

c) version 2.

$$\{ T \in \mathcal{L}(V, W) : Tu = 0 \text{ for every } u \in U \} \subseteq \text{range } \Gamma$$

▶ let $T \in \{ T \in \mathcal{L}(V, W) : Tu = 0 \text{ for every } u \in U \}$.

want to show $T \in \text{range } \Gamma$ i.e. there exists

$$\begin{aligned} & \Gamma : \mathcal{L}(V/U, W) \rightarrow \mathcal{L}(V, W) \text{ by} \\ & \Gamma(s) = s \circ \pi. \end{aligned} \quad \begin{aligned} & s \in \mathcal{L}(V/U, W) \\ & \text{s.t. } \Gamma(s) \\ & = s \circ \pi = T. \end{aligned}$$

▶ By 3E18 we know that given any

$$T \in \mathcal{L}(V, W), \quad Tu = 0 \text{ for all } u \in U,$$

there exist s such that $s \circ \pi = T$.

▶ Thus $T \in \text{range } \Gamma$,

$$\text{range } \Gamma \subseteq \{ T \in \mathcal{L}(V, W) : Tu = 0 \text{ for every } u \in U \}$$

▶ given $T \in \text{range } \Gamma$, $T = s \circ \pi$ we want to show

$T \in \mathcal{L}(V, W)$ (true by definition)

and $Tu = 0$ for every $u \in U$. This is true

by 3E18, as such T exists iff $U \subseteq \text{null } T$