- Thus letting $d_1 = 1$,

  $A_{j,k} = c_j d_k$ for

  every $j = 1, \dots, m$ and every $k = 1, \dots, n$.

Suppose there exist $(c_1, \dots, c_m) \in \mathbb{F}^m$ and $(d_1, \dots, d_n) \in \mathbb{F}^n$ such that $A_{j,k} = c_j d_k$

- $$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} c_1 d_1 & \cdots & c_1 d_n \\ \vdots & & \vdots \\ c_m d_1 & \cdots & c_m d_n \end{bmatrix}$$

Since $A \neq 0$, there is at least one of

$d_1, \dots, d_n$ that is not zero. WLOG, suppose $d_1 \neq 0$

- Let the first column be $A_1 = \begin{bmatrix} c_1 d_1 \\ \vdots \\ c_m d_1 \end{bmatrix}$

The second column $A_2 = \frac{d_1}{d_1} A_2 = \frac{d_1}{d_1} \begin{bmatrix} c_1 d_2 \\ \vdots \\ c_m d_2 \end{bmatrix} = \frac{d_2}{d_1} \begin{bmatrix} c_1 d_1 \\ \vdots \\ c_m d_1 \end{bmatrix} = \frac{d_2}{d_1} A_1$