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BSSE23058

Assignment # 3
Probability & Statistics

3.43

$$f(x,y) = \begin{cases} 4xy, & 0 < x < 1, \; 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) $P\left(0 \le X \le \frac{1}{2} \quad \& \quad \frac{1}{4} \le Y \le \frac{1}{2}\right)$

$$\int_{0}^{1/2} \int_{1/4}^{1/2} 4xy \; dy \; dx$$

$$4 \int_{0}^{1/2} x \int_{1/4}^{1/2} (y) \; dy \; dx$$

$$4 \int_{0}^{1/2} x \left(\left. \frac{y^2}{2} \right|_{1/4}^{1/2} \right) dx$$

$$4 \int_{0}^{1/2} x \times \left(\frac{(1/2)^2}{2} - \frac{(1/4)^2}{2} \right)$$

$$4 \int_{0}^{1/2} x \times \left(\frac{1}{8} - \frac{1}{32} \right)$$

$$4 \int_{0}^{1/2} \frac{3}{32} x$$

$$4 \left(\left. \frac{\frac{3x^2}{32}}{2} \right|_{0}^{1/2} \right) = 4 \left(\frac{3}{64} \left(\frac{1}{2}\right)^2 - \frac{3}{64} \left((0)\right)^2 \right)$$

$$= 4 \left(\frac{3}{256} - 0 \right)$$

$$= \frac{3}{64}$$

b) $P(X < Y) = \int_0^1 \int_0^y 4xy \, dx \, dy$

$$= 4 \int_0^1 y \int_0^y x \, dx \, dy$$

$$= 4 \int_0^1 yx \left(\left. \frac{x^2}{2} \right|_0^y \right) dy$$

$$= 4 * \int_0^1 yx \left(\frac{y^2}{2} - \frac{0^2}{2} \right) dy$$

$$= 4 \int_0^1 \frac{y^3}{2} \, dy$$

$$= \frac{4}{2} \int_0^1 y^3 \, dy$$

$$= 2 \int_0^1 y^3 \, dy$$

$$= 2 * \left(\left. \frac{y^4}{4} \right|_0^1 \right)$$

$$= 2 * \left(\frac{(1)^4}{4} - \frac{(0)^4}{4} \right)$$

$$= 2 \left(\frac{1}{4} \right) = \boxed{\frac{1}{2}}$$