Proof

- suppose $T' = 0$

ie given any $\varphi \in W'$

$$T'(\varphi) = \varphi \circ T = 0$$

- Suppose $T \neq 0$, ie $\exists\, v$ such that $Tv = w \neq 0$

- since W is finite dimensional,

$$w = c_1 w_1 + \cdots + c_m w_m$$

where $w_1, \dots, w_m$ is the basis and $c_1, \dots, c_m$ is some scalar

- since $w \neq 0$, at least one $c_i \neq 0$. pick φ such that $\varphi(w_i) \neq 0$, which must exist by 3.98

by definition   by basis definition

- $$0 = (T'\varphi)v = \varphi \circ Tv = \varphi(w) = \varphi(c_1 w_1 + \cdots c_m w_m)$$
$$= c_1 \varphi(w_1) + \cdots + c_m \varphi(w_m)$$

by hypothesis   by hypothesis   by linearity

- $\varphi(w_i) = 0 \Rightarrow$ contradict. Thus $T = 0$.