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Linear Optimization: Assignment 1

$$\begin{aligned}
\max \quad & z = x_1 + 12x_2 \\
\text{s.t.} \quad & 3x_1 + x_2 + 12x_3 \leq 5 \\
& x_1 + x_3 \leq 16 \\
& 15x_1 + x_2 = 14 \\
& x_j \geq 0, \quad j = 1, 2, 3.
\end{aligned}$$

1.17 (a)

$$\begin{aligned}
\min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
& x_1 + x_2 + x_3 + x_4 \leq M \\
& P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 \leq PM \\
& N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \leq NM \\
& x_j \geq 0, \quad j = 1, \ldots, 4
\end{aligned}$$

(b)

$$\begin{aligned}
\min \quad & c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq K \\
& x_1 + x_2 + x_3 + x_4 \leq M \\
& (P_1 - P)x_1 + (P_2 - P)x_2 + (P_3 - P)x_3 + (P_4 - P)x_4 \leq 0 \\
& (N_1x - N)_1 + (N_2 - N)x_2 + (N_3 - N)x_3 + (N_4 - N)x_4 \leq 0 \\
& x_j \geq 0, \quad j = 1, 2, 3, 4
\end{aligned}$$

1.18 (a)

$$\begin{aligned}
\min \quad & \textstyle\sum_{i=1}^4 c_i x_{1,i} + \sum_{i=1}^4 c_i x_{2,i} + \sum_{i=1}^4 c_i x_{3,i} \\
\text{s.t.} \quad & \textstyle\sum_{i=1}^4 x_{1,i} \geq K_A \\
& \textstyle\sum_{i=1}^4 x_{2,i} \geq K_B \\
& \textstyle\sum_{i=1}^4 x_{3,i} \geq K_C \\
& \textstyle\sum_{i=1}^4 x_{1,i} \leq M_1 \\
& \textstyle\sum_{i=1}^4 x_{2,i} \leq M_2 \\
& \textstyle\sum_{i=1}^4 x_{3,i} \leq M_1 + M_2 \\
& \textstyle\sum_{i=1}^4 P_i x_{1,i} \geq P_S M_1 \\
& \textstyle\sum_{i=1}^4 P_i x_{2,i} \geq P_B M_2 \\
& \textstyle\sum_{i=1}^4 P_i (x_{1,i} + x_{2,i} + x_{3,i}) \geq P_S(M_1 + M_2) \\
& \textstyle\sum_{i=1}^4 N_i x_{1,i} \geq N_S M_1 \\
& \textstyle\sum_{i=1}^4 N_i x_{2,i} \geq N_B M_2 \\
& \textstyle\sum_{i=1}^4 N_i (x_{1,i} + x_{2,i} + x_{3,i}) \geq N_S(M_1 + M_2) \\
& x_{i,j} \geq 0, \quad i = 1, 2, 3, j = 1, 2, 3, 4
\end{aligned}$$

(b) The c_i's, P_i's and N_i's will unique for each plant thus we will have $c_{p,i}$'s, $P_{p,i}$ and $N_{p,i}$ for $p \in \{A, B, C\}$.

1.20 We shall let t denote the $t+6 \mod 12$ month of the year, i.e. $t = 0$ is June and $t = 10$ is April. Let $x_t = x_t^+ - x_t^-$ denote the change in production from month t to month $t+1$ and d_t denote the sales forecast for month t. Letting the units to be in thousands below:

$$
\begin{array}{llllllll}
\min & 0.5\sum_{t=0}^{11} x_t^+ & + 0.25\sum_{t=0}^{11} x_t^- & & & & \\
\text{s.t.} \quad 4 & + & x_0 + & 2 - & d_1 & & \leq 10 \\
4 & + & x_0 + & 2 - & d_1 + 4 + \sum_{t=0}^{1} x_t & - d_2 & \leq 10 \\
2(4) & + \sum_{t=0}^{1}(2-t)x_t + & & 2 - \sum_{i=1}^{2} d_t + 4 + \sum_{t=0}^{2} x_t & & - d_3 & \leq 10 \\
3(4) & + \sum_{t=0}^{2}(3-t)x_t + & & 2 - \sum_{i=1}^{3} d_t + 4 + \sum_{t=0}^{3} x_t & & - d_4 & \leq 10 \\
\vdots & \vdots & & \vdots & & \vdots & \\
11(4) & + \sum_{t=0}^{10}(11-t)x_t + & & 2 - \sum_{i=1}^{11} d_t + 4 + \sum_{t=0}^{11} x_t & & - d_{12} & \leq 10 \\
& x_i^+, x_i^- \geq 0, \quad i = 1, \ldots, 12 & & & & &
\end{array}
$$

1.25 We first list down the different ways such that a 100-inch roll can be cut into combinations of 24-, 40-, and 32-inch widths. Let x_i denote the number of combination i used.

Combination	24	40	32	trim waste
1	4	0	0	4
2	0	2	0	20
3	0	0	3	4
4	1	1	1	4
5	2	1	0	12
6	2	0	1	20
7	1	0	2	12

$$
\begin{array}{ll}
\min & 4x_1 + 20x_2 + 4x_3 + 4x_4 + 12x_5 + 20x_6 + 12x_7 \\
\text{s.t.} & 4x_1 + x_4 + 2x_5 + 2x_6 + x_7 \geq 75 \\
& 2x_2 + x_4 + x_5 \geq 50 \\
& 3x_3 + x_4 + x_6 + 2x_7 \geq 110 \\
& x_j \geq 0, \quad j = 1, \ldots, 7
\end{array}
$$

2.7 Letting the units be in thousands below:

$$
\begin{array}{ll}
\max & 2x_1 + 1.8x_2 \\
\text{s.t.} & x_1 + x_2 \leq 10 \\
& 2x_1 + x_2 \leq 9 \\
& x_1, x_2 \geq 0
\end{array}
$$

2.8 Let the units be in pounds below, and x_1, x_2 and x_3 denoting amount of ingredient A, B and C used respectively.

$$
\begin{array}{ll}
\min & 4x_1 + 3x_2 + 2x_3 \\
\text{s.t.} & x_1 \geq 200 \\
& x_2 \geq 400 \\
& x_3 \leq 800 \\
& x_1, x_2, x_3 \geq 0
\end{array}
$$

The bounded-variable simplex method cannot be used to solve this problem as it will lead to a matrix A that is of not full rank.

2.9 Phase I:

$$
\begin{array}{ll}
\max & x_0 \\
\text{s.t.} & x_0 + x_6 + x_7 + x_8 = 0 \\
& + 2x_1 + 3x_2 - x_3 + x_4 + x_6 = 9 \\
& 2x_2 + x_3 - x_5 + x_7 = 4 \\
& + x_1 + x_3 + x_8 = 6 \\
& x_i \geq 0, \forall i
\end{array}
$$

2.12

2.13

$(-w)$	0	0	0	
x_6	1	0	0	9
x_7	1	0	0	4
x_8	1	0	0	6