

Statistics: Homework 3

10.5 Given $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$ and $Y = \max\{X_1, \dots, X_n\}$, we have the cdf of Y to be $F_Y(y) = (y/\theta)^n$ for $y \in [0, 1/2]$.

(a) When we choose to reject H_0 when $Y > c$, the power function is $\beta(\theta) = 1 - (c/\theta)^n$, $c \in [0, 1/2]$.

(b) Given size of the test to be .05, we need to solve,

$$1 - (2c)^n = .05$$

which gives us a solution of $c = 1/2(.95)^{1/n}$

(c) The size, $\alpha = \beta(1/2) = 1 - (2c)^n$, $c \in [0, 1/2]$. Thus, when $n = 20$, $Y = .48$, the p-value is

$$\inf\{\alpha : X^n \in R_\alpha\} = 1 - (2 \times .48)^{20} = 0.557997566$$

We would conclude that we do not reject H_0 with an approximate probability of 0.56, which does not give a strong evidence to reject H_0

(d) When $n = 20$, $Y = .52$, using the α formula in (c) gives us $1 - (2 \times .52)^{20} = -1.19112314$. But the given $Y = .52 > 1/2$ which is out of the defined boundaries of the size, i.e. $F_Y(0.52; \theta = 1/2) = 0$. Hence the p-value is 0. This allows us to conclude that H_0 is to be rejected as the p-value always lies in the criteria region; a very strong reason to reject H_0 .

10.7b Let $H_0 : F_T = F_S$ and $H_1 : F_T \neq F_S$, where the subscripts denote Twain and Snodgrass respectively. The observed value of the test statistic given by the absolute difference of their means, $|\bar{T} - \bar{S}|$ is

$$|0.231875 - 0.2097| = 0.022175$$

Have to do some simulation here.

Under this p-value, do we reject H_0 at a 5 percent level? How about 2.5 percent level?

10.8 (a) The size of this test with rejection region R is

$$\begin{aligned} \mathbb{P}(T(X^n) > c | \theta = 0) &= \mathbb{P}(\bar{X}_n > c) \\ &= \mathbb{P}(Z > c), \text{ } Z \text{ is the standard normal distribution} \\ &= 1 - \Phi(c), \text{ } \Phi \text{ is the cdf of the standard normal} \end{aligned}$$

for size α , the c is $\Phi^{-1}(1 - \alpha)$

(b) Under $H_1 : \theta = 1$, the power is $\beta(1) = \mathbb{P}(T(X^n) > c | \theta = 1) = 1 - \Phi(c - 1)$

(c)

10.12

11.3

11.4