Proof:

- Suppose $\exists\ S : V/U \to W$ such that $T = S \circ \pi : V \xrightarrow{\pi} V/U \xrightarrow{S} W$

- Let $u \in U$. Then $T(u) = S \circ \pi\,(u)$

$$= S(U) = S(0_{V/W})$$

$$= 0_W \quad \text{by 3-11 (Axler)}$$

Thus $U \subseteq \text{null } T$

- Suppose $U \subset \text{null } T$.

- Define $S : V/U \to W$ by

$$S(v + U) = Tv$$

S is linear (can be shown satisfying addivity and homogeneity)

- For all $v \in V$, $S \circ \pi(v) = S(v + U)$

$$= Tv$$

Thus $S \circ \pi = T$