Proof.

- Let $u \in V$ and $u \notin W$.
- Let $w_1, \ldots, w_m$ be basis of W

Claim: $u, w_1, \ldots, w_m$ is linearly independent.

- Suppose $c_0 u + c_1 w_1 + \cdots + c_m w_m = 0$ — (1)

$$c_1 w_1 + \cdots + c_m w_m = -c_0 u$$

- Since LHS is a combination of $w_1, \ldots, w_m$, it is an element of W. Meanwhile, $-c_0 u$ is not element of W unless $-c_0 = 0$.

- Thus, (1) is equal to $\cancel{c_0 u} + c_1 w_1 + \cdots + c_m w_m = 0$

Thus, independent since $w_1, \ldots, w_m$ is independent.

- Next, we extend $u, w_1, \ldots, w_m$ to basis of V,

$u, w_1, \ldots, w_m, w_{m+1}, \ldots, w_n$. The dual basis of of this basis is $\phi_u, \phi_{w_1}, \ldots, \phi_{w_m}, \phi_{w_{m+1}}, \ldots, \phi_{w_n}$. Note $\phi_u(u) = 1$ but $\phi_u(w_i) = 0$ for all $w_1, \ldots, w_m$

- Thus $\phi_u \in W^0$ but $\phi_u \notin U^0$, contradicting hypothesis