

Proof:

$$\text{Let } f: \mathcal{L}(V, W) \rightarrow \mathcal{L}(W', V')$$

$$\text{defined by } f(T) = T'.$$

show linear:

$$\text{Let } T_1, T_2 \in \mathcal{L}(V, W) \text{ and } v \in W',$$

$$\begin{aligned} \triangleright f(T_1 + T_2)(v) &= (T_1 + T_2)'v = v(T_1 + T_2) \quad \text{by definition} \\ &= vT_1 + vT_2 \quad \text{by distributivity.} \\ &= T_1'(v) + T_2'(v) \\ &= f(T_1)(v) + f(T_2)(v) \end{aligned}$$

$$\triangleright \text{Given } \alpha \in \mathbb{F},$$

$$\begin{aligned} f(\alpha T)(v) &= (\alpha T)'v = v \alpha T \\ &= \alpha vT \quad \text{by scalar commutativity.} \\ &= \alpha T'(v) \\ &= \alpha f(T)(v) \end{aligned}$$