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Algorithmic Game Theory: HW 2

1.

2. (a) Consider the utility maximizing game below starting with the the initial outcome (A_1, B_1), from which best-response dynamics cycles forever, avoiding the pure Nash of (A_3, B_2).

		P1		
		A_1	A_2	A_3
	B_1	$4,1$	$1,2$	$0,0$
P2	B_2	$0,0$	$0,0$	$5,5$
	B_3	$3,3$	$3,2$	$0,0$

(b) Consider the game below

		P1		
		A_1	A_2	A_3
	B_1	$0,0$	$0,0$	$5,5$
P2	B_2	$0,0$	$0,0$	$5,5$
	B_3	$5,5$	$5,5$	$5,5$

3.

4. Let $f_\epsilon(x) = (1-\epsilon)^x$ and $g_\epsilon(x) = 1 + \epsilon x$, then

$$
\begin{aligned}
&f_\epsilon(0) = 1 = g_\epsilon(0) \\
&f_\epsilon(1) = 1 - \epsilon = g_\epsilon(1) \\
&\left.\begin{aligned} f'_\epsilon(x) &= (1-\epsilon)^x \ln(1-\epsilon) \\ g'_\epsilon(x) &= \epsilon \end{aligned}\right\} f'_\epsilon(0) = \ln(1-\epsilon) < 0 = g'_\epsilon(0)
\end{aligned}
$$

also f_ϵ is a convex function as $f''_\epsilon(x) = (1-\epsilon)^x \left[\ln(1-\epsilon)\right]^2 > 0$ for $\epsilon \in (0, 1/2]$. This this proves $f_\epsilon(x) \leq g_\epsilon(x)$ since the initial gradient of f_ϵ is smaller then g_ϵ