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Stochastic Models: Exercise 3

1. (i)

   (ii)

$$\begin{aligned}\mathbb{P}(N(9.5) - N(8.5) = 0) &= e^{-(m(9.5)-m(8.5))}\frac{m(9.5) - m(8.5)^0}{0!} \\ &= e^{-10}\end{aligned}$$

   (iii)

$$\begin{aligned}\mathbb{E}(\text{number of arrivals from 8:30AM-9:30AM}) &= \sum_{k=0}^{\infty} k \cdot e^{-(m(9.5)-m(8.5))}\frac{(m(9.5) - m(8.5))^k}{k!} \\ &= 10\sum_{k=1}^{\infty} e^{-10}\frac{10^{(k-1)}}{(k-1)!} \\ &= 10\end{aligned}$$

2.

$$\begin{aligned}\mathbb{P}(N(I_1) = k_1, \ldots, N(I_n) = k_n \mid N(U) = k) &= \frac{\mathbb{P}(N(I_1) = k_1, \ldots, N(I_n) = k_n, N(u) = k)}{\mathbb{P}(N(U) = k)} \\ &= \left(\prod_{i=1}^{n} e^{-\lambda c_i}\frac{(\lambda c_i)^{k_i}}{k_i!}\right)\Bigg/ e^{-\lambda c}\frac{(\lambda c)^k}{k!} \\ &= \frac{k!}{k_1!k_2!\ldots k_n!}\left(\frac{c_1}{c}\right)^{k_1}\left(\frac{c_2}{c}\right)^{k_2}\ldots\left(\frac{c_n}{c}\right)^{k_n}\end{aligned}$$

3. Let N_i denote the number of families with number of member of size i migrating to Batan Island over a t week(s) period and let such an event be called a type-i event for $i = 1, 2, 3, 4$. Hence $N_i(t)$ is a Poisson process and $\mathbb{E}(N_i(t)) = \lambda t p_i = 10p_i$. Let $M(t) = \sum_i iN_i(t)$ denote the number of individuals migrating during a t-week period.

$$\begin{aligned}\mathbb{E}(M(t)) &= \sum_i i\mathbb{E}(N_i(t)) \\ &= (1+4)\frac{10}{6} + (2+3)\frac{10}{3} \\ &= 25\end{aligned}$$

   To find variance, we first find $\mathbb{E}(N_i(t)^2)$

$$\begin{aligned}\mathbb{E}(N_i(t)^2) &= \sum_{n=0}^{\infty} n^2 e^{-\lambda t p_i}\frac{(\lambda t p_i)^n}{n!} \\ &= \sum_{n=2}^{\infty} e^{-\lambda t p_i}\frac{(\lambda t p_i)^{n-2}}{(n-2)!} + \sum_{n=1}^{\infty} e^{-\lambda t p_i}\frac{(\lambda t p_i)^{n-1}}{(n-1)!} \\ &= \lambda t p_i + (\lambda t p_i)^2\end{aligned}$$

   and so $Var(N_i(t)) = \lambda t p_i$.

$$\begin{aligned}Var(M(t)) &= \sum_i i^2 Var(N_i(t)) \\ &= (1^2 + 4^2)\frac{10}{6} + (2^2 + 3^2)\frac{10}{3} = \frac{215}{3}\end{aligned}$$

4.

5.

6. (a) Let $N_1(t)$ and $N_2(t)$ be the type-I and type-II events where Irma Pince finds a misplaced book and fails to find a misplaced book respectively. Hence, the N_is are independent Poisson process with rate λp_i where $i = 1, 2$. This the misplacements found by Irma Pince follows a homogeneous Poisson process. For $t = 100$, $\mathbb{E}(N_1(100)) = 90\lambda$.

   (b)

$$\mathbb{P}(N_1(t+s) - N(t) = 1) = 0.9$$