$U^0 \subset \operatorname{span}(\varphi_{m+1}, \dots \varphi_n)$.

Let $\varphi \in U^0$

$$\varphi = c_1\varphi_1 + \dots + c_m\varphi_m + c_{m+1}\varphi_{m+1} + \dots + c_n\varphi_n$$

for some $c_1, \dots, c_n \in \mathbb{F}$.

Since $u_1 \in U$, and $\varphi \in U^0$:

$$0 = \varphi(u_1) = (c_1\varphi_1 + \dots + c_m\varphi_m + c_{m+1}\varphi_{m+1} + \dots + c_n\varphi_n)(u_1)$$
$$= c_1$$

Similar, $c_2 = \dots = c_m = 0$

Thus $\varphi = c_{m+1}\varphi_{m+1} + \dots + c_n\varphi_n \in \operatorname{span}(\varphi_{m+1}, \dots, \varphi_n)$

$\boxed{\text{We will show } \varphi_{m+1}, \dots, \varphi_n \text{ is independent}}$

Suppose $c_{m+1}, \dots, c_n \in \mathbb{F}$ and

$$c_{m+1}\varphi_{m+1} + \dots + c_n\varphi_n = 0$$

Let $c_1 = \dots = c_m = 0$. Thus,

$$c_1\varphi_1 + \dots + c_m\varphi_m + c_{m+1}\varphi_{m+1} + \dots + c_n\varphi_n = 0$$

Thus all c's must be zero since $\varphi_1, \dots \varphi_n$ independent

$$\boxed{\underbrace{\varphi_1, \dots, \varphi_m}_{\dim U}, \underbrace{\varphi_{m+1}, \dots, \varphi_n}_{\dim U^0}}$$