Proof

a) Given any $\varphi \in \mathcal{P}(\mathbb{R})'$, $p \in \mathcal{P}(\mathbb{R})$, $x \in \mathbb{R}$

$$T'(\varphi)p = \varphi \circ Tp = \varphi \circ (x^2 p(x) + p''(x))$$

$$= 2x\, p(x) + x^2 p'(x) + p'''(x) \Big|_{x=4}$$

$$= 8p(4) + 16p'(4) + p'''(4)$$

b) $(T'(\varphi))x^3$

$$\varphi T(x^3) = \varphi \circ (x^2 x^3 + 6x)$$

$$= \varphi \circ (x^5 + 6x)$$

$$= \int_0^1 x^5 + 6x \, dx$$

$$= \left[\frac{x^6}{6} + \frac{6x^2}{2} + c \right]_0^1$$

$$= \frac{1}{6} + \frac{6}{2}$$

$$= \frac{1}{6} + 3 = \frac{19}{6} \quad \blacksquare$$