3F20 Suppose U and W are subsets of V with $U \subset W$. Prove that $W^\circ \subset U^\circ$.

Proof

- Let $\varphi \in W^\circ \Rightarrow \varphi \in V'$ and $\varphi(w) = 0$ for all $w \in W$
- Given any $u \in U$, $u \in W$ (since $U \subset W$) (by definition of W°)

Thus $\varphi(u)$ for all $u \in U$ (by definition of W°)

- Note that:

$$U^\circ = \{ \phi \in V' : \phi(u) = 0 \text{ for all } u \}$$

Thus $\varphi \in U^\circ$ (by definition of U°)

$\Rightarrow W^\circ \subset U^\circ$