Assignment #03.
Discrete Structures.

Q1 1.

a. $\{x \mid x$ is a real number such that $x^2 = 1\}$.

$x = \{1, -1\}$ when $x = 1$, $(1)^2 = 1$

when $x = -1$, $(-1)^2 = 1$.

b. $\{x \mid x$ is a positive integer less than $12\}$.

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ are all positive integers that are less than 12.

c. $\{x \mid x$ is the square of an integer and $x < 100\}$.

$\{1, 4, 9, 16, 25, 36, 49, 64, 81\}$.

$(1)^2 = 1 < 100$ | $(2)^2 = 4 < 1$

$(3)^2 = 9 < 100$ | $(4)^2 = 16 < 1$

$(5)^2 = 25 < 100$ | $(6)^2 = 36 < 1$

$(7)^2 = 49 < 100$ | $(8)^2 = 64 < 1$

$(9)^2 = 81 < 100$ | $(10)^2 = 100 < 1$

d. $\{x \mid x$ is an integer such that $x^2 = 2\}$.

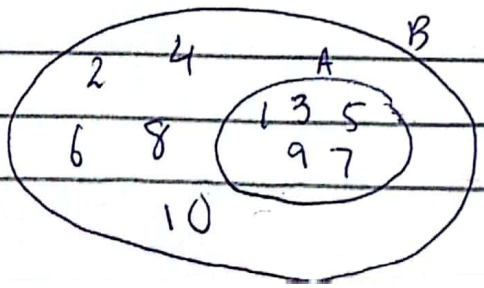
ϕ , no integer square that equals 2.

Q2 2.

$A = \{1, 3, 5, 7, 9\}$, set containing odd integers < 10

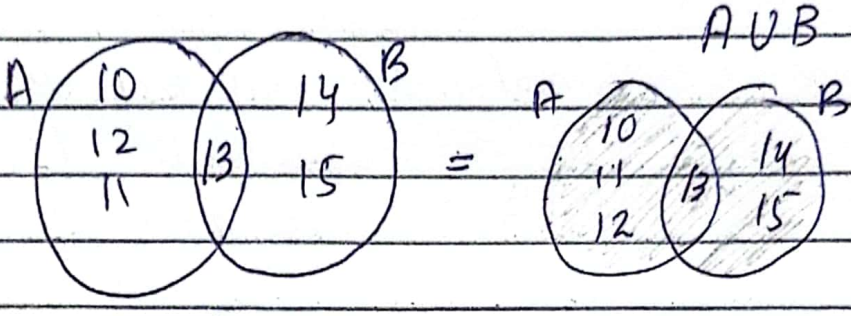
$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, set containing all positive integers < 10

$A \subset B$



Q2 1. $A = \{10, 11, 12, 13\}$. $B = \{13, 14, 15\}$.

$A \cup B = \{10, 11, 12, 13, 14, 15\}$.



2. $A = \{a, b\}$ $B = \{1, 2\}$.

$A \times B = \{(a,1), (a,2), (b,1), (b,2)\}$.

Q3 $A = \{$ sophomores in school $\}$ $B = \{$ students in DS $\}$

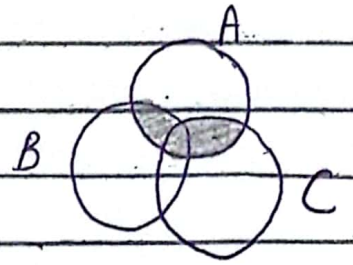
a. $A \cap B$.

b. $A - B$

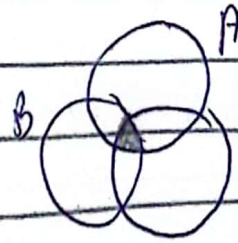
c. $A \cup B$

d. $(A \cup B)' = A' \cap B'$

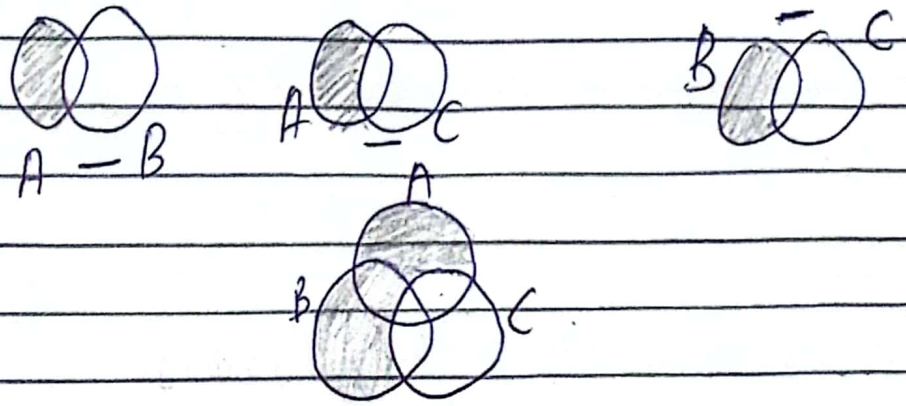
Q4 1. a. $A \cap (B \cup C)$.



b. $A \cap B \cap C$.



c. $(A-B) \cup (A-C) \cup (B-C)$



2. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a. $\{3, 4, 5\}$.

0011100000

b. $\{1, 3, 6, 10\}$.

1010010001

c. $\{2, 3, 4, 7, 8, 9\}$

0111001110.

3. Example of finite set could be - $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

a. a string with all zeros.

0000000000, would suggest either the set elements in the set do not match the U set eg: $A = \{11, 12, 3\}$ so the set would be an empty set $\emptyset$.

b. a string with all 1.

1111111111, would suggest a set contains all elements of the U set. example: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Q5 a. $\phi \in \{\phi\}$.

True, as the empty set is an element of the set containing only the empty set.

b. $\phi \in \{\phi, \{\phi\}\}$.

True, ϕ is an element of the set $\{\phi, \{\phi\}\}$.

c. $\{\phi\} \in \{\phi\}$.

False, The set $\{\phi\}$ is not element of the set $\{\phi\}$ but rather ϕ is an element of the set $\{\phi\}$.

d. $\{\phi\} \in \{\{\phi\}\}$.

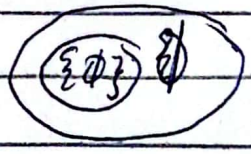
True, The set $\{\phi\}$ is an element of $\{\{\phi\}\}$.
element.

e. $\{\phi\} \subset \{\phi, \{\phi\}\}$.

True, The set $\{\phi\}$ is a subset of set $\{\phi, \{\phi\}\}$.

f. $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$.

True, the set $\{\{\phi\}$ is a subset of $\{\phi, \{\phi\}\}$.



g. $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$

False, The set $\{\{\phi\}\}$ is not a subset of set containing 2 separate sets, both containing empty set $\{\{\phi\}, \{\phi\}\}$.

1. a) $f(n) = \pm n$,

For both values of n, $+n^-$ the function is defined. The output is different for both the values. Thus f is not a function from Z to R.

b) $f(n) = \sqrt{n^2+1}$

for every integer n, f(n) produces ~~different~~ the same value, whether we put positive or negative,

$$\sqrt{(-n)^2+1} = \sqrt{(+n)^2+1}$$

$$\sqrt{n^2+1} = \sqrt{n^2+1}$$

In this case f is a function

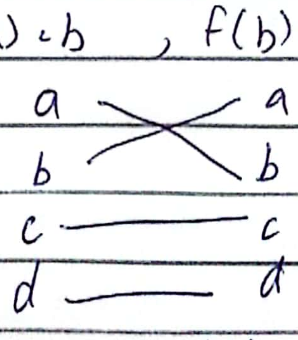
c. $f(n) = \frac{1}{n^2-4}$

$f(2) = \frac{1}{(2)^2-4}$ $\quad$ $f(-2) = \frac{1}{(-2)^2-4}$

$= \frac{1}{0} = \infty$ $\quad$ $= \frac{1}{0} = \infty$

f is not a function it cannot be defined as it is equal to infinity.

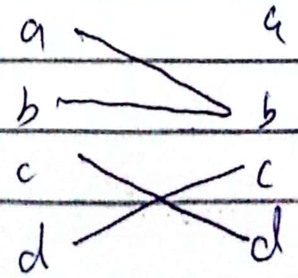
2. a. $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$



for each x value, there is one y value

One to one.

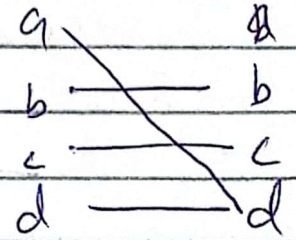
b. $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$



The value of $y = b$ has two x values

not one to one

2. $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$



not one to one, as value of $y = d$ has two x values.

3. $f(x) = x^2 + 1$ $\quad g(x) = x + 2$.

$f \circ g = (x+2)^2 + 1$

$= (x+2)(x+2) + 1$

$= x^2 + 2x + 2x + 4 + 1$

$= x^2 + 4x + 5$

$g \circ f = (x^2 + 1) + 2$

$= x^2 + 1 + 2$

$= x^2 + 3$