Proof:

$(U+W)^0 \subset U^0 \cap W^0$

Suppose $\varphi \in (U+W)^0 = \{\varphi \in V' : \varphi(u+w) = 0$ for all $u, w \in W\}$

Since $0 \in W$, $\varphi(u+0) = \varphi(u) = 0$ for all $u \in U$.

Similarly, since $0 \in U$, $\varphi(0+w) = \varphi(w) = 0$ for all $w \in W$.

Thus $\varphi \in U^0$ and $\varphi \in W^0 \Rightarrow \varphi \in U^0 \cap W^0$.

$\Rightarrow (U+W)^0 \subset U^0 \cap W^0$

$U^0 \cap W^0 \subset (U+W)^0$

Suppose $\varphi \in U^0 \cap W^0$

$\varphi(u) = 0$ for all $u \in U$ and $\varphi(w) = 0$ for all $w \in W$.

Let $u+w \in U+W$ $\therefore$ $\varphi(u+w)$

$= \varphi(u) + \varphi(w)$ by linearity

$= 0$

$\Rightarrow \varphi \in (U+W)^0 \Rightarrow U^0 \cap W^0 \subset (U+W)^0$