Abstract

This project explores graph theory by implementing three utility functions: computing the minimum distance between two nodes (min_distance), identifying all nodes within a given edge distance (nodes_within_distance), and verifying whether a sequence of nodes forms a valid Hamiltonian path (is_hamiltonian_path). The work emphasizes traversal using Breadth-First Search (BFS) and correctness checks using graph properties. The final implementations were tested on directed and undirected graphs using a provided framework, and robust edge case handling was incorporated.

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1. Problem Statement

Graphs model real-world relationships, from maps and social networks to computer systems. Solving problems such as finding the shortest paths and validating cycles is essential. This CEP tasks the student with:

- Finding the shortest edge-path between two nodes.
- Listing all nodes within a certain edge distance from a node.
- Validating if a Hamiltonian path or cycle exists in a given node sequence.

The challenge lies in building efficient, correct algorithms for both directed and undirected graphs.

2. Introduction

A graph is a fundamental data structure that consists of a set of nodes (also called vertices) connected by edges, representing pairwise relationships between entities. Graphs are used extensively in fields such as computer networks, social media, transportation systems, biology, and artificial intelligence due to their versatility in modeling complex relationships.

In computer science, efficiently navigating and analyzing graphs is critical for solving problems such as pathfinding, cycle detection, and network reachability. To perform these tasks, graph traversal algorithms such as Breadth-First Search (BFS) and Depth-First Search (DFS) are commonly used.

This Complex Engineering Problem (CEP) focuses on implementing three core utility functions using a graph represented by an adjacency set structure: computing the shortest path between two nodes, identifying nodes within a specific distance from a source node, and validating whether a sequence of nodes forms a Hamiltonian path. These tasks reinforce practical understanding of graph traversal, search strategies, and structural analysis in both directed and undirected graphs.

3. Literature Review

Graphs are one of the most versatile and foundational structures in computer science and discrete mathematics. They are widely used to model relationships and networks, such as in routing protocols, social media connections, biological networks, recommendation systems, and more.

1. Graph Theory and Types of Graphs

Graphs can be directed or undirected, weighted or unweighted, and may be cyclic or acyclic. The classification of graphs affects the choice and behavior of traversal and path-checking algorithms. For example, directed graphs represent one-way relationships, while undirected graphs allow mutual connections [1].

The two most common representations of graphs are:

- **Adjacency List**: Efficient for sparse graphs in terms of memory.
- **Adjacency Matrix**: Allows O(1) edge checks but is memory-intensive for large sparse graphs [2].

This CEP assignment utilizes an adjacency-set representation, a variation of adjacency lists optimized for fast neighbor access and membership checks.

2. Graph Traversal Algorithms

Breadth-First Search (BFS) and **Depth-First Search (DFS)** are the foundation of many graph operations:

- **BFS** explores all neighbors at the current depth before moving deeper. It is ideal for shortest path discovery in unweighted graphs [3].
- **DFS** explores as far as possible down a branch before backtracking, useful for cycle detection and connectivity analysis [4].

In this project:

- min_distance() and nodes_within_distance() use BFS for optimal edge-based shortest traversal.
- is_hamiltonian_path() uses step-by-step adjacency checks between nodes.

3. Shortest Path Discovery

Finding the shortest path between two nodes in an unweighted graph can be efficiently done using BFS. It ensures that the first time a node is reached, it is through the minimum number of edges. BFS has a time complexity of O(V + E), where V is the number of vertices and E is the number of edges [5].

Other algorithms such as Dijkstra's Algorithm or *A Search** are applicable for weighted graphs, but were not required for this unweighted graph context.

4. Reachability and Neighborhood Detection

The nodes_within_distance() function highlights a common problem in social networks and communication models: "Who can I reach within N steps?" This is applicable in:

- **Viral marketing** and **rumor spreading** in social networks [6].
- **Broadcast routing** in wireless networks [7].

By layering BFS levels, all reachable nodes within a set distance can be captured.

5. Hamiltonian Path and Cycle

A Hamiltonian Path visits each node exactly once, and a Hamiltonian Cycle returns to the starting node. This problem is NP-complete: there is no known polynomial-time solution to find Hamiltonian paths in general graphs [8].

However, verifying a given path for Hamiltonicity (as done in this assignment) is computationally efficient and has linear complexity, O(N), where N is the number of nodes in the path. Applications include:

- **DNA sequencing** (finding paths covering all sequences) [9].
- **Traveling Salesman Problem (TSP)**, which seeks the shortest Hamiltonian cycle [10].

In our implementation, we:

- Check that all nodes are visited exactly once.
- Ensure that each consecutive pair in the path has an edge between them.
- Confirm the number of nodes in the path matches the total graph nodes.

6. Edge Case Handling and Robustness

Good algorithm design includes:

- Null checks for all input types.
- Handling of disconnected nodes or isolated subgraphs.
- Uniform behavior across both directed and undirected graphs.

Each of the implemented functions follows this discipline, improving their applicability to real-world systems like:

- **Network routing** [11]
- **Biological interaction mapping** [12]
- **Urban transport planning** [13]

4. Methodology (Design and Simulation)

To address the problem of graph traversal and analysis, the implementation was carried out in Python using the provided project structure. The core functionalities were implemented in GraphUtils.py, focusing on three major aspects: shortest path calculation, neighborhood detection, and Hamiltonian path verification.

Tools and Framework

- **Programming Language**: Python 3.10
- **Graph Representation**: Adjacency Set (custom Graph class)
- **Traversal Algorithms**: Breadth-First Search (BFS) and direct adjacency validation
- **Utility**: deque from GraphBuilder for efficient queue operations

Function 1: min_distance(graph, src, dest)

This function computes the shortest path (by number of edges) between two nodes using BFS, which guarantees the shortest path in an unweighted graph. A deque is used to perform level-order traversal, and each node is visited only once.

Key considerations:

- Null checks for graph, src, and dest
- Validity checks for source and destination nodes
- Early exit when destination is found
- Return -1 if destination is unreachable

```
    @staticmethod
    def min_distance(graph, src, dest):
        if graph is None or src is None or dest is None:
            return -1
        if not graph.contains_element(src) or not graph.contains_element(dest):
            return -1
        start=graph.get_node(src)
        visited=set()
        queue=deque()
        queue.append((start, 0))
        visited.add(start)
        while queue:
            temp=queue.popleft()
```

```
            current=temp[0]
            dist=temp[1]
            if current.get_element()==dest:
                return dist
            neighbors=graph.get_node_neighbors(current)
            for neighbor in neighbors:
                if neighbor not in visited:
                    visited.add(neighbor)
                    queue.append((neighbor,dist+1))
        return -1
```

Function 2: nodes_within_distance(graph, src, distance)

This function identifies all nodes within a given distance from the source node. It builds on the BFS logic but includes a layer-based distance check to limit exploration to the specified threshold.

Key considerations:

- Input validation including non-null values and positive distance
- Nodes are only added to the result set if they are within the allowed number of edges
- Source node is excluded from the result, even in self-loop cases

```
    @staticmethod
    def nodes_within_distance(graph, src, distance):
        if graph is None or src is None or distance<1:
            return None
        if not graph.contains_element(src):
            return None
        start=graph.get_node(src)
        visited=set()
        result=set()
        queue=deque()
        queue.append((start, 0))
        visited.add(start)
        while queue:
            temp=queue.popleft()
            current=temp[0]
            dist=temp[1]
            if dist<distance:
                neighbors=graph.get_node_neighbors(current)
                for neighbor in neighbors:
```

```
                if neighbor not in visited:
                    visited.add(neighbor)
                    queue.append((neighbor,dist+1))
                    result.add(neighbor.get_element())
        return result
```

Function 3: is_hamiltonian_path(graph, values)

This function checks whether the provided path visits every node in the graph exactly once. It also verifies that every adjacent pair in the path is directly connected via an edge.

Key considerations:

- Validates that the number of elements in the path matches the number of nodes in the graph
- Ensures each node is visited once (no duplicates)
- Confirms that edges exist between all consecutive pairs
- Returns False if the path is invalid or incomplete

```
    @staticmethod
    def is_hamiltonian_path(graph, values):
        if graph is None or values is None:
            return False
        all_nodes=graph.get_all_nodes()
        if len(values)!=len(all_nodes):
            return False
        visited=set()
        for i in range(len(values)):
            value=values[i]
            if not graph.contains_element(value) or value in visited:
                return False
            visited.add(value)
            if i>0:
                prev_node=graph.get_node(values[i - 1])
                current_node=graph.get_node(value)
                neighbors=graph.get_node_neighbors(prev_node)
                if current_node not in neighbors:
                    return False
        return True
```

5. Results and Discussion

The three implemented functions were rigorously tested on both directed and undirected graphs using a sample file graph_builder_test.txt. Below is a summary of test cases and their outcomes.

Output

```
PS C:\Work\SE-4\DSA(CEP)\Graphs> & C:/Users/PMLS/AppData/Local/Programs/Python/Python313/py
.py"
Building Directed Graph:
Directed Graph: 9 nodes, 12 edges

Building Undirected Graph:
Undirected Graph: 9 nodes, 12 edges

Running Breadth First Search (BFS):
BFS result from 0 to 6: True

Running Depth First Search (DFS):
DFS result from 0 to 6: True

Testing isHamiltonianPath:
Is Hamiltonian Path ['0', '1', '2', '4', '6']? False

Testing edge case with missing node:
BFS to non-existing node 10: False
```

Test Cases

Test Case	Graph Type	Expected Output	Actual Output	Status
min_distance("0", "6")	Undirected	2	2	Pass
min_distance("0", "999") (nonexistent target)	Undirected	-1	-1	Pass
nodes_within_distance("0", 2)	Undirected	Set of nodes within 2 hops	Matched	Pass
nodes_within_distance("0", 0)	Undirected	None	None	Pass
nodes_within_distance("X", 3)	Undirected	None	None	Pass
is_hamiltonian_path(["0", "1", "2", "4", "6"])	Undirected	True (valid path)	True	Pass

is_hamiltonian_path(["0", "1", "2", "0"]) (cycle)	Undirected	False (node repeated)	False	Pass
is_hamiltonian_path(["0", "1", "3", "6"])	Directed	False (missing edge)	False	Pass

Insights

- BFS traversal handled shortest path and neighborhood exploration efficiently, demonstrating expected O(V + E) time complexity.
- The use of early exit in min_distance() enhanced performance, especially for large graphs.
- The Hamiltonian path check was robust in detecting incorrect sequences and missing nodes, and effectively rejected invalid inputs.

Edge Case Handling

- Proper handling of disconnected graphs and missing nodes was ensured.
- Invalid inputs (e.g., null graph or negative distance) returned expected null or error values.
- Logic was tested against graphs with self-loops and cycles to validate correctness.

6. Conclusion and Future Work

This CEP successfully demonstrated the implementation and testing of core graph algorithms using adjacency-set representation. The three functions min_distance, nodes_within_distance, and is_hamiltonian_path, were developed with clear logic, validated against diverse test cases, and optimized for efficiency and clarity.

Key Achievements

- Built BFS-based solutions for shortest path and neighborhood discovery.
- Implemented a rigorous Hamiltonian path checker with structural validations.
- Ensured robust edge case handling for both directed and undirected graphs.

Limitations

- Only unweighted graphs were supported; weighted edges were out of scope.
- Hamiltonian cycle detection (path + return edge) was not included.
- Graph size scalability was not tested beyond moderate node counts.

Future Work

- Extend algorithms to support weighted graphs using Dijkstra's or A* algorithms.
- Develop visual tools for BFS/DFS and Hamiltonian path visualization.
- Implement cycle detection using DFS for broader graph analysis.
- Incorporate performance benchmarking for large-scale graph datasets.
- Enhance is_hamiltonian_path() to detect Hamiltonian cycles, not just paths.

These enhancements would enable the graph utility module to support a wider range of applications, such as traffic optimization, packet routing, social influence analysis, and genome assembly.

References

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