

3.19. Suppose V is finite dimensional and U is a subspace of V . show that $U = V$ if and only if $U^\circ = \{0\}$

► Suppose $U = V$

$$\begin{aligned} \text{By 3.10b, } \dim U^\circ &= \dim V - \dim U \\ &= 0 \quad \checkmark \end{aligned}$$

► Suppose $U^\circ = \{0\}$

$$\dim U^\circ = 0$$

$$\text{By 3.10b, } \dim V = \dim U$$

Since U is a subspace of V , $U = V \quad \checkmark$