BSSE23058 - Zunaira Abdul Aziz.

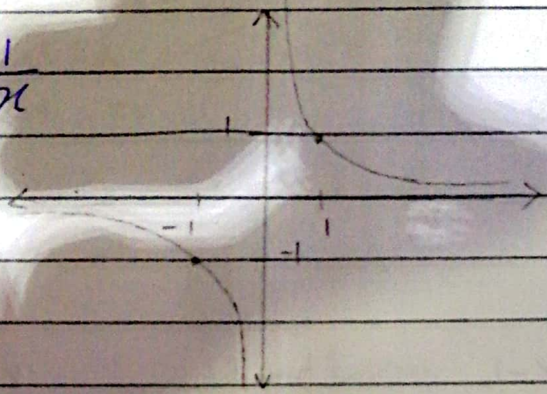
# Assignment #1.

## Calculus & Analytical Geometry.

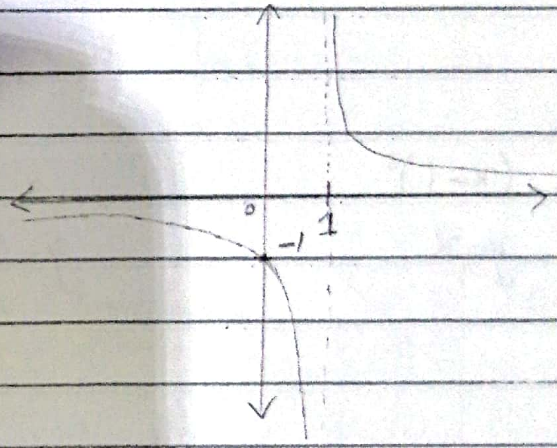
### Problem 1.

a. $y = \frac{1}{x-1}$

$y = \frac{1}{x}$

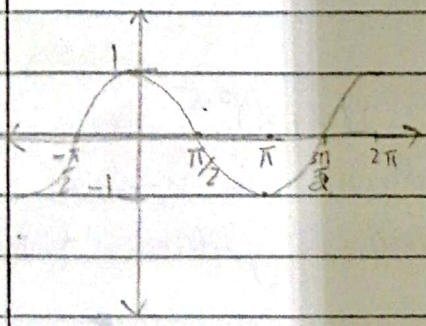



$y = \frac{1}{x-1}$ (horizontal translation 1 unit right)

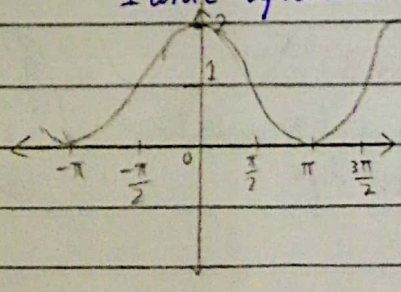



b. $y = \frac{1}{2}(\cos x + 1)$

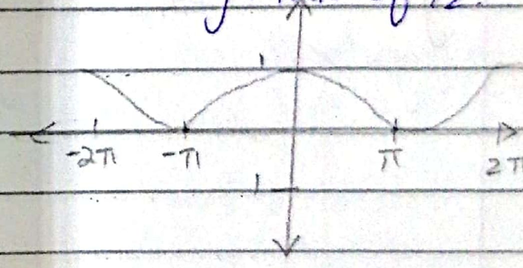
$y = \cos x$




$y = \cos x + 1$
vertical translation 1 unit upwards.

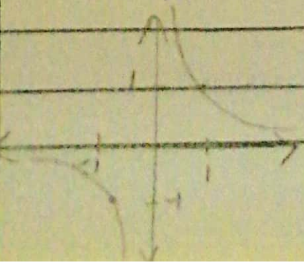



$y = \frac{1}{2}(\cos x + 1)$
squeeze in y direction by a factor of ½.

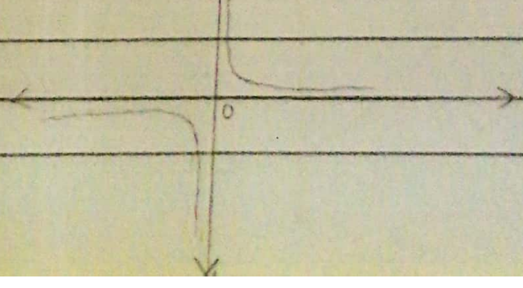



c. $y = \frac{1}{2x} - 3$

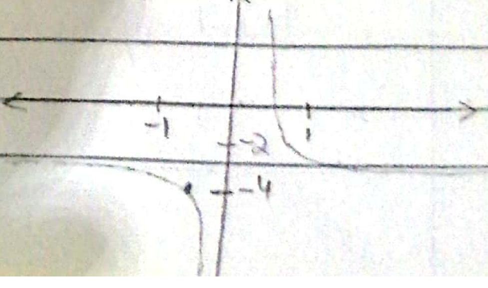
$y = \frac{1}{x}$



$y = \frac{1}{2x}$ squeeze in y-direction factor ½.

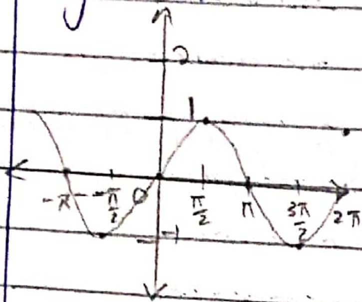



$y = \frac{1}{2x} - 3$ vertical translation. 3 units downward

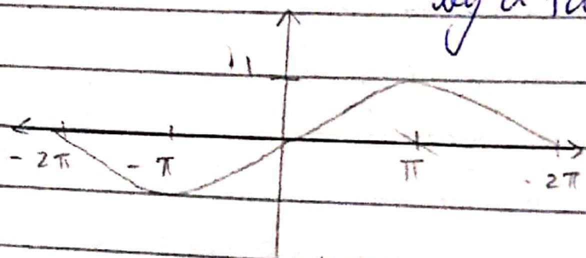


d. $y = \sin \frac{1}{2}x$

$y = \sin x$

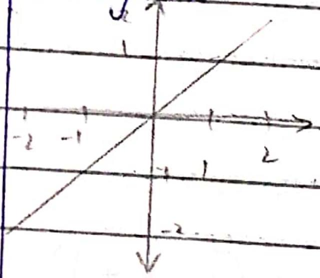


$y = \sin \frac{1}{2}x$ stretch in x-direction by a factor of 2.

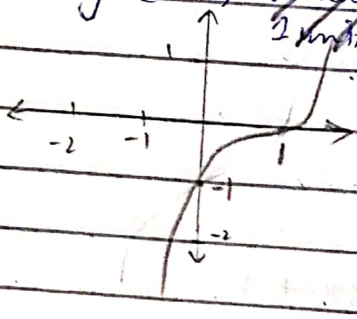


e. $y = (x-1)^3$

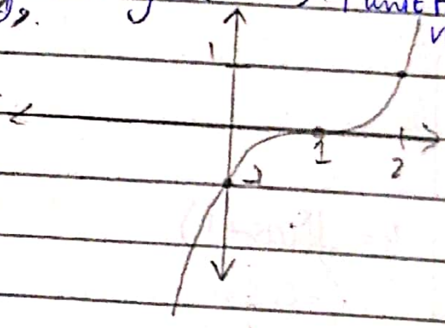
$y = (x)^3$



$y = (x-1)^3$ ~~vertical translation 1 unit downwards.~~

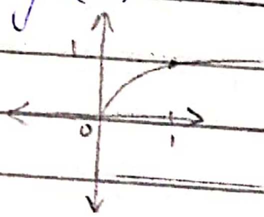


$y = (x-1)^3$ Horizontal translation. 1 unit to right.



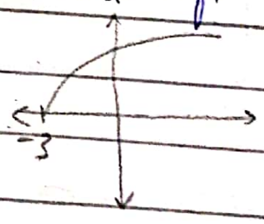
f. $y = 1 - 2\sqrt{x+3}$

$y = (x)^{\frac{1}{2}}$



$y = (x+3)^{0.5}$

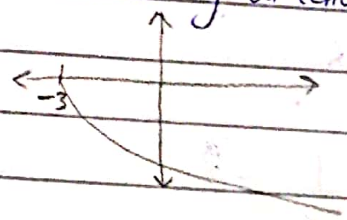
horizontal translation 3 units left



$y = -2(x+3)^{0.5}$

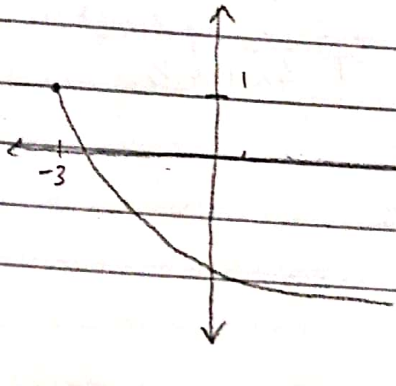
reflection in x-axis.

stretch in y-direction factor = 2



$y = 1 - 2(x+3)^{0.5}$

vertical translation 1 unit upwards.

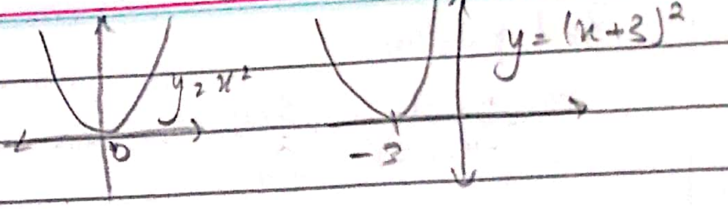


g. $y = x^2 + 6x + 4.$

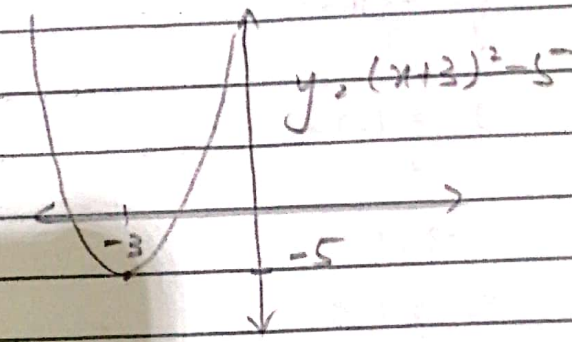
$y = x^2 + 6x + (3)^2 - (3)^2 + 4$

$y = (x+3)^2 - 9 + 4$

$y = (x+3)^2 - 5$

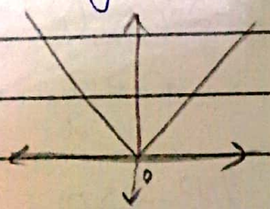



- Standard function $y = x^2$
- Horizontal translation $y = (x+3)^2$
  3 units left
- Vertical translation $y = (3+x)^2 - 5$
  3 units downwards

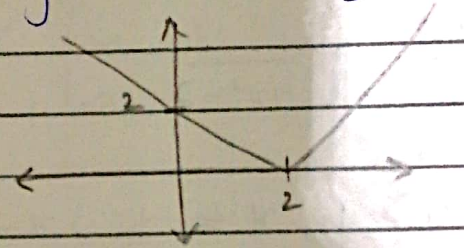



h. $y = |x-2|$

$y = |x|$




$y = |x-2|$ Horizontal translation 2 units to the right.




Problem 2.

a. $$y = \begin{cases} \cancel{-x+3} & \cancel{0 \le x < 3} \\ 2x-6 & 3 \le x \le 5 \end{cases}.$$

1st function

$m = \frac{0-3}{3-0} = -1$

$y = -x + 3$

2nd function.

$m = \frac{4-0}{5-3} = 2$

$y = 2x + c$

$0 = 2(3) + c$

$-6 = c$

$y = 2x - 6$

b. $$y = \begin{cases} \frac{-3}{2}x - 3 & -4 \le x < -2 \\ -\frac{1}{2}(x)^2 + 2. & -2 \le x < 2 \\ \frac{3}{2}x - 3 & 2 \le x \le 4 \end{cases}$$

1st function.

$m = \frac{3-0}{4-2} = \frac{3}{2}$

$y = \frac{3}{2}x + c.$

$0 = \frac{3}{2}(2) + c$

$-3 = c$

$y = \frac{3}{2}x - 3.$

3rd function

$m = \frac{+3-0}{-4-(-2)} = \frac{-3}{2}$

$y = \frac{-3}{2}x + c$

$0 = \frac{-3}{2}(-2) + c$

$-3 = c$

$y = \frac{-3}{2}x - 3$

2nd function

$y = x^2$.

$y = -x^2$ Reflection in y axis

$y = \frac{-1}{2}x^2$ stretch in y-direction & factor = $\frac{1}{2}$

$y = -\frac{1}{2}x^2 + 2$ Vertical translation 2 units upwards.

Problem 3

a. $f(x) = \sqrt{3-2x}$ $g(x) = \sqrt{4x^2-2}$.

$f+g = f(x) + g(x)$.

$= \sqrt{3-2x} + \sqrt{4x^2-2}$

domain of f+g.

$x \geq \frac{1}{\sqrt{2}}, x \leq \frac{-1}{\sqrt{2}}, x \geq \frac{3}{2}$.

$3-2x \geq 0$

$x \geq \frac{3}{2}$

$4x^2-2 \geq 0$

$x \geq \frac{1}{\sqrt{2}}$

$x \leq -\frac{1}{\sqrt{2}}$

b. $f-g = f(x) - g(x)$

$= \sqrt{3-2x} - \sqrt{4x^2-2}$

domain of f-g

$x \geq \frac{3}{2}, \frac{1}{\sqrt{2}}$ or $x \leq \frac{-1}{\sqrt{2}}$

c. $fg = f(x).g(x)$

$= \sqrt{3+2x} \times \sqrt{4x^2-2}$

domain of $f(x).g(x)$.

$x \leq \frac{-1}{\sqrt{2}}$ or $x \geq \frac{1}{\sqrt{2}}, \frac{3}{2}$.

d. $\dfrac{f}{g} = \dfrac{f(x)}{g(x)}$ domain of $\dfrac{f}{g}$ $\{x \mid x \neq \pm\frac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{3+2x}}{4x^2-2}$ denomiator shouldn't be equal to zero.

Problem 4

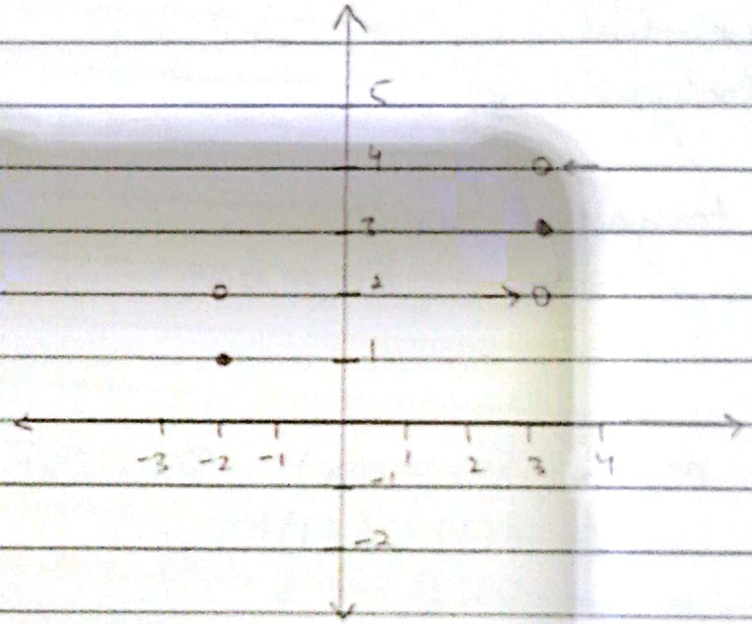
i. $\lim_{x \to 3^+} f(x) = 4$

$\lim_{x \to +3^-} f(x) = 2$

$\lim_{x \to -2} f(x) = 2$

$f(3) = 3$

$f(-2) = 1$




ii. f is discontinous at $-4, -2, 2,$ and $4$.
At $-4$, the function is undefined at $f(-4)$. The $f(-4)$ is continous from both sides.
At $f(-2)$, the function is defined but the left hand & right hand limits are not equal, so regular limit doesnt exist. $-2$ is continous from the left hand side.
At $f(+2)$, function is defined but regular limit doesn't exists. Limit is continous from the right hand side.
At $f(+4)$, the function is defined but regular limit doesn't exist. $f(4)$ is continous from the right hand side.

Problem 5

a. $y = 3 + 4x^2 - 3x^3$

$$\frac{dy}{dx} = \frac{d(3)}{dx} + \frac{d(4x^2)}{dx} - \frac{d(2x^3)}{dx}$$

$$= 0 + 4 \times 2x - 2(3)x^2$$

$$= 0 + 8x \cancel{+} - 6x^2$$

$$\frac{dy}{dx} = 8x - 6x^2$$

slope of tangent at $x = a$

$$m_T = 8a - 6a^2$$

b. point (1, 5)

$$m_T = 8a - 6a^2 = 8x - 6x^2$$

$$= 8(1) - 6(1)^2$$

$$= 8 - 6$$

$$= 2$$

Point (2, 3)

$$m_T = 8(2) - 6(2)^2$$

$$= 16 - 24$$

$$= -8$$

Equation at point (1, 5)

$y = 2x + c$

$5 = 2(1) + c$

$3 = c$

$y = 2x + 3$

Equation at Point (2, 3)

$y = -8x + c$

$\cancel{y}\ 3 = -8(2) + c$

$3 = -16 + c$

$3 + 16 = c$

$19 = c$

Equation at Point (1,5)

$y = \frac{-1}{2}x + c$

$5 = \frac{-1}{2} + c.$

$\frac{5 \times 2 + 1}{2} = c.$

$\frac{11}{2} = c$

$y = \frac{-1}{2}x + \frac{11}{2}.$

Equation at Point (2,3)

$y = \frac{1}{8}x + c.$

$3 = \frac{1}{8}(2) + c.$

$3 \times 4 = 1 + 4c$

$\frac{12 - 1}{4} = c.$

$\frac{11}{4} = c.$

$y = \frac{1}{8}x + \frac{11}{4}.$

Problem 6.

a. $y = 3x \Rightarrow G$
because it is the only linear graph. Satisfies the equation of line. $y = mx + c$. y intercept $= 0$.

b. $y = 3^x \Rightarrow f$
it is an exponential graph. The graph is asymptotic to the x-axis as x approaches $-\infty$. The range is greater than 0. $y > 0$.

c. $y = x^3 \Rightarrow F$
cubic graph. (polynomial) The graph has two turning points.

d. $y = \sqrt[3]{x} \Rightarrow g.$
The graph of $\sqrt[3]{x}$ is a less steep increasing curve than an original cuberoot graph.

Problem 7

a. $\lim_{x \to 2} \frac{x + 2}{x^2 + 5x + 6}.$

b. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

$$\frac{x+2}{x^2+3x+2x+6} = \frac{x+2}{x(x+3)+2(x+3)}$$

$$= \frac{\cancel{x+2}}{(\cancel{x+2})(x+3)}$$

$$= \frac{1}{x+3}$$

$$= \frac{1}{2+3} = \frac{1}{5}$$

$$\frac{x^2-1}{x+1}$$

$$= \frac{(x-1)(\cancel{x+1})}{(\cancel{x+1})}$$

$$= x-1$$

$$= -1-1$$

$$= -2$$

c. $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$

$$\frac{x-1}{\sqrt{x+3}-2} \times \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$\frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4}$$

$$\frac{(\cancel{x-1})(\sqrt{x+3}+2)}{(\cancel{x-1})}$$

$$\sqrt{x+3}+2$$

$$\sqrt{1+3}+2$$

$$\sqrt{4}+2$$

$$4$$

(d) $\lim_{x\to 1} \frac{x^2-1}{x+2}$

$$\frac{(x-1)(x+1)}{(x+2)}$$

$$\frac{(1-1)(1+1)}{(1+2)}$$

$$\frac{(0)(2)}{(3)}$$

$$= 0.$$

e. $\lim_{x\to -1} \frac{x^2+x}{x^2-x-2}$

$$\frac{x(x+1)}{x^2+x-2x-2}$$

$$\frac{x(x+1)}{x(x+1)-2(x+1)}$$

$$\frac{x(\cancel{x+1})}{(x-2)(\cancel{x+1})}$$

f. $\lim_{x\to 2} \frac{x^2-2x}{x^2-4}$

$$\frac{x(\cancel{x-2})}{(\cancel{x-2})(x+2)}$$

$$\frac{x}{x+2}$$

$$\frac{2}{2+2}$$

$\frac{x}{x-2} = \frac{-1}{-1-2}$

$= \frac{+1}{+3}$

$= \frac{1}{3}.$

$= \frac{2}{4}$

$= \frac{1}{2}.$

g. $\lim_{x \to \pi} x \sin x$

$\pi \times \sin \pi$

$\pi \times 0$

$0$

$0 - \sin x - 0$, $1$, $-1$

h. $\lim_{x \to 0} \frac{\tan x}{\sin x}.$

$\frac{\sin x}{\cos x} \div \sin x$

$-1 - \cos x - 1$, $0$, $0$

$\frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$\frac{1}{\cos x} = \frac{1}{\cos 0}$

$= \frac{1}{1}$

$= 1.$

i. $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

$\frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}.$

$\frac{1-\cos^2 x}{x^2(1+\cos x)}$

$\frac{\sin^2 x}{x^2(1+\cos x)}$

$\frac{\sin x}{x} \times \frac{\sin x}{x} \times \frac{1}{1+\cos x}.$

$1 \times 1 \times \frac{1}{2} = \frac{1}{2}.$

j. $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right)$

$0$, $-1 - \cos x - 1$, $0$

$\lim_{x \to 0} (-x^4) = 0^4 = 0.$

$-1 \leq \cos x \leq 1$

$\lim_{x \to 0} (x^4) = 0^4 = 0$

$\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$

due to sandwich theorm.

Problem 8:

a. $f(x) = \dfrac{2x+3}{5x+7}$

$$\lim_{x\to\infty} \frac{\frac{2x}{x}+\frac{3}{x}}{\frac{5x}{x}+\frac{7}{x}} = \frac{2+\frac{3}{\infty}}{5+\frac{7}{\infty}}$$

rule $\frac{1}{\infty} = 0$.

rule $\frac{1}{-\infty} = 0$

$$= \frac{2}{5}$$

$$\lim_{x\to-\infty} \frac{\frac{2x}{x}+\frac{3}{x}}{\frac{5x}{x}+\frac{7}{x}} = \frac{2+\frac{3}{-\infty}}{5+\frac{7}{-\infty}} = \frac{2}{5}$$

b. $f(x) = \dfrac{2x^3+7}{x^3-x^2+x+7}$

$$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{7}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}+\frac{x}{x^3}+\frac{7}{x^3}} = \frac{2+\frac{7}{x^3}}{1-\frac{1}{x}+\frac{1}{x^2}+\frac{7}{x^3}}$$

$$= \frac{2+\frac{7}{\infty^3}}{1-\frac{1}{\infty}+\frac{1}{\infty^2}+\frac{7}{\infty^3}}$$

$$= 2.$$

$$\lim_{x\to-\infty} \frac{2+\frac{7}{(-\infty)^3}}{1-\frac{1}{-\infty}+\frac{1}{(-\infty)^2}+\frac{7}{(-\infty)^3}} = 2.$$

c. $f(x) = \dfrac{x+1}{x^2+3}$ Rule ( Limit at infinity).

$\lim_{x\to\infty} \dfrac{\frac{x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{3}{x^2}} = \dfrac{\frac{1}{\infty} + \frac{1}{\infty^2}}{1 + \frac{3}{\infty^2}}$

$= 0$

$\lim_{x\to-\infty} \dfrac{\frac{x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{3}{x^2}} = \dfrac{\frac{1}{-\infty} + \frac{1}{(-\infty)^2}}{1 + \frac{3}{(-\infty)^2}}$

$= 0$

d. $f(x) = \dfrac{3x+7}{x^2-2}$

$\lim_{x\to\infty} \dfrac{\frac{3x}{x^2} + \frac{7}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \dfrac{\frac{3}{x} + \frac{7}{x^2}}{1 - \frac{2}{x^2}}$

$= 0$

$\lim_{x\to-\infty} \dfrac{\frac{3}{-\infty} + \frac{7}{(-\infty)^2}}{1 - \frac{2}{(-\infty)^2}}$

$= 0$

e. $h(x) = \dfrac{7x^3}{x^3 - 3x^2 + 6x}$

$\lim_{x\to\infty} \dfrac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3} - \frac{3x^2}{x^3} + \frac{6x}{x^3}}$

$= \dfrac{7}{1 - \frac{3}{\infty} + \frac{6}{(+\infty)^2}}$

$= 7$

$\lim_{x\to-\infty} \dfrac{7}{1 - \frac{3}{-\infty} + \frac{6}{(-\infty)^2}}$

$= 7$

f. $g(x) = \dfrac{1}{x^3 - 4x + 1}$

$\lim_{x\to\infty} \dfrac{\frac{1}{x^3}}{\frac{x^3}{x^3} - \frac{4x}{x^3} + \frac{1}{x^3}}$

$\lim_{x\to-\infty} = \dfrac{\frac{1}{(-\infty)^3}}{1 - \frac{4}{(-\infty)^2} + \frac{1}{(-\infty)^3}}$

$$\frac{\frac{1}{(\infty)^3}}{1-\frac{4}{\infty^2}+\frac{1}{\infty^3}} = 0.$$

$$= 0$$

g. $g(x) = \dfrac{10x^5 + x^4 + 31}{x^6}$

$$\lim_{x\to\infty} \frac{\frac{10x^5}{x^6} + \frac{x^4}{x^6} + \frac{31}{x^6}}{\frac{x^6}{x^6}}$$

$$= \frac{10}{\infty} + \frac{1}{\infty^2} + \frac{31}{\infty}$$

$$= 0 + 0 + 0$$

$$= 0$$

$$\lim_{x\to-\infty} \frac{\frac{10}{x} + \frac{1}{x^2} + \frac{31}{x^6}}{1}$$

$$\frac{10}{-\infty} + \frac{1}{(-\infty)^2} + \frac{31}{(-\infty)^6}$$

$$= 0$$

h. $h(x) = \dfrac{9x^4 + x}{2x^4 + 5x^2 - x + 6}.$

$$\lim_{x\to\infty} \frac{\frac{9x^4}{x^4} + \frac{x}{x^4}}{\frac{2x^4}{x^4} + \frac{5x^2}{x^4} - \frac{x}{x^4} + \frac{6}{x^4}}$$

$$\frac{9 + \frac{1}{(\infty)^3}}{2 + \frac{5}{(\infty)^2} - \frac{1}{(\infty)^3} + \frac{6}{(\infty)^4}}$$

$$= \frac{9}{2}$$

$$\lim_{x\to-\infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}}$$

$$= \frac{9 + \frac{1}{(-\infty)^3}}{2 + \frac{5}{(-\infty)^2} - \frac{1}{(-\infty)^3} + \frac{6}{(-\infty)^4}}$$

$$= \frac{9}{2}.$$

i. $h(x) = \dfrac{-2x^3 - 2x + 3}{3x^3 + 3x^2 - 5x}$

$$\lim_{x\to\infty} \frac{\frac{-2x^3}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}{\frac{3x^3}{x^3} + \frac{3x^2}{x^3} - \frac{5x}{x^3}}$$

$$\frac{-2 - \frac{2}{(\infty)^2} + \frac{3}{(\infty)^3}}{3 + \frac{3}{\infty} - \frac{5}{(\infty)^2}}$$

$$= -\frac{2}{3}$$

$$\lim_{x\to-\infty} \frac{-2 - \frac{2}{x^2} + \frac{3}{x^3}}{3 + \frac{3}{x} - \frac{5}{x^2}}$$

$$= \frac{-2 - \frac{2}{(-\infty)^2} + \frac{3}{(-\infty)^3}}{3 + \frac{3}{\infty} - \frac{5}{(-\infty)^2}}$$

$$= -\frac{2}{3}.$$

j. $h(x) = \dfrac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$

$$\lim_{x\to\infty} \frac{-\frac{x^4}{x^4}}{\frac{x^4}{x^4} - \frac{7x^3}{x^4} + \frac{7x^2}{x^4} + \frac{9}{x^4}}$$

$$\frac{-1}{1 - \frac{7}{x} + \frac{7}{x^2} + \frac{9}{x^4}}$$

$$\frac{-1}{1 - \frac{7}{\infty} + \frac{7}{\infty^2} + \frac{9}{\infty^4}}$$

$$\frac{-1}{1 + 0 - 0 + 0}$$

$$= -1$$

$$\lim_{x\to-\infty} \frac{-1}{1 - \frac{7}{x} + \frac{7}{x^2} + \frac{9}{x^4}}$$

$$\frac{-1}{1 - \frac{7}{(-\infty)} + \frac{7}{(-\infty)^2} + \frac{9}{(-\infty)^4}}$$

$$\frac{-1}{1 - 0 + 0 + 0}$$

$$\frac{-1}{1}$$

$$= -1$$