

## Proof

Suppose  $\text{rank } A = 1$

▶ Since  $\text{rank } A = 1$ , columns 2, 3, ..., n of matrix A are multiple of column 1.

▶ let the first column of A be  $A_1 = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}$ ,  
for some  $c_1, \dots, c_m \in \mathbb{F}$

▶ Then columns 2, 3, ..., n are

$$A_2 = d_2 A_1; \quad A_3 = d_3 A_1, \dots, \quad A_n = d_n A_1$$

for some  $d_2, \dots, d_n \in \mathbb{F}^n$

▶ Thus 
$$A = \begin{bmatrix} A_1 & d_2 A_1 & \dots & d_n A_1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \cdot c_1 & d_2 c_1 & \dots & d_n c_1 \\ \vdots & \vdots & \dots & \vdots \\ 1 \cdot c_m & d_2 c_m & \dots & d_n c_m \end{bmatrix}$$