# 40.752 Algorithmic Game Theory

## Project Proposal

## TO COOPERATE OR TO COMPETE: A GAME THEORETIC MODEL

Lai Zhangsheng        Nguyen Tan Thai Hung

## 1 Motivation

The idea for this project is drawn upon our own observation from past work experiences.

In the workplace, managers typically want their staff to cooperate in order to achieve a good performance for the department. However, staff appraisals are done according to a bell curve, which disincentify people to cooperate (If I help you, you'll get better and my chance of getting the top post is less). This appraisal mechanism is thus not incentive compatible.

In the school context, we observe a similar phenomenon. Students should study together and help each other, but the bell curve system can cause competition among students, leading to inefficiency.

## 2 Project description

In this project, we want to model what happens in the workplace from a game theoretic perspective. Then, we will attempt to figure out a way to quantify the performance, and propose a better mechanism.

### 2.1 The model

For the simplest case, we will model the game as a two-agent payoff-maximizing game. The payoff each agent receives depend on whether his performance is better than the other agent. Each agent's performance depends on whether they perform their task alone or with help from the other agent. We believe that under the ranking-base reward scheme, the equilibrium strategy is for each agent to work alone, whereas the socially optimal strategy is for them to cooperate. We will then compute the Price of Anarchy and its bounds.

### 2.2 Mechanism design

After determining the performance of the existing reward scheme, we will attempt to propose a different reward scheme such that cooperation becomes the dominant strategy. We draw our inspiration from real life games such as World of Warcraft and DOTA, where we have seen cooperative efforts to achieve the game objectives. Hence, we hope to transfer what was applied in the games to the workplace setting.

## 3 Experiment

This may be somewhat too ambitious, but we hope to conduct an experimental game on the ARENA website to verify our model.

# 4    Theoretical background

The problem was first mentioned in literature by Drago and Turnbull (1991). Since this seminal work, several scholars have investigated the problems in deeper details:

- Drago and Garvey (1998) and Kistruck et al. (2016) discuss different reward structures to encourage cooperation.

- Banerjee et al. (2014) and Chakravarti et al. (2015) also explore incentive schemes, but with focus on knowledge sharing among researchers and team members.

- Immorlica et al. (2011) discussed a dueling game and show how bad head-on competition can get.

- Landers et al. (2015) described an interesting gamification experiment to understand how people actually behave when the reward mechanism is changed.

# 5    Comparison with Prisoners' Dilemma

The Prisoners' Dilemma's payoff table is

Table 1: Prisoners' Dilemma's payoff table

|        | Silent | Betray |
|--------|--------|--------|
| Silent | 1, 1   | 3, 0   |
| Betray | 0, 3   | 2, 2   |

The payoff table of our game is still to be determined, but we believe that there are some differences, and we may want to tackle some or all of them

- In the Prisoners' Dilemma game, players can move only once and they are not allowed to talk to each other. In our game, both these constraints are not present.

- The Prisoners' Dilemma is symmetric, while it is reasonable to assume that our two office workers are different in many ways. For example, they have different skill sets.

Table 2: Workplace game's payoff table

|           | Cooperate | Compete |
|-----------|-----------|---------|
| Cooperate | ?, ?      | ?, ?    |
| Compete   | ?, ?      | ?, ?    |

# 6 The Model

We shall consider an office game where there are $k$ agents, each with its own project. Every agent is assumed to have:

- the same time budget $T$

- a daily schedule represented by a vector $t_i = (t_{i1}, t_{i2}, \ldots, t_{ik})$ with $\sum_k t_{ik} = T$.

For each project which is owned by a sole agent, there is a work output (performance) associated with it that is dependent on the total time (effort) invested into the project

$$p_j = \beta_j \log \left( t_{ii} + \sum_{i \neq j}^{k} \alpha_{ij} t_{ij} \right)$$

where $\alpha_{ij}$ is the effectiveness (skill) if agent $i$ when working on project $j$ (w.r.t the owner) and $\beta_j$ is the value of the project. Using the performance $p_j$, we can assign a rank to each agent and then derive its corresponding payoffs:

$$u_j = f(r_j) \times g(P)$$

where $P = \sum_i p_i$. The utility you derive depends on how well you perform relative to the other agents and how the company performs as a whole.

# References

Banerjee, S., Goel, A., and Kollagunta Krishnaswamy, A. (2014). Re-incentivizing discovery. In *Proceedings of the fifteenth ACM conference on Economics and computation - EC '14*, pages 149–166, New York, New York, USA. ACM Press.

Chakravarti, A., He, C., and Wagman, L. (2015). Inducing knowledge sharing in teams through cost-efficient compensation schemes. *Knowledge Management Research & Practice*, 13(1)(2015):1–20.

Drago, R. and Garvey, G. T. (1998). Incentives for Helping on the Job: Theory and Evidence. *Journal of Labor Economics*, 16(1):1–25.

Drago, R. and Turnbull, G. K. (1991). Competition and cooperation in the workplace. *Journal of Economic Behavior and Organization*, 15(3):347–364.

Immorlica, N., Kalai, A. T., Lucier, B., Moitra, A., Postlewaite, A., and Tennenholtz, M. (2011). Dueling Algorithms. *Stoc*, page 26.

Kistruck, G. M., Lount Jr, R. B., Smith, B. R., Bergman Jr, B. J., and Moss, T. W. (2016). Cooperation Vs. Competition: Alternative Goal Structures for Motivating Groups in a Resource Scarce Environment. *Academy of Management Journal*, 59(4):1174–1198.

Landers, R. N., Bauer, K. N., and Callan, R. C. (2015). Gamification of task performance with leaderboards: A goal setting experiment. *Computers in Human Behavior*, Article in:1–8.