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Algorithmic Game Theory: HW 2

1.

2.

3.

4. Let $f_\epsilon(x) = (1-\epsilon)^x$ and $g_\epsilon(x) = 1 + \epsilon x$, then

$$\left.\begin{array}{ll} f_\epsilon(0) = 1 = g_\epsilon(0) & \\ f_\epsilon(1) = 1 - \epsilon = g_\epsilon(1) & \\ f'_\epsilon(x) & = (1-\epsilon)^x \ln(1-\epsilon) \\ g'_\epsilon(x) & = \epsilon \end{array}\right\} f'_\epsilon(0) = \ln(1-\epsilon) < 0 = g'_\epsilon(0)$$

also f_ϵ is a convex function as $f''_\epsilon(x) = (1-\epsilon)^x \left[\ln(1-\epsilon)\right]^2 > 0$ for $\epsilon \in (0, 1/2]$. This this proves $f_\epsilon(x) \leq g_\epsilon(x)$ since the initial gradient of f_ϵ is smaller then g_ϵ