Jumana Abdul Aziz
BSSE23058-A

Assignment # 2
Linear Algebra.

Q1

a. $$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ y+z \end{bmatrix}$$

$ax + by + cz = x$
$(1)x + (0)y + (0)z = x$
$x + 0 + 0 = x$
$x = x$

$dx + ey + fz = y$
$(0)x + (1)y + (0)z = y$
$0 + y + 0 = y$
$y = y$

$gx + hy + iz = y + z$
$(0)x + (1)y + (1)z = y + z$
$0 + y + z = y + z$
$y + z = y + z$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ y+z \end{bmatrix}.$$

b. $$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z-x \end{bmatrix}$$

$ax + by + cz = x$
$(1)x + (0)y + (0)z = x$
$x + 0 + 0 = x$
$x = x$

$dx + ey + fz = y$
$0(x) + (1)y + (0)z = y$
$0 + y + 0 = y$
$y = y$

$gx + hy + iz = z - x$
$(-1)x + (0)y + (1)z = z - x$
$-x + 0 + z = z - x$
$z - x = z - x$

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$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z-x \end{bmatrix}$$

c. $$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 8 & 1 & 3 \\ 15 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 3 \\ 7 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

$a = 1$ $b = 0$ $c = 0$ as the first row is the same, no change.

$8a + 15b + 2c = 8 \Rightarrow 8(1) + 15(0) + 2(0) = 8$

$a + 2b + 2c = 1 \Rightarrow (1) + 2(0) + 2(0) = 1$

$3a + 5b + 0c = 3 \Rightarrow 3(1) + 5(0) + 0 = 3$

$g = 0$ $h = 0$ $i = 1$ as the last row is the same.

$8g + 15h + 2i = 2 \Rightarrow 8(0) + 15(0) + 2(1) = 2$

$g + 2h + 2i = 2 \Rightarrow 0 + 2(0) + 2(1) = 2$

$3g + 5h + i(0) = 0 \Rightarrow 3(0) + 5(0) + 1(0) = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 1 & 3 \\ 15 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 3 \\ 7 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

$8d + 15e + 2f = 7$	$d + 2e + 2f = 1$	$3d + 5e + 0f = 2$
$8\left(\frac{2-5e}{3}\right) + 15e + 2f = 7$	$\frac{2-5e}{3} + 2e + 2f = 1$	$3d = 2 - 5e$ $d = \frac{2-5e}{3}$
$16 - 40e + 45e + 6f = 21$	$2 - 5e + 6e + 6f = 3$	

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$16 + 5e + 6f = 21$ $\quad$ $2 + e + 6f = 3$ $\quad$ $d = \frac{2 - 5e}{3}$

$5e + 6f = 5$ $\quad$ $e + 6f = 1$

$$5e + 6f = 5$$
$$-(e + 6f = 1)$$
$$5e - e = 5 - 1$$
$$4e = 4$$
$$e = 1$$

$$d = \frac{2 - 5(1)}{3}$$
$$d = \frac{-3}{3}$$
$$d = -1.$$

$$8d + 15e + 2f = 7$$
$$8(-1) + 15(1) + 2f = 7$$
$$7 + 2f = 7$$
$$f = \frac{7 - 7}{2}$$
$$f = 0.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 1 & 3 \\ 15 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 3 \\ 7 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

d. $$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -4 \\ 0 & -2 & -6 \end{bmatrix}$$

$a = 1$ $\quad$ $b = 0$ $\quad$ $c = 0$ $\quad$ no change in 1st row

$a + 3b + 2c = 1$ $\quad$ $a + 3(0) + 2(0) = 1$

$2a + 2b + 2c = 2$ $\quad$ $2(1) + 2(0) + 2(0) = 2$

$3a + b5 + 0c = 3$ $\quad$ $3(1) + 5(0) + 0 = 3$

$$d + 3e + 2f = 0$$
$$2f + 2e + 2f = -4$$
$$3d + 5e + 0 = -4$$

Ans: $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$d = \frac{-4-5e}{3}$

$\frac{-4-5e}{3} + 3e + 2f = 0$

$-4-5e+9e+6f = 0$

$4e+6f = 4$

$2\left(\frac{-4-5e}{3}\right) + 2e + 2f = -4$

$-8-10e+6e+6f = -4\times 3$

$-4e+6f = -4$

$4e+6f = 4$

$-(-4e+6f = -4)$

$4e+4e = 4+4$

$8e = 8$

$\boxed{e = 1}$

$4(1)+6f = 4$

$\boxed{f = 0}$

$d = \frac{-4-5(1)}{3}$

$\boxed{d = -3}$

$g+3h+2i = 0$

$2g+2h+2i = -2$

$3g+5h = -6$

$g = \frac{-6-5h}{3}$; $\boxed{g = -2}$

$g+3h+2i = 0$

$-(2g+2h+2i = -2)$

$-g+h = 2$

$\frac{-6-5h}{3} + h = 2$

$-5h+3h = 6+6$

$\boxed{h = 0}$

$g+3h+2i = 0$

$-2+0+2i = 0$

$\boxed{i = \frac{2}{2} = 1}$

e. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

just shifting of :

R1 to R3 hence use of 100

R2 to R1 hence the use of 010

R3 to R2 use of 001

Q2

a. $2x - 3y + 1 = 2 \Rightarrow 2x - 3y + 0z = 2 - 1$

 $4x - 5y + z = 5 \qquad 2x - 3y + 0z = 1$

 $2x - y - 3z = 5$

Using Gyassian Elimination.

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & 1 \\ 4 & -5 & 1 & 5 \\ 2 & -1 & -3 & 5 \end{array}\right] \quad \begin{array}{l} R1 \leftarrow R1 + R3\left(\frac{-1}{2}\right) \\ R2 \leftarrow R2 + R3(-2) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -5/2 & 3/2 & -3/2 \\ 0 & -3 & 7 & -5 \\ 2 & -1 & -3 & 5 \end{array}\right] \quad R3 \leftarrow R3 + R1(-2)$$

$$\left[\begin{array}{ccc|c} 1 & -5/2 & 3/2 & -3/2 \\ 0 & -3 & 7 & -5 \\ 0 & 4 & -6 & 8 \end{array}\right] \quad R2 \leftarrow R2 + R3$$

$$\left[\begin{array}{ccc|c} 1 & -5/2 & 3/2 & -3/2 \\ 0 & 1 & 1 & 3 \\ 0 & 4 & -6 & 8 \end{array}\right] \quad R3 \leftarrow R3 + R2(-4)$$

$$\left[\begin{array}{ccc|c} 1 & -5/2 & 3/2 & -3/2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -10 & -4 \end{array}\right] \quad R3 \leftarrow \frac{R3}{-10}$$

$$\left[\begin{array}{ccc|c} 1 & -5/2 & 3/2 & -3/2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2/5 \end{array}\right] \quad \begin{array}{l} x - \frac{5}{2}y + \frac{3}{2}z = -3/2 \\ 0x + y + z = 3 \\ 0x + 0y + z = 2/5 \end{array}$$

$\boxed{z = 2/5}$

$y + z = 3$

$y + \frac{2}{5} = 3$

$\boxed{y = \frac{13}{5}}$

$x - \frac{5}{2}y + \frac{3}{2}z = -3/2$

$x - \frac{5}{2}\left(\frac{13}{5}\right) + \frac{3}{2}\left(\frac{2}{5}\right) = -3/2$

$10x - 65 + 6 = -\frac{3}{2} \times \frac{10}{1}$

$x = \frac{15 + 5[illegible]}{10}$

$\boxed{x = \frac{22}{5}}$

$x = \frac{22}{5}$, $y = \frac{13}{5}$, $z = \frac{2}{5}$

<u>Verify</u>

$2x - 3y + 1 = 2$

$2\left(\frac{22}{5}\right) - 3\left(\frac{13}{5}\right) + 1 = 2$

$\frac{44}{5} - \frac{39}{5} = 1$

$\frac{5}{5} = 1$

$1 = 1.$

$4x - 5y + z = 2$

$4\left(\frac{22}{5}\right) - 5\left(\frac{13}{5}\right) + \frac{2}{5} = 2$

$\frac{88}{5} - \frac{65}{5} + \frac{2}{5} = 2$

$2 = 2.$

<u>b.</u>

$x_1 + x_2 + x_3 = 1$

$2x_1 - x_2 + 3x_3 = 5$

$4x_1 + 5x_2 + x_3 = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 5 \\ 4 & 5 & 1 & 3 \end{array}\right] \quad R2 \leftarrow R2 + R1(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & 3 \\ 4 & 5 & 1 & 3 \end{array}\right] \quad R3 \leftarrow R3 + R1(-4)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & 3 \\ 0 & 1 & -3 & -1 \end{array}\right] \quad R2 \leftarrow R2 + R3(4)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -11 & -1 \\ 0 & 1 & -3 & -1 \end{array}\right] \quad R3 \leftarrow R3 - R2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -11 & -1 \\ 0 & 0 & 8 & 0 \end{array}\right] \quad R3 \leftarrow \frac{R3}{8} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -11 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x_3 = 0$

$x_2 - 11x_3 = -1$

$x_2 - 11(0) = -1$

$x_2 = -1$

$x_1 + x_2 + x_3 = 1$

$x_1 - 1 + 0 = 1$

$x_1 = 1+1$

$x_1 = 2$

$x_3 = 0$, $x_1 = 2$, $x_2 = -1$

<u>Verify</u>

- $x_1 + x_2 + x_3 = 1$
  $2 - 1 + 0 = 1$
  $1 = 1$

- $2x_1 - x_2 + 3x_3 = 5$
  $2(2) - (-1) + 3(0) = 5$
  $5 = 5.$

- $4x_1 + 5x_2 + x_3 = 3$
  $4(2) + 5(-1) + 0 = 3$
  $8 - 5 + 0 = 3$
  $3 = 3$

<u>Q3</u> Using gauss-Jordan method.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad A^{-1} = ?$$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R2 \leftarrow R2 + R1(-2)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R2 \leftarrow \frac{R2}{3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R2 \leftarrow R2 + R3\left(-\frac{1}{3}\right)$$

$$[I \mid A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2/3 & 1/3 & -1/3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & -1/3 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix} \qquad B^{-1} = ?$$

$$[B \mid I] = \left[\begin{array}{ccc|ccc} 3 & 5 & -9 & 1 & 0 & 0 \\ 4 & 2 & 6 & 0 & 1 & 0 \\ 8 & 9 & 7 & 0 & 0 & 1 \end{array}\right] \qquad R1 \leftarrow \frac{R1}{3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{5}{3} & -3 & \frac{1}{3} & 0 & 0 \\ 4 & 2 & 6 & 0 & 1 & 0 \\ 8 & 9 & 7 & 0 & 0 & 1 \end{array}\right] \qquad R2 \leftarrow R2 + R1(-4)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & 1/3 & 0 & 0 \\ 0 & -14/3 & 18 & -4/3 & 1 & 0 \\ 8 & 9 & 7 & 0 & 0 & 1 \end{array}\right] \qquad R3 \leftarrow R3 + R1(-8)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & \frac{1}{3} & 0 & 0 \\ 0 & -14/3 & 18 & -4/3 & 1 & 0 \\ 0 & -\frac{13}{3} & 31 & -\frac{8}{3} & 0 & 1 \end{array}\right] \qquad R_2 \leftarrow R2 \times \left(-\frac{3}{14}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & 1/3 & 0 & 0 \\ 0 & 1 & -27/7 & 2/7 & -3/14 & 0 \\ 0 & -13/3 & 31 & -8/3 & 0 & 1 \end{array}\right] \qquad \begin{array}{l} R1 \leftarrow R1 + R2\left(-\frac{5}{3}\right) \\ R3 \leftarrow R3 + R2\left(\frac{13}{3}\right) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{7} & -\frac{1}{7} & 5/14 & 0 \\ 0 & 1 & -\frac{27}{7} & 2/7 & -3/14 & 0 \\ 0 & 0 & \frac{100}{7} & -\frac{10}{7} & -13/14 & 1 \end{array}\right] \qquad R3 \leftarrow R3\left(\frac{7}{100}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{7} & -1/7 & 5/14 & 0 \\ 0 & 1 & -27/7 & 2/7 & -3/14 & 0 \\ 0 & 0 & 1 & -\frac{1}{10} & -13/200 & 7/100 \end{array}\right] \qquad \begin{array}{l} R1 \leftarrow R1 + R3\left(\frac{24}{7}\right) \\ R2 \leftarrow R2 + R3\left(\frac{27}{7}\right) \end{array}$$

$$[I|B^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 29/50 & -6/25 \\ 0 & 1 & 0 & -1/10 & -93/200 & 27/100 \\ 0 & 0 & 1 & -1/10 & -13/200 & 7/100 \end{array}\right] \quad B^{-1} = \begin{bmatrix} 1/5 & 29/50 & -6/25 \\ -1/10 & -93/200 & 27/100 \\ -1/10 & -13/200 & 7/100 \end{bmatrix}$$

Q4

a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$

Show that $(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0+0 & 5+0+0 & -9+0+0 \\ 6+12+8 & 10+6+9 & -18+18+7 \\ 0+0+8 & 0+0+9 & 0+0+7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & -9 \\ 26 & 25 & 7 \\ 8 & 9 & 7 \end{bmatrix}$$

$$[AB \mid I] = \left[\begin{array}{ccc|ccc} 3 & 5 & -9 & 1 & 0 & 0 \\ 26 & 25 & 7 & 0 & 1 & 0 \\ 8 & 9 & 7 & 0 & 0 & 1 \end{array}\right] \quad R_1 \leftarrow \frac{R_1}{3}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & 1/3 & 0 & 0 \\ 26 & 25 & 7 & 0 & 1 & 0 \\ 8 & 9 & 7 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 + R_1(-26) \\ R3 \leftarrow R3 + R1(-8) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & 1/3 & 0 & 0 \\ 0 & -55/3 & 85 & -26/3 & 1 & 0 \\ 0 & -13/3 & 31 & -8/3 & 0 & 1 \end{array}\right] \quad R_2 = \left(-\frac{3}{55}\right)R_2$$

$x - x = x - x$

$$= \left[\begin{array}{ccc|ccc} 1 & 5/3 & -3 & 1/3 & 0 & 0 \\ 0 & 1 & -51/11 & 26/55 & -3/55 & 0 \\ 0 & -13/3 & 31 & -8/3 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R1 \leftarrow R1 + R2\left(-\frac{5}{3}\right) \\ R3 \leftarrow R3 + R2\left(\frac{13}{3}\right) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 52/11 & -5/11 & 1/11 & 0 \\ 0 & 1 & -51/11 & 26/55 & -3/55 & 0 \\ 0 & 0 & 120/11 & -34/55 & -13/55 & 1 \end{array}\right] \quad R3 \leftarrow R3\left(\frac{11}{120}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 52/11 & -5/11 & 1/11 & 0 \\ 0 & 1 & -51/11 & 26/55 & -3/55 & 0 \\ 0 & 0 & 1 & -17/300 & -13/600 & 11/120 \end{array}\right] \quad \begin{array}{l} R2 \leftarrow R2 + R3\left(\frac{11}{51}\right) \\ R1 \leftarrow R1 + R3\left(-\frac{52}{11}\right) \end{array}$$

$$[I \,|\, (AB)^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -14/75 & 29/150 & -13/30 \\ 0 & 1 & 0 & 21/500 & -31/200 & 17/40 \\ 0 & 0 & 1 & -17/300 & -13/600 & 11/120 \end{array}\right]$$

$$AB^{-1} = B^{-1}A^{-1}$$

$$\begin{bmatrix} -14/75 & 29/150 & -13/30 \\ 21/500 & -31/200 & 17/40 \\ -17/300 & -13/600 & 11/120 \end{bmatrix} = \begin{bmatrix} 1/5 & 29/50 & -6/25 \\ -1/10 & -93/200 & 27/100 \\ -1/10 & -13/200 & 7/100 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & -1/3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 + \left(-\frac{29}{75}\right) + 0 & 0 + \frac{29}{150} + 0 & 0 + \left(\frac{29}{150}\right) + \left(-\frac{6}{25}\right) \\ -1/10 + \frac{31}{100} + 0 & 0 + \left(\frac{-31}{200}\right) + 0 & 0 + \left(\frac{31}{200}\right) + 27/100 \\ -1/10 + \frac{13}{300} + 0 & 0 - 13/600 + 0 & 0 + \left(13/600\right) + 7/100 \end{bmatrix}$$

$$\begin{bmatrix} -14/75 & 29/150 & -13/30 \\ 21/100 & -31/200 & 17/40 \\ -17/300 & -13/600 & 11/120 \end{bmatrix} = \begin{bmatrix} -\frac{14}{75} & \frac{29}{150} & -13/30 \\ 21/100 & -31/200 & 17/40 \\ -17/300 & -13/600 & 11/120 \end{bmatrix}$$

$$(AB)^{-1} = A^{-1}B^{-1}$$

they are equal, proved.

b) $(AB)(B^{-1}A^{-1}) = I$.

$$\begin{bmatrix} 3 & 5 & -9 \\ 26 & 25 & 7 \\ 8 & 9 & 7 \end{bmatrix} \begin{bmatrix} -\frac{14}{75} & 29/150 & -13/30 \\ 21/100 & -31/200 & 17/40 \\ -17/300 & -13/600 & 11/120 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{14}{25} + \frac{21}{20} + \frac{51}{100} & 3\times\frac{29}{150} + 5\times -\frac{31}{200} - 9\times -\frac{13}{600} & 3\times -13/30 + 5\times\frac{17}{40} - 9\times\frac{11}{120} \\ -\frac{364}{75} + 21/4 + \left(\frac{-119}{300}\right) & 26\times\frac{29}{150} + 25\times\left(-\frac{31}{200}\right) + 7\times\left(-\frac{13}{600}\right) & 26\times -13/30 + 25\times 17/40 + 7\times\frac{11}{120} \\ -\frac{112}{75} + \frac{189}{100} + \left(-\frac{119}{300}\right) & 8\times -\frac{29}{150} + 9\times -\frac{31}{200} + 7\times 11/120 & 8\times -13/30 + 9\times 17/40 + 7\times 11/120 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(AB)(A^{-1}B^{-1}) = I$

$$\begin{bmatrix} 3 & 5 & -9 \\ 26 & 25 & 7 \\ 8 & 9 & 7 \end{bmatrix} \begin{bmatrix} -\frac{14}{75} & 29/150 & -13/30 \\ 21/100 & -31/200 & 17/40 \\ -17/300 & -13/600 & 11/120 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence proved.

Q5 a. $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ 2 & 3 & 0 \end{bmatrix}$ $R2 \leftarrow R2 + R1(-4)$

$$\cancel{E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \quad \cancel{E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}} \quad \cancel{E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix} \qquad E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= E_{21}A$

$y = y$ $\quad x - x = x - x$

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$R3 \leftarrow R3 + R1(-2)$

$3 + 1 - 2$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= E_{21} E_{31} A \qquad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = E_{21} E_{31} E_{32} A$$

$R3 \leftarrow R3 + R2\left(-\frac{1}{2}\right)$

$1 + 2 \times -\frac{1}{2}$

$1 - \frac{1}{2}$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$E_{21} E_{31} E_{32} A = U$

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

6. $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{i \times j}$

i. $A^T = ?$

$A^T = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}_{j \times i}$

ii. Prove that $AA^T \neq A^TA$.

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$2 \times \boxed{3 \quad 3} \times 2 \qquad 3 \times \boxed{2 \quad 2} \times 3$

$$\begin{bmatrix} -1\times-1+1\times1+0\times0 & -1\times0+1\times-1+0\times1 \\ 0\times-1+-1\times1+1\times0 & 0\times0+1\times-1+1\times1 \end{bmatrix} \neq \begin{bmatrix} -1\times-1+0\times0 & -1\times1+0\times-1 & -1\times0+0\times1 \\ 1\times-1+0\times-1 & 1\times1+-1\times-1 & 1\times0+-1\times1 \\ 0\times-1+0\times1 & 0\times1+1\times-1 & 0\times0+1\times1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \neq \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Hence proved.

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