MODEL QUESIONS & ANSWERS MA(101)

(MATRICES)

1. Given $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$

(a) Compute that A+B and A-C

(b) Verify that A+(B+C) = (A+B)+C

(c) Compute AB, BA and AC^T

Solution: $A + B = \begin{pmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & 1 & 7 \end{pmatrix}$ and $A - C = \begin{pmatrix} -3 & 1 & -5 \\ 5 & -3 & 0 \\ 0 & 1 & -2 \end{pmatrix}$

$$(A + B) + C = A + (B + C) = \begin{pmatrix} 8 & 2 & 1 \\ 9 & 5 & 9 \\ 4 & -3 & 7 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{pmatrix} \quad BA = \begin{pmatrix} 0 & 4 & -9 \\ 19 & 3 & -3 \\ 5 & 1 & -3 \end{pmatrix}$$

$$AC^T = \begin{pmatrix} 0 & 0 & 12 \\ 24 & 4 & 11 \\ 5 & -1 & 6 \end{pmatrix}$$

Q(2). Evaluate :

$$\begin{pmatrix} 0 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 5 \\ 6 & 3 & 2 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 12 & 5 \\ 46 & 49 & 36 \\ 29 & 43 & 43 \end{pmatrix}$$

Q(3). If $A=\begin{pmatrix}1 & 2\\ 4 & -3\end{pmatrix}$, show that $A^2=\begin{pmatrix}9 & -4\\ -8 & 17\end{pmatrix}$ and $A^3=\begin{pmatrix}-7 & 30\\ 60 & -67\end{pmatrix}$

Q(4). If $A=\begin{pmatrix}-1 & -2 & -2\\ 1 & 2 & 1\\ 1 & -1 & 0\end{pmatrix}$, $B=\begin{pmatrix}3 & 2 & 2\\ -1 & 0 & -1\\ -3 & -3 & -2\end{pmatrix}$

Show that $\{kA+(1-k)B\}^2=I$, k being a scalar.

Solution: $\{kA+(1-k)B\}^2=k^2A^2+(1-k)^2B^2+k(1-k)AB+k(1-k)BA$

Now compute each matrix as follows;

$k^2A^2+(1-k)^2B^2+k(1-k)AB+k(1-k)BA=$

$$A^2=\begin{pmatrix}-3 & 0 & 0\\ 2 & 1 & 0\\ -2 & -4 & -3\end{pmatrix},\quad B^2=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\quad AB=\begin{pmatrix}5 & 4 & 4\\ -2 & -1 & -2\\ 4 & 2 & 3\end{pmatrix},BA=\begin{pmatrix}1 & -4 & -4\\ 0 & 3 & 2\\ -2 & 2 & 3\end{pmatrix}$$

Consider element a_{11}; of $k^2A^2+(1-k)^2B^2+k(1-k)AB+k(1-k)BA$

$-3k^2+(1-k)^2+5k(1-k)+k(1-k)=1-8k^2+4k\equiv 1$

Therefore k =1/2

When this value $\left(A^2+B^2+AB+BA\right)\dfrac{1}{4}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

Q(5). Find x, y such that $\begin{pmatrix}2 & -1\\ -3 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}8\\ 1\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$.

$$\begin{aligned}2x-y&=8\\ -3x+4y&=1\end{aligned}\quad x=\frac{31}{5},\ y=\frac{22}{5}$$

Q(6). Express the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ -3 & 1 & 4 \end{pmatrix}$ as the sum of symmetric and skew symmetric matrix.

$$S = \begin{pmatrix} 1 & 2 & 1/2 \\ 2 & 3 & 0 \\ 1/2 & 0 & 4 \end{pmatrix} \text{ and } T = \begin{pmatrix} & 0 & 7/2 \\ 0 & 0 & -1 \\ -7/2 & 1 & 0 \end{pmatrix}$$

Exercises 3.5

Q(1). If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, show that $A^3 = A^{-1}$.

If $A^3 = A^{-1}$ is satisfied then $AA^3 = AA^{-1}$ or $A^4 = I$

Try

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ therefore, } A^3 = A^{-1}$$

Q(2). Find the Inverse of (a) $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$,

Use direct method $A^{-1} \begin{pmatrix} 1/3 & -2/3 & 4/3 \\ -1/3 & -1/3 & 5/3 \\ 1/3 & 4/3 & -11/3 \end{pmatrix}$