Junaina Abdul Raz
BSSE23058

Assignment # 1
Linear Algebra

Q1 Let vector $\begin{pmatrix}1\\2\\3\\4\end{pmatrix} = p$

Assume $x = \begin{pmatrix}a\\b\\c\\d\end{pmatrix}$ $y = \begin{pmatrix}e\\f\\g\\h\end{pmatrix}$ $z = \begin{pmatrix}i\\j\\k\\l\end{pmatrix}$

$x \cdot p = 0$

$\begin{pmatrix}a\\b\\c\\d\end{pmatrix}\begin{pmatrix}1\\2\\3\\4\end{pmatrix} = 0$

$a(1) + b(2) + c(3) + d(4) = 0$

$a + 2b + 3c + 4d = 0$

Let $x = \begin{pmatrix}1\\1\\-1\\0\end{pmatrix}$

$1 + 2(1) + 3(-1) + 4(0) = 0$

$1 + 2 + (-3) + 0 = 0$

$0 = 0$

$x \cdot y = 0$

$\begin{pmatrix}1\\1\\-1\\0\end{pmatrix} \cdot \begin{pmatrix}e\\f\\g\\h\end{pmatrix} = 0$

$1e + f + g(-1) + h(0) = 0$

$e + f + (-g) + 0 = 0$

let $y = \begin{pmatrix}1\\0\\1\end{pmatrix}$

$(1) + 0 + (-1) + 0 = 0$

$1 + 0 - 1 + 0 = 0$

$0 = 0$

$z \cdot y = 0$

$\begin{pmatrix}i\\j\\k\\l\end{pmatrix} \cdot \begin{pmatrix}1\\0\\1\\1\end{pmatrix} = 0$

$i(1) + j(0) + k(1) + l(1) = 0$

$i + 0j + k + l = 0$

let $z = \begin{pmatrix}-1\\1\\0\\1\end{pmatrix}$

$-1 + (0) + (0) + (1)$

$-1 + 0 + 0 + 1 = 0$

$0 = 0$

Ans $x = \begin{pmatrix}1\\1\\-1\\0\end{pmatrix}$

$y = \begin{pmatrix}1\\0\\1\end{pmatrix}$

$z = \begin{pmatrix}-1\\1\\0\\1\end{pmatrix}$

Q2

$u = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ $v = \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $v + w = \begin{pmatrix} 0.3+2 \\ 0.4+1 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 1.4 \end{pmatrix}$

a)

$u \cdot v = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}$

$= 5 \times 0.3 + 4 \times 0.4$

$= 1.5 + 1.6$

$= 3.1$

$u \cdot (v+w) = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2.3 \\ 1.4 \end{pmatrix}$

$= 5 \times 2.3 + 4 \times 1.4$

$= 11.5 + 5.6$

$= 17.1$

$w \cdot v = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}$

$= 2 \times 0.3 + 1 \times 0.4$

$= 0.6 + 0.4$

$= 1.0$

b) $\|u\| = \sqrt{5^2 + 4^2}$
$= \sqrt{41}$

$\|v\| = \sqrt{0.3^2 + 0.4^2}$
$= 0.5$

$\|w\| = \sqrt{2^2 + 1^2}$
$= \sqrt{5}$

$\|v + w\| = \sqrt{2.3^2 + 1.4^2}$
$= \frac{\sqrt{29}}{2}$

$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$
$= \frac{3.1}{\sqrt{41} \times 0.5} = \frac{6.2}{\sqrt{41}}$
$= 0.097$

$\cos\theta = \frac{u \cdot (v+w)}{\|u\| \|v+w\|}$
$= \frac{17.1}{\sqrt{41} \times \frac{\sqrt{29}}{2}}$
$= 0.992$

$\cos\theta = \frac{w \cdot v}{\|w\|\|v\|}$
$= \frac{1.0}{\sqrt{5} \times 0.5}$
$= \frac{2\sqrt{5}}{5}$
$= 0.894$

c) $\hat{u}$nit vector in the direction of u $= \dfrac{u}{\|u\|}$

$$= \frac{\begin{bmatrix} 5 \\ 4 \end{bmatrix}}{\sqrt{41}}$$

$$= \begin{bmatrix} 5/\sqrt{41} \\ 4/\sqrt{41} \end{bmatrix}$$

Unit vector in the direction of v $= \dfrac{v}{\|v\|}$

$$= \frac{\begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}}{0.5}$$

$$= \begin{pmatrix} 0.3/0.5 \\ 0.4/0.5 \end{pmatrix}$$

$$= \begin{vmatrix} 0.6 \\ 0.8 \end{vmatrix}$$

Q3

$$xw_1 + yw_2 + zw_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{when } x = 1.$$

$$x\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + z\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + z\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$1 + 4y + 7z = 0$
$2 + 5y + 8z = 0$
$3 + 6y + 9z = 0$

$4y + 7z = -1$
$5y + 8z = -2$
$6y + 9z = -3$

$$y = \frac{-1-7z}{4}$$

$$5y + 8z = -2$$

$$5\left(\frac{-1-7z}{4}\right) + 8z = -2$$

$$\frac{-5-35z}{4} + 8z = -2$$

$$-5-35z+32z = -2\times 4$$

$$-3z = -8+5$$

$$z = \frac{-3}{-3}$$

$$z = 1$$

$$\Rightarrow z = 1$$

$$y = \frac{-1-7(1)}{4}$$

$$= \frac{-1-7}{4}$$

$$y = \frac{-8}{4} = -2$$

Ans $x = 1$, $y = -2$, $z = 1$

$$1w_1 - 2w_2 + 1w_3 = 0$$

$$\Rightarrow 2x + 5y + 8z = 0$$

$$2(1) + 5(-2) + 8(1) = 0$$

$$2 - 10 + 8 = 0$$

$$-10 + 10 = 0$$

$$0 = 0$$

$$\Rightarrow x + 4y + 7z = 0$$

$$1 + 4(-2) + 7(1) = 0$$

$$1 - 8 + 7 = 0$$

$$-7 + 7 = 0$$

$$0 = 0$$

$$\Rightarrow 3x + 6y + 9z = 0$$

$$3(1) + 6(-2) + 9(1) = 0$$

$$3 - 12 + 9 = 0$$

$$0 = 0.$$

linearly dependent as they are consistent equations

Q4

$$au + bv + cw = x$$

$$a\begin{pmatrix}1\\1\\2\end{pmatrix} + b\begin{pmatrix}1\\2\\3\end{pmatrix} + c\begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}2\\3\\7\end{pmatrix}$$

$$a + b + c = 2 \quad \text{— (1)}$$

$$a + 2b + c = 3 \quad \text{— (2)}$$

$$2a + 3b + 4c = 7 \quad \text{— (3)}$$

$$\cancel{a}+b+\cancel{c}=2$$
$$-(\cancel{a}+2b+\cancel{c}=3)$$
$$b-2b=2-3$$
$$\cancel{-}b=\cancel{-}1$$
$$b=1$$

$$a+b+c=2$$
$$a+c=2-1$$
$$a+c=1$$
$$\cancel{a+c=3-2(1)}$$
$$\cancel{a+c=1}$$

$$2a+3(1)+4c=7$$
$$2a+4c=7-3$$
$$2a+4c=4$$
$$-\ 2(a+c=1)$$
$$2a-2a+4c-2c=4-2$$
$$2c=2$$
$$c=1$$

$$a+c+b=2$$
$$a+1+1=2$$
$$a=2-2$$
$$a=0$$

Ans

$a=0$

$b=1$

$c=1$

Q5

a) $$\|a\|=\sqrt{(4)^2+0^2+(-3)^2+(1)^2}$$
$$=\sqrt{26}$$

$$\|b\|=\sqrt{(2)^2+(3)^2+(5)^2+(7)^2}$$
$$=\sqrt{87}$$

(b) $$a\cdot c=0$$
$$b\cdot c=0$$

$a \cdot c = 0$

$$\begin{pmatrix}4\\0\\-3\\1\end{pmatrix} \cdot \begin{pmatrix}a\\b\\c\\d\end{pmatrix} = 0$$

$4a + 0 + -3c + d = 0$

Let $a = 1$

$4 + 0 - 3c + d = 0$

$-3c + d = -4$

$d = -4 - 3c$

$d = -4 - 3(0)$

$d = -4$

$b \cdot c = 0$

$$\begin{pmatrix}2\\3\\5\\7\end{pmatrix} \cdot \begin{pmatrix}a\\b\\c\\d\end{pmatrix} = 0$$

$2a + 3b + 5c + 7d = 0$

$2(1) + 3b + 5c + 7(-4 - 3c) = 0$

$+2 + 3b + 5c - 28 - 21c = 0$

$3b - 16c = 26$

$b = \frac{26 - 16c}{3}$

$c = 0$

$b = \frac{26}{3}$

$$c = \begin{pmatrix}1\\26/3\\0\\-4\end{pmatrix}$$

c) $a \cdot d = 0$

$$\begin{pmatrix}4\\0\\-3\\1\end{pmatrix} \cdot \begin{pmatrix}e\\f\\g\\h\end{pmatrix} = 0$$

$4e + 0 - 3g + h = 0$

$4e - 3g + h = 0$

Let $e = 0$

$-3g + h = 0$

$-3g + \frac{13}{6} = 0$

$-18g + 13 = 0$

$g = +\frac{13}{18}$

$g = +\frac{13}{18}$

$b \cdot d = 0$

$$\begin{pmatrix}2\\3\\5\\7\end{pmatrix} \cdot \begin{pmatrix}e\\f\\g\\h\end{pmatrix} = 0$$

$2e + 3f + 5g + 7h = 0$

$0 + 3 + 5\left(\frac{+13}{18}\right) + 7\left(\frac{+13}{6}\right) = 0$

$3 + \frac{+65}{18} + \frac{+91}{6} = 0$

$c \cdot d = 0$

$$\begin{pmatrix}1\\26/3\\0\\-4\end{pmatrix} \cdot \begin{pmatrix}e\\f\\g\\h\end{pmatrix} = 0$$

$e + \frac{26}{3}f + 0 - 4h = 0$

$e + \frac{26}{3}f \quad 4h = 0$

let $e = 0$ and $f = 1$

$0 + \frac{26}{3} - 4h = 0$

$-12h = -26$

$h = \frac{13}{6}$

Q6

a) $(u+v)\cdot w = u\cdot w + v\cdot w$

let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ $w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$u+v = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1+v_1 \\ u_2+v_2 \end{pmatrix}$

$(u+v)\cdot w = \begin{pmatrix} u_1+v_1 \\ u_2+v_2 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$= (u_1+v_1)w_1 + (u_2+v_2)w_2$

$= (w_1u_1 + w_1v_1) + (w_2v_2 + w_2v_2)$ rearrange

$= (u_1w_1 + u_2w_2) + (w_1v_1 + w_2v_2)$

$= (u_1w_1 + u_2w_2) + (v_1w_1 + v_2w_2)$

$= u\cdot w + v\cdot w$

b) $\underset{\text{constant being multiplied}}{c}(u\cdot v)$ Scalar Multiplication $= u\cdot cv$.

$u\cdot v = \begin{pmatrix} u_1v_1 \\ u_2v_2 \end{pmatrix}$ $cu\cdot v = \begin{pmatrix} cu_1 \\ cu_2 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = cu_1v_1 + cu_2v_2$

$c(u\cdot v) = c\begin{pmatrix} u_1v_1 \\ u_2v_2 \end{pmatrix}$

$= c(u_1v_1) + c(u_2v_2)$ rearrange

$= u_1(cv_1) + u_2(cv_2)$

$= u\cdot cv$

Q

a) $Ax = b$

$$x\begin{pmatrix}-3\\5\\1\end{pmatrix} + y\begin{pmatrix}2\\7\\4\end{pmatrix} + z\begin{pmatrix}-6\\-5\\-2\end{pmatrix} = \begin{pmatrix}6\\6\\8\end{pmatrix}$$

$$Ax = b.$$

$$\underbrace{\begin{bmatrix}-3 & 2 & -6\\5 & 7 & -5\\1 & 4 & -2\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x\\y\\z\end{bmatrix}}_{x} = \underbrace{\begin{bmatrix}6\\6\\8\end{bmatrix}}_{b}$$

b)

determinant of A $= -3\begin{bmatrix}7 & -5\\4 & -2\end{bmatrix} - 2\begin{bmatrix}5 & -5\\1 & -2\end{bmatrix} + (-6)\begin{bmatrix}5 & 7\\1 & 4\end{bmatrix}$

$= -3\{(7\times-2)-(-5\times4)\} - 2\{(5\times-2)-(1\times-5)\} - 6\{(5\times4)-(1\times7)\}$

$= -3[6] - 2[-5] - 6[13]$

$= -86$

$$A^{-1} = \frac{1}{\text{determinant of } A}\,\mathrm{adj}(A).$$

$$\mathrm{Adj}(A) = \begin{bmatrix} \begin{bmatrix}7 & -5\\4 & -2\end{bmatrix} & -\begin{bmatrix}5 & -5\\1 & -2\end{bmatrix} & \begin{bmatrix}5 & 7\\1 & 4\end{bmatrix} \\ \begin{bmatrix}2 & -6\\4 & -2\end{bmatrix} & \begin{bmatrix}-3 & -6\\1 & -2\end{bmatrix} & -\begin{bmatrix}-3 & 2\\1 & 4\end{bmatrix} \\ \begin{bmatrix}2 & -6\\7 & -5\end{bmatrix} & -\begin{bmatrix}-3 & -6\\5 & -5\end{bmatrix} & \begin{bmatrix}3 & 2\\5 & 7\end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix}+ & - & +\\- & + & -\\+ & - & +\end{bmatrix}$$

$$= \begin{bmatrix} 7\times 2 - (-5\times 4) & -(5\times 2 - 1\times -5) & 5\times 7 - (1\times 4) \\ 2\times 2 - (-6\times 4) & (-3\times 2) - (-6\times 1) & -(-3\times 4 - 2\times 1) \\ 2\times 5 - (-6\times 7) & -(-3\times 5 - (-6\times 5)) & 3\times 7 - 5\times 2 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 6 & 5 & 13 \\ -20 & 12 & +14 \\ 32 & 45 & -31 \end{bmatrix} = \begin{bmatrix} 6 & -20 & 32 \\ 5 & 12 & 45 \\ 13 & 14 & -31 \end{bmatrix}$$

$$A^{-1} = \frac{1}{86}\begin{bmatrix} 6 & -20 & 32 \\ 5 & 12 & 45 \\ 13 & 14 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{43} & \frac{10}{43} & \frac{-16}{43} \\ \frac{-5}{86} & \frac{-6}{43} & \frac{45}{86} \\ \frac{-13}{86} & \frac{-7}{43} & \frac{31}{86} \end{bmatrix}$$

c)

$-3x + 2y - 6z = 6$

$5x + 7y - 5z = 6$ } add

$(x + 4y - 2z = 8) \times 3$

$3x + 12y - 6z = 24$

$+ \; -3x + 2y - 6z = 6$

$12y + 2y - 6z - 6z = 24 + 6$

$14y - 12z = 30$

$14(-4 + 7z) - 12z = 30$

$-56 + 98z - 12z = 30$

$86z = 86$

$z = 1.$

$5x + 7y - 5z = 6$

$+ (-3x + 2y - 6z = 6)$

$2x + 9y - 11z = 12$

$- \; 2(x + 4y - 2z = 8)$

$y - 7z = -4$

$y = -4 + 7z$

$y = -4 + 7(1)$

$= -4 + 7$

$= 3$

$x + 4(3) - 2(1) = 8$

$x = 8 - 10$

$x = -2$

Q8 Cramer Rule

determinant of A = -86

$$A = \begin{bmatrix} 3 & 2 & -6 \\ 5 & 7 & -5 \\ 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 8 \end{bmatrix}$$

$$x = \begin{bmatrix} 6 & 2 & -6 \\ 6 & 7 & -5 \\ 8 & 4 & -2 \end{bmatrix}.$$

determinant $|x|$ $= 6\begin{bmatrix} 7 & -5 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 6 & -5 \\ 8 & -2 \end{bmatrix} + (-6)\begin{bmatrix} 6 & 7 \\ 8 & 4 \end{bmatrix}$

$= 172$

$$x = \frac{|x|}{|A|} = \frac{172}{-86} = -2$$

~~determinant~~

$$y = \begin{bmatrix} 3 & 6 & -6 \\ 5 & 6 & -5 \\ 1 & 8 & -2 \end{bmatrix}$$

determinant $|y|$ $= 3\begin{bmatrix} 6 & -5 \\ 8 & -2 \end{bmatrix} - 6\begin{bmatrix} 5 & -5 \\ 1 & -2 \end{bmatrix} - 6\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$= -258$

$$y = \frac{|y|}{|A|} = \frac{-258}{-86} = 3$$

$$z = \begin{bmatrix} 3 & 2 & 6 \\ 5 & 7 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

$$\text{determinant } |Z| = 3\begin{vmatrix} 7 & 6 \\ 4 & 8 \end{vmatrix} - 2\begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix} + 6\begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix}$$

$$= -86$$

$$z = \frac{|Z|}{|A|} = \frac{-86}{-86} = 1$$

Q9

$$A = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

a) $$A + B = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+5 & 7-9 \\ 6+4 & 3+2 & 1+6 \\ 2+8 & 9+9 & 5+7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{bmatrix}$$

b) $$A - B = \begin{bmatrix} 4-3 & 6-5 & 7-(-9) \\ 6-4 & 3-2 & 1-6 \\ 2-8 & 9-9 & 5-7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 16 \\ 2 & 1 & -5 \\ -6 & 0 & -2 \end{bmatrix}$$

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c) $AB = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 4\times3+6\times4+7\times8 & 4\times5+6\times2+7\times9 & 4\times-9+6\times6+7\times7 \\ 6\times3+3\times4+1\times8 & 6\times5+3\times2+1\times9 & 6\times-9+3\times6+1\times7 \\ 2\times3+9\times4+5\times8 & 2\times5+9\times2+5\times9 & 2\times-9+9\times6+5\times7 \end{bmatrix}$$

d) $BA = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix} \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix}.$

$$= \begin{bmatrix} 3\times4+5\times6+-9\times2 & 3\times6+5\times3+-9\times9 & 3\times7+5\times1+-9\times5 \\ 4\times4+2\times6+6\times2 & 4\times6+2\times3+6\times9 & 4\times7+2\times1+6\times5 \\ 8\times4+9\times6+7\times2 & 8\times6+9\times3+7\times9 & 8\times7+9\times1+7\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & -48 & -19 \\ 40 & 84 & 60 \\ 100 & 138 & 100 \end{bmatrix}$$

e) Commutative law doesn't hold for multiplication of matrices as we saw the result of AB and BA, they are not the same whereas commutative law does hold for addition of Matrix $A+B = B+A$.

$A+B = \begin{bmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{bmatrix}$

$B+A = \begin{bmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{bmatrix}.$

Q1 a) $A^{-1}A$ $\quad \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix}$

determinant

$$|A| = 4\begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix} - 6\begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix} + 7\begin{bmatrix} 6 & 3 \\ 2 & 9 \end{bmatrix}$$

$$= 4(3\times5 - 9\times1) - 6(6\times5 - 2\times1) + 7(6\times9 - 2\times3)$$

$$= 24 - 168 + 336.$$

$$= 192.$$

$$\operatorname{adj}(A) = \begin{bmatrix} \begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix} & -\begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix} & \begin{bmatrix} 6 & 3 \\ 2 & 9 \end{bmatrix} \\ -\begin{bmatrix} 6 & 7 \\ 9 & 5 \end{bmatrix} & \begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix} & -\begin{bmatrix} 4 & 6 \\ 2 & 9 \end{bmatrix} \\ \begin{bmatrix} 6 & 7 \\ 3 & 1 \end{bmatrix} & -\begin{bmatrix} 4 & 7 \\ 6 & 1 \end{bmatrix} & \begin{bmatrix} 4 & 6 \\ 6 & 3 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -28 & 48 \\ -2 & 6 & -24 \\ -15 & +38 & -24 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -15 \\ -28 & 6 & +38 \\ 48 & -24 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{192}\begin{bmatrix} 0 & 2 & -15 \\ 28 & 6 & -38 \\ 48 & 24 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{192} & \frac{2}{192} & \frac{-15}{192} \\ \frac{28}{192} & \frac{6}{192} & \frac{-38}{192} \\ \frac{48}{192} & \frac{24}{192} & \frac{-24}{192} \end{bmatrix} = \begin{bmatrix} \frac{1}{32} & \frac{11}{64} & -\frac{5}{64} \\ \frac{-7}{48} & \frac{1}{32} & \frac{19}{96} \\ \frac{1}{4} & \frac{-1}{8} & \frac{-1}{8} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{1}{32} & \frac{11}{64} & -\frac{5}{64} \\ -\frac{7}{48} & \frac{1}{32} & \frac{19}{96} \\ \frac{1}{4} & -\frac{1}{8} & -\frac{1}{8} \end{bmatrix} \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{32}\times 4 + \frac{11}{64}\times 6 + -\frac{5}{64}\times 2 & \frac{1}{32}\times 6 + \frac{11}{64}\times 3 + -\frac{5}{64}\times 9 & \frac{1}{32}\times 7 + \frac{11}{64}\times 1 + -\frac{5}{64}\times 5 \\ -\frac{7}{48}\times 4 + \frac{1}{32}\times 6 + \frac{19}{96}\times 2 & -\frac{7}{48}\times 6 + \frac{1}{32}\times 3 + \frac{19}{96}\times 9 & -\frac{7}{48}\times 7 + \frac{1}{32}\times 1 + \frac{19}{96}\times 5 \\ \frac{1}{4}\times 4 + -\frac{1}{8}\times 6 + -\frac{1}{8}\times 2 & \frac{1}{4}\times 6 + -\frac{1}{8}\times 3 + -\frac{1}{8}\times 9 & \frac{1}{4}\times 7 + -\frac{1}{8}\times 1 + -\frac{1}{8}\times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $$AA^{-1} = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{32} & \frac{11}{64} & -\frac{5}{64} \\ -\frac{7}{48} & \frac{1}{32} & \frac{19}{96} \\ \frac{1}{4} & -\frac{1}{8} & -\frac{1}{8} \end{bmatrix}.$$

$$= \begin{bmatrix} 4\times\frac{1}{32} + 6\times -\frac{7}{48} + 7\times\frac{1}{4} & 4\times\frac{11}{64} + 6\times\frac{1}{32} + 7\times -\frac{1}{8} & 4\times -\frac{5}{64} + 6\times\frac{19}{96} + 7\times -\frac{1}{8} \\ 6\times\frac{1}{32} + 3\times -\frac{7}{48} + 1\times\frac{1}{4} & 6\times\frac{11}{64} + 3\times\frac{1}{32} + 1\times -\frac{1}{8} & 6\times -\frac{5}{64} + 3\times\frac{19}{96} + 1\times -\frac{1}{8} \\ 2\times\frac{1}{32} + 9\times -\frac{7}{48} + 5\times\frac{1}{4} & 2\times\frac{11}{64} + 9\times\frac{1}{32} + 5\times -\frac{1}{8} & 2\times -\frac{5}{64} + 9\times\frac{19}{96} + 5\times -\frac{1}{8} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

c) $B^{-1}B$.

determinant of $|B|$ $= 3\begin{vmatrix} 2 & 6 \\ 9 & 7 \end{vmatrix} - 5\begin{vmatrix} 4 & 6 \\ 2 & 7 \end{vmatrix} + (-9)\begin{vmatrix} 4 & 2 \\ 8 & 9 \end{vmatrix}$

$= 3(2\times 7 - 9\times 6) - 5(4\times 7 - 2\times 6) - 9(4\times 9 - 8\times 2)$

$= -200$

$$\operatorname{adj}(B) = \begin{bmatrix} \begin{vmatrix} 2 & 6 \\ 9 & 7 \end{vmatrix} & \begin{vmatrix} 4 & 6 \\ 8 & 7 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 8 & 9 \end{vmatrix} \\ \begin{vmatrix} 5 & -9 \\ 9 & 7 \end{vmatrix} & \begin{vmatrix} 3 & -9 \\ 8 & 7 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 8 & 9 \end{vmatrix} \\ \begin{vmatrix} 5 & -9 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 3 & -9 \\ 4 & 6 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -40 & -20 & 20 \\ 116 & 93 & -13 \\ 48 & 54 & -14 \end{bmatrix} = \begin{bmatrix} -40 & 116 & 48 \\ -20 & 93 & 54 \\ 20 & -13 & -14 \end{bmatrix}.$$

$$B^{-1} = \frac{1}{-200} \begin{bmatrix} -40 & 116 & 48 \\ -20 & 93 & 54 \\ 20 & -13 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & \frac{29}{50} & -6/25 \\ -1/10 & -93/200 & 27/100 \\ -1/10 & -13/200 & 7/100 \end{bmatrix}$$

$$B^{-1}B = \begin{bmatrix} 1/5 & 29/50 & -6/25 \\ -1/10 & -93/200 & 27/100 \\ -1/10 & -13/200 & 7/100 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

d) $$B B^{-1} = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix} \begin{bmatrix} 1/5 & 29/50 & -6/25 \\ -1/10 & -93/200 & 27/100 \\ -1/10 & -13/200 & 7/100 \end{bmatrix}$$

$$-1 + 0 + 0 + 1 = 0$$

$$0 = 0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

e) $(A + B)^2$.

$$\left(\begin{pmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{pmatrix} \right)^2.$$

$$\begin{pmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{pmatrix}^2$$

$$\begin{pmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{pmatrix} \begin{pmatrix} 7 & 11 & -2 \\ 10 & 5 & 7 \\ 10 & 18 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 139 & 96 & 39 \\ 190 & 261 & 99 \\ 370 & 416 & 250 \end{pmatrix}$$