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Asg 3
13SSE23058
Section A.

Assignment #3
Calculus & Analytical Geometry.

Q1

a. $\int 4x^6 - 2x^3 + 7x - 4 \, dx.$

$\frac{4x^7}{7} - \frac{2x^4}{4} + \frac{7x^2}{2} - 4x + C$

$\frac{4x^7}{7} - \frac{x^4}{2} + \frac{7x^2}{2} - 4x + C.$

b. $\int 6\cos z + \frac{4}{\sqrt{1-z^2}} \, dz$

$6\sin z + 4\sin^{-1} z + C.$

1c. $\int 2\cos w - \sec w \tan w \, dw$

$\int 2\cos w \, dw - \int \sec w \tan w \, dw$

$2\sin w - \sec w + C$

Q2

a. $\int 4x\cos(2-3x) \, dx.$

$4\left(x\int \cos(2-3x) - \int x' \int \cos(2-3x)\right) + C.$

$4\left(x \times \frac{\sin(2-3x)}{-3}\right) - \int (1) \frac{\sin(2-3x)}{-3} + C.$

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$$-\frac{4x\sin(2-3x)}{3} - 4\left(\frac{-\cos(2-3x)}{-3x-3}\right) + C$$

$$-\frac{4x\sin(2-3x)}{3} + \frac{4\cos(2-3x)}{9} + C$$

b. $\int_6^0 (2+5x)e^{\frac{1}{3}x}\,dx$

$$\Big|_6^0\ (2+5x)\int e^{\frac{1}{3}x} - \int (2+5x)'\int e^{\frac{1}{3}x}\Big|$$

$$\Big|_6^0\ \frac{(2+5x)e^{\frac{1}{3}x}}{1/3} - \int \frac{5e^{\frac{1}{3}x}}{1/3}\Big|$$

$$\Big|_6^0\ 3(2+5x)e^{\frac{1}{3}x} - 15\int e^{\frac{1}{3}x}\Big|$$

$$\Big|_6^0\ 3\times 2e^{\frac{1}{3}x} + 15xe^{\frac{1}{3}x} - \frac{15e^{\frac{1}{3}x}}{1/3}\Big|$$

$$\Big|_6^0\ 6e^{\frac{1}{3}x} + 15xe^{\frac{1}{3}x} - 45e^{\frac{1}{3}x}\Big|$$

$$\Big|_6^0\ 15xe^{\frac{1}{3}x} - 39e^{\frac{1}{3}x}\Big|$$

$$15(0)e^{\frac{1}{3}\times 0} - 39e^{\frac{1}{3}\times 0} - \left[15(6)e^{\frac{6}{3}} - 39e^{\frac{6}{3}}\right]$$

$$0 - 39 - 90e^{2} + 39e^{2}$$

$$-39 - 51e^{2}$$

$$\approx 416$$

Q3. a. $\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx.$

$\int \frac{7x}{\sqrt{1-25x^2}}\,dx + \int \frac{2}{\sqrt{1-25x^2}}\,dx$

$7\int \frac{x}{\sqrt{1-25x^2}}\,dx + 2\int \frac{1}{\sqrt{1-25x^2}}$

$u = 1-25x^2$

$\frac{du}{dx} = -25 \times 2x$

$= -50x$

$-\frac{1}{50x}\,du = dx$

$-\frac{7}{50}\int \frac{\cancel{x}}{\sqrt{u}} \times \frac{1}{\cancel{x}}\,du + 2\left(\frac{\sin^{-1}(5x)}{5}\right)$

$-\frac{7}{50}\int u^{-1/2}\,du + \frac{2\sin^{-1}(5x)}{5}$

$-\frac{7\times 2}{50}\,u^{1/2} + \frac{2\sin^{-1}(5x)}{5}$

$-\frac{7}{25}\sqrt{1-25x^2} + \frac{2\sin^{-1}(5x)}{5}$

b. $\int \sec^2(v)e^{1+\tan(v)}\,dv$

$u = 1+\tan v$

$\frac{du}{dv} = \sec^2 v.$

$\frac{1}{\sec^2 v}\,du = dv$

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$\int \cancel{\sec^2 v}\, e^u \times \frac{1}{\cancel{\sec^2 v}}\, du$

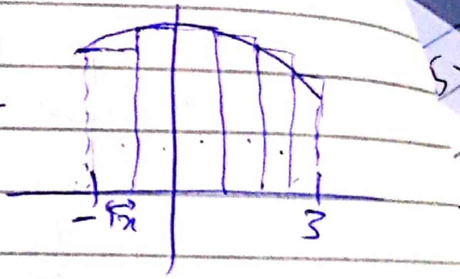
$\int e^u\, du$

e^u

$e^{1+\tan v}$

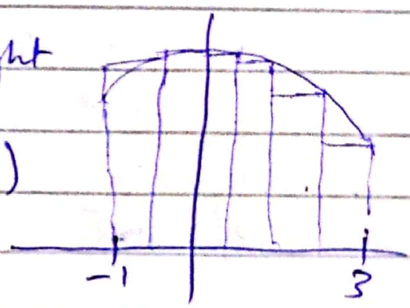
Q4 $f(x) = 4 - \sqrt{x^2+2}$ on $[-1, 3]$

1. Area $= \sum_{i=1}^{n} \Delta x\, f(x_i)$ Left



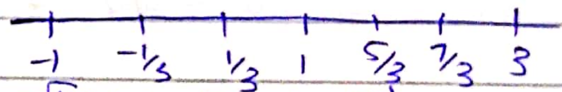
$\Delta x = \frac{b-a}{n}$ $n = 6$

$= \frac{3-(-1)}{6} = \frac{2}{3}$ Right



$\text{Area}_R = \frac{2}{3}\left(f(-\tfrac{1}{3}) + f(\tfrac{1}{3}) + f(1) + f(\tfrac{5}{3}) + f(\tfrac{7}{3}) + f(3)\right)$

~~$= \frac{2}{3}(12.7155)$~~



$= \frac{2}{3}\left(\frac{12-\sqrt{19}}{3} + \frac{12-\sqrt{19}}{3} + 4-\sqrt{3} + \frac{12-\sqrt{43}}{3} + \frac{12-\sqrt{67}}{3} + 4-\sqrt{11}\right)$

$= 7.42$

x	y
-1	$4-\sqrt{3}$
$-\frac{1}{3}$	$\frac{12-\sqrt{19}}{3}$
$\frac{1}{3}$	$\frac{12-\sqrt{19}}{3}$
1	$4-\sqrt{3}$
$\frac{5}{3}$	$\frac{12-\sqrt{43}}{3}$
$\frac{7}{3}$	$\frac{12-\sqrt{67}}{3}$
3	$4-\sqrt{11}$

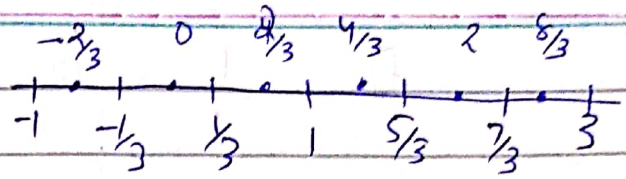
2. $\Delta x = \frac{2}{3}$

$\text{Area}_L = \frac{2}{3}\left(f(-1) + f(-\tfrac{1}{3}) + f(\tfrac{1}{3}) + f(1) + f(\tfrac{5}{3}) + f(\tfrac{7}{3})\right)$

$= \frac{2}{3}(12.7155)$

$= 8.48$

3. $A_m = \Delta x \sum_{i=1}^{4} f(x_i)$



$\Delta x = \frac{2}{3}$

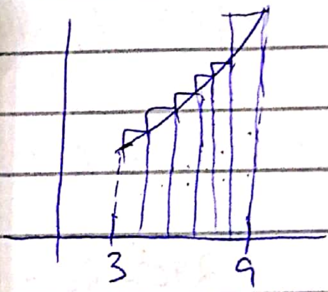
$A_m = \frac{2}{3}\left(f(-\tfrac{2}{3}) + f(0) + f(\tfrac{2}{3}) + f(\tfrac{4}{3}) + f(2) + f(\tfrac{8}{3})\right)$

$= \frac{2}{3}\left(\frac{12-\sqrt{22}}{3} + 4-\sqrt{2} + \frac{12-\sqrt{22}}{3} + \frac{12-\sqrt{34}}{3} + 4-\sqrt{6} + \frac{12-\sqrt{82}}{3}\right)$

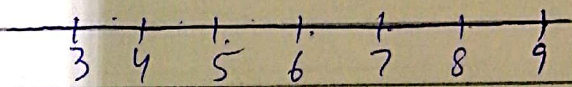
$= \frac{2}{3} \times (12.047)$

$= 8.03.$

Q5 $\Delta x = \cancel{\frac{9-3}{6}} / \cancel{\frac{3}{6}} = \frac{9-3}{6} = 1$



$\int_3^9 (x^2-1)\,dx.$



Area $R = 1\left[\cancel{f(3)} \cdot f(4) + f(5) + f(6) + f(7) + f(8) + f(9)\right)$

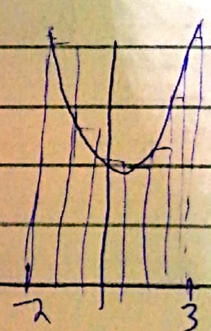
$= 1\left[15 + 24 + 35 + 48 + 63 + 80\right)$

$= 265$

Q6 $f(x) = 4x^2 - x + 5$ On $[-2, 3]$.

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{3-(-2)}\int_{-2}^{3} 4x^2 - x + 5$ Left



$= \frac{1}{5}\int_{-2}^{3} 4x^2 - x + 5$

$$f_{avg} = \frac{1}{5} \times \left| \frac{4x^3}{3} - \frac{x^2}{2} + 5x \right|_{-2}^{3}$$

$$= \frac{1}{5}\left[\left(\frac{4(3)^3}{3} - \frac{(3)^2}{2} + 5(3)\right) - \left(\frac{4(-2)^3}{3} - \frac{(-2)^2}{2} + 5(-2)\right)\right]$$

$$= \frac{1}{5}\left[\frac{93}{2} - \left(-\frac{68}{3}\right)\right]$$

$$= \frac{83}{6} \approx 13.8$$

$f(c) = f_{avg}$

$$4c^2 - c + 5 = \frac{83}{6}$$

$$24c^2 - 6c + 30 = 83$$

$$24c^2 - 6c - 53 = 0$$

$$c = \frac{+6 \pm \sqrt{(-6)^2 - 4(24)(-53)}}{2(24)}$$

$c = 1.62$ $\quad c = -1.37$

Q7 $\quad y = \frac{1}{x+2} \quad y = (x+2)^2 \quad x = -3/2, 1$

$$\text{Area} = \int_{-3/2}^{+1} \frac{1}{x+2} - (x+2)^2 \, dx$$

$$= \int \frac{1}{x+2}\, dx - \int (x+2)^2\, dx$$

$$= \left| \ln(x+2) - \frac{(x+2)^3}{3} \right|_{-3/2}^{1}$$

$$= \left(\ln(1+2) - \frac{(1+2)^3}{3} \right) - \left(\ln\left(\frac{-3}{2}+2\right) - \frac{\left(\frac{-3}{2}+2\right)^3}{3} \right)$$

$$= \left(\ln(3) - 9 - \ln\left(\frac{1}{2}\right) + \frac{1}{24} \right)$$

$$= \ln\left(3 \div \frac{1}{2}\right) - 9 + \frac{1}{24}$$

$$= \ln(6) - \frac{215}{24}$$

$$= -7.17.$$