

Proof

▶ dual map of identity map on V : if $i \in \mathcal{L}(V, V)$, then the dual map of i is the linear map $i' \in \mathcal{L}(V', V')$ defined by $i'(\varphi) = \varphi \circ i$ for $\varphi \in V'$ (by definition)

$$\begin{array}{ccccc} V & \xrightarrow{i} & V & \xrightarrow{\varphi} & \mathbb{F} \\ & \searrow & & \nearrow & \\ & & \varphi \circ i & & \end{array}$$

▶ For any $\varphi \in V' = \mathcal{L}(V, \mathbb{F})$, $i'(\varphi) = \varphi \circ i$
 $= \varphi$