

3F2. Give three distinct examples of linear functionals on $\mathbb{R}^{[0,1]}$.

1. Zero map

2. $\varphi(x) = -x$

$$\begin{aligned}\text{Given } x, y \in [0, 1], \quad \varphi(x+y) &= -(x+y) \\ &= -x + (-y) \\ &= \varphi(x) + \varphi(y) \quad \checkmark \\ \varphi(2x) &= -2x \\ &= 2(-x) \\ &= 2\varphi(x) \quad \checkmark\end{aligned}$$

3. $\varphi(x) = 2x$

$$\begin{aligned}\text{Given } x, y \in [0, 1] \Rightarrow \varphi(x+y) &= 2(x+y) = 2x + 2y \quad \checkmark \\ &= \varphi(x) + \varphi(y) \\ \varphi(2x) &= 2(2x) = 2(2x) = 2\varphi(x) \quad \checkmark\end{aligned}$$