Proof

- By 3.91: $V / \text{null}(\varphi) = \text{range } \varphi$
- we know that range φ is a subspace of $\mathbb{F}$ which has dimension 1.
- Thus range φ's dimension is either 1 or 0 (since a dimension of subspace cannot exceed that of the vector space)
- If $\dim(\text{range } \varphi) = 0$, then $\varphi = 0$. But this is not possible since $\varphi \neq 0$ by problem hypothesis.

Thus $\dim(\text{range } \varphi) = 1$