- $\Gamma(\varphi) = (\varphi(v_1), \dots, \varphi(v_m)) = 0$

  but $\varphi \neq 0$

- Γ is not injective

▶ $v_1, \dots, v_m$ is linearly independent $\Rightarrow$ Γ is surjective.

- Let $(a_1, \dots, a_m) \in \mathbb{F}^m$
- Extend $v_1, \dots, v_m$ into basis of V.
- By 3.5 Axler, there exists $\varphi : V \to \mathbb{F}$

  such that

  $\varphi(v_i) = a_i$ for $i = 1, \dots, m$ and

  $\varphi(v_i) = 0$ for $i > m$.

- Thus $\Gamma(\varphi) = (\varphi(v_1), \dots, \varphi(v_m))$

  $= (a_1, \dots, a_m)$

- Γ is surjective.