40.752 Algorithmic Game Theory

Project Proposal

TO COOPERATE OR TO COMPETE: A GAME THEORETIC MODEL

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1 Motivation

The idea for this project is drawn upon our own observation from past work experiences. In the workplace, managers typically want their staff to cooperate with one another in order to achieve a good overall performance for the company. However, staff appraisals are often done according to a bell curve, which disincentifies people to cooperate: if a worker helps his colleague, he faces stronger competition for the top ranks. This appraisal mechanism is thus not incentive compatible. A similar phenomenon is observed in the classroom context. Students should study together and help each other, but the bell curve system can cause competition among students, leading to poorer learning outcomes. In this project, we want to model what happens in the workplace from a game theoretic perspective. Then, we will attempt to figure out a way to quantify the performance, and propose a better mechanism.

Certainly, there are situations in which competition increases production, such as what was observed in the steel factory of Charles M. Schwab[1]. For each shift, Schwab wrote on the factory floor the amount of steel the previous shift had produced. The night shift workers, upon seeing the number from the day shift, worked themselves hard so as to break the record set by the day shift. Soon, the two shifts were trying their best to out perform each other for bragging rights, and production soared. Schwab's method may have worked back in those days, but may not be applicable in a modern day's context. In fact, as Dan Pink[2] pointed out, incentives may boost productivity if the task only requires linear thinking and individual focus. However, the workplace environment nowadays increasingly requires nonlinear thinking and collaborative efforts. This work considers a modern workplace setting where everyone receives, a part from their fixed remuneration, a variable reward (e.g. financial bonus or job promotion) that depends on his own performance relative to those of his colleagues, as well as the performance of the company as a whole. Everyone wants to rise up the ranks, and the strategy employed by the general population seems to move towards individualistic competition so as to outshine their colleagues.

2 Model

2.1 A general model

We shall consider a workplace reward maximizing game where there are k agents, each of whom is assigned a specific project. Every agent i is assumed to have the same time budget T, and he

[1]Netessine and Yakubovich. The Darwinian workplace. *Harvard Business Review.* `https://hbr.org/2012/05/the-darwinian-workplace`

[2]Dan Pink. The puzzle of motivation (TED Talk). `https://youtu.be/rrkrvAUbU9Y`

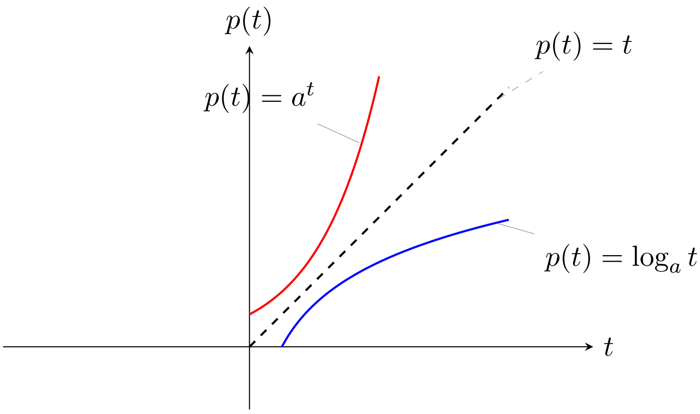

Figure 1. Comparison between diminishing return (blue), linear return (black) and exponential return (red).

dedicate some of his time to his own project and some to his colleagues'. As a result, he has a time distribution (from here on referred to as a *schedule)*, which is a vector $t_i = (t_{i1}, t_{i2}, \ldots, t_{ik})$ where t_{ii} denotes the time agent i allocates to his own project and t_{ij} represents the time allocated by agent i to agent j's project, and $\sum_{j=1}^{k} t_{ij} = T$.

For each project j, its work output (performance) is a function of the total time effort dedicated to it. To reflect the diminishing returns of time investment, we use the natural logarithm to express this relationship (Figure 4).

$$p_j = \beta_j \log \left(t_{jj} + \sum_{i \neq j}^{k} \alpha_{ij} t_{ij} \right) \tag{1}$$

where α_{ij} is the effectiveness (as a result of skill or experience) of agent i when working on project j, compared to that of the project owner, and β_j is a coefficient that reflects the value of the project. Agents are ranked according to his own project's performance. A higher rank means a higher reward. In addition, the reward is also affected by the company's performance as a whole.

$$u_i = f(r_i, P) \tag{2}$$

where

- r_i is the rank of agent i compared to his colleagues. r_i can be obtained by cardinal ranking, by means of a bell curve, or byt any other ranking methods, and
- P is the company performance, which, for simplicity, is taken as the total performance of all its employees, $P = \sum_i p_i$.

2.2 Routing game equivalent

We will prove that this game is equivalent to an atomic routing game. The time budget could be though of as the commodity of an agent and the agent needs to route all its commodity through a single source and sink network with parallel links. Each edge linking the source and sink represents a project and when an agent chooses to route some 'flow' through a particular edge, it is just same as saying that the agent is investing time into that project (Figure 2). This equivalence guarantees that a pure Nash equilibrium exists.

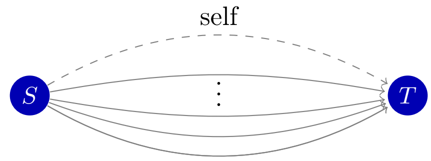

Figure 2. An equivalent routing game

2.3 A toy example

We will investigate the simplest instantiation of the model: a two-agent case, for which different scenarios concerning β_i and α_{ij} will be investigated. The optimal and equilibrium strategies will be determined and the Price of Anarchy as well as its bound will be analysed.

In a two-agent case, it is important to distinguish our model from the well-known Prisoners' Dilemma

- In the Prisoners' Dilemma game, players can move only once and they are not allowed to talk to each other. In our game, both these constraints are not present.
- The Prisoners' Dilemma is symmetric, while it is reasonable to assume that our two office workers are different in many ways. For example, they have different skill sets.

3 Mechanism design

After determining the performance of the existing reward scheme, we will attempt to propose a different reward scheme such that cooperation becomes the dominant strategy. We draw our inspiration from real life games such as World of Warcraft and DOTA, where we have seen cooperative efforts to achieve the game objectives. Hence, we hope to transfer what was applied in the games to the workplace setting.

3.1 Fitting into an atomic routing game scenario

The properties of an atomic routing game is well known to us and by trying to fit the model of our problem into the structure of a routing game, we hope to gain better insight and better understanding of the model of the office games we proposed in Section 2.

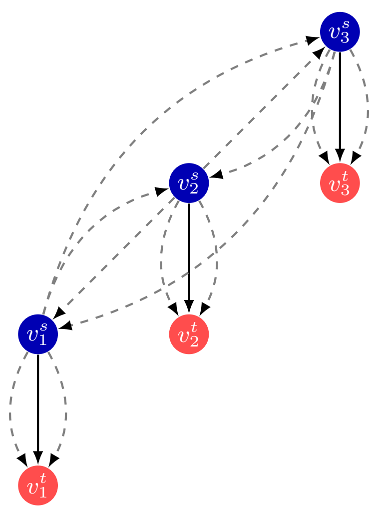

Figure 3. Office game seen as a routing game with 3 agents

In such a routing game, we have $G = (V, E)$, where V and E are the usual set of vertices and edges respectively. A usual routing game will require a player to route his commodity from s_i to t_i; ours is similar but with some constrains:

- an agent j is only allowed to route from a single source that is unique to him, this vertex v_j^s represents the agent himself or herself.
- any edge from v_j^s can only be directed to:
  - the unique vertex v_j^t which represents the agent's assigned project, or
  - a vertex v_k^s, that represents another agent, which then there must be another edge from v_k^s to v_k^t. This represents cooperation between v_j^s and v_k^s (agents k and j), with the first edge of the path from $v_j^s \to v_k^s$ represent some form of cooperation utility and the second $v_k^s \to v_k^t$ representing the contribution of agent j's time to agent's k project.

The solid directed edges in Figure 3 denotes an agent's investment of time into it's own project. The dotted lines represents the edges formed when cooperation occurs. With this idea, we want to see the correspondence between our office games and routing games to gain a better understanding of how bad office games can get.

4 Experiment

This may be somewhat too ambitious, but we hope to conduct an experimental game on the ArenaLabs website to verify our model.

5 Theoretical background

The problem was first mentioned in literature by Drago and Turnbull (1991). Since this seminal work, several scholars have investigated the problems in deeper details:

- Drago and Garvey (1998) and Kistruck et al. (2016) discuss different reward structures to encourage cooperation.
- Banerjee et al. (2014) and Chakravarti et al. (2015) also explore incentive schemes, but with focus on knowledge sharing among researchers and team members.
- Immorlica et al. (2011) discussed a dueling game and show how bad head-on competition can get.
- Landers et al. (2015) described an interesting gamification experiment to understand how people actually behave when the reward mechanism is changed.

References

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In the workplace, managers typically want their staff to cooperate in order to achieve a good performance for the department. However, staff appraisals are done according to a bell curve, which disincentify people to cooperate (If I help you, you'll get better and my chance of getting the top post is less). This appraisal mechanism is thus not incentive compatible.

In the school context, we observe a similar phenomenon. Students should study together and help each other, but the bell curve system can cause competition among students, leading to inefficiency.

7 Project description

In this project, we want to model what happens in the workplace from a game theoretic perspective. Then, we will attempt to figure out a way to quantify the performance, and propose a better mechanism.

7.1 The model

For the simplest case, we will model the game as a two-agent payoff-maximizing game. The payoff each agent receives depend on whether his performance is better than the other agent. Each agent's performance depends on whether they perform their task alone or with help from the other agent. We believe that under the ranking-base reward scheme, the equilibrium strategy is for each agent to work alone, whereas the socially optimal strategy is for them to cooperate. We will then compute the Price of Anarchy and its bounds.

7.2 Mechanism design

After determining the performance of the existing reward scheme, we will attempt to propose a different reward scheme such that cooperation becomes the dominant strategy. We draw our inspiration from real life games such as World of Warcraft and DOTA, where we have seen cooperative efforts to achieve the game objectives. Hence, we hope to transfer what was applied in the games to the workplace setting.

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10 Comparison with Prisoners' Dilemma

The Prisoners' Dilemma's payoff table is

Table 1. Prisoners' Dilemma's payoff table

	Silent	Betray
Silent	1, 1	3, 0
Betray	0, 3	2, 2

The payoff table of our game is still to be determined, but we believe that there are some differences, and we may want to tackle some or all of them

- In the Prisoners' Dilemma game, players can move only once and they are not allowed to talk to each other. In our game, both these constraints are not present.
- The Prisoners' Dilemma is symmetric, while it is reasonable to assume that our two office workers are different in many ways. For example, they have different skill sets.

Table 2. Workplace game's payoff table

	Cooperate	Compete
Cooperate	?, ?	?, ?
Compete	?, ?	?, ?

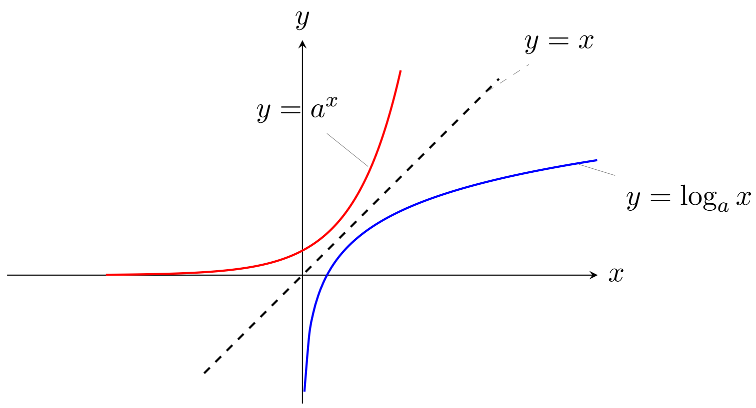

Figure 4. Diminishing returns

11 The Model

We shall consider an office game where there are k agents, each with its own project. Every agent is assumed to have:

- the same time budget T
- a daily schedule represented by a vector $t_i = (t_{i1}, t_{i2}, \ldots, t_{ik})$ with $\sum_k t_{ik} = T$.

For each project which is owned by a sole agent, there is a work output (performance) associated with it that is dependent on the total time (effort) invested into the project

$$p_j = \beta_j \log \left(t_{ii} + \sum_{i \neq j}^{k} \alpha_{ij} t_{ij} \right)$$

where α_{ij} is the effectiveness (skill) if agent i when working on project j (w.r.t the owner) and β_j is the value of the project. Here the log function is used to model the scenario when many agents choose to allocate their time to a particular project; the work output will reach saturation over time, see Fig. 4.

Using the performance p_j, we can assign a rank to each agent and then derive its corresponding payoffs:

$$u_j = f(r_j) \cdot g(P)$$

where $P = \sum_i p_i$. The utility you derive depends on how well you perform relative to the other agents and how the company performs as a whole.

12 Discussion

Further ideas on how we could model this problem was to think of it in the form of an atomic routing game. The time budget could be though of as the commodity of an agent and the agent needs to route all its commodity through a single source and sink network with parallel links. Each edge linking the source and sink represents a project and when an agent chooses to route some 'flow' through a particular edge, it is just same as saying that the agent is investing time into that project. This may be extended to a larger network

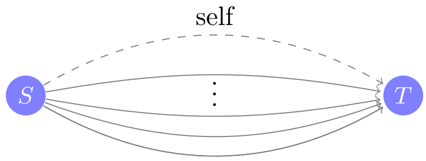

Figure 5. Further ideas

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