Proof

- **Big idea:** Let $\varphi \in (V_1 \times \cdots \times V_m)'$

$$= \mathcal{L}(V_1 \times \cdots \times V_m, \mathbb{F})$$

$$V_1 \times \cdots \times V_m \xrightarrow{\varphi} \mathbb{F}$$

- We are interested to see how φ can correspond to

$$\varphi_i \in V_i' = \mathcal{L}(V_i, \mathbb{F}), \quad V_i \xrightarrow{\varphi_i} \mathbb{F}$$

for each $V_1, \ldots, V_m$

- Thus for each $V_1, \ldots, V_m$, if we can find a linear mapping f_i such that $f_i : V_i \to V_1 \times \cdots \times V_m$,

$$\begin{tikzcd}
 & \varphi_i \arrow[dr] & \\
V_i \arrow[ur] \arrow[r, "f_i"] \arrow[dr] & V_1 \times \cdots \times V_m \arrow[r, "\varphi"] & \mathbb{F} \\
 & \varphi \circ f_i \arrow[ur] &
\end{tikzcd}$$

we can make the linkage between $\mathcal{L}(V_i, \mathbb{F})$ and $\mathcal{L}(V_1, \ldots, V_m, \mathbb{F})$