- Γ is surjective $\Rightarrow$ $v_1, \ldots, v_m$ is linearly independent

  - Suppose $v_1, \ldots, v_m$ is linearly dependent

    There exist $\alpha_1, \ldots, \alpha_m$, not all zero such that

    $0 = \alpha_1 v_1 + \cdots + \alpha_m v_m$

    Suppose $\alpha_i \neq 0$, thus v_i can be expressed as a linear combination of $v_1, \ldots, v_{i-1}, v_{i+1}, \ldots v_m$.

  - claim: $(0, \ldots, 1, \ldots, 0) \in \mathbb{F}^m$, where the element is zero everywhere, except 1 is in the i'th position, is not in range Γ.

    ↳ Proof:

    Suppose there is φ such that

  - $\Gamma(\varphi) = (\varphi(v_1), \ldots, \varphi(v_i), \ldots \varphi(v_m)) = (0, \ldots, 1, \ldots, 0)$ — (1)