

## Proof

9) Generally :

If  $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ , then the dual map of  $T$  is

the linear map  $T' \in \mathcal{L}(\mathbb{R}^{2'}, \mathbb{R}^{3'})$  defined by

$$T'(\varphi) = \varphi \circ T \quad \text{for } \varphi \in \mathbb{R}^{2'}$$

Specifically, for any  $(x, y, z) \in \mathbb{R}^3$

$$\begin{aligned} T'(\varphi_1)(x, y, z) &= \varphi_1 \circ T(x, y, z) = \varphi_1(4x + 5y + 6z, 7x + 8y + 9z) \\ &= 4x + 5y + 6z \end{aligned}$$

$$\begin{aligned} T'(\varphi_2)(x, y, z) &= \varphi_2 \circ T(x, y, z) = \varphi_2(4x + 5y + 6z, 7x + 8y + 9z) \\ &= 7x + 8y + 9z \end{aligned}$$