Information Technology University, Lahore, Pakistan

Linear Algebra (MT-121)

Assignment # 5

Spring 2024

BSCE2023

April 27, 2024

Submission Deadline: Friday, May 03, 2024 Maximum Marks: 100

- Late submissions will not be graded.
- This Assignment will be conducted under the rules and guidelines of the ITU Honour Code, and no cheating will be tolerated (i.e., no discussion about the Assignment with other students, no plagiarism at all). Each student must be able to justify his/her work.

Question 1

Draw Figure 1 to show each subspace correctly for [10]

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$$

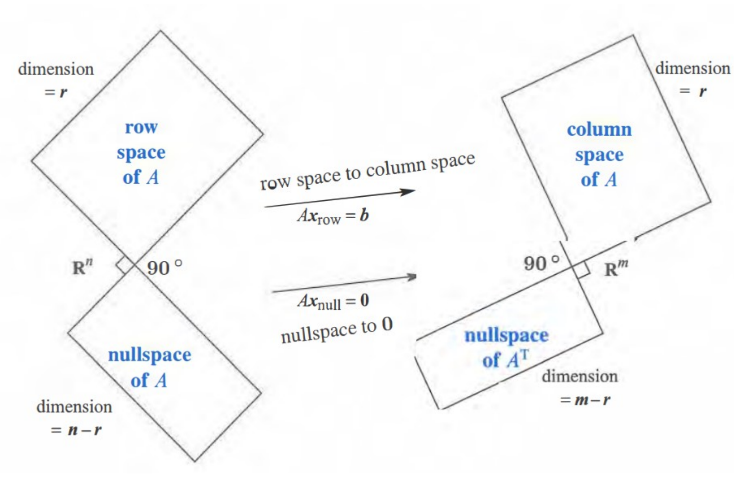

Figure 1: Two pairs of orthogonal subspaces. The dimensions add to n and add to m.

Question 2

Check that $e = b - p = (-1, 3, -5, 3)$ is perpendicular to both columns of the same matrix A. What is the shortest distance $\|e\|$ from b to the column space of A? [10]

Question 3

Given the matrix A = $\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$ [10 + 10 = 20]

(a) Find orthonormal vectors q1, q2, q3 such that q1,q2 span the column space of A.

(b) Solve Ax = (1, 2, 7) by least squares.

Question 4

Given the matrics [10 + 10 = 20]

A = $\begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$

(a) Find an orthonormal basis for the column space of A:

(b) Compute the projection of b onto that column space.

Question 5

Find the orthogonal vectors A, B, C by Gram-Schmidt from [10 + 10 = 20]

a = $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and b = $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and c = $\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

Question 6

Find a QR factorization of A = $\begin{bmatrix} 1 & 1 & 8 \\ 1 & 0 & 2 \\ 4 & 4 & 0 \\ 2 & 8 & 6 \end{bmatrix}$ [10]

Question 7

Given the matrix A = $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 1 \end{bmatrix}$ [10]

(a) Find the cofactors of A.

(b) Find A^{-1} from the cofactor formula $C^T/detA$.