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Stochastic Models: Exercise 4

1.

$$
\begin{aligned}
m(t) &= \sum_{n=1}^{\infty} F_n(t), \quad \text{where } F_n(t) \text{ is the } n\text{-fold convolution.} \\
&= F(t) + \sum_{n=2}^{\infty} F_n(t), \quad \text{since } F(t) = F_1(t) \\
&= F(t) + \sum_{n=2}^{\infty} F * F_{n-1}(t) \\
&= F(t) + \sum_{n=2}^{\infty} \int_0^t F_{n-1}(t-x)\, dF(x) \\
&= F(t) + \int_0^t \sum_{n=1}^{\infty} F_n(t-x)\, dF(x) \\
&= F(t) + \int_0^t m(t-x)\, dF(x)
\end{aligned}
$$

2. Let $\{N_D(t), t \geq 0\}$ be a given delay renewal process, then

$$
\begin{aligned}
P\left[S_{N_D(t)} \leq s\right] &= \sum_{n=0}^{\infty} P\left[S_n \leq s, S_{n+1} > t\right] \\
&= \bar{F}(t) + \sum_{n=1}^{\infty} P\left[S_n \leq s, S_{n+1} > t\right] \\
&= \bar{F}(t) + \sum_{n=1}^{\infty} \int_0^{\infty} P\left[S_n \leq s, S_{n+1} > t \mid S_n = y\right] dF_n(y) \\
&= \bar{F}(t) + \int_0^s \bar{F}(t-y)\, d\left(\sum_{n=1}^{\infty} F_n(y)\right) \\
&= \bar{F}(t) + \int_0^s \bar{F}(t-y)\, dm_D(y), \quad \text{since } F_1(y) = G(y)
\end{aligned}
$$

where $m_D(y) = \sum_{n=0}^{\infty} G * F_n(y)$.

3.

$$
\begin{aligned}
P\left[X_{N(t)+1} > x\right] &= \sum_{n=0}^{\infty} P\left[X_{N(t)+1} > x\right] \\
&= \bar{F}(x) + \sum_{n=1}^{\infty} P\left[X_{N(t)+1} > x\right]
\end{aligned}
$$

4.

5. Given the scenario, a new cycle starts each time the policyholder payment rate is reverted to r_1.

   (i) Let X_0 and X_1 denote the paying rate 0 and 1 in a cycle respectively.

   (ii)

6.

7.