

Statistics: Homework 1

- 1.19 Let X_1, X_2 and X_3 denote the computer owners that use Macintosh, Windows and Linux respectively and let V denote the event that the user's system is infected with the virus. We want to find $\mathbb{P}(X_2|V)$

$$\begin{aligned}\mathbb{P}(X_2|V) &= \frac{\mathbb{P}(V|X_2)\mathbb{P}(X_2)}{\sum_{i=1}^3 \mathbb{P}(V|X_i)\mathbb{P}(X_i)} \\ &= \frac{(.82)(.5)}{(.65)(.3) + (.82)(.5) + (.5)(.2)} \\ &= 0.581560284\end{aligned}$$

2.4 (a)

$$F_X(x) := \begin{cases} \frac{1}{4}x & 0 < x < 1 \\ \frac{3}{8}x - \frac{7}{8} & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$\begin{aligned}\mathbb{P}(Y \leq y) &= \mathbb{P}(1/X \leq y) \\ &= \mathbb{P}(X \geq 1/y) \\ &= 1 - \mathbb{P}(X \leq 1/y)\end{aligned}$$

From (a):

$$\begin{aligned}F_Y(y) &:= \begin{cases} \frac{15}{8} - \frac{3}{8y} & 1/5 < y < 1/3 \\ 1 - \frac{1}{4y} & y \geq 1 \\ 0 & \text{otherwise} \end{cases} \\ f_Y(y) &:= \begin{cases} \frac{3}{8y^2} & 1/5 < y < 1/3 \\ \frac{1}{4y^2} & y \geq 1 \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

- 2.11 (a) We see that $\mathbb{P}(X = 1) = p = \mathbb{P}(Y = 0)$. Since the state space contains is $\{H, T\}$, we have $1 - \mathbb{P}(X = 1, Y = 0) = 1 - p = \mathbb{P}(X = 0, Y = 1)$. But since

$$\mathbb{P}(X = 1)\mathbb{P}(Y = 0) = p^2 \neq p = \mathbb{P}(X = 1, Y = 0)$$

X and Y are dependent.

(b)

3.4

4.3

5.7