Information Technology University, Lahore, Pakistan

Department of Computer and Software Engineering

Linear Algebra

Assignment # 6, Spring 2024

Submission Deadline: Thursday May 16, 2024 Maximum Marks: 100

—Use blue or black ink only.

1. Given a matrix

$$A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$$

   (a) Find all the eigenvalues and corresponding eigenvectors. [5]

   (b) Find matrices X and Λ such that X is nonsingular and $\Lambda = X^{-1}AX$ is diagonal. [5]

2. Which of the following two matrices A and B is diagonalizable and why? If yes, find X such that Λ is diagonal. [10]

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 5 & 2 \\ 1 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$$

3. Find Λ and X to diagonalize A if [10]

$$A = \begin{bmatrix} 0.6 & 0.9 \\ 0.1 & 0.6 \end{bmatrix}$$

   What is $A^{10}u_o$ for these u_o?

$$u_o = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_o = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, u_o = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

4. Compute A^6 if [10]

$$A = \begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix}$$

5. Write as $X\Lambda X^{-1}$. [10]

$$A = \begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix}$$

   Multiply $Xe^{\Lambda t}X^{-1}$ to find the matrix exponential e^{At} and the derivative of e^{At} when $t = 0$.

6. Use Linear Algebra to solve the 1^{st} order differential equations: [10]

$$\frac{dy_1}{dt} = y_1 + y_2$$
$$\frac{dy_2}{dt} = 4y_1 - 2y_2$$

   with initial conditions $y_1(0) = 1$ and $y_2(0) = 6$.

7. Note that an n^{th} order differential equation can be solved by converting it into a system of n 1^{st} order differential equations, and then representing it in matrix form. The general solution can be found by using eigenvalues and eigenvectors and the particular solution is obtained by using the initial conditions as well.

   Now solve the following differential equations: [10]

   (a) $y'' - 5y' - 4y = 0$.

   (b) $y'' + 9y = 0$ if $u(0) = (3, 0)$.

8. (a) Find all eigenvalues of S.

   (b) Find nonzero orthonormal eigenvectors of S.

   (c) Find Q such that $\Lambda = Q^{-1}SQ$ is orthonormal.

[10]

$$S = \begin{bmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix}$$

9. Show that the matrix A is positive definite matrix if [10]

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

10. Find SVD of the following matrix A. [10]

$$A = \begin{bmatrix} 4 & 6 \\ 0 & 4 \end{bmatrix}$$