Question 1.

1.1

According to the formula on the slide:

$$R_\infty = 1 - S(0)e^{-R_0(R_\infty - R(0))}$$

$R_\infty = 0.6 \quad R(0) = 0 \quad S(0) = 0.98$

$$0.98^{-R_0(0.6-0)} = 0.4$$

$$R_0 = 75.59$$

1.2.1

$$P(t) = \frac{Y(t)\beta}{N}$$

1.2.2

$E[X(t+1)]$

$= E[X(t)] - P(t) \times E[X(t)]$

$= E[X(t)] - \dfrac{E[Y(t)] \cdot E[X(t)]\beta}{N}$

$E[Y(t+1)]$

$= E[Y(t)] - \gamma E[Y(t)] + P(t) \times E[X(t)]$

$= E[X(t)] - \gamma E[Y(t)] + \dfrac{E[Y(t)] \times E[X(t)]\beta}{N}$

$E[Z(t+1)]$

$= E[Z(t)] - \gamma E[Y(t)]$

1.2.3

Given $E[X(t)] \approx X(t)$

$$X(t+1) - X(t) = -\frac{Y(t)X(t)\beta}{N}$$

$$Y(t+1) - Y(t) = \frac{Y(t)X(t)\beta}{N} - \gamma Y(t)$$

$$Z(t+1) - Z(t) = \gamma Y(t)$$

1.2.4

$$X(t+\Delta t) - X(t) = -\frac{Y(t)X(t)\beta}{N} \times \Delta t$$

$$Y(t+\Delta t) - Y(t) = \frac{Y(t)X(t)\beta}{N} \cdot \Delta t - \gamma Y(t)\Delta t$$

$$Z(t+\Delta t) - Z(t) = \gamma Y(t)\Delta t$$

1.2.5

Divide by Δt at both side

$$\frac{X(t+\Delta t) - X(t)}{\Delta t} = -\frac{Y(t)X(t)\beta}{N}$$

$$Y(t+\Delta t) - Y(t) = \frac{Y(t)X(t)\beta}{N} - \gamma Y(t)$$

$$Z(t+\Delta t) - Z(t) = \gamma Y(t)$$

When $\lim_{\Delta t \to 0}$

$$\frac{dX}{dt} = -Y(t)X(t)\beta$$

$$\frac{dY}{dt} = Y(t)X(t)\beta - \gamma Y(t)$$

$$\frac{dZ}{dt} = \gamma Y(t)$$

As X, Y, Z correspond to SIR

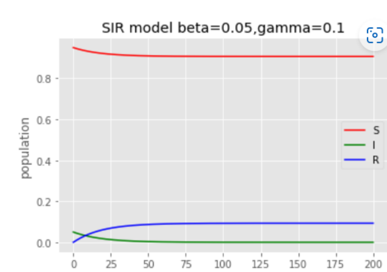
$$\frac{dS}{dt} = -SI\beta$$

$$\frac{dI}{dt} = SI\beta - \delta I$$

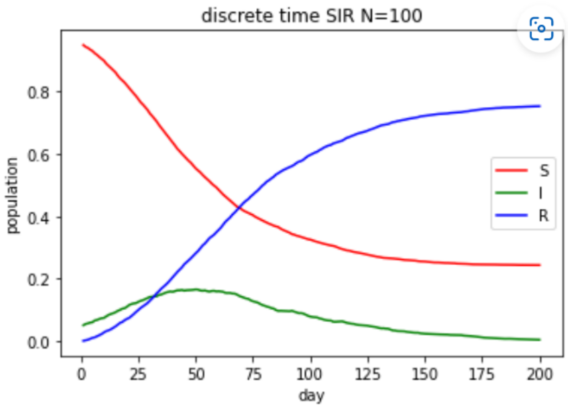
$$\frac{dR}{dt} = \partial I$$

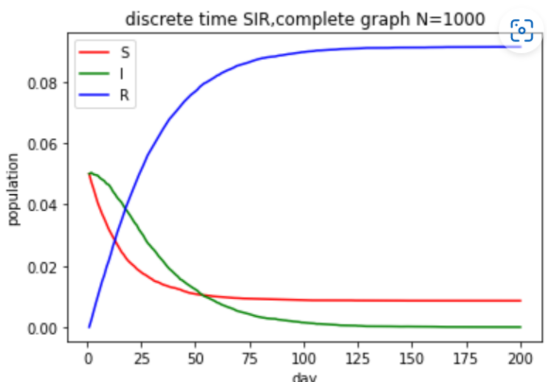
1.3.1

When $\beta > \gamma$ R will be greater than S at some point, and remain that way, but when $\gamma > \beta$, recover is faster than infection, so the virus would die out and R never greater than S, S remains high.

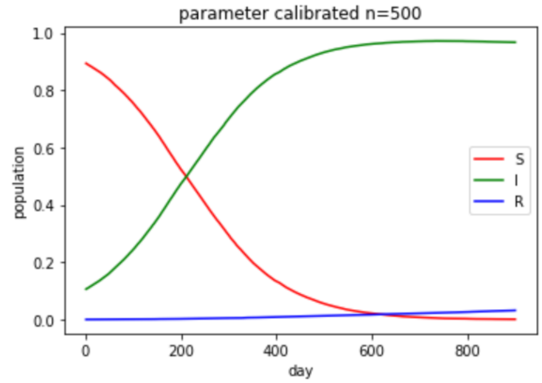


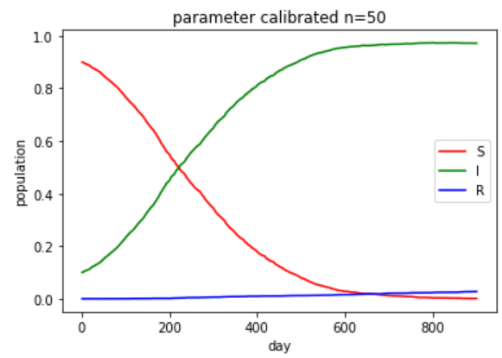
1.3.2





1.3.4



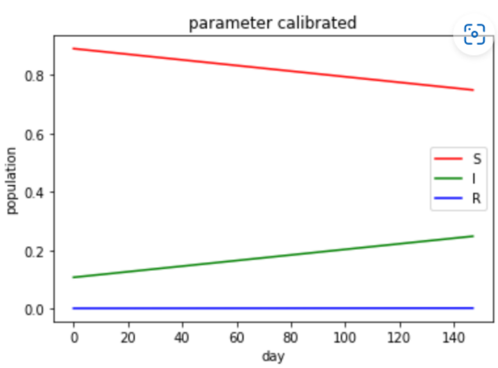


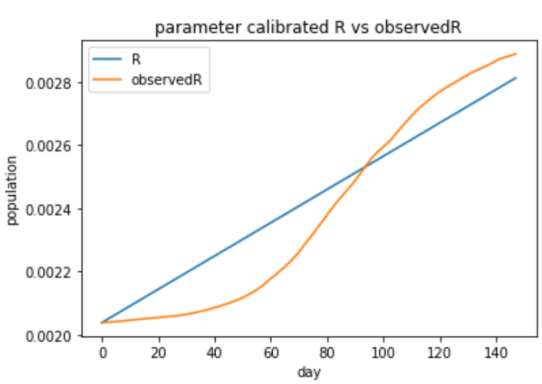
For different N, there are little difference, because using 50 nodes and 50 repeats are accurate enough. Between different ODEs, in 1.3.4 the graph is nonlinear, but in 1.3.3 the graph is linear.

1.3.3

$\beta = 0.01$

$\gamma = 4.8792e^{-5}$





Question 2

2.1

when $k=3$

d	num of node
1	3
2	6
3	12

Except at $d=1$, each step forward add num of previous step's nodes $\times$ num of new nodes reached from each previous depth nodes (that is $k-1$)

The general formula is $k(k-1)^{d-1}$

2.2

Consider the relationship between depth and diameter (use l for easier representation)

$l = 2d$

so now I need to express d in terms of k and n

$$n = \sum_{d=1} k(k-1)^{d-1}$$

$$n = k \sum_{d=1} (k-1)^{d-1}$$

$$= k\,\frac{1-(k-1)^d}{2-k}$$

$$(k-1)^d = 1 - \frac{(2-k)n}{k}$$

$$d = \frac{\ln\left(1-\frac{n(2-k)}{k}\right)}{\ln(k-1)}$$

$$l = \frac{2\ln\left(1-\frac{n(2-k)}{k}\right)}{\ln(k-1)}$$

2.3

Originally the diameter is l, adding a node and connect it to each other node cannot increase the longest path, so diameter remains the same

Adding an edge will maximumly create a $2d+1$ length of cycle

Question 3

3.1

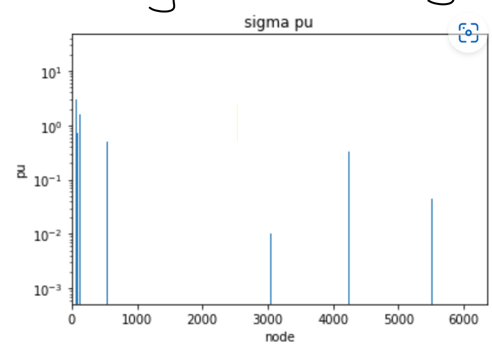
The best condition is given v and u are connected

and all u are not infected before v

so, $P^*_{vu} = \arg\max_{p_{vu}} L = \frac{A_{v2u}}{A_v}$

3.3

The weight on nodes is very uneven, shows spontaneous peaks.



Question 4

4.1

Goal is get minimum of

$$\left(40 - \frac{x}{200000}\right) \times \left(0.04 - \frac{y}{100000000}\right)$$

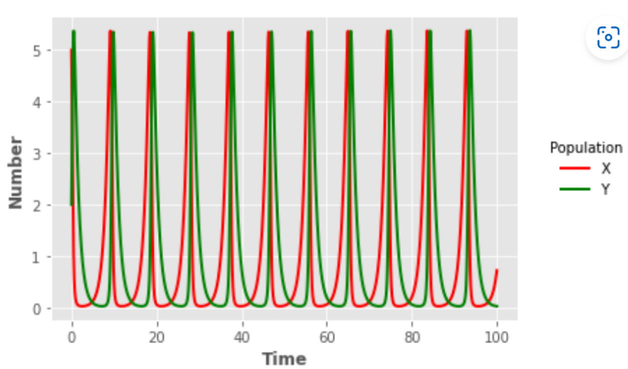
$$x + y = 2000000$$

$$\min \left(40 - \frac{x}{200000}\right) \times \left(0.04 - \frac{2000000 - x}{100000000}\right) \quad 0 \le x \le 2000000$$

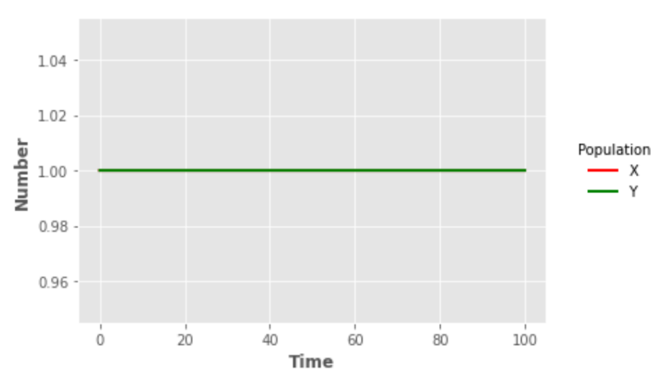
$x = 0 \quad y = 2000000$

Put all the money to control transmission

4.2.1



When $x(0) = y(0) = 1$



4.2.2

Fixed point is at $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$

$\alpha x = \beta y + \phi z$

$\gamma xy = \delta y$

$x = \frac{\delta}{\gamma}$ (if $y \neq 0$)

$\rho xz = \epsilon z$

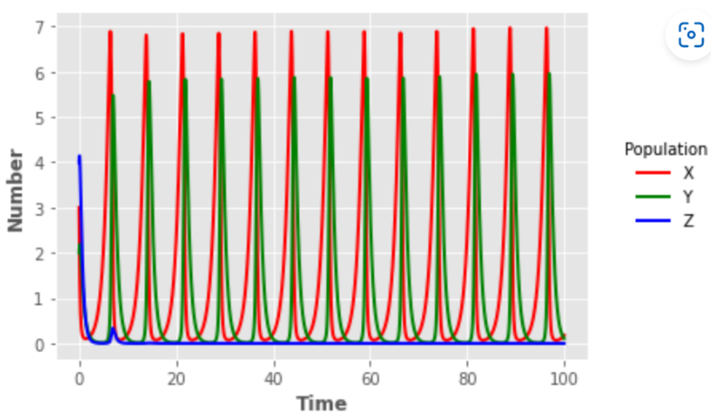
$x = \frac{\epsilon}{\rho}$ (if $z \neq 0$)

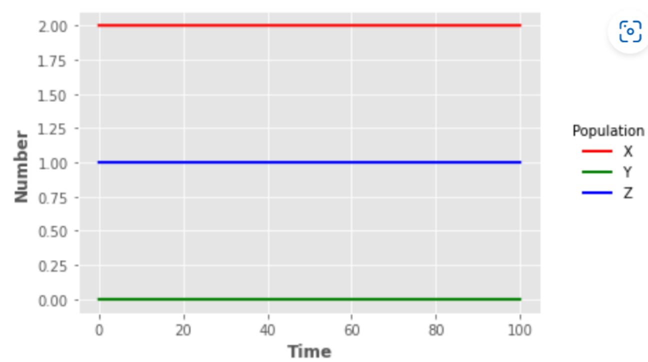
4.2.3

$x(0), y(0), z(0) = 2, 0, 1$

so that under the constants,

$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$





4.2.4

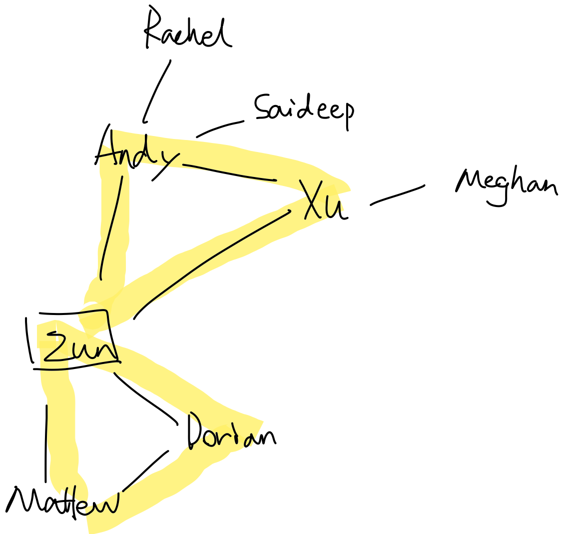
x and y are periodic, but z is not,

show variant 2 is not competitive

enough so it soon die out

Question J

1.



2. I am in 2 complete triangles

3. I am in 3. incomplete triangles