- Define $f_i : V_i \to V_1 \times \cdots \times V_m$

  by $f_i(v_i) = (0, 0, \ldots, v_i, \ldots, 0, 0)$ $\ast$

- Define $\emptyset : \mathcal{L}(V_1, \ldots, V_m, \mathbb{F}) \to \mathcal{L}(V_1, \mathbb{F}) \times \cdots \times \mathcal{L}(V_m, \mathbb{F})$

  $$\emptyset(\varphi) : (f_1'\varphi, \ldots, f_m'\varphi)$$

Note that the right hand side is an element of $\mathcal{L}(V_1, \mathbb{F}) \times \cdots \times \mathcal{L}(V_m, \mathbb{F})$ since $f_i'\varphi = \varphi f_i$ takes an element of V_i to $\mathbb{F}$.

- Next, we show <u>that $\emptyset$ is linear</u>

  Given $\varphi, \lambda \in (V_1 \times \cdots \times V_m)'$

$$
\begin{aligned}
f(\varphi + \lambda) &= (f_1'(\varphi + \lambda), \ldots, f_m'(\varphi + \lambda)) \\
&= ((\varphi + \lambda) f_1, \ldots, (\varphi + \lambda) f_m) \\
&= (\varphi f_1 + \lambda f_1, \ldots, \varphi f_m + \lambda f_m) \\
&= (\varphi f_1, \ldots, \varphi f_m) + (\lambda f_1, \ldots, \lambda f_m) \\
&= (f_1'\varphi, \ldots, f_m'\varphi) + (f_1'\lambda, \ldots, f_m'\lambda) \\
&= f(\varphi) + f(\lambda) \quad \checkmark
\end{aligned}
$$