Proof

$U^0 + W^0 \subset (U \cap W)^0$

Let $\varphi + \emptyset \in U^0 + W^0$

Given any $x \in (U \cap W)$

$$(\varphi + w)x = \varphi(x) + w(x)$$
$$= 0 + 0$$

thus $\varphi + w \in (U + W)^0$

show $\dim(U^0 + W^0) = (U \cap W)^0$

$\dim(U^0 + W^0) = (\dim U^0) + (\dim W^0) - (\dim(U^0 \cap W^0))$

(by 2.43. U^0 is subspace of finite-dimensional V'

W^0 is subspace of finite-dimensional V')

$$= (\dim V - \dim U) + (\dim V - \dim W)$$
$$- (\dim V - \dim(U + W))$$ (by 3.106)