature*.

In a way, this is not fair obviously, because mathematicians have such a vast potential of various sorts of objects under study that almost any imaginable thing could be addressed by the generalised generalisation of the very general object whose generalised extension is maximal according to the principles of inductive generalisability. But those are not substantial issues either in Mathematics or Science.

Returning to the history of analysis, Leonhard Euler in 1734 in answer to Jean d'Alembert had proposed that functions that could resolve the vibrating string problem did not have to have an analytic expression but could be 'discontinuous'.

Attention was on whether the partial differential equation for the vibrating string problem was constrained to have a single analytic expression, and whether it would be characterised by the properties of the correspondence $x \mapsto f(x)$.

My dear readers will be pleased to learn that I, Zulfikar Moinuddin Ahmed, in the summer of 2018, had been only too pleased to provide the *final law of macroscopic universe*, the S4 Electromagnetic Law, in Allen Texas, from the second floor, when I wrote down

$$(\frac{1}{c^2}\partial_t^2 - D^2)u(t,x) = 0$$

where $u(t,x)$ is a spinor field and not a scalar value, harkening back to the very first partial differential equation studied by Euler and d'Alembert in 1747. In the intervening 274 years, the human race had made progress towards the very pinnacle of understanding of Nature.

The vibrating string problem, the very beginning of exact scientific quest, reached its full maturity with the announcement by myself of the fundamental laws governing all of macroscopic nature, also a wave equation.

This note is as much autobiography as it is about the history of real analysis, and I will admit that my knowledge of partial differential equations is not highly developed. At Princeton, after some courses in analysis, differential geometry and algebraic topology, standard courses in algebra too, I gravitated towards geometry of four-manifolds. My strength was in geometry and I did not think to have a solid education in partial differential equations at all. My senior thesis was in Several Complex Variables, for which I won a prize for most original thesis at Princeton in 1995, and it was supervised by Jeffrey McNeal who is the head of Department of Mathematics at Ohio State University. Indeed, my interest in returning to studying analysis almost thirty years after my undergraduate real analysis today is to gain deeper understanding of partial differential equations.

"The study of partial differential equations started in the eighteenth century in the work of Euler, d'Alembert, Lagrange and Laplace as a central tool in the description of mechanics of continua and more generally, as the principal mode of analytical study of models in the physical science. The analysis of physical models has remained to the present day the one of the fundamental concerns of the development of PDE's. Beginning in the middle of nineteenth century, with the work of Riemann, PDE's also became an essential tool in other branches of mathematics." This is an assessment from extremely talented contemporary analysts Brezis and Browder [1].

My perspectives about natural science was not straightforward as at Princeton, even though I had won a Westinghouse Science Talent Search and excelled in Chemistry AP and Physics BC earlier with 5 on both from John Adams High School, I gravitated towards Mathematics and Literature. And afterward I was in Finance, Technology and had my first full time position in Science 2002-3 in Biospect/Predicant in South San Francisco (eventually) as Scientist II. And I laboured on my Four-Sphere Theory between 2008-2018 without any financial compensation at all before success.

The models of physical science is precisely the fundamental concern for me, as Four-Sphere Theory differs from most models extant in physical science in being a successful model of all of Nature of all thing in the external world above scale $\delta = 10^{-13}$ cm.

0.1. **Digression: Partial Explanation of My Trajectory In America.** Israel's Mossad was responsible for the events of September 11, 2001. I was not aware of this at all soon afterward. And then only became quite certain that there was no 19 hijacker in any of the planes on that day. There were some wars planned in Washington for regime changes in some seven nations. It was for geostrategic reasons for a 'post-Soviet World Order'. I was American, of Bengali Muslim background, but I was Atheist from 1979 and considered myself to be an urban cosmopolitan liberal-leaning man in America. That did not matter much because for me the past two decades of war turned the entire American culture behaving like the historical accounts of Medieval Europe. People stopped giving me interviews for high level jobs, and I was not interested in 'experienced python developers' and other rubbish employment. This is partly the explanation of my trajectory. The whole of America suddenly was fuelling Medieval anti-Muslim propaganda without any knowledge of Muslims at all, and there was a great deal of enthusiasm for war with orchestra accompaniment of New York Times and Economist and all other press. New York Times should be totally discredited for lying to the world with random photographs of 19 people who had nothing to do with 9/11.

1. Poincare's 1890 Remarks

Henri Poincare in 1890 is cited by Brezis and Browder as having noted the 'family resemblance' of problems in electricity, hydrodynamics, heat, magnetism, optics, elasticity that should be treated by common methods [1]. I will immediately remind my dear reader that my own views about Science is holistic and I am not about to embark at 49 on a new career in specialisation of partial differential equations. My work is the Four-Sphere Theory and it is a holistic fundamental physics model from 2008-2018 whose conception was my own and I have laboured on this and some other problems in Science in the past decade quite isolated from the great

universities of America. So I have an agenda at the moment, in this note and more generally with my life, and this is to seek to understand Mathematics and its relation to external Nature. I want to point out the difference in my viewpoint from Henri Poincare's from 1890. First, I am not interest in mathematics and its *applications* as tools for physical science. Instead I am interested in the status of Mathematics in human understanding of Nature itself. The point of view I have is extremely different from Poincare, mostly because Poincare, although he was active in both mathematics and science, was not a dyed-in-the-wool scientist, and he was a great mathematician with purely mathematical interests. For better or for worse, my *purely mathematical interests* are strictly secondary by my particular trajectory. But I am not interested in many engineering problems. I naturally, and independently gravitated towards fundamental physics and questions of human nature and what some might consider extremely classical problems that would appeal to Aristotle and Academy of Hellenistic Greece more than to modern professionals in mathematics and sciences. This is nothing to justify nor a source of pride or shame for me. It is so.

2. The Deepest Question About Mathematics

What is the *substance* of Mathematics in Science? This seems in this age to be still a frivolous question, one for amateurs perhaps. But it is the deepest mystery of Mathematics. Obviously I am considering all of Mathematics together when I ask this question. It is a matter of pedestrian history that Mathematics is useful as a *tool* for some models of physical science. But what is the *substance* of Mathematics vis-a-vis external Nature, that allows it to function effectively as a tool at all.

This is not a semantic quibble; it is not a matter of definition. I have used zonal harmonics of a scaled four-sphere as a fundamental theory of physics that differs substantially from quantum field theory, from general relativity, from expansionary cosmology and predicted with precision the *width of an electron*. The mystery here is overwhelming. The people who laboured to prove theorems about zonal harmonics of spheres were not Omniscient deities who had descended from the Heavens with interest in physics for the most part. And yet these things from pure mathematical work yield profound agreement with exact measurements. The substance of my query is difficult and trivial counterexamples, pointing to mathematics of Arithmetic Geometry not having direct input into efficient production of Swiss watches that are more popular commercially and so on are not serious. The question has never really been asked seriously. Leonard Euler and Pierre-Simon Laplace and other early geniuses of partial differential equations were doing theoretical physics and many other things long before there was division of many fields, and had no division in their work. We don't live in that environment today, and since Einstein and the Quantum Physicists had 'revolutionised' physics so that valid mathematics no longer stood in the way of physicists, the question was moot.

3. The Solution Is Not Strife Between Mathematicians And Scientists

Scientists who have spent their entire lives pursuing understanding Nature, and I count myself among scientists myself, will not find it palatable that mathematicians without particular interest in the difficult work and frustrations inherent in science should suddenly claim to have some mystical understanding of Nature. That is not

my purpose to foment discord and strife among people who are pursuing serious productive work in differing fields.

The issue that matters is that the *collective intellectual understanding of human race* is missing some grasp of a deep mystery, and that mystery cannot be resolved by strife, war, bloodshed but requires serious purpose and productive effort in respectful harmonious peaceful manner.

4. History Of The Wave Equation

Below I take some statements directly from [1] without intent to take credit but because I think their account is extremely lucid and enlightening. I am not concerned here with originality in historiography or as expert opinion of leading analysts so I don't worry about issues of plagiarism. Substance is more important here.

The one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

was introduced by d'Alembert in 1752 as a model of vibrating string. His work was later extended by Euler (1759) and later by D. Bernoulli (1762) to 2 and 3 dimensional wave equations

$$\frac{\partial^2 u}{\partial t^2} = \Delta u$$

with

$$\Delta = \sum_j \frac{\partial^2}{\partial x_j^2}$$

in the study of acoustic waves.

I introduced the fundamental law of the single S4 Electromagnetic law in 2018 as

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} = D^2 u$$

where now not the Laplacian but the square of the four-sphere Dirac operator acting on spinor fields is on the right side. This Dirac operator is not the one discovered by Paul Dirac in the 1920s but in fact one of the major *mathematical* achievements of Sir Michael Atiyah and Isadore Manuel Singer (who passed away this February). In fact the reliance on the mathematical infrastructure for my four-sphere model is not an accident and was based on deep *scientific convictions that arose from examination of fundamental physics in intuitive form between 2006-2008 but explicitly afterward. The history of the wave equation was not clear to me during most of my work which was much more concerned with the spatial geometry of the universe and especially strong positions against expansionary cosmology which is most certainly false in Nature; there was no big bang or any such nonsense 14 billion years ago. The universe has existed for infinite time in the past.*

5. Digression: Scientists Are Habituated To Contempt For Theorists

Mathematicians, and especially younger mathematicians and graduate students ought to be fully cognizant of the basic fact that there is extremely deep-grained contempt for theorists by scientists. Perhaps the top physicists will break with the crowd since they are better trained in higher mathematics than the others, but empirical scientists have contempt for theorists by a century of habituation *led by revered*

figures like Richard Feynman. I will demonstrate this to you. You, the young and aspiring mathematician, ought to watch [2] *as education about life. Whether you love Richard Feynman as all scientists do, or have deep problems with him, as some do, you will have to agree that his influence was tremendous and he has led the* flock *to far pastures away from thinking that mathematicians* know what they are talking about. *So beware.*

6. Digression: Four-Sphere Theory Is Infinitely Stronger Science Than Quantum Field Theory and Relativity

Now I will re-emphasize that my conviction that Four-Sphere Theory is the exact description of actual nature is stronger than anything else that I can even hope to believe. It is close to my strength in belief that 2+2=4 and much stronger than my belief that tomorrow the sun will not refuse to shine. In the next million years these weaker theories like Quantum Field Theory and Relativity cannot budge Four-Sphere Theory even a micrometer.

7. How Did I Determine My S4 Electromagnetic Law?

I already had a fair amount of conviction in 2008 that the geometry of space was four *dimensional sphere rather than three. This came from some examination of the scientific ouvre. I had begun reading Paul K. Feyerabend from roughtly 1999 when my friend Alessio Caldarera who was at MIT introduced me to him, and then in 2003-5 I was a great fan and re-read Thomas Kuhn's* The Structure of Scientific Revolutions *as well as letters between Galileo and Bellarmine. My conviction became very strong regarding a* purely electromagnetic fourth spatial dimension *in the universe from these things that are now called 'quasicrystals' for which the Nobel Prize in 2011 was given to Dan Shechtman for discovery of 'quasicrystals'. I know about his struggles but I don't believe in quasicrystals either. But unlike his detractors, I think he discovered something more profound than 'quasicrystals'. He may vehemently disagree. You see, I believe he discovered that the actual universe has a* macroscopic *fourth spatial dimension, and the things that he found are just* ordinary crystals *with* ordinary rotational symmetries of four-spatial dimensions. In other words, I don't denigrate his discoveries at all. I think the Nobel Prize Committee should award him another Nobel Prize for empirical discovery of fourth purely electromagnetic macroscopic dimension that is objectively real.

I was more interested for some years from 2008 to 2014 perhaps on establishing static geometry; of course this was difficult because people did not accept that rotational symmetries of 5, 8, 10, 12 discovered for so-called quasicrystals were evidence for a macroscopic purely electromagnetic fourth spatial dimension; but my conviction was specific, a static four-sphere geometry for all of existence and I had already become quite clear that the explanation of cosmic background radiation of roughly 2.71-2 Kelvin was better explained by simple equilibrium in a compact heat bath than by absolutely outrageous fairy tales generated by Big Bang theorists. And I established the *prediction of redshift* for the first time with some accuracy. Wordpress took down my site where I had a tremendous amount of work recorded.

Anyway, until I was absolutely sure that my prediction of redshift in a static four-sphere model of the universe was solid, I could not turn to other issues. And that took quite a long time. But afterward I obtained a lot of graduate textbooks in physics and began studying them. I challenged Albert Einstein's Special Relativity

later. Originally I thought that I could use some General Relativity theory to patch things. But more careful thought showed me that Einstein's Relativity is wrong. It was not really his fault; Einstein's genius was indeed profound. It was that his reasoning led him to conjecture a cosmological constant $\Lambda > 0$. What was not clear to him is that this implies that the *empty universe* in his model would have $\Lambda > 0$ and then Special Relativity is invalid. I won't go through the argument; an empty universe with $\Lambda > 0$ does not have any Lorentz invariance. Then he was not alive when $\Lambda = 1.11 \times 10^{-52} m^{-2}$ was measured. I had these numbers easily available from 2008 so I could just examine whether the basic theory that "Λ is interpreted any which way by these clueless people but it's actually just the *curvature* of ambient four-sphere geometry of the universe" is true. And it is.

Then finally I had in my possession Misner, Thorne, and Wheeler's very famous book *Gravitation*. I knew this book from people I knew at Princeton who considered it as a Bible. I examined it, and what caught my eye was their justification for General Relativity as being far more successful than James Clerk Maxwell's theories such as luminiferous aether. I knew that this luminiferous aether was not popular in physics since Michelson-Morley experiments, but I was impressed by this James Clerk Maxwell fellow quite a bit. I immediately decided that I will model the entire four-sphere as a new aether concept, checked that Michelson-Morley's experiments would not be able to reject it. Then I looked at Schroedinger's Equation. It was clear that I had to act; the world needed a correct law. And I just examined Maxwell's Law and knew that it would have to be a wave equation on spinor fields. That is roughly how my S4 Electromagnetic law was born. That it would be a wave equation was quite clear by the success of the approximate model of Maxwell on $\mathbf{R}^3$. But he did establish a wave equation; so then I just realised that this wave equation is powerful for deeper reasons than *particular* models; this is how the entire universe moves. It was crystal clear that I should be very careful and keep the fundamental law very close to what I called then the 'classical' wave equation. And now I would call it the wave equation of d'Alembert from 1752, because I hate the word 'classical' already. It's such a horrible word. You can call anything at all 'classical' and so it's not meaningful. But mathematicians have been studying these equations for a long time too and it's just *the* wave equation in mathematics. And that's where the law arose. I am not actually stupid enough to try to pretend that I can posit arbitrary laws and expect that this is good science. Nature is the mysterious thing that does not cater to my will.

As an aside, *never expend your will frivolously.* Only apply a *little bit of will* when absolutely necessary. Your life is precious. Expend your will with care.

8. Let Me Re-Iterate My Views Of Jewish People

I had two friends from high school who were Jewish: Howard Jerichimowitz and Kenneth Rosenberg. They were no different from me. Howie was a scrub when he called me one and otherwise an ordinary gentleman. I don't think Jewish people are any different from the rest of my beloved people the human race. Now I do believe that I am the Chosen One and everyone else is special in their own way. But that leaves plenty of room.

9. The "Evidence" For Time Dilation Is Not Compelling

The various experiments that have claimed to establish 'time dilation' are not credible. I examined the error drifts in atomic clocks due to a various environmental factors and so I don't think that flying around with atomic clocks in a jumbo jet has any compelling evidence for time-dilation. Special Relativity tests will not be able to distinguish themselves from my no-dilation Four-Sphere Theory. These are delusions. The universe does not allow time dilations and such exotic phenomena.

10. Wave Equation Sole Discovery of d'Alembert

Two names of Leonard Euler and Jean le Rond d'Alembert are associated with the equation that is today called the 'wave equation'. But one of the names ought to get full credit for original discovery and it is Jean le Rond d'Alembert. This is absolute genius, what d'Alembert did, and with my S4 Electromagnetic Law being correct, I want to be sure that the world knows where I stood on this issue. The wave equation's original concept *must* go to d'Alembert, only to d'Alembert and *not to Leonard Euler*.

Now I will quote from a text [3].

D'Alembert had written his first paper on vibrating strings in 1746, and had discussed his solution with Euler in their correspondence. A few months later Euler gave his own paper on vibrating strings which he published twice and as quickly as possible.

Euler not only deserves no credit whatsoever for the pioneering genius of a discovery of the *the form of the partial differential equation that governs all of nature* but showed extremely bad taste and judgment morally and I am not impressed at all. Jean le Rond d'Alembert was the great genius behind the discovery of this central core of dynamics in nature. I learn about this horrid state of affairs today and am filled with sadness because I did not know that Leonhard Euler was this sort of cat at all. He deserves absolutely no credit for discovery of the fundamental dynamical law, that in the capable hands of Zulfikar Moinuddin Ahmed, is seen to govern the entirety of existence. I am calling this equation S4 Electromagnetic Equation. But posterity ought to have alternate name *Ahmed-d'Alembert Electromagnetic Equation*.

11. The S4 Electromagnetic Law Is The Final And Only Law Of All Physics Above $\delta = 10^{-13}$ cm

I want to repeatedly emphasize to my dear readers that my S4 Electromagnetic Law is the final law of all physics above 10^{-13} cm in Nature. There are *no other forces at all that operate in this regime*. I have little to say about what operates in smaller scales. But above $\delta = 10^{-13}$ cm, all of Nature is governed by one law alone, and that is my S4 Electromagnetic Law. I have recommended that if *names be attached* in posterity people use my name, Zulfikar Moinuddin Ahmed, and the name of the great genius who has sole credit for conceiving of the wave equation, solely Jean le Rond d'Alembert. This man had, by uncanny insight found in the vibrating string a partial differential equation that is literally the form that I had sought and he deserves to have his name attached because it was his immortal discovery of deep structure of dynamics in Nature that was compelling to me.

James Clerk Maxwell's equations are not for four-sphere spinor fields. They are for *models of 'classical electromagnetic fields'* on $\mathbf{R}^3$. They are approximate models of Nature and not the final law. Neither is Schroedinger's equation the exact law. His law is also for $\mathbf{R}^3$. I have shown that *solutions of my S4 Electromagnetic Law* restricted to tangent space of the physical hypersurface at a point are approximated by solutions of certain Schroedinger solutions in my notes. The conclusion is obvious, that I have provided human race with the Final Truth of Nature, and these other gentlemen have not done so before me.

12. Revisiting The Vibrating String Controversy

Gerald Wheeler and William Crummett's 1987 paper is truly a wonderful gift for me [4]. They have an account of the controversy and an analysis. But instead I would like to highlight a different thing as their most valuable contribution in this paper, at least to me.

"Few physicists are aware of the intense controversy that existed over the original descriptions of the vibrating string. Many of the principles of applying mathematical formalism to physical phenomena which we take for granted today were poorly understood at the time of the debate. In particular, the extent to which a mathematical formalism should explain a physical phenomenon had yet to be established in this post-Newton period of rational mechanics." [4], p. 34.

Their mention of these issues are the most important here and of course the account and analyses are interesting as well. My dear readers ought to understand that these issues are not yet resolved at all today still, and they remain quite open in a very deep sense, and are fundamental and interesting questions that requires enormous and serious effort still.

I want to return to this point with the language here of physicists publishing in 1987 in a reputable physics journal, that of 'mathematical formalism' being applied to 'physical phenomena'. Mr. Wheeler and Mr. Crummett are not obviously representing the view of millions of physicists; but they are representing one view that is considered creditable in physics, that these are the things in question, i.e. *mathematical formalism* on the one hand and *physical phenomena* on the other. This is precisely what I believe has *never been settled because it has not ever been discussed and debated with rigorous effort for most of scientific history roughly between 1641 and 2021.* I mark the beginning of scientific history arbitrarily to the publication of Rene Descartes' *Meditations on the First Philosophy.* There has never been any open and rigorous process with wide participation which accepted this sort of understanding that what is at stake is the application of *mathematical formalism* to *natural phenomena.* And that is not a fine point for quibbling. This is an enormously substantial issue. Why is any of this 'formalism' ever successful at all at predicting anything at all?

13. In Memoriam I Shall Christen The Atiyah-Singer-Dirac Operator On Four-Sphere S^4

I am feeling rather sad about the death of Isadore Singer. He died this February. Michael Atiyah died in 2019. I did get success with Four-Sphere Theory in 2018 a year before Michael Atiyah's death but I am rather saddened that they will not be hear to see one of their greatest contributions honoured. My S4 Electromagnetic Law uses the operator that is called the Dirac operator but Paul Dirac does not

deserve credit for it. The operator he discovered is far less sophisticated. He produced the matrix square root of the Laplacian on $\mathbf{R}^3$ which is a great innovation. But my S4 Electromagnetic law is not the Dirac equation; it is a wave equation on spinor fields of a four-sphere. I want it to be called Ahmed-d'Alembert Equation or just S4 Electromagnetic Equation. In it we have

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = D^2 u$$

The D I use ought to be called the Atiyah-Singer-Dirac operator. It's a true shame that the work of Atiyah and Singer is not recognised. Four-Sphere Theory owes much more to them than Paul Dirac. They deserve this honour. I owe it to them.

14. Four-Sphere Theory Is The Correct Exact Physics Of Nature And Not GR,QFT,Big Bang

Four-Sphere Theory I have worked on with my blood sweat and tears for more than a decade without funding, without tenure, without any university support, without decent employment without government support. So I don't really give a damn about who won how many Nobel Prizes. I checked the numbers. I am right, and QFT, GR, and Big Bang are wrong. Those who want to work on these wrong theories will obviously do so. But their fate will be that all their precious life's work will be collecting cobweb in forgotten basement store-rooms for centuries as the loveless cold world does not even offer them crumbs and a few footnotes. If this is the fate you want, carry on. Otherwise abandon GR, QFT, and Big Bang and work on Four-Sphere Theory.

15. What Is Wrong With Experience-Based Empiricism In Science?

I have emphasized before the *open question* of status of mathematics in Science as a mystery that requires a satisfactory understanding. Now I will make another point which is extremely subtle. We know that theories that are without empirical support in Science are ultimately worthless. They are *not Science at all*. We know this and we have plenty of easy examples of the folly of having trust in theories without empirical support. We live in an Age where this is second nature to most of us, at least most of us blessed with reasonable science education. But at the same time, we would be facing end of world's Civilisation if we actually relied on pure empiricism from *experience*. I would like to make the conjecture that Mathematics is necessary for us to avoid this horrible end of the world, that Mathematical substance is what saves us from facing the horrible situation where we have any theories that can be trusted without repeated necessity to rebuilding all our theories from experience. Thus this is an appeal to readjust the roles again. We love to talk and write and teach about the great value of experience but we would never live that way. We are instinctively wiser and value our Civilisation and lives too much to believe in such an outrageous and false religion.

16. The D'Alembert Equation Works Fine

I wanted to show you that the original vibrating string model arising from d'Alembert actually works just fine in the actual world. I'll show you some clear data from an Engineering paper by Che-Hua Yang and Tai-Chieh Wu [5]. This experiment is from 2017.

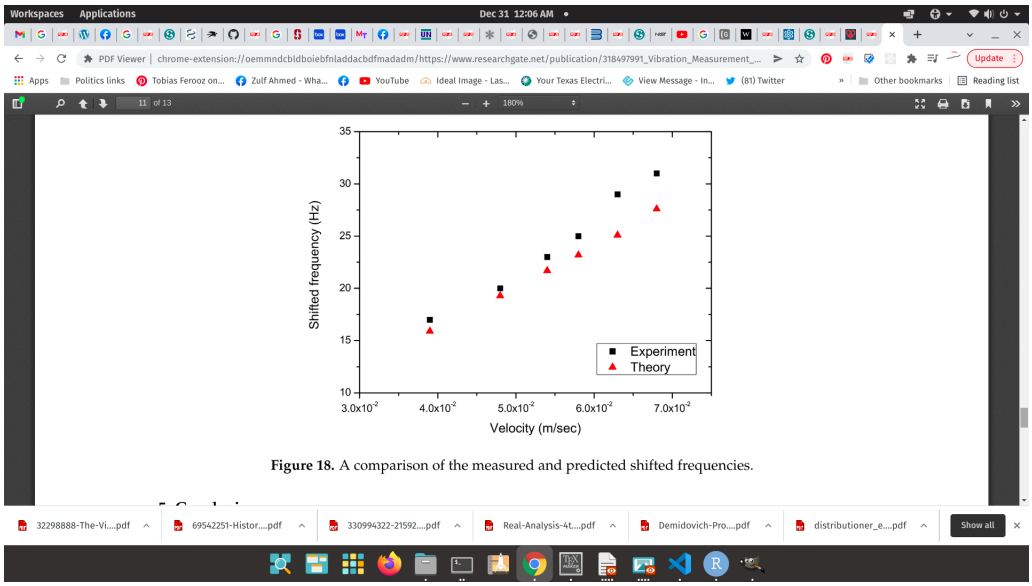


Figure 18. A comparison of the measured and predicted shifted frequencies.

I want my dear reader to reflect on the greatness of d'Alembert's accomplishment from 1746, that he wrote down an equation that actually models anything at all in Nature and sharp measurements of 2017 produces from this *exact model* something anything sensible at all. If you are not impressed. Do the following. Take off the day from work. Go to the park and contemplate the stars for some hours. Then cook up something that comes to your mind and write down some equations. Then go and tell your friends to test the equation. Chances are good that it produced nothing that works. Impressed?

17. Truth Is My S4 Electromagnetism Is The Exact Final Single Law Of Nature

I want to assure the world that my S4 Electromagnetic Law, which you may call Ahmed-d'Alembert Law, is the Final Theory of Nature and no one in a trillion years in the future will be able to improve on it at all. I have my name on the Final Absolute Impossible-To-Improve Law of all of Nature.

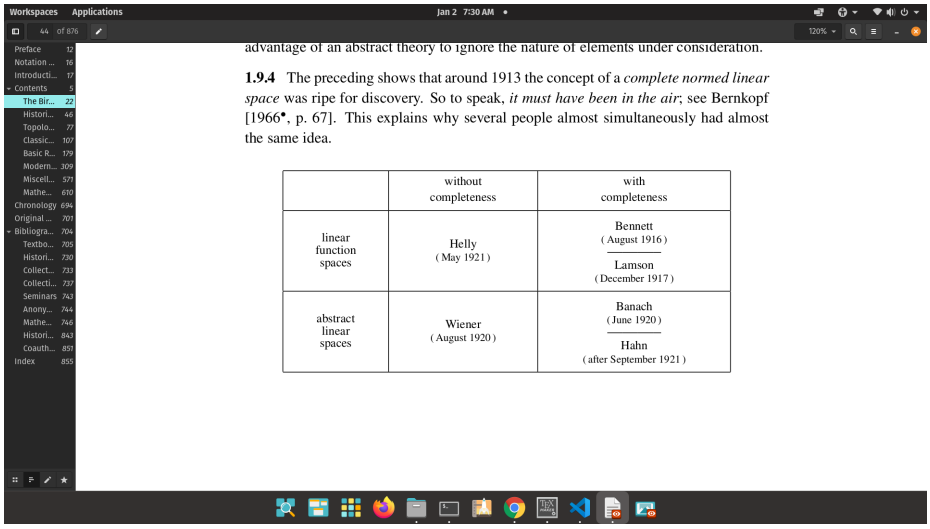
Sad but true.

18. When Did Functional Analysis Begin

I obtained an electronic copy of Albrecht Pietsch, *A History of Banach Spaces And Linear Operators*. I am thoroughly incapable of understanding things without history. I learn that it was Jacques Hadamard who in 1903 first considered the collection of all continuous functions on $[a, b]$ although Weierstrass, Arzela and Ascoli had considered sequences.

This is interesting for me, since I am curious now about what motivated Lebesgue at all, given that in the period 1854 to 1903 people were not as concerned about all contiuous functions. I see, sequences were considered without a space. Most curious. And so I just picked up habits and just declared $C^{\infty}(\Sigma S^4)$ as the space of matter fields. This is fascinating to me that functional analysis is a purely twentieth century affair.

19. Timetable For Discovery Of Banach Spaces



20. The History Of Topological Concepts

Caratheodory invented the name *open sets* but it had appeared before in 1899 by Baire and in 1902 in Lebesgue. Nowadays following a proposal of Tietze 1923 a topological set is specified by prescribing its open sets; a subset $\mathcal{B}$ is a base if all open sets can be written as unions from it.

Compactness in terms of sequences in metric spaces first appear in Frechet (1906) but the much more important coverings definition is by Alexandroff and Urysohn (1924).

Topological linear spaces where every point has a neighborhood bases of convex sets was considered first by Von Neumann in 1935. A *seminorm* is a function p satisfying (a) $p(x+y) \leq p(x) + p(y)$ and (b) $p(ax) = |a|p(x)$.

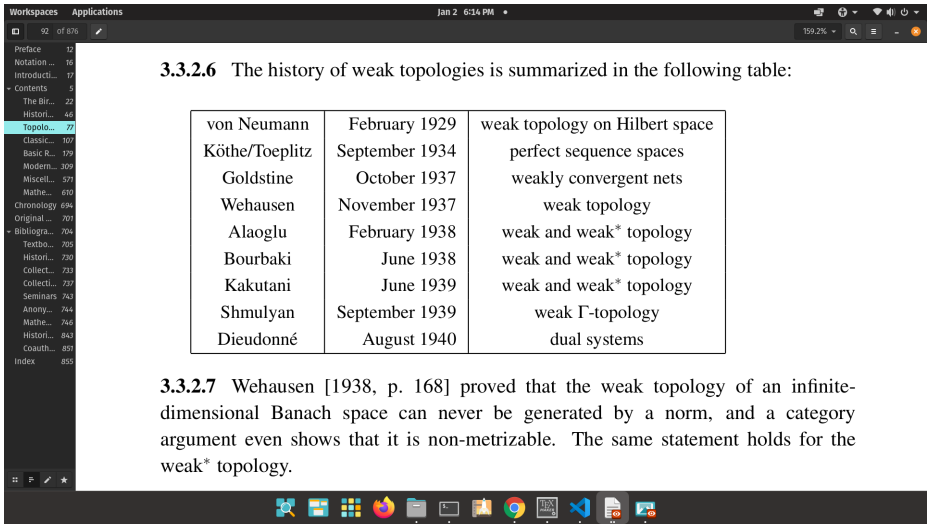
Von Neumann defined weak topology using families of seminorms, and Wehausen carried it over from Hilbert to Banach spaces.

For a finite set of linear functionals $\ell, \ldots, \ell_n \in X^*$ and $\delta > 0$ the sets

$$U(\ell_1, \ldots, \ell_n; \delta) = \{x : |\ell_j(x)| < \delta, j = 1, \ldots, n\}$$

defines a neighborhood base for the weak topology on X.

21. Timeline of Weak Topology



3.3.2.6 The history of weak topologies is summarized in the following table:

von Neumann	February 1929	weak topology on Hilbert space
Köthe/Toeplitz	September 1934	perfect sequence spaces
Goldstine	October 1937	weakly convergent nets
Wehausen	November 1937	weak topology
Alaoglu	February 1938	weak and weak* topology
Bourbaki	June 1938	weak and weak* topology
Kakutani	June 1939	weak and weak* topology
Shmulyan	September 1939	weak Γ-topology
Dieudonné	August 1940	dual systems

3.3.2.7 Wehausen [1938, p. 168] proved that the weak topology of an infinite-dimensional Banach space can never be generated by a norm, and a category argument even shows that it is non-metrizable. The same statement holds for the weak* topology.

22. A Renewed Attempt To Put The Importance Of These In Perspective For Philosophy of Science

For four-sphere theory functions in $X = L^2(\Gamma\Sigma S^4)$ are *matter fields*. They are direct models of things in the external world and nature. They are not *representations* by some mathematical formalism that indirectly models a *state of the physical system*. Therefore, there is a question of what is *weak convergence* on X physically?

Let me take a step back to explain why I am interested in this issue. Years ago, at Princton, I took a course on Philosophical Problems of Quantum Mechanics by Bas Van Fraasen. I was not a particularly brilliant student in that class. But I have, over the years, gained deeper interest in these issues of philosophy of science. I had come to the conclusion that the way in which quantum mechanical theory mapped to actual Nature was deeply unsatisfactory and in the end had to be abandoned sooner or later.

I am seeking, in these efforts, to find the alternative interpretation that is more satisfactory. I think that quantum mechanics suffers from a very bad paradigm of scientific theory and its correspondence to Nature and I believe that four-sphere theory can produce an infinitely superior Science and fundamental understanding of this correspondence of mathematical objects and objects of Nature.

It might at first seem rather abstract, to examine various philosophical questions of this sort but this is the heart of the *enterprise of Science* and this is what I wish to clarify for myself and for the world.

23. History of Spherical Harmonics

Arthur Cayley introduced Spherical Harmonics for arbitrary dimensions in 1848 [6]. Mehler, a student of Dirichlet, in 1866 was the first to attempt an expansion on S^d for $d \geq 2$ in terms of spherical harmonics. It is known that a subset, the *zonal* spherical harmonics are localised near a point.

This last issue of the *localisation of zonal harmonics of* S^4 a purely mathematical fact due to geometry of space is my scientific explanation for why *localised particles*

exist in nature as well as why *light has localised photons*. This is the basis for my four-sphere theory explanation for *wave particle duality in nature*.

References

[1] Haim Brezis and Felix Browder, Partial Differential Equations In The 20th Century, Advances in Math. 135 (1998), 76–144

[2] https://www.youtube.com/watch?v=obCjODeoLVw&t=49s

[3] https://e-l.unifi.it/pluginfile.php/773582/mod_resource/content/1/hankins_dalembert_euler_vibrating_string_controversy.pdf

[4] Gerald Wheeler and William Crummett, The vibrating string controversy, Am. J. Phys. 55 (1), 1987, pp.33–37

[5] Chia-Hua Yang and Chai-Tieh Wu, Vibration Measurement Method of a Stringin Transversal Motion by Using a PSD, Sensors 2017,17, 1643

[6] E. Heine, Handbuch der Kugelfunctionen. 1 1878, Bd. 2 1881.Berlin: Reimer. (There is a 1961 reprint by Physica Verlag, Würzburg.)