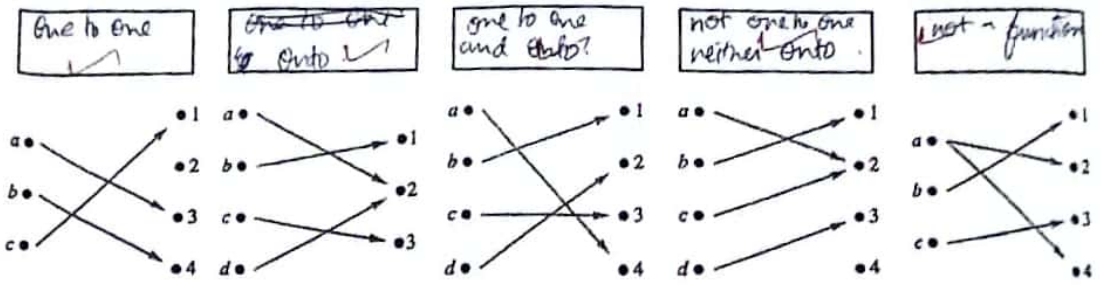
Section A (CLO 1, 20 points)

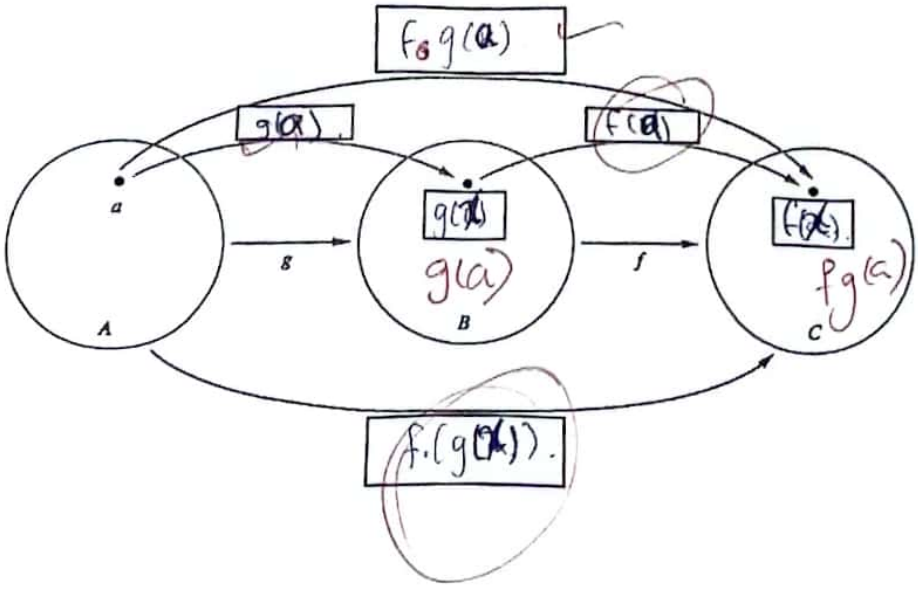
1. Prove that $(n+1)^3 \geq 3n$ if n is a positive integer with $n \leq 4$. *(4 points)*
2. Show that $|xy| = |x||y|$, where x and y are real numbers. (Recall that $|a|$, the absolute value of a, equals a when $a \geq 0$ and equals $-a$ when $a \leq 0$.) *(4 points)*
3. Show that there exist irrational numbers x and y such that xy, is rational using a nonconstructive existence proof. *(4 points)*
4. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." *(8 points)*

Section B (CLO 2, 24 points)

1. Write the type of correspondences in the boxes below. *(1 * 5 points)*



2. Write appropriate mathematical notations in the boxes below. *(0.5 * 6 points)*



3. Write the value of the functions given in the table below. *(1 * 8 points)* 18

Function	Value
$\lfloor 1 \rfloor$	1
$\lceil \frac{1}{2} \rceil$	1
$\lfloor -\frac{1}{2} \rfloor$	-1
$\lceil -\frac{1}{2} \rceil$	0
$\lfloor 3.1 \rfloor$	3
$\lceil 3.1 \rceil$	4
$\lfloor 7 \rfloor$	7
$\lceil 7 \rceil$	7.

4. Find the Fibonacci numbers f_2, f_3, f_4, f_5, and f_6. The initial conditions tell us that $f_0 = 0$ and $f_1 = 1$. *(4 points)*

5. Find $\sum_{k=50}^{100} k^2$. *(4 points)*

Section C (CLO 3, 56 points)

1. The following set of codes correspond to which basic counting principle? Write your answer in the boxes below each set of code. *(4 points)*

```
k := 0
for i1 := 1 to n1
    k := k + 1
for i2 := 1 to n2
    k := k + 1
    .
    .
    .
for im := 1 to nm
    k := k + 1
```

4

```
k := 0
for i1 := 1 to n1
   for i2 := 1 to n2
        .
        .
        .
      for im := 1 to nm
          k := k + 1
```

3

2. A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business? *(6 points)*

3. In a random experiment, 2 balls are withdrawn from an urn containing 2 red balls and 3 white balls, and the sequence of the colors is noted. *(3 + 3 points)*
   a. Find the probability that both the balls are red.
   b. Find the probability that second ball is red.

4. A bag contains fifteen balls distinguishable only by their colors; ten are blue and five are red. I reach into the bag with both hands and pull out two balls (one with each hand) and record their colors. *(2 + 2 + 2 points)*
   a. What is the random phenomenon?
   b. What is the sample space?
   c. Express the event that the ball in my left hand is red as a subset of the sample space.

5. Consider a binary communication channel in which user transmits 1 with a probability p and the channel's error transition probability is ε. *(2 + 3 points)*
   a. Find the probability that a received bit is 1.
   b. Find the probability that a received bit is in error.

6. Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities? *(4 points)*

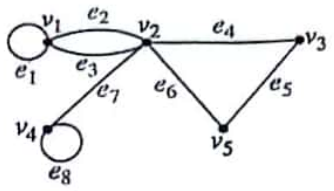
7. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)? *(4 points)*

8. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x - 3y)^{25}$? *(4 points)*

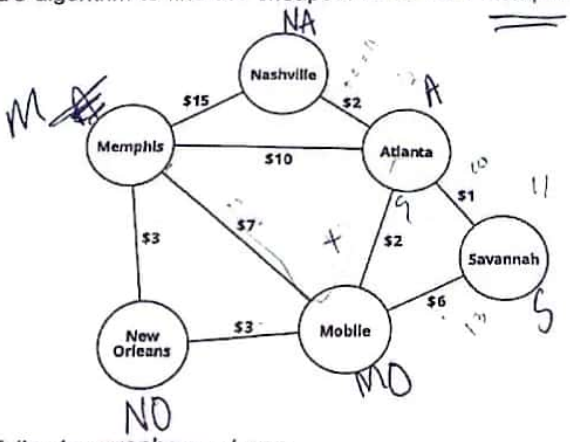
9. Draw the following graphs *(6 points)*
   a. K_7 b. C_7 c. W_7

$\binom{n}{r}(a)^r(b)^{n-r}$

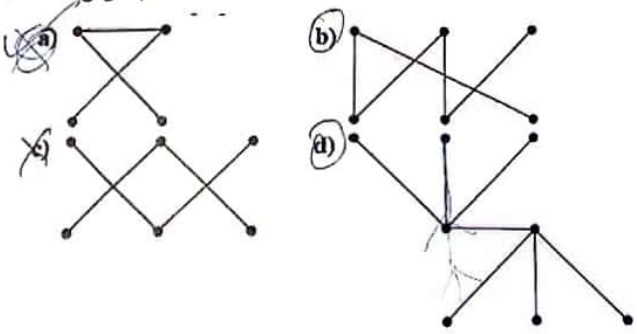
10. Represent the following graph using an incidence matrix. *(3 points)*



11. The following graph represent the travel cost between different cities in the South US. Use Dijkstra's algorithm to find the cheapest route from Memphis to all other cities. *(4 points)*



12. Which of the following graphs are trees. *(1 * 4 points)*



Good Luck!

$\frac{(n)(n+1)(2n+1)}{6}$