Proof

- Need to show that

$$\varphi_j(x^k) = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$$

- Suppose $j = k$

$$\varphi_j(x^k) = \frac{\left.\dfrac{d^j}{dx^j} x^k \right|_{x=0}}{j!} = \frac{\left.\dfrac{d^j}{dx^j} x^j \right|_{x=0}}{j!}$$

$$= \frac{j(j-1)\cdots(1)\, x^0 \big|_{x=0}}{j!}$$

$$= \frac{j!}{j!} = 1$$

- suppose $j > k$

$$\varphi_j(x^k) = \frac{d^j}{dx^j} x^k \Big|_{x=0}$$

$$= \frac{k(k-1)\dots(1)(0)}{j!}$$

$$= 0$$

- suppose $j < k$

$$\varphi_j(x^k) = \frac{\frac{d^j}{dx^j} x^k \Big|_{x=0}}{j!}$$

$$= \frac{k(k-1)\dots(k-j+1)\, x^{k-j} \Big|_{x=0}}{j!}$$

$$= \frac{k(k-1)\dots(k-j+1)\, 0}{j!}$$

$$= 0$$