

Example 2.1 MM 5ed

- Simplify the boolean functions:

1. $x(x' + y)$
2. $x + x'y$
3. $(x + y)(x + y')$
4. $xy + x'z + yz$
5. $(x + y)(x' + z)(y + z)$

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

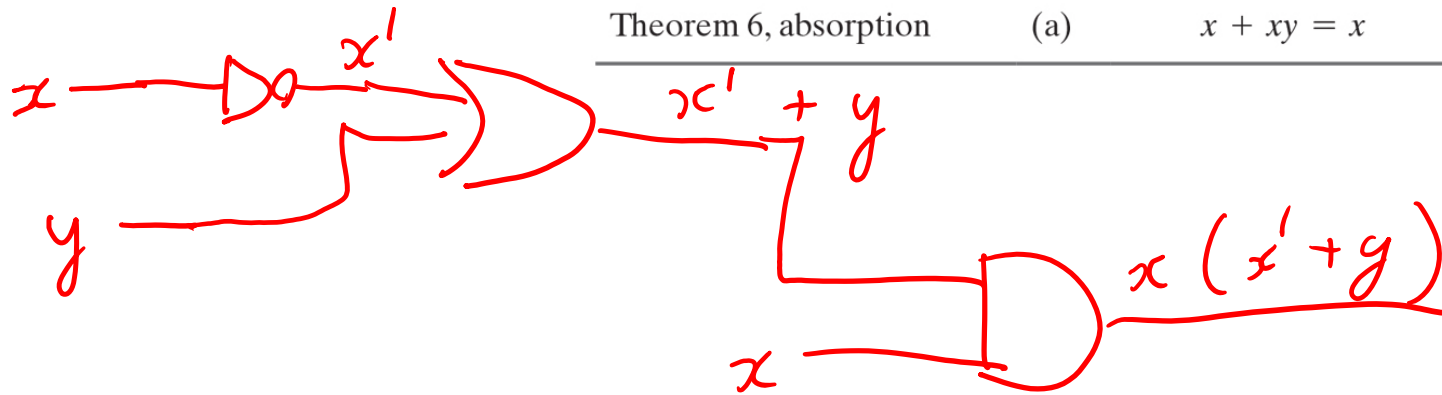
$$x(x' + y)$$

$$\downarrow$$

$$x \cdot x' + xy$$

$$\downarrow$$

$$0 \quad xy$$



2 gates

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$$x + x'y$$

$$(x + x')(x + y)$$

$$1(x + y)$$

$$x + y$$

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$$x + y = z$$

$$(x + y)(x + y')$$

$$(x + y) \cdot x + (x + y) \cdot y'$$

$$x \cdot x + x \cdot y + y' \cdot x + y' \cdot y$$

$$x + y' \cdot x + y' \cdot y$$

$$x + y' \cdot x$$

$$\Downarrow$$

$$x$$

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$$x + xy + y' \cdot x$$

$$x + x(y + y')$$

$$x + x(1)$$

$$\Downarrow$$

$$x$$

$$xy + x'z + yz$$

$$xy + x'z + yz(x + x')$$

$$xy + x'z + xyz + x'yz$$

$$xy(1 + z) + x'z(1 + y)$$

$$xy \cdot 1 + x'z \cdot 1$$

$$xy + x'z$$

Postulate 2

(a)

$$x + 0 = x$$

(b)

$$x \cdot 1 = x$$

Postulate 5

(a)

$$x + x' = 1$$

(b)

$$x \cdot x' = 0$$

Theorem 1

(a)

$$x + x = x$$

(b)

$$x \cdot x = x$$

Theorem 2

(a)

$$x + 1 = 1$$

(b)

$$x \cdot 0 = 0$$

Theorem 3, involution

$$(x')' = x$$

Postulate 3, commutative

(a)

$$x + y = y + x$$

(b)

$$xy = yx$$

Theorem 4, associative

(a)

$$x + (y + z) = (x + y) + z$$

(b)

$$x(yz) = (xy)z$$

Postulate 4, distributive

(a)

$$x(y + z) = xy + xz$$

(b)

$$x + yz = (x + y)(x + z)$$

Theorem 5, DeMorgan

(a)

$$(x + y)' = x'y'$$

(b)

$$(xy)' = x' + y'$$

Theorem 6, absorption

(a)

$$x + xy = x$$

(b)

$$x(x + y) = x$$

$$yz \cdot 1 = yz$$

$$yz(x + x')$$

$$(x + y)(x' + z)(y + z) = (x + y)(x' + z)$$

Ex 1



$$x \cdot y + x' \cdot z + y \cdot z$$

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$$xy + x'z + yz = xy + x'z$$

$$(x+y)(x'+z)(y+z) = (x+y) \cdot (x'+z)$$

Example 2.2 MM 5ed

- Find the complement of functions $F_1 = (x'yz' + x'y'z)$, $F_2 = x(y'z' + yz)$

$$\begin{aligned} F_1' &= (x'yz' + x'y'z)' \\ &= (x'yz')' \cdot (x'y'z)' \\ &= (x + y' + z) \cdot (x + y + z') \end{aligned}$$