

▶  $\pi$  is linear

composition of two linear maps

is again linear, we are done.

(true by page 56 Axler, line 3-4)

✓

▶  $U \subseteq \text{null } \pi$

$$\pi(U) = \pi_1 \circ \pi_2(U)$$

$$= \pi_1(U) \quad (\text{by definition of } \pi_2)$$

$$= 0 \quad (\text{need to define } \pi_1)$$

Need to construct  $\pi_1 : V/U \rightarrow U$  such that

$\pi_1$  is linear and  $\pi_1(U) = 0$

1 Since  $\dim V/U \leq 1$ , there exists basis

$v_1 + U$  of  $V/U$ , such that  $v_1 \notin U$ .

2 Define  $\pi_1 : V/U \rightarrow \mathbb{F}$  by :

$$\pi_1(\lambda v_1 + U) = \lambda, \quad \lambda \in \mathbb{F}.$$

3  $\pi_1$  is linear : let  $x, y \in V/U$