Proof:

Let $f: \mathcal{L}(V,W) \to \mathcal{L}(W',V')$

defined by $f(T) = T'$.

show linear:

Let $T_1, T_2 \in \mathcal{L}(V,W)$ and $\varphi \in W'$,

- $f(T_1+T_2)(\varphi) = (T_1+T_2)'\varphi = \varphi(T_1+T_2)$ by definition

  $= \varphi T_1 + \varphi T_2$ by distributivity.

  $= T_1'(\varphi) + T_2'(\varphi)$

  $= f(T_1)(\varphi) + f(T_2)(\varphi)$

- Given $\alpha \in \mathbb{F}$,

  $f(\alpha T)\varphi = (\alpha T)'\varphi = \varphi a T$

  $= a \varphi T$ by scalar commutability.

  $= a T'(\varphi)$

  $= a f(T)(\varphi)$