Literature Review: Real-Time Dynamic Pricing for Multiproduct Models with Time-Dependent Customer Arrival Rates

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1 Introduction

Introduction to talk about what we learnt in class (Single Product DP) and how the model in the paper we have chosen is a generalizing from single to multi-products.

In the single product dynamic pricing model that we were introduced in class, we looked at a monopolist seller which finite units x_0 of a single indivisible product with no replenishment over a finite and continuous horizon $[0, T)$. The unit price π_t is decided by the seller at each point of time $t \in [0, T)$ and customers product valuations follow a distribution over $\mathbb{R}_+$. However, practically, sellers often have a wide range of products that the customers can choose from, with the products having similar functionalities; catering to customers of varying purchasing power.

Thus we look to a multiproduct model [2]...

2 Related Work

[Optional] We might like to discuss other papers like [1] that might be related to the paper we are looking at, e.g. the 1997 paper Yiwei told us to look at.

3 The Multinomial Logit Model

The model of course.

3.1 Summary of the MNL model

The MNL model is a multiproduct dynamic pricing problem and assumes the products are nominal. It describes dynamic consumer choice preferences over substitute products as prices are varied. The customer makes choices from a range of products to maximise his or her utility. The utility of the i product is defined by the logit demand function $v^i(r^i) = \exp((q^i - r^i)/\mu)$ which is a positive function of the quality q^i, the price r^i of product i for $i = 1, \dots, n$ and μ is the constant representing the stochastic preference of the choice process. The customer expected demand probability of product i is defined as

$$P^i(r) = \frac{v^i(r^i)}{v^0 + \sum_{j=1}^{n} v^j(r^j)}, \quad i = 1, \dots n \tag{1}$$

and v^0 denotes the utility of not making any purchase.

The customer's arrivals are assumed to follow a nonhomogenous Poisson process with a time-dependent rate $\lambda(t)$ and in a small time interval δt, the probability of one arrival is $\lambda(t)\delta t$. The price of the products at time t is $r_t = (r_t^1, \dots, r_t^n) \in \mathbb{R}^n$ is decided according to the current inventory level. If some product i is sold out before the end of the selling season, as we do not allow replenishments, the price is set to $r_t^i = \infty$ for all t occuring after $c_t^i = 0$. If all the products are sold at at $t < T$, then the selling season ends at t. All unsold products are salvaged at $t = 0$.

Let $\mathbf{r} = \{r_t, t \in [T, 0]\} = (r^1, \dots, r^n)$ denote a pricing policy for the entire season, where r^i denotes the trend of the price of product i over the selling season. We denote the probabilty for a customer to arrive at time t to choose product $i \in \mathbf{n} = \{1, \dots, n\}$ by $P_t^i(r_t)$ and $P_t^0(r_t)$ denotes the no-purchase probability. Using (1) with the no-purchase utility $v^0 = \exp(u_0/\mu)$, we have the demand of product i and of no-purchase

$$P_t^i(r_t) = \frac{\exp((q^i - r_t^i)/\mu)}{\sum_{i \in \mathbf{n}} \exp((q_i - r_t^i)/\mu) + \exp(u_0/\mu)}$$
$$P_t^0(r_t) = \frac{\exp(u_0/\mu)}{\sum_{i \in \mathbf{n}} \exp((q_i - r_t^i)/\mu) + \exp(u_0/\mu)}$$

and by construction, $P_t^0(r_t) + \sum_{i \in \mathbf{n}} P_t^i(r_t) = 1$

3.2 Optimal control of the MNL model

4 Experiments

If we manage to find time to run any experiments.

5 Discussions

This is where we can add our comments and our inputs, how the model can be further improved or how we can find estimates for the solution.

6 Conclusions

Closing conclusions, futher areas that can be explored and research opportunities (for Yiwei only haha).

References

[1] G. Gallego and G. van Ryzin. A multiproduct dynamic pricing problem and its applications to network yield management. *Operations Research*, 45(1):24–41, 1997.

[2] J. S. Li and S. Chen. Real-time dynamic pricing for multiproduct models with time-dependent customer arrival rates. *Proceedings of the American Control Conference*, pages 2196–2201, 2009.