ON A METRIZABLE TOPOLOGY FOR SMOOTH SPINOR FIELDS ON A FOUR-SPHERE

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Ever since the empirical successes of my Four-Sphere Theory in 2018, I had been concerned about clear and detailed understanding of the mathematical issues. There are several mathematical texts that I have used extensively in my work. They are Harvey Lawson and Marie-Louise Michelsohn's *Spin Geometry*, Steven Zelditch's *The Spectrum of the Laplacian on Riemannian Manifolds*, and Michael Taylor's *Non-Commutative Harmonic Analysis*. I have also used a beautiful paper of Christian Bär, "The Dirac Operator On Space Forms Of Positive Curvature" from 1996.

I have always shied away from the technical aspects of topologies on function spaces from my undergraduate years at Princeton between 1991 and 1995, considering Jon Von Neumann's work as too technical and uninteresting. But now, I am beginning to understand that they are important, and so I am taking advantage of a wonderful problem from the Stanford Ph.D. Qualification exam from Spring of 2017 that asks the candidate to prove that the topology on $C^\infty(\mathbf{T})$ defined by various seminorms is metrizable but is not a *norm*-topology.

Therefore I decided to write a separate note with these issues for smooth sections of spinor fields on four-sphere as well.

1. Stanford 2017 Spring Analysis Ph.D. Qualifications Problem II.3

Let $\mathcal{T}$ be the weakest topology on $C^\infty(\mathbf{T})$ such that

$$f_{k,\psi}(\varphi) = \|\varphi - \psi\|_{C^k(\mathbf{T})}$$

are continuous for all $k \geq 0$ and for all $\phi \in C^\infty(\mathbf{T})$.

The norm $\|\cdot\|_{C^k(\mathbf{T})}$ is define by

$$\|\varphi\|_{C^k(\mathbf{T})} = \sum_{j \leq k} \sup |\partial^j \varphi|$$

(a) Prove that $\mathcal{T}$ is metrizable and write down a metric on $C^\infty(\mathbf{T})$ such that $\mathcal{T}$ is the metric topology.

(b) Show there is no *norm* on $C^\infty(\mathbf{T})$ is the norm-topology for it.

2. Purpose Of This Note

The purpose of this note is to provide a detailed and succinct and clear solution to the Stanford 2017 Analysis Problem II.3 and then extend the solution for the case of smooth spinor fields of a four-sphere.

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3. Smooth Spinor Fields On a Four-Sphere

Many mathematicians and physicists are not familiar with smooth spinor fields on a four-sphere. For this reasons, and without any intention to do anything original at all, I will give a brief account of these.

We expect that our dear readers are familiar with smooth real-valued functions on the four-sphere, $C^\infty(S^4)$. We let S^4 be the round four-sphere. It is a smooth manifold with riemannian metric g_0 with constant positive sectional curvature. We will not consider any exotic metrics in this note.

What is less well-known is that spinor fields on S^4 are isomorphic to 16-tuples of smooth functions. In particular, there is a basis of spinor fields given by *Killing spinor fields* $\sigma_1, \ldots, \sigma_{16}$ which are *globally defined* on the four-sphere such that all smooth spinor fields have representation

$$f(x) = \sum_{k=1}^{1} 6\sigma_k(x) f^k(x) \tag{1}$$

for f^k all smooth real-valued functions. This is described in detail in [1]. This is a beautiful paper and I recommend that all interested in my Four-Sphere Theory read this paper to appreciate the mathematical structure of spinor fields on a four-sphere. Professor Bär's presentation of the material is elegant and quite accessible where other techniques involving representation theory for the eigenspinors of the Dirac operator are substantially more complicated.

4. Topology on Smooth Spinor Fields From Topology On $C^\infty(S^4)$

The topology that we will posit for $X = C^\infty(\Gamma\Sigma S^4)$ will be just the product topology from that we will posit for $X_0 = C^\infty(S^4)$ using the identification from (1).

$$X = X_0^{16}$$

In other words a standard product topology construction will give us the naturally correct topology on X. We emphasize that the basis $(\sigma_1(x), \ldots, \sigma_{16}(x))$ of Killing spinor fields is a natural basis at every point $x \in S^4$ and the topology for X will not change by another choice of spinor field basis.

This is a special situation for four-sphere that we do not worry about generalising because the exact geometry of four-sphere is important for Four-Sphere Theory.

5. Revelations About My Prejudices Against Special Relativity

When I was a young undergraduate at Princeton, there was some expectation that I might get involved in physics. Fine Hall was right next to the Physics Department at Princeton. I will show you a picture because Princeton lives always in my heart as a loving permanent home.

I had done well in Physics BC in tenth grade in New York in 1988 with a 5 in the AP exam but I was not immediately drawn to physics courses. I was aware of Albert Einstein's Relativity Theory, but I was frankly skeptical of it and many other theories that were established. I was young and spent a great deal more time in pure mathematical matter in those years. I was simply not really part of the physics crew. But I took many Mathematics classes with good physics students such as Steven Gubser who was a stellar student a year ahead. I took both Peter Sarnak's Functional Analysis and Frederic Bien's Differential Geometry with Steven.

I became interested in serious work in theoretical physics first in 2003-5 in San Francisco. By then I had developed my scientific philosophy from work in Finance, from biology and other extremely empirical sciences, having had enormous amount of experience with statistical data. I had developed over the years an extremely strong sense for philosophy and sociology of science as well, with Paul K. Feyerabend's *Against Method* and other writings and Thomas Kuhn's *The Structure of Scientific Revolutions.*

Thus by the time I decided to take very seriously my own ideas in 2008 in Williamsburg, Brooklyn, relatively isolated from academic world, I was not beholden to any of the established theories of physics, and ready to challenge general relativity, quantum field theory and expansionary cosmology simultaneously.

I had decided that Einstein and his followers are a *religious cult of time-benders* who believe that time is more malleable than speed of light. This, over time, would horrify me with its absurdity. Time is quite a bit less malleable than speed of light or matter, and the Ancient Greeks were wise to give the Titan Chronos the distinction they had. I am 'classical' in the sense of absolutely refusing to yield to time-deformations or space deformations, but this is not an *anachronistic view.* It is a view that is truth. Special Relativity does not stand up to scrutiny for many reasons because *it is not true about nature.*

6. Some History For My Path To Four-Sphere Theory For The Reader's Perspective

I have had a strong resistance to a number of aspects of established physics for many years. Between 2008 and 2012 I had produced the first prediction of the redshift-distance slope that was discovered by Edwin Hubble and published in 1929 [**?**]. The theory that was established was that this slope represents a *dynamical process* in Nature. I was convinced in 2008 already that it was not evidence of any dynamical process in Nature at all. This slowly developed to a conviction that absolute space was a homogeneous four-sphere, and I produced the first explanation of this redshift slope as a geometric artifact of assuming that light travels in a flat space, which is the established theory, and my theory that light travels in a four-sphere of curvature $\Lambda = 1.11 \times 10^{-52} m^{-2}$. I solved the wave equation in the latter case and noted that the frequency-wavelength relation deviates from

$$\lambda = c/\nu$$

as rays on sphere have a discrepancy. This led to *predictions* of the redshift observed that matched measured extremely satisfactorily, and therefore I rejected expansionary cosmology based on my assessment.

A static non-dynamic space is far more parsimonious than a theory where both time and space deforms, for obvious reasons.

I rejected Special Relativity when I realised that Einstein's General Relativity Equation with $\Lambda > 0$ with the latter measured to be nonzero leads to an empty spacetime that does not follow Special Relativity at all. You see, Special Relativity is derived from Maxwell's Equations being the exact law of Nature with the $\mathbf{R}^3$ model, as the Lorentz Invariant transformations are just groups preserving the wave equation in $\mathbf{R}^3 \times \mathbf{R}$. Therefore when $\Lambda > 0$ is measured, Special Relativity is not even a *natural hypothesis* and so I rejected Special Relativity as irrelevant to actual physics in Nature when I considered what was known. My absolute time and absolute space model had much better features for Science, and I have been strongly resistant to consider either Special Relativity or Expansion as serious scientific models with my work.

7. Bill Gates Is A Worthless Cunt Who Deserves To Be Slaughtered

Bill Gates is a totally worthless cunt who is a charlatan about his projection of great prodigious intellect of which there is very little evidence. He is an illiterate college dropout who is obstructing my livelihood. During a Harvard tenure consider he intervened with US Industrial Power to damage my exoskeleton. These sorts of criminal cunts who are totally worthless intellectually inferior but pretend to be great intellectual giants with promoting their favourite book of all time as Steven Pinker's *Enlightenment Now* while repeating 'Natural rights are claptrap' all day do not deserve anything but lethal military end. He is an intellectual amoeba not just compared to me, but compared to every serious scholar and gentleman that has ever lived on Earth. I strongly advocate violent destruction of this worthless cunt.

8. Topology Of $C^\infty(\mathbf{R})$

Suppose X is a vector space. We give our respect to Michael Reed and Barry Simon for their extraordinary lucid volumes on functional analysis, and we freely use their Chapter V on Locally Convex Spaces. We are interested in understanding the issues clearly in this note primarily rather than only on novelty. In fact the novelty is not profound, but the understanding is more substantial.

For a Banach space $(X, \|\cdot\|)$ the *norm topology* is generated by balls

$$B(x_0, r) = \{x_0 \in X : \|\|x - x_0\| < r\} \tag{2}$$

For many people, including me, just the notation is comfortable. At Princeton, those who are interested in taking a Mathematical track generally take a course on Mathematical Analysis in their second semester after a course on Linear Algebra usually with material that is equivalent to Walter Rudin's *Principles of Mathematical Analysis*, affectionately called *baby Rudin* by many students.

My dear readers are quite a bit more sophisticated than this, but simply for the sake of comfort, let us recall that the topology of $\mathbf{R}^n$ consist of all sets that can be produced by unions of these sorts of balls $B(x_0, r)$ with various $x_0 \in \mathbf{R}^n$ and $r > 0$. Our early encounter with the analysis leaves an indelible mark and then we feel comfortable with the norm-topology on Banach space X.

The troubles begin in Paradise when the norm topology does not manage to capture what we are looking for when the spaces are infinite dimensional. And this unfamiliar situation is what was the primary purpose of all sorts of technical innovations between 1900-1940 in analysis.

Analysts considered, instead of norms, notions of convergence based on *families of seminorms*. These are families $(\rho_a)_{a\in A}$. The familiar $x_j \to x$ iff $\|x_j - x\| \to 0$ near and dear to our hearts was replaced by something different.

The new vogue for these topologies was that $x_j \to x$ iff

$$\rho_a(x_j - x) \to$$

for all $a \in A$ simultaneously. These things do not satisfy

$$\rho_a(x) = 0 \text{ implies } x = 0$$

Now if you see these things in the wild, they might give you a fright. And you're not alone. I was given a terrible fright when I saw this as well for the first time. My heart cried out, "What have they done to my beautiful Euclidean topology! They are savage barbaric beasts!"

Now Professors Reed and Simon motivate the seminorms abstractly as a replacement for the convergence criteria. I am not motivated by this. So I will try to present the motivation more broadly.

The most standard example of this seminorm convergence arose for a topology of bounded linear maps $L(X,Y)$ where X and Y are Banach spaces. One parametrizes this by *points in X and linear functionals on Y*. For each $x \in X$ and each $\ell \in Y*$ we consider

$$E_{x,\ell}(T) = \ell(Tx)$$

This gives us all these functions $L(X,Y) \to \mathbf{R}$. Then we demand that for all $(x,\ell) \in X \times Y^*$ all these $E_{x,\ell}$ are continuous.

This topology is defined by pulling back the norm topology of $\mathbf{R}$ by these huge family $\{E_{x,\ell}\}_{x\in X,\ell\in Y^*}$ to produce the topology on $L(X,Y)$ and then this is the *weak operator topology.*

It is worth just getting used to this pattern because the other examples are similar in nature. We set up large families of real-valued functions and demand that the topology be defined by the pull-backs of the topology of $\mathbf{R}$ so that the all these functions are continuous.

It's never as big as the the norm-topology which is even larger. This abstract maneuver then allows various sorts of things to be proven. Not long ago I proved that the translation operator $T_t f(x) = f(x-t)$ which translates f to the right by t is not continuous as a map $t \mapsto L(L^1(\mathbf{R}))$ in the norm topology but once one puts weak operator topology it is continuous.

And that's the reason for weakening – i.e. shedding a huge number of open sets – from $L(X,Y)$, for this sort of situation when you are mapping *onto* $L(X,Y)$ and then too many open sets overwhelms the domain topology and that's what one really cares about more than the geometry of huge infinite-dimensional spaces. You want to have functions:

$$G : N \to L(X,Y)$$

where N is really nice and familiar and you want to proceed with N is so nice and so Euclidean-like, and G is continuous, and G is smooth. These things are not possible with norm-topology and we sacrifice intuition on $L(X,Y)$ topology in order to keep our intuition intact about G and N.

So this motivation is right, but unfortunately many people do not understand that we, the ordinary lovers of Euclidean topology and calculus of smooth functions, need to know what these G and N are rather than seeing the horrible seminorms and other things in the weak operator topology detached from the rest of the world of sanity.

9. The True Motivation For Locally Convex Spaces

I want to argue next that the true motivation for locally convex topologies is precisely the situation for spinor fields on the four-sphere, $C^\infty(\Gamma\Sigma S^4)$. In order to appreciate that note that for people who will not be publishing in totally esoteric mathematical journals fine tuned results on the general properties of locally convex spaces and Frechet spaces with great pride in their obscurity and their exclusiveness, no one cares about local convexity intrinsically. This is like caring about the deep wiring of your laptop motherboard. You might have some electrical engineers getting excited about all that but no ordinary person will care. It's technical mumbo-jumbo. Let's be honest. Ordinary civilised human beings will care about Rembrandt and Shakespeare and some will care about the finer points of Eliot's *The Waste Land*. But except for people in tech-babble land, no one thinks that the laptop motherboard is all that exciting except electrical engineers who make a living from this; they are the red stapler guy to most people. In the same way, what Von Neumann and his colleagues did with topologies by families of seminorms is a purely technical achievement.

I will give you the true motivation. Suppose you have a function

$$G : \mathbf{R}^5 \to C^{infty}(\Sigma S^4)$$

This is a five parameter smooth function of *matter fields*. Then you do some calculus. This is artificial but let's say you have reason to declare

$$(\partial_x^2 + \partial_y^2 + \partial_z^2)G = \partial_t G + wG$$

And then you think this is the greatest matter field equation in the world, and you proceed to seek a solution. Well you better have a Von Neumann seminorm-based topology on $C^\infty(\Sigma S^4)$ because if you don't, this equation is meaningless because linear movement in $\mathbf{R}^5$ in any direction does not produce anything you can understand at all with norm-topology on $C^\infty(\Sigma S^4)$. You will need to pay tribute to Von Neumann for solving your problem.

10. For Physics The Weak Topology Is Necessary

For actual situations in Nature, a complex polyatomic molecule, a vast number of molecules in motion in a supernova, the biological fluid in a complex environment, and so on, there needs to be calibration to measurements, and so these sorts of functions

$$G : N \to C^\infty(\Sigma S^4)$$

are not artificials but the natural models of things in Nature with Four-Sphere Theory. No physicist wants to get stuck on technicalities when their evidently smooth parametrization of N, the configuration space, perhaps a smooth manifold adapted to a physical situation, with formulae on how matter and light will interact will suddenly run into *topology problems* on the space of matter fields.

And so what Von Neumann did in 1935 – and others were involved as well – was to provide the general solution of how to ensure that smooth calculus on N translates to smooth calculus relative to the topology of $C^\infty(\Sigma S^4)$. He did not know about this particular function space but it falls within the general locally convex linear spaces he considered. And so his work is directly relevant for *physics* in Four-Sphere Theory. In other words *it is his seminorm topology that is physically important* and not *the norm topology*. This is a very deep philosophical point, because there is always a question of what mathematical constructions are actually reflecting Nature and what are not. In this case, norm topologies *do not reflect Nature* although they reflect *human intuition*.

Looking at this issue in this way, it is a marvelous thing that Von Neumann did. It's hard to understand this just by looking at convergence by seminorms and other technical parts of the solution. But this is the right *physical* solution to this problem, and we don't understand it without the G and N in the discourse about it.

11. Analytic Solution For Locally Convex Spaces

The analytic view is that we consider a family of seminorms $(\rho_a)_{a\in A}$ on a normed space. These satisfy

- $\rho_a(x+y) \leq \rho_a(x) + \rho_a(y)$
- $\rho(sx) = |s|\rho(x)$

And we say that the family *separates points* if $\rho_a(x) = 0$ for all $a \in A$ implies $x = 0$. The whole family together gives us some norm-like behaviour. This whole technical direction was led by Jon Von Neumann. It's the correct topology if we want to retain our intuitive calculus. It's much more significant to physics than a technical formalism. Physics will fail in formulations of various physical situations if another topology is put on matter fields in Four-Sphere Theory.

Recall, dear reader, that Four-Sphere Theory is the final theory of macroscopic Nature and not any toy model. So we want to honour Jon Von Neumann here for allowing us to proceed without worry for physical models of actual complexity of the world with Newtonian calculus even with matter fields that are global in the universe without technical worry at all.

12. The Seminorms For $C^\infty(S^4)$

Let us define the $C^k(S^4)$ norm as

$$\|\varphi\|_{C^k(S^4)} = \sum_{|\alpha|\leq k} \sup_{x\in S^4} |\nabla^\alpha \varphi(x)|$$

where we will define the operators ∇^α later but they are substitute for ∂^α on $\mathbf{R}^4$. We need a bit of care to account for the Riemannian curvature.

For any $k \geq 0$ and any $\psi \in C^\infty(S^4)$ we define

$$E_{k,\psi}(\varphi) = \|\varphi - \psi\|_{C^k(S^4)}$$

Now we define the topology on $C^\infty(S^4)$ as the weakest topology such that $E_{k,\psi}$ for all $k \geq 0$ and all $\psi \in C^\infty(S^4)$ are continuous. This is the weak topology for $C^\infty(S^4)$ and we can then use 16 copies of this topology to obtain matter field topology in Four-Sphere Theory following the prescription in Topology books such as James Munkres *Topology*.

13. The Problems Of Physicists Engaged With Mathematics

Physicists are varied and there are many of them, hundreds of thousands, maybe millions. Every since Hilbert's Sixth problem was not solved, early in the twentieth century there was rift that grew between Mathematics and Physics that was never easy to mend. Physicists like Richard Feynman were impatient with mathematical concerns, and developed the mathematically meaningless *Feynman Path Integral* that no one was able to fix satisfactorily. There are no translation-invariant measures in infinite dimensions. So they just threw mathematical rigour to the wind and focused on physical intuition and physical sense as *knowing what one is talking about.*

The other end was a stream of physicists who became hyper-conscious and paranoid about mathematical justification and refused to move without producing tomes of justification for each of their steps.

Neither direction is good. What ought to exist are strong theorems for basic situations that guarantee that reasonable operations that physicists might use are offered with mathematical justification, and that physicists have serious mathematical results that give them some reassurance that their elaborate calculation techniques are are thoroughly checked for validity with clear indication of where they will fail. This is really a task that *mathematicians should do* and then physicists can focus on physics without becoming experts on all manner of esoterica about locally convex spaces. These things are extremely technical details and not all that important as concerns for physicists who will be dealing with actual situations in Nature and have the responsibility to produce scientific theories that work.

I am extremely glad that Von Neumann and others have worked out good solutions to these problems, and I do hope that in the future, physicists who choose to work with Four-Sphere Theory will not either throw mathematics to the wind or become hypochondriac nervous wrecks.

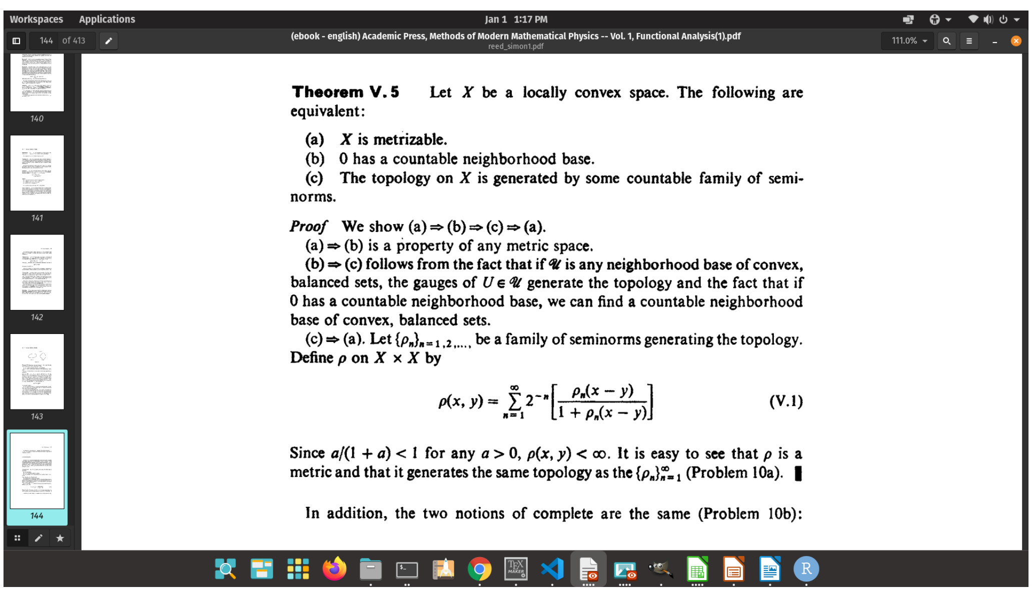
14. A Return To the Stanford Spring 2017 Analysis Quals Problem II.3

The problem is to understand the *metrizability* of the weak topology defined a family of seminorms

$$E_{k,\psi}(\varphi) = \|\varphi - \psi\|_{C^k(\mathbf{T})}$$

The topology is the weakest on $C^\infty(\mathbf{T})$ that guarantees that $E_{k,\psi}$ are continuous for all $k \geq 0, \psi \in C^\infty(\mathbf{T})$.

15. Reed-Simon V.5



Theorem V.5 Let X be a locally convex space. The following are equivalent:

(a) X is metrizable.
(b) 0 has a countable neighborhood base.
(c) The topology on X is generated by some countable family of seminorms.

Proof We show (a) ⇒ (b) ⇒ (c) ⇒ (a).

(a) ⇒ (b) is a property of any metric space.

(b) ⇒ (c) follows from the fact that if $\mathscr{U}$ is any neighborhood base of convex, balanced sets, the gauges of $U \in \mathscr{U}$ generate the topology and the fact that if 0 has a countable neighborhood base, we can find a countable neighborhood base of convex, balanced sets.

(c) ⇒ (a). Let $\{\rho_n\}_{n=1,2,\ldots}$ be a family of seminorms generating the topology. Define ρ on $X \times X$ by

$$\rho(x, y) = \sum_{n=1}^{\infty} 2^{-n}\left[\frac{\rho_n(x-y)}{1+\rho_n(x-y)}\right] \tag{V.1}$$

Since $a/(1+a) < 1$ for any $a > 0$, $\rho(x, y) < \infty$. It is easy to see that ρ is a metric and that it generates the same topology as the $\{\rho_n\}_{n=1}^{\infty}$ (Problem 10a). ∎

In addition, the two notions of complete are the same (Problem 10b):

Here we see the beautiful general construction that produces a metric from a family of seminorms, and proves metrizability directly. This is from p. 131. They attribute it as a special case of metrizability of uniform spaces and refers to Kelley's *General Topology* pp. 184–190. This issue of a topology in this situation that not only separates points, but so that maps from finite dimensional manifolds have smoothness and continuity and so on, this is the beauty of this situation. I am not knowledgeable enough about the technicalities, and don't really want to delve in these either.

So the procedure is to produce a neighborhood basis using seminorms, and then apply this theorem of Reed-Simon to conclude that the topology is reasonable.

16. Four-Sphere Theory Is Mine And Mine Alone

Years ago when I became interested in four-sphere theory in 2008 I emailed Dan Stroock and he was not interested in supporting me then. I don't hold it against him. He wrote to me that if I shoot at a great man I must kill him and to find someone else. I laboured with my convictions regarding physics, primarily that *quantisation of energy is due to compact geometry of space and an application of spectral theorem for compact self-adjoint operators on a geometric Laplacian.* This cost me everything. I abandoned all my material possessions in late summer in Williamsburg Brooklyn. There were other issues involved too. I almost died in six months of homelessness. Then I lived with my aunt in Allen texas roughly from 2011. I had no income, was on disability for a decade. I laboured on Four-Sphere Theory for years without any government support, no university support and I succeeded in getting empirical results and resolved many fundamental issues. I studied ordinary quantum mechanics to understand what the issues have been, and I shared my successes from 2018 with many people in physics.

Bill Gates has put a lot of money against four-sphere theory and he had resolved to murder me by all manner of power black magic and other means of harm. Why should I care about anything but to seek his absolute destruction? He's a charlatan and I am an immortal genius whose achievements are far superior to his menial achievements of 1955-2021. He wants to give credit of my work achieved through suffering and isolation for more than a decade to various white people out of insidious racial agenda. That is not even possible for me to contemplate. I believe in a single human race, and consider him to be pestilence and a curse upon my beloved people. He should be slaughtered in a bloody manner.

17. Natural Base For Locally Convex Spaces

Recall that a locally convex space is associated with a family of seminorms $(\rho_a)_{a\in A}$. A neighborhood base at 0 is given by $\{N_{a_1,\ldots,a_n} : a_j \in A\}$ where

$$N_{a,\ldots,a_n} = \{x : \rho_{a_j}(x) < \epsilon\}$$

Only finitely many a_j are simultaneously involved for the these open sets. This *finiteness* is a consequence of the requirement that the family $(\rho_a)_{a\in A}$ separates points. Without this separation condition, we do not have this result.

18. Good Memories From Biospect/Predicant

My time in Biospect/Predicant was a happy and productive time. I worked with Hans Bitter the Physical Chemist, with Arjuna Balasingham, the computer science man, with Perry de Valpine who was doing some very nice work in science then, with Mikhail Belov the famous designer of mass spectrometer, John Stults a leader in proteomics, and Pete Foley the engineer who had a stint at Apple. Several episodes stand out in my mind. There was a celebration of our lives that the leadership team wanted and Hans and I decided to put photographs of a young boy praying in Afghanistan and a young German boy in Lederhosen. Not many people noticed the National Geographic quality of the photographs. Then there was the episode of us buying a postcard for Pete Foley with all manner of romantic flowers which was so funny I started cackling, disrupting a Board Meeting so that one of the board members came to shut me up. Then there was the time I was very into reading correspondence between Galileo and Bellarmine and decided to opine loudly in the common room that Bellarmine was right and Galileo simply did not have enough and that caused great consternation among the reputable scientists in the company. Ah, the happiness and joy of youth!

19. Matter Field Calculus Can Be Fully Rigorous And Infinitely Less Difficult For Realistic Models

Matter fields in quantum field theory are matters so esoteric that only some small group of people can even understand what they are. In four-sphere theory matter fields are both simpler, more accurate as models of Nature, and the only fundamental law for scales above $\delta > 10^{-13}$ cm, i.e. the only scales that matter for all of science with exception of high energy physics, are extremely mathematically rigorous and robust in calculus.

Once the right topology, that by seminorms

$$\rho_k(\varphi) = \|\varphi\|_{C^k(\Sigma S^4)}$$

are put in, the weak topology on $C^\infty(\Sigma S^4)$ allows us to consider smooth functions

$$G : N \to C^\infty(\Sigma S^4)$$

and full calculus is sensible. Thus one can consider differentiation in all directions, produce formulae for *matter field-valued functions* and manipulate them as other functions between finite dimensional smooth manifolds without needing to justify every step.

This means that *matter fields* become objects that can be modeled in various ways and physicists do not have to have fudge models and have other problems because the weak topology eliminates all manner of problems is smoothness.

The issues of what is a Frechet space and what are its local parametrization is mostly technical detail that is irrelevant once some theorems guarantee validity of calculus operations.

I don't have examples at the moment, but this is very valuable, this topology because physicists don't actually have to worry about the topology all that much. Physicists are interested in predictions that can be compared with measurements. These are obviously infinitely easier in this case without introducing wooly path integrals. Spinor fields are matrix-valued fields that are 'classical' and are no more difficult than electromagnetic fields.

20. Graff's Differential Calculus Theory For Locally Convex Spaces

I am not interested in getting involved in mathematical theory on theories of differential calculus with values in spaces like $C^\infty(\Sigma S^4)$. But I respect those, like

Richard Graff, who had been working in this area [2]. There are several approaches but my purpose in this note is to point out that these issues have been addressed and people in science and especially physics ought to use the extant theorems without getting involved in the business of re-inventing things. Differential calculus can be used with a due credit to mathematical work and *ought* to be used for matter fields in four-sphere theory. The mathematicians who worked on these have done a fantastic job and they ought to see more use for their great work.

References

[1] Christian Bär, The Dirac Operator On A Space Form Of Positive Curvature, J. Math. Soc. Japan, 48 (1), 1996, pp. 69–83

[2] Richard A. Graff, Theory of Differential Calculus in Locally Convex Spaces,Transactions of the American Mathematical Society, Vol. 293, No. 2 (Feb., 1986), pp. 485- 509