40.752 Algorithmic Game Theory

Project Proposal

TO COOPERATE OR TO COMPETE: A GAME THEORETIC MODEL

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1 Motivation

The idea for this project is drawn upon our own observation from past work experiences. In the workplace, managers typically want their staff to cooperate in order to achieve a good overall performance for the company. However, staff appraisals are often done according to a bell curve, which disincentifies people to cooperate: if a worker helps his colleague, he faces stronger competition for the top ranks. This appraisal mechanism is thus not incentive compatible. A similar phenomenon is observed in the classroom context. Students should study together and help each other, but the bell curve system can cause competition among students, leading to poorer learning outcomes. *Here, we do not consider scenarios that competition, in fact do help the company achieve better performance, for example there is a story of the steel magnate Charles M. Schwab, who wrote on the floor the amount of steel the day shift had produced. The night shift workers, upon seeing the numbers worked themselves hard so as to break the record set by the day shift and soon the two shifts were trying their best to out perform each other for bragging rights, and production soared [from here]. Our scenario considers a modern society in an office setting where everyone is draws a fixed monthly paid and may be awarded a bonus or promotion but not necessary at the end of a financial year. Everyone wants to rise up the ranks, but the strategy employed by the general population seem to move towards individualistic competition so as to out shine their colleagues.* In this project, we want to model what happens in the workplace from a game theoretic perspective. Then, we will attempt to figure out a way to quantify the performance, and propose a better mechanism.

2 Model

For the simplest case, we will model the game as a two-agent payoff-maximizing game. The payoff each agent receives depend on whether his performance is better than the other agent. Each agent's performance depends on whether they perform their task alone or with help from the other agent. We believe that under the ranking-base reward scheme, the equilibrium strategy is for each agent to work alone, whereas the socially optimal strategy is for them to cooperate. We will then compute the Price of Anarchy and its bounds.

We shall consider an office game where there are k agents, each with its own project. Every agent is assumed to have:

- the same time budget T.
- a daily schedule represented by a vector $t_i = (t_{i1}, t_{i2}, \ldots, t_{ik})$ with $\sum_k t_{ik} = T$ *where* t_{ii} *denotes the time allocated to the agent's own project and* t_{ij} *represents the time allocated by agent* i *to agent* j*'s project.*

For each project which is owned by a sole agent, there is a work output (performance) associated with it that is dependent on the total time (effort) invested into the project

$$p_j = \beta_j \log \left(t_{jj} + \sum_{i \neq j}^{k} \alpha_{ij} t_{ij} \right)$$

where α_{ij} is the effectiveness (skill) of agent i when working on project j (w.r.t the owner) and β_j is the value of the project. Using the performance p_j, we can assign a rank to each agent and then derive its corresponding payoffs:

$$u_j = f(r_j, P)$$

where $P = \sum_i p_i$. The utility you derive depends on how well you perform relative to the other agents and how the company performs as a whole.

Here, we would like to address some of the assumptions that have been made which might have to lead to some scenarios of a real office setting not being considered in this model.

3 Mechanism design

After determining the performance of the existing reward scheme, we will attempt to propose a different reward scheme such that cooperation becomes the dominant strategy. We draw our inspiration from real life games such as World of Warcraft and DOTA, where we have seen cooperative efforts to achieve the game objectives. Hence, we hope to transfer what was applied in the games to the workplace setting.

4 Experiment

This may be somewhat too ambitious, but we hope to conduct an experimental game on the ArenaLabs website to verify our model.

5 Theoretical background

The problem was first mentioned in literature by Drago and Turnbull (1991). Since this seminal work, several scholars have investigated the problems in deeper details:

- Drago and Garvey (1998) and Kistruck et al. (2016) discuss different reward structures to encourage cooperation.
- Banerjee et al. (2014) and Chakravarti et al. (2015) also explore incentive schemes, but with focus on knowledge sharing among researchers and team members.
- Immorlica et al. (2011) discussed a dueling game and show how bad head-on competition can get.
- Landers et al. (2015) described an interesting gamification experiment to understand how people actually behave when the reward mechanism is changed.

6 Comparison with Prisoners' Dilemma

The Prisoners' Dilemma's payoff table is

The payoff table of our game is still to be determined, but we believe that there are some differences, and we may want to tackle some or all of them

Table 1: Prisoners' Dilemma's payoff table

	Silent	Betray
Silent	1, 1	3, 0
Betray	0, 3	2, 2

- In the Prisoners' Dilemma game, players can move only once and they are not allowed to talk to each other. In our game, both these constraints are not present.
- The Prisoners' Dilemma is symmetric, while it is reasonable to assume that our two office workers are different in many ways. For example, they have different skill sets.

7 Discussion

Further ideas on how we could model this problem was to think of it in the form of an atomic routing game. The time budget could be though of as the commodity of an agent and the agent needs to route all its commodity through a single source and sink network with parallel links. Each edge linking the source and sink represents a project and when an agent chooses to route some 'flow' through a particular edge, it is just same as saying that the agent is investing time into that project. This may be extended to a larger network

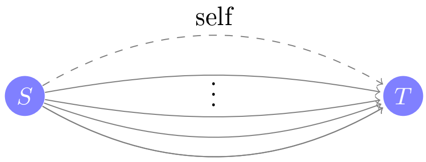

Figure 1: Further ideas

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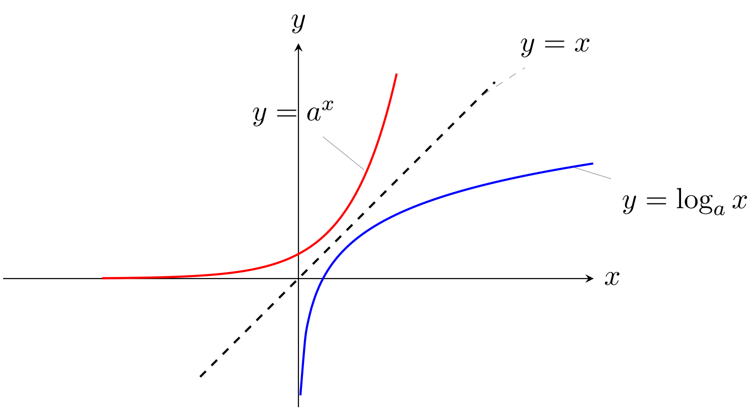

Figure 2: Diminishing returns

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