Proof

a) Independence:

let $0 = a_0(1) + a_1(x-5) + \dots + a_m(x-5)^m$

→ Since LHS has no $x^m$, but RHS has one $x^m$ term,

$a_m = 0$.

→ Since LHS has no $x^{m-1}$, but RHS has one $x^{m-1}$ term,

$a_{m-1} = 0$

→ Proceeding the same way, and we'll obtain

$a_0 = a_1 = \dots = a_m = 0$.

Since $\dim \mathcal{P}_m(\mathbb{R}) = m+1$,

and the list above has length $m+1$,

it is a basis. (by 2.39 Axler).

b) The dual basis of the basis $1, x-5, \ldots, (x-5)^m$ of $\mathcal{P}_m(\mathbb{R})$ is $\varphi_0, \varphi_1, \ldots, \varphi_m$, where $\varphi_j(p) = \dfrac{p^{(j)}(5)}{j!}$. Here $p^{(j)}$ denotes the $j^{th}$ derivative

Similar procedure to previous question.