Eigen decomposition

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Eigen decomposition

Eigen decomposition is the process of decomposing a matrix into its eigenvectors and eigenvalues. We can also transform a matrix into an Eigen basis

- Eigen values

If A is a square matrix of order n , then a non-zero vector X in R^n is called eigenvector of A if $AX = \lambda X$ for some scalar . The scalar λ is called an eigenvalue of A, and X is said to be an eigenvector of A corresponding to λ.

Remark: Eigen values are also called proper values or characteristic values.

- If A is a square matrix of order n and is a real number, then λ is an eigenvalue of A if and only if $|A - \lambda \mathrm{I}| = 0$.

Proof:

If λ is an eigenvalue of A, the there exist a non-zero X a vector in R^n such that $AX = \lambda X$.

$AX = \lambda X$

$AX = \lambda IX$ where I is an identity matrix of order n.

$(A - \lambda I)X = 0$

- The equation has trivial solution when if and only if $|A| = 0$. The equation has non-zero solution if and only if $|A - \lambda \mathrm{I}| = 0$.
- Conversely , if $|A - \lambda \mathrm{I}| = 0$ then by the result there will be a non-zero solution for the equation,

Exercise 1:

Find the eigen values of the matrixes

i. A=$\begin{pmatrix} 2 & 7 \\ 1 & -2 \end{pmatrix}$

ii. B=$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$

- If A is an $n \times n$ matrix and λ is a real number, then the following are equivalent

i. λ is an eigenvalue of A

ii. The system of equations $(A - \lambda I)X = 0$ has non-trivial solutions.

iii. There is a non-zero vector X in R^n such that c

Eigen vector:

Let A be an $n \times n$ matrix and λ be the eigen value of A. The set of all vectors X in R^n which satisfy the identity $AX = \lambda X$ is called the eigen space of a corresponding to λ . This is denoted by $\mathrm{E}(\lambda)$.

To find the Eigenvector of a matrix, the following steps are emplyed:

1. The eigenvalues for matrix A are found by using the formula, det $(A - \lambda I) = 0$. Here, ‘I’ is defined as the equivalent of the order of the matrix identity ‘A’. Further, eigenvalues can be denoted as λ_1, λ_2, and λ_3.
2. $AX = \lambda_1 X$ is the formula used to substitute the above values.
3. The value of eigenvector X is calculated.
4. The above steps are repeated to obtain the remaining eigenvectors by using other eigen values.

Find the eigen vectors for the following matrix?

i. $A=\begin{pmatrix} -5 & 2 \\ -7 & 4 \end{pmatrix}$

ii. $B=\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$