STANFORD SPRING 2012 ANALYSIS PROBLEM II.4

ZULFIKAR MOINUDDIN AHMED

1. Stanford Analysis 2013 Problem II.4

Let X be an uncountable set with discrete topology, let $\hat{X}$ be the one-point compactification and $C(\hat{X})$ the continuous functions.

(a) Find the Baire and Borel sets for $\hat{X}$.

(b) Find a sigma-algebra $\mathcal{B}$ that is bigger than Baire and smaller than Borel where there are two distinct measures such that $\int f d\mu_1 = \int f d\mu_2$ and explain why this does not contradict the Riesz Representation theorem.

2. I Have No Feel For This Problem

This is my third day, and I made sense of nine problems out of ten in the past two days for Stanford Analysis 2013 Ph.D. Qual Problems. I simply do not have any sense for this problem at all. So this will be a learning exercise for me, and while in the other nine problems I did not really look at many texts, in this one I will disclaim that I will look at any and all texts. Stanford Examiners should not treat this particular note as a fair examination at all. I plan to just educate myself on these issues with a lot of texts I will consult.

3. The Differing Addressing I Use With Bill Gates In Meta

I have various appellations for Bill Gates in meta. I will tell you about three of them, and they are very comfortable for me, and I make no apologies for them at all.

- Cunt: e.g. "Speak for yourself, cunt."
- Billy Boy Old Chap: e.g. "Things are not looking so good for you now, Billy Boy Old Chap."
- Demon: e.g. "No Demon has ever defeated an Angel in an Angelic World in millions of years, Demon. How will you survive now, huh?"

4. What Is The Riesz Representation Theorem Exactly?

I look at Stein-Shakarji III pp. 181-182 first. Every *continuous linear functional* on a Hilbert space arises as an inner product.

There is a slightly more abstract version. Theorem 4.3 of p. 290.

Suppose μ is a σ-finite positive measure on measure space $(X, \mathcal{M})$ and ν a σ-finite signed measure. Then there is a decomposition $\nu = \nu_a + \nu_s$ where ν_a is absolutely ontinuous with respet to μ and ν_s is singular with respect to μ. Furthermore there is a μ-integrable f such that

$$\nu_a(E) = \int_E f d\mu$$

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Fine. Let us return to this later.

5. Measurability In $\hat{X}$

I am used to one-point compactification of $\mathbf{C}$ to obtain the Riemann sphere. Never had I considered the barbaric situation of a discrete topology on an uncountable set X in my life. I better try to examine this strange situation.

Now in the discrete topology, only finite unions of points are compact in X. This has to be one of the major points of the problem. Now why would one want such a dreadful thing one does not know. I keep away from such horrible things generally.

Let $\hat{X} = X \cup \{\infty\}$. The neighborhoods of ∞ will only be those subsets $U \subset \hat{X}$ whose complements are finite sets.

What are the Baire and Borel sets? I don't know either. You see, if isn't too close to $\mathbf{R}^n$ with metric topology it generally is lost to me. This involves chasing down technicalities and looking things up.

Baire sets are the smallest σ-algebra where every *compactly supported* functions are *measurable*.

Borel sets are those defined by the topology, so all the subsets of X and then the open neighborhoods of ∞.

The whole space is compact, and all the unions of finite points. In this σ-algebra you can't even have any compactly supported function that is not zero except for finite number of points or constants.

6. This Is Not An Insightful Test Of Riesz Representation Theorem

I don't understand why this is an interesting problem. No one cares about these technical conditions that I have ever heard off. Who cares if Riesz Representation fails in a distant constellation dying in the corner of the sky, as the camera moves in slow motion, as we look to a song, as we look to a song. These are the days of miracle and wonder, and this is a long distance call. And I believe: these are the days of lasers in the jungle, lasers in the jungle somewhere. Staccato signals of constant information;a loose affiliation of millionaires and billionaires and baby. These are the days of miracle and wonder. This is the long distance call. The way the camera follows us in slo-mo. The way we look to us all, oh yeah.

I just had to get this off my chest.

7. This Problem Is Bad Mathematics

See, I am not exactly *laissez faire liberal* about mathematics. This is example of bad mathematics because it is asking someone to examine what happens in bizarro situation that no would care about in a million years for any serious work at all, and asking us to see how Riesz Representation would not be unique there. This does not provide fundamental insight about what could go wrong in situations that we do care about at all. Who cares if Riesz Representation failes *if you put discrete topology on an uncountable set*? The real question is why would you do such a horrible thing? Even raving lunatics and Jesus freaks out on the street selling tickets to God have enough sense never to put discrete topology on an uncountable space and try to do anything. Who cares if other things such as Riesz Representation does not work there?

See, if I thought that someone spent a lot of time working with discrete topologies on uncountable sets, I would think the person looks like this.

Who else would do such bizarre things? I was trained in geometry and topology. I need these things before I encounter such horrors.

Anyway. Let's get back to (b) now. Consider the Lebesgue measure μ on $[0, 1]$. We're going to try to totally mess up the topology of $[0, 1]$ by the process in (a) now all the open sets are complements of finite sets including the end points, and our $\mathcal{B}$ will be generated by ordinary Borel sets in the metric topology (sane topology).

Now $f \in C(\hat{X})$ is pulling back from ordinary Borel sets from $\mathbf{R}$. Maybe this will not help.

Dec 20 2021. 8:57 PM and I don't have two measures with $\int f d\mu_1 = \int f d\mu_2$ for all $f \in C(\hat{X})$ yet. This is a very difficult problem for me because I don't have any intuition for sigma-algebras that are not from the ordinary topology from metric space open sets.

I will leave this alone and give up for now.

8. Good Taste: Frigyes Riesz 1880-1956

I feel a bit better knowing that there is a paragon of good taste in mathematics, and that is Frigyes Riesz. See these sorts of problems seeking failure of Riesz Representation theorem do not fully appreciate the good fortune that the world had of having Frigyes Riesz develop the central pathways of functional analysis.

F. Riesz is the pioneer who found the dual of Hilbert spaces, L^p spaces, and $C([a, b])$ and proved the spectral theorem for compact self-adjoint operators. His great contributions are from 1905-1935. He named L^p after Lebesgue and H^p after Hardy.

That's right ladies and gentlemen, it is not good mathematics to show that the marvelous and beautiful results of Frigyes Riesz do not apply in distant constellations. Before he invented the path to Riesz Representations no one knew that it mattered at all. And just look at the beautiful spaces he considered. Genuine $C([a, b])$. Truly beautiful. Do you see any pathological topology in $[a, b]$? Who cares if his results don't work on all manner of rubbish topologies? They work on topologies that *matter* and that is fantastic new mathematics. Everyone lives and breathes these spaces in Analysis. No one gives a hoot about discrete topology one-point compactifications. Those are getting derailed from proper cultivation in great mathematics. If they mattered, F. Riesz would have told us between 1905-1935. He knew better.

9. Canonical Teachers Of Canon Are Christian Theologians

Both Eastern and Wester Church Fathers are roughly from 350 AD. They worked on traditions that are still going after 1700 years. Compare this with the entire history of Mathematical Analysis. Compare that with a puny 360 years since Newton invented calculus. I think that Mathematics will find, in around 1350 years that it ought to have learned something about keeping to centrality of issues in the Canon. Christianity has 2 billion faithful today. Mathematics will be lucky if retains a hundred million ever, and certainly not if Mathematical Canon sways with the wind and keeps trying to radically refocus attention on the totally esoteric in the obsession for novelty. Stick with great paths of great mathematicians. Otherwise generations of mathematicians will have no taste and no tradition and no sense of what is substantial and what is trivia.

10. Marvelous Little Poster