INFORMATION TECHNOLOGY UNIVERSITY

FACULTY OF ENGINEERING
Faculty of Engineering
Mid Exam – Spring 2025
MT202T: BS-SE 2023 (A&B),BS-SE 2024
Probability and Statistics

Total Allowed Time: 1.5 Hours **Total Marks: 50**

Question No. 1 (10 marks) A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective.What is the probability that two defectives exist in the lot?

Question No. 2 (10 marks) The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of these proportions is given by

$$f(x) = \begin{cases} c(1-x)^4, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Find value of c so that above $f(x)$ is a valid density function. What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution controls?

Question No. 3 (10 marks) A service facility operates with two service lines. On a randomly selected day, let *X* be the proportion of time that the first line is in use whereas *Y* is the proportion of time that the second line is in use. Suppose that the joint probability density function for (X, Y) is

$$f(x,y) = \begin{cases} 1.5(x^2 + y^2) & 0 \le x, y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute the probability that neither line is busy more than half the time.
Find the probability that the first line is busy more than 75% of the time.

Question No. 4 (10 marks)

An elevator in a tower starts with 4 people. It stops at 7 floors. Suppose each passenger is equally likely to exit at any floor. What is the probability that all passengers exit on the same floor? What is the probability no two passengers exit on the same floor ?

Question No. 5 (10 marks)

Kittu and Billu are professional computer programmers. The number of programming mistakes each makes when writing a code are random variables. On the average Kittu makes 5 errors per 100 lines of coder and Billu makes 2 errors per 100 lines of code. Anarkali Software House hires Kittu and Billu to write a 200 line code for the company. If only one of them write the program, there is a 0.75 probability that Kittu is the one who writes the code. What is the probability that code contains no errors ?

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Formula Sheet

Additive Rule for Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B).$

Conditional Probability: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}, \quad \text{provided} \quad P(A) > 0.$

Bayes Theorem:
$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for } r = 1, 2, \ldots, k.$$

Total Probability Theorem :
$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A|B_i).$$

Chain Rule:
$$P(A_1 \cap A_2 \cap \cdots \cap A_k) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots P(A_k|A_1 \cap A_2 \cap \cdots \cap A_{k-1}).$$

Probability from pdf: $P(a < X < b) = \int_a^b f(x)\, dx. \qquad \int_{-\infty}^{\infty} f(x)\, dx = 1.$

Probability from joint pdf: $P[(X, Y) \in A] = \iint f(x, y)\, dx\, dy$, for any region A in the xy plane

Binomial pdf: $b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n.$

Poisson pdf: $p(x; \lambda t) = \dfrac{e^{-\lambda t}(\lambda t)^x}{x!}, \quad x = 0, 1, 2, \ldots,$

Hypergeometic pdf: $h(x; N, n, k) = \dfrac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}, \quad \max\{0, n - (N - k)\} \le x \le \min\{n, k\}.$

Negative Binomial pdf: $b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \ldots.$