

Stochastic Models: Exercise 4

1.

$$\begin{aligned}
 m(t) &= \sum_{n=1}^{\infty} F_n(t), \quad \text{where } F_n(t) \text{ is the } n\text{-fold convolution.} \\
 &= F(t) + \sum_{n=2}^{\infty} F_n(t), \quad \text{since } F(t) = F_1(t) \\
 &= F(t) + \sum_{n=2}^{\infty} F * F_{n-1}(t) \\
 &= F(t) + \sum_{n=2}^{\infty} \int_0^t F_{n-1}(t-x) dF(x) \\
 &= F(t) + \int_0^t \sum_{n=1}^{\infty} F_n(t-x) dF(x) \\
 &= F(t) + \int_0^t m(t-x) dF(x)
 \end{aligned}$$

2. Let $\{N_D(t), t \geq 0\}$ be a given delay renewal process, then

$$\begin{aligned}
 P[S_{N_D(t)} \leq s] &= \sum_{n=0}^{\infty} P[S_n \leq s, S_{n+1} > t] \\
 &= \bar{F}(t) + \sum_{n=1}^{\infty} P[S_n \leq s, S_{n+1} > t] \\
 &= \bar{F}(t) + \sum_{n=1}^{\infty} \int_0^{\infty} P[S_n \leq s, S_{n+1} > t \mid S_n = y] dF_n(y) \\
 &= \bar{F}(t) + \int_0^s \bar{F}(t-y) d\left(\sum_{n=1}^{\infty} F_n(y)\right) \\
 &= \bar{F}(t) + \int_0^s \bar{F}(t-y) dm_D(y), \quad \text{since } F_1(y) = G(y)
 \end{aligned}$$

where $m_D(y) = \sum_{n=0}^{\infty} G * F_n(y)$.

3.

$$\begin{aligned}
 P[X_{N(t)+1} > x] &= \sum_{n=0}^{\infty} P[X_{N(t)+1} > x] \\
 &= \bar{F}(x) + \sum_{n=1}^{\infty} P[X_{N(t)+1} > x]
 \end{aligned}$$

4.

5.

6.