ZULF GEARS UP FOR STANFORD ANALYSIS QUALS 2013

ZULFIKAR MOINUDDIN AHMED

Spring 2013 Stanford Analysis Problems. I see immediate solutions for I.1-3 and unclear about I.4,5. I see that II.1-2,3,5 are reasonable for me and II.4 is unfamiliar. Now I will begin to get more serious and spend some time resolving all the problems.

1. Comments

As you know, I am 49, and I have nothing to prove at all to anyone. I am comfortable generally with my understanding of analysis, and I generally really could not give a damn who evaluates me how. I am not a student any more. I am an immortal genius for my work not just in Four-Sphere Theory but also my pioneering work in Universal Human Moral Nature. I do understand that people like politeness, but I want to ensure that my dear readers understand that I am not trying to prove my worth to anyone at all. I am just fine the way that I am. I am doing Stanford Mathematics Ph.D. Qualifying exams for my own benefit. I am familiar with Princeton Honour Code from 1991-1995 and will be quite open about where I have used assistance and give references. I am doing these in Allen Texas, in my aunt's house, and not in any academic setting and have no assistance.

Having removed these introductory issues, let me immediately point out that the spectral theorem of compact self-adjoint operators, Problem II.3 is by far the most important problem in the ten. This is a result that I originally learned in Peter Sarnak's Functional Analysis course in 1993-4 which was also attended by Terry Tao and Steven Gubser. I have used this result extensively in Four-Sphere Theory directly to explain quantum phenomena in nature. I will spend some time on this problem hoping to understand the issues better.

2. II.3 Spectral Theorem For Compact Self-Adjoint Operators

Suppose $T \in L(X)$, and X is a separable Hilbert space. Suppose both that it is self-adjoint, $T^* = T$ and that it is compact.

(a) Show that there is a complete orthogonal set of eigenvectors of T.

(b) Find non-selfadjoint operator $T \in L(X)$ that is compact with spectrum $\{0\}$ and no eigenvectors.

This is by far one of the most important results in mathematics, and, thanks to my work of immortal genius, one of the most important results in all of physics. For these reasons, I will be quite willing to examine this situation.

Let me give you context for this problem. You could be misguided and think: "Well, this looks like a good technical result based on these functional analysis concepts." Wrong. Wrong. And wrong.

The right answer is that when we see that the energy in the actual universe is *quantised*, we could either be quite in error and model the universe as an infinite

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number of *harmonic oscillators* like Max Planck, or we could be right like Zulfikar Moinuddin Ahmed, and say, "No, Max Planck. You're confused. The phenomena of quantisation of energy is a consequence of global geometry of space, for absolute space in external existence is not *flat*, as you seem to believe, but a fixed radius four-sphere of radius $R = 3075.69$ Mpc. And it is the spectral theorem for compact self-adjoint operators applied to the *resolvent of the operator D, the Dirac operator on spinor fields of four-sphere, which is compact and self-adjoint, that tell you exactly the quantisation of energy in the actual universe." Once you are properly respectful of how nature is described thanks to the spectral theorem for compact self-adjoint operator, then you suddenly realise that all aspects leading to this result are great advances in man's intellectual quest for understanding nature.*

2.1. **Where Did This Beautiful Result Come From?** *I take a trek back to freshman year at Princeton. John Horton Conway teaches us linear algebra in 1991. And he is a phenomenal teacher. His primal screams allow us to have any memory of what makes a set a basis in a finite dimensional vector space, of linear independence. He screams a set of vectors $x_1, \dots, x_K$ are linearly dependent when $a_1x_1 + \dots a_Kx_K = 0$ where $a_j \in \mathbf{R}$ are* not all zero*. He screams and screams* not all zero*. I am young and new, and I have dreams of screams of* not all zero*,* not all zero*. And that is how I learn something. Some weeks pass, and we are looking at symmetric matrices and their diagonalisation. Years later, whenever I see a symmetric matrix I diagonalise it out of sheer habit. I can't even imagine not diagonalising every symmetric matrix I ever encounter. You think this is some sort of OCD, Obsessive Compulsive Disorder Symptom. That could be, but I am not suing Princeton for this. I am just fine with this OCD. A symmetric matrix that remains undiagonalised ought to disturb anyone I think.*

That's right. The spectral theorem for compact self-adjoint operators on Hilbert space is a result that generalises this absolutely crucial truth, that finite dimensional symmetric matrices are diagonalisable.

2.2. **Intuitive Approach.** *I want to try to examine this problem from the most clear and elementary path because the result is so important that clear insights are more valuable than sophisticated techniques. I have a strong feeling for diagonalisability of symmetric real matrices, and I assume this is known.*

I will try to reduce the spectral theorem for compact selfadjoint operators to the finite dimensional symmetric matrix diagonalisability.

Suppose $\epsilon > 0$ is a small number. The idea is this. We start with an orthonormal complete set $\{x_k\}$ in X. Then we will consider the set $\{Tx_k\}$. Then using compactness we will want to extract a finite set *after relabeling, $\{x_1, \dots, x_N\}$ such that for $1 \le k \le N$ have $\|Tx_k\| \ge \epsilon$ but $\|Tx_k\| < \epsilon$ for $k \ge N+1$.*

Then we define T^ϵ by $T^\epsilon x_k = Tx_k$ for $1 \le k \le N$ and $T^\epsilon x_k = 0$ for $k \ge N+1$.

Now T^ϵ is then a projection to $\mathrm{span}(x_1, \dots, x_N)$ *followed by a finite dimensional symmetric matrix. We can then apply the symmetric diagonalisation of elementary linear algebra on it.*

This process will not exactly give us the spectral theorem for compact selfadjoint operators yet, but it will give us some sense of what the substance is in the phenomena.

So what happens as we change the $\epsilon > 0$ a bit? Define

$$X_\epsilon = \{x \in X : \|x\| = 1, \|Tx\| \ge \epsilon\}$$

These are all finite dimensional, but they grow, perhaps quite quickly as $\epsilon \downarrow 0$.

Suppose $y_1, \ldots, y_N$ *are eigenvectors of the symmetric matrix* T^ϵ. *Then*

$$Ty_k = T^\epsilon y_k = \lambda_k^\epsilon y_k.$$

Therefore y_k *is an eigenvector of* T. *This is very nice. This means that we have managed to find some of the eigenvectors of* T *in the finite dimensional vector space* X_ϵ. *Now we just forget about* X_ϵ *and consider its* closed orthogonal complement *and repeat the procedure.*

3. History That Is New To Me

Augustin Louis Cauchy innovated eigenvalues and diagonalisation first in 1826, and it was Cauchy who proved that symmetric matrices are diagonalisable. And the spectral theorem for compact self-adjoint operators was the work of Frigyes Riesz.

The concept of an algebra of operators made its appearance in series of articles culminating in a 1913 book by Frigyes Riesz, where Riesz studied the algebra of bounded operators on the Hilbert space l2. Riesz representation, orthogonal projectors, and spectral integrals made their first appearance in this work. In 1916 Riesz created the theory of what he called "completely continuous" operators, now more familiarly compact operators. Since he wrote this in Hungarian, wide recognition came only two years later with a translation into German. Riesz's spectral theorem for compact operators made abstract, greatly extended, and largely supplanted Fredholm's work [1].

This is the crucial result, the spectral theorem for compact *self-adjoint operators, of F. Riesz. Others made more powerful and general spectral theorems, Marshall Stone and Jon Von Neumann, but it is the 1913 result that I have found most powerful in four-sphere theory.*

4. Four-Sphere Theory Vastly Advances Physics Beyond Schroedinger's Theory

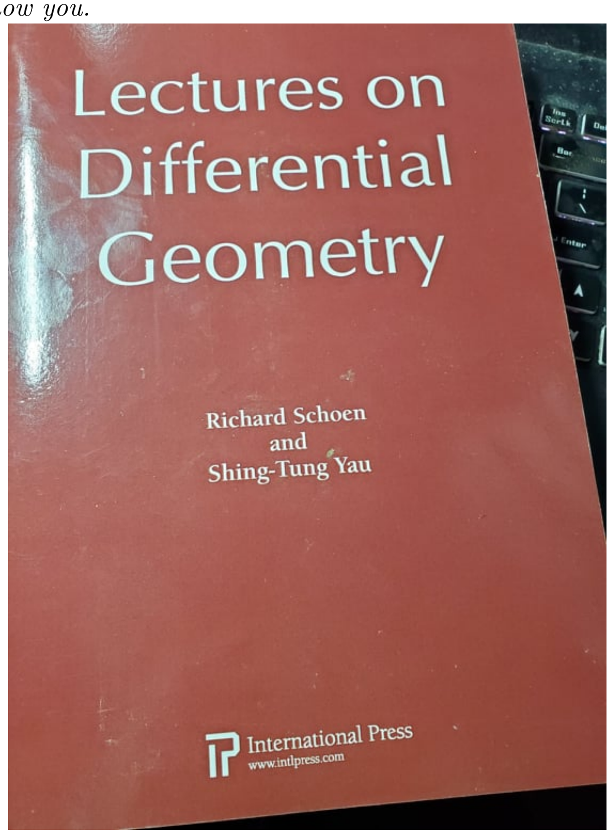
Four-Sphere Theory is physics; it is not mathematics. It is a scientific theory about actual nature. It happens to be extremely focused on mathematical issues, but it is not a theory for mathematicians. It is simply a theory of nature. The scope of four-sphere theory is all natural phenomena down to scale $\delta = 10^{-13}$ *cm. I won't go into details here but the work is archived* [2]. *My S4 Electromagnetic Equation is in the theory the replacement for both Schroedinger's Equation and Maxwell's Equations, and it is empirically superior fit to data than these. For Schroedinger's Theory, which is good for simple atoms, there are vast pathologies for polyatomic molecules that are addressed by Barry Simon and others in their work on perturbation theory. These all disappear in my Four-Sphere Theory.*

The troubles of Schroedinger and Maxwell theories is that $\mathbf{R}^3$ *model of space is wrong. Space is actually* S^4, *and so the complexities introduced by non-compactness of* $\mathbf{R}^3$ *had hindered progress in physics for more than a century. These I have resolved.*

5. Digression On How Four-Sphere Theory Developed

In 2007, I told Chris Thorpe when I was working at Babel that I thought electromagnetism was four dimensional. Then I was fired from Babel and I had a loft in Williamsburg Brooklyn in 2008. That's when I began working on Four-Sphere

Theory. I had no supervisors but I was sure that four-sphere theory would be right. Four-dimensionality I checked with examining the anomalous rotational symmetries of orders 5, 8, 10, 12 found from 1980s by crytallographers. These are exactly the rotational symmetries of four-dimensional crystals. They Nobel prize was given for 'quasi-crystals' in 2011 but I was sure that's wrong. They show that the actual universe has four *rather than three spatial dimensions. Then I looked at Einstein's gravitational equations, and I had then good command of differential geometry. I'll show you.*



I knew this book and so I was quite convinced that Einstein's equations are not telling us something about dynamics but they are embedding of the physical universe *in a constant curvature ambient space. I immediately guessed that quantisation of the universe is due to spectral theory. But I had trouble gaining attention then of people. I was not in a physics career.*

For a decade I worked on Four-Sphere Theory until the confidence I gained that it was the only true physics *became extremely strong after I had checked many measurements. Now I am sure that all other physics theories that are not Four-Sphere Theory are wrong.*

6. Commutation and Anticommutation Relations of Quantum Theory Far Simpler In Four-Sphere

These are not my ideas, so I won't claim credit for them. Peter Woit at Columbia has nice work on quantum mechanics from which these became clear to me. You see quantum theory has difficulties because the space is $\mathbf{R}^3$*. Let me show you how* *things are much clearer without the mysticism of quantum mechanics and quantum field theory. Four-Sphere Theory removes the mysticism.*

Take the cotangent space of the four-sphere, letting the radius of the universe be 1 instead of $R = 3075.69$ *Mpc. What do you see here?*

$$T_x^* S^4$$

Now introduce the exterior algebra.

$$\bigwedge T_x^* S^4$$

So that's nice, we have differential forms. Now look at the Clifford multiplication, on these, and they don't preserve degrees but even and oddness. All this is in Spin Geometry *of Harvey Lawson and Marie-Louis Michelsohn, so I won't go into the nitty gritty.*

It is an exercise to see that the canonical commutation and anticommutation relations of quantum theory are exhibited in the Clifford algebra $C\ell(T_x^* S^4)$*. Atiyah, Bott, Shapiro had examined the fundamentals here many years ago. In other words, all the quantum woo woo is not necessary. Four-Sphere Theory does not have all those funky things. Spinor fields are canonical and the mathematics is sharp, and the science begins when we take spinor fields on four-sphere as the fundmental objects of nature.*

Particles appear in nature from zonal harmonics *of a four-sphere that have localisation. Quantisation occurs because spectrum of the Dirac is* $\mathbf{Z} - \{0, \pm 1\}$*. The scale of the universe is inverse of Planck constant roughly. Everything works here without mystical quantum fields that are operator-valued distributions. Spinor fields are the objects of nature, not quantum fields. Special relativity is not true in nature. Time and space are separate and unrelated. No one will ever displace my Four-Sphere Theory in infinite time in the future because this is how nature is. There are no alternatives.*

7. What Is Interesting Here In S4 Physics?

The most interesting is that established chemistry is very deficient *because the dynamics of chemical bonds and interactions is missing some symmetry, roughly, that is not seen in the physical hypersurface. That is my terminology for the three dimensional world that we experience directly. And it is this missing symmetry of spinor fields of four-sphere, that is the reason the* protein folding *and other problems of complex biomolecules had been difficult. In fact biomolecular dynamics problems have much easier methods with my S4 Electromagnetic law. I have done some work on Spark implementations for simulations but unfortunately Bill Gates hurt my eyes with racial power and black magic with a foolish effort to destroy and kill me because of his illiterate hick tribal genocidal attitudes towards non-white people.*

That's not the only interesting aspect. Others include possibilities for revolutions in semiconductor technology, ferromagnetism, and other interesting areas.

8. Problem I.1

(a) Show that if $X^* = X^{***}$ *then* $X = X^{**}$

(b) Show that for $x_0 \in \mathbf{R}^n$ *and* $\epsilon > 0$ *there is a function* $\varphi \in C_0^\infty(\mathbf{R}^n)$ *with the properties that*

$$\operatorname{supp}(\varphi) \subset \{x \in \mathbf{R}^n : |x - x_0| < \epsilon\}$$

and $\varphi(x_0) = 1$.

I will do the second problem. We let $n = 1$ *and* $x_0 = 0$ *and* $\epsilon = 1$. *We claim*

$$f(x) = \exp(1 - 1/(1 - |x|))$$

for $x \in (-1, 1)$ *and*

$$f(x) = 0$$

for $|x| \geq 0$ *has the requisite properties. In the exponent, as* $|x| \to 1$ *we get* $-\infty$ *so the function is zero at the boundaries,* $f(0) = 1$ *is clear. It's smooth in the interior* $x \in (-1, 1)$ *is obvious.*

Then what we do is take this single-dimensional function and put $|x|_{\mathbf{R}^n}$ *instead of absolute value and translate it to center* x_0.

9. Problem I.2

Suppose f, g *are measureable and positive with* $f(x)g(x) \geq 1$ *on* $[0, 1]$. *Show that*

$$\int f dx \int g dx \geq 1$$

For this we just apply Cauchy-Scharz or Hölder with $p = 1/2$ *to* $\sqrt{fg}$. *We have*

$$\sqrt{fg} \geq 1$$

Now integrate and apply Cauchy-Schwarz.

10. Probem I.3

Suppose $(X, \mathcal{B}, \mu)$ *be* σ*-finite, and* K *measurable on* $X \times X$ *with*

$$\int |K(x, y)| d\mu(x) \leq C$$

and

$$\int |K(x, y)| d\mu(y) \leq C$$

Show that $Af(x) = \int K(x, y)f(x)d\mu(x)$ *is bounded and has norm bounded by* C. *For this, note*

$$\int \int |K(x, y)|^2 d\mu(x)d\mu(y) \leq C^2$$

Now apply Cauchy-Schwarz on $\int |Af(x)|^2 d\mu(x)$ *as follows*

$$\begin{aligned}
\int |Af(x)|^2 d\mu(x) &= \int (\int K(x, y)f(y)d\mu(y))^2 d\mu(x) \\
&\leq \int (\int |K(x, y)|^2 d\mu(x)d\mu(y) \int |f(y)|^2 d\mu(y) \\
&\leq C^2 \|f\|_2^2
\end{aligned}$$

11. Problem II.2

Suppose $1 < p < \infty$ and $f_n \in L^p([0,1])$ with $\|f_n\|_p \leq 1$ and $f_n \to f$ pointwise. Show $f_n \to f$ weakly. Show $\|f\|_p \leq 1$.

Apply Lebesgue's dominated convergence theorem to f_n^p and since 1 is integrable, conclude that the limit is L^p with $\|f\|_p \leq 1$.

Let $\phi \in L^q$ the dual. Apply Holder's inequality to $(f_n - f)\phi$ and get weak convergence.

Oops. No just pull out $\|f_n - f\|_\infty$ to get upper bound, not Holder here.

12. Ruminations About Mathematics

I was aware already early in Princeton that Mathematics is rarely about cleverness. It is about gaining insight into phenomena. In the long run, that is what I had talent in, when the phenomena is from nature. The issues of mathematics do not require more cleverness but sharper understanding of certain types of phenomena that take time and experience *to understand. What I had learned is that practice allows one to see clearly some aspects of phenomena. I am amused by people who are continuously attempting to seem clever; they are rare among mature mathematicians.*

You see, I am examining human nature now, and there is far more to understand. No one before me had even known that there is a universal human moral nature that spans the globe. Immanuel Kant and Friedrich Nietzsche could not have even guessed so.

13. Problem II.5

Define, for integers $m \geq 1$,

$$H^m(\mathbf{T}^n) = \{f \in L^2(\mathbf{T}^n) : \sum_{k \in \mathbf{Z}^n} (1 + |k|^2)^m |\hat{f}(k)|^2 < \infty\}$$

Let

$$P(\xi) = \sum_{|\alpha| \leq m} a_\alpha \xi^\alpha$$

be a polynomial. Assume $P(\xi)$ is an elliptic symbol, i.e. $P(\xi) \neq 0$ whenever $\xi \neq 0$. Now let P be the operator $P = P(D)$.

(a) Show that as linear operator $P : H^m \to L^2$, the nullspace is finite dimensional and consist of infinitely differentiable functions.

(b) Show that P is invertible if and only if P is injective.

13.1. Smoothness of nullspace.

If $Pf = 0$, then for any $q = 1, 2, 3, \ldots$ we have $f \in Dom(P^q)$. And then

$$f \in \bigcap_{m \geq 1} H^m = C^\infty(\mathbf{T}^n)$$

13.2. The Embedding Is Compact.

For $m \geq 1$, the embedding $i : H^m \to L^2$ is compact. To see this, take the unit ball in H^m and by continuity of i, and for a bounded sequence f_k in H^m,

$$\|f_r - f_s\|_m \leq 2B$$

and

$$\|f_r - f_s\|_2 \leq C\|f_r - f_s\|_m$$

I will return to this problem.

13.3. **Finite dimensional nullspace.** *I want to do the following. I want to take* $X = H^m$ *and break it up into subspaces* $X = A \oplus B$ *where* A *is the nullspace and* B *is the orthogonal complement of the nullspace. Then I want to apply the inverse mapping theorem on* $P|B$ *to conclude that it is invertible. Then I want to consider the inverse map* Q *on the range* $P(B)$*. This* Q *will be bounded.*

14. The Ideas Are Not Right Yet Here

The inverse mapping theorem is the key to (b) and the Sobolev embedding theorem the key to (a) but I am not yet clear enough about this. I will resolve this clearly in the next days. Here my rustiness comes from lack of practice.

15. Zulf's Strong Views About Recording Failures

For a year or more, I had been playing Starcraft II. I am roughly at the AI level Very Hard 80% or 70% penalty on the AI. I slowly got better with practice. Some people don't like to show their errors. I record them, especially when I find my limit. I do not think it is a mature approach to hide errors. Those are the frontiers of one's skills and they need to be recorded and examined even more than successes. Some people don't like to be judged for their errors. I am really far too old to even consider that sort of attitude serious. All serious and respectful people of good character were never deceptive about where they are strong and where they are weak. I don't really think it is shameful at all to make errors and failures and record them even for public. They have great value to me.

You know, you live once in this life that you can be sure of living. You have to live your life being comfortable with yourself. You're good at what you're good at. You're not good at what you're not good at. You will get better at things that you love to do and enjoy. And it does not even matter a tiny bit who else is good at doing what. Who cares? If someone else is better than you at something, good. That's their issue, not yours. You still need to be yourself and do the things you were born to do.

I just don't understand people who are giving up precious moments of their only life worrying about who else is good at what and wasting their time. Life will end, so do what you love to do and you will be as good or bad as you will be. Your self-esteem should never ever depend on some sort of thing that is far from the substance within you, your virtues, your character, your love, things that are much more central to who you are deep in your soul. Who cares who does not think highly of your work? That's their headache. They're not going to live your life for you, so fuck them.

I will give two examples of great mathematicians failing. Marcel Riesz told Laurent Schwartz that he was wasting his time and then said that solutions to general constant coefficient differential equations would be work of another century. With Schwartz' distribution theory Ehrenpreis and someone else solved it in a few years. Von Neumann told everyone that self-adjointness of polyatomic Hamiltonians would be unreachable for quantum mechanics , and Tosio Kato resolved it not long after.

Those examples are not rare. It does not matter who thinks what; even the greatest geniuses with vast accomplishments do not have the right answers about things they did not work on. I think this needs to be just understood. You should *not deal with people who don't respect you, period. It does not matter how good or important they are. Your life is precious and you should find people to talk to who respect you and cut out all those who do not. They don't belong in your life. Let them do their thing with someone else.*

16. Bill Gates Is Talentless Charlatan

You see Bill Gates was always a con man charlatan without talent. He likes to PR that he had an SAT of 1590 while it was 1290, and his IQ is 120 not 160. He never did any serious work at all and his coding is not good. His mathematical level is high school. He pretends to be a great genius at all sorts of things but he's a charlatan. I said, let's compete and I'll do some of these Stanford Mathematics Ph.D. Quals problem and you do them too and we'll see who gets the right answer. As expected, he avoids it by bird-brain responses like "It's trivia." He's a proven total charlatan and always was.

17. What's Left?

I.3,4,5 and II.1, 4. Then I want to understand how to get the nullspace of P to be the image of a compact operator in II.5. I will examine these in the next days. That's not so bad. December 18 2021, four problems reasonably done, and six that will require some clearer understanding.

References

[1] `http://www.mathphysics.com/opthy/OpHistory.html`
[2] `https://github.com/zulf73/S4TheoryNotes`