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Linear Optimization: Assignment 1

$$\begin{aligned}
\max \quad & z = x_1 + 12x_2 \\
\text{s.t.} \quad & 3x_1 + x_2 + 12x_3 \leq 5 \\
& x_1 + x_3 \leq 16 \\
& 15x_1 + x_2 = 14 \\
& x_j \geq 0, \quad j = 1, 2, 3.
\end{aligned}$$

1.17 (a)

$$\begin{aligned}
\min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
& x_1 + x_2 + x_3 + x_4 \leq M \\
& P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 \leq KP/M \\
& N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \leq KN/M \\
& x_j \geq 0, \quad j = 1, \ldots, 4
\end{aligned}$$

(b)

$$\begin{aligned}
\min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
& x_1 + x_2 + x_3 + x_4 \leq M \\
& P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 \leq P \\
& N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \leq N \\
& x_j \geq 0, \quad j = 1, 2, 3, 4
\end{aligned}$$

1.18 (a)

$$\begin{aligned}
\min \quad & \textstyle\sum_{i=1}^4 c_ix_{1,i} + \sum_{i=1}^4 c_ix_{2,i} + \sum_{i=1}^4 c_ix_{3,i} \\
\text{s.t.} \quad & \textstyle\sum_{i=1}^4 c_ix_{1,i} \geq K_A \\
& \textstyle\sum_{i=1}^4 c_ix_{2,i} \geq K_B \\
& \textstyle\sum_{i=1}^4 c_ix_{3,i} \geq K_C \\
& \textstyle\sum_{i=1}^4 c_ix_{1,i} \leq M_1 \\
& \textstyle\sum_{i=1}^4 c_ix_{2,i} \leq M_2 \\
& \textstyle\sum_{i=1}^4 c_ix_{3,i} \leq M_1 + M_2 \\
& \textstyle\sum_{i=1}^4 P_ix_{1,i} \geq K_AP_S/M_1 \\
& \textstyle\sum_{i=1}^4 P_ix_{2,i} \geq K_BP_B/M_2 \\
& \textstyle\sum_{i=1}^4 P_ix_{3,i} \geq K_CP_S/(M_1 + M_2) \\
& \textstyle\sum_{i=1}^4 N_ix_{1,i} \geq K_AN_S/M_1 \\
& \textstyle\sum_{i=1}^4 N_ix_{2,i} \geq K_BN_B/M_2 \\
& \textstyle\sum_{i=1}^4 N_ix_{3,i} \geq K_CN_S/(M_1 + M_2) \\
& x_{i,j} \geq 0, \quad i = 1, 2, 3, j = 1, 2, 3, 4
\end{aligned}$$

(b) The c_i's, P_i's and N_i's will unique for each plant thus we will have $c_{p,i}$'s, $P_{p,i}$ and $N_{p,i}$ for $p \in \{A, B, C\}$.

1.20 Let $x_t = x_t^+ - x_t^-$ denote the change in production from month t to month $t+1$ and d_t denote the sales forecast for month t. Letting the units to be in thousands below:

$$\begin{aligned}
\min \quad & 0.5 \textstyle\sum_{t=1}^{12} x_t^+ + 0.25 \sum_{t=1}^{12} x_t^- \\
\text{s.t.} \quad & x_6 + 4 + 2 - 4 \leq 10 \\
& x_6 + 4 + 2 - 4 \leq 10 \\
& x_i^+, x_i^- \geq 0, \quad i = 1, \ldots, 12
\end{aligned}$$

1.25