Introduction to vectors and vector operations

K. Sajuran

Consultant/Lecturer

VECTORS AND SCALARS

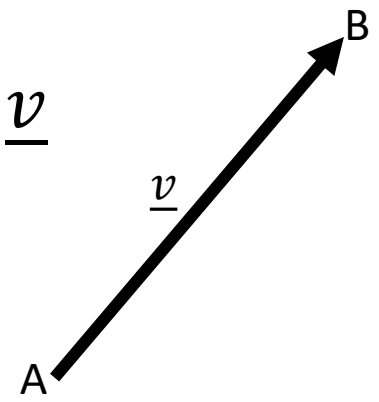
In Data Science , we deal with many different measured quantities. Some quantities

- need only a number and a unit of measurement. Examples are mass, length, speed and time. These quantities are scalars. They have magnitude only.
- Other quantities need both magnitude and direction. Examples of these are force, displacement, velocity and acceleration. These quantities are called vectors.
- Vectors are usually represented by arrows (the length of the arrow shows the magnitude, and the direction of the vector is indicated by the arrowhead.

Notation

The vector shown in the diagram at right can be written in a number of ways:

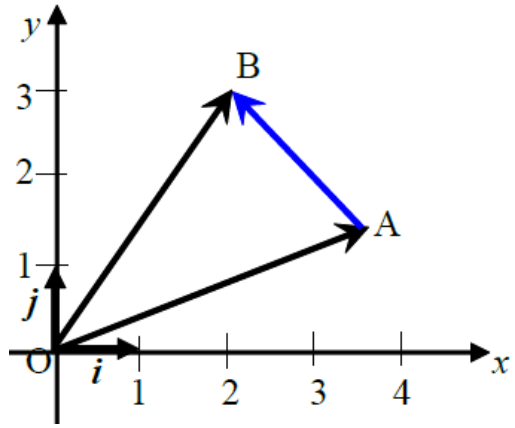
- Uppercase letters with an arrow above it : $\overrightarrow{AB}$
- A single lowercase letter with a tilde below it: $\underset{\sim}{v}$
- Bold uppercase letters : AB
- A single bold lowercase letter : v



POSITION VECTORS

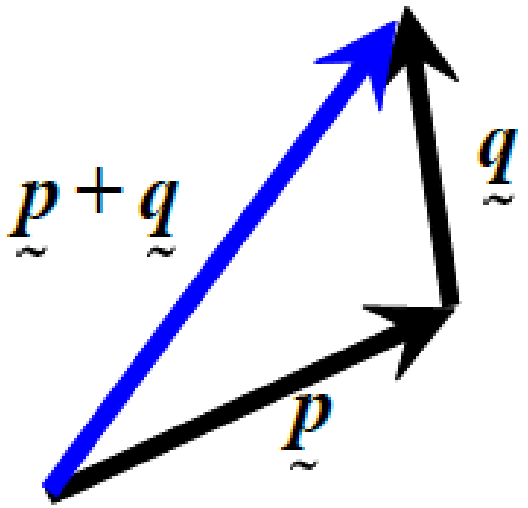
2-dimensional plane

- The position of any point A on a Cartesian plane (relative to the origin O) can be represented by the position vector. The vector OA starts at O and ends at A.
- Similarly, position vector $\overrightarrow{OB}$ represents the position of the point B relative to the origin.
- The position vector $\overrightarrow{AB}$ represents the position of point B relative to point A.



- On the2-dimensional plane above, we see that $\overrightarrow{OA}$ + $\overrightarrow{AB} = \overrightarrow{OB}$

- So (where) $\overrightarrow{AB}$ = $\overrightarrow{OB} - \overrightarrow{OA}$

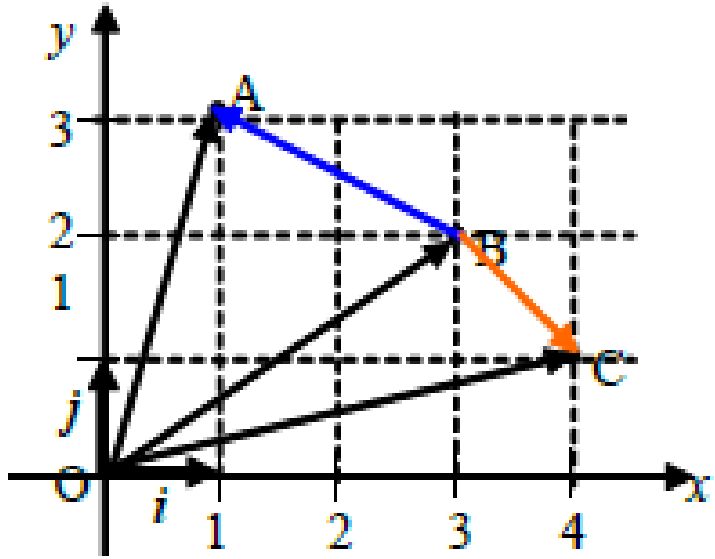


Unit vectors on 2-dimensional plane

- i is a unit vector in the *x*-direction (it has magnitude of 1 unit).
- j is a unit vector in the *y*-direction (it has a magnitude of 1 unit).

So, the position of any point on a two-dimensional plane can be represented in terms of unit vectors.

On the diagram shown :



- A (1,3) would have a position vector

$$\overrightarrow{OA} = \underline{a} = \underline{i} + 3\underline{j}$$

- B (3,2) would have a position vector

$$\overrightarrow{OB} = \underline{b} = 3\underline{i} + 2\underline{j}$$

- C (4,1) would have a position vector

$$\overrightarrow{OC} = \underline{c} = 4\underline{i} + \underline{j}$$

- We can add (or subtract) vectors written in ***i***, ***j*** notation by simply adding (or subtracting) the ***i*** and ***j*** components.

From above:

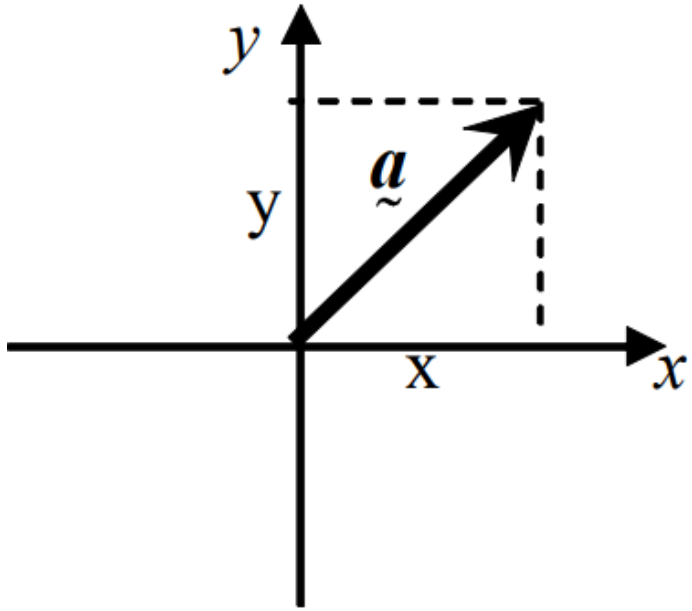
- $\overrightarrow{OA}+\overrightarrow{OB} = \underline{a}+\underline{b}= \underline{i} + 3\underline{j}+3\underline{i} + 2\underline{j} = 4\underline{i} + 5\underline{j}$
- $\overrightarrow{OA}+\overrightarrow{OB} = \underline{c}+\underline{a}= \underline{4i} + \underline{j}+\underline{i} + 3\underline{j} = 5\underline{i} + 4\underline{j}$
- $\overrightarrow{OA} - \overrightarrow{OB}= \underline{a} - \underline{b}= \underline{i} + 3\underline{j} -3\underline{i} + 2\underline{j} = -2\underline{i} + \underline{j}$

Magnitude of a 2-dimensional vector

- A general 2-d vector is shown at right

For the vector $\underline{a}$= x $\underline{i}$+ y$\underline{j}$.

- Magnitude of $\underline{a}$ =$|a|$=$\sqrt{x^2 + y^2}$



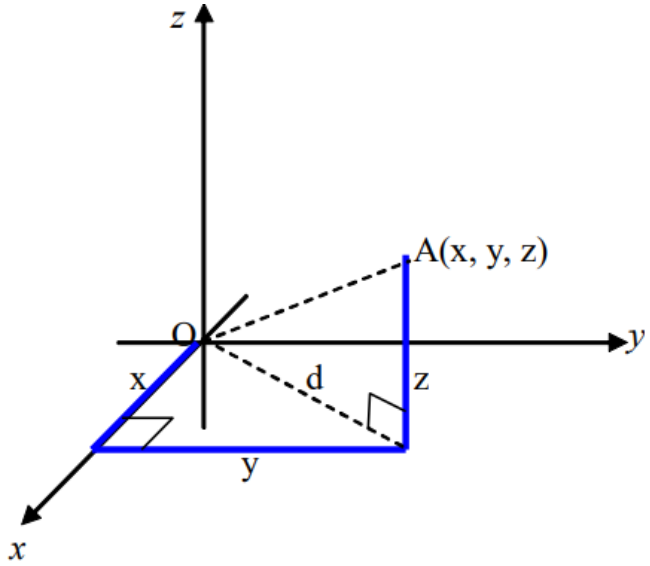
Magnitude of a 3-dimensional vector

- The magnitude of any 3-dimensional vector can be calculated by using Pythagoras theorem

$d^2 = x^2 + y^2$ and $OA^2 = d^2 + z^2 = z^2 + x^2 + y^2$

$\underline{a} = x\,\underline{i} + y\underline{j} + z\underline{k}$

Magnitude of $\underline{a} = |a| = \sqrt{x^2 + y^2 + z^2}$



Exercise 1

For points D(2, 3, –1) and E(–1, 2, – 4) ;

find $|d|, |e|, \left|\overrightarrow{DE}\right|$

UNIT VECTOR $\hat{a}$

$\underline{a} = \underline{i} + 3\underline{j}$

$|\underline{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

In general, for any vector $\underline{a}$ the unit vector $\hat{a}$ is given by

$\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|}$

$\hat{a} = \frac{(\underline{i} + 3\underline{j})}{\sqrt{10}}$

Exercise 2

For vectors $\underline{a}$ and $\underline{b}$ where $\underline{a} = -2\underline{i} + 3\underline{j}$ and $\underline{b} = 4\underline{i} - 5\underline{j}$

Plot the points on graph paper, and draw and label position vectors OP = $\underline{a}$, OQ = $\underline{b}$

1. $|\underline{a}|$
2. $|\underline{b}|$
3. $\hat{\underline{a}}$
4. $\hat{\underline{b}}$