- $v_i$ is a linear combination of $v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_m$

$$v_i = \frac{-\alpha_1 v_1 - \cdots - \alpha_{i-1} v_{i-1} - \alpha_{i+1} v_{i+1} - \cdots - \alpha_m v_m}{\alpha_i}$$

$$= \lambda_1 v_1 + \cdots + \lambda_{i-1} v_{i-1} + \lambda_{i+1} v_{i+1} + \cdots + \lambda_m v_m \quad — ②$$

- $\varphi(v_i) = 1$ from ①

$= \varphi(\lambda_1 v_1 + \cdots + \lambda_{i-1} v_{i-1} + \lambda_{i+1} v_{i+1} + \cdots \lambda_m v_m)$

from ②

$= \lambda_1 \varphi(v_1) + \cdots + \lambda_{i-1} \varphi(v_{i-1}) + \lambda_{i+1} \varphi(v_{i+1})$

$+ \cdots \lambda_m \varphi(v_m)$ by linearity

$= 0$ from ①

- Not possible. $\Gamma$ is not surjective.