Assignment #2
Discrete Structures.

Q1

a. $\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$

$\forall x$: for every student x.

$C(x)$: x has a computer.

$\exists y$: There exists a student y.

$C(y)$: y has a computer

$F(x,y)$: x & y are friends.

For every student x, x has a computer or there ~~is a~~ exists a student y. such that y has a computer and x and y are friends.
Every student has a computer or is friends with a student who has a computer.

b. $\forall x \exists y (x = -y)$

$\forall x$: for all x $\Rightarrow$ $\neg \forall x$: there exists an x, which is equal to $\exists x$

$\exists y$: There exists a y $\Rightarrow$ $\neg \exists y$: for all y, which is equal to $\forall y$

$x = -y$: x is equal to $-y$ $\Rightarrow$ $x \neq -y$: x is not equal to $-y$.

Negation of $\forall x \exists y (x = -y)$ is $\exists x \forall y (x \neq -y)$.
There exists an x such that for all y, x is not equal to negative y.

Q2

a. $x = y + 1$

$Z(1,3)$ $\qquad 1 \neq 3 + 1$

The statement $Z(1,3)$ is false.

The statement $Z(2,1)$ is the proposition "$2 = 1+1$", which is true.

b. $\forall x \forall y (x+y = y+x)$.
$\forall x$: for all real numbers x
$\forall y$: for all real numbers y
$x+y = y+x$: x plus y is equal to y plus x.

For all real numbers x and y, x plus y is equal to y plus x.

c. $\forall x \exists y (x = -y)$
$\forall x$: for all real numbers x
$\exists x$: There exists a real number y
$x = -y$: x equals negative y.

For all real numbers x, there exists a real number y such that x equals negative y.

Q3

m : I go to the movies.
h : I do my homework.
$\sim h$: I didn't do my homework.
$\sim m$: I didn't go to the movies.

$$\begin{array}{l} m \to \sim h \\ \underline{h \qquad\quad} \\ \therefore \sim m. \end{array}$$

This is valid by modus tollens

$$\begin{array}{ll} T & m^F \to \sim h^F \\ \underline{T} & \underline{h^T \qquad\quad} \\ T & \therefore \sim m^T \end{array}$$

→ If h is true, then $\sim h$ is false. If $\sim h$ than to make this argument $m \to \sim h$, true, then m would be false. meaning $\sim m$ is true.
This argument is valid.

Q4

1. A : Allen is a good boy
~A : Allen is a bad boy
H : Hillary is a good girl.
D : David is happy.

$\sim A \vee H$
$A \vee D$
$\therefore H \vee D$. argument is valid according to Resolution Rule.

2.

a. $\forall x$: Everyone
$P(x)$: x enrolled in university.
$Q(x)$: x lives in the dormitory.

Step	Reason
(1) $\forall x(P(x) \to Q(x))$	Premise.
(2) $P(Mia) \to Q(Mia)$	Universal instantiation from (1)
(3) $\sim Q(Mia)$	Premise.
(4) $\sim P(Mia)$	Modus tollens from (2) and (3).

$\forall x\,(P(x) \to Q(x))$
$\sim Q(Mia)$
$\therefore \sim P(Mia)$, this is therefore valid.
The argument is correct.

b. c : convertible car
f : fun to drive.
~c : not a convertible car.
$C(x)$: x is a convertible car
$F(x)$: x is fun to drive.

(1) $\forall x(C(x) \to F(x))$ premise
(2) $\forall C(Issac) \to F(Issac)$ universal instantiation from (1)
(3) $\sim C(Issac)$ Premise.

$[(p \to q) \wedge \sim p] \to \sim q$

This argument is not correct. After applying the universal instantiation, it contains the fallacy of denying the hypothesis.

c. $A(x)$: x is an action movie.
$Q(x)$: Quincy likes x movies.
$\forall x$: all movies.

$\forall x(A(x) \to Q(x))$
T T $A(\text{Eight men Out}) \to Q(\text{Eight}...)$ T
T $Q(\text{Eight men Out})$
F $\therefore A(\text{Eight men Out})$

(1) $\forall x\ (A(x) \to Q(x).)$ Premise.
(2) $A(\text{Eight men out}) \to Q(\text{Eight men out})$. Universal instantiation from (1).
(3) $Q(\text{Eight men out})$ Premise.

Incorrect Argument because it is in the form of $[(p \to q) \wedge q] \to p$ which uses the fallacy of ~~stating~~ affirming the conclusion.

d. $\forall x$: All x.
$L(x)$: ~~lob~~ x lobsterman
$T(x)$: sets x downtraps.

(1) $\forall(x)\ (L(x) \to T(x))$ Premise
(2) $L(\text{Hamilton}) \to T(\text{Hamilton})$ universal instantiation from (1)
(3) $L(\text{Hamilton})$ Premise.
(4) $T(\text{Hamilton})$ Modus ponens from (2) and (3).

$\forall x(L(x) \to T(x))$
$L(\text{Hamilton})$
$\therefore T(\text{Hamilton})$ This argument is correct.

(a) $\exists x P(x,3)$.

$P(1,3) \vee P(2,3) \vee P(3,3)$.

Inorder for this expression $\exists x P(x,3)$ to be true, any one of the following propositional functions has to be true.

(b) $\forall x P(1,y)$.

$P(1,1) \wedge P(1,2) \wedge P(1,3)$.

In order for the expression $\forall x P(1,y)$ to be true, all of the propositional functions has to be true.

(c) $\exists y \neg(P(2,y))$

$\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

In order for the $\exists x \neg P(2,y)$ to be true., any one of the propositional function has to be true.

(d) $\forall x \neg P(x,2)$

$\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

For the $\forall x \neg P(x,2)$ to be true, all of the propositional function has to be true.

Q6 1. $P(x,y)$: student x has taken class y.

a. $\exists x \exists y P(x,y)$

$\exists x$: There is a student in your class.

$\exists y$: ~~There exist~~ that has taken a computer science course at your school.

There exists a student in your class who has taken a computer science course at your school.

b. $\exists x \forall y\, P(x,y)$
There exists a student in your class who has taken all the computer science courses at your school.

c. $\forall x \exists y\, P(x,y)$
For every ~~All the~~ student x in your class, there exists a student that at least one has taken all the computer science courses at your school.

d. $\exists y \forall x\, P(x,y)$
~~There exists a student in your class who has taken all the computer science courses at your school.~~
There exists a computer science course ~~such~~ at your school such that ~~e.~~ ~~$\forall x$~~ all the students in your class has taken that course.

e. $\forall y \exists x\, P(x,y)$
For every computer science course at your school, there exists at least one student in your class that has taken the course.

f. $\forall x \forall y\, P(x,y)$
For every student x in your class and for every computer science course y at your school, student x has taken course y.

2.

$\exists x$: There exists an integer x.
$\exists x(x>0)$: There exists a positive integer x.
sum of three integers : $a^2+b^2+c^2$
$x \neq a^2+b^2+c^2$: positive integer x that is not the sum of the three squares

$$\exists x\,[(x>0) \wedge \forall a \forall b \forall c\,(x \neq a^2+b^2+c^2)]$$

There exists an integer x that is positive & not equal to the sum of the three squares a^2, b^2, c^2.