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Linear Optimization: Assignment 1

$$\begin{array}{ll}\max & z = x_1 + 12x_2 \\ \text{s.t.} & 3x_1 + x_2 + 12x_3 \le 5 \\ & x_1 + x_3 \le 16 \\ & 15x_1 + x_2 = 14 \\ & x_j \ge 0, \quad j = 1,2,3.\end{array}$$

1.17 (a)

$$\begin{array}{ll}\min & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\ \text{s.t.} & x_1 + x_2 + x_3 + x_4 \ge K \\ & x_1 + x_2 + x_3 + x_4 \le M \\ & P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 \le PM \\ & N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \le NM \\ & x_j \ge 0, \quad j = 1,\dots,4\end{array}$$

(b)

$$\begin{array}{ll}\min & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\ \text{s.t.} & x_1 + x_2 + x_3 + x_4 \ge K \\ & x_1 + x_2 + x_3 + x_4 \le M \\ & (P_1 - P)x_1 + (P_2 - P)x_2 + (P_3 - P)x_3 + (P_4 - P)x_4 \le 0 \\ & (N_1x - N)_1 + (N_2 - N)x_2 + (N_3 - N)x_3 + (N_4 - N)x_4 \le 0 \\ & x_j \ge 0, \quad j = 1,2,3,4\end{array}$$

1.18 (a)

$$\begin{array}{ll}\min & \sum_{i=1}^4 c_i x_{1,i} + \sum_{i=1}^4 c_i x_{2,i} + \sum_{i=1}^4 c_i x_{3,i} \\ \text{s.t.} & \sum_{i=1}^4 x_{1,i} \ge K_A \\ & \sum_{i=1}^4 x_{2,i} \ge K_B \\ & \sum_{i=1}^4 x_{3,i} \ge K_C \\ & \sum_{i=1}^4 x_{1,i} \le M_1 \\ & \sum_{i=1}^4 x_{2,i} \le M_2 \\ & \sum_{i=1}^4 x_{3,i} \le M_1 + M_2 \\ & \sum_{i=1}^4 P_i x_{1,i} \ge P_S M_1 \\ & \sum_{i=1}^4 P_i x_{2,i} \ge P_B M_2 \\ & \sum_{i=1}^4 P_i (x_{1,i} + x_{2,i} + x_{3,i}) \ge P_S(M_1 + M_2) \\ & \sum_{i=1}^4 N_i x_{1,i} \ge N_S M_1 \\ & \sum_{i=1}^4 N_i x_{2,i} \ge N_B M_2 \\ & \sum_{i=1}^4 N_i (x_{1,i} + x_{2,i} + x_{3,i}) \ge N_S(M_1 + M_2) \\ & x_{i,j} \ge 0, \quad i = 1,2,3, j = 1,2,3,4\end{array}$$

(b) The c_i's, P_i's and N_i's will unique for each plant thus we will have $c_{p,i}$'s, $P_{p,i}$ and $N_{p,i}$ for $p \in \{A, B, C\}$.

1.20 We shall let t denote the $t+6 \mod 12$ month of the year, i.e. $t = 0$ is June and $t = 10$ is April. Let $x_t = x_t^+ - x_t^-$ denote the change in production from month t to month $t+1$ and d_t denote the sales forecast for month t. Letting the units to be in thousands below:

$$
\begin{array}{llllll}
\min & 0.5\sum_{t=0}^{11} x_t^+ & +0.25\sum_{t=0}^{11} x_t^- & & & \\
\text{s.t.} \quad 4 & + \quad x_0 + & 2 - & d_1 & & \leq 10 \\
4 & + \quad x_0 + & 2 - & d_1 + 4 + \sum_{t=0}^{1} x_t & - d_2 & \leq 10 \\
2(4) & + \sum_{t=0}^{1}(2-t)x_t + & 2 - \sum_{i=1}^{2} d_t + 4 + \sum_{t=0}^{2} x_t & & - d_3 & \leq 10 \\
3(4) & + \sum_{t=0}^{2}(3-t)x_t + & 2 - \sum_{i=1}^{3} d_t + 4 + \sum_{t=0}^{3} x_t & & - d_4 & \leq 10 \\
\vdots & \vdots & \vdots & & \vdots & \\
11(4) & + \sum_{t=0}^{10}(11-t)x_t + & 2 - \sum_{i=1}^{11} d_t + 4 + \sum_{t=0}^{11} x_t & & - d_{12} & \leq 10 \\
& x_i^+, x_i^- \geq 0, \quad i = 1, \ldots, 12 & & & &
\end{array}
$$

1.25 We first list down the different ways such that a 100-inch roll can be cut into combinations of 24-, 40-, and 32-inch widths. Let x_i denote the number of combination i used.

Combination	24	40	32	trim waste
1	4	0	0	4
2	0	2	0	20
3	0	0	3	4
4	1	1	1	4
5	2	1	0	12
6	2	0	1	20
7	1	0	2	12

$$
\begin{array}{lll}
\min & 4x_1 + 20x_2 + 4x_3 + 4x_4 + 12x_5 + 20x_6 + 12x_7 + 24x_8 + 40x_9 + 32x_{10} & \\
\text{s.t.} & 4x_1 + x_4 + 2x_5 + 2x_6 + x_7 + x_8 & = 75 \\
& 2x_2 + x_4 + x_5 + x_9 & = 50 \\
& 3x_3 + x_4 + x_6 + 2x_7 + x_{10} & = 110 \\
& x_j \geq 0, \quad j = 1, \ldots, 10 &
\end{array}
$$

2.7 Letting the units be in thousands below:

$$
\begin{array}{ll}
\max & 2x_1 + 1.8x_2 \\
\text{s.t.} & x_1 + x_2 \leq 10 \\
& 2x_1 + x_2 \leq 9 \\
& x_1, x_2 \geq 0
\end{array}
$$

2.8 Let the units be in pounds below, and x_1, x_2 and x_3 denoting amount of ingredient A, B and C used respectively.

$$
\begin{array}{ll}
\min & 4x_1 + 3x_2 + 2x_3 \\
\text{s.t.} & x_1 + x_2 + x_3 = 2000 \\
& x_1 \geq 200, x_2 \geq 400, 0 \leq x_3 \leq 800
\end{array}
$$

The bounded variable index method can be used to solve this problem.

2.9 Phase I:

$$
\begin{array}{ll}
\max & x_0 \\
\text{s.t.} & (-x_0) + x_6 + x_7 + x_8 = 0 \\
& + 2x_1 + 3x_2 - x_3 + x_4 + x_6 = 9 \\
& 2x_2 + x_3 - x_5 + x_7 = 4 \\
& + x_1 + x_3 + x_8 = 6 \\
& x_i \geq 0, \forall i
\end{array}
$$

2.12 (a)

$$
\begin{array}{ll}
\min & x_6 + x_7 + x_8 \\
\text{s.t.} & x_1 - 6x_2 + x_3 - x_4 + x_6 = 5 \\
& x_1 - 2x_2 + 2x_3 - 3x_4 - x_5 + x_7 = 3 \\
& -3x_1 + 2x_3 - 4x_4 + x_8 = 1 \\
& x_j \geq 0, \quad j = 1, \ldots, 8.
\end{array}
$$

$(-w)$	0	0	0	
x_6	1	0	0	9
x_7	1	0	0	4
x_8	1	0	0	6

(b)

$$
\begin{aligned}
\max \quad & -x_6 - x_7 \\
\text{s.t.} \quad & -3x_1 - 2x_2 + x_3 - x_4 + x_6 = 3 \\
& x_1 + x_2 - 2x_3 - x_5 + x_7 = 1 \\
& x_j \geq 0, \quad j = 1, \dots, 7.
\end{aligned}
$$

$$
\begin{aligned}
\max \quad & -4 - 2x_1 - x_2 - x_3 - x_4 - x_5 \\
\text{s.t.} \quad & -3x_1 - 2x_2 + x_3 - x_4 + x_6 = 3 \\
& x_1 + x_2 - 2x_3 - x_5 + x_7 = 1 \\
& x_j \geq 0, \quad j = 1, \dots, 7.
\end{aligned}
$$

thus since all the coefficients of the objective function is ≤ 0 the system contains no feasible solution.

2.13 (a)

(b)

(c)

(d)