Information Technology University, Lahore, Pakistan

Linear Algebra (MT-121)

Assignment # 1, Fall 2024

Submission Deadline:
Tuesday September 03, 2024 (BSSE23-A) at the start of the class.
Wednesday September 04, 2024 (BSSE23-B) at the start of the class.
Maximum Marks: 100

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1. Find nonzero vectors **x, y, z** that are perpendicular to the vector (1, 2, 3, 4) and each other. [10]

2. Given vectors

$$\mathbf{u} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

   (a) Calculate dot products $\mathbf{u}.\mathbf{v}, \mathbf{u}.(\mathbf{v}+\mathbf{w})$ and $\mathbf{w}.\mathbf{v}$ for the above vectors. [3]

   (b) Find $\cos\theta$ for each case. [3]

   (c) Find unit vectors in the direction of **u** and **v**. [4]

3. Given vectors

$$\mathbf{w1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w2} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad \mathbf{w3} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

   Find $x\mathbf{w1} + y\mathbf{w2} + z\mathbf{w3}$ that gives zero vector with $x = 1$. Also, check whether they are dependent or independent. [10]

4. Given vectors [10]

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

   Find a, b, and c such that

$$a\mathbf{u} + b\mathbf{v} + c\mathbf{w} = \mathbf{x}$$

5. Given the vectors **a** and **b**:

$$\mathbf{a} = \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix},$$

   (a) Find the norm (i.e., the length) of the column vectors **a** and **b**. [2]

   (b) Find a vector **c** that is perpendicular to both the vectors **a** and **b**. [4]

   (c) Find a vector **d** that is perpendicular to all the three vectors **a**, **b** and **c**. [4]

6. Prove the following properties of the dot product.

   (a) $(\mathbf{u}+\mathbf{v}).\mathbf{w} = \mathbf{u}.\mathbf{w} + \mathbf{v}.\mathbf{w}$ [5]

   (b) $c\mathbf{u}.\mathbf{v} = c(\mathbf{u}.\mathbf{v}) = \mathbf{u}.c\mathbf{v}$ [5]

7. Given the system of linear equations

$$\begin{cases} -3x + 2y - 6z = 6 \\ 5x + 7y - 5z = 6 \\ x + 4y - 2z = 8 \end{cases}$$

   (a) Write it in the matrix form as Ax = b. [2]
   (b) Find the inverse of the matrix A. [4]
   (c) Solve for the variables x, y and z. [4]

8. Solve the system of linear equations of Q. 7 using Cramer's Rule. [10]

9. Consider the following Matrices:

$$A = \begin{bmatrix} 4 & 6 & 7 \\ 6 & 3 & 1 \\ 2 & 9 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 2 & 6 \\ 8 & 9 & 7 \end{bmatrix}$$

   Find

   (a) The sum matrix A + B [2]
   (b) The difference matrix (A − B) [2]
   (c) The product matrix AB [2]
   (d) The product matrix BA [2]
   (e) Does the commutative law hold for addition of matrices? Does it hold for multiplication of matrices? [2]

10. For the matrices of Q. 9, find

   (a) $A^{-1}A$ [2]
   (b) AA^{-1} [2]
   (c) $B^{-1}B$ [2]
   (d) BB^{-1} [2]
   (e) $(A + B)^2$ [2]