Inverse Matrix Questions

Inverse Matrix Questions and Answers

1. Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

.

Solution:

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Let us find the determinant of A.

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 2 \times 2 - 3 \times 1 = 4 - 3 = 1$$

Here, |A| ≠ 0, so the inverse of A exists.

$$adjA = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Now, A^{-1} = adjA/|A|

Therefore,

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

2. What is the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$$

?

Solution:

Given,

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$$

Let us calculate the determinant of A.

$$|A| = \begin{vmatrix} 1 & 2 \\ -3 & 0 \end{vmatrix} = 1 \times 0 - 2 \times (-3) = 6$$

Here, |A| ≠ 0, so the inverse of A exists.

Now,

$$adjA = \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix}$$

As we know, A^{-1} = adjA/|A|

Hence,

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

3. If

$$A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

and $A^{-1} = A^T$, find the value of θ.
Solution:

Given,

$$A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

Also, given that,

$A^{-1} = A^T$

⇒ $AA^{-1} = AA^T$

⇒ $I = AA^T$

Now,

$$A^T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

$$AA^T = I$$

$$\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} . \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} 10 \\ 01 \end{bmatrix}$$

$$\begin{bmatrix} cos^2\theta + sin^2\theta & -cos\theta sin\theta + cos\theta sin\theta \\ -cos\theta sin\theta + cos\theta sin\theta & cos^2\theta + sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From the above,

$\cos^2\theta + \sin^2\theta = 1$

This is one of the trigonometric identities and is true for all real values of θ.

4. Calculate the inverse of the matrix

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

.

Solution:

Given,

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

First, find the determinant of matrix A.

$$A = \begin{vmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{vmatrix}$$

= 2(12 + 10) – 4(28 – 5) – 6(-14 – 3)

= 2(22) – 4(23) – 6(-17)

= 44 – 92 + 102

= 54 ≠ 0

Thus, the inverse matrix exists.

Thus, the minor matrix of A

$$= \begin{bmatrix} 22 & 23 & -17 \\ 4 & 14 & -8 \\ 38 & 52 & -22 \end{bmatrix}$$

Cofactor matrix of A

$$= \begin{bmatrix} 22 & -23 & -17 \\ -4 & 14 & 8 \\ 38 & -52 & -22 \end{bmatrix}$$

Also, adjA

$$= \begin{bmatrix} 22 & -4 & 38 \\ -23 & 14 & -52 \\ -17 & 8 & -22 \end{bmatrix}$$

Therefore, A^{-1} = adjA/|A|

$$A^{-1} = \frac{1}{54} \begin{bmatrix} 22 & -4 & 38 \\ -23 & 14 & -52 \\ -17 & 8 & -22 \end{bmatrix} = \begin{bmatrix} \frac{11}{27} & -\frac{2}{27} & \frac{19}{27} \\ -\frac{23}{54} & \frac{7}{27} & -\frac{26}{27} \\ -\frac{17}{54} & \frac{4}{27} & -\frac{11}{27} \end{bmatrix}$$

5. If

$$A = \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$$

, show that $(A^{-1})^{-1} = A$.
Solution:

Given,

$$A = \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 2 & 1 \\ 7 & 2 \end{vmatrix} = 4 - 7 = -3$$

Here, matrix A is non-singular.

$$adjA = \begin{bmatrix} 2 & -1 \\ -7 & 2 \end{bmatrix}$$

Thus,

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{-3}\begin{bmatrix} 2 & -1 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{3} & -\frac{2}{3} \end{bmatrix}$$

Let $A^{-1} = B$

So,

$$B = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{3} & -\frac{2}{3} \end{bmatrix}$$

And

$$|B| = \begin{vmatrix} -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{3} & -\frac{2}{3} \end{vmatrix}$$

= (-⅔)(-⅔) – (⅓)(7/3)

= (4/9) – (7/9)

= (4 – 7)/9

= -3/9

= -⅓

Now,

$$adjB = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & -\frac{2}{3} \end{bmatrix}$$

Also,

$$B^{-1} = \frac{adjB}{|B|} = \frac{1}{-\frac{1}{3}} \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & -\frac{2}{3} \end{bmatrix} = -3 \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$$

That means $B^{-1} = (A^{-1})^{-1} = A$

6. Find x, y, z if

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

satisfies $A^T = A^{-1}$.
Solution:

Given,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$A^T = A^{-1}$

$\Rightarrow AA^T = AA^{-1}$

$\Rightarrow AA^T = I$ {since $A^{-1}A = AA^{-1} = I$}

Now,

$$A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$AA^T = I$$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By performing multiplication on the LHS, we get:

By equating the corresponding elements, we have:

$4y^2 + z^2 = 1$ …(1)

$x^2 + y^2 + z^2 = 1$ …(2)

$2y^2 – z^2 = 0$ …(3)

Adding equations (1) and (3), we get:

$4y^2 + z^2 + 2y^2 – z^2 = 1 + 0$

$6y^2 = 1$

$y^2 = 1/6$

$\Rightarrow y = \pm 1/\sqrt{6}$

Substituting the value of y in equation (3), we get:

$z^2 = 2y^2$

$z^2 = 2(1/6)$

$z^2 = 1/3$

$\Rightarrow z = \pm 1/\sqrt{3}$

Substituting the values of y and z in equation (2), we get:

$x^2 = 1 – y^2 – z^2$

$x^2 = 1 – (1/6) – (1/3)$

$x^2 = (6 – 1 – 2)/6$

$x^2 = 3/6$

$x^2 = 1/2$

$\Rightarrow x = \pm 1/\sqrt{2}$

Therefore, $x = \pm 1/\sqrt{2}$, $y = \pm 1/\sqrt{6}$ and $z = \pm 1/\sqrt{3}$.

7. Find the value of x for which the matrix

$$A = \begin{bmatrix} 2 & 0 & 10 \\ 0 & x+7 & -3 \\ 0 & 4 & x \end{bmatrix}$$

is invertible.
Solution:

$$A = \begin{bmatrix} 2 & 0 & 10 \\ 0 & x+7 & -3 \\ 0 & 4 & x \end{bmatrix}$$

Let us find the determinant of the given matrix.

$$|A| = \begin{vmatrix} 2 & 0 & 10 \\ 0 & x+7 & -3 \\ 0 & 4 & x \end{vmatrix}$$

= 2[(x + 7)x – (-3)(4)] – 0 + 10(0 – 0)

= $2(x^2 + 7x + 12)$

We know that a matrix is invertible if and only if its determinant is not equal to 0.

Let |A| = 0

$2(x^2 + 7x + 12) = 0$

⇒ $x^2 + 7x + 12 = 0$

⇒ $x^2 + 3x + 4x + 12 = 0$

⇒ x(x + 3) + 4(x + 3) = 0

⇒ (x + 3)(x + 4) = 0

⇒ x + 3 = 0, x + 4 = 0

⇒ x = -3, x = -4

Thus, for x = -3 and -4, the given matrix is invertible.

8. Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

using row operations.
Solution:

Given,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

Let us write the augmented matrix [A | I] such that I is a square matrix of the order same as A.

$R_3 \to R_3 - 3R_1$

$R_2 \to R_2 - 2R_1$

Now, interchange R_2 and R_3.

$R_2 \to (-1).R_2$ and $R_3 \to (-1).R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$R_2 \to R_2 - 3R_3$

$R_1 \to R_1 - 2R_2$

$R_1 \to R_1 - 3R_3$

This is of the form [I | B].

Here, B is the inverse of A.

Therefore,

$$B = A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

9. Determine the formula for the inverse of matrix

$$A = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$

, where p, q, r, s ≠ 0.
Solution:

Given,

$$A = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$

Let us write the augmented matrix [A | I].

$$[\,A \mid I\,] = \left[\begin{array}{cccc|cccc} p & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & q & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \rightarrow (1/p)\, R_1$

$R_2 \rightarrow (1/q)\, R_2$

$R_3 \rightarrow (1/r)\, R_3$

$R_4 \rightarrow (1/s)\, R_4$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{p} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{q} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{s} \end{array}\right]$$

Hence, the inverse of A is:

$$A^{-1} = \begin{bmatrix} \frac{1}{p} & 0 & 0 & 0 \\ 0 & \frac{1}{q} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{s} \end{bmatrix}$$

10. If A is 3 × 3 invertible matrix, then show that for any scalar k (non-zero), kA is invertible and $(kA)^{-1} = (1/k)A^{-1}$.

Solution:

Consider $(kA)\,[(1/k)\,A^{-1}]$

$= [k\,(1/k)]\,(A\,A^{-1})$

$= 1.\,(AA^{-1})$

$= I$ {since $AA^{-1} = A^{-1}A = I$}

That means kA is the inverse of $(1/k)A^{-1}$.

Therefore, $(kA)^{-1} = (1/k)\,A^{-1}$

Practice Questions on Inverse Matrix

1. Find the inverse of the matrix

   $$A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$$

   .
2. If

   $$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

   , find A^{-1} exists.
3. Using elementary row operations, find the inverse of the matrix

   $$\begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

   .

4. Calculate the inverse matrix for

$$B = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$$

.

5. What is the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ -1 & 1 & 2 & 0 \\ -2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$