

Proof

a) Given any $\psi \in \mathcal{P}(\mathbb{R})'$, $p \in \mathcal{P}(\mathbb{R})$, $x \in \mathbb{R}$

$$\begin{aligned} T'(\psi)p &= \psi \circ T_p = \psi \circ (x^2 p(x) + p''(x)) \\ &= 2x p(x) + x^2 p'(x) + p'''(x) \Big|_{x=4} \\ &= 8p(4) + 16p'(4) + p'''(4) \end{aligned}$$

b) $(T'(\psi_1))x^3$

$$\begin{aligned} \psi T(x^3) &= \psi \circ (x^2 x^3 + 6x) \\ &= \psi \circ (x^5 + 6x) \\ &= \int_0^1 x^5 + 6x \, dx \\ &= \left[\frac{x^6}{6} + \frac{6x^2}{2} + c \right]_0^1 \\ &= \frac{1}{6} + \frac{6}{2} \\ &= \frac{1}{6} + 3 = \frac{19}{6} \quad \square \end{aligned}$$