Proof:

Let $\varphi \in \mathcal{L}(V, \mathbb{F})$.

Then range φ is a subspace of $\mathbb{F}$. (3.19 Axler)

Thus $\dim(\text{range } \varphi) \leq \dim \mathbb{F} = 1$ (2.38 Axler)

ie. dim range $\varphi = 1$ or 0

ie range $\varphi = \mathbb{F}$ or range $\varphi = \{0\}$.