3:1

21 2025 .

1.

1.1.

3:1 . , . , , , , , , , , , , .

. , , A, J, S (, ,). :

$$\begin{cases} m_S\ddot{\boldsymbol{r}}_{\boldsymbol{S}} = \frac{Gm_Sm_J}{|\boldsymbol{r_J}-\boldsymbol{r_S}|^3}(\boldsymbol{r_J}-\boldsymbol{r_S}) + \frac{Gm_Sm_A}{|\boldsymbol{r_A}-\boldsymbol{r_S}|^3}(\boldsymbol{r_A}-\boldsymbol{r_S}), \\ m_J\ddot{\boldsymbol{r}}_{\boldsymbol{J}} = \frac{Gm_Jm_S}{|\boldsymbol{r_S}-\boldsymbol{r_J}|^3}(\boldsymbol{r_S}-\boldsymbol{r_J}) + \frac{Gm_Jm_A}{|\boldsymbol{r_A}-\boldsymbol{r_J}|^3}(\boldsymbol{r_A}-\boldsymbol{r_J}), \\ m_A\ddot{\boldsymbol{r}}_{\boldsymbol{A}} = \frac{Gm_Am_S}{|\boldsymbol{r_S}-\boldsymbol{r_A}|^3}(\boldsymbol{r_S}-\boldsymbol{r_A}) + \frac{Gm_Am_J}{|\boldsymbol{r_J}-\boldsymbol{r_A}|^3}(\boldsymbol{r_J}-\boldsymbol{r_A}), \end{cases}$$

m_k $(k \in \{A, J, S\})$ - k- , r_k - -, G - . , , , , . 9 . , , , 6, . . , , . , , , , . , :

$$m_A \ll m_J \ll m_S.$$

, m_A, , , :

$$\begin{cases} \ddot{\boldsymbol{r}}_{\boldsymbol{S}} = \frac{Gm_J}{|\boldsymbol{r_J}-\boldsymbol{r_S}|^3}(\boldsymbol{r_J}-\boldsymbol{r_S}), \\ \ddot{\boldsymbol{r}}_{\boldsymbol{J}} = \frac{Gm_S}{|\boldsymbol{r_S}-\boldsymbol{r_J}|^3}(\boldsymbol{r_S}-\boldsymbol{r_J}), \\ \ddot{\boldsymbol{r}}_{\boldsymbol{A}} = \frac{Gm_S}{|\boldsymbol{r_S}-\boldsymbol{r_A}|^3}(\boldsymbol{r_S}-\boldsymbol{r_A}) + \frac{Gm_J}{|\boldsymbol{r_J}-\boldsymbol{r_A}|^3}(\boldsymbol{r_J}-\boldsymbol{r_A}). \end{cases}$$

, , , . [?], . , . . [?], S J:

$$\boldsymbol{r} = \boldsymbol{r_J} - \boldsymbol{r_S}, \quad \boldsymbol{\rho} = \frac{m_S}{m_J + m_S}\boldsymbol{r} - \boldsymbol{r_J} + \boldsymbol{r_A}, \quad \boldsymbol{r_c} = \boldsymbol{r_J} - \frac{m_S}{m_J + m_S}\boldsymbol{r}.$$

, , :

$$\begin{cases} \ddot{\boldsymbol{r}}_{\boldsymbol{c}} = 0, \\ \ddot{\boldsymbol{r}} = -\frac{G(m_S+m_J)}{r^3}\boldsymbol{r}, \\ \ddot{\boldsymbol{\rho}} = [\boldsymbol{\rho} \times \frac{d\boldsymbol{\Omega}}{dt}] + 2[\dot{\boldsymbol{\rho}} \times \boldsymbol{\Omega}] + [\boldsymbol{\Omega} \times [\boldsymbol{\rho} \times \boldsymbol{\Omega}]] + \frac{1}{m_A}\frac{Gm_Am_S}{|\boldsymbol{r_S}-\boldsymbol{r_A}|^3}(\boldsymbol{r_S}-\boldsymbol{r_A}) + \frac{Gm_Am_J}{|\boldsymbol{r_J}-\boldsymbol{r_A}|^3}(\boldsymbol{r_J}-\boldsymbol{r_A}), \end{cases}$$

$$\boldsymbol{\Omega} = \frac{[\boldsymbol{r_c} \times \dot{\boldsymbol{r}}_{\boldsymbol{c}}]}{r_c^2}.$$

, :

$$G = 1, \quad m_j = \nu, \quad m_s = 1 - \nu, \quad |\boldsymbol{\Omega}| = 1.$$

, , :

$$\begin{cases} \ddot{\rho_x} = +2\dot{\rho_y} + \rho_x - \frac{\partial U}{\partial \rho_x}, \\ \ddot{\rho_y} = -2\dot{\rho_x} + \rho_y - \frac{\partial U}{\partial \rho_y}, \end{cases}$$

$$U(r, \boldsymbol{\rho}) = -\frac{1-\nu}{\sqrt{\left(\rho_x + \nu r\right)^2 + \rho_y^2}} - \frac{\nu}{\sqrt{\left(\rho_x - (1-\nu)r\right)^2 + \rho_y^2}}.$$

:

$$L = \frac{\dot{\rho_x}^2 + \dot{\rho_y}^2}{2} + \frac{\rho_x{}^2 + \rho_y{}^2}{2} + \rho_x\dot{\rho_y} - \rho_y\dot{\rho_x} - U(r, \boldsymbol{\rho}).$$

:

$$x = \rho_x, \quad y = \rho_y, \quad p_x = \dot{x} - y, \quad p_y = \dot{y} + x,$$

:

$$H = \frac{p_x^2 + p_y^2}{2} + p_x y - p_y x + U(r, \boldsymbol{\rho}).$$

$r = |\boldsymbol{r}|$ ( ), :

$$\ddot{\boldsymbol{r}} = -\frac{G(m_S + m_J)}{r^3}\boldsymbol{r},$$

. , .

, ($e_J \approx 0.05$), . $e_J \to 0$ r . . , , .. , - :

$$\dot{\rho}_x^2 + \dot{\rho}_y^2 - \rho_x^2 - \rho_y^2 - 2U(\rho_x, \rho_y) = \text{const}.$$

, [?, ?], e_J . $e_J > 0$.

1.2.

, [?]:

$$H = -\frac{(1-\nu)^2}{2L^2} - \nu R(L, \rho_1, \rho_2, l, \omega_1, \omega_2),$$

R :

$$R = \sum_{s-j-k+p-m-n=0, s\geq 0} K^{sjkpmn}(L, \rho_1, \rho_2) \cos(sl + pl_J + j\omega_1 + k\omega_2 + m\omega_{1J} + n\omega_{2J}).$$

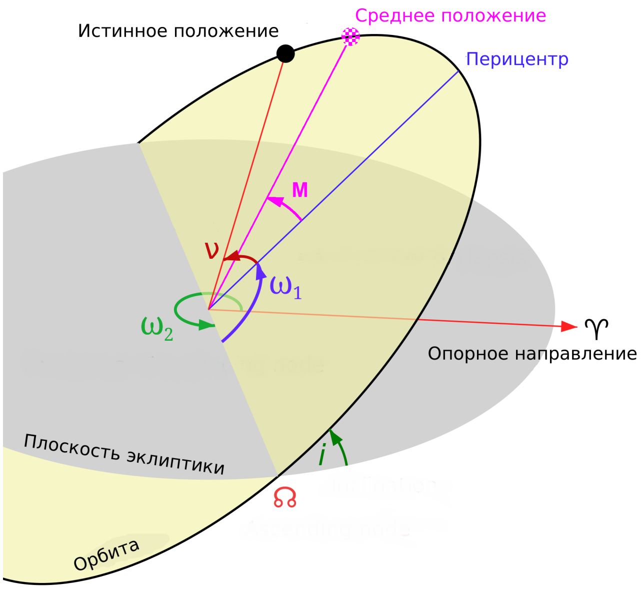
$(1-\nu)$ - , $\nu \approx \frac{1}{1000}$ - , . , , J, , , , . :

$$L = \sqrt{(1-\nu)a},$$

$$\rho_1 = \sqrt{(1-\nu)a}(1 - \sqrt{1-e^2}),$$

$$\rho_2 = \sqrt{(1-\nu)a(1-e^2)}(1 - \cos i),$$

a - , e - , i - , l - , ω_1 - , ω_2 - .



. 1. $l = \omega_1 + \omega_2 + M$, M -

[?]:

$$sl + pl_J + j\omega_1 + k\omega_2 + m\omega_{1J} + n\omega_{2J} = \text{const}.$$

, l , ( ):

$$s\dot{l} + p\dot{l}_J \approx 0,$$

:

$$\dot{l} \approx \frac{-p}{s}\dot{l}_J \equiv \frac{s+q}{s}\dot{l}_J.$$

q 3:1 $q = 2,\ s = 1.$

, , , $i = 0,\ \rho_2 = 0,\ \omega_2 = 0.$ e_J, [?]:

$$H = -\frac{(1-\nu)^2}{2L^2} - \nu R_{sec}(\rho_1, \omega_1) - \nu R_{res}(L, l, \rho_1, \omega_1),$$

$R_{sec}(\rho_1, \omega_1) = -2\rho_1 F - e_J G\sqrt{2\rho_1}\cos\omega_1$ - , ,
$R_{res}(L, l, \rho_1, \omega_1) = 2\rho_1 C\cos(l - \omega_1 - 3l_J) + e_J^2 E\cos(l - 3l_J)$ - , .
[?]:

$$a_J = 1, \dot{l}_J = 1.$$

:

$$\Lambda = L - L_{res},$$

$$\lambda = l - 3l_J,$$

$$x = \sqrt{2\rho_1}\cos\omega_1,$$

$$y = \sqrt{2\rho_1}\sin\omega_1,$$

Λ, :

$$H = \alpha\frac{\Lambda^2}{2} + \nu\left(C(x^2 - y^2) + e_J Dx + e_J^2 E\right)\cos\lambda + \nu(2Cxy + e_J Dy)\sin\lambda + \nu e_J Gx + \nu F(x^2 + y^2), \tag{1}$$

$L_{res} = \left(\frac{(1-\nu)^2}{3}\right)^{\frac{1}{3}}$ - ,

$$\dot{\lambda} = \frac{\partial H}{\partial L}|_{L=L_{res}} = 0,$$

a α:

$$\alpha = -\frac{3(1-\nu)^2}{L_{res}^4} < 0.$$

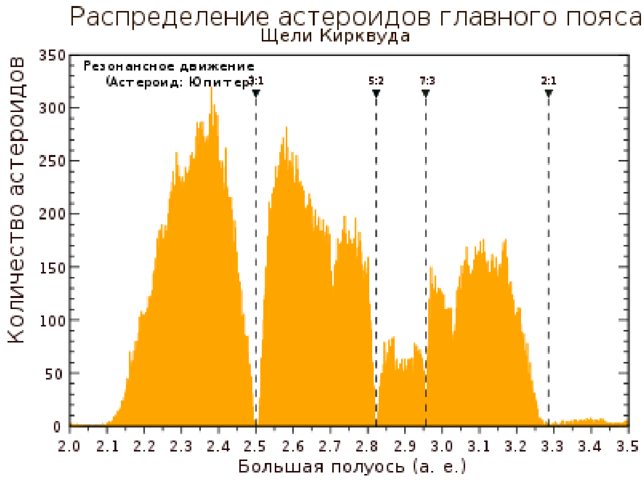
, y λ , x L . , λ, , :

$$(x, y, \Lambda, \lambda) \in \mathbb{R}^3 \times S^1.$$

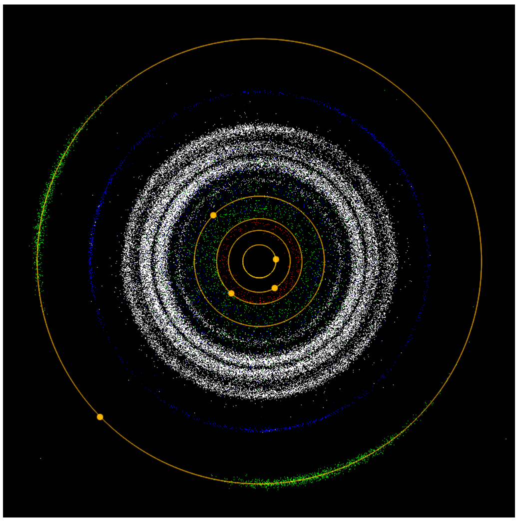
F, G, C, D, E, e_J , - , , [?].
, (1), 3:1.

1.3.

– , . 1866 . . , . : 2:1, 3:1, 5:2, 7:3.



. 2.



. 3.

. , . [?]:

$$
\begin{cases}
\frac{d\Lambda}{dt} = \nu\big(U(x,y)\sin\lambda - V(x,y)\cos\lambda\big),\\
\frac{d\lambda}{dt} = \alpha\Lambda,\\
\frac{dx}{dt} = -\nu\left(2Fy + \frac{\partial U(x,y)}{\partial y}\cos\lambda + \frac{\partial V(x,y)}{\partial y}\sin\lambda\right),\\
\frac{dy}{dt} = \nu\left(2Fx + e_J G + \frac{\partial U(x,y)}{\partial x}\cos\lambda + \frac{\partial V(x,y)}{\partial x}\sin\lambda\right),
\end{cases} \tag{2}
$$

$$
U(x,y) = C(x^2 - y^2) + e_J D x + e_J^2 E,
$$
$$
V(x,y) = 2Cxy + e_J D y.
$$

$\frac{dx}{dt}$ $\frac{dy}{dt}$ ν, x y ν^{-1}, λ ($\nu^{-\frac{1}{2}}$). , x y - : $x = \overline{x} + \xi$, $y = \overline{y} + \eta$, :

$$
\begin{cases}
\frac{d\overline{x}}{dt} = -\nu\left(2F\overline{y} + \frac{\partial U(\overline{x},\overline{y})}{\partial \overline{y}} < \cos\lambda > + \frac{\partial V(\overline{x},\overline{y})}{\partial \overline{y}} < \sin\lambda >\right),\\
\frac{d\overline{y}}{dt} = \nu\left(2F\overline{x} + e_J G + \frac{\partial U(\overline{x},\overline{y})}{\partial \overline{x}} < \cos\lambda > + \frac{\partial V(\overline{x},\overline{y})}{\partial \overline{x}} < \sin\lambda >\right),
\end{cases} \tag{3}
$$

T:

$$< \cos\lambda >= \frac{1}{T}\int_0^T \cos\lambda dt = \begin{cases} \frac{2E(k_L)}{K(k_L)} - 1, & -\sqrt{U^2+V^2} < H < \sqrt{U^2+V^2}, \\ \frac{2E(k_C)}{k_C^2 K(k_C)} + 1 - \frac{2}{k_C^2}, & H < -\sqrt{U^2+V^2}, \end{cases}$$

$$< \sin\lambda >= \frac{1}{T}\int_0^T \sin\lambda dt = 0,$$

$$k_L = \sqrt{\frac{\nu\sqrt{U^2+V^2} - H}{2\nu\sqrt{U^2+V^2}}}, \quad k_C = \sqrt{\frac{2\nu\sqrt{U^2+V^2}}{\nu\sqrt{U^2+V^2} - H}},$$

$K(k)$ - 1 , $E(k)$ - 2 .
1 , - .
, . , λ $\overline{x}$ $\overline{y}$. $\frac{d\lambda}{dt} = O(\nu)$, , $\frac{dx}{dt}$ $\frac{dx}{dt}$ ν. , $U(x,y)\sin\lambda - V(x,y)\cos\lambda$ () .

2. - ,

2.1. 3:1 -

, (1) - . $\Lambda = \sqrt{\nu}\Lambda_{new}$, $t_{new} = \sqrt{\nu}t$, :

$$\begin{cases} \dfrac{d\Lambda_{new}}{dt_{new}} = U\sin\lambda - V\cos\lambda, & \dfrac{d\lambda}{dt_{new}} = \alpha\Lambda_{new}, \\ \dfrac{dx}{dt_{new}} = -\sqrt{\nu}\big(2Fy + \dfrac{\partial U}{\partial y}\cos\lambda + \dfrac{\partial V}{\partial y}\sin\lambda\big), & \dfrac{dy}{dt_{new}} = \sqrt{\nu}\big(2Fx + e_J G + \dfrac{\partial U}{\partial x}\cos\lambda + \dfrac{\partial V}{\partial x}\sin\lambda\big). \end{cases}$$

new. $\varepsilon = \sqrt{\nu}$. (t):

$$\begin{cases} \dot{\Lambda} = U\sin\lambda - V\cos\lambda, & \dot{\lambda} = \alpha\Lambda, \\ \dot{x} = -\varepsilon\big(2Fy + \dfrac{\partial U}{\partial y}\cos\lambda + \dfrac{\partial V}{\partial y}\sin\lambda\big), & \dot{y} = \varepsilon\big(2Fx + e_J G + \dfrac{\partial U}{\partial x}\cos\lambda + \dfrac{\partial V}{\partial x}\sin\lambda\big). \end{cases} \tag{4}$$

λ Λ , x y .

2.2.

(4) :

$$M_0 = \{(x,y,\lambda,\Lambda) : \Lambda = 0, u(x,y)\sin\lambda = v(x,y)\cos\lambda\}.$$

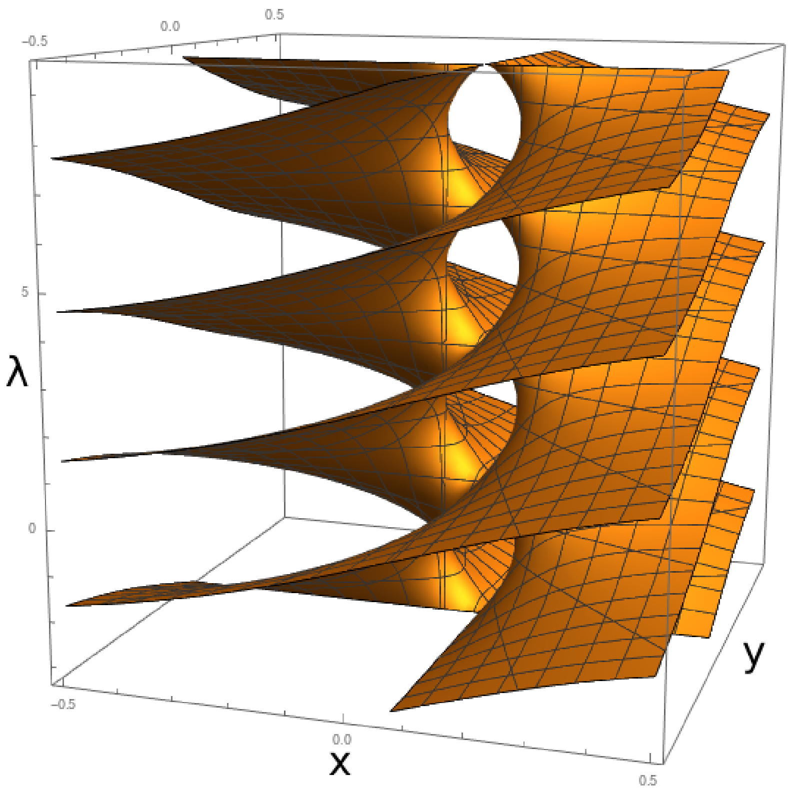
$U\sin\lambda = V\cos\lambda$ $\lambda = \arctan(V/U) + \pi k$, $k \in \mathbb{Z}$. $U = 0$ $\lambda_\pm : \mathbb{R}^2 \setminus \{U = V = 0\} \to S^1$, π:

$$\lambda_+(x,y) \equiv \begin{cases} \arctan\frac{V}{U}, & U > 0, V > 0, \\ \arctan\frac{V}{U} + \pi, & U < 0, \\ \arctan\frac{V}{U} + 2\pi, & U > 0, V < 0, \end{cases} \qquad \lambda_-(x,y) \equiv \begin{cases} \arctan\frac{V}{U} + \pi, & U > 0, V > 0, \\ \arctan\frac{V}{U} + 2\pi, & U < 0, \\ \arctan\frac{V}{U} + 3\pi, & U > 0, V < 0. \end{cases}$$

, $U = V = 0$, :

$$y = 0, \quad x = x_b^{\pm} \equiv \frac{-e_J D \pm e_J \sqrt{D^2 - 4EC}}{2C}, \quad \Lambda = 0, \quad \lambda \in S^1.$$

, $\tan\lambda = y/(x - x_b^{\pm})$.



. 4.

, $M_{0,\pm}$, $M_{0,b}^{\pm}$:

$$\begin{aligned} M_0 &= M_{0,+} \cup M_{0,-} \cup M_{0,b}^{+} \cup M_{0,b}^{-}, \\ M_{0,\pm} &= \{\lambda = \lambda_{\pm}(x,y), \Lambda = 0, (x,y) \in \mathbb{R}^2 \setminus \{U = V = 0\}\}, \\ M_{0,b}^{\pm} &= \{y = 0, x = x_b^{\pm}, \Lambda = 0, \lambda \in S^1\}. \end{aligned}$$

$M_{0,\pm}$, $M_{0,b}^{\pm}$ - .

2.3.

(4) ε , "" :

$$\begin{cases} \dot\Lambda = U(x,y)\sin\lambda - V(x,y)\cos\lambda, & \dot\lambda = \alpha\Lambda, \\ \dot x = 0, & \dot y = 0. \end{cases} \tag{5}$$

(x,y) U, V . λ t $\dot\Lambda$, :

$$\ddot\lambda - \alpha U \sin\lambda + \alpha V \cos\lambda = 0,$$

($\alpha < 0$) :

$$\ddot{\lambda} - \alpha\beta \sin\big(\lambda - \lambda_+(x,y)\big) = 0, \quad \beta(x,y) \equiv \sqrt{U^2+V^2} > 0.$$

, ( ) "" , $(\lambda,\Lambda) = (\lambda_\pm(x,y),0)$. (5) :

$$\begin{pmatrix} U\sin\lambda - V\cos\lambda \\ \alpha\Lambda \end{pmatrix} = \begin{pmatrix} 0 & \alpha \\ \pm\beta(x,y) & 0 \end{pmatrix} \begin{pmatrix} \lambda - \lambda_\pm \\ \Lambda \end{pmatrix} + O((\lambda-\lambda_\pm)^2, \Lambda^2).$$

:

$M_{0,+}$:

$$\zeta = \pm i\sqrt{|\alpha|\beta},$$

$M_{0,-}$:

$$\zeta = \pm\sqrt{|\alpha|\beta}.$$

, - .

, $M_{0,+}$, $M_{0,-}$ - . , :

$$\lambda(t,t_0) = \pm 2\arctan\sinh\big(\alpha\beta(t-t_0)\big) + \lambda_-(x,y),$$
$$\Lambda(t,t_0) = \frac{\pm 2\beta}{\cosh\big(\alpha\beta(t-t_0)\big)},$$

$t_0 \in \mathbb{R}$, .

2.4.

$\tau = \varepsilon t$ (4), (τ):

$$\begin{cases} \varepsilon\Lambda' = U\sin\lambda - V\cos\lambda, & \varepsilon\lambda' = \alpha\Lambda, \\ x' = -\big(2Fy + \dfrac{\partial U}{\partial y}\cos\lambda + \dfrac{\partial V}{\partial y}\sin\lambda\big), & y' = 2Fx + e_J G + \dfrac{\partial U}{\partial x}\cos\lambda + \dfrac{\partial V}{\partial x}\sin\lambda. \end{cases} \tag{6}$$

ε , "" :

$$\begin{cases} 0 = U\sin\lambda - V\cos\lambda, & 0 = \alpha\Lambda, \\ x' = -\big(2Fy + \dfrac{\partial U}{\partial y}\cos\lambda + \dfrac{\partial V}{\partial y}\sin\lambda\big), & y' = 2Fx + e_J G + \dfrac{\partial U}{\partial x}\cos\lambda + \dfrac{\partial V}{\partial x}\sin\lambda. \end{cases} \tag{7}$$

(7) , :

$$\Big\{ x' = -\big(2Fy \pm \frac{\partial\beta}{\partial y}\big), \qquad y' = 2Fx + e_J G \pm \frac{\partial\beta}{\partial x}. \tag{8}$$

"+" "−" $M_{0,+}$ $M_{0,-}$.

:

$$H_S = F(x^2+y^2) + e_J G x \pm \beta(x,y),$$

, .

:

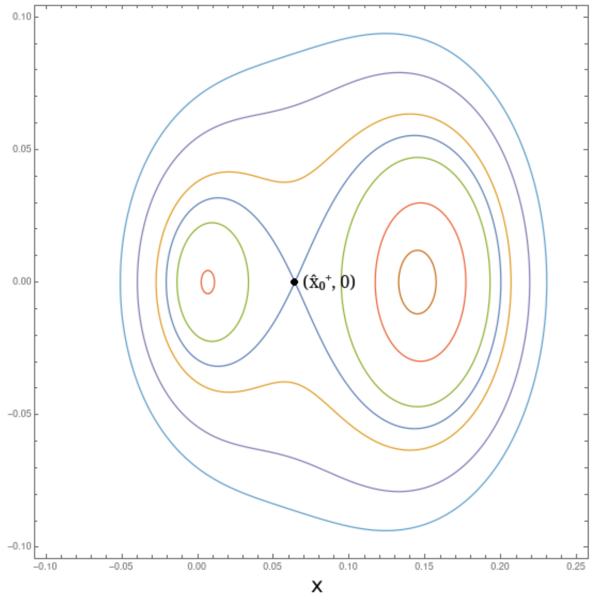
$$y_0^\pm = 0, \quad x_0^\pm = \frac{e_J(\pm D - G)}{2(F \mp C)}.$$

, :

$$\zeta = \pm\sqrt{-\left(2F \pm 2C \pm \frac{2Cx_0^\pm + e_J D}{|U(x_0^\pm, 0)|}\right)\big(2F \mp 2C\big)} \in \mathbb{R},$$

, , , .
, , , $\tau \to \pm\infty$. $H_S(x,y) = H_S(x_0^\pm, y_0^\pm)$, :

$$H_S(x_0^\pm, y_0^\pm) = F(x^2+y^2) + e_J G x \pm \beta(x,y).$$



. 5. H_S $M_{0,+}$. $M_{0,-}$

$x = x_0^\pm + h$, :

$$(F^2 - C^2)(h^2+y^2)^2 + 2(h^2+y^2)\big(U(x_0^\pm, y_0) + h(2Cx_0^\pm + e_J D)\big)(\pm F - C) + \\ + y^2\big(4U(x_0^\pm, y_0^\pm)C - (2Cx_0^\pm + e_J D)^2\big) = 0. \quad (9)$$

: $X = \frac{h}{h^2+y^2}$, $Y = \frac{y}{h^2+y^2}$. (9) :

$$(F^2 - C^2) + AX^2 + 2BX + (A+\delta)Y^2 = 0,$$

$$A = 2U(x_0^\pm, y_0^\pm)(\pm F - C),$$
$$B = (2Cx_0^\pm + e_J D)(\pm F - C),$$
$$\delta = -(2Cx_0^\pm + e_J D)^2.$$

:

$$\xi = \left(X + \frac{B}{A}\right)\sqrt{\frac{A}{F^2 - C^2 - \frac{B^2}{A}}} \equiv \omega\left(X + \frac{B}{A}\right),$$

$$\eta = Y\sqrt{\frac{-(A+\delta)}{F^2 - C^2 - \frac{B^2}{A}}} \equiv \sigma Y,$$

:

$$\xi^2 - \eta^2 = 1. \quad (10)$$

, (ξ, η) $H_S(x,y) = H_S(x_0^\pm, y_0^\pm)$. (x,y), .. , "" (. . 5).

f:

$$\xi = \operatorname{ch} f,$$
$$\eta = \operatorname{sh} f,$$
$$\xi^2 - \eta^2 = \operatorname{ch}^2 f - \operatorname{sh}^2 f = 1.$$

f τ. :

$$x' = \frac{\partial x}{\partial X}X' + \frac{\partial x}{\partial Y}Y' = \frac{Y^2 - X^2}{(X^2+Y^2)^2}X' + \frac{-2XY}{(X^2+Y^2)^2}Y' = f' \operatorname{sh} f \frac{Q(\operatorname{ch} f)}{P^2(\operatorname{ch} f)}, \tag{11}$$

$$P(\operatorname{ch} f) \equiv X^2 + Y^2 = \operatorname{ch}^2 f \left(\frac{1}{\omega^2} + \frac{1}{\sigma^2}\right) + \frac{-2B}{\omega A}\operatorname{ch} f + \left(\frac{B^2}{A^2} - \frac{1}{\sigma^2}\right),$$

$$Q(\operatorname{ch} f) \equiv -\frac{\operatorname{ch}^2 f}{\omega}\left(\frac{1}{\omega^2} + \frac{1}{\sigma^2}\right) + \frac{2B}{A}\operatorname{ch} f\left(\frac{1}{\sigma^2} + \frac{1}{\omega^2}\right) - \frac{1}{\omega}\left(\frac{1}{\sigma^2} + \frac{B^2}{A^2}\right).$$

, (9) (24), :

$$x' = -\left(2Fy \pm \frac{\partial \beta}{\partial y}\right) = -2Fy \mp \frac{1}{\beta}\left(U\frac{\partial U}{\partial y} + V\frac{\partial V}{\partial y}\right) =$$

$$= \frac{y\Big(2(C^2 - F^2)(x^2+y^2) + 2x(2Cx_0 + e_J D)(C \mp F) + (2Cx_0 + e_J D)^2 + 2U(x_0, y_0)(-C \mp F)\Big)}{F(x^2+y^2) \pm U(x_0, y_0) \pm x(2Cx_0 + e_J D)}.$$

f

$$x' = \frac{\operatorname{sh} f}{\sigma P(\operatorname{ch} f)}\frac{R(\operatorname{ch} f)}{S(\operatorname{ch} f)}, \tag{12}$$

$$R(\operatorname{ch} f) \equiv P(\operatorname{ch} f)\left(2U(x_0,y_0)(-C \mp F) + (2Cx_0 + e_J D)^2\right) +$$
$$+2\left(\frac{\operatorname{ch} f}{\omega} - \frac{B}{A}\right)(2Cx_0 + e_J D)(C \mp F) + 2(C^2 - F^2),$$

$$S(\operatorname{ch} f) \equiv \pm U(x_0, y_0)P(\operatorname{ch} f) \pm \left(\frac{\operatorname{ch} f}{\omega} - \frac{B}{A}\right)(2Cx_0 + e_J D) + F.$$

(11) (12), f:

$$f' = \frac{1}{\sigma}\frac{R(\operatorname{ch} f)P(\operatorname{ch} f)}{S(\operatorname{ch} f)Q(\operatorname{ch} f)}. \tag{13}$$

, P, Q, R, S $\operatorname{ch} f$. (13), :

$$\tau - \tau_0 = \sigma \int_{f_0}^{f} \frac{S(\operatorname{ch} f)Q(\operatorname{ch} f)}{R(\operatorname{ch} f)P(\operatorname{ch} f)}df =$$

$$= (f - f_0)\frac{\mp 2U(x_0^{\pm}, y_0^{\pm})\frac{\sigma}{\omega}}{2U(x_0^{\pm}, y_0^{\pm})(-C \mp F) + (2Cx_0^{\pm} + e_J D)^2} + \sigma\int_{f_0}^{f}\frac{L(\operatorname{ch} f)}{R(\operatorname{ch} f)P(\operatorname{ch} f)}df, \tag{14}$$

L 3 .

1. $M_{0,k}, k=+,-$ (24) $(x,y)=(x_0^{\pm},y_0^{\pm})$. , :

$$x(\tau,\tau_0)=x_0+\frac{\frac{\operatorname{ch}\left(f(\tau-\tau_0)\right)}{\omega}-\frac{B}{A}}{P\Big(\operatorname{ch}\left(f(\tau-\tau_0)\right)\Big)},$$

$$y(\tau,\tau_0)=\frac{\operatorname{sh}\left(f(\tau-\tau_0)\right)}{\sigma P\Big(\operatorname{ch}\left(f(\tau-\tau_0)\right)\Big)},$$

$\tau_0\in\mathbb{R}$, $f(\tau)$ (14).

2.5.

, $(\hat{x}_0^-,0,\lambda_-(\hat{x}_0^-,0),0)$ (..), . , - (, ,).
$(\hat{x}_0^+,0,\lambda_+(\hat{x}_0^+,0),0)\in M_{0,+}$, , (4). , - $(\pm i\omega,\pm\xi)$.

$$\omega=\frac{\sqrt{\sqrt{p}-q}}{\sqrt{2}}=\sqrt{|\alpha U(\hat{x}_0^+,0)|}+\mathcal{O}(\varepsilon^2),$$

$$\xi=\frac{\sqrt{\sqrt{p}+q}}{\sqrt{2}}=\mathcal{O}(\varepsilon),$$

$$p=(4\varepsilon^2(C-F)(C+F)+\alpha U(\hat{x}_0^+,0))^2+8\alpha\varepsilon^2(F-C)V_y(\hat{x}_0^+,0)>,0$$

$$q=4\varepsilon^2(C-F)(C+F)-\alpha U(\hat{x}_0^+,0)<0.$$

- . , , . ε, , , .

3.

3.1.

3.1.1.

. .

1. , , C^r-, $M\subset R^n$ () $\theta_t(x)$, $\frac{dx}{dt}=F(x)$ ($x\in\mathbb{R}^n$, $F(x)$ - C^r-), $p\in M$ $\theta_t(p)\in M$ $t\le 0$ ($t\ge 0$) $p\in\partial M$ $-F(p)$ M.
, M $\theta_t(x)$, .

2. , C^r-, $M\subset\mathbb{R}^n$ $\theta_t(x)$, $p\in M\setminus\partial M$ (t_1,t_2), $t_1<0$, $t_2>0$, $t\in(t_1,t_2)$ $\theta_t(p)\in M$.

3. M , $x_0\in M$ $\lambda_{1,2}>0$, :

$$T_{x_0}\mathbb{R}^n=T_{x_0}M\oplus E^u_{x_0}\oplus E^s_{x_0},$$

$$\forall v\in E^s_{x_0},(T_{x_0}\theta_t)v\in E^s_{\theta_t(x_0)}\Rightarrow||(T_{x_0}\theta_t)v||\le e^{-\lambda_1 t}||v||,\ t\ge 0,$$

$$\forall v\in E^u_{x_0},(T_{x_0}\theta_t)v\in E^u_{\theta_t(x_0)}\Rightarrow||(T_{x_0}\theta_t)v||\le e^{\lambda_2 t}||v||,\ t\le 0.$$

1. (-):
M - , , :

$$W^s(M)=\{x:\operatorname{dist}\big(\theta_t(x),M\big)\to 0,t\to+\infty\},$$

$$W^u(M)=\{x:\operatorname{dist}\big(\theta_t(x),M\big)\to 0,t\to-\infty\}.$$

:

4. M , $x_0 \in M$ $\lambda_{1,2} > 0$, :

$$T_{x_0}\mathbb{R}^n = T_{x_0}M \oplus E^u_{x_0} \oplus E^s_{x_0},$$

$$\forall v \in E^s_{x_0}, (T_{x_0}\theta_t)v \in E^s_{\theta_t(x_0)} \Rightarrow ||(T_{x_0}\theta_t)v|| \le e^{-\lambda_1 t}||v||, 0 \le t < t_2,$$
$$\forall v \in E^u_{x_0}, (T_{x_0}\theta_t)v \in E^u_{\theta_t(x_0)} \Rightarrow ||(T_{x_0}\theta_t)v|| \le e^{\lambda_2 t}||v||, t_1 < t \le 0.$$

- [?]

2. M - , , :

$$W^s(M) = \{x : \mathrm{dist}(\theta_t(x), M) \le \mathrm{dist}(x, M), 0 \le t < t_2\},$$

$$W^u(M) = \{x : \mathrm{dist}(\theta_t(x), M) \le \mathrm{dist}(x, M), t_1 < t \le 0\}.$$

, ( ) ( ).

5. M_1, M_2 - () , $W^{s,u}(M_{1,2})$ - . $x_0 \in W^u(M_1) \cap W^s(M_2)$, $M_1 = M_2$, $M_1 \ne M_2$. $\theta_t(x_0) \in W^u(M_1) \cap W^s(M_2)$ () -.

6. () x_0 , :

$$T_{x_0}\mathbb{R}^n = T_{x_0}W^s(M_1) \oplus T_{x_0}W^u(M_2).$$

:

3. (, [?]):
M_0 $\theta_t^{(0)}$, F_0. $\delta > 0$ $W^{(s,u)}_{loc}(M_0) = W^{(s,u)}_{loc}(M_0) \cap U_\delta(M_0)$. $\varepsilon_0 > 0$, F_ε, $|F_\varepsilon - F_0| < \varepsilon < \varepsilon_0$ M_ε, ε M_0, $O(\varepsilon)$ $W^{s,u}_{loc}(M_0)$.

4. (, [?]):
M_0 ( ) (4), (??).
$\varepsilon > 0$ M_ε (4) , ε M_0.

, , .
- $W^s(M_0) \cap W^u(M_0)$. , , M_0 , . , $W^s(M_\varepsilon) \cap W^u(M_\varepsilon)$ - , .
, , .

3.2.

. , , .
. :

$$\begin{cases} \frac{dX}{dt} = JD_X H_1(X, I) + \varepsilon g^X(X, I), \\ \frac{dI}{dt} = \varepsilon g^I(X, I), \end{cases} \tag{15}$$

$$X \in \mathbb{R}^2, I \in \mathbb{R}^m,$$

D_X, D_I - , J - :

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

5. ([?]):

$V \subset M_0$ - M_0 (15), V_ε ε- V, .

$$\Gamma = W^s(V) \cap W^u(V) \setminus V$$

$X_0^I(t-t_0, I)$ - , (15).

$W^s(V_\varepsilon)$ $W^u(V_\varepsilon)$ $p \in \Gamma$:

$$d^I(p, t_0, \varepsilon) = \varepsilon \frac{M^I(p, t_0)}{||D_X H(X_0^I, I)||} + O(\varepsilon^2),$$

M^I - , :

$$M^I(p, t_0) = \int_{-\infty}^{\infty} \Big(< D_X H_1, g^X > + < D_X H_1, (D_I J D_X) \int_{t_0}^{t} g^I dt > \Big) \big(X_0^I(t-t_0, I), I\big) dt =$$

$$= \int_{-\infty}^{\infty} \Big(< D_X H_1, g^X > + < D_I H_1, g^I > \Big) \big(X_0^I(t-t_0, I), I\big) dt -$$

$$- < D_I H_1\big(v(I), I\big), \int_{-\infty}^{\infty} g^I \big(X_0^I(t-t_0, I), I\big) dt >,$$

$X = v(I)$ - V, $<,>$ - $\mathbb{R}^2$.

[?]: $p \in \Gamma = W^s(V) \cap W^u(V) \setminus V$ Π_p $D_X H_1(p)$.

(3) $p_\varepsilon^u \equiv (X_\varepsilon^u, I_\varepsilon^u) \in W^u(V_\varepsilon) \cap \Gamma \cap \Pi_p$ $p_\varepsilon^s \equiv (X_\varepsilon^s, I_\varepsilon^s) \in W^s(V_\varepsilon) \cap \Gamma \cap \Pi_p$, $I_\varepsilon^s = I_\varepsilon^u$.

$W^s(V_\varepsilon)$ $W^u(V_\varepsilon)$:

$$d(p, t_0, \varepsilon) = ||p_\varepsilon^u - p_\varepsilon^s|| = ||X_\varepsilon^u - X_\varepsilon^s|| = \frac{< D_X H_1\big(X_0^I(t-t_0, I), I\big), X_\varepsilon^u - X_\varepsilon^s >}{||D_X H_1\big(X_0^I(t-t_0, I), I\big)||}.$$

ε , $d(p, 0) = 0$, $p \in W^s(V) \cap W^u(V)$, :

$$d(p, t_0, \varepsilon) = \varepsilon \frac{< D_X H_1\big(X_0^I(t-t_0, I), I\big), \frac{\partial X_\varepsilon^u}{\partial \varepsilon}\big|_{\varepsilon=0} - \frac{\partial X_\varepsilon^s}{\partial \varepsilon}\big|_{\varepsilon=0} >}{||D_X H_1\big(X_0^I(t-t_0, I), I\big)||} + O(\varepsilon^2).$$

:

$$M^I(p, t_0) = < D_X H_1\big(X_0^I(t-t_0, I), I\big), \frac{\partial X_\varepsilon^u}{\partial \varepsilon}\Big|_{\varepsilon=0} - \frac{\partial X_\varepsilon^s}{\partial \varepsilon}\Big|_{\varepsilon=0} > .$$

$$M(t, t_0) = < D_X H_1\big(X_0^I(t-t_0, I), I\big), \frac{\partial X_\varepsilon^u(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} - \frac{\partial X_\varepsilon^s(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} >= \Delta^u(t) - \Delta^s(t),$$

$$\Delta^{u,s}(t) = < D_X H_1\big(X_0^I(t-t_0, I), I\big), \frac{\partial x_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} > .$$

ε :

$$\frac{d}{dt} \frac{\partial X_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} = J D_X^2 H_1 \frac{\partial X_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} + D_I J D_X H_1 \frac{\partial I_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} + g^X\big(X_0^I(t-t_0, I), I\big),$$

$$\frac{d}{dt} \frac{\partial I_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0} = g^I\big(X_0^I(t-t_0, I), I\big).$$

:

$$x_1^{u,s}(t) = \frac{\partial X_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0},$$

$$I_1^{u,s}(t) = \frac{\partial I_\varepsilon^{u,s}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0},$$

:

$$\frac{d}{dt}\Delta^{u,s}(t) =< \frac{d}{dt}\Big(D_X H_1\big(X_0^I(t-t_0,I),I\big)\Big), x_1^{u,s} > + < D_X H_1\big(X_0^I(t-t_0,I),I\big), \frac{d}{dt}x_1^{u,s} >=$$

$$=< D_X H_1, (JD_X^2 H_1)x_1^{u,s} > + < D_X H_1, (D_I J D_X H_1) I_1^{u,s} > +$$
$$+ < D_X H_1, g^X > + < (D_X^2 H_1)(J D_X H_1), x_1^{u,s} > .$$

$$< D_X H_1, (JD_X^2 H_1)x_1^{u,s} > + < (D_X^2 H_1)(J D_X H_1), x_1^{u,s} >= 0,$$

:

$$\frac{d}{dt}\Delta^{u,s}(t) =< D_X H_1, (D_I J D_X H_1) I_1^{u,s} > + < D_X H_1, g^X > .$$

:

$$I_1^s(t) = I_1^u(t) = \int_{t_0}^t g^I\big(X_0^I(t-t_0,I),I\big)dt.$$

t:

$$\Delta^u(0) - \Delta^u(-T^u) = \int_{-T^u}^0 \Big(< D_X H_1, g^X > + < D_X H_1, (D_I J D_X) \int_{t_0}^t g^I dt > \Big)\big(X_0^I(t-t_0,I),I\big)dt,$$

$$\Delta^s(T^s) - \Delta^s(0) = \int_0^{T^s} \Big(< D_X H_1, g^X > + < D_X H_1, (D_I J D_X) \int_{t_0}^t g^I dt > \Big)\big(X_0^I(t-t_0,I),I\big)dt.$$

$W^{s,u}(V)$ $T^{s,u}$, , . , $\delta > 0$ ε_0 , $\varepsilon < \varepsilon_0$ $T^{s,u} > \frac{1}{\delta}$.
, :

$$M^I(p,t_0) = M^I(t=0,t_0),$$

:

$$M^I(p,t_0) = M^I(t=0,t_0) \approx$$

$$\approx \int_{-T^u}^{T^s} \Big(< D_X H_1, g^X > + < D_X H_1, (D_I J D_X) \int_{t_0}^t g^I dt > \Big)\big(X_0^I(t-t_0,I),I\big)dt + \Delta^u(-T^u) - \Delta^s(T^s).$$

6. (, [?]):
:

$$M^I = 0,$$
$$\nabla M^I \neq 0.$$

$\varepsilon > 0$ $W^s(V_\varepsilon)$ $W^u(V_\varepsilon)$.
, ε d^I .

3.3.

1,2

2. (4) :

1.

$$M_0, M_{0,+}, M_{0,-}, M_{0,b}^{\pm}$$

.

2. $M_{0,-}$.
3. $\Big(x(t), y(t), 0, \lambda_-\big(x(t), y(t)\big), \Lambda = 0\Big)$ - (7) $M_{0,-}$, xy .

. (4) (15):

$$X = \begin{pmatrix} \Lambda \\ z \end{pmatrix}, I = \begin{pmatrix} x \\ y \end{pmatrix}.$$

z :

$$z = \lambda - \lambda_-(x, y).$$

$M_{0,-}$ $\tilde{M}_{0,-}$:

$$\tilde{M}_{0,-} = \{(\Lambda, z, x, y) : \Lambda = 0, \ z = 0\},$$

:

$$z_{sep}(t, t_0) = \pm 2 \arctan \operatorname{sh}\big(\alpha\beta(t - t_0)\big), \Lambda_{sep}(t, t_0) = 2\beta / \operatorname{ch}\big(\alpha\beta(t - t_0)\big).$$

:

$$D_X = \begin{pmatrix} \frac{\partial}{\partial \Lambda} \\ \frac{\partial}{\partial z} \end{pmatrix}, D_I = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix},$$

$$H_1 = \alpha \frac{\Lambda^2}{2} + \beta(x, y)(1 - \cos z), \quad \beta(x, y) = \sqrt{u(x, y)^2 + v(x, y)^2},$$

$$g^I = \begin{pmatrix} -2Fy + \frac{\partial \beta}{\partial y} \cos z + \beta(x, y) \frac{\partial}{\partial y} \arctan(\frac{v}{u}) \sin z \\ 2Fx + e_J G - \frac{\partial \beta}{\partial x} \cos z - \beta(x, y) \frac{\partial}{\partial x} \arctan(\frac{v}{u}) \sin z \end{pmatrix},$$

$$g^X = \frac{-1}{\varepsilon} \begin{pmatrix} 0 \\ \frac{d}{dt} \arctan(\frac{v}{u}) \end{pmatrix} = \frac{-(u\dot{v} - v\dot{u})}{\varepsilon\beta^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{-\Big(\dot{x}\big(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\big) + \dot{y}\big(u\frac{\partial v}{\partial y} - v\frac{\partial u}{\partial y}\big)\Big)}{\varepsilon\beta^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{\big(2Fy + \frac{\partial u}{\partial y}\cos\lambda + \frac{\partial v}{\partial y}\sin\lambda\big)\big(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\big) - \big(2Fx + e_J G + \frac{\partial u}{\partial x}\cos\lambda + \frac{\partial v}{\partial x}\sin\lambda\big)\big(u\frac{\partial v}{\partial y} - v\frac{\partial u}{\partial y}\big)}{\beta^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

3.4.

$V = \tilde{M}_{0,-}$ 5. :

$$< D_I H_1, g^I >= (1 - \cos z)\Bigg(-2Fy\frac{\partial \beta}{\partial x} + (2Fx + e_J G)\frac{\partial \beta}{\partial y} + \frac{\sin z}{\beta}\bigg(\frac{\partial \beta}{\partial y}\Big(v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y}\Big) - \frac{\partial \beta}{\partial x}\Big(v\frac{\partial u}{\partial x} - u\frac{\partial v}{\partial x}\Big)\bigg)\Bigg),$$

$$< D_X H_1, g^X >= \frac{-\beta \sin z}{\varepsilon} \frac{d}{dt}\Big(\arctan\frac{v}{u}\Big),$$

$$D_I H_1\big(v(I), I\big) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

:

$$M^I(x,y) = \int_{-\infty}^{\infty} \big(1 - \cos z_{sep}(t)\big) dt \Big(-2Fy \frac{\partial \beta}{\partial x} + (2Fx + e_J G) \frac{\partial \beta}{\partial y} \Big) +$$

$$+ \frac{1}{\beta} \int_{-\infty}^{\infty} \sin z_{sep}(t) dt \bigg(\Big(v \frac{\partial u}{\partial y} - u \frac{\partial v}{\partial y} \Big) \Big(-\frac{\partial \beta}{\partial x} + 2Fx + e_J G \Big) + \Big(v \frac{\partial u}{\partial x} - u \frac{\partial v}{\partial x} \Big) \Big(\frac{\partial \beta}{\partial y} - 2Fy \Big) \bigg).$$

z_{sep}, :

$$\int_{-\infty}^{\infty} \big(1 - \cos z_{sep}(t)\big) dt = \int_{-\infty}^{\infty} \frac{2}{\operatorname{ch}^2(\alpha\beta t)} dt = \frac{4}{\alpha\beta},$$

$$\int_{-\infty}^{\infty} \sin z_{sep}(t) dt = \int_{-\infty}^{\infty} \frac{2 \operatorname{sh}(\alpha\beta t)}{\operatorname{ch}^2(\alpha\beta t)} dt = 0.$$

,

$$M^I(x,y) = \frac{4}{\alpha\beta} \left(-2Fy \frac{\partial \beta}{\partial x} + (2Fx + e_J G) \frac{\partial \beta}{\partial y} \right) =$$

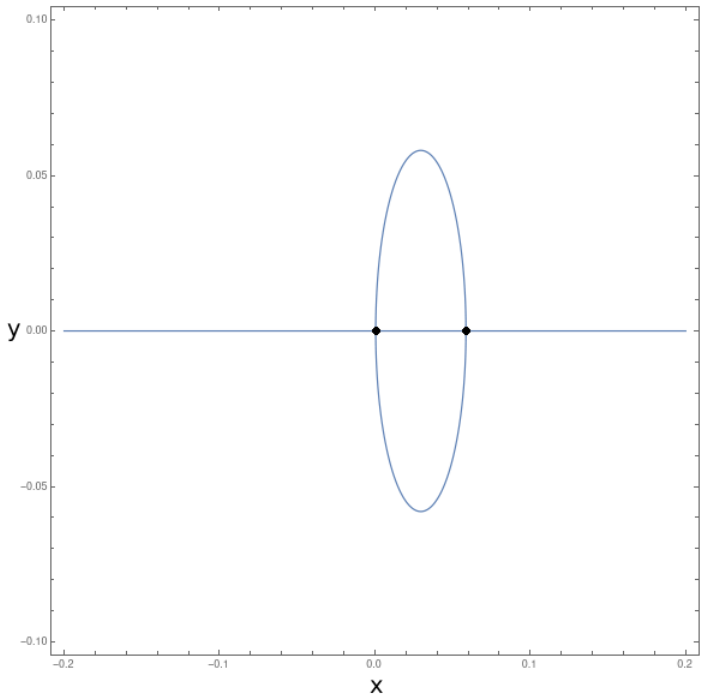
$$= \frac{4}{\alpha\beta^2} \bigg(u \Big((2Fx + e_J G) \frac{\partial u}{\partial y} - 2Fy \frac{\partial u}{\partial x} \Big) + v \Big((2Fx + e_J G) \frac{\partial v}{\partial y} - 2Fy \frac{\partial v}{\partial x} \Big) \bigg) =$$

$$= \frac{4Cy}{\alpha\beta^2} \Big(-2u\big((2Fx + e_J G) + F(2x + \tilde{\alpha})\big) + C(2x + \tilde{\alpha})\big((2Fx + e_J G)(2x + \tilde{\alpha}) - 4Fy^2\big) \Big).$$

:

$$0 = y \left((2x + \tilde{\alpha})^2 + y^2 - (2x + \tilde{\alpha}) \frac{4FK}{C(e_J G - F\tilde{\alpha})} - \frac{K}{C} \right),$$

$y = 0$ (. . 6).



. 6. .

$D_I M^I$:

$$y_M = 0,$$

$$x_M = \frac{-\tilde{\alpha}}{2} + \frac{FK}{C(e_J G - F\tilde{\alpha})} \pm \sqrt{\frac{F^2K^2}{C^2(e_J G - F\tilde{\alpha})} + \frac{K}{4C}}.$$

,

3. 1. $\{\big(x(\tau), y(\tau), 0, \lambda_-\big(x(\tau), y(\tau)\big)\big), \tau \in \mathbb{R}\} \in M_{0,-}$ - (7) $M_{0,-}$, D_0 - δ- $M_{0,-}$, $(x_b^{\pm}, y_b)$. $\varepsilon > 0$ D_ε (4), .

2. $\varepsilon > 0$ $\big(x_0^-, y_0^-, 0, \lambda_-(x_0^-, y_0^-)\big)$ (7). , $\varepsilon > 0$, , .

:

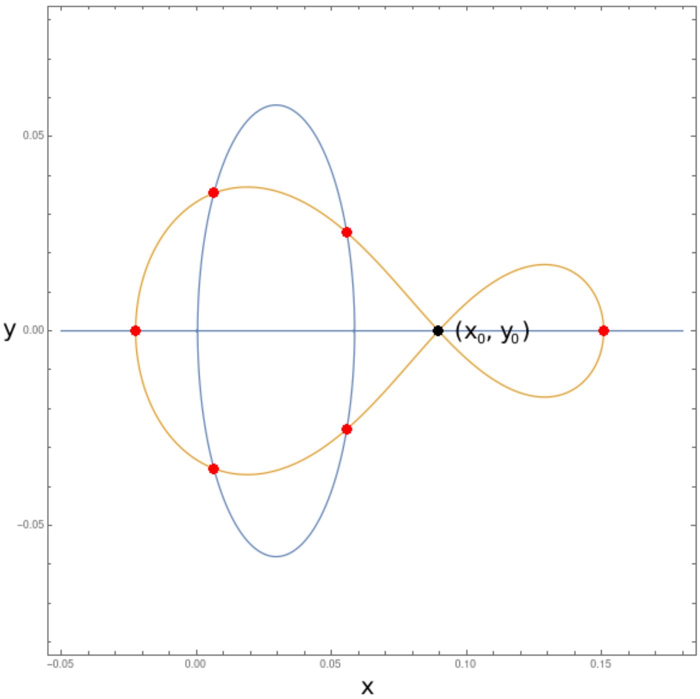
D_0 $M_{0,-}$. $\varepsilon > 0$ D_ε, ε D_0. , , .

,

$$\big(x_0^-, y_0^-, 0, \lambda_-(x_0^-, y_0^-)\big)$$

(24), (4). , , .

, , (. . 7), .



. 7. $M_{0,-}$

3.5.

, μ (1), . , .

$(x, y, \Lambda, \lambda) = \big(x_0^-, y_0^-, 0, \lambda_-(x_0^-, y_0^-)\big) \in M_{0,-}$, , , $t \to \pm\infty$. (1).

4.

.

$(\hat{x}_0^+, 0, \lambda_+(\hat{x}_0^+, 0), 0) \in M_{0,+}$, , (4). , - :

$$(\pm i\omega, \pm\xi),$$

$$\omega = \frac{\sqrt{\sqrt{p} - q}}{\sqrt{2}} = \sqrt{|\alpha U(\hat{x}_0^+, 0)|} + \mathcal{O}(\varepsilon^2),$$
$$\xi = \frac{\sqrt{\sqrt{p} + q}}{\sqrt{2}} = \mathcal{O}(\varepsilon),$$

$$p = (4\varepsilon^2(C - F)(C + F) + \alpha U(\hat{x}_0^+, 0))^2 + 8\alpha\varepsilon^2(F - C)V_y(\hat{x}_0^+, 0) >, 0$$
$$q = 4\varepsilon^2(C - F)(C + F) - \alpha U(\hat{x}_0^+, 0) < 0.$$

- . , , . ε, , , .

4.1.

. , , : , .

- 0, :

$$\Lambda \to \Lambda,$$
$$\lambda \to \lambda - \lambda_+(\hat{x}_0, 0) = \lambda - \pi,$$
$$x \to x - \hat{x}_0,$$
$$y \to y.$$

:

$$\begin{cases} \dot{\Lambda} = -U \sin\lambda + V \cos\lambda \equiv g^\Lambda(\Lambda, \lambda, x, y), \\ \dot{\lambda} = \alpha\Lambda \equiv g^\lambda(\Lambda, \lambda, x, y), \\ \dot{x} = -\varepsilon\big(2Fy - \frac{\partial U}{\partial y}\cos\lambda - \frac{\partial V}{\partial y}\sin\lambda\big) \equiv g^x(\Lambda, \lambda, x, y), \\ \dot{y} = \varepsilon\big(2F(x + \hat{x}_0) + e_J G - \frac{\partial U}{\partial x}\cos\lambda - \frac{\partial V}{\partial x}\sin\lambda\big) \equiv g^y(\Lambda, \lambda, x, y), \end{cases} \tag{16}$$

$$U(x, y) = U_0 + x(2C\hat{x}_0 + e_J D) + C(x^2 - y^2),$$
$$V(x, y) = 2Cxy + y(2C\hat{x}_0 + e_J D).$$

, (16)

$$\Omega = d\Lambda \wedge d\lambda + \varepsilon^{-1} dx \wedge dy.$$

:

$$H = \frac{\alpha\Lambda^2}{2} - U(x, y)\cos\lambda - V(x, y)\sin\lambda + F\left((x + \hat{x}_0)^2 + y^2\right) + e_J Gx. \tag{17}$$

4.1.1.

(16), $W^{s,u}(0)$. (16) :

- $W^s(0)$:

$$\begin{cases} \Lambda^s(+\infty) = 0, \\ \lambda^s(+\infty) = 0, \\ x^s(+\infty) = 0, \\ y^s(+\infty) = 0, \end{cases}$$

- $W^u(0)$:

$$\begin{cases} \Lambda^u(-\infty) = 0, \\ \lambda^u(-\infty) = 0, \\ x^u(-\infty) = 0, \\ y^u(-\infty) = 0. \end{cases}$$

(16) :

$$\begin{cases} \Lambda^{s,u}(\tau) = \sum_{k=0}^{\infty} \varepsilon^k \Lambda_k^{s,u}(\tau) \\ \lambda^{s,u}(\tau) = \sum_{k=0}^{\infty} \varepsilon^k \lambda_k^{s,u}(\tau) \\ x^{s,u}(\tau) = \sum_{k=0}^{\infty} \varepsilon^k x_k^{s,u}(\tau) \\ y^{s,u}(\tau) = \sum_{k=0}^{\infty} \varepsilon^k y_k^{s,u}(\tau) \end{cases} \tag{18}$$

, $\tau = \varepsilon t$. ε, , , 2 , $(\lambda_k, \Lambda_k), k \geq 1$:

$$\begin{cases} \Lambda_k(\tau) = \frac{1}{\alpha}\lambda'_{k-1}, \\ \lambda_k(\tau) = \frac{1}{\beta}\Big(\alpha^{-1}\lambda''_{k-2} - G_k^{\Lambda} + x_k\big((2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 - 2Cy_0\cos\lambda_0\big) + \\ \qquad + y_k\big(-2Cy_0\sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\big)\Big), \\ \lambda_{-1} = 0, \end{cases} \tag{19}$$

$(x_k, y_k), k \geq 1$ - :

$$\frac{d}{d\tau}\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} a(\tau) & b(\tau) \\ c(\tau) & -a(\tau) \end{pmatrix}\begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} \tilde{G}_k^x(\tau) \\ \tilde{G}_k^y(\tau) \end{pmatrix} \equiv \mathcal{A}(\tau)\begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} \tilde{G}_k^x(\tau) \\ \tilde{G}_k^y(\tau) \end{pmatrix}, \tag{20}$$

:

$$\tilde{G}_k^x = G_k^x - \beta^{-1}(\alpha^{-1}\lambda''_{k-2} - G_k^{\Lambda})\big(-2Cy_0\sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\big),$$
$$\tilde{G}_k^y = G_k^y - \beta^{-1}(\alpha^{-1}\lambda''_{k-2} - G_k^{\Lambda})\big(-(2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 + 2Cy_0\cos\lambda_0\big).$$

a, b, c - $x_0(\tau), y_0(\tau), \lambda_0(\tau), \Lambda_0(\tau)$:

$$a(\tau) = \frac{(e_J^2 D^2 - 4CEe_J^2)\sin\lambda_0\cos\lambda_0}{\beta(x_0, y_0)},$$

$$b(\tau) = -2F - 2C\cos\lambda_0 - \frac{1}{\beta(x_0, y_0)}\big((2Cy_0)\sin\lambda_0 + (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\big)^2,$$

$$c(\tau) = 2F - 2C\cos\lambda_0 + \frac{1}{\beta(x_0, y_0)}\big((2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 - (2Cy_0)\cos\lambda_0\big)^2.$$

4. 1) $G_k^{\rho}, \rho \in \{\lambda, \Lambda, x, y\}$, (20), :

$$G_k^{\rho} = \sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^{\rho}}{j!}, \quad k \geq 2,$$

g^{ρ} - , $\rho \in \{\lambda, \Lambda, x, y\}$,

$$\mathcal{L}_k^{(j)} = \sum_{k_1 + \ldots + k_j = k} \mathfrak{D}_{k_1} \cdot \ldots \cdot \mathfrak{D}_{k_j},$$

$$\mathfrak{D}_k = \left(\begin{pmatrix} \lambda_k \\ \Lambda_k \\ x_k \\ y_k \end{pmatrix}, \begin{pmatrix} \frac{\partial}{\partial \lambda}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial \Lambda}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial x}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial y}|_{\lambda_0,\Lambda_0,x_0,y_0} \end{pmatrix} \right),$$

$(\cdot,\cdot)$ – $\mathbb{R}^4$.

2) :

$$\mathcal{F}_s = \left\{ f\colon \mathbb{R} \to \mathbb{R} \;\middle|\; \exists C > 0 : \; \forall x \in \mathbb{R} \Rightarrow \; |f(x)| \le C e^{-s|x|} \right\}, s > 0$$

$$\mathcal{F}_{-s} = \left\{ f\colon \mathbb{R} \to \mathbb{R} \;\middle|\; \exists C > 0 : \; \forall x \in \mathbb{R} \Rightarrow \; |f(x)| \ge C e^{s|x|} \right\}, s > 0$$

$\lambda_j, \Lambda_j, x_j, y_j \in \mathcal{F}_s \; \forall \; 0 \le j \le k, \;\; g^\rho \in C_b^\infty(\mathbb{R}^4)$ ($C_b^\infty(\mathbb{R}^4)$,).
$G_{k+1}^\rho \in \mathcal{F}_{2s}$

3)

$$G_k^\lambda = 0 \quad \forall k \ge 1$$

:

1) $g^\rho(\lambda, \Lambda, x, y)$ (18). :

$$g^\rho(\lambda, \Lambda, x, y) = g^\rho(\lambda_0, \Lambda_0, x_0, y_0) + \sum_{j=1}^{+\infty} \frac{\mathcal{T}^j g^\rho}{j!},$$

$\mathcal{T}$ - :

$$\mathcal{T} = \left(\sum_{k=1}^{+\infty} \lambda_k \varepsilon^k \right) \frac{\partial}{\partial \lambda}\bigg|_{\lambda_0,\Lambda_0,x_0,y_0} + \left(\sum_{k=1}^{+\infty} \Lambda_k \varepsilon^k \right) \frac{\partial}{\partial \Lambda}\bigg|_{\lambda_0,\Lambda_0,x_0,y_0} + \\ + \left(\sum_{k=1}^{+\infty} x_k \varepsilon^k \right) \frac{\partial}{\partial x}\bigg|_{\lambda_0,\Lambda_0,x_0,y_0} + \left(\sum_{k=1}^{+\infty} y_k \varepsilon^k \right) \frac{\partial}{\partial y}\bigg|_{\lambda_0,\Lambda_0,x_0,y_0}.$$

$$\mathfrak{D}_k = \left(\begin{pmatrix} \lambda_k \\ \Lambda_k \\ x_k \\ y_k \end{pmatrix}, \begin{pmatrix} \frac{\partial}{\partial \lambda}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial \Lambda}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial x}|_{\lambda_0,\Lambda_0,x_0,y_0} \\ \frac{\partial}{\partial y}|_{\lambda_0,\Lambda_0,x_0,y_0} \end{pmatrix} \right),$$

:

$$\mathcal{T} = \sum_{k=1}^{+\infty} \varepsilon^k \mathfrak{D}_k.$$

$\mathcal{T}^j$, , :

$$\mathcal{T}^j = \left(\sum_{k=1}^{+\infty} \varepsilon^k \mathfrak{D}_k \right)^j = \sum_{k=j}^{+\infty} \varepsilon^k \mathcal{L}_k^{(j)},$$

:

$$\mathcal{L}_k^{(j)} = \sum_{k_1+\ldots+k_j=k} \mathfrak{D}_{k_1} \cdot \ldots \cdot \mathfrak{D}_{k_j}.$$

ε :

$$g^\rho(\lambda, \Lambda, x, y) = g^\rho(\lambda_0, \Lambda_0, x_0, y_0) + \sum_{k=1}^{+\infty} \varepsilon^k \left(\sum_{j=1}^{k} \frac{\mathcal{L}_k^{(j)} g^\rho}{j!} \right).$$

2 , $\mathcal{L}_k^{(1)} = \mathfrak{D}_k$:

$$g^\rho(\lambda, \Lambda, x, y) = g^\rho(\lambda_0, \Lambda_0, x_0, y_0) + \varepsilon \mathfrak{D}_1 g^\rho + \sum_{k=2}^{+\infty} \varepsilon^k \left(\mathcal{L}_k^{(1)} g^\rho + \sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^\rho}{j!} \right) =$$

$$= g^\rho(\lambda_0, \Lambda_0, x_0, y_0) + \varepsilon \mathfrak{D}_1 g^\rho + \sum_{k=2}^{+\infty} \varepsilon^k \left(\mathfrak{D}_k g^\rho + \underbrace{\sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^\rho}{j!}}_{G_k^\rho} \right)$$

- $\mathfrak{D}_k g^\rho$ - $(\lambda_k, \Lambda_k, x_k, y_k)$;
- G_k^ρ - $(\lambda_j, \Lambda_j, x_j, y_j)$, $j \le k-1$ (20).

G_k^ρ $k \ge 2$, $k = 1$ $G_1^\rho = 0$. :

$$G_k^\rho = \sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^\rho}{j!}, \quad k \ge 2, \tag{21}$$

$$G_1^\rho \equiv 0,$$

$$g^\rho(\lambda, \Lambda, x, y) = g^\rho(\lambda_0, \Lambda_0, x_0, y_0) + \sum_{k=1}^{\infty} \varepsilon^k \mathfrak{D}_k g^\rho + \sum_{k=2}^{+\infty} \varepsilon^k G_k^\rho.$$

2) $g^\rho, \rho \in \{\lambda, \Lambda, x, y\}$, (21) $j = 2$, G_{k+1}^ρ 2 ($\le k$) $\mathfrak{D}_i \mathfrak{D}_j g^\rho, i + j \le k$. $\le k$ $\mathcal{F}_s$, , , G_{k+1}^ρ $\mathcal{F}_{2s}$.

3) $g^\lambda = \alpha \Lambda$, 0. $G_k^\lambda = 0$ $\forall k \ge 1$. ■

$\varepsilon = 0$. , $W^{s,u}(0)$, :

$$\begin{cases} \Lambda_0(\tau) = 0, \\ \lambda_0(\tau) = \lambda_-(x_{sep}(\tau), y_{sep}(\tau)), \\ x_0(\tau) = x_{sep}(\tau) - \hat{x}_0, \\ y_0(\tau) = y_{sep}(\tau). \end{cases}$$

5. 1) $(\Lambda_1, \lambda_1, x_1, y_1)$ (Λ_1, λ_1):

$$\begin{cases} \Lambda_1(\tau) = \lambda_0', \\ \lambda_1(\tau) = \frac{1}{\beta}\Big(x_1\big((2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 - 2Cy_0\cos\lambda_0\big) + \\ + y_1\big(- 2Cy_0 \sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\big)\Big), \end{cases}$$

(x_1, y_1):

$$\frac{d}{d\tau}\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \mathcal{A}(\tau)\begin{pmatrix} x_k \\ y_k \end{pmatrix}. \tag{22}$$

2) (22) :

$$\begin{cases} x_1(\tau) = x_1^{s,u}(\tau) = \dfrac{dx_0}{d\tau}, \\ y_1(\tau) = y_1^{s,u}(\tau) = \dfrac{dy_0}{d\tau}. \end{cases} \tag{23}$$

:

1) 4 - :

$$\begin{cases} \dot{\Lambda}(t) = g^{\Lambda}(\Lambda, \lambda, x, y), \\ \dot{\lambda}(t) = g^{\lambda}(\Lambda, \lambda, x, y), \\ \dot{x}(t) = \varepsilon g^{x}(\Lambda, \lambda, x, y), \\ \dot{y}(t) = \varepsilon g^{y}(\Lambda, \lambda, x, y), \end{cases}$$

$\tau = \varepsilon t$:

$$\begin{cases} \varepsilon\Lambda'(\tau) = g^{\Lambda}(\Lambda, \lambda, x, y), \\ \varepsilon\lambda'(\tau) = g^{\lambda}(\Lambda, \lambda, x, y), \\ x'(\tau) = g^{x}(\Lambda, \lambda, x, y), \\ y'(\tau) = g^{y}(\Lambda, \lambda, x, y), \end{cases}$$

$\boldsymbol{X} = (\Lambda, \lambda, x, y)$: $\boldsymbol{X} = \sum_{k=0}^{\infty} \varepsilon^k \boldsymbol{X}_k$. ε :

$$\begin{cases} 0 = g^{\Lambda}(\boldsymbol{X}_0), \\ 0 = g^{\lambda}(\boldsymbol{X}_0), \\ x_0'(\tau) = g^{x}(\boldsymbol{X}_0), \\ y_0'(\tau) = g^{y}(\boldsymbol{X}_0), \end{cases} \tag{24}$$

$$\begin{cases} \Lambda_0'(\tau) = (\nabla g^{\Lambda}(\boldsymbol{X}_0), \boldsymbol{X}_1), \\ \lambda_0'(\tau) = (\nabla g^{\lambda}(\boldsymbol{X}_0), \boldsymbol{X}_1), \\ x_1'(\tau) = (\nabla g^{x}(\boldsymbol{X}_0), \boldsymbol{X}_1), \\ y_1'(\tau) = (\nabla g^{y}(\boldsymbol{X}_0), \boldsymbol{X}_1), \end{cases} \tag{25}$$

, (25) (Λ_1, λ_1), :

$$\begin{pmatrix}(\nabla g^\Lambda)_\Lambda & (\nabla g^\Lambda)_\lambda \\ (\nabla g^\lambda)_\Lambda & (\nabla g^\lambda)_\lambda\end{pmatrix}\begin{pmatrix}\Lambda_1\\ \lambda_1\end{pmatrix} = A\begin{pmatrix}\Lambda_1\\ \lambda_1\end{pmatrix} = \begin{pmatrix}\Lambda_0' - (\nabla g^\Lambda)_x x_1 - (\nabla g^\Lambda)_y y_1 \\ \lambda_0' - (\nabla g^\lambda)_x x_1 - (\nabla g^\lambda)_y y_1\end{pmatrix},$$

$$A = \begin{pmatrix}(\nabla g^\Lambda)_\Lambda & (\nabla g^\Lambda)_\lambda \\ (\nabla g^\lambda)_\Lambda & (\nabla g^\lambda)_\lambda\end{pmatrix}.$$

:

$$\begin{pmatrix}\Lambda_1\\ \lambda_1\end{pmatrix} = M\begin{pmatrix}x_1\\ y_1\end{pmatrix} + \boldsymbol{C},$$

$$M = \frac{1}{\det A}\begin{pmatrix}-(\nabla g^\lambda)_\lambda(\nabla g^\Lambda)_x + (\nabla g^\Lambda)_\lambda(\nabla g^\lambda)_x & -(\nabla g^\lambda)_\lambda(\nabla g^\Lambda)_y + (\nabla g^\Lambda)_\lambda(\nabla g^\lambda)_y \\ (\nabla g^\lambda)_\Lambda(\nabla g^\Lambda)_x - (\nabla g^\Lambda)_\Lambda(\nabla g^\lambda)_x & (\nabla g^\lambda)_\Lambda(\nabla g^\Lambda)_y - (\nabla g^\Lambda)_\Lambda(\nabla g^\lambda)_y\end{pmatrix},$$

$$\boldsymbol{C} = \frac{1}{\det A}\begin{pmatrix}(\nabla g^\lambda)_\lambda\Lambda_0' - (\nabla g^\Lambda)_\lambda\lambda_0' \\ -(\nabla g^\lambda)_\Lambda\Lambda_0' + (\nabla g^\Lambda)_\Lambda\lambda_0'\end{pmatrix}.$$

(25) :

$$\begin{cases} x_1'(\tau) = \Big((\nabla g^x)_x + \big((\nabla g^x)_\Lambda M_{11} + (\nabla g^x)_\lambda M_{21}\big)\Big)x_1 + \\ \qquad + \Big((\nabla g^x)_y + \big((\nabla g^x)_\Lambda M_{12} + (\nabla g^x)_\lambda M_{22}\big)\Big)y_1 + \\ \qquad + \left(\begin{pmatrix}(\nabla g^x)_\Lambda \\ (\nabla g^x)_\lambda\end{pmatrix}, \boldsymbol{C}\right), \\ \\ y_1'(\tau) = \Big((\nabla g^y)_x + \big((\nabla g^y)_\Lambda M_{11} + (\nabla g^y)_\lambda M_{21}\big)\Big)x_1 + \\ \qquad + \Big((\nabla g^x)_y + \big((\nabla g^y)_\Lambda M_{12} + (\nabla g^y)_\lambda M_{22}\big)\Big)y_1 + \\ \qquad + \left(\begin{pmatrix}(\nabla g^y)_\Lambda \\ (\nabla g^y)_\lambda\end{pmatrix}, \boldsymbol{C}\right). \end{cases} \tag{26}$$

$(\cdot,\cdot)$ – $\mathbb{R}^n$.

, , , (16) . , :

$$\Lambda_0 = 0 \quad \Rightarrow \quad \boldsymbol{C} = \begin{pmatrix}\frac{\lambda_0'}{\alpha} \\ 0\end{pmatrix}$$

, g^x, g^y Λ, , (26) :

$$\begin{pmatrix}(\nabla g^x)_\Lambda \\ (\nabla g^x)_\lambda\end{pmatrix} = \begin{pmatrix}0\\ *\end{pmatrix},$$

$$\begin{pmatrix}(\nabla g^y)_\Lambda \\ (\nabla g^y)_\lambda\end{pmatrix} = \begin{pmatrix}0\\ *\end{pmatrix}.$$

(26) :

$$\left(\begin{pmatrix}(\nabla g^x)_\Lambda\\(\nabla g^x)_\lambda\end{pmatrix},\boldsymbol{C}\right)=\left(\begin{pmatrix}(\nabla g^y)_\Lambda\\(\nabla g^y)_\lambda\end{pmatrix},\boldsymbol{C}\right)=0.$$

, (16) (x_1, y_1) .

2) (24). x_0.

x_0 , , Λ_0, λ_0 x_0, y_0:

$$\begin{aligned} x_0'' = (\nabla g^x)_x x_0' + (\nabla g^x)_y y_0' + (\nabla g^x)_\Lambda \Lambda_0' + (\nabla g^x)_\lambda \lambda_0' = \\ = \left((\nabla g^x)_x + (\nabla g^x)_\Lambda \frac{\partial \Lambda_0}{\partial x} + (\nabla g^x)_\lambda \frac{\partial \lambda_0}{\partial x}\right) x_0' + \\ + \left((\nabla g^x)_y + (\nabla g^x)_\Lambda \frac{\partial \Lambda_0}{\partial y} + (\nabla g^x)_\lambda \frac{\partial \lambda_0}{\partial y}\right) y_0'. \end{aligned} \tag{27}$$

, $\Lambda_0(x_0, y_0), \lambda_0(x_0, y_0)$:

$$\begin{cases} 0 = g^\Lambda(\boldsymbol{X}_0), \\ 0 = g^\lambda(\boldsymbol{X}_0). \end{cases} \tag{28}$$

x:

$$\begin{cases} (\nabla g^\Lambda)_x + (\nabla g^\Lambda)_y \underbrace{y_x}_{0} + (\nabla g^\Lambda)_\Lambda \cdot \Lambda_x + (\nabla g^\Lambda)_\lambda \cdot \lambda_x = 0, \\ (\nabla g^\lambda)_x + (\nabla g^\lambda)_y \underbrace{y_x}_{0} + (\nabla g^\lambda)_\Lambda \cdot \Lambda_x + (\nabla g^\lambda)_\lambda \cdot \lambda_x = 0. \end{cases}$$

:

$$\begin{pmatrix}(\nabla g^\Lambda)_\Lambda & (\nabla g^\Lambda)_\lambda \\ (\nabla g^\lambda)_\Lambda & (\nabla g^\lambda)_\lambda\end{pmatrix}\begin{pmatrix}\Lambda_x\\ \lambda_x\end{pmatrix} = -\begin{pmatrix}(\nabla g^\Lambda)_x\\ (\nabla g^\lambda)_x\end{pmatrix}.$$

(Λ_x, λ_x) :

$$\Lambda_x = \frac{(\nabla g^\Lambda)_\lambda(\nabla g^\lambda)_x - (\nabla g^\Lambda)_x(\nabla g^\lambda)_\lambda}{(\nabla g^\Lambda)_\Lambda(\nabla g^\lambda)_\lambda - (\nabla g^\Lambda)_\lambda(\nabla g^\lambda)_\Lambda},$$
$$\lambda_x = \frac{(\nabla g^\Lambda)_x(\nabla g^\lambda)_\Lambda - (\nabla g^\Lambda)_\Lambda(\nabla g^\lambda)_x}{(\nabla g^\Lambda)_\Lambda(\nabla g^\lambda)_\lambda - (\nabla g^\Lambda)_\lambda(\nabla g^\lambda)_\Lambda}.$$

, M:

$$\frac{\partial \Lambda_0}{\partial x} = M_{11},$$
$$\frac{\partial \lambda_0}{\partial x} = M_{21},$$

:

$$\frac{\partial \Lambda_0}{\partial y} = M_{12},$$
$$\frac{\partial \lambda_0}{\partial y} = M_{22}.$$

(27) :

$$\begin{aligned} (x_0')' = (\nabla g^x)_x x_0' + (\nabla g^x)_y y_0' + (\nabla g^x)_\Lambda \Lambda_0' + (\nabla g^x)_\lambda \lambda_0' = \\ = \left((\nabla g^x)_x + (\nabla g^x)_\Lambda M_{11} + (\nabla g^x)_\lambda M_{21}\right) x_0' + \\ + \left((\nabla g^x)_y + (\nabla g^x)_\Lambda M_{12} + (\nabla g^x)_\lambda M_{22}\right) y_0'. \end{aligned} \tag{29}$$

, (16) (26) , (29) (26) .

, $\tau \to \pm\infty$:

$$\begin{cases} x_0(\tau) = \dfrac{\text{const}}{\cosh(f_\pm \cdot \tau)} + \mathcal{O}(e^{-2f_\pm|\tau|}), \quad \tau \to \pm\infty, \\ y_0(\tau) = \dfrac{\text{const} \cdot \sinh(f_\pm \cdot \tau)}{\cosh^2(f_\pm \cdot \tau)} + \mathcal{O}(e^{-2f_\pm|\tau|}), \quad \tau \to \pm\infty, \end{cases}$$

$f_\pm = \lim_{\tau\to\pm\infty} \frac{df^\pm(\tau)}{d\tau} = \frac{J^+\chi^+}{\sigma^+} \lim_{\tau\to\pm\infty} \frac{P^+(f^+(\tau))}{S^+(f^+(\tau))}$. $P^\pm$ $S^\pm$ $\cosh f^\pm$, :

$$f_+ = f_- = \frac{J^+\chi^+}{\sigma^+} \cdot \frac{1}{U_0} > 0$$

:

$$x_0, y_0 \in \mathcal{F}_{s_0}, \quad s_0 = f_+ = f_- > 0$$

:

$$\begin{cases} x_1(\pm\infty) = 0, \\ y_1(\pm\infty) = 0. \end{cases}$$

$$x_1, y_1 \in \mathcal{F}_{s_0}$$

, (x_1, y_1) $W^s(0)$, $W^u(0)$, s, u .

■

(22) ( ) $(\tilde{x}_1(\tau), \tilde{y}_1(\tau))$. .. $\mathcal{A}$ 0, , -, , , :

$$W(\tau) = x_1(\tau)\tilde{y}_1(\tau) - y_1(\tau)\tilde{x}_1(\tau) = 1$$

(x_1, y_1) $(\tilde{x}_1, \tilde{y}_1)$.

:

$$\boldsymbol{u}_1(\tau) = \begin{pmatrix} x_1(\tau) \\ y_1(\tau) \end{pmatrix}$$

$$\tilde{\boldsymbol{u}}_1(\tau) = \begin{pmatrix} \tilde{x}_1(\tau) \\ \tilde{y}_1(\tau) \end{pmatrix}$$

$\mathcal{A}(\tau)$ $|\tau|$:

$$\mathcal{A}(\tau) = \underbrace{\begin{pmatrix} 0 & -2F - 2C + \frac{(2C\hat{x}_0 + e_J D)^2}{U_0} \\ 2F - 2C & 0 \end{pmatrix}}_{\mathcal{A}_0} + \mathcal{O}(e^{-s_0|\tau|}) \equiv \begin{pmatrix} 0 & b_0 \\ c_0 & 0 \end{pmatrix} + \mathcal{O}(e^{-s_0|\tau|}) \tag{30}$$

, , $\pm\sqrt{b_0 c_0}$ $\mathcal{A}_0$. , b_0, c_0, , :

$$\sqrt{b_0 c_0} = s_0.$$

τ:

$$\boldsymbol{u}_1(\tau) = \text{const} \begin{pmatrix} b_0\sqrt{c_0} \\ c_0\sqrt{b_0} \end{pmatrix} e^{-s_0\tau} + \mathcal{O}(e^{-2s_0\tau}),$$

$$\tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau) = \mathrm{const}\begin{pmatrix} b_0\sqrt{c_0} \\ -c_0\sqrt{b_0}\end{pmatrix} e^{s_0\tau} + \mathcal{O}(1).$$

τ:

$$\boldsymbol{u}_1(\tau) = \mathrm{const}\begin{pmatrix} b_0\sqrt{c_0} \\ -c_0\sqrt{b_0}\end{pmatrix} e^{s_0\tau} + \mathcal{O}(e^{2s_0\tau}),$$

$$\tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau) = \mathrm{const}\begin{pmatrix} b_0\sqrt{c_0} \\ c_0\sqrt{b_0}\end{pmatrix} e^{-s_0\tau} + \mathcal{O}(1).$$

:

$$\tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau) \in \mathcal{F}_{-s_0}.$$

$\mathcal{A}$, . , , :

$$\begin{cases} \boldsymbol{u}_k^u = \boldsymbol{u}_1(\tau)\displaystyle\int_{\tau_k^u}^{\tau}\Big(\tilde{y}_1(\tau)\tilde{G}_k^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_k^y(\tau)\Big)\,d\tau + \tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau)\displaystyle\int_{-\infty}^{\tau}\Big(x_1(\tau)\tilde{G}_k^y(\tau) - y_1(\tau)\tilde{G}_k^x(\tau)\Big)\,d\tau, \\ \boldsymbol{u}_k^s = \boldsymbol{u}_1(\tau)\displaystyle\int_{\tau_k^s}^{\tau}\Big(\tilde{y}_1(\tau)\tilde{G}_k^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_k^y(\tau)\Big)\,d\tau - \tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau)\displaystyle\int_{\tau}^{+\infty}\Big(x_1(\tau)\tilde{G}_k^y(\tau) - y_1(\tau)\tilde{G}_k^x(\tau)\Big)\,d\tau. \end{cases} \tag{31}$$

2, (31) $\tau \to \pm\infty$. , (31) τ .
$\mathbf{u}_1(\tau)$ $\tau \to \pm\infty$, $\mathbf{u}_k^u$ $\mathbf{u}_k^s$ $\tau_k^u, \tau_k^s \in \mathbb{R}$.
$\tilde{\mathbf{u}}_1(\tau)$ $\tau \to \pm\infty$. $\mathbf{u}_k^u$ $\mathbf{u}_k^s$, (31), . $(x_1\tilde{G}_k^y - y_1\tilde{G}_k^x) \in \mathcal{F}_{3s_0}$, , , $\tilde{\mathbf{u}}_1$.

6. :

$$\begin{cases} \Lambda^s(\tau) = \sum_{k=0}^{\infty}\varepsilon^k\Lambda_k(\tau) \\ \lambda^s(\tau) = \sum_{k=0}^{\infty}\varepsilon^k\lambda_k(\tau) \\ x^s(\tau) = \sum_{k=0}^{\infty}\varepsilon^k x_k(\tau) \\ y^s(\tau) = \sum_{k=0}^{\infty}\varepsilon^k y_k(\tau) \end{cases} \tag{32}$$

(x_1, y_1) (20):

$$\begin{pmatrix} x_1(\tau) \\ y_1(\tau)\end{pmatrix} = \mathrm{const}\cdot\boldsymbol{u}_1.$$

τ_k^s .
(16), $W^s(0)$, , . (32).
(16), $W^u(0)$, .

:

$$|\boldsymbol{u}_k^u(\tau) - \boldsymbol{u}_k^s(\tau)| = \Bigg|\boldsymbol{u}_1(\tau)\int_{\tau_k^u}^{\tau_k^s}\Big(\tilde{y}_1(\tau)\tilde{G}_k^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_k^y(\tau)\Big)\,d\tau + \\ + \tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau)\int_{-\infty}^{+\infty}\Big(x_1(\tau)\tilde{G}_k^y(\tau) - y_1(\tau)\tilde{G}_k^x(\tau)\Big)\,d\tau\Bigg| \tag{33}$$

τ . $W^s(0)$ $W^u(0)$ $\boldsymbol{u}_k^s$ $\boldsymbol{u}_k^u$ τ.
, $|\boldsymbol{u}_k^u(\tau_1) - \boldsymbol{u}_k^s(\tau_2)| = 0$ τ_1, τ_2. , , $\tau_1 = \tau_2 = \tau$.

7. G_k^{ρ} :

$$G_{k+1}^{\rho} = \frac{1}{2}\sum_{m=1}^{k} \mathfrak{D}_m \mathfrak{D}_{k+1-m} g^{\rho} + \sum_{m=1}^{k} \mathfrak{D}_m \hat{G}_{k+1-m}^{\rho},$$

$\hat{G}_{k+1}^{\rho}$ G_{k+1}^{ρ} :

$$G_k^{\rho} = \sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^{\rho}}{j!},$$

$$\hat{G}_k^{\rho} = \sum_{j=2}^{k} \frac{\mathcal{L}_k^{(j)} g^{\rho}}{(j+1)!},$$

$$\rho \in \lambda, \Lambda, x, y.$$

:

$$\mathcal{L}_{k+1}^{(j)} g^{\rho} = \sum_{m=1}^{k+2-j} \mathfrak{D}_m \mathcal{L}_{k+1-m}^{(j-1)} g^{\rho}$$

$$G_{k+1}^{\rho} = \sum_{j=2}^{k+1} \frac{\mathcal{L}_{k+1}^{(j)} g^{\rho}}{j!} = \sum_{j=2}^{k+1} \sum_{m=1}^{k+2-j} \frac{\mathcal{L}_{k+1-m}^{(j-1)} g^{\rho}}{j!} = \sum_{m=1}^{k} \mathfrak{D}_m \sum_{j=2}^{k+2-m} \frac{\mathcal{L}_{k+1-m}^{(j-1)} g^{\rho}}{j!} =$$

$$= \sum_{m=1}^{k} \mathfrak{D}_m \sum_{l=1}^{k+1-m} \frac{\mathcal{L}_{k+1-m}^{(l)} g^{\rho}}{(l+1)!} = \sum_{m=1}^{k} \mathfrak{D}_m \left(\sum_{l=2}^{k+1-m} \frac{\mathcal{L}_{k+1-m}^{(l)} g^{\rho}}{(l+1)!} + \frac{1}{2}\mathcal{L}_{k+1-m}^{(1)} g^{\rho} \right) =$$

$$= \sum_{m=1}^{k} \mathfrak{D}_m \left(\hat{G}_{k+1-m}^{\rho} + \frac{1}{2}\mathfrak{D}_{k+1-m} g^{\rho} \right) \blacksquare$$

7. , $f(\tau)$, $f(\tau)$, τ.

8. 1) $\lambda_j, \Lambda_j, x_j, y_j,$ $j \le k-1$ τ ,
$G_k^x(\tau), \tilde{G}_k^x(\tau), G_k^{\Lambda}(\tau), G_k^y(\tau)$. $G_k^x(\tau), \tilde{G}_k^x(\tau), G_k^{\Lambda}(\tau)$, $G_k^y(\tau), \tilde{G}_k^y(\tau)$ $G_k^x(\tau)$.

2) $G_2^x(\tau), \tilde{G}_2^x(\tau), G_2^{\Lambda}$ - , $G_2^y(\tau), \tilde{G}_2^y(\tau)$ - .

:

1) . .. $x_0(\tau)$ – , $y_0(\tau), \lambda_0(\tau)$ – :

$$g^{\Lambda}|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} = U(x_0,y_0)\sin\lambda_0 - V(x_0,y_0)\cos\lambda_0 \quad \text{- ,}$$

$$g^{x}|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} = -\varepsilon\left(2Fy_0 - \frac{\partial U}{\partial y}\cos\lambda_0 - \frac{\partial V}{\partial y}\sin\lambda_0\right) \quad \text{- ,}$$

$$g^{y}|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} = \varepsilon\left(2F(x_0+\hat{x}_0) + e_J G - \frac{\partial U}{\partial x}\cos\lambda_0 - \frac{\partial V}{\partial x}\sin\lambda_0\right) \quad \text{- .}$$

:

- (x_0) .
- $(y_0 \quad \lambda_0)$.

, $+1$ -1 - , :

$$\left.\frac{\partial^{p+q+m} g^{\Lambda}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} \quad \Leftrightarrow \quad -(-1)^{q+m},$$

$$\left.\frac{\partial^{p+q+m} g^{x}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} \quad \Leftrightarrow \quad -(-1)^{q+m}, \tag{34}$$

$$\left.\frac{\partial^{p+q+m} g^{y}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} \quad \Leftrightarrow \quad (-1)^{q+m}.$$

, $G_k^{\Lambda}(\tau), G_k^x(\tau), G_k^y(\tau)$:

$$G_k^{\Lambda}(\tau) = \sum_{p_1,q_1,m_1,\dots} \left.\frac{\partial^{p+q+m} g^{\Lambda}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} (x_{p_1} y_{q_1} \lambda_{m_1} \dots),$$

$$G_k^{x}(\tau) = \sum_{p_1,q_1,m_1,\dots} \left.\frac{\partial^{p+q+m} g^{x}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} (x_{p_1} y_{q_1} \lambda_{m_1} \dots),$$

$$G_k^{y}(\tau) = \sum_{p_1,q_1,m_1,\dots} \left.\frac{\partial^{p+q+m} g^{y}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} (x_{p_1} y_{q_1} \lambda_{m_1} \dots),$$

, .

, (.. $\le k-1$), $G_k^{\Lambda}(\tau), G_k^x(\tau), G_k^y(\tau)$. , g^{Λ} g^x , g^y , . $\tilde{G}_k^x, \tilde{G}_k^y$ $G_k^{\Lambda}(\tau), G_k^x(\tau), G_k^y(\tau)$.

2) , $x_0(\tau), y_1(\tau), \tilde{x}_1, \lambda_1$ – , $y_0(\tau), x_1(\tau), \tilde{y}_1$ – .

$$G_2^{\rho}(\tau) = \sum_{p+q+m=2, \quad p,q,m\ge 0, \quad p,q,m\ne 2} \left.\frac{\partial^2 g^{\rho}}{\partial x^p \partial y^q \partial \lambda^m}\right|_{\lambda_0(\tau),\Lambda_0(\tau),x_0(\tau),y_0(\tau)} x_1^p y_1^q \lambda_1^m \tag{35}$$

, $x_1^p y_1^q \lambda_1^m$ $(-1)^p$. , (34), (35) $-(-1)^{p+q+m}$ $\rho \in \{\Lambda, x\}$ $(-1)^{p+q+m}$ $\rho = y$. $p+q+m=2$, : $G_2^x(\tau), G_2^{\Lambda}$ - , $G_2^y(\tau)$ - . $\tilde{G}_2^x, \tilde{G}_2^y$ $G_2^{\Lambda}(\tau), G_2^x(\tau), G_2^y(\tau)$. ■

1. $x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)$ τ :

$$\int_{-\infty}^{+\infty} \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau = 0.$$

1 , , :

$$\int_{-\infty}^{\tau} \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau \quad \text{- ,}$$

$$\int_{\tau}^{+\infty} \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau \quad \text{- .}$$

$$\int_{\tau_2^u}^{0} \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau \ne 0,$$

$$\int_{\tau_2^s}^{0} \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau \ne 0,$$

$$\int_{\tau_2^u}^{\tau} \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau = \underbrace{\int_{\tau_2^u}^{0} \left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right) d\tau}_{\text{const}} + \underbrace{\int_{0}^{\tau} \overbrace{\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)}^{} d\tau}_{},$$

$$\int_{\tau^s}^{\tau} \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau = \underbrace{\int_{\tau_2^s}^{0} \left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right) d\tau}_{\text{const}} + \underbrace{\int_{\tau}^{0} \overbrace{\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)}^{} d\tau}_{}.$$

.

$\boldsymbol{u}_2^u(\tau)$ $\boldsymbol{u}_2^s(\tau)$ $\tau_2^{s,u}$.

4 , (31) $W^s(s)$ $W^s(s)$, . $\tau_k^s = \tau_k^u = 0$ $\forall k \geq 2$.

:

$$|\boldsymbol{u}_2^u(\tau) - \boldsymbol{u}_2^s(\tau)| = \Bigg|\boldsymbol{u}_1(\tau) \underbrace{\int_0^0 \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau}_{0} +$$
$$+\tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau) \underbrace{\int_{-\infty}^{+\infty} \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau}_{0} \Bigg| = 0$$

$$|\boldsymbol{u}_k^u(\tau) - \boldsymbol{u}_k^s(\tau)| = \Bigg|\boldsymbol{u}_1(\tau) \underbrace{\int_0^0 \left(\tilde{y}_1(\tau)\tilde{G}_k^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_k^y(\tau)\right) d\tau}_{0} +$$
$$+\tilde{\boldsymbol{u}}_{\mathbf{1}}(\tau) \int_{-\infty}^{+\infty} \left(x_1(\tau)\tilde{G}_k^y(\tau) - y_1(\tau)\tilde{G}_k^x(\tau)\right) d\tau \Bigg|, \quad k \geq 3.$$

4.1.2.

, 0.

, :

$$\boldsymbol{u}_1^0(\tau) \equiv \begin{pmatrix} x_1^0(\tau) \\ y_1^0(\tau) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$(\lambda_1^0(\tau), \Lambda_1^0(\tau)) = (0, 0)$. :

$$\mathfrak{D}_1 = 0 \cdot \quad - \quad 0,$$

$$G_2^\rho = \frac{1}{2}\mathfrak{D}_1^2 g^\rho = \frac{1}{2}(x_1^0)^2 g_{xx}^\rho + \frac{1}{2}x_1^0 y_1^0 g_{xy}^\rho + ... = 0,$$

$$\tilde{G}_2^x = \frac{1}{\alpha\beta}\lambda_0''(\tau)\left(-2Cy_0\sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\right) \ - \ ,$$

$$\tilde{G}_2^y = \frac{1}{\alpha\beta}\lambda_0''(\tau)\left(-(2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 + 2Cy_0\cos\lambda_0\right) \ - \ .$$

$\boldsymbol{u}_2^u, \boldsymbol{u}_2^s$ $\tau_2^u = \tau_2^s = 0$:

$$\begin{cases} \boldsymbol{u}_2^u = \boldsymbol{u}_1(\tau) \displaystyle\int_0^\tau \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau + \tilde{\boldsymbol{u}}_{\boldsymbol{1}}(\tau) \displaystyle\int_{-\infty}^\tau \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau, \\ \boldsymbol{u}_2^s = \boldsymbol{u}_1(\tau) \displaystyle\int_0^\tau \left(\tilde{y}_1(\tau)\tilde{G}_2^x(\tau) - \tilde{x}_1(\tau)\tilde{G}_2^y(\tau)\right) d\tau - \tilde{\boldsymbol{u}}_{\boldsymbol{1}}(\tau) \displaystyle\int_\tau^{+\infty} \left(x_1(\tau)\tilde{G}_2^y(\tau) - y_1(\tau)\tilde{G}_2^x(\tau)\right) d\tau. \end{cases}$$

1 $\boldsymbol{u}_2^s(\tau) = \boldsymbol{u}_2^u(\tau) \equiv \boldsymbol{u}_2^0(\tau)$

.. G_k^ρ $\hat{G}_k^\rho$. :

$$G_3^\rho = \frac{1}{2}(\mathfrak{D}_1\mathfrak{D}_2 + \mathfrak{D}_2\mathfrak{D}_1)g^\rho + \mathfrak{D}_1\hat{G}_2^\rho = 0,$$

$$\tilde{G}_3^x = \frac{1}{\alpha\beta} \underbrace{(\lambda_1^0)''(\tau)}_{0} (-2Cy_0 \sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0) = 0,$$

$$\tilde{G}_3^y = \frac{1}{\alpha\beta} \underbrace{(\lambda_1^0)''(\tau)}_{0} (-(2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 + 2Cy_0\cos\lambda_0) = 0.$$

3 . :

$$\boldsymbol{u}_3^0(\tau) \equiv \begin{pmatrix} x_3^0(\tau) \\ y_3^0(\tau) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$(\lambda_3^0(\tau), \Lambda_3^0(\tau)) = (0, 0).$$

9. $k \geq 0$

$$G_{2k+1}^\rho = 0, \rho \in \{\Lambda, \lambda, x, y\},$$
$$G_{2k+2}^x, y_{2k+2} - ,$$
$$G_{2k+2}^y, x_{2k+2} - ,$$
$$(\lambda_{2k+1}^0, \Lambda_{2k+1}^0, x_{2k+1}^0, y_{2k+1}^0) = (0, 0, 0, 0).$$

:

. :

$$G_{2k-1}^\rho = 0,$$
$$G_{2k}^x, y_{2k} - ,$$
$$G_{2k}^y, x_{2k} - ,$$
$$(\lambda_{2k-1}^0, \Lambda_{2k-1}^0, x_{2k-1}^0, y_{2k-1}^0) = (0, 0, 0, 0).$$

5

$$G_{2k+1}^\rho = \frac{1}{2}\sum_{m=1}^{2k} \mathfrak{D}_m\mathfrak{D}_{2k+1-m}g^\rho + \sum_{m=1}^{2k} \mathfrak{D}_m\hat{G}_{2k+1-m}^\rho. \tag{36}$$

, $j \leq 2k-1$ $\mathfrak{D}_j$ 0.
(36) $\mathfrak{D}_m\mathfrak{D}_{2k+1-m}g^\rho$, $2k+1$. , $\mathfrak{D}_m$, $\mathfrak{D}_{2k+1-m}$. $\mathfrak{D}_m\mathfrak{D}_{2k+1-m}g^\rho = 0$ $\forall m \leq 2k$.
(36) $\mathfrak{D}_m$, $\hat{G}_{2k+1-m}^\rho$. $j \leq 2k-1$ $\hat{G}_j^\rho \equiv 0$, 0.

(36) :

$$\tilde{G}^x_{2k+1} = \frac{1}{\alpha\beta}\underbrace{(\lambda^0_{2k-1})''(\tau)}_{0}(-2Cy_0\sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0) = 0,$$

$$\tilde{G}^y_{2k+1} = \frac{1}{\alpha\beta}\underbrace{(\lambda^0_{2k-1})''(\tau)}_{0}(-(2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 + 2Cy_0\cos\lambda_0) = 0.$$

:

$$(\lambda^0_{2k+1}, \Lambda^0_{2k+1}, x^0_{2k+1}, y^0_{2k+1}) = (0,0,0,0).$$

, $\mathfrak{D}_{2k}$:

$$\mathfrak{D}_{2k}g^\rho = \underbrace{x^0_{2k}}_{\cdot}\, g^\rho_x|_{\lambda_0,\Lambda_0,x_0,y_0} + \underbrace{y^0_{2k}}_{\cdot}\, g^\rho_y|_{\lambda_0,\Lambda_0,x_0,y_0} + \underbrace{\lambda^0_{2k}}_{\cdot}\, g^\rho_\lambda|_{\lambda_0,\Lambda_0,x_0,y_0} + \underbrace{\Lambda^0_{2k}}_{(\lambda^0_{2k-1})'=0}\, g^\rho_\Lambda|_{\lambda_0,\Lambda_0,x_0,y_0}$$

$\frac{\partial}{\partial x}|_{\lambda_0,\Lambda_0,x_0,y_0}$, $\frac{\partial}{\partial y}|_{\lambda_0,\Lambda_0,x_0,y_0}$, $\frac{\partial}{\partial \lambda}|_{\lambda_0,\Lambda_0,x_0,y_0}$.

, 5, G^ρ_{2k} G^ρ_{2k-2}. ■

:

$$\begin{cases} \lambda^0(\tau) = (\lambda_0(\tau) - \pi) + \sum_{k=1}^{+\infty}\varepsilon^{2k}\lambda^0_{2k}(\tau), \\ \Lambda^0(\tau) = \frac{1}{\alpha}\sum_{k=1}^{+\infty}\varepsilon^{2k}(\lambda^0_{2k-1})'(\tau), \\ x^0(\tau) = x_0(\tau) + \sum_{k=1}^{+\infty}\varepsilon^{2k}x^0_{2k}(\tau), \\ y^0(\tau) = y_0(\tau) + \sum_{k=1}^{+\infty}\varepsilon^{2k}y^0_{2k}(\tau), \end{cases}$$

$$\begin{cases} x^0_{2k}(\tau) = x_1(\tau)\int_0^\tau \left(\tilde{y}_1(\tau)\tilde{G}^x_{2k}(\tau) - \tilde{x}_1(\tau)\tilde{G}^y_{2k}(\tau)\right)d\tau + \tilde{x}_1(\tau)\int_{-\infty}^\tau \left(x_1(\tau)\tilde{G}^y_{2k}(\tau) - y_1(\tau)\tilde{G}^x_{2k}(\tau)\right)d\tau, \\ y^0_{2k}(\tau) = y_1(\tau)\int_0^\tau \left(\tilde{y}_1(\tau)\tilde{G}^x_{2k}(\tau) - \tilde{x}_1(\tau)\tilde{G}^y_{2k}(\tau)\right)d\tau + \tilde{y}_1(\tau)\int_{-\infty}^\tau \left(x_1(\tau)\tilde{G}^y_{2k}(\tau) - y_1(\tau)\tilde{G}^x_{2k}(\tau)\right)d\tau, \end{cases}$$

$$\tilde{G}^x_{2k} = \frac{1}{\alpha\beta}(\lambda^0_{2k-2})''(\tau)\left(-2Cy_0\sin\lambda_0 - (2Cx_0 + 2C\hat{x}_0 + e_J D)\cos\lambda_0\right),$$

$$\tilde{G}^y_{2k} = \frac{1}{\alpha\beta}(\lambda^0_{2k-2})''(\tau)\left(-(2Cx_0 + 2C\hat{x}_0 + e_J D)\sin\lambda_0 + 2Cy_0\cos\lambda_0\right).$$

7. (16):

- $(\Lambda^s(\tau), \lambda^s(\tau), x^s(\tau), y^s(\tau)) \in W^s(0)$,
- $(\Lambda^u(\tau), \lambda^u(\tau), x^u(\tau), y^u(\tau)) \in W^u(0)$,

ε $(\Lambda^0(\tau), \lambda^0(\tau), x^0(\tau), y^0(\tau))$. $\exists$ $c > 0$:

$$(\Lambda^s(\tau), \lambda^s(\tau), x^s(\tau), y^s(\tau)) = (\Lambda^0(\tau), \lambda^0(\tau), x^0(\tau), y^0(\tau)) + \mathcal{O}(e^{-\frac{c}{\varepsilon}}),$$

$$(\Lambda^u(\tau), \lambda^u(\tau), x^u(\tau), y^u(\tau)) = (\Lambda^0(\tau), \lambda^0(\tau), x^0(\tau), y^0(\tau)) + \mathcal{O}(e^{-\frac{c}{\varepsilon}}).$$

:

$$\int_0^\tau \Big(\tilde{y}_1(\tau)\tilde{G}^x_{2k}(\tau) - \tilde{x}_1(\tau)\tilde{G}^y_{2k}(\tau)\Big)\, d\tau,$$

$\forall k \in \mathbb{N}$, .. $\mathcal{F}_{s_0}$.
(7) 0:

$$\int_{-\infty}^{+\infty} \Big(x_1(\tau)\tilde{G}^y_{2k}(\tau) - y_1(\tau)\tilde{G}^x_{2k}(\tau)\Big)\, d\tau, \quad k \in \mathbb{N}$$

, $\mathcal{F}_{3s_0}$,

$$\int_{-\infty}^{\tau} \Big(x_1(\tau)\tilde{G}^y_{2k}(\tau) - y_1(\tau)\tilde{G}^x_{2k}(\tau)\Big)\, d\tau$$

$\mathcal{F}_{2s_0}$. $\tilde{\boldsymbol{u}}_{\boldsymbol{1}}$.
, $(\Lambda^0(\tau), \lambda^0(\tau), x^0(\tau), y^0(\tau))$ $\tau \to \pm\infty$.
, $(g^\Lambda, g^\lambda, g^x, g^y)$ (16) (Λ, λ, x, y) $\varepsilon > 0$. , $(g^\Lambda, g^\lambda, g^x, g^y)$ (Λ, λ, x, y) . .
, $W^s(0)$ $W^u(0)$ $(\Lambda^0(\tau), \lambda^0(\tau), x^0(\tau), y^0(\tau))$. .
■

2. - (16) .

4.2.

[?], - . [?] [?, ?]. .

4.2.1.

(16) 0.

$$\boldsymbol{F}(\Lambda, \lambda, x, y) = \begin{pmatrix} -U(x,y)\sin\lambda + V(x,y)\cos\lambda \\ \alpha\Lambda \\ -\varepsilon\big(2Fy - \frac{\partial U}{\partial y}\cos\lambda - \frac{\partial V}{\partial y}\sin\lambda\big) \\ \varepsilon\big(2F(x+\hat{x}_0) + e_J G - \frac{\partial U}{\partial x}\cos\lambda - \frac{\partial V}{\partial x}\sin\lambda\big) \end{pmatrix}.$$

:

$$\begin{pmatrix} 0 & -U_0 & 0 & (2C\hat{x}_0 + e_J D) \\ \alpha & 0 & 0 & 0 \\ 0 & \varepsilon(2C\hat{x}_0 + e_J D) & 0 & -\varepsilon(2F+2C) \\ 0 & 0 & \varepsilon(2F-2C) & 0 \end{pmatrix}.$$

($\pm i\omega, \pm\xi$ -):

$$R = \begin{pmatrix} \frac{i\omega(\xi^2+\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)} & -\frac{i\omega(\xi^2+\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)} & -\frac{\xi(\omega^2-\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)} & \frac{\xi(\omega^2-\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)} \\ \frac{\xi^2+\alpha U_0}{(2C\hat{x}_0+e_J D)} & \frac{\xi^2+\alpha U_0}{(2C\hat{x}_0+e_J D)} & -\frac{\omega^2-\alpha U_0}{(2C\hat{x}_0+e_J D)} & -\frac{\omega^2-\alpha U_0}{(2C\hat{x}_0+e_J D)} \\ -i\omega\varepsilon & i\omega\varepsilon & -\xi\varepsilon & \xi\varepsilon \\ 2\varepsilon^2(C-F) & 2\varepsilon^2(C-F) & 2\varepsilon^2(C-F) & 2\varepsilon^2(C-F) \end{pmatrix}. \tag{37}$$

(z, η, a, b), :

$$\begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix} = R^{-1} \begin{pmatrix} \Lambda \\ \lambda \\ x \\ y \end{pmatrix}. \tag{38}$$

:

$$\frac{d}{dt}\begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix} = \operatorname{diag}(i\omega, -i\omega, \xi, -\xi) \cdot \begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix} + \boldsymbol{\mathcal{G}}(z, \eta, a, b),, \tag{39}$$

$$\boldsymbol{\mathcal{G}} \equiv R^{-1}\boldsymbol{F}\left(R\begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix}\right) - \operatorname{diag}(i\omega, -i\omega, \xi, -\xi) \cdot \begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix},$$

$\boldsymbol{\mathcal{G}}$ - .
$\phi(t), \psi(t)$, , [?]. :

$$\varphi(t) = \varphi_0 e^{i\theta(\varphi_0, \psi_0)t},$$
$$\psi(t) = \varphi_0 e^{-i\theta(\varphi_0, \psi_0)t}.$$

$\theta(\varphi_0, \psi_0)$ φ_0, ψ_0:

$$\theta(\varphi_0, \psi_0) = \omega + \sum_{k=1}^{+\infty} \theta_k \cdot (\varphi_0\psi_0)^k.$$

:

$$\begin{cases} z(\varphi, \psi) = \varphi + \sum_{k,j=1}^{+\infty} \{z\}_{k,j}\varphi^k\psi^j, \\ \eta(\varphi, \psi) = \psi + \sum_{k,j=1}^{+\infty} \{\eta\}_{k,j}\varphi^k\psi^j, \\ a(\varphi, \psi) = \sum_{k,j=1}^{+\infty} \{a\}_{k,j}\varphi^k\psi^j, \\ b(\varphi, \psi) = \sum_{k,j=1}^{+\infty} \{b\}_{k,j}\varphi^k\psi^j. \end{cases} \tag{40}$$

, z $(\varphi\psi)^k\varphi$, η $(\varphi\psi)^k\psi$. φ, ψ [?].
(39) :

$$\begin{cases} ((p-q)i\omega - i\omega)\{z\}_{p,q} + \sum_{r=1}^{+\infty}(p-q)i\theta_r\{z\}_{p-r,q-r} = \{\boldsymbol{\mathcal{G}}_z(z, \eta, a, b)\}_{p,q}, \\ ((p-q)i\omega + i\omega)\{\eta\}_{p,q} + \sum_{r=1}^{+\infty}(p-q)i\theta_r\{\eta\}_{p-r,q-r} = \{\boldsymbol{\mathcal{G}}_\eta(z, \eta, a, b)\}_{p,q}, \\ ((p-q)i\omega - \xi)\{a\}_{p,q} + \sum_{r=1}^{+\infty}(p-q)i\theta_r\{a\}_{p-r,q-r} = \{\boldsymbol{\mathcal{G}}_a(z, \eta, a, b)\}_{p,q}, \\ ((p-q)i\omega + \xi)\{b\}_{p,q} + \sum_{r=1}^{+\infty}(p-q)i\theta_r\{b\}_{p-r,q-r} = \{\boldsymbol{\mathcal{G}}_b(z, \eta, a, b)\}_{p,q}. \end{cases} \tag{41}$$

$\{\boldsymbol{\mathcal{G}}_\rho(z, \eta, a, b)\}_{p,q}, \rho \in \{z, \eta, a, b\}$ $\varphi^p\psi^q$ $\boldsymbol{\mathcal{G}}$.
$p = q+1, p+q > 1$ z , $\{z\}_{p,q} = 0$. $q = p+1$ η , $\{\eta\}_{p,q} = 0$ ($(\varphi\psi)^k\varphi$ z $(\varphi\psi)^k\psi$ η). $\{z\}_{1,0} = \{\eta\}_{0,1} = 1$. ,

$$\begin{cases} \theta_p = \{\boldsymbol{\mathcal{G}}_z\}_{p,q} \quad p = q+1 > 1, \\ \theta_q = -\{\boldsymbol{\mathcal{G}}_\eta\}_{p,q} \quad q = p+1 > 1. \end{cases}$$

, :

$$\begin{cases} \{z\}_{1,1} = -\{\eta\}_{1,1} = i\dfrac{(\xi^2+\alpha U_0)(\omega^2-\alpha U_0)(\xi^2+\alpha U_0-8\varepsilon^2 C(C-F))}{4\omega(C-F)(\xi^2+\omega^2)(2C\hat{x}_0+e_J D)\varepsilon} = \mathcal{O}(\varepsilon)\in i\mathbb{R}, \\ \{a\}_{1,1} = -\{b\}_{1,1} = \dfrac{-(\xi^2+\alpha U_0)(\xi^2+\alpha U_0-8\varepsilon^2 C(C-F))}{4\xi(C-F)(\xi^2+\omega^2)(2C\hat{x}_0+e_J D)\varepsilon} = \mathcal{O}\left(\dfrac{1}{\varepsilon^2}\right)\in\mathbb{R}, \end{cases}$$

$$\begin{aligned}
\theta_1 =& \frac{(\xi^2+\alpha U_0)}{8(2C\hat{x}_0+e_J D)^2\omega(C-F)\left(\xi^2+\omega^2\right)^2}\times \\
&\Bigg[\alpha U_0\Bigg(-4\alpha\xi^2(2C\hat{x}_0+e_J D)\varepsilon^2(C-F)\left(8c^2\hat{x}_0 - C(2C\hat{x}_0+e_J D)+3f(2C\hat{x}_0+e_J D)\right) \\
&\quad + 2\omega^2\left(\xi^4(C+2F)-6\alpha(2C\hat{x}_0+e_J D)^2\varepsilon^2(C-F)(C+F)+2\alpha\xi^2(2C\hat{x}_0+e_J D)^2\right) \\
&\quad + \xi^4\left(8C\varepsilon^2(C-F)(C+3F)+\alpha(2C\hat{x}_0+e_J D)(6C\hat{x}_0+(2C\hat{x}_0+e_J D))\right) \\
&\quad + 4\omega^4\left(\xi^2(C+F)-2C\varepsilon^2(C-F)(C+3F)\right)-4C\xi^6-2C\omega^6\Bigg) \\
&+\alpha\xi^2(2C\hat{x}_0+e_J D)\Bigg(-4\xi^2\varepsilon^2(C-F)\left(4C^2\hat{x}_0+C(2C\hat{x}_0+e_J D)+3f(2C\hat{x}_0+e_J D)\right) \\
&\quad + 16C(2C\hat{x}_0+e_J D)\varepsilon^4(C-F)^2(C+3F)+\xi^4(2C\hat{x}_0+(2C\hat{x}_0+e_J D))\Bigg) \\
&+\alpha^2U_0^2\Bigg(8C^3\varepsilon^2\left(\xi^2-2\alpha(2C\hat{x}_0+e_J D)\hat{x}_0+\omega^2\right) \\
&\quad + 8C^2\varepsilon^2\left(2F\left(\xi^2+\omega^2\right)+2\alpha F(2C\hat{x}_0+e_J D)\hat{x}_0+\alpha(2C\hat{x}_0+e_J D)^2\right) \\
&\quad - 2C\left(12F^2\varepsilon^2\left(\xi^2+\omega^2\right)+4\alpha F(2C\hat{x}_0+e_J D)^2\varepsilon^2+4\xi^4+2\xi^2\omega^2-3\alpha\xi^2(2C\hat{x}_0+e_J D)\hat{x}_0-2\omega^4\right) \\
&\quad + 2F\omega^2\left(\xi^2+\omega^2\right)-\alpha(2C\hat{x}_0+e_J D)^2\left(\xi^2-2\omega^2\right)\Bigg) \\
&+\alpha De_J(2C\hat{x}_0+e_J D)\left(\xi^2+\alpha U_0\right)^2\left(8C\varepsilon^2(F-C)+\xi^2+\alpha U_0\right) \\
&+2\omega^4\left(2\alpha C(2C\hat{x}_0+e_J D)^2\varepsilon^2(C-F)-4C\xi^2\varepsilon^2(C-F)(C+3F)+F\xi^4\right) \\
&+2\omega^2\Bigg(\xi^6(C+F)+8\alpha C(2C\hat{x}_0+e_J D)^2\varepsilon^4(C-F)^2(C+3F) \\
&\quad + \xi^4\left(\alpha(2C\hat{x}_0+e_J D)^2-4C\varepsilon^2(C-F)(C+3F)\right)-2\alpha\xi^2(2C\hat{x}_0+e_J D)^2\varepsilon^2(C-F)(2C+3F)\Bigg) \\
&-2C\xi^2\omega^6-\alpha^3U_0^3\left(4C\left(\xi^2+\omega^2\right)-2\alpha C(2C\hat{x}_0+e_J D)\hat{x}_0+\alpha(2C\hat{x}_0+e_J D)^2\right)\Bigg] = \mathcal{O}(1)\in\mathbb{R}.
\end{aligned}$$

R^{-1}, , ε:

$$\Lambda = (z-\eta)\underbrace{\frac{i\omega(\xi^2+\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)}}_{\mathcal{O}(1)} + (a-b)\underbrace{\frac{-\xi(\omega^2-\alpha U_0)}{\alpha(2C\hat{x}_0+e_J D)}}_{\mathcal{O}(\varepsilon^3)},$$

$$\lambda = (z+\eta)\underbrace{\frac{(\xi^2+\alpha U_0)}{(2C\hat{x}_0+e_J D)}}_{\mathcal{O}(1)} + (a+b)\underbrace{\frac{-(\omega^2-\alpha U_0)}{(2C\hat{x}_0+e_J D)}}_{\mathcal{O}(\varepsilon^2)},$$

$$x = (z-\eta)\underbrace{(-i\omega\varepsilon)}_{\mathcal{O}(\varepsilon)} + (a-b)\underbrace{(-\varepsilon\xi)}_{\mathcal{O}(\varepsilon^2)},$$

$$y = 2(C-F)\varepsilon^2(z+\eta+a+b).$$

(Λ, λ, x, y), , :

$$\varphi_0 = \psi_0 \equiv h.$$

4.2.2.

.

8. , $f(x)$, - 0,

$$g(s) = \sum_{n=0}^{+\infty} g_n s^n, \quad s \geq 0,$$

$$g_n \geq 0 \quad \forall n \geq 0,$$

:

$f(x)$:

$$f(x) = \sum_{n=0}^{+\infty} f_n x^n.$$

$$\exists C > 0 : |f_n| \leq C \cdot g_n \quad \forall n \geq 0.$$

$$f \prec g.$$

$\varkappa$:

$$\varkappa = \max\left(\sup_{p,q\in\mathbb{R}}\left(\left|\frac{1}{(p-q)i\omega-\xi}\right|\right), \sup_{p,q\in\mathbb{R}}\left(\left|\frac{p-q}{(p-q)i\omega-\xi}\right|\right),\right.$$
$$\left.\sup_{p,q\in\mathbb{R}}\left(\left|\frac{1}{(p-q)i\omega-i\omega}\right|\right), \sup_{p,q\in\mathbb{R}}\left(\left|\frac{p-q}{(p-q)i\omega-i\omega}\right|\right)\right) =$$
$$= \max\left(\frac{1}{|\xi|}, \frac{1}{|\omega|}\right) = \frac{1}{|\xi|} = O\left(\frac{1}{\varepsilon}\right) \qquad \varepsilon.$$

, (41), :

$$\begin{cases} |\{z\}_{p,q}| \leq \varkappa|\{\boldsymbol{\mathcal{G}}_z(z,\eta,a,b)\}_{p,q}| + \varkappa\sum_{r=1}^{+\infty}|(p-q)\theta_r\{z\}_{p-r,q-r}|, \\ |\{\eta\}_{p,q}| \leq \varkappa|\{\boldsymbol{\mathcal{G}}_\eta(z,\eta,a,b)\}_{p,q}| + \varkappa\sum_{r=1}^{+\infty}|(p-q)\theta_r\{\eta\}_{p-r,q-r}|, \\ |\{a\}_{p,q}| \leq \varkappa|\{\boldsymbol{\mathcal{G}}_a(z,\eta,a,b)\}_{p,q}| + \varkappa\sum_{r=1}^{+\infty}|(p-q)\theta_r\{a\}_{p-r,q-r}|, \\ |\{b\}_{p,q}| \leq \varkappa|\{\boldsymbol{\mathcal{G}}_b(z,\eta,a,b)\}_{p,q}| + \varkappa\sum_{r=1}^{+\infty}|(p-q)\theta_r\{b\}_{p-r,q-r}|. \end{cases} \tag{42}$$

[?] $\boldsymbol{\mathcal{G}}$

$$\boldsymbol{\mathcal{G}} \prec \frac{1}{1-s},$$

$s = |z| + |\eta| + |a| + |b|$.

$\boldsymbol{\mathcal{G}}$.

.

(Λ, λ, x, y) (z, η, a, b) R, s:

$$s = |z| + |\eta| + \varepsilon(|a| + |b|).$$

(4.2.1) :

$$\begin{cases} \Lambda \prec s, \\ \lambda \prec s, \\ x \prec \varepsilon s, \\ y \prec \varepsilon s. \end{cases} \tag{43}$$

(43) , 0 , (16):

$$\begin{cases} -U\sin\lambda + V\cos\lambda \prec (\varepsilon s + \varepsilon^2 s^2)e^s, \\ \alpha\Lambda \prec s, \\ -\varepsilon\big(2Fy - \frac{\partial U}{\partial y}\cos\lambda - \frac{\partial V}{\partial y}\sin\lambda\big) \prec \varepsilon(\varepsilon s + \varepsilon^2 s^2)e^s, \\ \varepsilon\big(2F(x + \hat{x}_0) + e_J G - \frac{\partial U}{\partial x}\cos\lambda - \frac{\partial V}{\partial x}\sin\lambda\big) \prec \varepsilon(\varepsilon s + \varepsilon^2 s^2)e^s. \end{cases}$$

R^{-1}, :

$$R^{-1}\boldsymbol{F}\left(R\begin{pmatrix} z \\ \eta \\ a \\ b \end{pmatrix}\right) \prec \begin{pmatrix} (\varepsilon s + \varepsilon^2 s^2)e^s + s \\ (\varepsilon s + \varepsilon^2 s^2)e^s + s \\ \frac{1}{\varepsilon}(\varepsilon s + \varepsilon^2 s^2)e^s + s \\ \frac{1}{\varepsilon}(\varepsilon s + \varepsilon^2 s^2)e^s + s \end{pmatrix}. \tag{44}$$

(44) , $\boldsymbol{\mathcal{G}}$:

$$\begin{cases} \boldsymbol{\mathcal{G}}_{z,\eta} & \prec (\varepsilon(s + \varepsilon s^2)e^s - \varepsilon s), \\ \boldsymbol{\mathcal{G}}_{a,b} & \prec ((s + \varepsilon s^2)e^s - s). \end{cases} \tag{45}$$

, $-s$ $(s + \varepsilon s^2)e^s$ $(s + \varepsilon s^2)e^s - s$.

h, φ, ψ, :

$$||h|| = \sum_{p,q} |\{h\}_{p,q}|\varphi^p\psi^q$$

. :

$$\begin{cases} z^* = z - \varphi, \\ \eta^* = \eta - \psi, \\ a^* = a, \\ b^* = b, \\ \theta^* = \theta - \omega. \end{cases}$$

(42) ε a b, $\varphi^p\psi^q$. :

$$(\varphi + \psi)||\theta^*|| + S \prec \varkappa\big(||\boldsymbol{\mathcal{G}}_z|| + ||\boldsymbol{\mathcal{G}}_\eta|| + \varepsilon(||\boldsymbol{\mathcal{G}}_a|| + ||\boldsymbol{\mathcal{G}}_b||) + ||\theta^*||S\big), \tag{46}$$

$S = ||z^*|| + ||\eta^*|| + \varepsilon(||a^*|| + ||b^*||)$.

, S $||\theta^*||$ $\varphi = 0, \psi = 0,$ $\varphi = \psi > 0.$

(45) (46):

$$2\varphi||\theta^*|| + S \prec \varkappa\left(\left(\varepsilon(s + \varepsilon s^2)e^s - \varepsilon s\right) + ||\theta^*||S\right).$$

s :

$$s \prec ||s|| \prec 2\varphi + S.$$

S , Y :

$$Y = 2||\theta^*|| + \frac{1}{\varphi}S.$$

S,

$$S = \varphi Y - 2\varphi||\theta^*|| \prec \varphi U.$$

$||\theta^*||S.$

$$||\theta^*||S = \frac{\varphi^2 Y^2}{4\varphi} - \frac{S^2}{4\varphi} - \frac{4\varphi^2||\theta^*||^2}{4\varphi} \prec \frac{\varphi^2 Y^2}{4\varphi}.$$

:

$$Y \prec \frac{\varkappa}{4}\left(Y^2 + \varepsilon\left((2+Y) + \varepsilon\varphi(2+Y)^2\right)e^{\varphi(2+Y)} - (2+Y)\right). \tag{47}$$

, Y S $||\theta^*||$, , Y φ. :

$$\hat{Y} = \frac{\varkappa}{4}\left(\hat{Y}^2 + \varepsilon\left((2+\hat{Y}) + \varepsilon\varphi(2+\hat{Y})^2\right)e^{\varphi(2+\hat{Y})} - (2+\hat{Y})\right),$$

:

$$\hat{Y} = \hat{Y}(\varphi) = \sum_{k=1}^{+\infty}\gamma_k\varphi^k.$$

R :

$$R = O\left(\frac{1}{\varepsilon\varkappa^2}\right) = \mathcal{O}(\varepsilon).$$

4.3.

-.

9. Γ_h (h -), .
:

$$W^s(\Gamma_h) = \{\boldsymbol{X} \in \mathbb{R}^n : \quad \mathrm{dist}\big(\Gamma_h, \theta^t(\boldsymbol{X})\big) \to 0, t \to +\infty\},$$
$$W^u(\Gamma_h) = \{\boldsymbol{X} \in \mathbb{R}^n : \quad \mathrm{dist}\big(\Gamma_h, \theta^t(\boldsymbol{X})\big) \to 0, t \to -\infty\},$$

θ^t - , n - .

$\gamma(t)$ Γ_h, :

$$\gamma(t) \in W^s(\Gamma_h) \cap W^u(\Gamma_h), \quad \gamma(t) \notin \Gamma_h.$$

Γ_{h_1} Γ_{h_2} :

$$\gamma(t) \in W^s(\Gamma_{h_2}) \cap W^u(\Gamma_{h_2}).$$

, $\gamma(t)$ Γ_h $t \to +\infty$, $t \to -\infty$ (Γ_{h_1} $t \to +\infty$ Γ_{h_2} $t \to -\infty$).

(??) :

$$\Gamma_h(t) = (\Lambda^*(h,t), \lambda^*(h,t), x^*(h,t), y^*(h,t)).$$

, $W^{s,u}(\Gamma_h)$, :

$$\begin{cases} \Lambda^{s,u}(h,t) = \Lambda^*(h,t) + z_\Lambda^{s,u}(h,t), \\ \lambda^{s,u}(h,t) = \lambda^*(h,t) + z_\lambda^{s,u}(h,t), \\ x^{s,u}(h,t) = x^*(h,t) + z_x^{s,u}(h,t), \\ y^{s,u}(h,t) = y^*(h,t) + z_y^{s,u}(h,t), \end{cases} \tag{48}$$

(48) (16). :

- $W^s(\Gamma_h)$:

$$\begin{cases} z_\Lambda^s(k,+\infty) = 0, \\ z_\lambda^s(k,+\infty) = 0, \\ z_x^s(k,+\infty) = 0, \\ z_y^s(k,+\infty) = 0. \end{cases}$$

- $W^u(\Gamma_h)$:

$$\begin{cases} z_\Lambda^u(k,-\infty) = 0, \\ z_\lambda^u(k,-\infty) = 0, \\ z_x^u(k,-\infty) = 0, \\ z_y^u(k,-\infty) = 0. \end{cases}$$

$\tilde{\tau} = \frac{\theta(h)}{\omega}\tau = (1+\mathcal{O}(h^2))\tau$. , h h $\frac{\omega}{\varepsilon}$. :

$$H_{old} \to H = \frac{\omega}{\theta(h)} H_{old} = H_{old} + \mathcal{O}(h^2).$$

$\frac{\omega}{\varepsilon} = \mathcal{O}(1/\varepsilon)$ $\frac{\xi}{\varepsilon} = \mathcal{O}(1)$, . $f(\Lambda, \lambda, x, y)$:

$$\langle f \rangle = \int_0^{\frac{2\pi\varepsilon}{\omega}} f\left(\Lambda^*(\tau), \lambda^*(\tau), x^*(\tau), y^*(\tau)\right) d\tau.$$

(17) (48):

$$\begin{aligned} H(\Gamma_h + \boldsymbol{z}) = \frac{\omega}{\theta(h)} \Bigg(& \frac{\alpha(\Lambda^*)^2}{2} + \alpha\Lambda^* z_\Lambda + \frac{\alpha z_\Lambda^2}{2} + F\left((x^*)^2 + (y^*)^2\right) + F\left((z_x + \hat{x}_0)^2 + z_y^2\right) + \\ & + 2Fx^*(z_x + \hat{x}_0) + 2Fy^* z_y + e_J G x^* + e_J G z_x - \\ & - \left(U(\Gamma_h) + U(\boldsymbol{z}) + (2Cx^* z_x - 2Cy^* z_y - U_0)\right)(\cos\lambda^* \cos z_\lambda - \sin\lambda^* \sin z_\lambda) - \\ & - \left(V(\Gamma_h) + V(\boldsymbol{z}) + (2Cy^* z_x + 2Cz^* z_y)\right)(\sin\lambda^* \cos z_\lambda + \cos\lambda^* \sin z_\lambda) \Bigg) \end{aligned}$$

$\boldsymbol{z}$. , .
. :

$$\langle U(x^*, y^*)\cos\lambda^* \rangle = U_0 \langle \cos\lambda^* \rangle + (2C\hat{x}_0 + e_J D)\langle x^* \cos\lambda^* \rangle + C\left\langle (x^*)^2 \cos\lambda^* \right\rangle - C\left\langle (y^*)^2 \cos\lambda^* \right\rangle.$$

Γ_h $\cos\lambda^*$:

$$\langle\cos\lambda^*\rangle = \left\langle 1 - \frac{h^2}{2}\left(\frac{2(\xi^2-\alpha U_0)}{(2C\hat{x}_0+e_J D)}\right)^2\cos^2\left(\frac{\tilde{\tau}\omega}{\varepsilon}\right) + \mathcal{O}(h^3)\right\rangle =$$

$$= 1 - h^2\underbrace{\frac{1}{4}\left(\frac{2(\xi^2-\alpha U_0)}{(2C\hat{x}_0+e_J D)}\right)^2}_{\mathcal{O}(1)} + \mathcal{O}(h^4),$$

$$\langle x^*\cos\lambda^*\rangle = \left\langle h^2\left((-2i\varepsilon\omega)\{z\}_{1,1} + (-2\varepsilon\xi)\{a\}_{1,1}\right) + \mathcal{O}(h^3)\right\rangle =$$

$$= h^2\left(\underbrace{(-2i\varepsilon\omega)\{z\}_{1,1}}_{\mathcal{O}(\varepsilon^2)} + \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)}\right) + \mathcal{O}(h^4),$$

$$\left\langle (x^*)^2\cos\lambda^*\right\rangle = \left\langle h^2(2\varepsilon\omega)^2\sin^2\left(\frac{\tilde{\tau}\omega}{\varepsilon}\right) + \mathcal{O}(h^3)\right\rangle = h^2\underbrace{\frac{1}{2}(2\varepsilon\omega)^2}_{\mathcal{O}(\varepsilon^2)} + \mathcal{O}(h^4),$$

$$\left\langle (y^*)^2\cos\lambda^*\right\rangle = \left\langle h^2\left(4\varepsilon^2(C-F)\right)^2\cos^2\left(\frac{\tilde{\tau}\omega}{\varepsilon}\right) + \mathcal{O}(h^3)\right\rangle = h^2\underbrace{\frac{1}{2}\left(4\varepsilon^2(C-F)\right)^2}_{\mathcal{O}(\varepsilon^4)} + \mathcal{O}(h^4).$$

:

$$\langle y^*\sin\lambda^*\rangle = h^2\underbrace{\frac{1}{2}\left(4\varepsilon^2(C-F)\right)\left(\frac{2(\xi^2-\alpha U_0)}{(2C\hat{x}_0+e_J D)}\right)}_{\mathcal{O}(\varepsilon^2)} + \mathcal{O}(h^4),$$

$$\langle y^*\cos\lambda^*\rangle, \langle x^*y^*\cos\lambda^*\rangle, \langle\sin\lambda^*\rangle, \langle x^*\sin\lambda^*\rangle, \langle x^*y^*\sin\lambda^*\rangle, \langle (x^*)^2\sin\lambda^*\rangle, \langle (y^*)^2\sin\lambda^*\rangle = \mathcal{O}(h^4).$$

, :

$$\left\langle \sin^n\left(\frac{\tilde{\tau}\omega}{\varepsilon}\right)\cos^m\left(\frac{\tilde{\tau}\omega}{\varepsilon}\right)\right\rangle = 0 \qquad n \quad m,$$

h^3 (h^3 $n+m=3, n+m=1$ –).
(..), :

$$\langle H\rangle = \frac{\omega}{\theta(h)}\Bigg(H_{old}(\boldsymbol{z}) + \alpha\langle\Lambda^*\rangle z_\Lambda + 2F\langle x^*\rangle z_x - \langle\cos\lambda^* - 1\rangle\Big(U(z_x,z_y)\cos z_\lambda + V(z_x,z_y)\sin z_\lambda\Big) -$$
$$-\cos z_\lambda\Big((2C\hat{x}_0+e_J D)\langle y^*\sin\lambda^*\rangle + z_x 2C\left(\langle x^*\cos\lambda^*\rangle + \langle y^*\sin\lambda^*\rangle\right) + \left\langle\left(U(x^*,y^*)-U_0\right)\cos\lambda^*\right\rangle\Big) -$$
$$-\sin z_\lambda\Big(z_y 2C\left(\langle x^*\cos\lambda^*\rangle + \langle y^*\sin\lambda^*\rangle\right)\Big)\Bigg) + \mathcal{O}(h^4), \quad (49)$$

$$N = 2C\left(\langle x^*\cos\lambda^*\rangle + \langle y^*\sin\lambda^*\rangle\right).$$

, h^2:

$$\langle H\rangle_0 = H_{old}(\boldsymbol{z}) - h^2\theta_1 H_{old}(\boldsymbol{z}) + \alpha\langle\Lambda^*\rangle_0 z_\Lambda + 2F\langle x^*\rangle_0 z_x -$$
$$- \langle\cos\lambda^* - 1\rangle_0\Big(U(z_x, z_y)\cos z_\lambda + V(z_x, z_y)\sin z_\lambda\Big) -$$
$$-\cos z_\lambda\Big((2C\hat{x}_0 + e_J D)\langle y^*\sin\lambda^*\rangle_0 + z_x 2C\left(\langle x^*\cos\lambda^*\rangle_0 + \langle y^*\sin\lambda^*\rangle_0\right) + \big\langle\big(U(x^*, y^*) - U_0\big)\cos\lambda^*\big\rangle_0\Big) -$$
$$- \sin z_\lambda\Big(z_y 2C\left(\langle x^*\cos\lambda^*\rangle_0 + \langle y^*\sin\lambda^*\rangle_0\right)\Big). \quad (50)$$

$\langle\cdot\rangle_0$ h^2, .

10. 1) h (50)

$$(\hat{\Lambda}_0^*(h), \hat{\lambda}_0^*(h), \hat{x}_0^*(h), \hat{y}_0^*(h))$$

- $\mathcal{O}(h^2)$,

2) h (49)

$$(\hat{\Lambda}_0^{**}(h), \hat{\lambda}_0^{**}(h), \hat{x}_0^{**}(h), \hat{y}_0^{**}(h))$$

- $\mathcal{O}(h^2)$.

:

1,2) :

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}), \quad (51)$$

$\mathbf{x} \in \mathbb{R}^4$ – , $\mathbf{p} \in \mathbb{R}^m$ – .

, :

) $\mathbf{p} = \mathbf{p}_0$ - $\mathbf{x} = 0$, $A_0 = D_\mathbf{x}\mathbf{f}(0, \mathbf{p}_0)$:

$$(\pm i\omega, \pm\xi), \quad \omega > 0, \xi > 0$$

$$\omega \neq l\xi, \quad l \in \mathbb{Z}$$

) :

$$\mathbf{p} = \mathbf{p}_0 + h^2\mathbf{q} + \mathcal{O}(h^4), \quad 0 < h \ll 1$$
$$\mathbf{q} = \mathrm{const}.$$

:

$$\mathbf{f}(\mathbf{x}, \mathbf{p}_0 + h^2\mathbf{q}) = 0.$$

, $\mathbf{f}(0, \mathbf{p}_0) = 0$ ( ) $\det D_\mathbf{x}\mathbf{f}(0, \mathbf{p}_0) = \det A_0 \neq 0$ (0). $\mathbf{x}^*(h) = \mathcal{O}(h^2)$. h (51) $\mathbf{x}^*(h)$, $\mathcal{O}(h^2)$ 0.

:

$$A(h) = A_0 + h^2\mathcal{B} + \mathcal{O}(h^4),$$

$\mathcal{B}$ – .

[?, ?] h:

- -,
- .

(50) (49), .

, $\tilde{U}, \tilde{V}$, U, V :

$$\begin{cases} \tilde{U}(x,y) = \underbrace{C\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)}_{C+\mathcal{O}(h^2)}(x^2-y^2) + \\ +x\underbrace{\left((2C\hat{x}_0 + e_J D)\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right) + 2C\left(\langle x^*\cos\lambda^*\rangle_0 + \langle y^*\sin\lambda^*\rangle_0\right)\right)}_{(2C\hat{x}_0+e_JD)+\mathcal{O}(h^2)} + \\ +\underbrace{\left(U_0\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right) + (2C\hat{x}_0 + e_J D)\langle y^*\sin\lambda^*\rangle_0 + \left\langle\left(U(x^*,y^*) - U_0\right)\cos\lambda^*\right\rangle_0\right)}_{U_0+\mathcal{O}(h^2)} \\ \\ \tilde{V}(x,y) = 2C\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)xy + \\ +y\big((2C\hat{x}_0 + e_J D)\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right) + 2C\left(\langle x^*\cos\lambda^*\rangle_0 + \langle y^*\sin\lambda^*\rangle_0\right)\big). \end{cases}$$

$\langle H\rangle_0$:

$$\langle H\rangle_0 = \frac{\alpha\omega z_\Lambda^2}{2\theta(h)} + \frac{\omega}{\theta(h)}\alpha\langle\Lambda^*\rangle_0 z_\Lambda - \tilde{U}(z_x,z_y)\cos z_\lambda - \tilde{V}(z_x,z_y)\sin z_\lambda + \\ + \frac{F\omega}{\theta(h)}\left((z_x+\hat{x}_0)^2 + z_y^2\right) + (e_J G + 2F\langle x^*\rangle_0)z_x.$$

, $(\hat{\Lambda}_0^*(h), \hat{\lambda}_0^*(h), \hat{x}_0^*(h), \hat{y}_0^*(h))$ -:

$$\begin{cases} \Lambda_0^*(h) = -\dfrac{\omega\alpha\langle\Lambda^*\rangle_0}{\theta(h)}, \\ \lambda_0^*(h) = 0, \\ x_0^*(h) = \mathcal{O}(h^2), \\ y_0^*(h) = 0. \end{cases} \tag{52}$$

0, :

$$\langle H\rangle_0 = \frac{\alpha\omega z_\Lambda^2}{2\theta(h)} - \hat{U}(z_x,z_y)\cos z_\lambda - \hat{V}(z_x,z_y)\sin z_\lambda + \frac{F\omega}{\theta(h)}\left((z_x+\hat{x}_0+\hat{x}_0^*(h))^2 + z_y^2\right) + \\ + \frac{\omega}{\theta(h)}\left(e_J G + 2F\langle x^*\rangle_0 + 2F\hat{x}_0^*(h)\right)z_x, \tag{53}$$

$$\begin{cases} \hat{U}(x,y) = \underbrace{C\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)}_{C+\mathcal{O}(h^2)}(x^2-y^2) + \\ +x\underbrace{\left((2C\hat{x}_0 + e_J D)\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right) + 2C\left(\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)\hat{x}_0^*(h) + T\right)\right)}_{(2C\hat{x}_0+e_JD)+\mathcal{O}(h^2)} + \\ +\big((2C\hat{x}_0 + e_J D)\langle y^*\sin\lambda^*\rangle_0 + \langle U(x^*,y^*)\cos\lambda^*\rangle_0 + T\hat{x}_0^*(h) + C\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)(\hat{x}_0^*(h))^2 + \\ +(2C\hat{x}_0 + e_J D)\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)x_0^*(h)\big), \\ \\ \hat{V}(x,y) = 2C\langle\cos\lambda^*\rangle_0 xy + \\ +y\big((2C\hat{x}_0 + e_J D)\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right) + 2C\left(\langle\cos\lambda^*\rangle_0 - h^2\theta_1\right)\hat{x}_0^*(h) + T\big), \end{cases}$$

$$T \equiv 2C\left(\langle x^*\cos\lambda^*\rangle_0 + \langle y^*\sin\lambda^*\rangle_0\right).$$

, (16) $\mathcal{O}(h^2)$.

, h

11. 1) 0 - (53) , .

:
(53) (16) h (53) 2.1. , .

4.3.1. W^s W^u

$\mathcal{O}(\varepsilon)$ $(|h| \le R(\varepsilon) = \mathcal{O}(\varepsilon))$, μ:

$$h = \mu\varepsilon$$

$$|\mu| \le \frac{R(\varepsilon)}{\varepsilon} = \mathcal{O}(1)$$

(53):

$$\langle H \rangle_0(\boldsymbol{z}) = H_{old}(\boldsymbol{z}) + \hat{H}(\boldsymbol{z}, h, \varepsilon),$$

$$\hat{H}(\boldsymbol{z}, h, \varepsilon) \equiv -h^2\theta_1 H_{old}(\boldsymbol{z}) + 2F\langle x^* \rangle_0 z_x - \langle \cos\lambda^* - 1 \rangle_0 \Big(U(z_x, z_y)\cos z_\lambda + V(z_x, z_y)\sin z_\lambda \Big) - \\ -\cos z_\lambda \Big((2C\hat{x}_0 + e_J D)\langle y^* \sin\lambda^* \rangle_0 + 2Cz_x \left(\langle x^* \cos\lambda^* \rangle + \langle y^* \sin\lambda^* \rangle \right) + \Big\langle \big(U(x^*, y^*) - U_0 \big)\cos\lambda^* \Big\rangle_0 \Big) - \\ - \sin z_\lambda \Big(2Cz_y \left(\langle x^* \cos\lambda^* \rangle + \langle y^* \sin\lambda^* \rangle \right) \Big) + \mathcal{O}(h^4). \quad (54)$$

(54) μ ε:

$$\hat{H}(\boldsymbol{z}, h, \varepsilon) \equiv -\mu^2\varepsilon^2\theta_1 H_{old}(\boldsymbol{z}) + 2F \underbrace{\langle x^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^2)} z_x - \underbrace{\langle \cos\lambda^* - 1 \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^2)} \Big(U(z_x, z_y)\cos z_\lambda + V(z_x, z_y)\sin z_\lambda \Big) - \\ -\cos z_\lambda \Big((2C\hat{x}_0 + e_J D) \underbrace{\langle y^* \sin\lambda^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^4)} + 2Cz_x \left(\underbrace{\langle x^* \cos\lambda^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^2)} + \underbrace{\langle y^* \sin\lambda^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^4)} \right) + \underbrace{\Big\langle \big(U(x^*, y^*) - U_0 \big)\cos\lambda^* \Big\rangle_0}_{\mu^2\mathcal{O}(\varepsilon^2)} \Big) - \\ - \sin z_\lambda \Big(2Cz_y \left(\underbrace{\langle x^* \cos\lambda^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^2)} + \underbrace{\langle y^* \sin\lambda^* \rangle_0}_{\mu^2\mathcal{O}(\varepsilon^4)} \right) \Big) = \\ = \varepsilon^2\mu^2 \Bigg(- H_{old}(\boldsymbol{z}) + 2Fz_x \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)} - \\ - \underbrace{\frac{1}{4}\left(\frac{2(\xi^2 - \alpha U_0)}{(2C\hat{x}_0 + e_J D)} \right)^2}_{\mathcal{O}(1)} \Big(U(z_x, z_y)\cos z_\lambda + V(z_x, z_y)\sin z_\lambda \Big) - \\ - \cos z_\lambda \left(2Cz_x + (2C\hat{x}_0 + e_J D) \right) \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)} - \sin z_\lambda 2Cz_y \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)} \Bigg) + \mathcal{O}(\varepsilon^4).$$

$[\cdot]_0$ ε $H_1(\boldsymbol{z})$, μ ε:

$$H_1(\boldsymbol{z}) \equiv \Bigg[-\theta_1 H_{old}(\boldsymbol{z}) + 2Fz_x \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)} - \underbrace{\frac{1}{4}\left(\frac{2(\xi^2-\alpha U_0)}{(2C\hat{x}_0+e_J D)}\right)^2}_{\mathcal{O}(1)} \Big(U(z_x,z_y)\cos z_\lambda + V(z_x,z_y)\sin z_\lambda\Big) -$$

$$- \cos z_\lambda \left(2Cz_x + (2C\hat{x}^0+e_J D)\right) \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)} - \sin z_\lambda 2Cz_y \underbrace{(-2\varepsilon\xi)\{a\}_{1,1}}_{\mathcal{O}(1)}\Bigg]_0 .$$

:

$$\langle H\rangle_0 = H_{old}(\boldsymbol{z}) + \varepsilon^2\mu^2 H_1(\boldsymbol{z}) + \mathcal{O}(\varepsilon^4). \tag{55}$$

$$\hat{H}(\boldsymbol{z}) = \varepsilon^2\mu^2 H_1(\boldsymbol{z}) + \mathcal{O}(\varepsilon^4).$$

(55) (18):

$$\begin{cases} z_\Lambda^{s,u}(\tau,\mu) = \sum_{k=0}^{\infty} \varepsilon^k z_{\Lambda,k}^{s,u}(\tau,\mu), \\ z_\lambda^{s,u}(\tau,\mu) = \sum_{k=0}^{\infty} \varepsilon^k z_{\lambda,k}^{s,u}(\tau,\mu), \\ z_x^{s,u}(\tau,\mu) = \sum_{k=0}^{\infty} \varepsilon^k z_{x,k}^{s,u}(\tau,\mu), \\ z_y^{s,u}(\tau,\mu) = \sum_{k=0}^{\infty} \varepsilon^k z_{y,k}^{s,u}(\tau,\mu), \end{cases} \tag{56}$$

:

- $W^s(0)$:

$$\begin{cases} z_\Lambda^s(+\infty) = 0, \\ z_\lambda^s(+\infty) = 0, \\ z_x^s(+\infty) = 0, \\ z_y^s(+\infty) = 0, \end{cases}$$

- $W^u(0)$:

$$\begin{cases} z_\Lambda^u(-\infty) = 0, \\ z_\lambda^u(-\infty) = 0, \\ z_x^u(-\infty) = 0, \\ z_y^u(-\infty) = 0. \end{cases}$$

(56) (55)

$$\begin{cases} \dot{z}_\Lambda = -U\sin z_\lambda + V\cos z_\lambda - \varepsilon^2\mu^2 \dfrac{\partial H_1}{\partial z_\lambda} + \mathcal{O}(\varepsilon^4), \\ \dot{z}_\lambda = \alpha z_\Lambda + \varepsilon^2\mu^2 \dfrac{\partial H_1}{\partial z_\Lambda} + \mathcal{O}(\varepsilon^4), \\ \dot{z}_x = -\varepsilon\left(\left(2Fz_y - \dfrac{\partial U}{\partial z_y}\cos z_\lambda - \dfrac{\partial V}{\partial z_y}\sin z_\lambda\right) + \varepsilon^2\mu^2\dfrac{\partial H_1}{\partial z_y} + \mathcal{O}(\varepsilon^4)\right), \\ \dot{z}_y = \varepsilon\left(\left(2F(z_x+\hat{x}_0) + e_J G - \dfrac{\partial U}{\partial z_x}\cos z_\lambda - \dfrac{\partial V}{\partial z_x}\sin z_\lambda\right) + \varepsilon^2\mu^2\dfrac{\partial H_1}{\partial x} + \mathcal{O}(\varepsilon^4)\right), \end{cases} \tag{57}$$

ε.

. :

$$\begin{cases} z_{\Lambda,0}(\tau) = 0, \\ z_{\lambda,0}(\tau) = \lambda_-(x_{sep}(\tau), y_{sep}(\tau)), \\ z_{x,0}(\tau) = x_{sep}(\tau) - \hat{x}_0, \\ z_{y,0}(\tau) = y_{sep}(\tau). \end{cases}$$

$(z_{\Lambda,1}, z_{\lambda,1}, z_{x,1}, z_{y,1})$ $(z_{\Lambda,1}, z_{\lambda,1})$:

$$\begin{cases} z_{\Lambda,1}(\tau) = z'_{\lambda,0}, \\ z_{\lambda,1}(\tau) = \frac{1}{\beta}\Big(z_{x,1}\big((2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\sin z_{\lambda,0} - 2Cz_{y,0}\cos z_{\lambda,0}\big) + \\ + z_{y,1}\big(-2Cz_{y,0}\sin z_{\lambda,0} - (2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\cos z_{\lambda,0}\big)\Big), \end{cases}$$

$(z_{x,1}, z_{y,1})$:

$$\frac{d}{d\tau}\begin{pmatrix} z_{x,1} \\ z_{y,1} \end{pmatrix} = \mathcal{A}(\tau)\begin{pmatrix} z_{x,1} \\ z_{y,1} \end{pmatrix}$$

$\mathcal{A}(\tau)$ - (20). - $\boldsymbol{u}_1(\tau), \tilde{\boldsymbol{u}}_1(\tau)$.
10 W^s W^u , . :

$$(z_{\Lambda,1}(\tau), z_{\lambda,1}(\tau), z_{x,1}(\tau), z_{y,1}(\tau)) \equiv (0,0,0,0)$$

, μ, μ .
$(z_{x,2}, z_{y,2})$, μ:

$$\frac{d}{d\tau}\begin{pmatrix} z_{x,2} \\ z_{y,2} \end{pmatrix} = \mathcal{A}(\tau)\begin{pmatrix} z_{x,2} \\ z_{y,2} \end{pmatrix} + \begin{pmatrix} \tilde{G}_2^x(\tau) \\ \tilde{G}_2^y(\tau) \end{pmatrix} + \mu^2\begin{pmatrix} \mathfrak{G}_2^x(\tau) \\ \mathfrak{G}_2^y(\tau) \end{pmatrix}, \tag{58}$$

:

$$\mathfrak{G}_2^x = \frac{\partial H_1}{\partial z_y}\Big|_{z_{\Lambda,0}, z_{\lambda,0}, z_{x,0}, z_{y,0}} - \\ - \beta^{-1}\left(\frac{\partial H_1}{\partial z_\lambda}\Big|_{z_{\Lambda,0}, z_{\lambda,0}, z_{x,0}, z_{y,0}}\right)\big(-2Cz_{y,0}\sin z_{\lambda,0} - (2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\cos z_{\lambda,0}\big),$$

$$\mathfrak{G}_2^y = \frac{\partial H_1}{\partial z_x}\Big|_{z_{\Lambda,0}, z_{\lambda,0}, z_{x,0}, z_{y,0}} - \\ - \beta^{-1}\left(\frac{\partial H_1}{\partial z_\lambda}\Big|_{z_{\Lambda,0}, z_{\lambda,0}, z_{x,0}, z_{y,0}}\right)\big(-(2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\sin z_{\lambda,0} + 2Cz_{y,0}\cos z_{\lambda,0}\big).$$

$$\begin{cases} z_{\Lambda,2}(\tau) = \frac{1}{\alpha}z'_{\lambda,1}(\tau), \\ z_{\lambda,2}(\tau) = \frac{1}{\beta}\Big(\alpha^{-1}z''_{\lambda,0} + \frac{\partial H_1}{\partial z_\lambda}\Big|_{z_{\Lambda,0}, z_{\lambda,0}, z_{x,0}, z_{y,0}} + \\ + z_{x,2}\big((2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\sin z_{\lambda,0} - 2Cz_{y,0}\cos z_{\lambda,0}\big) + \\ + z_{y,2}\big(-2Cz_{y,0}\sin z_{\lambda,0} - (2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\cos z_{\lambda,0}\big)\Big). \end{cases}$$

, $\mathfrak{G}_2^x(\tau)$ – , $\mathfrak{G}_2^y(\tau)$ – .
, $\tilde{G}_2^x, \tilde{G}_2^y$ $\boldsymbol{z}_0, \boldsymbol{z}_1$:

$$\tilde{G}_2^x = -\beta^{-1}\left(\alpha^{-1}z''_{\lambda,0}\right)\big(-2Cz_{y,0}\sin z_{\lambda,0} - (2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\cos z_{\lambda,0}\big),$$

$$\tilde{G}_2^y = -\beta^{-1}\left(\alpha^{-1} z''_{\lambda,0}\right)\left(-(2Cz_{x,0} + 2C\hat{x}_0 + e_J D)\sin z_{\lambda,0} + 2Cz_{y,0}\cos z_{\lambda,0}\right).$$

, (58) (20) , (58) :

$$\begin{cases} z_{x,2}(\tau,\mu) = z^0_{x,2}(\tau) + \hat{z}_{x,2}(\tau,\mu), \\ z_{y,2}(\tau,\mu) = z^0_{y,2}(\tau) + \hat{z}_{y,2}(\tau,\mu), \end{cases}$$

$(z^0_{x,2}, z^0_{y,2})$ - (20), $(\hat{z}_{x,2}, \hat{z}_{y,2})$ - :

$$\frac{d}{d\tau}\begin{pmatrix}\hat{z}_{x,2}\\ \hat{z}_{y,2}\end{pmatrix} = \mathcal{A}(\tau)\begin{pmatrix}\hat{z}_{x,2}\\ \hat{z}_{y,2}\end{pmatrix} + \mu^2\begin{pmatrix}\mathfrak{G}_2^x(\tau)\\ \mathfrak{G}_2^y(\tau)\end{pmatrix},$$

$\boldsymbol{u}_2 = (x_2, y_2)$, $\boldsymbol{u}_2^0 = (x_2^0, y_2^0)$, $\hat{\boldsymbol{u}}_{\boldsymbol{2}} = (\hat{x}_2, \hat{y}_2)$. $\hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{s},\boldsymbol{u}}$ (s u):

$$\begin{cases} \hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{u}} = \mu^2 \boldsymbol{u}_1(\tau)\displaystyle\int_{\tau^u}^{\tau}(\tilde{y}_1(\tau)\mathfrak{G}_2^x(\tau) - \tilde{x}_1(\tau)\mathfrak{G}_2^y(\tau))\,d\tau + \mu^2\tilde{\boldsymbol{u}}_{\boldsymbol{1}}(\tau)\displaystyle\int_{-\infty}^{\tau}(x_1(\tau)\mathfrak{G}_2^y(\tau) - y_1(\tau)\mathfrak{G}_2^x(\tau))\,d\tau, \\ \\ \hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{s}} = \mu^2 \boldsymbol{u}_1(\tau)\displaystyle\int_{\tau^s}^{\tau}(\tilde{y}_1(\tau)\mathfrak{G}_2^x(\tau) - \tilde{x}_1(\tau)\mathfrak{G}_2^y(\tau))\,d\tau - \mu^2\tilde{\boldsymbol{u}}_{\boldsymbol{1}}(\tau)\displaystyle\int_{\tau}^{+\infty}(x_1(\tau)\mathfrak{G}_2^y(\tau) - y_1(\tau)\mathfrak{G}_2^x(\tau))\,d\tau. \end{cases}$$

, $W^u(\Gamma_{h_1})$ $W^s(\Gamma_{h_2})$,

$$\boldsymbol{u}_2^u(\tau_1,\mu_1) - \boldsymbol{u}_2^s(\tau_2,\mu_2) = \left(\boldsymbol{u}_2^{0,u}(\tau_1) - \boldsymbol{u}_2^{0,s}(\tau_2)\right) + \left(\hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{u}}(\tau_1,\mu_1) - \hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{s}}(\tau_2,\mu_2)\right). \tag{59}$$

$\hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{u}}, \hat{\boldsymbol{u}}_{\boldsymbol{2}}^{\boldsymbol{s}}$ (59) :

$$B(\tau_1,\tau_2)\cdot\begin{pmatrix}\mu_1^2\\ \mu_2^2\end{pmatrix} = \begin{pmatrix}f_1(\tau_1,\tau_2)\\ f_2(\tau_1,\tau_2)\end{pmatrix}, \tag{60}$$

$$B_{1,1} = x_1(\tau_1)\int_{\tau^u}^{\tau_1}(\tilde{y}_1\mathfrak{G}_2^x - \tilde{x}_1\mathfrak{G}_2^y)\,d\tau + \tilde{x}_1(\tau_1)\int_{-\infty}^{\tau_1}(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x)\,d\tau,$$

$$B_{1,2} = -x_1(\tau_2)\int_{\tau^s}^{\tau_2}(\tilde{y}_1\mathfrak{G}_2^x - \tilde{x}_1\mathfrak{G}_2^y)\,d\tau + \tilde{x}_1(\tau_2)\int_{\tau_2}^{+\infty}(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x)\,d\tau,$$

$$B_{2,1} = y_1(\tau_1)\int_{\tau^u}^{\tau_1}(\tilde{y}_1\mathfrak{G}_2^x - \tilde{x}_1\mathfrak{G}_2^y)\,d\tau + \tilde{y}_1(\tau_1)\int_{-\infty}^{\tau_1}(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x)\,d\tau,$$

$$B_{2,2} = -y_1(\tau_2)\int_{\tau^s}^{\tau_2}(\tilde{y}_1\mathfrak{G}_2^x - \tilde{x}_1\mathfrak{G}_2^y)\,d\tau + \tilde{y}_1(\tau_2)\int_{\tau_2}^{+\infty}(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x)\,d\tau,$$

$$\begin{aligned} f_1 = &-x_2^{0,u}(\tau_1) + x_2^{0,s}(\tau_1) = \\ &= -x_1(\tau_1)\int_{\tau_2^u}^{\tau_1}\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)d\tau - \tilde{x}_1(\tau_1)\int_{-\infty}^{\tau_1}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau + \\ &\quad + x_1(\tau_2)\int_{\tau_2^s}^{\tau_2}\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)d\tau - \tilde{x}_1(\tau_2)\int_{\tau_2}^{+\infty}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau, \end{aligned}$$

$$\begin{aligned} f_2 = &-y_2^{0,u}(\tau_1) + y_2^{0,s}(\tau_1) = \\ &= -y_1(\tau_1)\int_{\tau_2^u}^{\tau_1}\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)d\tau - \tilde{y}_1(\tau_1)\int_{-\infty}^{\tau_1}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau + \\ &\quad + y_1(\tau_2)\int_{\tau_2^s}^{\tau_2}\left(\tilde{y}_1\tilde{G}_2^x - \tilde{x}_1\tilde{G}_2^y\right)d\tau - \tilde{y}_1(\tau_2)\int_{\tau_2}^{+\infty}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau. \end{aligned}$$

, (60) $\mu_1^2, \mu_2^2,$ $\{\tau_1, \tau_2, \tau^s, \tau^u, \tau_2^s, \tau_2^u\}.$

(60) $\det B(\tau_1, \tau_2) \neq 0.$

$\tau^u = \tau_1,\ \tau^s = \tau_2,$:

$$\det B(\tau_1,\tau_2) = \int_{-\infty}^{\tau_1} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau \int_{\tau_2}^{+\infty} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau\, (\tilde{x}_1(\tau_1)\tilde{y}_1(\tau_2) - \tilde{x}_1(\tau_2)\tilde{y}_1(\tau_1)) \equiv$$

$$\equiv F(\tau_1,\tau_2) \int_{-\infty}^{\tau_1} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau \int_{\tau_2}^{+\infty} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau,$$

$$B_{1,1} = \tilde{x}_1(\tau_1) \int_{-\infty}^{\tau_1} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau,$$

$$B_{1,2} = \tilde{x}_1(\tau_2) \int_{\tau_2}^{+\infty} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau,$$

$$B_{2,1} = \tilde{y}_1(\tau_1) \int_{-\infty}^{\tau_1} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau,$$

$$B_{2,2} = \tilde{y}_1(\tau_2) \int_{\tau_2}^{+\infty} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau.$$

, $\mu_1, \mu_2 > 0,$ (60) $\mu_1^2 > 0, \mu_2^2 > 0.$:

$$\frac{B_{1,1}f_2 - B_{2,1}f_1}{\det B} > 0,$$
$$\frac{B_{2,2}f_1 - B_{1,2}f_2}{\det B} > 0,$$

$$B_{1,1}f_2 - B_{2,1}f_1 = \int_{-\infty}^{\tau_1} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau \Bigg(\int_{\tau_2^u}^{\tau_1} \left(\tilde{y}_1 \tilde{G}_2^x - \tilde{x}_1 \tilde{G}_2^y\right) d\tau \Big(\underbrace{x_1(\tau_1)\tilde{y}_1(\tau_1) - \tilde{x}_1(\tau_1)y_1(\tau_1)}_{W(\boldsymbol{u}_1, \tilde{\boldsymbol{u}}_1)=1} \Big) +$$
$$+ \int_{\tau_2^s}^{\tau_2} \left(\tilde{y}_1 \tilde{G}_2^x - \tilde{x}_1 \tilde{G}_2^y\right) d\tau \Big(\tilde{x}_1(\tau_1)y_1(\tau_2) - \tilde{y}_1(\tau_1)x_1(\tau_2)\Big) +$$
$$+ \int_{\tau_2}^{+\infty} \left(x_1 \tilde{G}_2^y - y_1 \tilde{G}_2^x\right) d\tau \Big(\underbrace{\tilde{y}_1(\tau_1)\tilde{x}_1(\tau_2) - \tilde{x}_1(\tau_1)\tilde{y}_1(\tau_2)}_{-F(\tau_1,\tau_2)} \Big) \Bigg),$$

$$B_{2,2}f_1 - B_{1,2}f_2 = \int_{\tau_2}^{+\infty} (x_1 \mathfrak{G}_2^y - y_1 \mathfrak{G}_2^x)\, d\tau \Bigg(\int_{\tau_2^u}^{\tau_1} \left(\tilde{y}_1 \tilde{G}_2^x - \tilde{x}_1 \tilde{G}_2^y\right) d\tau \Big(\tilde{x}_1(\tau_2)y_1(\tau_1) - \tilde{y}_1(\tau_2)x_1(\tau_1)\Big) +$$
$$+ \int_{\tau_2^s}^{\tau_2} \left(\tilde{y}_1 \tilde{G}_2^x - \tilde{x}_1 \tilde{G}_2^y\right) d\tau \Big(\underbrace{x_1(\tau_2)\tilde{y}_1(\tau_2) - \tilde{x}_1(\tau_2)y_1(\tau_2)}_{W(\boldsymbol{u}_1, \tilde{\boldsymbol{u}}_1)=1} \Big) +$$
$$+ \int_{-\infty}^{\tau_1} \left(x_1 \tilde{G}_2^y - y_1 \tilde{G}_2^x\right) d\tau \Big(\underbrace{\tilde{y}_1(\tau_1)\tilde{x}_1(\tau_2) - \tilde{x}_1(\tau_1)\tilde{y}_1(\tau_2)}_{-F(\tau_1,\tau_2)} \Big) \Bigg).$$

, :

$$\frac{B_{1,1}f_2 - B_{2,1}f_1}{\det B} = -\frac{\int_{\tau_2}^{+\infty}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau}{\int_{\tau_2}^{+\infty}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} + \frac{\int_{\tau_2^u}^{\tau_1}\left(\tilde{x}_1\tilde{G}_2^y - \tilde{y}_1\tilde{G}_2^x\right)d\tau}{F(\tau_1,\tau_2)\int_{\tau_2}^{+\infty}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} + \\ + \frac{\int_{\tau_2^s}^{\tau_2}\left(\tilde{x}_1\tilde{G}_2^y - \tilde{y}_1\tilde{G}_2^x\right)d\tau\left(\tilde{x}_1(\tau_1)y_1(\tau_2) - \tilde{y}_1(\tau_1)x_1(\tau_2)\right)}{F(\tau_1,\tau_2)\int_{\tau_2}^{+\infty}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} > 0,$$

$$\frac{B_{2,2}f_1 - B_{1,2}f_2}{\det B} = -\frac{\int_{-\infty}^{\tau_1}\left(x_1\tilde{G}_2^y - y_1\tilde{G}_2^x\right)d\tau}{\int_{-\infty}^{\tau_1}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} + \frac{\int_{\tau_2^s}^{\tau_2}\left(\tilde{x}_1\tilde{G}_2^y - \tilde{y}_1\tilde{G}_2^x\right)d\tau}{F(\tau_1,\tau_2)\int_{-\infty}^{\tau_1}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} + \\ + \frac{\int_{\tau_2^u}^{\tau_1}\left(\tilde{x}_1\tilde{G}_2^y - \tilde{y}_1\tilde{G}_2^x\right)d\tau\left(\tilde{x}_1(\tau_2)y_1(\tau_1) - \tilde{y}_1(\tau_2)x_1(\tau_1)\right)}{F(\tau_1,\tau_2)\int_{-\infty}^{\tau_1}\left(x_1\mathfrak{G}_2^y - y_1\mathfrak{G}_2^x\right)d\tau} > 0,$$

$$F(\tau_1,\tau_2) \equiv \tilde{y}_1(\tau_1)\tilde{x}_1(\tau_2) - \tilde{x}_1(\tau_1)\tilde{y}_1(\tau_2).$$

5.