11.6.1: Eigenvalues and Eigenvectors (Exercises)

In Exercises 11.6.1.1 - 11.6.1.6, a matrix A and one of its eigenvectors are given. Find the eigenvalue of A for the given eigenvector.

? Exercise 11.6.1.1

$$A = \begin{bmatrix} 9 & 8 \\ -6 & -5 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

Answer

$\lambda = 3$

? Exercise 11.6.1.2

$$A = \begin{bmatrix} 19 & -6 \\ 48 & -15 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer

$\lambda = 1$

? Exercise 11.6.1.3

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Answer

$\lambda = 0$

? Exercise 11.6.1.4

$$A = \begin{bmatrix} -11 & -19 & 14 \\ -6 & -8 & 6 \\ -12 & -22 & 15 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

Answer

$\lambda = -5$

? Exercise 11.6.1.5

$$A = \begin{bmatrix} -7 & 1 & 3 \\ 10 & 2 & -3 \\ -20 & -14 & 1 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

Answer

$\lambda = 3$

? Exercise 11.6.1.6

$$A = \begin{bmatrix} -12 & -10 & 0 \\ 15 & 13 & 0 \\ 15 & 18 & -5 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Answer

$\lambda = -2$

In Exercises 11.6.1.7 – 11.6.1.11, a matrix A and one of its eigenvalues are given. Find an eigenvector of A for the given eigenvalue.

$$A = \begin{bmatrix} 16 & 6 \\ -18 & -5 \end{bmatrix} \quad \lambda = 4$$

Answer

$$\vec{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 6 \\ -9 & 13 \end{bmatrix} \quad \lambda = 7$$

Answer

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} -16 & -28 & -19 \\ 42 & 69 & 46 \\ -42 & -72 & -49 \end{bmatrix} \quad \lambda = 5$$

Answer

$$\vec{x} = \begin{bmatrix} 3 \\ -7 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & -5 & -10 \\ 6 & 2 & -6 \\ 2 & -5 & -5 \end{bmatrix} \quad \lambda = -3$$

Answer

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 5 & -3 \\ -7 & -8 & 3 \\ 1 & -5 & 8 \end{bmatrix} \quad \lambda = 2$$

Answer

$$\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

In Exercises 11.6.1.12 – 11.6.1.28, find the eigenvalues of the given matrix. For each eigenvalue, give an eigenvector.

$$\begin{bmatrix} -1 & -4 \\ -3 & -2 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 \text{ with } \vec{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 2 \text{ with } \vec{x_2} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 72 \\ -1 & 13 \end{bmatrix}$$

Answer

$$\lambda_1 = 4 \text{ with } \vec{x_1} = \begin{bmatrix} 9 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 5 \text{ with } \vec{x_2} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -12 \\ 2 & -8 \end{bmatrix}$$

Answer

$$\lambda_1 = -4 \text{ with } \vec{x_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix};$$

$$\lambda_2 = -2 \text{ with } \vec{x_2} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 12 \\ 1 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -3 \text{ with } \vec{x_1} = \begin{bmatrix} -2 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 5 \text{ with } \vec{x_2} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 9 \\ -1 & -5 \end{bmatrix}$$

Answer

$$\lambda_1 = -4 \text{ with } \vec{x_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 4 \text{ with } \vec{x_2} = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

Answer

$$\lambda_1 = 2 \text{ with } \vec{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 4 \text{ with } \vec{x_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

? Exercise 11.6.1.18

$$\begin{bmatrix} 0 & 1 \\ 25 & 0 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 \text{ with } \vec{x_1} = \begin{bmatrix} -1 \\ 5 \end{bmatrix};$$

$$\lambda_2 = 5 \text{ with } \vec{x_2} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

? Exercise 11.6.1.19

$$\begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -1 \text{ with } \vec{x_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix};$$

$$\lambda_2 = -3 \text{ with } \vec{x_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

? Exercise 11.6.1.20

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 3 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -1 \text{ with } \vec{x_1} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix};$$

$$\lambda_2 = 1 \text{ with } \vec{x_2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 3 \text{ with } \vec{x_3} = \begin{bmatrix} 5 \\ -8 \\ 2 \end{bmatrix}$$

? Exercise 11.6.1.21

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & 4 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

Answer

$$\lambda_1 = 3 \text{ with } \vec{x_1} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix};$$

$$\lambda_2 = 4 \text{ with } \vec{x_2} = \begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 5 \text{ with } \vec{x_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 12 \\ 2 & -5 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 \text{ with } \vec{x_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} ;$$

$$\lambda_2 = -2 \text{ with } \vec{x_2} = \begin{bmatrix} -12 \\ -8 \\ 3 \end{bmatrix}$$

$$\lambda_3 = 5 \text{ with } \vec{x_3} = \begin{bmatrix} 15 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -18 \\ -4 & 3 & -1 \\ 1 & 0 & -8 \end{bmatrix}$$

Answer

$$\lambda_1 = -5 \text{ with } \vec{x_1} = \begin{bmatrix} 24 \\ 13 \\ 8 \end{bmatrix} ;$$

$$\lambda_2 = -2 \text{ with } \vec{x_2} = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 3 \text{ with } \vec{x_3} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 18 & 0 \\ 1 & 2 & 0 \\ 5 & -3 & -1 \end{bmatrix}$$

Answer

$$\lambda_1 = -4 \text{ with } \vec{x_1} = \begin{bmatrix} -6 \\ 1 \\ 11 \end{bmatrix} ;$$

$$\lambda_2 = -1 \text{ with } \vec{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 5 \text{ with } \vec{x_3} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 5 & -2 \end{bmatrix}$$

Answer

$$\lambda_1 = -2 \text{ with } \overrightarrow{x_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 1 \text{ with } \overrightarrow{x_2} = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}$$

$$\lambda_3 = 5 \text{ with } \overrightarrow{x_3} = \begin{bmatrix} 28 \\ 7 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

Answer

$$\lambda_1 = 2 \text{ with } \overrightarrow{x_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$\lambda_2 = 3 \text{ with } \overrightarrow{x_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 7 \text{ with } \overrightarrow{x_3} = \begin{bmatrix} -1 \\ 15 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & -5 \\ -2 & 3 & 2 \\ -2 & 5 & 0 \end{bmatrix}$$

Answer

$$\lambda_1 = -2 \text{ with } \overrightarrow{x_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix};$$

$$\lambda_2 = 3 \text{ with } \overrightarrow{x_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix};$$

$$\lambda_3 = 5 \text{ with } \overrightarrow{x_3} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Answer

$\lambda_1 = 0$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$;

$\lambda_2 = -1$ with $\vec{x_2} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$;

$\lambda_3 = 2$ with $\vec{x_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

⊙ Toggle block-level attributions