$$
\begin{tikzcd}
 & \pi_i \arrow[rrd, bend left] & & \\
V_i \arrow[ru] \arrow[r, "f_i"] \arrow[rd] & V_1 \times \cdots \times V_m \arrow[rr, "\varphi"] & & \mathbb{F} \\
 & \varphi \circ f_i \arrow[rru] & &
\end{tikzcd}
$$

- we want to show for each π_i

  there exist $\varphi \in \mathcal{L}(V_1 \times \cdots \times V_m, \mathbb{F})$

  such that $\varphi \circ f_i = \pi_i$

For arbitrary $v_1 \in V_1, v_2 \in V_2, \ldots, v_m \in V_m$,

denote $\pi_1(v_1) = s_1,\ \pi_2(v_2) = s_2, \ldots \ \pi_m(v_m) = s_m$

- Select $\varphi \in \mathcal{L}(V_1 \times \cdots \times V_m, \mathbb{F})$ such that

$$\varphi(v_1, 0, \ldots 0) = s_1$$
$$\varphi(0, v_2 \ldots 0) = s_2$$
$$\vdots$$
$$\varphi(0, 0, \ldots v_m) = s_m$$

- Define f_i as $*$

- Given any $v_i \in V_i$,

$$\varphi \circ f_i(v_i) = \varphi(0, \ldots v_i, \ldots 0) = s_i = \pi_i(v_i)$$

$\Rightarrow \varphi f_i = \pi_i$, since v_i is arbitrary