Proof

- Suppose $U = \{0\}$

  Then given any $\varphi \in V'$, $\varphi(u) = 0$ by 3.11.

  Thus $V' \subset U^0$

  But $U^0 \subset V'$ since U^0 is a subspace of V'.

  $\therefore U^0 = V'$

- suppose $U^0 = V'$

  Let $u_1, \dots, u_m$ be the basis

  By contradiction, suppose $U \neq \{0\}$

  Then at least one $u_i \neq 0$

  Then $\varphi_i(u_i) = 1 \neq 0$

  But $\varphi_i \in V' = U^0$, hence $\varphi_i(u_i) = 0$ by hypothesis.

  Contradiction.