A SIMPLE EXAMINATION OF D'ALEMBERT'S REASONING FOR ONE DIMENSIONAL WAVE EQUATION

ZULFIKAR MOINUDDIN AHMED

I am tremendously impressed by Jean le Rond d'Alembert's first derivation of the wave equation. There are various aspects of this simplest situation from 1740s the first great discovery of the wave equation that is tremendously important for my understanding of my fundamental law of all of Nature, the S4 Electromagnetic Law.

I am attempting to get an understanding from multiple sources, and quite likely the understanding will deepen and grow with time. It is very clear to me that Peter J. Olver's viewpoint is Isaac Newton-centric [1]. I will start here because the book is good, but I will tell it slightly differently.

Olver tells us the story of d'Alembert's derivation of the wave equation thus. D'Alembert applied Newton's Second Law, $F = ma$ to a string in equilibrium. The string we draw on the page as a horizonal line on the (x, y) Cartesian coordinates. Let's say the horizontal line $y = 0$ and $0 \leq x \leq 1$. D'Alembert is interested in the string have *mass density* $\rho(x)$ and *tension* $\kappa(x)$. D'Alembert lets $u(x, t)$ be the vertical displacement in the y axis. He then writes

$$F = \rho(x)a(x,t) = \rho(x)\partial\partial^2 u(x,t)\partial x^2$$

where we just consider the vertical acceleration. Then d'Alembert sets it equal to

$$\frac{\partial}{\partial x}(\kappa(x)\frac{\partial u(t,x)}{\partial x})$$

This is one of the most profound discoveries in the history of Science.

You see, I do appreciate the importance of Isaac Newton, whose calculus and whose law are both used to *derive* this equation. You have two derivatives in time just from Newtonian acceleration here. You also have derivatives in the horizontal dimension due to a *restoring force brought about from the tension of the string.*

I think this equation is the *most important discovery about Nature in history.* It's much deeper a discovery than Newton's Second Law. And that is a perspective I will keep repeating, that there is something far more profound in d'Alembert's discovery of wave equation *regarding Nature itself* than the entire ouvre of Isaac Newton. Most people will disagree. This is deeper law and far transcends vibrating strings. You won't understand what I say here immediately. But it is true.

My S4 Electromagnetic Law is formulated as fundamental law of all of Nature, and it is a wave equation on *Spinor Fields of a Four-Sphere*, and there the notions of acceleration and tension do not directly occur, but this is the source of all dynamics of the entire universe.

Date: January 8, 2022.

My S4 Electromagnetic Law, the Final Law of all of Nature, is identical in form.

$$\frac{1}{c^2}\frac{\partial^2\varphi(t,x)}{\partial t^2} = D^2\varphi(t,x)$$

but the right hand side is not due directly to *tension* of a string. And this is precisely one of the most remarkable things about this law. One could *interpret* the law in terms of *tension of fundamental fabric of existence* because the law is for spinor fields of all of existence.

That is interesting speculation but it is intuitive at the moment and worth investigating as a potential. Four-Sphere Theory considers the fabric of four-sphere as a medium that is an exact four-sphere with a tension in a way, but it is much deeper sort of tension than the type in the vibrating string.

The intuition that quantisation of Nature ought to be exactly identical to vibrating four-dimensional membrane occurred to me originally not today but in the summer of 2008. And for this I have influence from two sources, that are both very vivid to me: one is an extremely sophisticated course that Peter Sarnak taught at Princeton in 1993 for which I worked very hard and only obtained an A-. I had worked through then the spectral theorem for compact self-adjoint operators on $L^2(G)$ where G is a compact Lie group. We had covered the Peter-Weyl theorem in the course. And of course Peter Sarnak was interested in the issue of *isospectral* manifolds for which John Milnor provided the first examples. The question goes back to a beautiful paper of Mark Kac, who was Daniel Stroock's official doctoral advisor. Kac's paper of 1966 'Can One Hear The Shape of A Drum?' I had the insight that regularity of spacing for energy in Planck's Law $E = h\nu$ could have a geometric explanation based on the global *compact* geometry of nature that is homogeneous. It is known that for a four-sphere, the Dirac spectrum does determine the geometry. So my immediate excitement was based on a set of conjectures which were not quite right, but the universe as a vibrating sphere made infinitely more sense even in the period before I actually had crisp prediction of the redshift and electron width and had calibrated $R = 3075.69$ Mpc and diligently examined distances observed by telescopes to ensure that they in line with this radius.

I was not immediately worried about the mathematics of spinor fields. I knew the book *Spin Geometry* of Harvey Lawson and Marie-Louise Michelsohn for many years by 2008, and so did not think that it was a serious problem as I was not actually intending then to challenge Maxwell and Schroedinger and Einstein. I was then more interested in doing a reasonable model to solve protein folding and also get a reasonable model of unification of gravity and electromagnetism. It was much later that I had refuted existence of gravity after I was very sure that $\Lambda > 0$ was just the curvature of *empty space*. See the membrane surface that is vibrating in the actual universe is itself something very mysterious; is is simultaneously aether, and something that is fundamentally homogeneous. And three dimensional universe emerges from this – and I believe this happens by magnetic dipoles and magnetic monopoles are not in the three dimensional universe.

What is important of course that my Four-Sphere Theory is almost guaranteed to produce sharper predictions than quantum field theory and relativity in *all Natural phenomena* because it is superior science and simultaneously rigorous mathematics.

1. The Maxwell's Equation In QED

The Maxwell equation in QED is summarised nicely in the Wiki entry [3].

Extension to quantum electrodynamics [edit]

Canonical quantization of the electromagnetic fields proceeds by elevating the scalar and vector potentials; $\varphi(\mathbf{x})$, $\mathbf{A}(\mathbf{x})$, from fields to field operators. Substituting $1/c^2 = \varepsilon_0\mu_0$ into the previous Lorenz gauge equations gives:

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J}$$

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

Here, $\mathbf{J}$ and ρ are the current and charge density of the *matter field*. If the matter field is taken so as to describe the interaction of electromagnetic fields with the Dirac electron given by the four-component Dirac spinor field ψ, the current and charge densities have form:[2]

$$\mathbf{J} = -e\psi^\dagger\boldsymbol{\alpha}\psi \quad \rho = -e\psi^\dagger\psi\,,$$

where $\boldsymbol{\alpha}$ are the first three Dirac matrices. Using this, we can re-write Maxwell's equations as:

Maxwell's equations (*QED*)

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \mu_0 e\psi^\dagger\boldsymbol{\alpha}\psi$$

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{1}{\varepsilon_0}e\psi^\dagger\psi$$

which is the form used in quantum electrodynamics.

My approach is to simply take the equation formally from QED with *spinor fields* involved and replace ∇ with the Atiyah-Singer-Dirac operator again removing Special relativity. There is substantial enough difference from Maxwell's physical theory that I do not have difficulties considering the result to be Ahmed-d'Alembert Law. It's not Maxwell's law *physically* at all. So we consider a spinor field that is formally similar to electromagnetic potential (A, φ).

I do not consider it unfair that the dynamical law be called after me as Ahmed-d'Alembert law rather than Maxwell law because there are significant physical differences that allow me to address protein folding and other problems using my law that are not possible with Maxwell on $\mathbf{R}^3$. The law is not what QED and Maxwell proposed physically. It's my law on four-sphere geometry that I described.

2. Why S4 Electromagnetic Law Should Not Be Credited to Maxwell

James Clerk Maxwell was a great physicist and he deserves fully the credit for his work for a *different paradigm of physics* than the one I am establishing. His *realm* is $\mathbf{R}^3$. In fact his realm is so much $\mathbf{R}^3$ that his name is synonymous with electromagnetism for almost two centuries. Michelson-Morley experiments were specifically testing for luminiferous aether of his particular ideas of the medium within which light travels.

He *does not deserve credit* for dynamical laws I am proposing in a different paradigm than either the $\mathbf{R}^3$ paradigm of his or the established QED paradigm that is established today.

Therefore it does not matter whether formally my equations are even the same as some of his. The physics is not the same at all. I am a Scientific Revolutionary, like Copernicus, and like Galileo. I am not extending Maxwell's theory in any way he would have recognised. In particular it is not fair to me to call the dynamical law with James Clerk Maxwell's name. It's my law and I discovered the fundamental geometry of absolute space that would have been inconceivable to Maxwell.

Therefore I don't want S4 Electromagnetic Law, *regardless of its formal similarities to Maxwell law of* $\mathbf{R}^3$ to be called by his name. It is Ahmed-d'Alembert Law. I have earned this right by successes in showing evidence that four-sphere geometry is correct. The law is formulated for *spinor fields on four-sphere of fixed radius.* Maxwell did not do all this work; I did. I deserve to have S4 Electromagnetic Law called Ahmed-d'Alembert Law.

3. Peter Olver's Presentation Of d'Alembert's Wave Equation Solution

Let $\Box = \partial_t^2 - c^2\partial_x^2$. We consider the wave equation on $[0,\infty)\times\mathbf{R}$:

$$\Box u(t,x) = 0 \tag{1}$$

The theorem is that every solution of (1) on $[0,\infty)\times\mathbf{R}$ can be represented as a sum of two traveling waves

$$u(t,x) = p(x-ct) + q(x+ct)$$

where $p, q \in C^2(\mathbf{R})$. To see that arbitrary p produce solution, use the factorisation

$$\Box = (\partial_t - c\partial_x)(\partial_t + c\partial_x)$$

The factorisation is due to equality of mixed partials and constancy of c. For any $p \in C^2(\mathbf{R})$ we see $p(x-ct)$ solves

$$(\partial_t + c\partial_x)p(x-ct) = 0$$

and therefore $\Box p(x-ct) = 0$. Similarly we have the $(\partial_t - c\partial_x)q(x+tc) = 0$. The theorem is that *every* solution of (1) on the line is representable as a superposition in this way which requires some effort. This is d'Alembert's original argument from 1740s.

Let me tell you the way that I had learned about the wave equation, from Princeton years 1991-1995. I don't know where I learned it first, but Dym-McKean surely has something like this.

Let's solve the equation on a circle S^1. I use separation of variables in a basis expansion of $L^2[-\pi,\pi]$. Assume

$$u(t,x) = \sum_{n=-\infty}^{\infty} a_n(t)e^{inx}$$

Then ignore all problems of summability. The ignoring all summability issues is key to understanding any analysis at all. This is not sufficiently taught at all in the mathematics. So let me assure you, *analysis is primarily about ignoring all summability and other possible problems.* I have learned this the hard way. Never be careful in analysis otherwise you will be just a nervous wreck and not learn anything at all. Always *ignore all possible problems and just play fast and loose.* Then after you know something, then *pretend that you are extremely meticulous to fend off all the people who are closet red stapler guys and will ruin your life if they find any limit unjustified. Do that just to keep them off your case. Otherwise, never worry about a thing. You see, I am not right now teaching a course so I don't really give a damn anymore and tell you the truth. No one ever did any great mathematics worrying about justifying all sorts of little things. Let things blow up and make no sense, just keep going without worry.*

$$\Box u(t,x) = \sum_{n=-\infty}^{\infty} a_n''(t)e^{inx} - c^2 a_n(t)(in)^2 e^{inx}$$

Just without any justification go right to

$$e^{inx}(a_n''(t) + c^2n^2a_n(t)) = 0$$

So

$$a_n''(t) = (icn)^2 a_n(t)$$

which we solve without worry about boundary conditions to obtain

$$a_n(t) = c_n e^{icnt}$$

for some constants c_n *And so*

$$u(t,x) = \sum_{n=-\infty}^{\infty} c_n e^{icnt+inx}$$

And these solve the wave equation. This solution has a history too, but it was neither of d'Alembert nor of Euler. Let me recall the history. It might have been Joseph Louis Lagrange in 1759 who calculated the Fourier coefficients for vibrating string first.

This particular solution was exceedingly important for me to discover the true source of the observed redshift *as I used this approach on a four-sphere and then discovered a different relationship between wavelengths and frequency that was powerful enough to* predict *redshift slope without any expansion.*

4. Solution Of Wave Equation On Four-Sphere

Here I won't try to be rigorous at all because the substance is not there. You take the unit four-sphere, and suppose the eigenvalues of the Laplacian are $0 = \nu_0^2, \nu_1^2, \nu_2^2, \dots$. *What is different about spheres is that eigenspaces are no longer one-dimensional. Stein-Weiss* Fourier Analysis On Euclidean Spaces *has a nice account of spherical harmonics. Those are the eigenfunctions but that does not matter.*

The scalar wave equation in this case is

$$\Box u(t,x) = (\partial_t^2 - c^2 \Delta_{S^4}) u(t,x) = 0$$

I won't worry about boundary conditions. That's not interesting. What is interesting is that separation of variables leads and the same reasoning as the cirle leads to

$$u(t,x) = \sum_{n=0}^{\infty} \sum_{\alpha_n} c_{\alpha_n} e^{i\nu_n^2 t} \varphi_{n,\alpha_n}$$

where everything is the same as the circle case except you have eigenspaces with multiple dimensions so you need to consider the contributions from all of them indexed by α_n.

This is serious work in Science: The key to explaining the redshift is a careful understanding of the exact relationship of frequency and wavelength in this situation. *This is what I did for the first time successfully around 2012. I remember the day. I was ecstatic and walking around on Mission Street and Valencia because I knew that I had killed Expansionary Cosmology and Big Bang theory totally that day.*

You see, in 1929 Edwin Hubble published his empirical discovery of a linear relationship between distance and redshift. For whatever reasons, various people worked to accomodate the discovery. Between 1929 and 2012 no one in the world had produced a clear explanation of redshift before my geometric explanation in a four-sphere of radius $R = 3075.69$ *Mpc. That was strong. Of course once you have a clear and valid explanation of the redshift slope in a* static cosmological model, *it is stronger Science than the entire established Expansionary Cosmology and instantaneously you can dismiss Big Bang event 14 billion years ago.*

5. Digression: Why Did Cosmologists Believe That Redshift Implied Expansion IN The 1920s?

Now I will take a step away from mathematics and tell you about something more serious about Science, something in which I have a great deal of experience and I have maturity *here. You see for a Mathematician, the important issue is to ensure that the various statements are true and justified without the Mathematical World. What are the actual eigenvalues of the Laplacian of a four-sphere of radius R? Well virtually every mathematician will appreciate that when you scale the sphere $S^4(1)$, i.e. the unit sphere, to $S^4(R)$ where $R >> 1$ the eigenvalues of $\Delta_{S^4(R)}$ and those of $\Delta_{S^4(1)}$ will be related by $1/R^2$. Let $\nu_{n,R}^2$ be the eigenvalues of $S^4(R)$ and let ν_n be those of $S^4(1)$; then*

$$\nu_{n,R}^2 = \frac{\nu_n^2}{R^2}$$

Thus we can just find the mathematicians' works that carefully calculated these and find that the eigenvalues are

$$\nu_n^2 = n(n+3)$$

They can be found even here [4]. *Now let's think about this carefully. This is what I had to do myself, between 2008-2012, so I am telling you my own reasoning. Let's assume that absolute space is actually $S^4(R)$ and not $\mathbf{R}^3$. What do I expect? I expect that the in the actual universe the relationship between frequency ν and wavelength λ of actual light is not*

$$\lambda = c/\nu$$

Instead it is something different. Now the formula I have for the sperical harmonics tell us the form of the difference. I define the discrepancy function

$$\delta(k) = \frac{1}{k} - \frac{1}{\sqrt{k(k+3)}}$$

for $k \geq 1$, integral values. This seems small but it is not small at all at cosmological scales. *And that's what explained the redshift slope. These small disrepancies for various frequencies add up as the light traveled for millions of light years.*

In 1920s all of science assumed that

$$\lambda = c/\nu$$

was truth. And the formula is in the guts of every instrument and in the interpretation of every measurement of wavelength of various emission spectra and so on. The resolution offered was space is not static but expands. *That is totally wrong. I gave the world the first correct explanation, that there is a curved background geometry with curvature Λ, i.e. $S^4(R)$ and since all our instruments computed λ the wavelength using the formula*

$$\lambda = c/\nu$$

there was a distortion from the actual relationship between wavelength and frequency. Cosmologists came to a wrong consensus, and so expansion became the consensus view.

In fact the universe was always static and never expanding all along, for trillions of years in the past, and Big Bang Cosmology is wrong.

6. Ok Bill Gates Why Do You Have \$131 Billion While So Many White People Are Homeless In America?

I was hobo in New York and San Fran and I collected food for others, many white too. I shared my coffee and cigarettes. In San Fran, I took some white hobos to cafe and bought them latte and treated them well. If you think that whites are so superior, why don't you resolve the problem of white homeless people in America?

You're a selfish stupid illiterate liar and total cunt Bill Gates. Your "whites are superior" are not worth a dime. You don't do all that much for white people.

7. Peter Olver's Valuable Fourier Series Convergence Theorem

I really like Peter Olver's decision to address piecewise $C^1[-\pi,\pi]$ *functions with only a finite set of jump discontinuities. I will tell you why. You see, for me a function is a* smooth *function. That's right, for me, in my mind, a function is a smooth function with maybe a little bit of problems somewhere. That's the part where my mind ignores it. I wave my hand dismissively about those things: "Who cares? Who cares? Epsilon measure, small part where things are bad. We don't care about that. Now where were we with that Taylor expansion with 25 terms right there?" See that's Zulf right there. Zulf does do some jump discontinuities. I work with sample paths of stochastic processes too, but I don't actually think of them as genuine functions. They are noisy fizz. No one in their sane mind actually invites these fizzy noise to their living rooms and gives them any refreshments and various tea and biscuits. They are noisy fizz. They might be functions technically but they are not presentable at all. What would my family say? What about my lovers? "Zulf, how could you do this to us? You brought fizzy noise and you told us you'd bring a function by? What happened to the derivatives? This is so horrible!"*

The unit step function is

$$s(x) = 1_{[0,\infty)}(x)$$

Let's calculate its Fourier series on $[-\pi,\pi]$.

$$c_n = \int_{-\pi}^{\pi} e^{-inx} s(x) dx$$

That's just

$$c_n = \int_0^{\pi} e^{-inx} dx = (-in)^{-1} e^{-inx}|_0^{\pi} = (-in)^{-1}[e^{-in\pi} - 1]$$

Let's see if we can compute this thing at all. It should be

$$e^{-in\pi} = (e^{i\pi})^{-n} = (-1)^n$$

This is the Euler formula $e^{i\pi} = -1$. *I always forget that when ordinary people see this they think it is mystical with all manner of transcendental numbers and imaginary numbers involved. Little do they know that if they do enough series expansions, all the mysticism will be replaced with joy that something is not left to approximation by iterative schemes in the computer and* true inscrutibility *which are all sorts of numbers like* 4.6739×10^{-2}. *See these types of numbers may look innocuous but they require you to pay attention to all manner of issues of whether the code had a bug and whether they are small or big; that's hard labour.*

Good. Olver has an account of Jean-Baptiste Joseph Fourier and how he declared Fourier series in 1800 studying heat propagation. His claim was arbitrary functions could be represented by Fourier series. This is extremely important for me, since *this is precise what is going on in the universe as well, that there is quantisation of energy and localisation of particles* because the universe is a homogeneous four-sphere. *Now it is a wonder whether Max Planck could have just used Ferdinand Mehler's work of 1866 on spherical harmonics expansions. For whatever reasons, in 1900, flat empty space was set into religious faith, and it had been even in 2000. In fact I do not know if anyone before me had made significant effort at all to ensure that that* empty space has curvature Λ *is promoted in Science. Fourier's original and prophetic claims about Fourier series were not accepted by the leading mathematicians of his time.*

The relationship of smoothness of functions and decay of Fourier coefficients is most important for Four-Sphere Theory.

8. What Is Wrong With Quantum Mechanics?

In order to understand deeply what is wrong with quantum mechanics, I think you will have to compare with what quantum mechanics was doing about phenomena compared to four-sphere theory. Quantum mechanics starts with two features that are seriously flawed – as models of Nature, and of course that is part of set of convictions that has driven me for more than a decade, so it is important to realise that my bias here is dyed-in-the-wool. I will not think you have any understanding of Nature whatever if you disagree with me here. I do not play this game of pretense of objectivity. Quantum mechanics, like humanity since Euclid, had taken for granted that space is $\mathbf{R}^3$ *and this is* totally wrong *from my viewpoint. Empty space is curved in the actual universe. Second, quantum mechanics introduced probabilistic ideas of where particles exist. This is, as most people in physics know, the interpretation of Max Born of Schroedinger wavefunctions.*

I will repeat that these wavefunctions *and these* quantum fields *on* $\mathbf{R}^3$ *are totally wrong. Spinor fields on* $S^4(R = 3075.69 Mpc)$ *are the only right objects of physics, and there are infinite distance bewteen these.*

9. Zonal Harmonics Are Exact Particle Profiles In Nature

With the right point of view we can see the confusions in quantum theory clearly. The only possible localised objects in nature at least above $\delta = 10^{-15}$ *cm scale that are natural have* exact descriptions *with high frequency zonal harmonics. They are themselves not particles but they tell us about what the profiles are spatially for particles.*

When you appreciate this deeply, then it becomes easier to see the confusions in quantum mechanics about the status of objects. Nature does not have pure point particles at all. An electron is localised but its support is the entire cosmos. The decay is exactly calculable from just the (a) the energy level of the particle, and (b) the radius $R = 3075.69$ *Mpc of the universe. The entire universe has these sorts of particles. The profile of an electron is not probabilistic at all. The dynamics of spinor fields is always deterministic and there is no time deformation possible. There is no space deformation possible. This will put all of macroscopic physics on solid accurate ground that is mathematically perfectly coherent and this will* always produce better match to experiments than quantum field theory and general relativity and expansionary cosmology. *This is a significant advance to human understanding of Nature with precision that cannot be imagined by the founders of these other theories.*

References

[1] Peter J. Olver, *Introduction To Partial Differential Equations*, Springer, 2014
[2] https://en.wikipedia.org/wiki/Hearing_the_shape_of_a_drum
[3] https://en.wikipedia.org/wiki/Mathematical_descriptions_of_the_electromagnetic_field
[4] https://en.wikipedia.org/wiki/Spherical_harmonics#Higher_dimensions