contradicting that

$v_1 + U, \ldots, v_m + U$ is independent

(therefore a basis) in V/U.

- Thus $v_1, \ldots, v_m$ is a basis of W.

$\Rightarrow \dim(W) = m = \dim(V/U)$

- Next, we show $V = W \oplus U$

Given any v,

$$v = \left(v - (a_1 v_1 + \cdots + a_m v_m)\right) + (a_1 v_1 + \cdots + a_m v_m)$$

$\in U + W$

by $*$ and $**$

Thus $V \subseteq U + W$.