Aziz
BSSE23058

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Assignment # 5
Linear Algebra.

Q1 $\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\text{proj}_a \vec{b} = \frac{a^T b}{a^T a} \vec{a}$$

$$= \frac{\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \frac{3+8}{9+16} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \frac{11}{25} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 33/55 \\ 44/55 \end{bmatrix} = \begin{bmatrix} \frac{33}{55} \\ \frac{44}{55} \end{bmatrix}$$

Q2 $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$P = A(A^T A)^{-1} A^T$$

$$A^T A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{2\times2 - 1\times1} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$A(A^T A)^{-1} A^T = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}}_{3\times2} \underbrace{\begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{2\times3}$$

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$$= \begin{bmatrix} 2/3 & -1/3 \\ 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 3$

$$P = \begin{bmatrix} 2/3 & 1/3 & -1/3 \\ 1/3 & 2/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix}$$

$3 \times 3.$

Q3 a. $\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$

$$\text{proj}_w \vec{x} = \frac{w^T x}{w^T w} \vec{w}$$

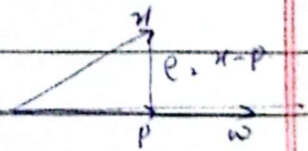
$$= \frac{\begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{2-2+6}{1+4+4} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{6}{9} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{2}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 \\ 4/3 \\ 4/3 \end{bmatrix}.$$



b. $e = b - p$

$$e = x - \text{proj}_w \vec{x}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 4/3 \\ 4/3 \end{bmatrix}$$

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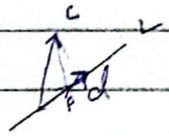
$$e = \begin{bmatrix} 4/3 \\ -7/3 \\ 5/3 \end{bmatrix}$$

$$\|e\| = \sqrt{(4/3)^2 + (-7/3)^2 + (5/3)^2}$$
$$= \sqrt{\frac{16}{9} + \frac{49}{9} + \frac{25}{9}}$$
$$= \sqrt{90/9}$$
$$= \sqrt{10}$$
$$= 3.16$$

Q4 a. $\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\vec{d} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ on L



$$\text{proj}_d \vec{c} = \frac{\vec{d}^T \vec{c}}{d^T d} \vec{d}$$
$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \frac{4+3}{1+1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \frac{7}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 7/2 \end{bmatrix}$$

b. When we project a vector onto a line in an orthogonal way it means we are dropping the vector straight down onto a line at a 90° angle. ~~Imagi~~ This is the shortest distance from the vector to the line. For example in first part vector c drops onto the line L at the point $P = \begin{bmatrix} 7/2 \\ 7/2 \end{bmatrix}$. This point represents the part of $\vec{c}$ that lines up with the direction of L.

Q5
$$2x + 3y = 5$$
$$4x + y = 6$$
$$x - y = 2.$$

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a. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & -1 \end{bmatrix}$. $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}$.

$$Ax \approx b$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}.$$

b. Multiply A^T on both sides to solve this. under-mined problem.

$$Ax \approx b$$

$$A^T A \hat{x} = A^T b$$

$$\underset{2\times 3}{\begin{bmatrix} 2 & 4 & 1 \\ 3 & 1 & -1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & -1 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \underset{2\times 3}{\begin{bmatrix} 2 & 4 & 1 \\ 3 & 1 & -1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}}$$

$$\begin{bmatrix} 21 & 9 \\ 9 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 36 \\ 19 \end{bmatrix}$$

c. $\left[\begin{array}{cc|c} 21 & 9 & 36 \\ 9 & 11 & 19 \end{array}\right]$ $R2 \leftarrow \left(\frac{-3}{4}\right) R1 + R2$

$$\left[\begin{array}{cc|c} 21 & 9 & 36 \\ 0 & \frac{50}{7} & \frac{25}{7} \end{array}\right]$$

$\frac{50}{7} y = \frac{25}{7}$ $21x + 9y = 36$

$y = \frac{25}{50}$ $21x + 9\left(\frac{1}{2}\right) = 36$

$42x + 9 = 72$

$y = 1/2$ $x = \frac{63}{42}$

$x = \begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$ $x = 3/2$

Q6 a. $(1, 2)$ $(2, 3)$ $(3, 5)$

$y = mx + c.$

$2 = m + c$ $3 = 2m + c$ $5 = 3m + c$

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$m + c = 2$

$2m + c = 3$

$3m + c = 5$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$ $x = \begin{bmatrix} m \\ c \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$m > n$ (No solution)

b. $A^T A x = A^T b$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 23 \\ 10 \end{bmatrix} \quad R2 \leftarrow R2 + R1\left(-\frac{3}{7}\right)$$

c. $14m + 6c = 23$

$6m + 3c = 10$

$$\left[\begin{array}{cc|c} 14 & 6 & 23 \\ 6 & 3 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 14 & 6 & 23 \\ 0 & 3/7 & 1/7 \end{array}\right]$$

$\frac{3}{7}\cancel{y}c = \frac{1}{7}$

$3c = 1$

$c = \frac{1}{3}$

$14m + 6c = 23$

$14m + 6\left(\frac{1}{3}\right) = 23$

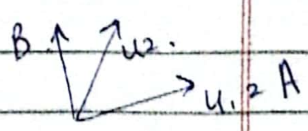
$14m + 2 = 23$

$m = \frac{23-2}{14}$

$m = 3/2$

$$y = \frac{3}{2}x + \frac{1}{3}$$

Q2 a. $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\vec{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$



Make orthogonal vectors A & B.

$\vec{u}_1 = A$ $B = u_2 - \text{proj}_{u_1} u_2$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{u_1^T u_2}{u_1^T u_1}\vec{u}_1 =$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

Make it into orthonormal basis. q_1 & q_2

$$q_1 = \frac{A}{\|A\|} \qquad q_2 = \frac{B}{\|B\|}$$

$$= \frac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{(1)^2+(1)^2+(0)^2}} \qquad = \frac{\begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix}}{\sqrt{(1/2)^2+(-1/2)^2+(1)^2}}$$

$$= \frac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{2}} \qquad = \frac{\begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix}}{\sqrt{3/2}}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \qquad = \begin{bmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{1}{\sqrt{3/2}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix} \qquad Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \cdots \\ \frac{1}{\sqrt{2}} & \cdots \\ 0 & \cdots \end{bmatrix}$$

b. Orthogonal. $q_1 \cdot q_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{6}}{6} + \frac{1}{\sqrt{2}} \times -\frac{\sqrt{6}}{6} + 0 \times \frac{\sqrt{6}}{3}$$

$$= \frac{\sqrt{6}}{12} - \frac{\sqrt{6}}{12} + 0 = 0.$$ Hence orthogonal.

Normalized. $\|q_1\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(+\frac{1}{\sqrt{2}}\right)^2 + (0)^2}$

$= \sqrt{1}$

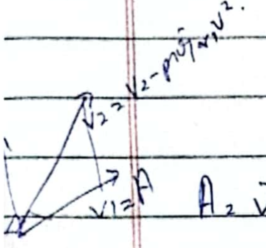
$= 1.$

$\|q_2\| = \sqrt{\left(\frac{\sqrt{6}}{6}\right)^2 + \left(-\frac{\sqrt{6}}{6}\right)^2 + \left(\frac{\sqrt{6}}{3}\right)^2}$

$= \sqrt{1} = 1.$

$\|q_1\|$ and $\|q_2\|$ are hence normalized.

Q8 a. $\vec{V_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\vec{V_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{V_3} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$



$A = \vec{V_1}$ $\quad B = \vec{V_2} - proj_{v_1} V^2$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{V_1^T V_2}{V_1^T V_1} \vec{V_1}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

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$$C = \vec{V_3} - \text{proj}_{v_1} v_3 - \text{proj}\, v^3$$

$$= \vec{V_3} - \frac{V_1^T V_3}{V_1^T V_1}\vec{V_1} - \frac{B^T V_3}{B^T B}\vec{B}$$

$$= \begin{bmatrix}0\\1\\1\end{bmatrix} - \frac{[1\,0\,1]\begin{bmatrix}0\\1\\1\end{bmatrix}}{[1\,0\,1]\begin{bmatrix}1\\0\\1\end{bmatrix}}\begin{bmatrix}1\\0\\1\end{bmatrix} - \frac{[0\,1\,0]\begin{bmatrix}0\\1\\1\end{bmatrix}}{[0\,1\,0]\begin{bmatrix}0\\1\\0\end{bmatrix}}\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\1\\1\end{bmatrix} - \frac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix} - \frac{1}{1}\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\1\\1\end{bmatrix} - \begin{bmatrix}1/2\\0\\1/2\end{bmatrix} - \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}-1/2\\0\\1/2\end{bmatrix}.$$

$$q_1 = \frac{A}{\|A\|} \qquad q_2 = \frac{B}{\|B\|} \qquad q_3 = \frac{C}{\|C\|}$$

$$= \frac{\begin{bmatrix}1\\0\\1\end{bmatrix}}{\sqrt{1^2+0^2+1^2}} \qquad = \frac{\begin{bmatrix}0\\1\\0\end{bmatrix}}{\sqrt{0^2+1^2+0^2}} \qquad = \frac{\begin{bmatrix}-1/2\\0\\1/2\end{bmatrix}}{\sqrt{(-\frac{1}{2})^2+(0)^2+(\frac{1}{2})^2}}$$

$$= \frac{\begin{bmatrix}1\\0\\1\end{bmatrix}}{\sqrt{2}} \qquad = \frac{\begin{bmatrix}0\\1\\0\end{bmatrix}}{\sqrt{1}} \qquad = \frac{\begin{bmatrix}-1/2\\0\\1/2\end{bmatrix}}{1/\sqrt{2}}$$

$$= \begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\1/\sqrt{2}\end{bmatrix}. \qquad = \begin{bmatrix}0\\1\\0\end{bmatrix}. \qquad = \begin{bmatrix}-\frac{\sqrt{2}}{2}\\0\\\frac{\sqrt{2}}{2}\end{bmatrix}$$

$$Q = \begin{bmatrix}\frac{1}{\sqrt{2}} & 0 & -\frac{\sqrt{2}}{2}\\ 0 & 1 & 0\\ \frac{1}{\sqrt{2}} & 0 & \frac{\sqrt{2}}{2}\end{bmatrix}.$$

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b. The Gram-Schmidt process takes a set of vectors & makes them orthogonal by removing overlapping parts the projections with previous vectors. This keeps each new vector perpendicular to the others. Afterward, each vector is scaled to have a length of 1, making them orthonormal. The process doesn't change the space they cover, so the new set spans the same subspace as the original vectors. This gives an orthonormal basis for the subspace.

Q9

$$A = \begin{array}{cc} \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \\ u_1 \quad u_2 \end{array}.$$

a. $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad \vec{u}_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}.$

$A = \vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$B = \vec{u_2} - \text{proj}_A \vec{u_2}$$

$$= \vec{u_2} - \frac{A^T \vec{u_2}}{A^T A} \vec{A}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} - \frac{[1\ 1\ 0]\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}}{[1\ 1\ 0]\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2/2 \\ 2/2 \\ 0 \end{bmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

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$q_1 = \dfrac{A}{\|A\|}$ $\qquad$ $q_2 = \dfrac{B}{\|B\|}$

$= \dfrac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{1^2+1^2+0^2}}$ $\qquad$ $= \dfrac{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}}{\sqrt{1^2+(-1)^2+2^2}}$

$= \dfrac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{2}}$ $\qquad$ $= \dfrac{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}}{\sqrt{6}}$

$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$ $\qquad$ $= \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix}$

$$Q = \begin{bmatrix} q_1 & q_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{6}}{3} \end{bmatrix}.$$

b. $Q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{6}}{3} \end{bmatrix}.$$

$R = Q^T A$

$$= \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \end{bmatrix}}_{2\times 3} \cdot \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}_{3\times 2}$$

$$= \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{6} \end{pmatrix}$$

$$A = QR$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{6}}{3} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{6} \end{bmatrix}$$

3×2 $\quad$ 2×2

Q10 a.

$$A = \begin{bmatrix} 1 & 1 & 8 \\ 1 & 0 & 2 \\ 4 & 4 & 0 \\ 2 & 8 & 6 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix} \quad c = \begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix}$$

$$X = a = \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} \qquad Y = b - proj_x b.$$

$$= \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix} - \frac{X^T b}{X^T X} \vec{X}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix} - \frac{[1\ \cancel{1}\ 4\ 2] \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix}}{[1\ 1\ 4\ 2] \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix} - \frac{33}{22} \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 \\ 0 \\ 4 \\ 8 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 3/2 \\ 6 \\ 3 \end{bmatrix}.$$

$$= \begin{bmatrix} -1/2 \\ -3/2 \\ -2 \end{bmatrix}$$

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$$Z = \vec{c} - \text{proj}_x \vec{c} - \text{proj}_y \vec{c}$$

$$= \vec{c} - \frac{x^T \vec{c}}{x^T x}\vec{x} - \frac{y^T \vec{c}}{y^T y}\vec{y}$$

$$= \begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix} - \frac{[1\ 1\ 4\ 2]\begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix}}{[1\ 1\ 4\ 2]\begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}}\begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} - \frac{[-\frac{1}{2}\ -\frac{3}{2}\ -2\ 5]\begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix}}{[-\frac{1}{2}\ -\frac{3}{2}\ -2\ 5]\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}}\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix} - \frac{22}{22}\begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} - \frac{23}{63/2}\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} - \frac{46}{63}\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 2 \\ 0 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix} - \begin{bmatrix} -\frac{23}{63} \\ -\frac{23}{21} \\ -\frac{92}{63} \\ \frac{230}{63} \end{bmatrix} = \begin{bmatrix} \frac{464}{63} \\ \frac{44}{21} \\ -\frac{160}{63} \\ \frac{22}{63} \end{bmatrix}$$

$$q_1 = \frac{X}{\|X\|} \qquad q_2 = \frac{Y}{\|Y\|} \qquad q_3 = \frac{Z}{\|Z\|}$$

$$= \frac{\begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}}{\sqrt{1^2+1^2+4^2+2^2}} \qquad = \frac{\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}}{\sqrt{(-\frac{1}{2})^2+(-\frac{3}{2})^2+(-2)^2+(5)^2}} \qquad = \frac{\begin{bmatrix} \frac{464}{63} \\ \frac{44}{21} \\ -\frac{160}{63} \\ \frac{22}{63} \end{bmatrix}}{\frac{2\sqrt{7189}}{21}}$$

$$= \frac{\begin{bmatrix} 1 \\ 1 \\ 4 \\ 2 \end{bmatrix}}{\sqrt{22}} \qquad \frac{\begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ -2 \\ 5 \end{bmatrix}}{\frac{3\sqrt{14}}{2}}$$

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$$q_1 = \begin{bmatrix} \frac{\sqrt{22}}{22} \\ \frac{\sqrt{22}}{22} \\ \frac{2\sqrt{22}}{11} \\ \frac{\sqrt{22}}{11} \end{bmatrix} \qquad q_2 = \begin{bmatrix} -\frac{\sqrt{14}}{42} \\ -\frac{\sqrt{14}}{14} \\ -\frac{2\sqrt{14}}{21} \\ \frac{5\sqrt{14}}{21} \end{bmatrix} \qquad q_3 = \begin{bmatrix} \frac{232\sqrt{7189}}{21567} \\ \frac{22\sqrt{7189}}{7189} \\ -\frac{80\sqrt{7189}}{21567} \\ \frac{11\sqrt{7189}}{21567} \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{\sqrt{22}}{22} & -\frac{\sqrt{14}}{42} & \frac{232\sqrt{7189}}{21567} \\ \frac{\sqrt{22}}{22} & -\frac{\sqrt{14}}{14} & \frac{22\sqrt{7189}}{7189} \\ \frac{2\sqrt{22}}{11} & -\frac{2\sqrt{14}}{21} & -\frac{80\sqrt{7189}}{21567} \\ \frac{\sqrt{22}}{11} & \frac{5\sqrt{14}}{21} & \frac{11\sqrt{7189}}{21567} \end{bmatrix}$$

$$\frac{22\sqrt{7189}\times 3}{7189\times 3} = \frac{66\sqrt{7189}}{21567}$$

$$R = Q^T A$$

$$= \begin{bmatrix} \frac{\sqrt{22}}{22} & \frac{\sqrt{22}}{22} & \frac{2\sqrt{22}}{11} & \frac{\sqrt{22}}{11} \\ -\frac{\sqrt{14}}{42} & -\frac{\sqrt{14}}{14} & -\frac{2\sqrt{14}}{21} & \frac{5\sqrt{14}}{21} \\ \frac{232\sqrt{7189}}{21567} & \frac{22\sqrt{7189}}{7189} & -\frac{80\sqrt{7189}}{21567} & \frac{11\sqrt{7189}}{21567} \end{bmatrix} \begin{bmatrix} 1 & 1 & 8 \\ 1 & 0 & 2 \\ 4 & 4 & 0 \\ 2 & 8 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{22} & \frac{3\sqrt{22}}{2} & \sqrt{22} \\ 0 & \frac{3\sqrt{14}}{2} & \frac{23\sqrt{14}}{21} \\ 0 & 0 & \frac{2\sqrt{7189}}{21} \end{bmatrix}$$

b. $A = QR$

$$= \begin{bmatrix} \frac{\sqrt{22}}{22} & -\frac{\sqrt{14}}{42} & \frac{232\sqrt{7189}}{21567} \\ \frac{\sqrt{22}}{22} & -\frac{\sqrt{14}}{14} & \frac{22\sqrt{7189}}{7189} \\ \frac{2\sqrt{22}}{11} & -\frac{2\sqrt{14}}{21} & -\frac{80\sqrt{7189}}{21567} \\ \frac{\sqrt{22}}{11} & \frac{5\sqrt{14}}{21} & \frac{11\sqrt{7189}}{21567} \end{bmatrix} \begin{bmatrix} \sqrt{22} & \frac{3\sqrt{22}}{2} & \sqrt{22} \\ 0 & \frac{3\sqrt{14}}{2} & \frac{23\sqrt{14}}{21} \\ 0 & 0 & \frac{2\sqrt{7189}}{21} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 1 & 0 & 2 \\ 4 & 4 & 0 \\ 2 & 8 & 6 \end{bmatrix}$$

Correct got A. back.