## 11.6.1: Eigenvalues and Eigenvectors (Exercises)

In Exercises 11.6.1.1 - 11.6.1.6, a matrix A and one of its eigenvectors are given. Find the eigenvalue of A for the given eigenvector.

#### **?** Exercise 11.6.1.1

$$A = \left[egin{array}{cc} 9 & 8 \ -6 & -5 \end{array}
ight] \quad ec{x} = \left[egin{array}{c} -4 \ 3 \end{array}
ight]$$

Answer

$$\lambda = 3$$

## **?** Exercise 11.6.1.2

$$A = egin{bmatrix} 19 & -6 \ 48 & -15 \end{bmatrix} \quad ec{x} = egin{bmatrix} 1 \ 3 \end{bmatrix}$$

Answer

$$\lambda = 1$$

## **?** Exercise 11.6.1.3

$$A = egin{bmatrix} 1 & -2 \ -2 & 4 \end{bmatrix} \quad ec{x} = egin{bmatrix} 2 \ 1 \end{bmatrix}$$

Answer

$$\lambda = 0$$

## ? Exercise 11.6.1.4

$$A = \begin{bmatrix} -11 & -19 & 14 \\ -6 & -8 & 6 \\ -12 & -22 & 15 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

Answer

$$\lambda = -5$$

## **?** Exercise 11.6.1.5

$$A = \left[ egin{array}{ccc} -7 & 1 & 3 \ 10 & 2 & -3 \ -20 & -14 & 1 \end{array} 
ight] \quad ec{x} = \left[ egin{array}{c} 1 \ -2 \ 4 \end{array} 
ight]$$

Answer

$$\lambda = 3$$

#### **?** Exercise 11.6.1.6

$$A = egin{bmatrix} -12 & -10 & 0 \ 15 & 13 & 0 \ 15 & 18 & -5 \end{bmatrix} \quad \vec{x} = egin{bmatrix} -1 \ 1 \ 1 \end{bmatrix}$$

**Answer** 

$$\lambda = -2$$

In Exercises 11.6.1.7 – 11.6.1.11, a matrix A and one of its eigenvalues are given. Find an eigenvector of A for the given eigenvalue.

$$A = egin{bmatrix} 16 & 6 \ -18 & -5 \end{bmatrix} \quad \lambda = 4$$

Answer

$$ec{x} = \left[ egin{array}{c} -1 \ 2 \end{array} 
ight]$$

## **?** Exercise 11.6.1.8

$$A = \begin{bmatrix} -2 & 6 \\ -9 & 13 \end{bmatrix}$$
  $\lambda = 7$ 

Answer

$$ec{x} = \left[ egin{matrix} 2 \ 3 \end{smallmatrix} 
ight]$$

## **?** Exercise 11.6.1.9

$$A = egin{bmatrix} -16 & -28 & -19 \ 42 & 69 & 46 \ -42 & -72 & -49 \end{bmatrix} \quad \lambda = 5$$

Answer

$$ec{x} = \left[egin{array}{c} 3 \ -7 \ 7 \end{array}
ight]$$

#### **?** Exercise 11.6.1.10

$$A = egin{bmatrix} 7 & -5 & -10 \ 6 & 2 & -6 \ 2 & -5 & -5 \end{bmatrix} \quad \lambda = -3$$

Answer

$$ec{x} = egin{bmatrix} 1 \ 0 \ 1 \end{bmatrix}$$

#### **?** Exercise 11.6.1.11

$$A = \begin{bmatrix} 4 & 5 & -3 \\ -7 & -8 & 3 \\ 1 & -5 & 8 \end{bmatrix} \quad \lambda = 2$$

Answer

$$ec{x} = egin{bmatrix} -1 \ 1 \ 1 \end{bmatrix}$$

In Exercises 11.6.1.12 – 11.6.1.28, find the eigenvalues of the given matrix. For each eigenvalue, give an eigenvector.

## **?** Exercise 11.6.1.12

$$\begin{bmatrix} -1 & -4 \\ -3 & -2 \end{bmatrix}$$

$$\lambda_1 = -5 ext{ with } \overrightarrow{x_1} = \left[egin{array}{c} 1 \ 1 \end{array}
ight];$$

$$\lambda_2=2$$
 with  $\overset{
ightarrow}{x_2}=egin{bmatrix} -4\ 3 \end{bmatrix}$ 

$$\begin{bmatrix} -4 & 72 \\ -1 & 13 \end{bmatrix}$$

Answer

$$\lambda_1=4 ext{ with } \overrightarrow{x_1}=\left[egin{array}{c} 9 \ 1 \end{array}
ight];$$

$$\lambda_2=5$$
 with  $\overrightarrow{x_2}=\left[egin{array}{c}8\1\end{array}
ight]$ 

## **?** Exercise 11.6.1.14

$$\begin{bmatrix} 2 & -12 \\ 2 & -8 \end{bmatrix}$$

Answer

$$\lambda_1 = -4 ext{ with } \overrightarrow{x_1} = \left[egin{array}{c} 2 \ 1 \end{array}
ight];$$

$$\lambda_2 = -2$$
 with  $\overrightarrow{x_2} = \left[egin{array}{c} 3 \ 1 \end{array}
ight]$ 

## **?** Exercise 11.6.1.15

$$\begin{bmatrix} 3 & 12 \\ 1 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -3 ext{ with } \overrightarrow{x_1} = egin{bmatrix} -2 \ 1 \end{bmatrix};$$

$$\lambda_2=5$$
 with  $\overrightarrow{x_2}=\left[egin{array}{c}6\1\end{array}
ight]$ 

## **?** Exercise 11.6.1.16

$$\begin{bmatrix} 5 & 9 \\ -1 & -5 \end{bmatrix}$$

Answer

$$\lambda_1 = -4 ext{ with } \overrightarrow{x_1} = \left[egin{array}{c} -1 \ 1 \end{array}
ight];$$

$$\lambda_2=4$$
 with  $\overrightarrow{x_2}=\left[egin{array}{c} -9 \ 1 \end{array}
ight]$ 

## **?** Exercise 11.6.1.17

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\lambda_1=2$$
 with  $\overset{
ightarrow}{x_1}=\left[egin{array}{c}1\1\end{array}
ight];$ 

$$\lambda_2=4$$
 with  $\overrightarrow{x_2}=\left[egin{array}{c} -1 \ 1 \end{array}
ight]$ 

$$\begin{bmatrix} 0 & 1 \\ 25 & 0 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 ext{ with } \overrightarrow{x_1} = egin{bmatrix} -1 \ 5 \end{bmatrix};$$

$$\lambda_2=5$$
 with  $\overrightarrow{x_2}=\left[egin{array}{c}1\5\end{array}
ight]$ 

## **?** Exercise 11.6.1.19

$$\begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -1 ext{ with } \overrightarrow{x_1} = \left[egin{array}{c} 1 \ 2 \end{array}
ight];$$

$$\lambda_2 = -3$$
 with  $\overrightarrow{x_2} = \left[egin{array}{c} 1 \ 0 \end{array}
ight]$ 

## **?** Exercise 11.6.1.20

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 3 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -1 ext{ with } \overrightarrow{x_1} = egin{bmatrix} 3 \ 0 \ 2 \end{bmatrix};$$

$$\lambda_2=1 ext{ with } \overrightarrow{x_2}=egin{bmatrix}1\0\0\end{bmatrix}$$

$$\lambda_3=3$$
 with  $\overset{
ightarrow}{x_3}=\left[egin{array}{c}5\-8\2\end{array}
ight]$ 

## **?** Exercise 11.6.1.21

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & 4 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\lambda_1=3$$
 with  $\overrightarrow{x_1}=egin{bmatrix} -3\ 0\ 2 \end{bmatrix};$ 

$$\lambda_2=4$$
 with  $\overrightarrow{x_2}=egin{bmatrix} -5 \ -1 \ 1 \end{bmatrix}$ 

$$\lambda_3=5$$
 with  $\overrightarrow{x_3}=egin{bmatrix}1\0\0\end{bmatrix}$ 

$$\begin{bmatrix} 1 & 0 & 12 \\ 2 & -5 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 ext{ with } \overrightarrow{x_1} = egin{bmatrix} 0 \ 1 \ 0 \end{bmatrix};$$

$$\lambda_2 = -2 ext{ with } \overrightarrow{x_2} = egin{bmatrix} -12 \ -8 \ 3 \end{bmatrix}$$

$$\lambda_3=5$$
 with  $\overset{
ightarrow}{x_3}=egin{bmatrix}15\3\5\end{bmatrix}$ 

## **?** Exercise 11.6.1.23

$$\begin{bmatrix} 1 & 0 & -18 \\ -4 & 3 & -1 \\ 1 & 0 & -8 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 ext{ with } \overrightarrow{x_1} = egin{bmatrix} 24 \ 13 \ 8 \end{bmatrix};$$

$$\lambda_2 = -2 ext{ with } \overrightarrow{x_2} = egin{bmatrix} 6 \ 5 \ 1 \end{bmatrix}$$

$$\lambda_3=3$$
 with  $\overset{
ightarrow}{x_3}=egin{bmatrix}0\1\0\end{bmatrix}$ 

#### **?** Exercise 11.6.1.24

$$\begin{bmatrix} -1 & 18 & 0 \\ 1 & 2 & 0 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\lambda_1 = -4 ext{ with } \overrightarrow{x_1} = egin{bmatrix} -6 \ 1 \ 11 \end{bmatrix};$$

$$\lambda_2 = -1 ext{ with } \overrightarrow{x_2} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix}$$

$$\lambda_3=5 ext{ with } \overrightarrow{x_3}=egin{bmatrix}3\1\2\end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 5 & -2 \end{bmatrix}$$

Answer

$$\lambda_1 = -2 ext{ with } \overrightarrow{x_1} = egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix};$$

$$\lambda_2=1$$
 with  $\overrightarrow{x_2}=egin{bmatrix}0\3\5\end{bmatrix}$ 

$$\lambda_3=5 ext{ with } \overrightarrow{x_3}=egin{bmatrix} 28 \ 7 \ 1 \end{bmatrix}$$

## **?** Exercise 11.6.1.26

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

Answer

$$\lambda_1=2 ext{ with } \overrightarrow{x_1}=egin{bmatrix}1\0\0\end{bmatrix};$$

$$\lambda_2=3$$
 with  $\stackrel{
ightarrow}{x_2}=egin{bmatrix} -1\ 1\ 0 \end{bmatrix}$ 

$$\lambda_3=7$$
 with  $\overrightarrow{x_3}=egin{bmatrix} -1\ 15\ 10 \end{bmatrix}$ 

#### **?** Exercise 11.6.1.27

$$\begin{bmatrix} 3 & 5 & -5 \\ -2 & 3 & 2 \\ -2 & 5 & 0 \end{bmatrix}$$

**Answer** 

$$\lambda_1 = -2 ext{ with } \overrightarrow{x_1} = egin{bmatrix} 1 \ 0 \ 1 \end{bmatrix};$$

$$\lambda_2=3 ext{ with } \overrightarrow{x_2}=egin{bmatrix}1\1\1\end{bmatrix};$$

$$\lambda_3=5$$
 with  $\overrightarrow{x_3}=egin{bmatrix}0\1\1\end{bmatrix}$ 

#### **?** Exercise 11.6.1.28

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Answer 
$$\lambda_1=0$$
 with  $\overrightarrow{x_1}=egin{bmatrix}1\3\1\end{bmatrix}; \ \lambda_2=-1$  with  $\overrightarrow{x_2}=egin{bmatrix}2\2\1\end{bmatrix}; \ \lambda_3=2$  with  $\overrightarrow{x_3}=egin{bmatrix}1\1\1\end{bmatrix}$ 

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