

Proof :

►  $\dim V/U = 1 \iff$  isomorphic to  $\mathbb{F}$   
(by 3.59 Axler)

invertible  
There exists  $\wedge$  linear map  $\pi_1 : V/U \rightarrow \mathbb{F}$

(by definition of  
isomorphism, 3.58  
Axler)

► Meanwhile, there exists a quotient map

$$\pi_2 : V \rightarrow V/U$$

$\pi_2(v) = v + U$ , which is linear  
(see 3.88)

and  $\text{null } \pi_2 = U$

► Thus, define  $\pi = \pi_1 \circ \pi_2 : V \xrightarrow{\pi_2} V/U \xrightarrow{\pi_1} \mathbb{F}$

► We need to show that  $\pi$  is linear and  
 $\text{null } \pi = U$