Matrix determinant, inverse solving of systems of equations

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Matrix Determinant

- Let A = $(a_{ij})_{n\times n}$ be a square matrix of order n , then the number $|A|$called determinant of the matrix A.

i. Determinant of 2$\times$ 2 matrix

Let A= $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then $|A|$=$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$= $a_{11}.\, a_{22}$- $a_{12}.\, a_{21}$

ii. Determinant of 3$\times$ 3 matrix

Let B=$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $|B|$ = $a_{11}(a_{22}\, .\ a_{33}$-$a_{23}.\, a_{32})$ -$a_{12}(a_{21}\, .\ a_{33}$-$a_{23}.\, a_{31})$+ $a_{13}(a_{21}\, .\ a_{32}$-$a_{31}.\, a_{22})$

Exercise 1

Calculate the determinants of the following matrices

- A= $\begin{bmatrix} 2 & 3 & 7 \\ 5 & 6 & 9 \\ -3 & 2 & 8 \end{bmatrix}$ B= $\begin{bmatrix} 4 & 2 & 1 \\ 2 & 6 & 2 \\ 2 & 9 & 7 \end{bmatrix}$ C= $\begin{bmatrix} 4 & 2 \\ 5 & 6 \end{bmatrix}$

Properties of the Determinant

A. The determinant of a matrix A and its transpose A^t are equal $|A|$=$|A^t|$
B. Let A be a square matrix
   1. If A has row(Column) of zeros, then $|A|$=0
   2. If A has two identical rows(or columns) then $|A|$=0
C. If A is triangular matrix, then $|A|$ is product of diagonal elements
D. If A is square matrix of order n and K is scaler, then $|KA| = k^n|A|$

- Minor and Cofactors

$A = (a_{ij})_{n \times n}$ be a square matrix .Then M_{ij} denote a sub matrix of A with order $(n-1) \times (n-1)$ obtained by deleting its i^{th} row and j^{th} column. The determinant $|M_{ij}|$ is called the minor of the element a_{ij} of A.

The cofactor of a_{ij} denoted by A_{ij} and is equal to $(-1)^{i+j} |M_{ij}|$

$C= \begin{bmatrix} 4 & 2 \\ 5 & 6 \end{bmatrix}$

$A= \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

Co factor matrix of (C) = $\begin{bmatrix} 6 & -5 \\ -2 & 4 \end{bmatrix}$

Co factor matrix of (A) (= $\begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$

❑ Singular Matrix

If A is square matrix of order n, the A is called singular matrix when $|A| = 0$ and non-singular otherwise.

❑ Adjoin Matrix

The transpose of the matrix of cofactors of the element of A denoted by *adj* A is called adjoin of matrix A.

For any square matrix A

A(*adj* A)= (*adj* A) A= $|A|$ I where I is the identity matrix of same order.

Inverse of a Matrix

Inverse of a Matrix

- If A and B are two matrices such that $AB = BA = I$, then each is said to be inverse of the other. The inverse of A is denoted by A^{-1} .

Existence of the Inverse

The necessary and sufficient condition for a square matrix A to have an inverse is that $|A| \neq 0$ (That is A is nonsingular).

Reversal law of the inverse of product

If A and B are two non-singular matrices of order n, then (AB) is also nonsingular

$$(AB)^{-1} = B^{-1}A^{-1}$$

If A is a non-singular matrix, then

$$A(A)^{-1} = I$$

$$|A^{-1}| = \frac{1}{|A|}$$

Solution of System of Linear Equation by Matrix Method

Solution of the linear system AX= B

$a_{11}x_{1+}a_{12}x_2$+…………………$a_{1n}x_n$=b_1

$a_{21}x_{1+}a_{22}x_2$+…………………$a_{2n}x_n$=b_2

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$a_{n1}x_{1+}a_{n2}x_2$+…………………$a_{nn}x_n$=b_n

Solution of System of Linear Equation by Matrix Method

- The above system can be put in the matrix form as

A=$\begin{bmatrix} a11 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ an1 & \cdots & ann \end{bmatrix}$ X=$\begin{pmatrix} x_1 \\ x_2 \\ \dots.. \\ x_n \end{pmatrix}$ b=$\begin{pmatrix} b_1 \\ b_2 \\ \dots.. \\ b_n \end{pmatrix}$

- Method of inversion:

$$Ax = B$$

$$(A)^{-1}(Ax) = A^{-1}B$$

$$(AA^{-1})x = A^{-1}B$$

$$\boldsymbol{x = A^{-1}B}$$

Exercise 3:

Solve the below equations using inversion method

$x_{1+}2x_2$+$3x_3$=5

$2x_{1+}5x_2$+$3x_3$=3

x_1 $+8x_3$=17