Introduction to matrices matrix multiplication

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Introduction of Matrices

- Definition

A rectangular arrangement of m×n numbers, in m rows and n columns and enclosed within a bracket is called a matrix. We shall denote matrices by capital letters as A,B, C etc.

$$A=\begin{bmatrix} a11 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ am1 & \cdots & amn \end{bmatrix}=(a_{ij})_{\mathrm{m} \times n}$$

- A is a matrix of order m×n, i^{th} row j^{th} column element of the matrix denoted by a_{ij}

- Remark: A matrix is not just a collection of elements, but every element has assigned a definite position in a particular row and column

Special Types of Matrices

1. Square matrix : A matrix in which numbers of rows are equal to number of columns is called a square matrix.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2 & 5 & -8 \\ 0 & -3 & -4 \\ 6 & 8 & 9 \end{bmatrix}$$

2. Diagonal matrix : A square matrix A = =$(a_{ij})_{n \times n}$ is called a diagonal matrix if each of its non-diagonal element is zero. That is $a_{ij}=0$ if $i \neq j$ and at least one element $a_{ii} \neq 0$

Example:

$$A= \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \qquad B=\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

3. Identity Matrix

A diagonal matrix whose diagonal elements are equal to 1 is called identity matrix and denoted by I_n.

That is $a_{ij} = \begin{cases} 0, & if\ i \neq j \\ 1, & if\ i = j \end{cases}$

Example:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Upper Triangular matrix:

A square matrix said to be an Upper triangular matrix if $a_{ij} = 0$ if $i > j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -2 & 5 \\ 0 & 0 & 7 \end{bmatrix}$$

Special Types of Matrices Continue....

5. lower Triangular matrix:

A square matrix said to be a Lower triangular matrix if a_{ij} =0 if i<j

Example:

A=$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ B=$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 5 & 0 \\ 3 & -4 & 8 \end{bmatrix}$

6. Symmetric Matrix:

A square matrix A = $(a_{ij})_{n \times n}$ said to be a symmetric if a_{ij} =a_{ji} for all i and j.

Example:

A=$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ B=$\begin{bmatrix} 8 & -2 & 7 \\ -2 & -9 & 3 \\ 7 & 3 & 5 \end{bmatrix}$

Special Types of Matrices Continue….

7. Skew- Symmetric Matrix :

A square matrix A = $(a_{ij})_{n\times n}$ said to be a skew-symmetric if a_{ij}= -a_{ji} for all i and j.

Example:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B=\begin{bmatrix} 8 & -2 & 7 \\ 2 & -9 & 3 \\ -7 & -3 & 5 \end{bmatrix}$$

8. Zero Matrix:

A matrix whose all elements are zero is called as Zero Matrix and order $\mathrm{n} \times m$ Zero matrix denoted by $0_{n\times \mathrm{m}}$.

Example:

$$B_{3\times 2}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Special Types of Matrices Continue....

9. Row Vector:

A matrix consists a single row is called as a row vector or row matrix

Example:

A=$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ **A=**$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}$ A=$\begin{bmatrix} 8 & -2 & 7 \\ 2 & -9 & 3 \\ -7 & -3 & 5 \end{bmatrix}$ **A=**$\begin{bmatrix} 8 & -2 & 7 \end{bmatrix}$

10. Column Vector : A matrix consists a single column is called a column vector or column matrix

Example:

A=$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ A=$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$ A=$\begin{bmatrix} 8 & -2 & 7 \\ 2 & -9 & 3 \\ -7 & -3 & 5 \end{bmatrix}$ A=$\begin{bmatrix} 8 \\ 2 \\ -7 \end{bmatrix}$

Matrix Algebra

1. Equality of two matrices:

Two matrices A and B are said to be equal if

i. They are same order

ii. Their corresponding elements are equal.

$A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ then $a_{ij} = b_{ij}$ for all i and j.

2. Scalar multiple of a matrix:

Let k be a scalar then scalar product of matrix $A = (a_{ij})_{m\times n}$ given denoted by kA and given by $kA = (ka_{ij})_{m\times n}$ or

$$kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

Matrix Algebra continue….

3. Addition of two matrices:

Let A = $(a_{ij})_{m \times n}$ and B = $(b_{ij})_{m \times n}$ are two matrices with same order then sum of the two matrices are given by

$A + B = (aij)_{m \times n} + (b_{ij})_{m \times n} = (aij + b_{ij})_{m \times n}$

4.Subraction of two matrices:

Let A = $(a_{ij})_{m \times n}$ and B = $(b_{ij})_{m \times n}$ are two matrices with same order then sum of the two matrices are given by

$A - B = (aij)_{m \times n} - (bij)_{m \times n} = (aij - b_{ij})_{m \times n}$

Matrix Algebra continue

4. Multiplication of two matrices:

Two matrices *A* and *B* are said to be confirmable for product *AB* if number of columns in *A* equals to the number of rows in matrix *B*. Let *A* = $(a_{ij})_{m\times n}$ and *B* = $(b_{ij})_{n\times r}$ are two matrices the product matrix *C*= *AB*, is matrix of order m×r where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

C= *AB*

$$C = \begin{bmatrix} a_{11} \times b_{11} + a_{12} \times b_{21} & a_{11} \times b_{12} + a_{12} \times b_{22} \\ a_{21} \times b_{11} + a_{22} \times b_{21} & a_{21} \times b_{12} + a_{22} \times b_{22} \end{bmatrix}_{2\times 2}$$

Matrix Algebra continue

5. Transpose:

The transpose of matrix A = $(a_{ij})_{m \times n}$, written A^t is the matrix obtained by writing the rows of A in order as columns.

$A^{t} = (A_{ij})_{n \times m}$

Properties of Transpose:

- $(A + B)^t = A^t + B^t$
- $(A^t)^t = A$
- $(kA)^t = kA^t$
- $(AB)^t = (B^t A^t)$

Exercise1:

$$A = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -5 & -2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 \\ -5 & 0 \\ 6 & -2 \\ -1 & -3 \end{bmatrix}$$

1.Calculate AB

2.Calculate $A^t + \mathrm{B}$

3.Check whether $\mathrm{AB} = \mathrm{B}A$

Exercise2:

- $A = \begin{bmatrix} 4 & 5 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ B= $\begin{bmatrix} 7 & 8 & -4 \\ -2 & 5 & 4 \\ 1 & 0 & -7 \end{bmatrix}$ C= $\begin{bmatrix} 4 & 5 \\ -4 & 0 \\ -2 & 9 \\ 3 & -1 \end{bmatrix}$

Find the followings:

1.AB

2.CA

3.(CA)B

$4.A^t$B

Exercise 3:

1. Find the Values of x can take, given that

A=$\begin{bmatrix} x^2 & 3 \\ 1 & 3x \end{bmatrix}$ B=$\begin{bmatrix} 3 & 6 \\ 2 & x \end{bmatrix}$ such that AB=BA