

Proof

a) Independence :

$$\text{let } 0 = a_0(1) + a_1(x-5) + \dots + a_m(x-5)^m$$

→ Since LHS has no x^m , but RHS has one x^m term,

$$a_m = 0.$$

⇒ Since LHS has no x^{m-1} , but RHS has one x^{m-1} term,

$$a_{m-1} = 0$$

→ Proceed, the same way, and we'll obtain

$$a_0 = a_1 = \dots = a_m = 0.$$

Since $\dim \mathcal{P}_m(\mathbb{R}) = m+1$,

and the list above has length $m+1$,

it is a basis. (by 2.39 Axiom)