- $a \to b$

Suppose T is invertible

- Consider linear combination of columns of $\mathcal{M}(T)$:

  given $c_1, \dots, c_n$, $c_1 \mathcal{M}(T)_{\cdot,1} + \dots + c_n \mathcal{M}(T)_{\cdot,n} = 0$.

- This is equivalent to

$$\mathcal{M}(T)\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = 0$$

- If the only choice of $c_1, \dots c_n$ that yields the above identity is $c_1 = \dots = c_n = 0$, then the columns are independent and we are done.

- Suppose at least one $c \neq 0$. Let $v = c_1 v_1 + \dots c_n v_n$, thus $v \neq 0$ and $\mathcal{M}(v) \neq 0$

- Then $\mathcal{M}(T)\mathcal{M}(v) = 0$ and $v \neq 0$

- This is equivalent to $\mathcal{M}(Tv) = 0$ and $v \neq 0$

- Since T is invertible, and $v \neq 0$, $Tv \neq 0 \in V$ *

  Let $Tv = v'$

- $\mathcal{M}(v') = 0 \iff v' = c_1' v_1 + \dots + c_n' v_n$ where $c_1' = \dots = c_n' = 0$

  A contradiction since v' cannot be 0