Baye's Theorem

1. **In a city, 5% of emails are spam. If 90% of spam emails contain the word 'Free', and 10% of non-spam emails also contain 'Free', what is the probability an email is spam if it contains 'Free'?**

Given:

- $P(\text{Spam}) = 0.05$
- $P(\text{Not Spam}) = 0.95$
- $P(\text{Free}|\text{Spam}) = 0.90$
- $P(\text{Free}|\text{Not Spam}) = 0.10$

We want: $P(\text{Spam}|\text{Free})$.

Step 1: Bayes' Formula

$$P(\text{Spam}|\text{Free}) = \frac{P(\text{Free}|\text{Spam}) \cdot P(\text{Spam})}{P(\text{Free})}$$

Where

$$P(\text{Free}) = P(\text{Free}|\text{Spam}) \cdot P(\text{Spam}) + P(\text{Free}|\text{Not Spam}) \cdot P(\text{Not Spam})$$

Step 2: Calculate $P(\text{Free})$

$$P(\text{Free}) = (0.90)(0.05) + (0.10)(0.95)$$

$$= 0.045 + 0.095 = 0.14$$

Step 3: Apply Bayes

$$P(\text{Spam}|\text{Free}) = \frac{0.90 \times 0.05}{0.14}$$

$$= \frac{0.045}{0.14} \approx 0.3214$$

✅ **Final Answer:** The probability the email is spam given it contains "Free" is about **32.14%**.

2. **A medical test for a rare disease is 99% accurate for detecting the disease when it's present and 95% accurate for detecting when it's absent. If 0.5% of the population has the disease, what's the probability a person actually has it if they test positive?**

Given:

- $P(\text{Disease}) = 0.005$ (0.5% have disease)
- $P(\text{No Disease}) = 0.995$
- $P(\text{Positive}|\text{Disease}) = 0.99$ (true positive rate)
- $P(\text{Positive}|\text{No Disease}) = 0.05$ (false positive rate, since 95% accurate)

We want:

$$P(\text{Disease}|\text{Positive})$$

Step 1: Bayes' Formula

$$P(\text{Disease}|\text{Positive}) = \frac{P(\text{Positive}|\text{Disease}) \cdot P(\text{Disease})}{P(\text{Positive})}$$

Where

$$P(\text{Positive}) = P(\text{Positive}|\text{Disease}) \cdot P(\text{Disease}) + P(\text{Positive}|\text{No Disease}) \cdot P(\text{No Disease})$$

Step 2: Calculate $P(\text{Positive})$

$$P(\text{Positive}) = (0.99)(0.005) + (0.05)(0.995)$$

$$= 0.00495 + 0.04975 = 0.0547$$

Step 3: Apply Bayes

$$P(\text{Disease}|\text{Positive}) = \frac{0.99 \times 0.005}{0.0547}$$

$$= \frac{0.00495}{0.0547} \approx 0.0905$$

✅ **Final Answer:** If a person tests positive, the probability they actually have the disease is about **9.05%**.

3. A company has two factories: Factory A produces 60% of products, Factory B produces 40%. 3% of Factory A's products are defective, while

5% of Factory B's are defective. If a product is defective, what is the probability it came from Factory B?

Given:

- $P(A) = 0.6$ (Factory A produces 60%)
- $P(B) = 0.4$ (Factory B produces 40%)
- $P(D|A) = 0.03$ (3% defective from A)
- $P(D|B) = 0.05$ (5% defective from B)

We want:

$$P(B|D) \quad \text{(probability product came from B given it is defective)}$$

Step 1: Bayes' Formula

$$P(B|D) = \frac{P(D|B) \cdot P(B)}{P(D|A) \cdot P(A) + P(D|B) \cdot P(B)}$$

Step 2: Calculate denominator $P(D)$

$$P(D) = (0.03)(0.6) + (0.05)(0.4)$$

$$= 0.018 + 0.020 = 0.038$$

Step 3: Apply Bayes

$$P(B|D) = \frac{0.05 \times 0.4}{0.038}$$

$$= \frac{0.020}{0.038} \approx 0.5263$$

✅ **Final Answer:** If a product is defective, there's about **52.63% chance it came from Factory B.**

3. **A website has two types of visitors: new (70%) and returning (30%). New visitors click on ads 5% of the time, returning visitors click 15% of the time. If someone clicked an ad, what's the probability they are a returning visitor?**

Given:

- $P(\text{New}) = 0.7$
- $P(\text{Returning}) = 0.3$
- $P(\text{Click}|\text{New}) = 0.05$
- $P(\text{Click}|\text{Returning}) = 0.15$

We want:

$$P(\text{Returning}|\text{Click})$$

Step 1: Bayes' Formula

$$P(\text{Returning}|\text{Click}) = \frac{P(\text{Click}|\text{Returning}) \cdot P(\text{Returning})}{P(\text{Click})}$$

Where

$$P(\text{Click}) = P(\text{Click}|\text{New}) \cdot P(\text{New}) + P(\text{Click}|\text{Returning}) \cdot P(\text{Returning})$$

Step 2: Compute $P(\text{Click})$

$$P(\text{Click}) = (0.05)(0.7) + (0.15)(0.3)$$

$$= 0.035 + 0.045 = 0.08$$

Step 3: Apply Bayes

$$P(\text{Returning}|\text{Click}) = \frac{0.15 \times 0.3}{0.08}$$

$$= \frac{0.045}{0.08} = 0.5625$$

✅ **Final Answer:** If someone clicked an ad, the probability they are a **returning visitor** is **56.25%.**

5. In a university, 40% of students study Data Science, and 60% study other fields. 80% of Data Science students own laptops, while 50% of others own laptops. If a randomly chosen student owns a laptop, what is the probability they study Data Science?

Given:

- $P(\text{Data Science}) = 0.4$
- $P(\text{Other}) = 0.6$
- $P(\text{Laptop}|\text{Data Science}) = 0.8$
- $P(\text{Laptop}|\text{Other}) = 0.5$

We want:

$$P(\text{Data Science}|\text{Laptop})$$

Step 1: Bayes' Formula

$$P(\text{Data Science}|\text{Laptop}) = \frac{P(\text{Laptop}|\text{Data Science}) \cdot P(\text{Data Science})}{P(\text{Laptop})}$$

Where

$$P(\text{Laptop}) = P(\text{Laptop}|\text{Data Science}) \cdot P(\text{Data Science}) + P(\text{Laptop}|\text{Other}) \cdot P(\text{Other})$$

Step 2: Compute $P(\text{Laptop})$

$$P(\text{Laptop}) = (0.8)(0.4) + (0.5)(0.6)$$

$$= 0.32 + 0.30 = 0.62$$

Step 3: Apply Bayes

$$P(\text{Data Science}|\text{Laptop}) = \frac{0.8 \times 0.4}{0.62}$$

$$= \frac{0.32}{0.62} \approx 0.5161$$

✅ **Final Answer:** If a student owns a laptop, the probability they study **Data Science** is about **51.61%**.

6. In a hospital, 1% of patients have diabetes. A test correctly identifies 95% of people with diabetes and 90% without it. If a patient tests positive, what's the probability they have diabetes?

Given:

- $P(\text{Diabetes}) = 0.01$
- $P(\text{No Diabetes}) = 0.99$
- $P(\text{Positive}|\text{Diabetes}) = 0.95$
- $P(\text{Positive}|\text{No Diabetes}) = 0.10$ (since 90% correctly test negative)

We want:

$$P(\text{Diabetes}|\text{Positive})$$

Step 1: Bayes' Formula

$$P(\text{Diabetes}|\text{Positive}) = \frac{P(\text{Positive}|\text{Diabetes}) \cdot P(\text{Diabetes})}{P(\text{Positive})}$$

Where

$P(\text{Positive}) = P(\text{Positive}|\text{Diabetes}) \cdot P(\text{Diabetes}) + P(\text{Positive}|\text{No Diabetes}) \cdot P(\text{No Diabetes})$

Step 2: Compute $P(\text{Positive})$

$$P(\text{Positive}) = (0.95)(0.01) + (0.10)(0.99)$$

$$= 0.0095 + 0.099 = 0.1085$$

Step 3: Apply Bayes

$$P(\text{Diabetes}|\text{Positive}) = \frac{0.95 \times 0.01}{0.1085}$$

$$= \frac{0.0095}{0.1085} \approx 0.0875$$

✅ **Final Answer:** If a patient tests positive, the probability they actually have diabetes is about **8.75%**.

7. A marketing campaign reaches two groups: teens (40%) and adults (60%). Teens respond 20% of the time, adults 5% of the time. If someone responds, what's the probability they are a teen?

Given:

- $P(\text{Teen}) = 0.4$
- $P(\text{Adult}) = 0.6$
- $P(\text{Respond}|\text{Teen}) = 0.2$
- $P(\text{Respond}|\text{Adult}) = 0.05$

We want:

$$P(\text{Teen}|\text{Respond})$$

Step 1: Bayes' Formula

$$P(\text{Teen}|\text{Respond}) = \frac{P(\text{Respond}|\text{Teen}) \cdot P(\text{Teen})}{P(\text{Respond})}$$

Where

$$P(\text{Respond}) = P(\text{Respond}|\text{Teen}) \cdot P(\text{Teen}) + P(\text{Respond}|\text{Adult}) \cdot P(\text{Adult})$$

Step 2: Compute $P(\text{Respond})$

$$P(\text{Respond}) = (0.2)(0.4) + (0.05)(0.6)$$

$$= 0.08 + 0.03 = 0.11$$

Step 3: Apply Bayes

$$P(\text{Teen}|\text{Respond}) = \frac{0.2 \times 0.4}{0.11}$$

$$= \frac{0.08}{0.11} \approx 0.7273$$

✅ **Final Answer:** If someone responds, the probability they are a **teen** is about **72.73%**.

8. In a tech store, 30% of customers buy smartphones, 50% buy laptops, and 20% buy both. If a customer bought a laptop, what's the probability they also bought a smartphone?

Given:

- $P(\text{Smartphone}) = 0.3$
- $P(\text{Laptop}) = 0.5$
- $P(\text{Both}) = 0.2$

We want:

$$P(\text{Smartphone}|\text{Laptop})$$

Step 1: Conditional Probability Formula

$$P(\text{Smartphone}|\text{Laptop}) = \frac{P(\text{Both})}{P(\text{Laptop})}$$

Step 2: Apply Values

$$P(\text{Smartphone}|\text{Laptop}) = \frac{0.2}{0.5} = 0.4$$

✅ **Answer:** If a customer bought a laptop, the probability they also bought a **smartphone** is **40%**.

9. In a factory, Machine X makes 70% of items and Machine Y makes 30%. Machine X's items are defective 2% of the time, Machine Y's are defective 4% of the time. If an item is defective, what's the probability it came from Machine Y?

Given:

- $P(X) = 0.7, P(Y) = 0.3$
- $P(D|X) = 0.02, P(D|Y) = 0.04$

We want:

$$P(Y|D)$$

Step 1: Bayes' Formula

$$P(Y|D) = \frac{P(D|Y) \cdot P(Y)}{P(D|X) \cdot P(X) + P(D|Y) \cdot P(Y)}$$

Step 2: Compute Denominator $P(D)$

$$P(D) = (0.02)(0.7) + (0.04)(0.3) = 0.014 + 0.012 = 0.026$$

Step 3: Apply Bayes

$$P(Y|D) = \frac{0.04 \cdot 0.3}{0.026} = \frac{0.012}{0.026} \approx 0.4615$$

✅ **Answer:** If an item is defective, the probability it came from **Machine Y** is about **46.15%**.

10. At a bank, 60% of transactions are deposits, 40% withdrawals. Deposits have a 1% error rate, withdrawals 2%. If a transaction had an error, what's the probability it was a withdrawal?

Given:

- $P(\text{Deposit}) = 0.6$, $P(\text{Withdrawal}) = 0.4$
- $P(\text{Error}|\text{Deposit}) = 0.01$, $P(\text{Error}|\text{Withdrawal}) = 0.02$

We want:

$$P(\text{Withdrawal}|\text{Error})$$

Step 1: Bayes' Formula

$$P(\text{Withdrawal}|\text{Error}) = \frac{P(\text{Error}|\text{Withdrawal}) \cdot P(\text{Withdrawal})}{P(\text{Error})}$$

Where

$$P(\text{Error}) = P(\text{Error}|\text{Deposit}) \cdot P(\text{Deposit}) + P(\text{Error}|\text{Withdrawal}) \cdot P(\text{Withdrawal})$$

Step 2: Compute $P(\text{Error})$

$$P(\text{Error}) = (0.01)(0.6) + (0.02)(0.4) = 0.006 + 0.008 = 0.014$$

Step 3: Apply Bayes

$$P(\text{Withdrawal}|\text{Error}) = \frac{0.02 \cdot 0.4}{0.014} = \frac{0.008}{0.014} \approx 0.5714$$

✅ **Answer:** If a transaction had an error, the probability it was a **withdrawal** is about **57.14%**.

11. A school has 55% female and 45% male students. 10% of female students wear glasses, and 5% of male students wear glasses. If a student is wearing glasses, what's the probability they are female?

Given:

- $P(\text{Female}) = 0.55, P(\text{Male}) = 0.45$
- $P(\text{Glasses}|\text{Female}) = 0.10, P(\text{Glasses}|\text{Male}) = 0.05$

We want:

$$P(\text{Female}|\text{Glasses})$$

Step 1: Bayes' Formula

$$P(\text{Female}|\text{Glasses}) = \frac{P(\text{Glasses}|\text{Female}) \cdot P(\text{Female})}{P(\text{Glasses})}$$

Where

$$P(\text{Glasses}) = P(\text{Glasses}|\text{Female}) \cdot P(\text{Female}) + P(\text{Glasses}|\text{Male}) \cdot P(\text{Male})$$

Step 2: Compute $P(\text{Glasses})$

$$P(\text{Glasses}) = (0.10)(0.55) + (0.05)(0.45) = 0.055 + 0.0225 = 0.0775$$

Step 3: Apply Bayes

$$P(\text{Female}|\text{Glasses}) = \frac{0.10 \cdot 0.55}{0.0775} = \frac{0.055}{0.0775} \approx 0.7097$$

✅ **Answer:** If a student is wearing glasses, the probability they are **female** is about **70.97%**.

12. In a call center, 80% of calls are customer service and 20% are technical support. 90% of customer service calls are resolved in under 5 minutes, but only 60% of technical support calls are. If a call was resolved in under 5 minutes, what's the probability it was customer service?

Given:

- $P(\text{Customer}) = 0.8$, $P(\text{Technical}) = 0.2$
- $P(<5\text{min}|\text{Customer}) = 0.9$
- $P(<5\text{min}|\text{Technical}) = 0.6$

We want:

$$P(\text{Customer}|<5\text{min})$$

Step 1: Bayes' Formula

$$P(\text{Customer}|<5\text{min}) = \frac{P(<5\text{min}|\text{Customer}) \cdot P(\text{Customer})}{P(<5\text{min})}$$

Where

$$P(<5\text{min}) = P(<5\text{min}|\text{Customer}) \cdot P(\text{Customer}) + P(<5\text{min}|\text{Technical}) \cdot P(\text{Technical})$$

Step 2: Compute $P(<5\text{min})$

$$P(<5\text{min}) = (0.9)(0.8) + (0.6)(0.2) = 0.72 + 0.12 = 0.84$$

Step 3: Apply Bayes

$$P(\text{Customer}|<5\text{min}) = \frac{0.9 \cdot 0.8}{0.84} = \frac{0.72}{0.84} \approx 0.8571$$

✅ **Answer:** If a call was resolved in under 5 minutes, the probability it was a **customer service** call is about **85.71%.**

13. A product review site finds that 40% of products are electronics, 60% non-electronics. 50% of electronics get 5-star ratings, but only 30% of non-electronics do. If a product got 5 stars, what's the probability it's electronics?

Given:

- $P(\text{Electronics}) = 0.4$, $P(\text{Non-Electronics}) = 0.6$
- $P(\text{5-Star}|\text{Electronics}) = 0.5$
- $P(\text{5-Star}|\text{Non-Electronics}) = 0.3$

We want:

$$P(\text{Electronics}|\text{5-Star})$$

Step 1: Bayes' Formula

$$P(\text{Electronics}|\text{5-Star}) = \frac{P(\text{5-Star}|\text{Electronics}) \cdot P(\text{Electronics})}{P(\text{5-Star})}$$

Where

$$P(\text{5-Star}) = P(\text{5-Star}|\text{Electronics}) \cdot P(\text{Electronics}) + P(\text{5-Star}|\text{Non-Electronics}) \cdot P(\text{Non-Electronics})$$

Step 2: Compute $P(\text{5-Star})$

$$P(\text{5-Star}) = (0.5)(0.4) + (0.3)(0.6) = 0.2 + 0.18 = 0.38$$

Step 3: Apply Bayes

$$P(\text{Electronics}|\text{5-Star}) = \frac{0.5 \cdot 0.4}{0.38} = \frac{0.2}{0.38} \approx 0.5263$$

✅ **Answer:** If a product got 5 stars, the probability it's **electronics** is about **52.63%.**

14. A sports store sells 65% running shoes and 35% casual shoes. 25% of running shoes are red, while 10% of casual shoes are red. If a shoe is red, what's the probability it's a running shoe?

Given:

- $P(\text{Running}) = 0.65$, $P(\text{Casual}) = 0.35$
- $P(\text{Red}|\text{Running}) = 0.25$
- $P(\text{Red}|\text{Casual}) = 0.10$

We want:

$$P(\text{Running}|\text{Red})$$

Step 1: Bayes' Formula

$$P(\text{Running}|\text{Red}) = \frac{P(\text{Red}|\text{Running}) \cdot P(\text{Running})}{P(\text{Red})}$$

Where

$$P(\text{Red}) = P(\text{Red}|\text{Running}) \cdot P(\text{Running}) + P(\text{Red}|\text{Casual}) \cdot P(\text{Casual})$$

Step 2: Compute $P(\text{Red})$

$$P(\text{Red}) = (0.25)(0.65) + (0.10)(0.35) = 0.1625 + 0.035 = 0.1975$$

Step 3: Apply Bayes

$$P(\text{Running}|\text{Red}) = \frac{0.25 \cdot 0.65}{0.1975} = \frac{0.1625}{0.1975} \approx 0.8228$$

✅ **Answer:** If a shoe is red, the probability it's a **running shoe** is about **82.28%**.

15. A streaming platform has 75% movie viewers and 25% series viewers. 40% of movie viewers watch comedies, while 60% of series viewers do. If someone is watching a comedy, what's the probability they are a series viewer?

Given:

- $P(\text{Movie}) = 0.75$, $P(\text{Series}) = 0.25$
- $P(\text{Comedy}|\text{Movie}) = 0.40$
- $P(\text{Comedy}|\text{Series}) = 0.60$

We want:

$$P(\text{Series}|\text{Comedy})$$

Step 1: Bayes' Formula

$$P(\text{Series}|\text{Comedy}) = \frac{P(\text{Comedy}|\text{Series}) \cdot P(\text{Series})}{P(\text{Comedy})}$$

Where

$$P(\text{Comedy}) = P(\text{Comedy}|\text{Movie}) \cdot P(\text{Movie}) + P(\text{Comedy}|\text{Series}) \cdot P(\text{Series})$$

Step 2: Compute $P(\text{Comedy})$

$$P(\text{Comedy}) = (0.40)(0.75) + (0.60)(0.25) = 0.30 + 0.15 = 0.45$$

Step 3: Apply Bayes

$$P(\text{Series}|\text{Comedy}) = \frac{0.60 \cdot 0.25}{0.45} = \frac{0.15}{0.45} = 0.3333$$

✅ **Answer:** If someone is watching a comedy, the probability they are a **series viewer** is about **33.33%.**