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# Statistics: Homework 2

6.3 Given $\hat{\theta} = 2\overline{X}_n$ and $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$,

$$\begin{aligned}
\text{bias}(\hat{\theta}) &= \mathbb{E}(2\overline{X}_n) - \theta \\
&= 2n^{-1}\mathbb{E}\left(\sum_{i=1}^{n} X_i\right) - \theta \\
&= 2n^{-1}\sum_{i=1}^{n}\mathbb{E}\left(X_i\right) - \theta \\
&= 2n^{-1}\frac{n\theta}{2} - \theta = 0 \\
\text{se}(\hat{\theta})^2 &= \mathbb{V}(2\overline{X}_n) \\
&= 4\mathbb{V}(\overline{X}_n) \\
&= 4n^{-2}\mathbb{V}\left(\sum_{i=1}^{n} X_i\right) \\
&= 4n^{-2}\sum_{i=1}^{n}\mathbb{V}\left(X_i\right) \\
&= 4n^{-2}\frac{n\theta^2}{12} = \frac{\theta^2}{3n} \\
\text{MSE}(\hat{\theta}) &= \text{bias}(\hat{\theta})^2 + \text{se}(\hat{\theta})^2 = \frac{\theta^2}{3n}
\end{aligned}$$

7.2

$$\text{se} = \mathbb{V}(\overline{X}_n)$$

7.9

8.7

9.2

9.6