Information Technology University, Lahore, Pakistan

Linear Algebra (MT-121)

Assignment # 4

Spring 2024

BSCE2023

March 18, 2024

Submission Deadline: Friday, March 22, 2024 Maximum Marks: 100

- Late submissions will not be graded.
- This Assignment will be conducted under the rules and guidelines of the ITU Honour Code, and no cheating will be tolerated (i.e., no discussion about the Assignment with other students, no plagiarism at all). Each student must be able to justify his/her work.

Question 1

Find the complete solution of $\begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 0 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ [10]

Question 2

What are the special solutions to Rx = 0 and $y^T R = 0$ for these R? [10]

(a) R = $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(b) R = $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Question 3

Given the matrices A = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ [10]

(a) Describe a subspace of M that contains A but not B.

(b) If a subspace of M does contain A and B, must it contain I?

(c) Describe a subspace of M that contains no nonzero diagonal matrices.

Question 4

The complete solution to Ax = $\begin{bmatrix}1\\2\\3\end{bmatrix}$ is $x = \begin{bmatrix}1\\0\\5\end{bmatrix} + \text{c}\begin{bmatrix}0\\1\\5\end{bmatrix}$. Find A. [10]

Question 5

The matrix A = $\begin{bmatrix}2 & -2\\2 & -2\end{bmatrix}$ is a "vector" in the space M of all 2 by 2 matrices. [10]

(a) Write down the zero vector in this space.

(b) The vector half of A.

(c) The vector $-A$.

(d) What matrices are in the smallest subspace containing A?

Question 6

Decide the dependence and independence of [10]

(a) The vectors (1, 3, 2), (2, 1, 3) and (3, 2,1).

(b) The vectors (1, -3, 2), (2, 1, -3) and (-3, 2,1).

Question 7

Without using elimination, find dimensions and bases for the four subspaces for [10]

$$\text{A} = \begin{bmatrix}0 & 3 & 3 & 3\\0 & 0 & 0 & 0\\0 & 1 & 0 & 1\end{bmatrix}, \quad \text{B} = \begin{bmatrix}1\\4\\5\end{bmatrix}$$

Question 8

Are the vectors $v_1 = \begin{bmatrix}1 & 0 & 1 & 2\end{bmatrix}$, $v_2 = \begin{bmatrix}0 & 1 & 1 & 2\end{bmatrix}$, and $v_3 = \begin{bmatrix}1 & 1 & 1 & 3\end{bmatrix}$ in R^4 linearly dependent or independent? [10]

Question 9

Are the vectors $v_1 = \begin{bmatrix}2 & 1\\0 & 1\end{bmatrix}$, $v_2 = \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$, $v_3 = \begin{bmatrix}0 & -3\\-2 & 1\end{bmatrix}$ in M_{22} linearly independent? [10]

Question 10

Describe the subspace of R^3 (is it a line or plane or R^3?) spanned by [10]

(a) the two vectors (1, 1, -1) and (-1, -1, 1)

(b) the three vectors (0, 1, 1) and (1, 1, 0) and ((0, 0, 0)

(c) all vectors in R^3 with whole number components

(d) all vectors with positive components