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Statistics: Homework 1

1.19 Let X_1, X_2 and X_3 denote the computer owners that use Macintosh, Windows and Linux respectively and let V denote the event that the user's system is infected with the virus. We want to find $\mathbb{P}(X_2|V)$

$$\begin{aligned}\mathbb{P}(X_2|V) &= \frac{\mathbb{P}(V|X_2)\mathbb{P}(X_2)}{\sum_{i=1}^{3}\mathbb{P}(V|X_i)\mathbb{P}(X_i)} \\ &= \frac{(.82)(.5)}{(.65)(.3)+(.82)(.5)+(.5)(.2)} \\ &= 0.581560284\end{aligned}$$

2.4 (a)

$$F_X(x) := \begin{cases} \frac{1}{4}x & 0 < x < 1 \\ \frac{3}{8}x - \frac{7}{8} & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$\begin{aligned}\mathbb{P}(Y \le y) &= \mathbb{P}(1/X \le y) \\ &= \mathbb{P}(X \ge 1/y) \\ &= 1 - \mathbb{P}(X \le 1/y)\end{aligned}$$

From (a):

$$F_Y(y) := \begin{cases} \frac{15}{8} - \frac{3}{8y} & 1/5 < y < 1/3 \\ 1 - \frac{1}{4y} & y \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) := \begin{cases} \frac{3}{8y^2} & 1/5 < y < 1/3 \\ \frac{1}{4y^2} & y \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

2.11 (a) We see that $\mathbb{P}(X = 1) = p = \mathbb{P}(Y = 0)$. Since the state space contains is $\{H, T\}$, we have $1 - \mathbb{P}(X = 1, Y = 0) = 1 - p = \mathbb{P}(X = 0, Y = 1)$. But since

$$\mathbb{P}(X = 1)\mathbb{P}(Y = 0) = p^2 \ne p = \mathbb{P}(X = 1, Y = 0)$$

X and Y are dependent.

(b)

3.4 Let Y_i denote the jump of the particle at the ith unit. Then $X_n = \sum_{i=1}^{n} Y_i$. The Y_i's are iid, with $\mathbb{E}(Y_i) = 1-2p$ and $\mathbb{V}(Y_i) = 1 - (1-2p)^2 = 4p(1-p)$ for $i = 1, 2, \ldots, n$.

$$\begin{aligned}\mathbb{E}(X_n) &= \sum_{i=1}^{n}\mathbb{E}(Y_i) = n(1-2p) \\ \mathbb{V}(X_n) &= \sum_{i=1}^{n}\mathbb{V}(Y_i) = n \cdot 4p(1-p)\end{aligned}$$

4.3 Using Chebyshev's and Hoeffding's inequality we have

$$\begin{aligned}\mathbb{P}(|\overline{X}_n - p| > \epsilon) &= \frac{p(1-p)}{n\epsilon^2} \\ \mathbb{P}(|\overline{X}_n - p| > \epsilon) &= 2e^{-2n\epsilon^2}\end{aligned}$$

The inequality $(1+x)^r \leq e^{rx}$ for $r > 0, x > 0$, thus for $r = 1$

$$\begin{aligned} x < 1 + x &\leq e^x \\ 1/x &> e^{-x} \\ \frac{1}{2n\epsilon^2} &> e^{-2n\epsilon^2} \\ \frac{1}{n\epsilon^2} &> 2e^{-2n\epsilon^2} \end{aligned}$$

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