Information Technology University, Lahore, Pakistan

Linear Algebra (MT-121)

Assignment # 4, Fall 2024

Submission Deadline: Friday October 04, 2024 Maximum Marks: 100

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1. (a) Find an LU decomposition of the matrix [8]

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

   (b) Exapress A in the form $A = LDU$. [2]

2. Consider the following matrix.

$$A = \begin{bmatrix} 0 & 0 & 6 & 2 & -4 & -8 \\ 0 & 0 & 3 & 1 & -2 & -4 \\ 2 & -3 & 1 & 4 & -7 & 1 \\ 6 & -9 & 0 & 11 & -19 & 3 \end{bmatrix}$$

   (a) Bring A in row echelon form. [4]

   (b) Find the rank of A. [2]

   (c) Find the null space of A. [4]

3. Consider the system of equations

$$\begin{aligned} x + 2y + z &= b \\ 2x + y + 2z &= 2 \\ 3x + 3y + az &= 3 \end{aligned}$$

   (a) For which values of a and b, does this system has a unique solution (if any)? Give solution for any such values of a and b. [4]

   (b) For which values of a and b, does this system has no solution? [3]

   (c) For which values of a and b, does this system has infinitely many solutions? [3]

4. For which vectors $\mathbf{b}$, the system has a solution?

   (a) [5]

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (b) [5]

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

5. Given the matrices A and B as

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

   (a) For each of A and B, compute the rank and decide whether the matrix is invertible. If so, find the inverse. [4]

   (b) Find a vector in b in $\mathbb{R}^3$ for which $\mathbf{Ax} = \mathbf{b}$ has many solutions, or explain why the existence of such a vector is impossible. [3]

   (c) Find a vector in b in $\mathbb{R}^3$ for which $\mathbf{Bx} = \mathbf{b}$ has many solutions, or explain why the existence of such a vector is impossible. [3]

6. Reduce $\mathbf{Ax} = \mathbf{b}$ to $\mathbf{Ux} = \mathbf{c}$ (Gaussian Elimination) and then to $\mathbf{Rx} = \mathbf{d}$ (Gauss-Jordan) if

$$\text{A=} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 0 & 4 & 9 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix}$$

   Find a particular solution x_p and all homogeneous solutions x_n. [10]

7. Describe the subspace of $\mathbb{R}^3$ (is it a line, or plane or $\mathbb{R}^3$?) spanned by
   (a) the two vectors (1, 1, -1) and (-1, -1, 1). [2]
   (b) the three vectors (0, 1, 1), (1, 1, 0) and (0, 0, 0). [2]
   (c) all vectors in $\mathbb{R}^3$ with whole number components. [3]
   (d) all vectors with positive components. [3]

8. (a) Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent vectors. [5]
   $$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$
   Hint: Solve $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$ or $\mathbf{Ax} = \mathbf{0}$.
   (b) If w_1, w_2, w_3 are independent vectors, show that v_1, v_2, v_3 are also independent where $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$ and $v_3 = w_1 + w_2$. [5]
   Hint: Write $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$ in terms of the w's. (Find and solve equations for the c's to show they are zero).

9. Given the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 3 & 5 & 7 & -1 \\ 1 & 4 & 2 & 7 \end{bmatrix}$$

   (a) Find the linearly independent columns of A. [2]
   (b) Write the basis set of the column space of A and its dimension. [2]
   (c) Write the basis set of the null space of A and its dimension. [2]
   (d) Write the basis set of the row space of A and its dimension. [2]
   (e) Write the basis set of the left null space of A and its dimension. [2]

10. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors.
   (a) (1, 1, -4, -3), (2, 0, 2, -2), (2, -1, 3, 2) [5]
   (b) (1, 1, 0, 0), (0, 0, 1, 1), (-2, 0, 2, 2), (0, -3, 0, 3) [5]