Let $s \in \mathbb{F}$, then

$$f(s\varphi) = (f_1'(s\varphi), \ldots, f_m'(s\varphi))$$
$$= (s\varphi f_1, \ldots, s\varphi f_m)$$
$$= s(\varphi f_1, \ldots, \varphi f_m)$$
$$= s(f_1'\varphi, \ldots, f_m'\varphi)$$
$$= sf(\varphi) \checkmark$$

Next, show $\emptyset$ is injective

Let $f(\varphi) = 0$

$$(f_1'(\varphi), \ldots, f_m'(\varphi)) = 0$$

$$(\varphi f_1, \ldots, \varphi f_m) = 0$$

Thus for any $v_i \in V_i$ and $f_i \in \mathcal{L}(V_i, V_1 \times \cdots \times V_m)$

$$\varphi f_i v_i = \varphi(f_i(v_i)) = 0$$

$\Rightarrow \varphi$ is a zero map.

Next, show $\emptyset$ is surjective

Let $(\eta_1, \ldots, \eta_m) \in \mathcal{L}(V_1, \mathbb{F}) \times \cdots \times \mathcal{L}(V_m, \mathbb{F})$