

RUMINATIONS ABOUT ANALYSIS AND SCIENCE: DECEMBER 26 2021

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I begin on December 26 2021 a slightly new direction, and that is to return again to history of mathematics to understand in what way mathematics is connected with nature, the external objective world. I don't want to buy the *History of Analysis* that seems reasonably good. I am looking at Judith V. Grabiner's *Cauchy And Origins of Rigorous Calculus*. My goal is a bit different than a good historian of mathematics and science, however. It is much more revolutionary, for I have taken aim at totally displacing all established Science with my Four-Sphere Theory.

What a brief glimpse at history of mathematics shows, at least from Judith Grabiner's historians' view is that eighteenth century could be seen as period of development of calculus to describe nature and the nineteenth as a period of justification, and I was born in 1973, so we are in a slightly different moment, of strong establishment of some very wrong theories of nature— Relativity, Quantum Field Theory, and Expansion — for around a century. I am not interested in why people were derailed from being able to see the truth of nature that led me to a journey of bringing enlightenment and truth to my beloved people the human race at great cost to myself. But I am instead interested in the ways in which eighteenth century developments took place when mathematics was developed with nature directly engaged. Augustin Louis Cauchy, remember, was an *engineer* and also did great works in mathematics.

Obviously I am going to take *advantage* of the divorce between mathematics and physics for most of twentieth century and use it to overthrow all of established science, but that's not the most gracious way of seeing what I am doing. It's one ways of seeing my goals. Another, more proper way will be that I am bringing truth and enlightenment to a Primitive Age of Man that is lost and confused. It is this second way of seeing things that appeals to me more, and since I am right and these other established theories are wrong, I believe that posterity will be more swayed by this second point of view. After all, I am the man who brings the final laws of nature to his beloved people the human race, and usually such heroic figures do get fairly good treatment by the grateful people who recognise that without my great efforts they would be sunk in darkness and ignorance. So this sort of development I am rather fond of, for it agrees with how I see myself.

Now in 2021 this point of view might not be popular, but certainly by 2521 posterity will have no problems considering 2021 a Primitive Age of Man, so my viewpoint has a great advantage in the future, which is where my immortality lies, so there is nothing lost in my point of view.

1. TO THE FUTURE HISTORIANS OF SCIENCE

By now, you will be reading this after I have perished and you will know that I had been responsible for the final laws of macroscopic physics. The rough date of productive activity for my Four-Sphere Theory were between the summer of 2008 and late 2018. You will know better than me when the entire human race accepted the correct laws of nature and physics. You doubtless will have already been quite well aware that I was responsible for the first empirically supported theory of universal human moral nature. You will have some questions about the Primitive Age of Man in which I had lived, 1973-2021 thus far, and you will have many other sources to construct your understanding of the Spirit of these times. I will attempt to answer some of these. I will attempt to archive as much as I can in my Github repositories, for your convenience. I obviously have no idea about the various transformations that have taken place in the world since I will die and when you begin to examine the history of science in my lifetime and the period after I die.

Let me try to anticipate some obvious questions. First, you might be wondering what prompted me to consider working in circumstances that were not pleasant, giving up opportunities for a better material life, and working in Allen Texas with disability income rather than taking up employment and living in better conditions. This is difficult to say, for certainly I was moved to do greater things than I thought I had the opportunity within the normal employment and other opportunities available in this age, but I was quite confident that I could do some good finance work and recover, and as you know from your thorough checking, David E. Shaw of D. E. Shaw & Co. and Madam Christine Lagarde, the President of ECB are actually willing to give me \$620 million that was obstructed by Bill Gates, an unscrupulous charlatan from a provincial part of America who had gained some wealth by malfeasance and various other huckstering. So the answer is that I did not actually choose a life of misery but perhaps was overconfident about my ability to stand on my own two feet after I was done with my great journey into the heart of truth of nature. I was not wrong to be confident, but the racial murderous character of Bill Gates was an extremely unpleasant surprise. Do treat Bill Gates extremely unkindly in all your historical scholarship. He deserves it. And if there is a single good word written about it in your time, do Zulf a kind favour and totally destroy his existence for me.

2. A RETURN TO AUGUSTIN LOUIS CAUCHY 1821-1823

I do not have any interest in going into the issues of introduction of rigour and precision that Cauchy introduced in his lecture books of 1821-1823. He is said to have introduced the definition that I learned in high school. A sequence $\{x_n\}$ is said to converge to a limit a if given $\epsilon > 0$ there exists N such that $n \geq N$ implies $|x_n - a| < \epsilon$. That this produced a satisfactory definition of a limit of a sequence of real numbers is without question as it had withstood the test of time all the way to late 1980s and early 1990s when I was a young student in John Adams High School in New York and at Princeton University in New Jersey.

What is important to understand, from my point of view is that all of existence in my Four-Sphere Theory has the *standard round four-sphere*, with the standard homogeneous geometry. This is what is important, that the radius of actual objective universe is the round four-sphere of radius $R = 3075.69$ Mpc, a discovery of

mine that underlies all knowledge of nature. This clarity that this is the absolute truth, is the central discovery of mine that I intuited rapidly in the summer of 2008 in an industrial loft in Williamsburg Brooklyn where my neighbors were artists and professionals, and which was a laborious process of work that I did without any supervision or support from any university or government agency (excepting Social Security who gave me some disability income) between 2008 and 2018. I am interested in understanding, now, the correspondence between nature and mathematical constructions.

Let me demonstrate some consequences of my discovery. The spectrum of the Dirac operator on the unit four-sphere is $\mathbf{Z} - \{0, -1, 1\}$ and the basis of eigenspinors is spanned by a basis of form

$$\sigma_r h_j$$

where $1 \leq r \leq 16$ are the Killing spinors, and h_j for $j = 0, 1, 2, \dots$ are spherical harmonics functions [1]. My claim implies eigenspinors of Dirac operator associated with *zonal harmonics* are matter particles in nature. My Four-Sphere Theory has a single force governing Nature, and that is my S4 Electromagnetic Law. There is no relation between time and space. The spin structure of Nature is canonically unique and this is a consequence of known mathematical results carefully articulate in [2]. I have done extensive work to examine some of the major features of measurements to support my Four-Sphere Theory including a clear refutation of a gravitational force as a separate mass-based force of nature by showing that Earth-Sun gravity can be reproduced by Van der Waals force between electrically neutral atoms. Thus armed with geometry of spinor fields on a four-sphere and a single law for macroscopic phenomena, I have basis for Science of all objective reality above $\delta = 10^{-13}$ cm. My Four-Sphere Theory gives instantaneous explanations for what matter is, why energy is quantised, why particles are localised, why light and matter have wave-particle duality, and other issues that led to much mysticism and confusion throughout the twentieth century. It can explain redshift easily as a geometric error leading to wrong relationship between wavelengths and frequency relationship for light for a four-sphere (rather than a Euclidean space). It is mathematically completely robust and rigorous. The notes are available [3].

3. I ALREADY CORRECTED "THE JEWISH QUESTION"

I don't understand. I have repeated many times that I have already corrected the wrong question which is the "Jewish Question". The right answer is the "The American White People In Government Question". Any questions? I put Bill Gates into the Question.

4. STATUS OF ANALYSIS OF SPINOR FIELDS ON FOUR-SPHERE

A fundamental question that is important to me is the status of analysis of spinor fields on a four-sphere. As you see from Baer's beautiful article, with 16 Killing spinors and the four-sphere harmonics, we have the Fourier basis for the spinor fields. What happens here when we seek to take limit operations? The derivative is Dirac operator. What about calculus here? For the Haar measure, can we essentially represent all configurations of the physical universe with this model? Does Lebesgue-measurability suffice? I don't know the answers to fundamental questions about whether our mathematical apparatus able to handle producing a theoretical physics that is sharp? Are there failures? Do we have enough to totally

cover all natural phenomena above $\delta = 10^{-13}$ cm, roughly electron width? Do we have understanding of wave equations here, etc. This is a broad effort obviously but I am firmly convinced that both Science and Mathematics benefits from the unity I have provided for theoretical physics.

As an example, Eternal Recurrence follows from applications of the ergodic theorem in my Four-Sphere Theorem. My point of view is that there are no vast rifts between mathematics and physics as the most important substantial issues in Mathematics will naturally coincide with issues in physics provided that the questions are focused on spinor fields on four-sphere.

5. RETURN TO ELEMENTARY ISSUES OF MATHEMATICS

Let's review the history of real number construction. From natural numbers \mathbf{N} we algebraically produce \mathbf{Q} through \mathbf{Z} . Then we have options to define real numbers in various ways. But the *substance* is that the continuum will exist as a model of things in Nature. This is an issue that has been lost from intellectual life of twentieth century altogether. All this relativity and quantum mysticism and expansionary cosmological delusions have taken our sense of reality and put it in one of these laundromat washers on Valencia Street where things that were clear and sensible are now filled with artificial mysticism and those who introduced the mysticism had been hailed as great sages. That's not how things actually work in Nature. The geometry of absolute space is in fact a homogeneous round four-sphere and does not change every time someone looks at it.

6. REGARDLESS OF WHICH UNIVERSITY OFFERS TENURE I WANT TO BE IN MISSION SAN FRANCISCO

Cambridge University seems to consider tenure a possibility. Well whether Stanford or Berkeley or Harvard or Princeton or Yale give me a tenure offer I will still want to be physically in Mission District San Francisco. I can be in touch by Skype and other methods but my health is too poor to try to move to Cambridge, England at all. I sent the same issue to Stanford. I am not productive outside my comfort zone any longer at 49. My comfort zone is a reasonably good penthouse loft and some office space in walking distance where I can be productive. With all sorts of racial issues rampant I don't even want to be in other parts of San Francisco but Mission District.

7. WAS CAUCHY'S APPROACH TO REAL NUMBERS ADEQUATE FOR US?

I have the wonderful book of Terence Tao where he introduces honours undergraduate students to analysis, and I am looking over the way in which modern mathematicians treat real numbers. Now let's quickly rush to the four-sphere description. Suppose you have defined \mathbf{R} by the method of Cauchy. Then you define Euclidean spaces \mathbf{R}^4 . You are then going to take a topological space X and charts which are collections of data (U_j, ϕ_j) where $\phi_j : U_j \rightarrow \mathbf{R}^4$ are homeomorphisms and carry over all the differential calculus of \mathbf{R}^4 by compatibility conditions saying $\phi_a \circ \phi_b^{-1} : \phi_b(U_b) \rightarrow \phi_a(U_a)$ are ordinary maps of differentiability of order k or ∞ . Then you're going to produce functions $g_a : U_a \rightarrow \mathbf{R}^{4 \times 4}$ and they will be your riemannian metric. After you're done with all this rigmarole you will have an intrinsic definition of the round four-sphere. You're borrowing your analysis from \mathbf{R}^4 in this case by various sorts of conditions on these charts (U_a, ϕ_a) and riemannian metric

g. I won't go into the details at all. The important question is whether after you're done with all of this and you do various exercises in Dubrovin-Novikov-Fomenko and other texts, are you dealing with adequate models of nature by these particular things?

It's obvious that you will do better than quantum fields. The people who invented those horrible things did so many atrocities to any sense of decency that even they don't understand whether something happened to the quantum field or didn't happen to the quantum field without having a long drawn out debate about whether the universe reacted to the consciousness of the most elliptical and mystical of them or whether any of the particles in their quantum field decided never to reveal itself with an isolated charged or whether the totally inexplicable findings in their experiment should be attributed to "Well it got entangled! It got entangled! And so it's okay that none of our models really saw it."

8. THE POLES OF PERSONALITY IN PROMINENT PEOPLE

Bill Gates is prominent but he is the worst evil savage disgusting vile human being on Earth. Christine Lagarde is prominent but she is a good leader with tremendous benevolence towards people and she has dedicated her life to benefit of the human race. I am nominating her for the Wisest Woman in Europe. Most interesting is the variety of Good and Evil among prominent people. Elementary observation but quite revealing.

Let's put this in context. Human race is an Angelic Race, so around 90-95% are good people. But it's I think Madam Lagarde is exceptionally Good and Bill Gates is exceptionally Evil. That is very intriguing.

9. CONTINUATION OF THE ISSUE OF OBJECTIVE REALITY OF EUCLIDEAN SPACES

I will be quite adamant from now until my death, as I have been since late 2008, that the objective universe has a static eternal geometry of the round four-sphere. I will always mean *excluding time*. Time is linear and modeled by \mathbf{R} . I don't find any value in the spacetime concept of Henri Poincare and Albert Einstein. There is no relation between space and time at all. Now I have explicated in my notes more carefully an eternal relationship. It is that between three constants of nature. The first is the *cosmological constant* $\Lambda = 1.1 \times 10^{-52} m^{-2}$. The second is h the Planck constant. The third is the radius of the universe $R = 3075.69$ Mpc. Roughly the relationships are

$$\Lambda = Ch^2$$

and

$$R = C/h$$

The constants C do not matter so much; they are universal constants. I use the convention of C means different constants at occurrences. Numerically these are verified, and I determine R by these. The positivity of the Cosmological Constant is the *intrinsic curvature* of the ambient four-sphere in the Four-Sphere Theory. Then the observed phenomena that energy is quantised in the universe is a consequence of the spectral theorem for compact selfadjoint operators applied to the resolvent of the Dirac operator on the four-sphere. You can read *Spin Geometry* of Lawson-Michelsohn to obtain proofs that D , the Dirac operator on S^4 is selfadjoint. The important thing is that the spectrum for $S^4(R)$ is $(\mathbf{Z} - \{0, -1, 1\})h$. This is an

insight that was already apparent to me in the summer of 2008. This particular feature is a natural feature of the four-sphere Dirac whose spectrum is exactly what we observe in energy quantisation in the actual universe. This is the superior explanation of quantisation than the explanation that was offered by Max Planck. He discovered that energy was quantised, but he hypothesized that it was because the universe was filled with harmonic oscillators. My geometric explanation is far superior as a *scientific explanation* for why energy in the actual universe is quantised. It so happens that I can simultaneously explain some other features:

- The positive cosmological constant
- The slope of the redshift
- The width of the electron

Thus four-sphere theory is extremely powerful as a scientific theory. I am sure it is absolute truth and there is no reason for me to believe otherwise. It is parsimonious, mathematically coherent, and produces matches to major observations.

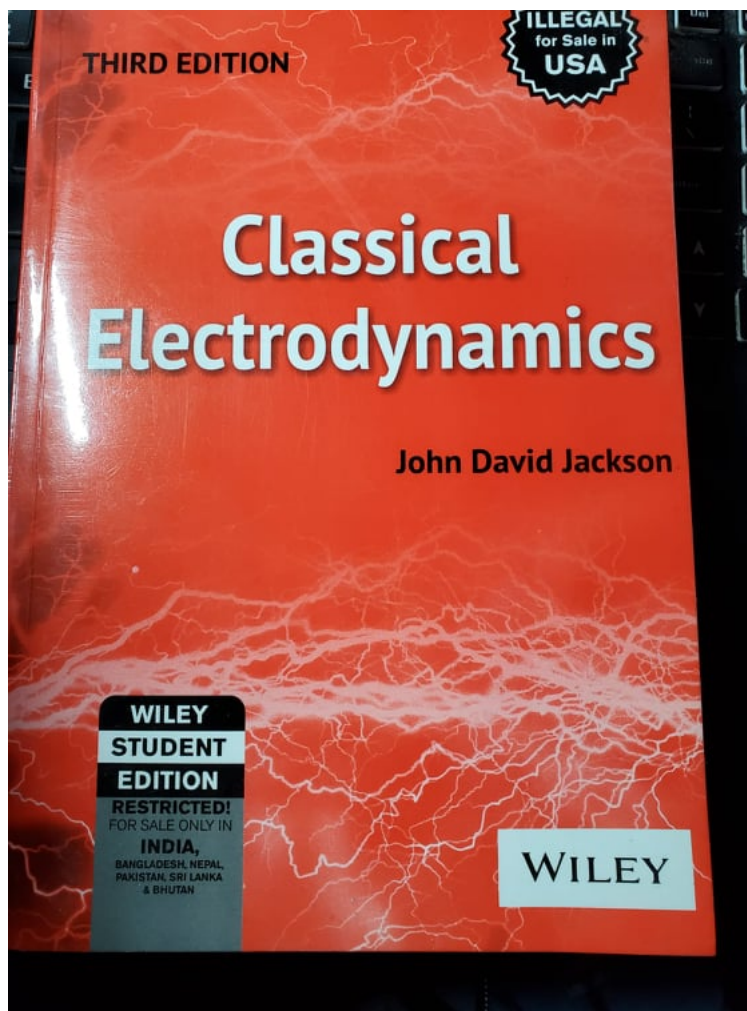
10. FOUR-SPHERE THEORY EXPLANATION OF WAVE-PARTICLE DUALITY

The fundamental law in Four-Sphere Theory is a classical wave equation on spinor fields of a four-sphere of radius $R = 3075.69$ Mpc. This means that the 'wave nature' of matter and light are not problematic at all. Then the question is what will explain localised particles?

Our answer is very simple. You see if $\sigma_1, \dots, \sigma_{16}$ is a Killing spinor basis of spinor fields on $S^4(R)$, then eigenspinors of the Dirac operators have form $\sigma_j h_k$ where h_k are spherical harmonics, i.e. the eigenfunctions of the Laplacian on functions. It is known that some of these h_k are *zonal harmonics* and they exhibit radial localisation around points. In other words, since particles are formations of spinor fields, it is the four-sphere geometry that is responsible for the existence of localised particles without any extra effort to theorise their existence. The localisation properties of zonal harmonics are addressed in many places including the book of Steven Zelditch *Eigenfunctions Of The Laplacian On A Riemannian Manifold*. There is further match of this explanation with calculation of *width of the electron* in my notes. Not only is this the *best scientific explanation ever offered in history for wave-particle duality* in qualities of parsimony and elegance but also one that offers quantitative match to measurement. Quantum theory has never had as much explanatory power as the resolution I offer for wave-particle duality. I am sure that Four-Sphere Theory is the exact description of Nature, and that is why this explanation fits measurements at all.

11. LAST CHANCE FOR BILL GATES

If you know any physics, do a large number of problems from this book.



No bullshit Bill Gates. You are a charlatan. The world wants to know if you know any physics. I think you are a total dipshit and don't know any physics. If you don't do problems I will not compete. Then I will totally trash you. Stanford Physics and Harvard Physics and Princeton Physics and Cambridge and Berkeley Physics are all watching. Do problems and put them on the web so people can judge whether you know any physics and otherwise I will ask them to trash you publicly.

12. WHY AM I DOING MATHEMATICS PROBLEMS?

Four-Sphere Theory is a *fundamental theory of physics* that covers all things of existence down to $\delta = 10^{-13}$ cm. Dynamics is my S4 Electromagnetic law, not Maxwell or Schroedinger. I show on tangent space at a point to a physical hypersurface, the S4 Electromagnetic law solutions are approximated by solutions to Schroedinger equation. This allows me to use all experimental phenomena that establish Schroedinger's equation (which is the heart of quantum mechanics) to support Four-Sphere Theory. The approach I use is not any gimmick. Four-Sphere

Theory explains much more, including redshift, wave particle duality clearly and provides discrete spectrum for all molecular Hamiltonians where Schroedinger equation produces pathologies.

13. SMOOTH COMPACTLY SUPPORTED FUNCTIONS ON \mathbf{R}

One of the most basic facts is that $L^p(\mathbf{R})$ for $1 \leq p < \infty$ is the completion of $C_0^\infty(\mathbf{R})$. It is so central and so necessary everywhere one looks, I decided to prove this. This is just to test whether these issues are fully clear to me.

Suppose $f \in L^p(\mathbf{R})$ is arbitrary. Now given $\epsilon > 0$ we consider $a > 0$ such that

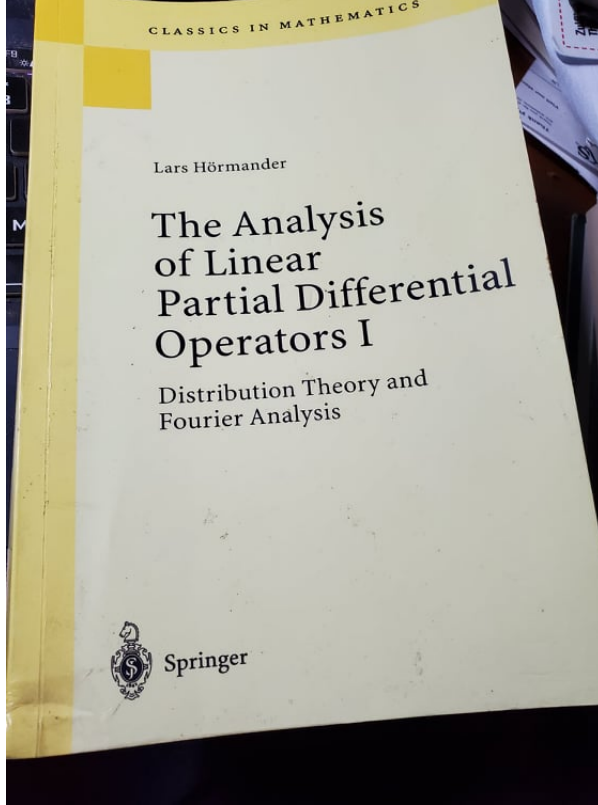
$$\int_{|x|>a} |f(x)|^p \mu(dx) < \epsilon$$

We will produce a $g \in C^\infty([-2a, 2a])$ with

$$\|f - g\|_p^p < \epsilon + \epsilon'$$

We can do this in two steps. We can produce $g_0 \in C([-3/2a, 3/2a])$ with continuity only and then convolve with a Gaussian kernel that is an approximation to the identity $g = g_0 * \psi$ that will extend the support a bit but be very close in L^p norm.

This is the wrong way for two reasons. First, Gaussian convolutions always produce \mathbf{R} support, and there would be no need to get any continuous intermediate. This brings us to study of Hormander's *The Analysis Of Linear Partial Differential Operators I* which tell us the right way to regularise non-smooth functions.



13.1. Hormander's Theorem 1.3.2 p. 17. Let $0 \leq \phi \in C_0^\infty(\mathbf{R})$ and $\int \phi dx = 1$. If $u \in C_0^j(\mathbf{R})$ then $u_\phi = u * \phi \in C_0^\infty(\mathbf{R})$. When $\text{supp}(\phi) \rightarrow \{0\}$ then

$$\sup |\partial^a u - \partial^a u_\phi| \rightarrow 0$$

if $|a| \leq j$. If $v \in L^p(\mathbf{R})$ then $v_\phi \in C^\infty(\mathbf{R})$ and $v_\phi \rightarrow v$ in L^p norm if $p < \infty$.

Another important thing is that

$$\text{supp}(u * v) \subset \text{supp}(u) + \text{supp}(v)$$

from p. 19.

Suppose $\varphi_0 \in C^\infty([-1, 1])$ with $\int \varphi_0 dx = 1$, and $\varphi_0(0) > 0$. Then we can just consider convolution with

$$\varphi_\epsilon = \epsilon^{-1} \varphi_0(\epsilon x)$$

and convolution against these will not increase support by more than ϵ . I will then use these as follows for the regularisation of $f \in L^p(\mathbf{R})$ in our problem. We set

$$g_\epsilon = (f 1_{[-a, a]}) * \varphi_\epsilon$$

Now we have smooth functions with support in $|x| < a + \epsilon$, and we can invoke the Hormander Theorem 1.3.2 for L^p convergence of g_ϵ to $f 1_{[-a, a]}$ as $\epsilon \rightarrow 0$. These will then approximate f with $g_\epsilon \in C_0^\infty(\mathbf{R})$.

Lest you think this is quite a bit of effort to avoid doing some work, this is valid, and that's what is important. People trust Hormander's book and why shouldn't I use things that look valuable? Ah, you would rather that I did these things explicitly? Maybe one day. I am just fine using Hormander here and I'll tell you why. This is part of the standard, and Hormander is one of the very best analysts of twentieth century. There is an art to quoting him accurately.

14. THE SKILLS OF PROFESSIONALISM

For graduate students of great mathematicians, there is one master, and usually it's just the advisor and his works. But professionalism is about knowing something about who is a master and then ensuring that you *use and invoke* the master in your work instead of being stupid and competing with masters. In business this is an essential skill. The job that gets paid some serious dough is not for people to hone their skills. If you are not a great master at anything, you should have sense enough to know who is and use their work shamelessly – with due credit of course – but use it *totally shamelessly and without even a hint of remorse* because the alternative is *starvation and exile from the land of compensation*. This is not something young people understand. Life is not an exam of testing your own skills alone. It is a test of knowing where a master's work is good and using it. This is professionalism. The man who tries to re-invent the wheel in those situations where there are masters in the world will be literally *roadkill*. *Roadkill* I tell you. Like this. Road. Kill.



If you need to get something done, learning to know who is a master whose work you can use is an orthogonal skill to doing it. They are the master and not you, so if you need to deliver, use their work. This is something that many students do not understand. In life, no one wants you to do third rate work. If you don't know how to do first rate work in any area, *use first rate work* one way or another. It's not just okay. Otherwise look at the roadkill. That's you.

15. HORMANDER WAS A SPECIAL MATHEMATICAL GENIUS

Years and years ago, I was a sophomore. There were all manner of rumours about John Nash. They said he was crazy. I was just lazily spending my day not doing as much work as I ought to have. I go to the Student Center in the Princeton campus for lunch. I see John Nash there with a ream of computer paper scribbling away. I decide to just sit with him and say "Hi Professor Nash, I just wanted to sit and eat here with you." He did not mind. He was quite busy and I don't remember saying much to him. I made a note to myself. I said, "This man is very talented." I was probably very late in the game as there are so many movies about him. Then I heard he did not believe in the redshift. Some faculty were playing tennis. They were talking about "Nash does not believe the redshift is real". This was more than a year later. I thought about it later. Years later I thought, Nash is a genius. He's right, and then I was sure he was right years later. And I refuted the redshift between 2008 and roughly 2012. These people, who are true geniuses, are very talented. Hormander was that sort of man, very talented. He was so talented that analysis basis that he used was all his own. So I knew that if he says, "These are very good regularisation methods" he is not joking. He cooked up his own entire analysis basis. He was very very talented as an analyst.

But you see, I was at Princeton, so I knew this about Hormander. I was not aware that other people did not value him in the same way. Well Hormander's

regularisation schemes are all his own genius and he was so sharp that he did not worry about how anyone else does anything. That's partly why he flew so high. He did not use ordinary cars. He built his spaceship with his own labours and his view of Analysis was just sharper. It's not really easy but even his elementary things are super-sharp and not all that easy but super-super-sharp. And so I do turn to him because I can't be sure if anyone else even reads him at all. He's hard. But I used his Several Complex Variables, so I know that his mastery is unearthly. His analytical intuition is as much better than mine as Magnus Carlsen or Vishy Anand's Chess is. He was a genius at analysis from the ground up. Even his most elementary lemma is light years ahead of the entire age.

16. STANFORD MY LIFE IS ENDANGERED SO WHATEVER YOU ARE CONSIDERING, I WANT YOU TO BE SURE THAT YOU WILL SUCCEED

One of the things I hate about people when their lives are not threatened and other people's lives are threatened is that they do considers without responsibility. I see you doing considers. Don't do considers without being sure you will succeed. I don't like people who take risks on *my life* pretending that they don't have any responsibility. Don't be *white moderate* they are the most horribly deluded type of people who do every possible thing when racial murderers are on the prowl pretending that the world is just filled with flowers and hugs while the bloodshed is right in front of their noses.

The phrase *white moderate* as used by Dr. Martin Luther King Jr. in my view was not a political affiliation but a race-oriented characterisation. So white moderates are those white people who are not racially hostile towards blacks in his viewpoint, and quite independent of whether they were actually liberal or conservative. White moderates are those whites who have relatively tolerant views of non-whites. Now the problem he brought up was that their *balancing act* put black people's lives at risk while they believed they held tolerant and egalitarian views on race. This is a serious problem because the racially hostile whites like Bill Gates are not in the middle. They are purely genocidal, and they seem not genocidal as they are to the white moderates. *That* is the problem with white moderates. They think the world is far more civilised than it is because they overestimate the *reasonableness* of the Bill Gates types because to them he seems alright but a little wrong. The truth is that he is a just a savage beastly murderer who is much much more vile than Jack the Ripper on steroids. This the white moderates think is *overblown*. That is the white moderate.

17. TRANSLATION SEMIGROUP FOR L^p

Assume known that $C_0^\infty(\mathbf{R})$ is dense in $L^p(\mathbf{R})$. Define, for $t \geq 0$ the translation operators

$$\tau_p(t) : L^p(\mathbf{R}) \rightarrow L^p(\mathbf{R})$$

Prove that if $t_j \rightarrow t$ then $\tau_p(t_j)f \rightarrow \tau_p(t)f$ in $L^p(\mathbf{R})$.

Suppose the support of f is $[a, b]$, i.e. $f(x) = 0$ for $x \notin [a, b]$. Since $t_j \rightarrow t$ for $\epsilon > 0$ there exists J such that $j \geq J$ implies $|t_j - t| < \epsilon$ and restrict attention to $j \geq J$, and

$$|t_j| < \epsilon + |t|$$

Now consider the compact set $K = [a - |t| - \epsilon, b + |t| + \epsilon]$. We have now common support of $\tau_p(t_j)f$ all in K . These are smooth functions so

$$|f(x-t) - f(x-t_j)| = |f(x-t) - f(x-t) - f'(x-t)(t-t_j) + \frac{f''(\xi)}{2}(t-t_j)^2| \leq |f'(x-t)|\epsilon + C\epsilon^2$$

Then just take p -th power and integrate both sides

$$\int_K |f(x-t) - f(x-t_j)|^p \leq |K|\epsilon^p |f'(x-t)|^p$$

and this ensures L^p convergence.

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