Information Technology University, Lahore, Pakistan

Linear Algebra (MT-121)

Assignment # 2, Fall 2024

Submission Deadline: Friday September 13, 2024 Maximum Marks: 100

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1. Find the values the variables a, b, c, d, e, f, g, h and i in each case below.

   (a) [4]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ y+z \end{bmatrix}$$

   (b) [4]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z-x \end{bmatrix}$$

   (c) [4]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 8 & 1 & 3 \\ 15 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 3 \\ 7 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

   (d) [4]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -4 \\ 0 & -2 & -6 \end{bmatrix}$$

   (e) [4]

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

2. Apply Guassian elimination and back substitution to solve the set of linear equations:

   (a) [10]

$$\begin{aligned} 2x - 3y + 1 &= 2 \\ 4x - 5y + z &= 5 \\ 2x - y - 3z &= 5 \end{aligned}$$

   (b) [10]

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 - x_2 + 3x_3 &= 5 \\ 4x_1 + 5x_2 + x_3 &= 3 \end{aligned}$$

3. Inverse matrices **A** and **B** using the Gauss-Jordan method. [10 + 10 = 20]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

4. Given matrices **A** and **B**,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

show that

(a) $(AB)^{-1} = B^{-1}A^{-1}$ [10]

(b) $(AB)(B^{-1}A^{-1}) = I$ [10]

5. (a) Which three matrices E_{21}, E_{31}, E_{32} put **A** into triangular form U and $(E_{32}E_{31}E_{21})\mathrm{A} = \mathrm{U}$? [12]

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

(b) Given a matrix

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

i. Find its transpose A^T. [2]

ii. Prove that $\mathrm{A}A^T \neq A^T A$. [6]