Linear Algebra

(MT-121T)

AFTAB ALAM

LECTURE # 4

(FRIDAY, FEBRUARY 16, 2024)

The Idea of Elimination

$$\begin{aligned} x \ - 2y &= 1 \\ 3x + 2y &= 11 \end{aligned}$$

Elimination:

1) Multiply Equation 1 by 3
2) Subtract (3 x Equation 1) from Equation 2 to eliminate $3x$

$$\begin{aligned} x \ - 2y &= 1 \\ 8y &= 8 \end{aligned}$$

$$(x, y) = (3,1)$$

Example 1: Permanent Failure with no Solution

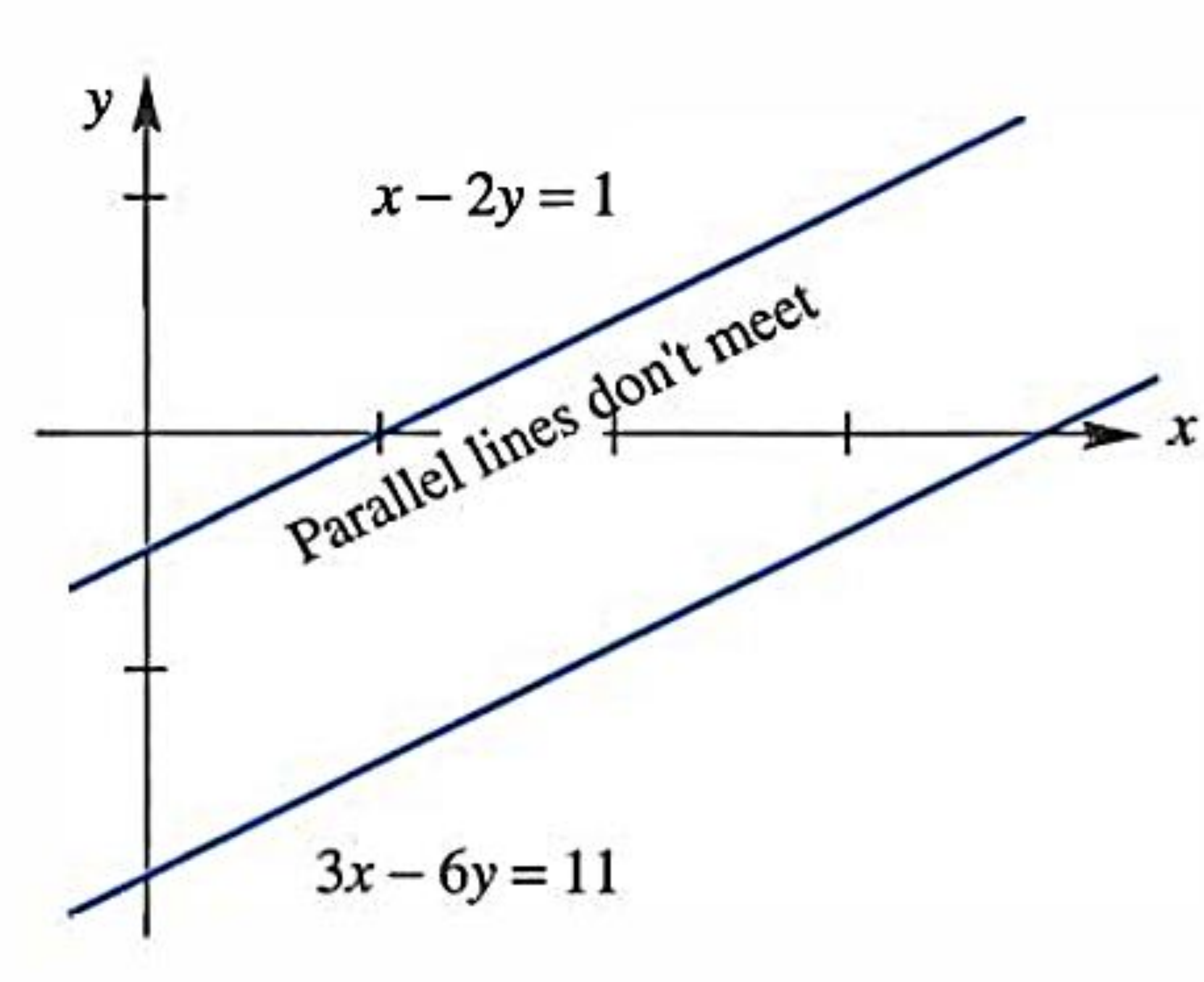
$$\begin{aligned} x \ -2y &= 1 \\ 3x \ -6y &= 11 \end{aligned}$$

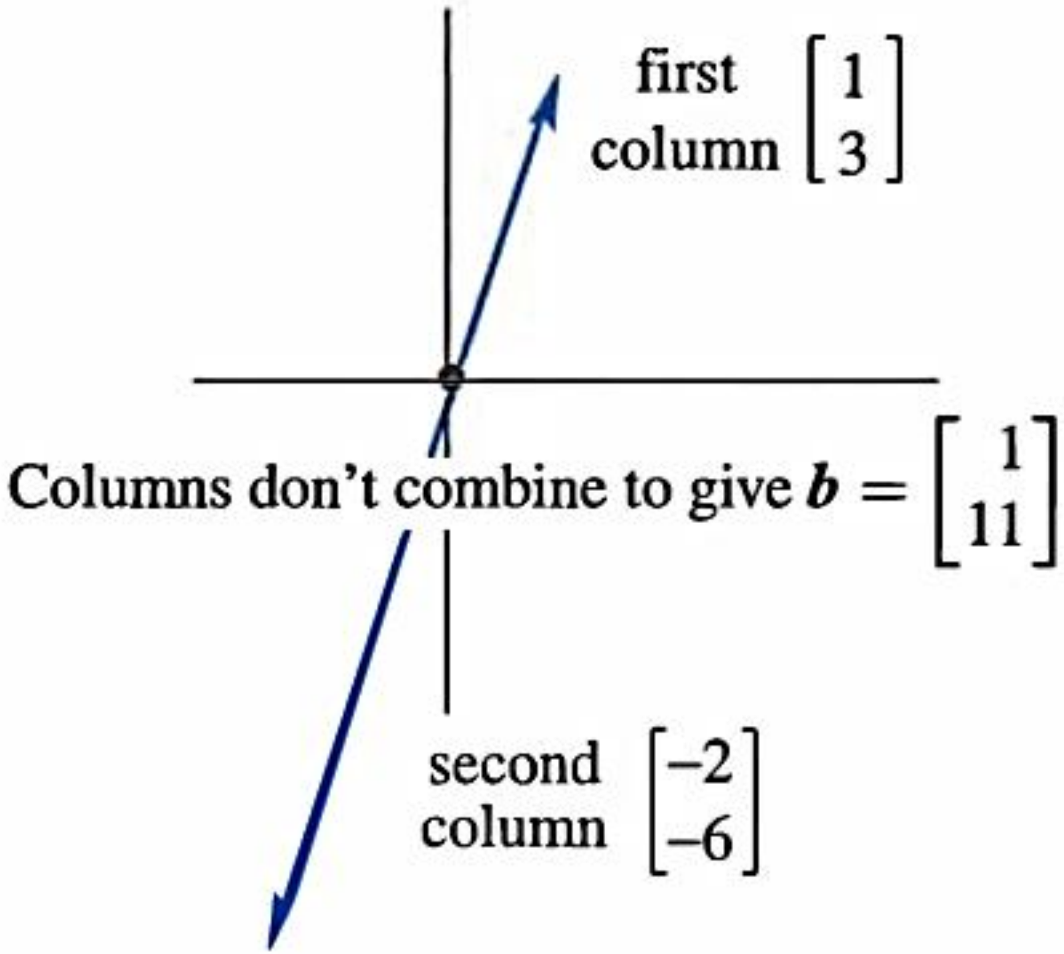
Subtract 3 times Eq. 1 from Eq. 2

$$\begin{aligned} x \ -2y &= 1 \\ 0y &= 8 \end{aligned}$$

Zero is never allowed as a **pivot.**

All combinations of the columns lie along a line. But the column from the right side is in a different direction (1,11). No combination of the columns can produce this right side – therefore NO SOLUTION.





Row Picture and Column Picture for Example 1: ***No Solution***

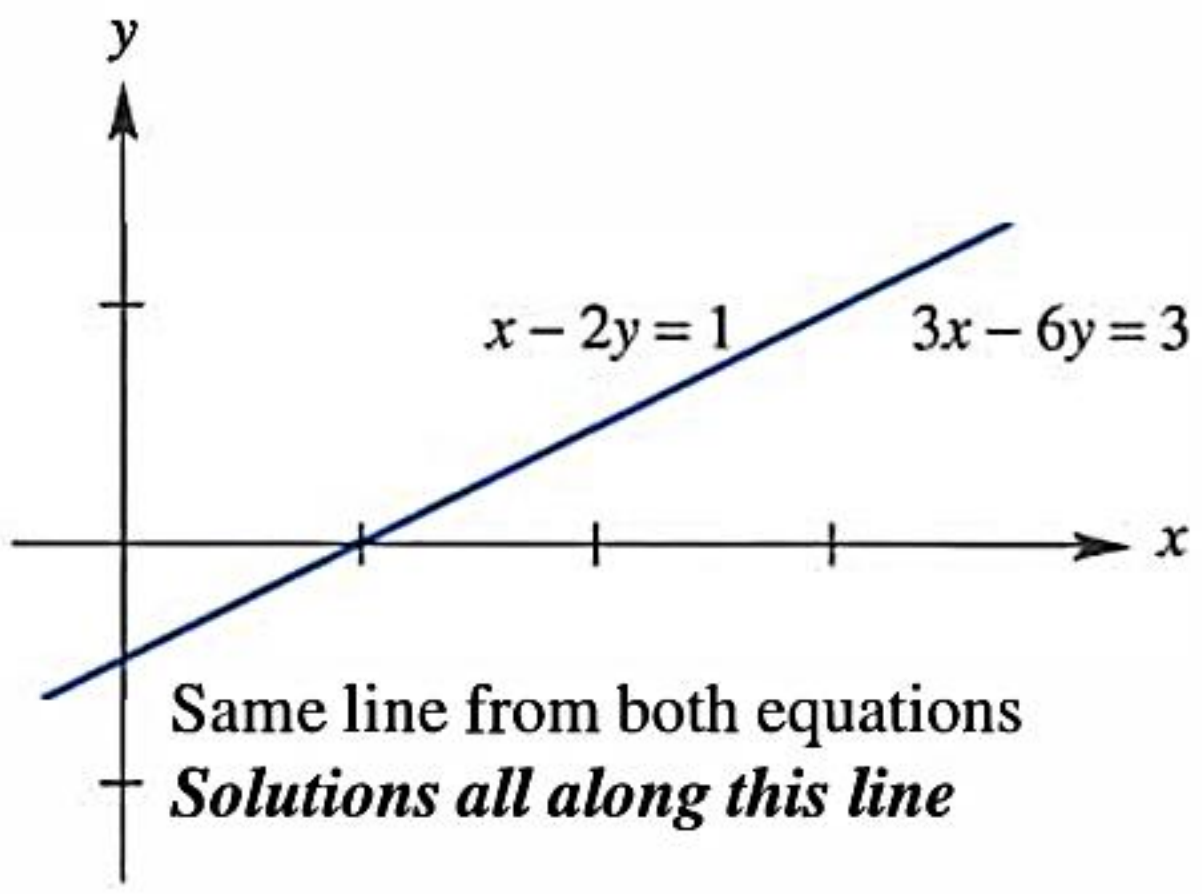
Example 2: Failure with Infinitely Many Solutions

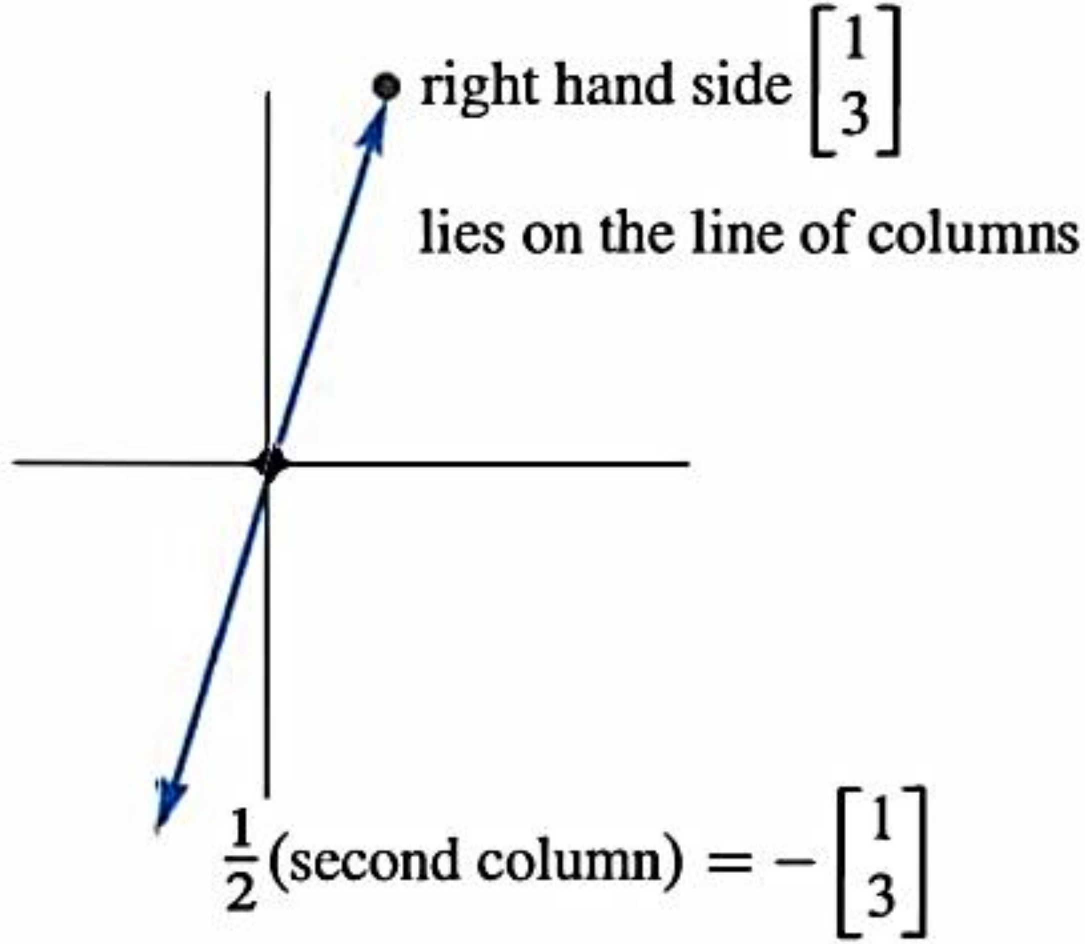
$$\begin{aligned} x \;\; - 2y &= 1 \\ 3x \;\; - 6y &= 3 \end{aligned}$$

Subtract 3 times Eq. 1 from Eq. 2

$$\begin{aligned} x \;\; - 2y &= 1 \\ 0y &= 0 \end{aligned}$$

The unknown y is **“free variable”.**





Row Picture and Column Picture for Example 2: ***Infinitely Many Solutions***

Example 3: Temporary Failure (zero in pivot). A Row Exchange Produces Two Pivots

$$
\begin{aligned}
0x + 2y &= 4 \\
3x \; - 2y &= 5
\end{aligned}
$$

Exchange the two equations

$$
\begin{aligned}
3x \; - 2y &= 5 \\
0x + 2y &= 4
\end{aligned}
$$

Three Equations in Three Unknowns

$$2x + 4y - 2z = 2$$
$$4x + 9y - 3z = 8$$
$$-2x - 3y + 7z = 10$$

After Elimination; $Ax = b$ has become $Ux = c$

$$\mathbf{2}x + 4y - 2z = 2$$
$$\mathbf{1}y + 1z = 4$$
$$\mathbf{4}z = 8$$

Solution: $(x, y, z) = (-1,2,2)$

Steps to Follow in Elimination

1. A linear system ($A\boldsymbol{x} = \boldsymbol{b}$) becomes **upper triangular** ($U\boldsymbol{x} = \boldsymbol{c}$) after elimination.

2. We **subtract** ℓ_{ij} times equation j from equation i, to make the (i, j) entry zero.

3. The **multiplier** is $\ell_{ij} = \dfrac{\text{entry to eliminate in row } i}{\text{pivot in row } j}$. **Pivots** can not be zero!

4. When zero is in the pivot position, **exchange rows** if there is a nonzero below it.

5. The upper triangular $U\boldsymbol{x} = \boldsymbol{c}$ is solved by **back substitution** (starting at the bottom).

When **breakdown** is permanent, $A\boldsymbol{x} = \boldsymbol{b}$ has no solution or infinitely many.

Follow Up Question:

Use elimination and back substitution to find the solution (if possible). Identify the pivots (never zero). Exchange equations when necessary.

a) $x + y + z = 7$

$x + y - z = 5$

$x - y + z = 3$

b) $x + y + z = 7$

$x + y - z = 5$

$-x - y + z = 3$