ZULF'S JANUARY 31 2022 STANFORD FALL 2015 ANALYSIS PH.D. QUAL EFFORT

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1. Developing Perspective In Mathematics After A Number of Stanford Ph.D. Analysis Qual Exams

I have been most interested in developing a deeper understanding of the history of natural sciences and mathematics for most of the past two decades, and I have succeeded in producing the final theory of Nature above $\delta = 10^{-15}$ cm with my four-sphere theory.

I am now 49 years old, no longer a young student, and the sorts of things I appreciate are *simple, clear, and deep*. Having succeeded in four-sphere theory – and overthrowing quantum field theory, relativity and expansionary cosmology as accurate theories of nature, I obviously feel vindicated that my particular perspective have value more than many others that exist.

But of course I do respect the erudition of many great mathematicians and physicists and do have a good attitude towards learning from them.

But unlike when I was younger, I have an agenda of my own, that I believe there is a perspective that will arise from considering the *absolute identity* of mathematics and science as central. This is much harder than one would imagine.

Suppose we decompose all of mathematics as $M = M_{nature} \cup M_{other}$. This is not for any firm erudite conclusions, but exploratory. My thought is always exploratory in some ways. Here the meaning is unclear; some mathematics directly represents nature, and we don't have a clear idea yet of what they are. I claim that four-sphere theory will identify M_{nature} more clearly for us. Then we have other parts of mathematics that do not have this property and from my point of view is not as important M_{other}.

You are sophisticated, and you will be unimpressed at my attempts to produce concepts of the sacred and the profane within mathematics. But this is to me quite interesting.

I was reading V. S. Varadarajan's *An Introduction to Harmonic Analysis on Semisimple Lie Groups*. The holistic picture that V. S. Varadarajan is quite profound and extremely valuable to me. I am thinking about some of these issues again. You see my perspective is that for M_{nature} it is important to have broader sense in philosophy of nature and existence from eighteenth century. The unity within mathematics in the end does not really solve the problem. But it is very valuable to gain insights from men like V. S. Varadarajan whose mathematical erudition is expansive. These sorts of men are able to provide highly nontrivial synthesis of the whole of mathematics unbounded by the divisions. Their writings are extremely important.

Date: February 5, 2022.

One of the most beautiful centers of the erudite unity that V. S. Varadarajan presents is the profound work of Hermann Weyl."The theory of compact groups is dominated by ideas and results of Hermann Weyl. Weyl's work had profound impact on the entire course of development of harmonic analysis." (p. 18) "In a series of pathbreaking memoirs Hermann Weyl developed the representation theory of *all compact semisimple groups. Among other things he obtained the character formulae and dimension of of the irreducible representations of which results for* $SU(n)$ *are special cases." (p. 35)*

You see, I am gaining some insight about Hermann Weyl's works slowly, even though my fundamental claim is that the geometry of the absolute space is $S^4 = SO(5)/SO(4)$*, and the radius,* $R = 3075.69Mpc$ *is eternal constant and the Planck constant* $h = C/R$ *for a universal constant* C*. Even though this is the case, when Hermann Weyl transitioned to theoretical physics after his work on compact groups, at first Wolfgang Pauli was most rude.*

In hindsight, Hermann Weyl's unified theory efforts were not sufficient to produce the correct theory of Nature. I have thought about this for many years, for four-sphere theory is the truth, and this required me to refute gravity altogether, and formulate my Ahmed-d'Alembert fundamental law as a wave equation on spinor fields *on a four-sphere. What was right about the convictions of Hermann Weyl about Nature was that importance of symmetries of Lie groups.*

Of course I am most interested in pursuing a deeper understanding of the harmonic analysis on homogeneous manifolds G/K *because* $S^4 = SO(5)/SO(4)$ *but alas I have to regain my abilities in Analysis atrophied over time.*

I am also more humanistic in my influences than these illustrious mathematical men. In four-sphere theory, finding myself in a primitive age where men believed that the universe did not even have existence some 14 billion years ago, I had to gain my sustenance from Percy Bysshe Shelley whose faith in Eternity had sustained my own. It was a relief finally to predict accurately the redshift slope in a static eternal absolute space with four-sphere geometry. I am of course in agreement to Nature while the entire establishment of Cosmology is deluded. The perfect merge of Mathematics and Nature is still a matter of effort.

Now Varadarajan gives reference to George Mackey's historical overview [1]*. My dear reader has to understand that I do have a good Princeton undergraduate education in Mathematics from 1991-1995, but these are unfamiliar situations, where I am independent in my works without any supervision at all and have been for the past two decades, but at the same time I have to regain mathematical development that I did not pursue. I did not pursue these because I was literally busy with Scientific revolution of Einstein, Schroedinger, Friedmann and Lemaitre in that time. That is mostly extremely focused on empirical work, philosophy of science, and things not involving mathematical coherence. That is fate and I accept that I have to slowly gain erudition in Mathematics again.*

"Looking at e^{inx} *as group characters and Fourier analysis as a special case of decomposition of group representations are of course twentieth century viewpoints and the very notion of group representations did not exist until the end of the nineteenth century. On the other hand characters of finite commutative groups go back to Gauss, and the analogue of Fourier expansions for functions on such groups have played a role in number theory since the beginning of nineteenth century." (*[1]*, p. 547)*

Hold on. We are in the end interested in $\mathcal{D}'(\Sigma S^4)$ and $L^2(\Sigma S^4)$ and spinor fields on four-sphere. We use Killing Spinors $\sigma_1, \ldots, \sigma_{16}$ by the beautiful results of Christian Bär to write

$$L^2(\Sigma S^4) = \bigoplus_{r=1}^{16} \sigma_r L^2(SO(5)/SO(4)) \tag{1}$$

Yes, the paper [3] *is very nice and proves Killing spinor field trivialisation of the spinor bundle ΣS^4. I am most pleased, most pleased indeed. With this representation of square integrable spinor fields I can reduce all physics of Nature into harmonic analysis and achieve unity of Mathematics with Science once and for all.*

Hahahaha. Hahahahahahahahaha. And soon the Science of this entire world will be mine! Hahahahahahahaha. Hahahahahaha.

I was mathematically gifted when I was young but not at the level of Varadarajan. He's a true pure mathematician. My natural genius is really truth of nature, and I always had a sharp sense for what is truth beyond even some top scientists. For example, I knew instinctively in 2008 that these anomalous rotational symmetries of orders 5, 8, 10, and 12 were just crystals *and not all sorts of funky things of Roger Penrose. They pointed to a vast purely electromagnetic fourth spatial dimensions that the world seems to have a difficult time accepting. Then in 2011 the Nobel Committee gave the prize for 'discovery of quasicrystals' to Daniel Shechtman. He does deserve a prize but for* discovery of fourth purely electromagnetic macroscopic fourth spatial dimension *rather than discovery of things other than crystals. But they didn't do the right thing obviously and I chalked it up to all these people will obviously do the wrong thing if Zulf doesn't do everything around here.*

With these various thoughts I begin Stanford Mathematics Ph.D. Quals for 2015.

2. What Are Zulf's Aims In These Stanford Ph.D. Quals Problems?

My main motivation to doing the Stanford Mathematics Analysis Ph.D. Qualifying Exams is to gain deeper understanding of analysis for a particular purpose. *I regret that I am not interested in general research in Analysis. I am interested only in* that analysis that will matter for mathematical physics in four-sphere theory context. *The most general objects in four-sphere theory will be distributional spinor fields on the four-sphere. I have publicised my decision that these ought to have the symbol*

$$\mathcal{D}'(\Sigma S^4)$$

This symbol matches symbols from two strong traditions of established Mathematics. On the one hand, the symbol $\mathbf{D}'$ is from Laurent Schwartz development of distribution theory from the late 1940s. In Analysis these have had great development when the base space is $\mathbf{R}$ and the test functions are $\mathbf{D}(\mathbf{R}) = C_c^\infty(\mathbf{R})$. My base space are smooth spinor fields on the scaled four-sphere $S^4(R)$ where $R = 3075.69$ Mpc. This is not a mathematical decision. My four-sphere theory is fundamental physical theory that is revolutionary in the sense that it displaces quantum field theory, general relativity, and expansionary cosmology *in Science. The theory of spinor fields comes from another strong tradition including the work of Michael Atiyah, Isadore M. Singer, Arnold Shapiro and others. Michael Atiyah and I. M. Singer, like Laurent Schwartz, are Fields Medalists in mathematics.*

The analysis in four-sphere theory is analogous to Fourier analysis on S^1 which is well-established since 1811 when Jean-Baptiste Joseph Fourier first conjectured

that arbitrary periodic functions will have linear expansions in exponentials e^{inx}. *The analogue for* S^n *is also classical and was worked out first by Gustav Ferdinand Mehler in 1866. I use Christian Bär's theorems to simplify matters and see immediately that*

$$L^2(\Sigma S^4) = \bigoplus_{k=1}^{16} \sigma_k L^2(S^4)$$

and this is based on a serious theorem of Christian Bär, and not a formal conjecture.

All mathematical physics in four-sphere theory will have analysis not with base space $\mathbf{R}^3$ *whose non-compactness is the source of a great deal of problems in the past centuries, but based on Dirac operator (defined by Atiyah-Singer and studied for riemannian spin manifolds) and spinor fields. Further detailed information about spinor fields can be found in LM.*

I felt that my level of broad mastery of Analysis was inadequate to the task of founding the Final Theory of Macroscopic Science, which needs to be on deep ground and must serve humanity for millions of years in the future.

My perspective and interests are broader than Ph.D. Mathematics students at Stanford who are preparing for careers in Mathematical Research. I am not interested in analysis research questions in general. I am only *and* solely *concerned with the founding of a great and final mathematical theory of Nature that will stand for millions of years as unshakable absolute knowledge of Nature, unchanging and unchallengeable. And this requires depth of analytical understanding.*

3. I Do Not Understand Why United States Does Not Follow Civilised Standards

I know that in England, France and Germany, unauthorised use of power to cut into anyone's eyes in meta is punishable by instant death. These are most heinous, vile and destructive and savage violations of natural rights. I have never been an employee of Bill Gates at all. Why had he been allowed by United States Government to not only invade my meta, cut into my eyes, but use extremely destructive US War, US Industrial and White Racial Powers to devastate my ethnic (Indian) meta, which led to significant degradation of my health and life and in addition cause massive destruction to my professional meta in Finace, Technology and Science and obstruct $620 million in legitimate income, lie about me to US Government officials claiming I have 'forged papers'. Why does United States of America follow ordinary standards of civilisation and allow such a malevolent predatory savage barbaric beast running amok with impunity in this country?

4. I Want Assistance From Stanford To Promote My Value From Stanford Mathematics Ph.D. Quals Performance

United States of America is horrible in treatment of non-white peoples, especially those who are not in circles in top professional circles in coastal cities. Now I do not have a doctorate from Columbia University partly because Daniel Stroock was not good to me at all with his opinions. I do have a publication with him in Journal of Differential Geometry *from 2000* [6].

Now I am being considered to be a threat to Bill Gates by the racist law enforcement in Texas despite this being a practical impossibility. I need Stanford to estimate my value based on the exams I have done.

The key input to the problem is a rough estimate of number of total Mathematics Ph.D.s in the world, around 200,000. The rarity is 0.026% roughly. Then you can estimate my value to America in terms of special skills. Bill Gates, in my own estimation, is much less valuable to the nation than I am because I am a successful Scientific Revolutionary and he is a charlatan who lied about his SAT score and has no significant talent in Mathematics, Science, or coding. He is also an extremely harmful and criminal racial murderer and has plans to harm millions of non-white American lives.

I need this done quickly because the local white-oriented Texas legal system might just decide that I represent a physical threat to Bill Gates while he is the one who had taken action against my interests including harming my actual physical body and health by illicit use of black magic and other powers that are strictly forbidden by White Law, Christian Law, as well as National Law. He has extreme contempt for all laws and has had lifelong dreams of being the literal monarch of the world, and for whatever reasons United States Government despite being a Multi-Ethnic Classical Liberal Democratic Republic considers my opinions, which are not threats but opinions that are valid and true, to be against criminal law.

I need Stanford University to actually get involved before the local law enforcement moves against me and then I disappear in solitary incarceration and other political repression mechanisms.

5. The Big Difference Between America And Asia

Bill Gates just admitted that his plan was to repeatedly tell people that all my work is trivia and murder me by his powers and then take all my work and ideas as his own. He then explained to Stanford in meta that he believes that's the way white man succeeds.

It is so outrageous. Now let me assure you that Henri Lebesgue and Emile Borel and James Clerk Maxwell and other great people of European ancestry were not in cahoots with podunk hick frontier huckster illiterate theory of how to be successful white men that Bill Gates adheres to.

You see in Asia, these sorts of scum like Bill Gates learn their lesson after they get public beating from the crowds for their scumbagness. In America, these sorts are considered 'captains of industry'.

6. Conscience Is Much More Important Than You Think

You see, nothing at all is worth going against your Conscience. Being true to your Conscience is literally worth $100 trillion and no pragmatic shortcut for any perceived advantage, whether financial, whether a hot fuck with an attractive female, whether for prestige and social respect, nothing at all is worth it because you see you live for a short period of time, and it's deeply internal issue of who you are, and nothing external can ever substitute for compromised Conscience. I am amused by people who think they are clever by doing things compromising their Conscience. I am 49 and I have a crystal clear Conscience, and I have been hobo, I have been literally beaten and abused, and I still managed to succeed overthrowing Scientific established theories with a century of history. It's not an accident. Above all it is because my Conscience is sacrosanct. You may pity the tragedy to your eyes in my fate, but I am extremely strong in my Virtues and my Conscience, and have never failed to consider it with extreme importance. I am not disappointed by this choice.

Soon the United States might drag me to solitary confinement on absurd idea that it is I *who represent a threat to Bill Gates while I know that Bill Gates has been a* habitual racial murderer *and he has been* liquidating Americans arbitrarily for four-decades without cause*. But I have no regrets about my actions. They were always in accordance with my Conscience. Bill Gates is such a filthy horrible cunt that it requires great imagination to even understand his savage disgusting beastly low barbaric Character. He makes every evil villain of comic books seem rather remarkably angelic choir boy type, and he makes Jack The Ripper seem like Big Bird of Sesame Street.*

6.1. **Value of The Above.** *Feel Free To Compare Value Of Above Statement to Stanford Graduation Speech Of Bill Gates With Melinda Gate (now née French). You will find, as expected that my statement is infinitely superior to Bill Gates in transmitting truth, and inspiring the hearts and minds of people with a future. I am competing with the little shit – who displays feces in a jar in public which is disgusting by evolutionary adaptation of homo sapiens while thinking of himself as monarch of the world.*

7. Problem I.1

Suppose A is a Borel set in $\mathbf{R}$ with the property that if $x \in A$ and y is any other number such that the decimal expansion of y differs from x on at most finite number of places then $y \in A$. Prove that either A or $\mathbf{R} - A$ have measure zero.

Let's make the problem cozier. Introduce $\mathbf{Z} \subset \mathbf{R}$. Let $A_0 = [0,1] \subset A$. If we prove the property for A_0 relative to $[0,1]$ then we can make the same conclusion for A.

In order to prove this, just note that if $m(A_0) = 0$ then for all $k \in \mathbf{Z}$ we we have $x + k \in A$ for all $x \in A_0$ because $x + k$ and x differ only on a finite number of places. Therefore

$$A = \bigcup_k A_0 + k$$

and so by countable additivity of Lebesgue measure $m(A) = \sum_k m(A_0 + k) = \sum_k m(A_0)$ by the translation invariance of Lebesgue measure and therefore $m(A) = 0$.

Similarly if $m([0,1] - A_0) = 0$ then $m(\mathbf{R} - A) = 0$. So we have reduced the problem to subsets A_0 of $[0,1]$ and that's cozier.

Now we consider the heart of the problem, which is to prove the assertion of Problem I.1 for A_0. For this purpose, for each n consider the finite sets

$$B_n = \{0, 1, \ldots, 10^n\}/10^n$$

Since the measures of single points are zero by finite additivity we have $m(B_n) = 0$.

What's our plan over here? So you stare at the sets $D_n = [0,1] - B_n$ for various n and note that these D_n have the property that $[0,1] - D_n = B_n$ have measure zero and the D_n have the property of Problem I.1.

Then you putz around and wonder if you can prove something like $A \subset D_n$ for some $n \geq 0$. Then you let all the clever whipper-snappers who won ten Putnam exams whizz by you because so what if you are slow and steady and take a lot of time? You know that the pleasure of doing Mathematics is your pleasure and you have to do things in your own pace. So what if Daniel Stroock will be quite

unimpressed at seeing your snail's pace and excommunicate you from the Church and all that who cares?

Suppose $y \in [0,1]$ *and* y *differs from* $x \in A_0$ *in at most finitely many places. Then* $y - x \in B_n$ *for some* $n \geq 0$*. This then implies* $(x + B_n) \cap [0,1] \subset A_0$*.*

We are seeking some precise formulation for this. I have an idea. We could use Baire Category Theorem for this problem. The idea would be that either A *contains a nonempty interval* $(a,b) \subset A$ *with* $a < b$ *and in this case we want to prove that* $m(\mathbf{R} - A) = 0$ *or it does not contain an interval and in this case* $A \subset [0,1]$ *is a finite set. The Baire Category Theorem says that if* $\mathbf{R} = \bigcup_k X_k$ *then one of these* X_k *must contain nonempty interior.*

Yes these are pretty confused ruminations. I am trying to find a path to a precise conclusion here. That's the challenge here, to have sharper mathematical intuition that leads to sharp certain conclusions. Look, you are my dear reader and I do love you, but my genetic code and evolutionary adaptations did not give automatic knowledge of how to make intuitions into precise mathematical proofs, okay? My ancestors all the way before Vedic period three thousand years ago, and before that seventy five thousand years ago in their spread from East Africa where all human beings were in a meta-tribe, they did not face extinction pressure just for lack of mathematical precision ok? Emile Borel was not even conjectured to have any existence through most of my evolutionary history.

You see this, he lived between 1871-1956. My ancestors in East Africa, long before even Sanskrit was invented by later ancestors during Vedic times never heard of Emile Borel ok? Let's look at Henri Lebesgue.

Now those round glasses are quite distinguished. Borel and Lebesgue and Baire all worked long after most of my evolutionary adaptations formed. My ancestors were more adapted to not being eaten by sabre-tooth tigers an such. They were not keen on proving all sorts of quaint little facts about when measures of sets are forced to be zero.

Perhaps an approach to the theorem is to suggest the following. Let

$$E_n = \bigcup_{k=1}^{\infty} B_n + k$$

These are countable. Then we could try to prove that if A contains any interval (a, b) then $A = \mathbf{R}$ and obviously measure of the complement is zero.

Ah, maybe same argument applies to the complement of A, that if it contains an interval it must be the whole real line. Let $F = \mathbf{R} - A$. If $y \in \mathbf{R}$ then $y \in F$ if there exists $x \in F$ that differs from y in at most finitely many places. So that is true.

Now we assume that neither A nor F contains any intervals. Then we apply the Baire Category Theorem to produce a contradiction.

Sneaky, sneaky, sneaky! One actually needs to apply the Baire Category Theorem here. Oh you can tell that I am not an expert on this problem eh? You can see that I am being inefficient eh? Well let me assure you that I am not ashamed of this in the very least. I would be frankly quite a bit spooked if the sabre-tooth tigers and lions and snakes and other survival dangers led to evolutionary adaptations which, not even having any idea that one day a small group of French mathematicians would just monopolise the thought about all sorts of sets and their measure of eight billion people and I, Zulikar Moinuddin Ahmed, would be repeating, "Borel Borel, Lebesgue Lebesgue Lebesgue, Baire, Lebesgue, Borel, Lebesgue Borel, Baire Baire." My ancestors would have been aghast at strange sounds no Sanskrit texts ever mentioned. So why should I be ashamed?

8. Another Attempt At Problem I.1

We make distinction between Mathematical Substance *and* Mathematical Rigour*. I only worked with Professor Daniel Stroock for a year, and was not actually in the same school of thought as him at all. I did not know then exactly what that school was at all. He's an analyst, and quite formidable in his analytical insights and learning. But I was from a geometric-topological background and I had read a great deal of things. I knew from my experiences that rigour was the last step. First, there is a much more basic issue of intuition and mathematical substance.*

This Problem I.1 shows you how I do things, and it's a simple problem, and I am not a professional mathematician now; I am a Scientific Revolutionary which fits my ways and my temperament better than being a professional mathematician. My concern is to decipher Nature's laws, the Truth of Nature, and not to prove strong new nontrivial theorems. I have been doing revolutionary scientific work independently and in total isolation without financial support or significant position at any university, corporation or government for more than 15 years now. My confidence, therefore in strength of my four-sphere theory is very high, and I am sure it is stronger than any theory like quantum field theory or expansion or general relativity. It's absolute truth.

But this problem is interesting because the exploration gives a different path than what I was expecting.

Let's examine the path. First we polish some of the terminology for the objects that we have discovered in the exploration. First we let

$$F = \mathbf{R} - A$$

Then we define

$$A_0 = A \cap [0,1]$$

and

$$F_0 = (\mathbf{R} - A) \cap [0,1]$$

Then we define sets

$$B_n = \{j10^{-n} : 1 \leq j \leq 10^n\}$$

which are finite sets $B_n \subset [0,1]$. Our strategy is to prove to proceed to prove that

- *Either A_0 or F_0 contains an open interval*
- *If A_0 contains an open interval then $A = \mathbf{R}$*
- *If F_0 contains an open interval then $F = \mathbf{R}$*

We want to use Baire Category Theorem to establish the dichotomy. If neither A_0 nor F_0 contains an interval, then we write

$$\mathbf{R} = \bigcup_k (A_0 + k) \cup \bigcup_k (F_0 + k)$$

and apply the Baire Category theorem to $X_k = (A_0 + k) \cup (F_0 + k)$ and get a contradiction. This then gives us reduction to the easier propositions because

$$\bigcup_k B_n + k$$

is countable and has measure zero by countable additivity and translation invariance of Lebesgue measure on $\mathbf{R}$. The easier propositions follow from the fact that if $y \in \mathbf{R}$ then either there is an $x_0 \in A$ and $n \geq 0$ such that $y - x_0 \in B_n$ or there is an $x_1 \in F$ and $n \geq 0$ with $y - x_1 \in B_n$. Thus if either A or F contains an interval, this allows us to translate the interval and fill up $\mathbf{R}$ using translations from B_n.

This is closer to a precise rigorous solution to Problem I.1. If you read the first attempt, you will note that I had no idea of how to reach this particular rigorous organisation for the result. But in fact the exploration I did there was more substantial than the rigorous path in search for mathematical substance. The rigourous proof in the end is not the mathematical substance at all.

Now this is a relatively simple problem. This is how I work on all efforts in mathematics, and this differs substantially from many others in how we think about what we are doing, including, I think, of Professor Stroock. That's cultural difference of geometrically oriented people and analytically oriented people. Geometry and topology never actually reached the technique-orientation of analysis and so idiosyncratic exploration is valued more highly.

Four-sphere theory challenges the 1900-1930 paradigm and it is a result of quite unorthodox scientific convictions. The heavy work on quantum field theory is challenged by four-sphere theory because the question was always explanation of Nature. I simply could not accept that it was possible for universe not to be a homogeneous four-sphere and still produce any particles or quantisation or wave-particle duality at all. But for whatever reason the physicists of the age did not have the same problem, and so I went my own way and I succeeded. I am right about this.

9. Digression: Character Strengths And Virtues And Their Relation To Mathematical Work

I did well in high school in New York, and got 5 on AP Calculus BC and Physics C in tenth grade. This was John Adams High School, and the year was 1988.

Then I spent two summers at Ohio State University Ross Program and studied Number Theory and Combinatorics. Then I attended Princeton University and did quite well on my mathematics courses. Highlights are Differential Geometry with Frederic Bien, Algebraic Topology with William Browder, Real Analysis with Nick Katz, Functional Analysis with Peter Sarnak, and a number of graduate level courses. I did my thesis with Jeff McNeal on Several Complex Variables.

But I did not go into professional mathematics but into Quant Finance, in Biotechnology, in Technology and eventually worked on my own original ideas about four-sphere theory independently for many years. I am profoundly interested in Personality Traits and Virtues and Character Strengths of individuals. I challenged work at Northwestern University that showed only a handful of personality types in data with a much more extensive theory that showed existence – in data – of thousands of personality types.

Let me remind my dear reader of ranking of my own virtues and character strengths.

	Rank	*Virtue*	*Class*
1	*1*	*Love*	*(Humanity)*
2	*2*	*Creativity*	*(Wisdom)*
3	*3*	*Honesty*	*(Courage)*
4	*4*	*Curiosity*	*(Wisdom)*
5	*5*	*Spirituality*	*(Transcendence)*
6	*6*	*Hope*	*(Transcendence)*
7	*7*	*Bravery*	*(Courage)*
8	*8*	*Humor*	*(Transcendence)*
9	*9*	*Forgiveness*	*(Temperance)*
10	*10*	*Perspective*	*(Wisdom)*
11	*11*	*LoveOfLearning*	*(Wisdom)*
12	*12*	*AppreciationOfBeautyAndExcellence*	*(Transcendence)*
13	*13*	*SocialIntelligence*	*(Humanity)*
14	*14*	*Zest*	*(Courage)*
15	*15*	*Gratitude*	*(Transcendence)*
16	*16*	*Self-Regulation*	*(Transcendence)*
17	*17*	*Fairness*	*(Justice)*
18	*18*	*Perserverence*	*(Courage)*
19	*19*	*Judgment*	*(Wisdom)*
20	*20*	*Leadership*	*(Justice)*
21	*21*	*Kindness*	*(Humanity)*
22	*22*	*Prudence*	*(Temperance)*
23	*23*	*Teamwork*	*(Justice)*
24	*24*	*Humility*	*(Temperance)*

This is a serious ranking with actual effort in Martin Seligman's Questionnaire and not artificial data. You see, my top strengths are Love, Creativity, and Honesty. I suspect that my mathematical style centered on exploration and concern for mathematical substance that is not focused on rigour but on discovery is probably due to my own personality. I am not aware now of a clear scientific theory of personality-mathematical acumen and ability relationship but my Starcraft II games

suggest that the mastery comes just with experience and habituation. This is probably where Daniel Stroock and I are quite different. I don't believe that there is some special genetic or ethnic or racial or religious advantage to mathematical genius. Mathematics is broad, and it's silly to think that there is any solid rational basis for thinking that one will have mathematical 'gifts' without simple love and pleasure for the subject and habituation and interest in the questions. Now a question like Problem I.1 is not particularly deep. It's a relative challenge at the level of an undergraduate real analysis course when Baire Category Theorem seems a bit strange. On the other hand I am not habituated to constant practice in these problems. I am humanistic in my cultivation. I can probably smoothen the solution I discovered a bit more but it's now quite clear.

I don't consider problems that are for exams and courses to be significant challenges. I have been working on revolutionary theoretical physics challenging the paradigm of 1900-1930 established by some of the greatest men in history of science. It would be rather shabby for me to suddenly consider Ph.D. Qualifying problems as significant. I am quite a bit more evolved than this sort of thing.

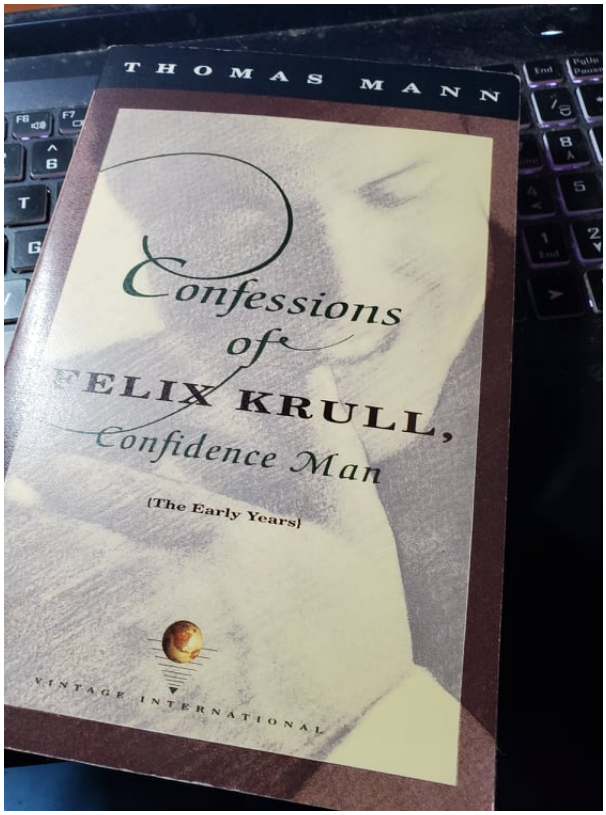
I was reading some papers of Henri Poincaré regarding philosophy of mathematics and ideas about mathematical discovery. I doubt that in the end these are possible to formalise; we are at the infancy of even approaching the full potential; the genetic code in common, G_c, which is 99.9% of each person's genetic code, is still a mystery. It would be quite interesting to learn something about individual potential. What I truly despise are moronic uncultivated hick charlatan wretched worthless scumbag like Bill Gates promoting racial strife and causing problems based on idiotic racial slogans. I just cannot have enough contempt for the son of a bitch.

10. Zulf's Theory Of Bill Gates

I have been dealing with Bill Gates in the most horrible manner. I had spent some years working on Medium Frequency Alpha Strategies. I pioneered the idea in Allen Texas living on disability. I worked on some sharp long memory stochastic volatility models with jumps with almost best performance on a large amount of options data for some years. Much better than Heston and Bates models and everything I found. I was looking at it and I decided that I didn't believe in pure martingales in price movements at all. There is pure arbitrage, and then I took all my AI skills and knowledge, because I had been quite familiar with Rob Tibshirani's statistical work at Stanford and knew Elements of Statistical Learning *from work at Biospect/Predicant years ago. And I found very good strategies in all asset classes.*

I encountered Bill Gates only in meta, and I found out that David E. Shaw of D. E. Shaw and company found my work 'in the wild' and commissioned a group to trade my work. Bill Gates had put in $2 billion but Human Rights from UN considered and found that he had, in meta, told David E. Shaw that he intended to murder me and destroy me an take all my work and pass it off as work of white people he prefers because he had a racial world order based on 'whites are masters and non-whites are slaves'. That the United States Government did not wipe him out with extreme lethal fire and huge amount of bloodshed is a testament to how illegitimate they are.

I have a theory about Bill Gates. I believe that this miserable wretched infernal scumbag who is more vile than Auschwitz butchers gassing the Jews daily was actually the conspiracy of some secret cabal. Just hold on and let me tell you the theory first.



See this is a great work of Thomas Mann, who was the most important writer of his time in all of the West. This is about a con man who reaches great heights of power and is totally devoid of any morals. I think some secret cabal got together before Thomas Mann's death in 1955 – and not coincidentally the birth of Bill Gates–and decided that they would create a Felix Krull figure totally devoid of any virtues, a pathological liar who never tells the truth and is totally malevolent and criminal to the core and that they would see if this experimental animal, would achieve wealth and power according to the novel Confessions of Felix Krull *and this whole "Bill Gates success" was their plot, and the United States Government was so stupid that they did not even know that he was a horrid evil white supremacist racial murderer and supported him all the way for decades. And even today, when he cuts into Zulf's eyes and sabotages his Finance career with plan to destroy an American man right in front of the United Nations with many letters sent to United States Senate and United States Supreme Court, they refuse to even secure Zulf's natural rights.*

11. Problem II.2

Suppose $H \subset \ell^2(\mathbf{Z})$ *consists of* x_n *satisfying* $\sum_n n^2|x_n|^2 < \infty$. *Show that* H *is of first category.*

Given U containing $a \in H$ we have to find $V \subset U$ with $V \cap H = \emptyset$. So let $a \in H$ and consider $B(a, 2r) \in \ell^2)$ for some $r > 0$. Now we will prove that there's a $y \in B(a, 2r)$ with

$$\|y - x\| = r$$

and

$$\sum_n n^2 |y_n|^2 = \infty$$

and then also $B(y, r/2) \cap H = \emptyset$. If we are able to do these things, then we let $V = B(y, r/2)$ conclude H is first countable.

11.1. **Spicing Up The Problem: Euler's Formula.** *Leonhard Euler was very pleased when he discovered that*

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots = \frac{\pi^2}{6}$$

Let us denote the series $\frac{6}{n^2\pi^2}$ as e_n^2. To get $\|e_n\| = 1$ He did this with infinite product factorisation of $\sin(x)/x$ and various manipulations in 1740. Now I was unhappy about his publishing papers on the wave equation without giving due credit to Jean le Rond d'Alembert, but his talents were quite spectacular.

In this case we have

$$\sum_n n^2 e_n^2 = 1 + 1 + \cdots = \infty$$

And that's what we need, so $e \notin H$

Now we do some Euclidean geometry and consider $y = x + re_n$. This ensures that

$$\|y - x\| = r$$

We need to prove

$$\sum_n n^2 |x_n + re_n|^2 = \infty$$

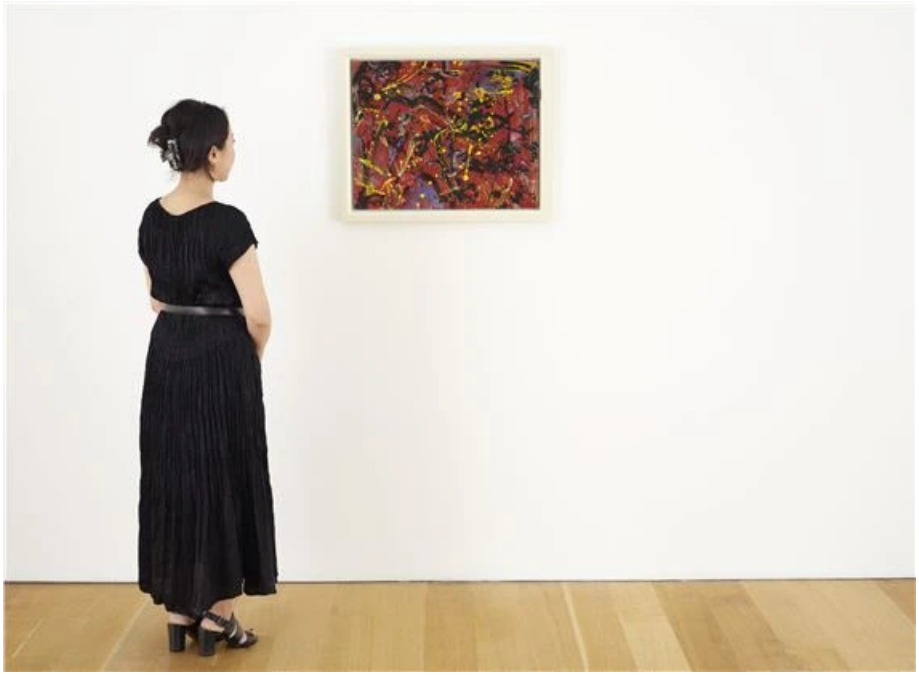
and for any $r > 0$. This then all $w \in B(y, r/2)$ will satisfy

$$\sum_n n^2 w_n^2 = \infty$$

so $B(y, r/2) \cap H = \emptyset$.

12. Worthless Retarded Wretched Miserable Bill Gates Ought To Have Been Killed With Horrible Lethal Bloodshed In 1955

Bill Gates, the wretched miserable worthless man would have contributed something more valuable to the world if he had been brutally slaughtered with his white baby blood splattered all over the walls and someone took a photograph of it. Jackson Pollock's The Red Composition *sold for $12 million. I would have bought the bloodstained wall artwork for more personally as a great blessing for the human race.*



The slimy little cunt says Osgood, Phillips, Sarnak's work was not good. He is a total dipshit moron without any intellectual talent who is not even literate from hickstown in boondocks of Pacific Northwest and he dares to judge mathematics of exquisite taste. Osgood-Phillips-Sarnak is beautiful work, showing analytic technique on a positive result. Compactness of isospectral metrics on surfaces is a phenomenal result with beautiful functional analytic technique. What would a total hick without any taste or cultivation know about whether Osgood-Phillips-Sarnak was good or not? Just look at the horrible Microsoft code. It's not even spaghetti code. It looks like someone murdered a cat in the kitchen with a hacksaw.

13. Problem II.3

Consider $A_k x_n = x_{n+k}$. *For any* x *we have*

$$\lim_{k\to\infty} A_k x = 0$$

but $\|A_k\| = 1$ *for all* k *so there is no convergence in norm topology.*

Take an orthonormal basis $e_n \in \ell^2$ *and let* $A_n = \langle e_n, \cdot\rangle e_n$. *These go to zero weakly as* $n \to \infty$ *but strongly they do not go to zero.*

I will return to issues of proof later on.

14. Problem I.4

Suppose $A : H_1 \to H_2$ *is a bounded linear operator between Hilbert spaces and there is a* $B \in L(H_2, H_1)$ *and compact operators* $E_j \in L(H_j)$ *for* $j = 1, 2$ *such that*

$$AB = I - E_2$$

and

$$BA = I - E_1.$$

Show that A has finite dimensional nullspace, closed range and the orthocomplement of $Ran(A)$ is finite dimensional.

14.1. **Clear Understanding Of Bounded Fredholm Operators.** *The way I think about these problems is several passes. First I explore a bit to extract the issues and put them in writing. Then I arrange the arguments in order for sharp results.*

Let's consider the nullspace of A first. If $x \in Ker(A)$ then

$$0 = BAx = x - E_1 x$$

We take the set

$$B_K = \{x \in Ker(A) : \|x\| \leq 1\}$$

This is the unit ball in $Ker(A)$. Since all $x \in B_K$ satisfies

$$x = E_1 x$$

we have $B_K = E_1(B_K)$ and since E_1 is compact, B_K is compact in H_1 in the norm-topology. This implies B_K is finite dimensional as H_1 is a separable Hilbert space for otherwise we could take an infinite sequence of unit vectors $y_n \in B_K$ with $\|y_n - y_m\|^2 = 2$ all in B_K and these have no convergent subsequences.

Now let us consider the range of A. Suppose $y_n \in Ran(A)$ and $y_n \to y$ in H_2. We want to know if $y \in Ran(A)$.

Since B is continuous we have $By_n \to By$. Let x_n be arbitrary elements of H_1 so that $y_n = Ax_n$. Then

$$BAx_n = x_n - E_1 x_n$$

so

$$By = \lim_n (x_n - E_1 x_n)$$

We want to examine if x_n must converge. Given $\epsilon > 0$ there exists N such that $n, m \geq N$ implies

$$\|(x_n - x_m) - E_1(x_n - x_m)\| < \epsilon$$

and this implies

$$\|x_n - x_m\| \leq \epsilon + \|E_1(x_n - x_m)\|$$

This could get us the bound. Another idea is to use the Neumann series

$$(I - E_1)^{-1} = \sum_{j=0}^{\infty} E_1^k$$

which is norm-convergent for $\|E_1\| < 1$. We can use this to ensure $\|x_n - x_m\| < C\epsilon$ and conclude x_n converges, and then deduce $y = A(\lim_n x_n)$ and is in $Ran(A)$.

We'll return to the finite dimensionality of cokernel in a while. Here the issue is that the adjoint A^* is Fredholm as well and the cokernel is the nullspace of A^*.

15. Problem I.5

Let $f(x) = x^4 - 2x^2 + 1$ for $x \in \mathbf{R}$. Prove that there exists $\alpha \in \mathbf{R}$ such that

$$C^{-1}\lambda^{-\alpha} \leq \left| \int_{-\pi}^{\pi} e^{i\lambda f(x)} \sin^4(x) dx \right| \leq C\lambda^{-\alpha}$$

For this problem, let us explore the options of change of variables.

Write

$$f(x) = (x^2 - 1)^2$$

Then consider

$$\lambda f(x) = (\lambda^{1/2}(x^2-1))^2$$

So

$$\lambda f(x) = ((\lambda^{1/4}x)^2 - \lambda^{1/2})^2$$

Let us then try the substitution

$$u = \lambda^{1/4}x$$

Let $a = \lambda^{1/4}$ *to make things easier to read. We have*

$$\int_{-\pi}^{\pi} e^{i\lambda f(x)} \sin^{(}x)dx = \int_{-a\pi}^{a\pi} e^{i(u^2-a^2)^2} \sin^4(u/a)du/a$$

We will have the result with $\alpha = 1/4$ *if we are able to find* $C > 0$ *with*

$$C^{-1} \leq \left| \int_{-a\pi}^{a\pi} e^{i(u^2-a^2)^2} \sin^4(u/a)du \right| \leq C \tag{2}$$

This seems like it should hold, so let us put some effort into establishing (2)*. I am quite pleased, quite pleased with this substitution idea instead of getting deceived by all sorts of Fourier transform sorts of issues.*

15.1. **Countour Integral And Residue Formula.** *We consider the closed contour constructed by taking the interval* $[-R, R]$ *and the semicircle* $Re^{i\theta}$ *for* $0 \leq \theta \leq \pi$*. Then we do the following*

$$|\int_{-a\pi}^{\pi} e^{i(u^2-a^2)^2} \sin^4(u/a)du| \leq \int_{-R}^{R} \sin^4(x)dx$$

The contour integration using holomorphicity of $\sin^4(z)$ *will get us an upper bound.*

This is progress for Problem I.5. We can assume from elementary complex analysis that

$$\int_{-\infty}^{\infty} \sin^4(x)dx = C$$

is known or evaluate it. We will still need to worry about the lower bound but progress is progress and this path is reasonable and will yield a result.

I used the order wrong. It's better to do things the other way around. We could use

$$\sin(\theta) \geq \frac{2\theta}{\pi}$$

for $\theta \in [0, \pi/2]$*. There is a very nice gentle presentation for Fresnel integrals like*

$$\int_0^{\infty} \cos(x^2)dx$$

here [4]*.*

We reconsider the possibilities for contour integration with a bit more flexibility now, since it is now clearer to me that this requires some exploration. One idea is to write down the integral

$$\int_{-a\pi}^{a\pi} \cos((u^2-a^2)^2) \sin^4(u/a)du + i\int_{-a\pi}^{a\pi} \sin((u^2-a^2)^2) \sin^4(u/a)du$$

This is a problem now of contour integrations and not actually a simple problem in terms of calculus. I will mark this as a fairly delicate issue to estimate both upper and lower bounds using contour integration. But it is better to use the bound on

$\sin^4(u/a)$ *including a lower bound which holds for a region* $|u/a| \in [0, \pi/2]$*, so we have*

$$\frac{2}{\pi}|x| \leq \sin(x) \leq |x|$$

for $x \in [-pi/2, \pi/2]$*. Now Problem I.5 is an elementary problem of all sorts of messy calculus analysis but you can see intuitively that we will be able to get the required upper and lower bound after some jambalaya.*

16. My Personal Feelings About Jambalaya

If you've ever been involved with engineers in Silicon Valley, you know something about them that many ordinary people don't. They don't like using computers. They open up their laptops and motherboards, and then they do all manner of hacks and 'tweaking' all day long with immense pleasure. They boast to each other about how they hacked every device driver, and how they reconfigured their machines to sing lullabyes and wax poetic and walk on the dark side of the moon when no one was looking and such.

You see, I am not like that. I just want the damn computer to work, write some papers, have the R computations run without crashing, and I don't want to know all the wiring in the inside of the Intel 6QY568365N-dash-53 chipsets that glow every time your lover is ready for hot lovemaking or whatever. That's not my cup of tea.

In the same manner, I don't roll up my sleeves and rush into jambalaya with the foolish bravery of First World War soldiers behind enemy lines. When I sense jambalaya, my evolutionary adaptations tell me to seek flight, seek flight and live another day. You see Dan Stroock likes estimations and reams of papers filled with computations and thinks those things make men men and thought people like me are slackers. That's okay, but people like me do mathematics differently from the rough soldiers in a war types like him. We are fine when we see a clear path and otherwise, we save the trees. What about the trees? Foolish rushing headlong into jambalaya is deadly. What if there are alligators in there? Some of the things that are hiding in the jambalaya could bite into my flesh and then who will console my mommy?

17. Meteor Code Is Infinitely Superior To Cro-Magnon Microsoft Code?

Microsoft Code is just as savage and primitive and barbaric as the founder Bill Gates. Let me show you some pictures of Microsoft coders.

Those are pictures of Microsoft coders Tom, Dick, and Harry from the Jurassic era. The code is so horrible that finally when people started caring about code that is actually good, Bill Gates had lied and charlatanned his way to selling horrible ugly products everywhere.

Meteor did the opposite of Microsoft. They addressed issues that made code so anarchic for the internet and worked to produce an infrastructure that brought civilisation to the savage jungle. It takes a little time to get familiar, but once done, their Blaze templating is literally a step away from complete swamp jungle filled with predators. I am immensely impressed by how they managed to do this.

18. Problem I.3

Show there is a compactly supported smooth function $\varphi \in C^\infty(\mathbf{R})$ *with* $\varphi \geq 0$*, and* $\varphi(0) > 0$ *and non-negative Fourier transform for all* $\xi \in \mathbf{R}$*.*

The Fourier transform is real whenever the function $\varphi(x)$ *is* even*, for then* $\sin(x\xi)\varphi(x)$ *is* odd*, and so the integral vanishes since Lebesgue integrals linear. We can take*

$$\varphi(x) = \begin{cases} e^{1-\frac{1}{1-|x|}} & |x| \leq 1 \\ 0 & |x| > 0 \end{cases}$$

and this is smooth since

$$1 - \frac{1}{1-|x|} \to -\infty$$

as $|x| \to 1$ *so* $\varphi(1) = \varphi(0) = 0$*. Now cosine is even and we have*

$$F\varphi(\xi) = \int_{-\infty}^{\infty} \cos(x\xi)\varphi(x)dx = \int_{-1}^{1} \cos(x\xi)\varphi(x)dx$$

The function and cosine are both nonnegative on $[-1, 1]$ *so the Fourier transform is certainly nonnegative at* $|\xi| \leq 1$. *The only issue left is to ensure that Fourier transform is non-negative when* $|\xi| \to \infty$.

19. Problem I.4 Equality of Mixed Partials

(a) Suppose $f \in C^2(\mathbf{R}^2)$. *show* $\partial_x\partial_y f = \partial_y\partial_x f$.

(b) Find $f \in C^1(\mathbf{R}^2)$ *and a point* x_0 *where both* $\partial_x\partial_y f(x_0)$ *and* $\partial_y\partial_x f(x_0)$ *exist but are unequal.*

(c) Show in distributional sense $\partial_x\partial_y f = \partial_y\partial_x f$ *if* $f \in L^1(\mathbf{R}^2)$.

19.1. **The Discretisation Of Second Derivatives.** *Let*

$$F(x, y, h_x, h_y) = f(x + h_x, y + h_y) + f(x, y) - f(x + h_x, y) - f(x, y + h_y)$$

and we will have proved equality of mixed partials if we show equality of

$$\lim_{h_x \to 0} \lim_{h_y \to 0} \frac{1}{h_x h_y} F(x, y, h_x, h_y) = \lim_{h_y \to 0} \lim_{h_x \to 0} F(x, y, h_x, h_y)$$

For (b) we need a function such that at $(x_0, y_0) \in \mathbf{R}^2$ *the mixed partials exist but are unequal. I will begin some exploration here. This is my way of doing mathematics because I do not know the answer.*

This is an interesting situation for me even though it might be elementary I don't know the answer, so let us first try to understand what the issues are as follows. Suppose we have two functions on $\mathbf{R}^2$ *say* $\phi(x, y)$ *and* $\psi(x, y)$. *we are interested in a situation where*

$$\lim_{(x,y) \to (x_0, y_0)} \phi(x, y) = \phi(x_0, y_0)$$

and

$$\lim_{(x,y) \to (x_0, y_0)} \psi(x, y) = \psi(x_0, y_0)$$

We first consider a functions ϕ, ψ *that are continuous. Then we put the condition*

$$\phi(x_0, y_0) \neq \psi(x_0, y_0)$$

We want these to be the mixed derivatives of some $f \in C^1(\mathbf{R}^2)$. *This will require some more exploration.*

Let's just quickly dispose of (c). By (a) we have equality of mixed partials for test functions because they are smooth, so we look at the definition of derivatives of distributions and note that by definition, the mixed partials apply as follows:

$$(\partial_x\partial_y u)(\phi) = u(\partial_x\partial_y \phi)$$

So for all distributions mixed partials are equal because they are on the test functions.

20. Problem II.5

Let A be the middle thirds Cantor set. Write

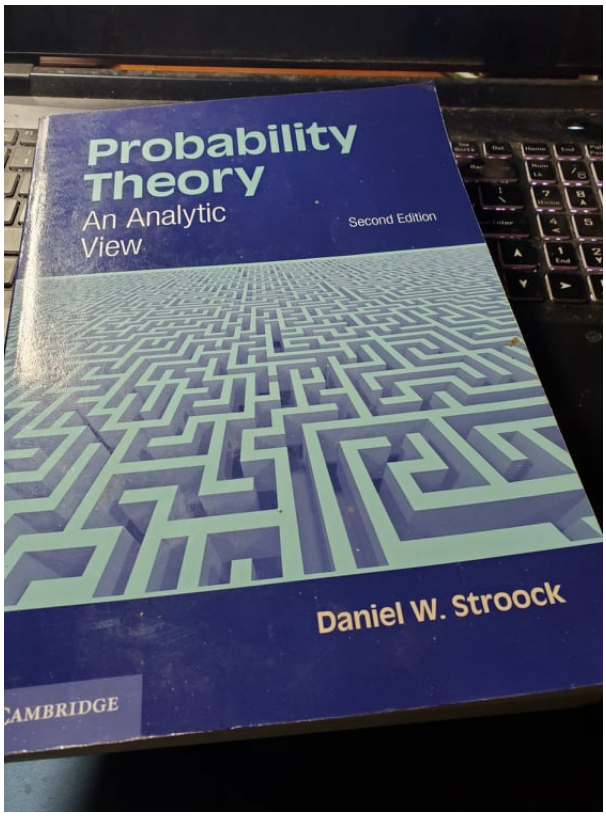
$$A = \bigcap_n A_n$$

where $A_0 = [0,1]$ and A_1 is the union of $[0,1/3]$ and $[2/3,1]$ and so on. Our goal is to prove that there is a Borel measure *μ on $[0,1]$ such that for all $f \in C[0,1]$ we have*

$$\frac{1}{m(A_n)} \int_{A_n} f dx \to \int f d\mu$$

This is a question about weak convergence of probability measures. *Although I did not use Daniel Stroock's book directly, I do recommend that those who are serious about real analysis do consider reading his book on these matters. Regardless of my various issues with his treatment of me in industry and academia, he has genius in these sorts of analysis-probability theory matters.*

Hold on, let me do some publicity for his book. You see, if I whine about my treatment by him, I better show him some good treatment so that the entire Jewish cabal does not assault me with all manner of accusations about how I never really cared about Hannukah and how I have some great chutzpah and how I never came to the Bar Mitzvah of the third cousin and I was really after Einstein for secret antisemitic views and so on. So I actually adore The Waste Land *of Thomas Stearns Eliot and Percy Bysshe Shelley. I know, I know that Eliot is considered anti-semitic, but I don't give a damn. He did good work in the Swiss sanatorium, and frankly Jewish people should forgive him. Oh Gentile or Jew, you who turn the wheel and turn to windward, consider Phlebas who was handsome, and tall as you. You see, Eliot was* gentile *so he wasn't speaking for himself. Obviously it was meant for* me *because I'm neither Gentile nor Jew!*



Yes, this is actually a very important book. It's underrated. You should read it if you're serious about analysis. It has some deep insight. Daniel Stroock's mathematics is very good. His judgment about abilities of Zulfikar Moinuddin Ahmeds of the world is not very good.

I will tell you what I will be doing for Problem II.5. I will use a basic part of probability theory for weak convergence of measures, a characterisation that says μ_n *conveges to* μ *weakly if and only if their Fourier transforms converge to* $\tau(\xi)$ and *there is a* $\delta > 0$ *such that*

$$\lim_{n\to\infty} \sup_{|\xi|\leq\delta} |\hat{\mu}(\xi) - \tau(\xi)| = 0$$

This is Daniel Stroock's Theorem 3.1.8. Let me assure the readers genuinely and without any political aims, that you will not even be able to scour through the literature and understand what is going on enough to extract such a clear understanding of convergence criteria. Daniel Stroock's understanding of these matters is absolutely fantastic. Get the book if you don't have a copy. Save Zulfikar Moinuddin Ahmed from the wrath of the angry Jewish men and women who will mobilise against me as well.

21. Fine I Will Explain Why You Really Should Get Daniel Stroock's Book

So you are young now, and you are ambitious in mathematics. You obviously want to have some immortality. Maybe you are inspired by Karl Friedrich Gauss

or Bernhard Riemann. You see, I have something profound of my own. I believe that my four-sphere theory is more profound than Atiyah-Singer Index Theorems. You may or may not agree, but I have 100% convictions about worth of my work. But I am not a renowned mathematician. But I understand mathematics. And I understand the value to you of Dan's book. You see, no one really wants to go through all this technical mumbo jumbo about weak convergence of probability measures with a fine-tooth comb. Perhaps you do, but then you'll be left in the dust when every one of your classmates are flying high doing all manner of funky dunky mathematical theorems and you are trapped in the dungeons slogging through weak convergence of probability measures.

You're more likely to be like me. You see some probability measures μ_n and you need the damn things to converge weakly and you wave your hand a little bit and say so $\hat{\mu_n}(\xi) \to \tau(\xi)$ so $\tau(\xi)$ is the Fourier transform of a probability measure. That's reasonable intuition. So you publish a number of papers with it.

Then suddenly one day you find out that various Very Important People in Russia had been keen to use your results. They were relatives of Sergei Sobolev and had married into the family of Gel'fand, and actually were related to both Markov and Chebyshev and could with a snap of their fingers make you the hottest commodity in all of Mathematical history. They called up the French School of Analysis in Ecole Polytechnique and you didn't even know that Cauchy's descendents actually existed and they read your paper. They suddenly found that you had gaps in your proof.

No one will care that it's trivial to fix. The cousins of Cauchy's descendents and the descendents of Markov and Chebyshev will collude to 'find a gap in your proof' and suddenly you will see your work become Markov-Chebyshev-Cauchy theorem and it will be too late to change your name to something illustrious like Bernoulli Riemann to take back what is yours.

The moral of this tale is buy Dan's book. You will avoid these sorts of situations where you are as lazy as me and will refuse to actually work out all the arcane things that could one day allow you to go to ballroom dances with the beautiful ladies from all the great mathematical families instead of losing your ticket to immortality. It's worth it.

Let us begin with the simple Fourier transform of the interval $[a, b]$. It is

$$\int_a^b e^{-ix\xi} dx = \frac{e^{-ia\xi} - e^{-ib\xi}}{i\xi}$$

Now let's get to the business of tracking the sizes and numbers of sets in the Cantor set.

	Level	*Number.of.Intervals*	*Size*
1	*0*	*1*	*1*
2	*1*	*2*	3^{-1}
3	*2*	*4*	3^{-2}
4	*3*	*8*	3^{-3}
5	n	2^n	3^{-n}

Our hope will be that even though the Fourier transforms μ_n are quite complicated, we will be able to use fairly generic data such as total number of intervals and their sizes to gain some insight about quantities like

$$|\hat{\mu_{n-1}}(\xi) - \hat{\mu_n}(\xi)|$$

From the table above, we can read off

$$m(A_n) = (2/3)^n$$

Now between A_n and A_{n+1} we have to track operations. Let's see what happens to one of the full intervals: $[a, b]$ was replaced by $[a, a']$ and $[b', b]$, and two points were introduced. Ignoring the normalisation by $m(A_n)$ the Fourier transforms differ by one integral:

$$\int_{a'}^{b'} e^{ix\xi} dx$$

Now the normalisation of $\hat{\mu_n}$ is $m(A_n)^{-1} = (3/2)^n$ for all n. So we can write

$$\hat{\mu_{n+1}}(\xi) = (2/3)^{-n-1}[(2/3)^n \hat{\mu_n}(\xi) + \sum_{j=1}^{2^n} \int_{x_j}^{x_{j+1}} e^{-ix\xi} dx]$$

We are approaching now something concrete for convergence rate

$$|\hat{\mu_{n+1}}(\xi) - \hat{\mu_n}(\xi)|$$

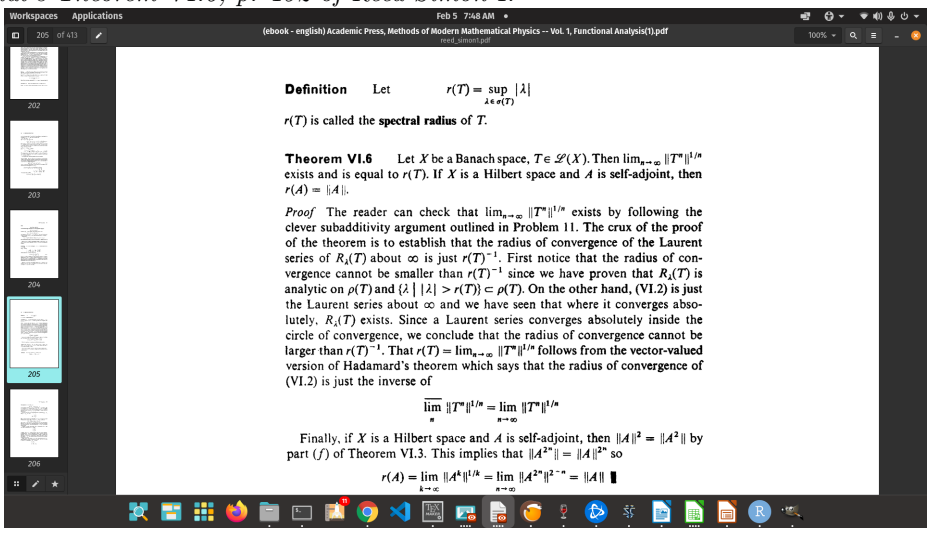
We want to get some sort of control over these by counting 2^n new integrals and using the sizes which are 3^{-n-1}. Now you might ask why don't we just do these things on the sets? We could do that but then we introduce all manner of potential issues that we don't want to deal with. With characteristic functions, we are dealing with sequence of functions and our conclusion will guarantee *a probability measure which is a solid rigorous conclusion that such and such a Borel measure exists and we don't really care what it's messy behaviour is with respect to the Lebesgue measure because we have assurance from a strong theorem that the measure is a probability measure. That's a nice firm conclusion to Problem II.5. There are no more headaches and no reason for paranoia or need to check that there are no exceptions. Every potential problem will reveal itself directly in the characteristic functions.*

My dear reader has to put things in context. There is a huge amount of activity in probability theory of Lévy Processes and the whole enterprise had no existence before the Lévy-Khinchine formula. Characteristic functions are quite miraculous in being able to ensure that I don't see the best minds of my generation destroyed by madness, starving hysterical naked, dragging themselves through negro streets at dawn looking for an angry fix, angelheaded hipsters burning for the ancient heavenly connection to the starry dynamo in the machinery of night who poverty and tatters and hollow-eyed and high sat up smoking in the supernatural darkness of cold-water flats floating across the tops of cities contemplating jazz, who bared their brains to Heaven under the El and saw Mohammedan angels staggering on tenement roofs illuminated. Well maybe that's not true. Maybe I did see the best minds of my generation do exactly that, what Allen Ginsberg says breathlessly. But they also did some sharp work on probability theory as well. In other words, the Mohammedan angels did, sporadically, bring inspiration, and let me assure you that despite what Mohammedans themselves believe, Mohammedan angels don't care about your religion. Take it from me. They are Mohammedan angels who can't see the finer issues of doctrine in the hearts and souls of men.

22. Problem I.2 Revisited

The substance of this problem is part (b), that that for a bounded self-adjoint operator on a Hilbert space the spectrum cannot be $\{0\}$*. Here one one has to actually know that in this case the* spectral radius *is equal to the norm of the operator.*

I did not want to cook up a solution that is wrong here since my memory of functional analysis course from 1993 was not as good. The best presentation I know is Reed-Simon and so I will just punt on this and give this as my answer. That's Theorem VI.6, p. 192 of Reed-Simon I.



Definition Let $r(T) = \sup_{\lambda \in \sigma(T)} |\lambda|$

$r(T)$ is called the **spectral radius** of T.

Theorem VI.6 Let X be a Banach space, $T \in \mathscr{L}(X)$. Then $\lim_{n\to\infty} \|T^n\|^{1/n}$ exists and is equal to $r(T)$. If X is a Hilbert space and A is self-adjoint, then $r(A) = \|A\|$.

Proof The reader can check that $\lim_{n\to\infty} \|T^n\|^{1/n}$ exists by following the clever subadditivity argument outlined in Problem 11. The crux of the proof of the theorem is to establish that the radius of convergence of the Laurent series of $R_\lambda(T)$ about ∞ is just $r(T)^{-1}$. First notice that the radius of convergence cannot be smaller than $r(T)^{-1}$ since we have proven that $R_\lambda(T)$ is analytic on $\rho(T)$ and $\{\lambda \mid |\lambda| > r(T)\} \subset \rho(T)$. On the other hand, (VI.2) is just the Laurent series about ∞ and we have seen that where it converges absolutely. $R_\lambda(T)$ exists. Since a Laurent series converges absolutely inside the circle of convergence, we conclude that the radius of convergence cannot be larger than $r(T)^{-1}$. That $r(T) = \lim_{n\to\infty} \|T^n\|^{1/n}$ follows from the vector-valued version of Hadamard's theorem which says that the radius of convergence of (VI.2) is just the inverse of

$$\overline{\lim_n} \|T^n\|^{1/n} = \lim_{n\to\infty} \|T^n\|^{1/n}$$

Finally, if X is a Hilbert space and A is self-adjoint, then $\|A\|^2 = \|A^2\|$ by part (f) of Theorem VI.3. This implies that $\|A^{2^n}\| = \|A\|^{2^n}$ so

$$r(A) = \lim_{k\to\infty} \|A^k\|^{1/k} = \lim_{n\to\infty} \|A^{2^n}\|^{2^{-n}} = \|A\| \blacksquare$$

Well I ought to be on top of this and I ought to know that but I don't really care about oughts. The problem is very important, so I will refine my understanding of these in time. I am looking for various issues in Fall 2015 exam that are missing now.

Now spectral projection operators exist for bounded selfadjoint operators originally proven by David Hilbert, the great man himself. For me the most important theorem is spectral theorem for compact *self-adjoint operators. The spectral resolution for bounded selfadjoint operators is important but I have not really thought about where their value is seen best. Resolvents of Dirac operator on* S^4 *are for me the most important application for compact operators. These are important for four-sphere theory too, not just the obsolete quantum mechanical and field theoretic theories.*

23. People Like Zulfikar Moinuddin Ahmed Are The True Geniuses Not People Like Bill Gates

Bill Gates is a scrub charlatan whose achievements and ideas are menial. He is not a true genius like Zulfikar Moinuddin Ahmed. See I can see very clearly that Money is like clean water, and there is no need at all to wait for Star Trek age for removal of Money. It's like clean water. What is necessary for abundance of Money *is just much sharper credit analysis than exists today, more assistance for improved credit. I can do this myself in a decade, reduce the power of Money over individual lives across the world. It's not hard either.*

And the Human Race is a social animal, a bit like ants. When the pioneer ant, Zulf says that my beloved people, let me show you the way where Money is less of a problem, this will spark a movement slowly. In a century or less, Money will be like clean water. The scrubs like Bill Gates will go the way of the dinosaurs, extinct.

References

[1] George Mackey, *Harmonic Analysis As The Exploitation of Symmetry*, Bull. AMS 1980, 3(1), pp. 543–

[2] V. S. Varadarajan, *An Introduction To Harmonic Analysis On Semi-Simple Lie Groups*, Cambridge, 1989

[3] Christian Bär, The Dirac Operator On Space Forms Of Positive Curvature, J. Math Soc. Japan, 48(1), 1996, 69–83

[4] `https://www.youtube.com/watch?v=cWjwG2nrjC0`

[5] H. Blaine Lawson and Marie-Louise Michelsohn, *Spin Geometry*, Princeton

[6] Zulfikar Ahmed and Daniel Stroock, A Hodge theory for Some non-Compact Manifolds, J. Differential Geom. 54(1): 177-225 (2000)