

Moreover,  $U + W \subseteq W$  by 1.39 Axler.

Therefore  $U + W = W$ .

► Next, we show  $U \cap W = \{0\}$ .

Let  $x \in U \cap W$ .

Since  $x \in W$ ,  $x = b_1 v_1 + \dots + b_m v_m$

Moreover, observe that  $x - 0 = x \in U$ .

By 3.85,  $x + U = 0 + U$

Thus,  $0 + U = x + U$

$$= (b_1 v_1 + \dots + b_m v_m) + U$$

$$= (b_1 v_1 + U) + \dots + (b_m v_m + U)$$

since  $v_1 + U, \dots, v_m + U$  is linearly indep.,

$$b_1 = \dots = b_m = 0 \Rightarrow x = 0$$

$$\Rightarrow U + W \subseteq \{0\}$$