

Proof ▶ Consider any list with arbitrary length  $n$  consisting of elements of  $\mathbb{F}^\infty / U$ .

For each element in this list, it takes the form of

$$(v, (x_1, x_2, \dots)) , v \in \mathbb{F}^\infty \text{ and}$$

$$(x_1, x_2, \dots) \in U \text{ where}$$

finitely many  $m$  of  $x_i$ 's are nonzero.

▶ Call  $m$  the degree of this element.

▶ Then every element in the span of this list has degree at most  $nm$ .

▶ Thus element of  $\mathbb{F}^\infty / U$  of degree  $nm+1$  is not in the span of our list.

Hence no list spans  $\mathbb{F}^\infty / U$