

# Literature Review: Real-Time Dynamic Pricing for Multiproduct Models with Time-Dependent Customer Arrival Rates

Ha Thi Phuong Thao      Huang Xia      Zhangsheng Lai

April 13, 2018

## 1 Introduction

Introduction to talk about what we learnt in class (Single Product DP) and how the model in the paper we have chosen is a generalizing from single to multi-products.

In the single product dynamic pricing model that we were introduced in class, we looked at a monopolist seller which finite units  $x_0$  of a single indivisible product with no replenishment over a finite and continuous horizon  $[0, T)$ . The unit price  $\pi_t$  is decided by the seller at each point of time  $t \in [0, T)$  and customers product valuations follow a distribution over  $\mathbb{R}_+$ . However, practically, sellers often have a wide range of products that the customers can choose from, with the products having similar functionalities; catering to customers of varying purchasing power.

Thus we look to a multiproduct model [2]...

## 2 Related Work

[Optional] We might like to discuss other papers like [1] that might be related to the paper we are looking at, e.g. the 1997 paper Yiwei told us to look at.

## 3 The Multinomial Logit Model

The model of course.

### 3.1 Summary of the MNL model

The MNL model is a multiproduct dynamic pricing problem and assumes the products are nominal. It describes dynamic consumer choice preferences over substitute products as prices are varied. The customer makes choices from a range of products to maximise his or her utility. The utility of the  $i$  product is defined by the logit demand function  $v^i(r^i) = \exp((q^i - r^i)/\mu)$  which is a positive function of the quality  $q^i$ , the price  $r^i$  of product  $i$  for  $i = 1, \dots, n$  and  $\mu$  is the constant representing the stochastic preference of the choice process. The customer expected demand probability of product  $i$  is defined as

$$P^i(r) = \frac{v^i(r^i)}{v^0 + \sum_{j=1}^n v^j(r^j)}, \quad i = 1, \dots, n \quad (1)$$

and  $v^0$  denotes the utility of not making any purchase.

The customer's arrivals are assumed to follow a nonhomogenous Poisson process with a time-dependent rate  $\lambda(t)$  and in a small time interval  $\delta t$ , the probability of one arrival is  $\lambda(t)\delta t$ . The price of the products at time  $t$  is  $r_t = (r_t^1, \dots, r_t^n) \in \mathbb{R}^n$  is decided according to the current inventory level. If some product  $i$  is sold out before the end of the selling season, as we do not allow replenishments, the price is set to  $r_t^i = \infty$  for all  $t$  occurring after  $c_t^i = 0$ . If all the products are sold at  $t < T$ , then the selling season ends at  $t$ . All unsold products are salvaged at  $t = 0$ .

Let  $\mathbf{r} = \{r_t, t \in [T, 0]\} = (r^1, \dots, r^n)$  denote a pricing policy for the entire season, where  $r^i$  denotes the trend of the price of product  $i$  over the selling season. We denote the probability for a customer to arrive at time  $t$  to choose product  $i \in \mathbf{n} = \{1, \dots, n\}$  by  $P_t^i(r_t)$  and  $P_t^0(r_t)$  denotes the no-purchase probability. Using (1) with the no-purchase utility  $v^0 = \exp(u_0/\mu)$ , we have the demand of product  $i$  and of no-purchase

$$P_t^i(r_t) = \frac{\exp((q^i - r_t^i)/\mu)}{\sum_{i \in \mathbf{n}} \exp((q_i - r_t^i)/\mu) + \exp(u_0/\mu)} \quad (2)$$

$$P_t^0(r_t) = \frac{\exp(u_0/\mu)}{\sum_{i \in \mathbf{n}} \exp((q_i - r_t^i)/\mu) + \exp(u_0/\mu)} \quad (3)$$

and by construction,  $P_t^0(r_t) + \sum_{i \in \mathbf{n}} P_t^i(r_t) = 1$ .

The revenue rate at time  $t$  is

$$\Phi_t(r_t) := \lambda(t) \sum_{i=1}^n r_t^i P_t^i(r_t) \quad (4)$$

By setting the price  $r_t$ , the retailer controls the probability of customer choice process so the revenue rate is optimized. From (2) and (3) we obtain  $P_t^i(r_t)/P_t^0 = \exp((q^i - r_t^i - u_0)/\mu)$ .

$$r_t^i = q^i - u_0 - \mu \ln P_t^i + \mu \ln P_t^0, \quad i = 1, \dots, n \quad (5)$$

which is a mapping from the sales probability space to the price space, and in (2) and (3) we have a mapping in the opposite direction. However, these two mappings are not inverses of each other as the mapping from the price space to the sales probability space is not injective and hence cannot be a bijection. This can be easily observed for given pricing  $r_t = (r_t^1, \dots, r_t^n)$ , choosing  $r'_t = (r_t^1 \pm \ln k, \dots, r_t^n \pm \ln k)$  for some positive real constant  $k$  will produce the same demand probability of product. We can think of this increase or decrease in the pricing as some form of inflation or deflation of the prices that preserve the customers demand probability of the products.

Thus, without factoring inflation or deflation, we have a one-to-one mapping from the price space to the sales probability space. The revenue rate as in (4) can be rewritten as

$$\Phi_t(P_t) = \lambda(t) \sum_{i=1}^n [q^i - u_0 - \mu \ln P_t^i + \mu \ln P_t^0] P_t^i \quad (6)$$

where  $P_t = (P_t^1, \dots, P_t^i)$

### 3.2 Optimal control of the MNL model

## 4 Experiments

If we manage to find time to run any experiments.

## 5 Discussions

This is where we can add our comments and our inputs, how the model can be further improved or how we can find estimates for the solution.

## 6 Conclusions

Closing conclusions, further areas that can be explored and research opportunities (for Yiwei only haha).

## References

- [1] G. Gallego and G. van Ryzin. A multiproduct dynamic pricing problem and its applications to network yield management. *Operations Research*, 45(1):24–41, 1997.
- [2] J. S. Li and S. Chen. Real-time dynamic pricing for multiproduct models with time-dependent customer arrival rates. *Proceedings of the American Control Conference*, pages 2196–2201, 2009.