Information Technology University, Lahore, Pakistan

Linear Algebra (CE)

Assignment # 1, Spring 2024

February 5, 2024

Submission Deadline: Thursday, February 15, 2024 Maximum Marks: 100

- Late submissions will not be graded.
- This Assignment will be conducted under the rules and guidelines of the ITU Honour Code, and no cheating will be tolerated (i.e., no discussion about the Assignment with other students, no plagiarism at all). Each student must be able to justify his/her work.

Question 1

Find nonzero vectors **x, y, z** that are perpendicular to the vector (3, 3, 3, 3) and each other. [10]

Question 2

Given vectors

$$\mathbf{u} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

1. Calculate dot products $\mathbf{u}.\mathbf{v}, \mathbf{v}.\mathbf{w}, \mathbf{u}.(\mathbf{v}+\mathbf{w})$ and $\mathbf{w}.\mathbf{v}$ for the above vectors. [5]
2. Find $\cos\theta$ for each case. [5]
3. Find unit vectors in the direction of $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$. [5]

Question 3

Given vectors

$$\mathbf{w1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w2} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad \mathbf{w3} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

Find $x\mathbf{w1} + y\mathbf{w2} + z\mathbf{w3}$ that gives zero vector with $x = 1$. Also, check whether they are dependent or independent. [10]

Question 4

Given vectors [10]

$$\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

Find u, v, and w such that

$$u\mathbf{a} + v\mathbf{b} + w\mathbf{c} = \mathbf{x}$$

Question 5

Give the proof of the inequality $|u \cdot U| \leq 1$ for unit vectors (u1, u2) = (0.6,0.8) and (U1, U2) = (0.8, 0.6) also find $\cos\theta$. [20]

$|u \cdot U| \leq |u1||U1| + |u2||U2| \leq \frac{u_1^2+U_1^2}{2} + \frac{u_2^2+U_2^2}{2} = 1$

Question 6

The given system of equations is: [20]

$$\begin{cases} -3x + 2y - 6z = 6 \\ 5x + 7y - 5z = 6 \\ x + 4y - 2z = 8 \end{cases}$$

Solve the above equations using Matrix Multiplication.

Question 7

Consider the following Matrices:

$$A = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

Find

1. $A \times B$ [5]
2. $B^{-1} \times B$ [5]
3. $(A + B)^2$ [5]