

## Linear Optimization: Assignment 1

$$\begin{aligned}
 \max \quad & z = x_1 + 12x_2 \\
 \text{s.t.} \quad & 3x_1 + x_2 + 12x_3 \leq 5 \\
 & x_1 + x_3 \leq 16 \\
 & 15x_1 + x_2 = 14 \\
 & x_j \geq 0, \quad j = 1, 2, 3.
 \end{aligned}$$

1.17 (a)

$$\begin{aligned}
 \min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
 \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
 & x_1 + x_2 + x_3 + x_4 \leq M \\
 & P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 \leq PM \\
 & N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \leq NM \\
 & x_j \geq 0, \quad j = 1, \dots, 4
 \end{aligned}$$

(b)

$$\begin{aligned}
 \min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
 \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
 & x_1 + x_2 + x_3 + x_4 \leq M \\
 & (P_1 - P)x_1 + (P_2 - P)x_2 + (P_3 - P)x_3 + (P_4 - P)x_4 \leq 0 \\
 & (N_1x - N)_1 + (N_2 - N)x_2 + (N_3 - N)x_3 + (N_4 - N)x_4 \leq 0 \\
 & x_j \geq 0, \quad j = 1, 2, 3, 4
 \end{aligned}$$

1.18 (a)

$$\begin{aligned}
 \min \quad & \sum_{i=1}^4 c_i x_{1,i} + \sum_{i=1}^4 c_i x_{2,i} + \sum_{i=1}^4 c_i x_{3,i} \\
 \text{s.t.} \quad & \sum_{i=1}^4 x_{1,i} \geq K_A \\
 & \sum_{i=1}^4 x_{2,i} \geq K_B \\
 & \sum_{i=1}^4 x_{3,i} \geq K_C \\
 & \sum_{i=1}^4 x_{1,i} \leq M_1 \\
 & \sum_{i=1}^4 x_{2,i} \leq M_2 \\
 & \sum_{i=1}^4 x_{3,i} \leq M_1 + M_2 \\
 & \sum_{i=1}^4 P_i x_{1,i} \geq P_S M_1 \\
 & \sum_{i=1}^4 P_i x_{2,i} \geq P_B M_2 \\
 & \sum_{i=1}^4 P_i (x_{1,i} + x_{2,i} + x_{3,i}) \geq P_S (M_1 + M_2) \\
 & \sum_{i=1}^4 N_i x_{1,i} \geq N_S M_1 \\
 & \sum_{i=1}^4 N_i x_{2,i} \geq N_B M_2 \\
 & \sum_{i=1}^4 N_i (x_{1,i} + x_{2,i} + x_{3,i}) \geq N_S (M_1 + M_2) \\
 & x_{i,j} \geq 0, \quad i = 1, 2, 3, j = 1, 2, 3, 4
 \end{aligned}$$

(b) The  $c_i$ 's,  $P_i$ 's and  $N_i$ 's will be unique for each plant thus we will have  $c_{p,i}$ 's,  $P_{p,i}$  and  $N_{p,i}$  for  $p \in \{A, B, C\}$ .

1.20 We shall let  $t$  denote the  $t+6 \pmod{12}$  month of the year, i.e.  $t = 0$  is June and  $t = 10$  is April. Let  $x_t = x_t^+ - x_t^-$  denote the change in production from month  $t$  to month  $t+1$  and  $d_t$  denote the sales forecast for month  $t$ . Letting the units to be in thousands below:

$$\begin{array}{ll}
\min & 0.5 \sum_{t=0}^{11} x_t^+ + 0.25 \sum_{t=0}^{11} x_t^- \\
\text{s.t.} & 4 + x_0 + 2 - d_1 \leq 10 \\
& 4 + x_0 + 2 - d_1 + 4 + \sum_{t=0}^1 x_t - d_2 \leq 10 \\
& 2(4) + \sum_{t=0}^1 (2-t)x_t + 2 - \sum_{i=1}^2 d_t + 4 + \sum_{t=0}^2 x_t - d_3 \leq 10 \\
& 3(4) + \sum_{t=0}^2 (3-t)x_t + 2 - \sum_{i=1}^3 d_t + 4 + \sum_{t=0}^3 x_t - d_4 \leq 10 \\
& \vdots \\
& 11(4) + \sum_{t=0}^{10} (11-t)x_t + 2 - \sum_{i=1}^{11} d_t + 4 + \sum_{t=0}^{11} x_t - d_{12} \leq 10 \\
& x_i^+, x_i^- \geq 0, \quad i = 1, \dots, 12
\end{array}$$

1.25 We first list down the different ways such that a 100-inch roll can be cut into combinations of 24-, 40-, and 32-inch widths. Let  $x_i$  denote the number of combination  $i$  used.

Combination	24	40	32	trim waste
1	4	0	0	4
2	0	2	0	20
3	0	0	3	4
4	1	1	1	4
5	2	1	0	12
6	2	0	1	20
7	1	0	2	12

$$\begin{array}{ll}
\min & 4x_1 + 20x_2 + 4x_3 + 4x_4 + 12x_5 + 20x_6 + 12x_7 \\
\text{s.t.} & 4x_1 + x_4 + 2x_5 + 2x_6 + x_7 \geq 75 \\
& 2x_2 + x_4 + x_5 \geq 50 \\
& 3x_3 + x_4 + x_6 + 2x_7 \geq 110 \\
& x_j \geq 0, \quad j = 1, \dots, 7
\end{array}$$

2.7 Letting the units be in thousands below:

$$\begin{array}{ll}
\max & 2x_1 + 1.8x_2 \\
\text{s.t.} & x_1 + x_2 \leq 10 \\
& 2x_1 + x_2 \leq 9 \\
& x_1, x_2 \geq 0
\end{array}$$

2.8 Let the units be in pounds below, and  $x_1, x_2$  and  $x_3$  denoting amount of ingredient  $A, B$  and  $C$  used respectively.

$$\begin{array}{ll}
\min & 4x_1 + 3x_2 + 2x_3 \\
\text{s.t.} & x_1 + x_2 + x_3 = 2000 \\
& x_1 \geq 200, x_2 \geq 400, 0 \leq x_3 \leq 800
\end{array}$$

The bounded variable index method can be used to solve this problem.

2.9 Phase I:

$$\begin{array}{ll}
\max & x_0 \\
\text{s.t.} & x_0 + x_6 + x_7 + x_8 = 0 \\
& + 2x_1 + 3x_2 - x_3 + x_4 + x_6 = 9 \\
& 2x_2 + x_3 - x_5 + x_7 = 4 \\
& + x_1 + x_3 + x_8 = 6 \\
& x_i \geq 0, \forall i
\end{array}$$

2.12

2.13

$(-w)$	0	0	0	
$x_6$	1	0	0	9
$x_7$	1	0	0	4
$x_8$	1	0	0	6