Proof: Let $u_1, \dots, u_m$ be basis of U.

Since $U \not\subset V$,

we can extend $u_1, \dots, u_m, u_{m+1}, \dots, u_n$

into a list of basis of V.

Define dual basis of this list as

$\varphi_1, \dots, \varphi_m, \varphi_{m+1}, \dots \varphi_n$ where

$$\varphi_j(u_k) = \begin{cases} 1 & \text{if } k = j, \\ 0 & \text{if } k \neq j. \end{cases}$$

By 3.98 Axler, the above dual basis is a basis V', $V' = \text{span}(\varphi_1, \dots, \varphi_n)$.

Take $\varphi \in V'$ such that

$$\varphi = 0\varphi_1 + \dots + 0\varphi_m + a_1\varphi_{m+1} + \dots + a_n\varphi_n$$

where not all $a_1, \dots, a_n$ are zero.

Thus $\varphi(u) = 0$ for all $u \in U$, yet $\varphi \neq 0$