Recommender Systems

Lecture 4

Previous lecture

Lifecycle of a recsys experiment

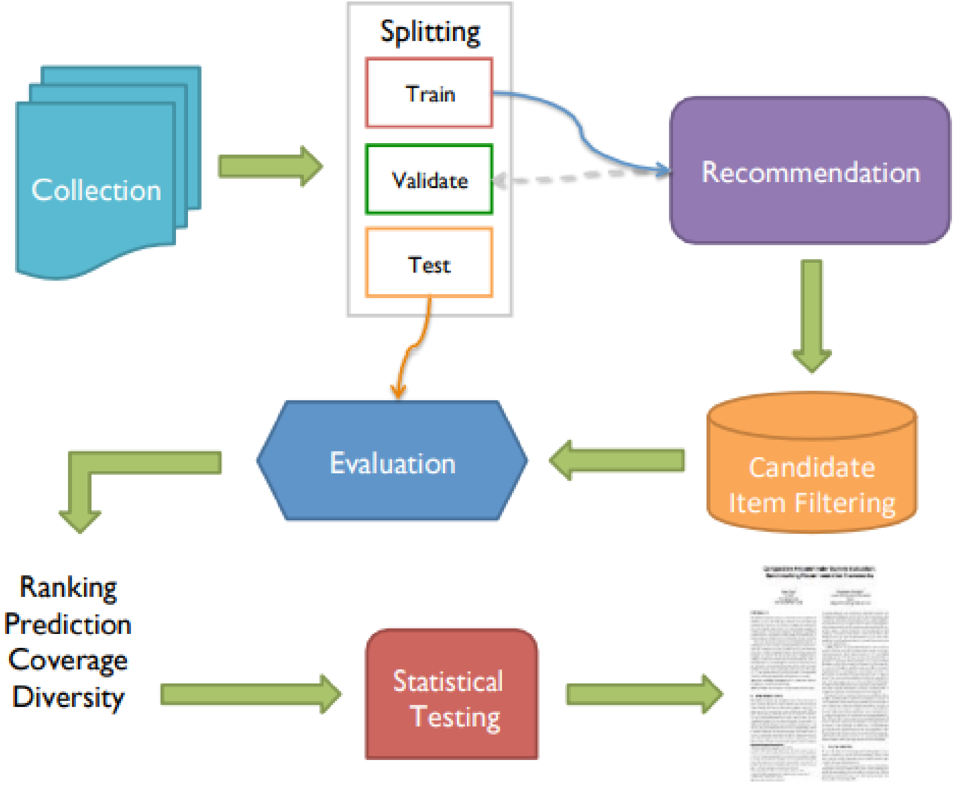
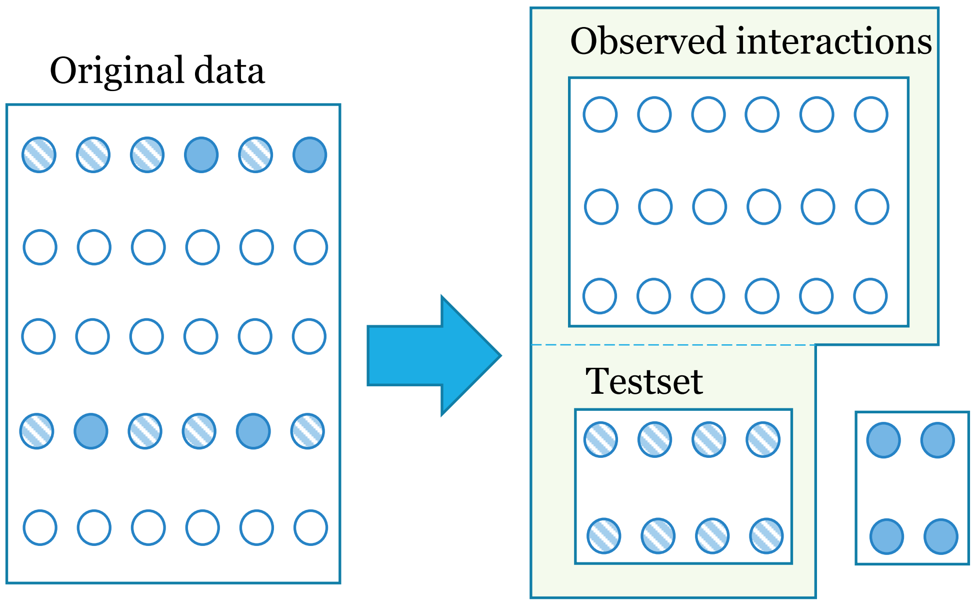


Image source: Bellogín, Alejandro, and Alan Said. "*Improving accountability in recommender systems research through reproducibility*." User Modeling and User-Adapted Interaction (2021): 1-37.

Data splitting



Splitting options:

- entry- or user-wise
- warm start
  - strong/weak generalization

Holdout sampling

Strategies

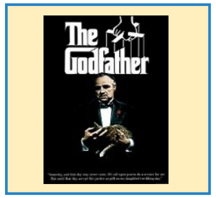
- Random
- Rating-based (e.g., top-rated)
- Temporal (e.g., most recent)

Sample size

- Fixed number of items (e.g., 1)
- Fixed percentage of items

Previous lecture

User

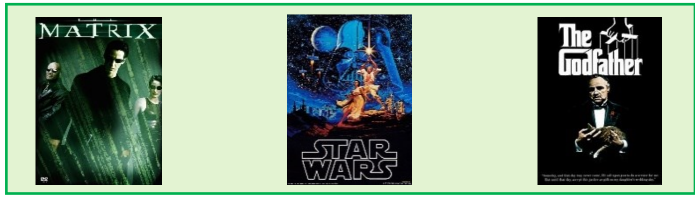


holdout

known user preferences

recommendations

Algorithm:



$$\mathrm{HR} = \frac{1}{\#(\text{test users})} \sum_{\substack{\text{test}\\ \text{users}}} \text{hit}$$

$$\text{hit} = \begin{cases} 1 & \text{if holdout item is in recommended items,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{MRR} = \frac{1}{\#(\text{test users})} \sum_{\substack{\text{test}\\ \text{users}}} \frac{1}{\text{hit rank}}$$

hit rank = position of the item in the recommendations list

Typically computed: metric@n, where n = #recommended items, e.g. Recall@n, MRR@n, etc.

Task

Consider top-2 recommender for 10 users from 20-items (2 items per user).

What's better:
1) Correctly recommend 2 items to each of 5 users?
or
2) 1 item to each of 10 users?

Why?

Use standard definition of precision and recall.

Today's lecture

Collaborative filtering

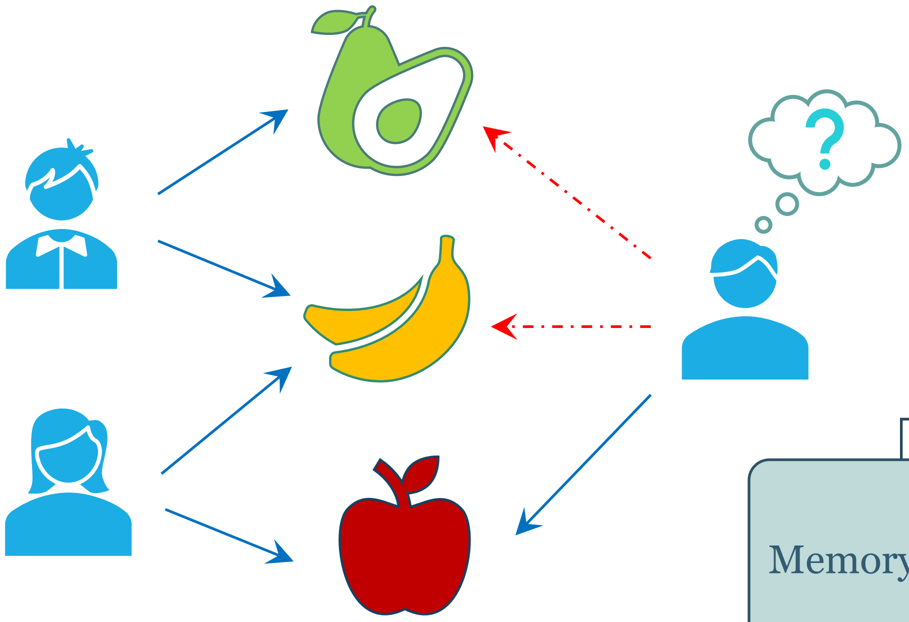
- memory-based approach
  - frequent pattern mining
  - nearest neighbors models

Previously: content-based approach

- **Issues**
  - Hard to gather information
  - **Over-specialization**
    - *too general* (e.g. only best-selling books)
    - *too narrow* (e.g., only movies of one director)
  - May require expert knowledge
  - Outperformed by hybrid systems

Collaborative Filtering: "wisdom of crowds"

- **Non-personalized**
  - popularity models
  - baseline predictors



- **Personalized**
  - Memory-based
    - item-based
    - user-based
  - Model-based
    - latent factors models

General workflow

Goal: predict user preferences based on prior user feedback and collective user behavior.

collect data

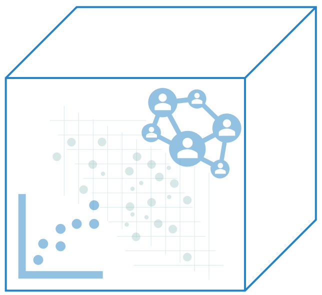
user-movie matrix A of size $M \times N$

a_{ij} is a rating of i^{th}user for j^{th} movie

? - missing (unknown) values

build model

f_U: User × Item → Relevance



generate recommendations

unknown user

"Customers who like … also like …"

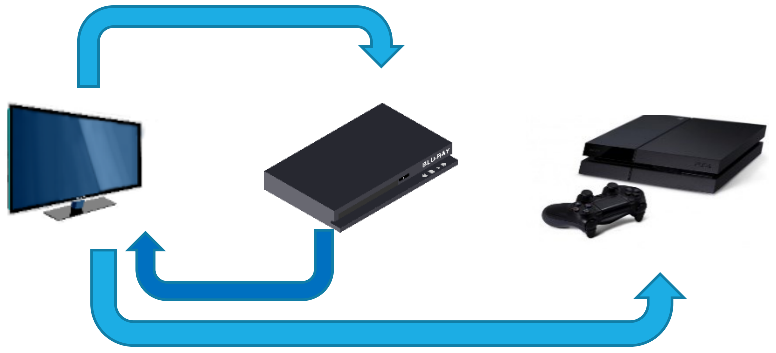
How do we implement that logic?

Pure item-to-item (I2I)

Typical transactions log:

user id	item id	transact.
0	575	view
0	1881	view
0	846	basket
1	1878	purchase
1	576	view
…	…	…

Count co-occurrence of items:



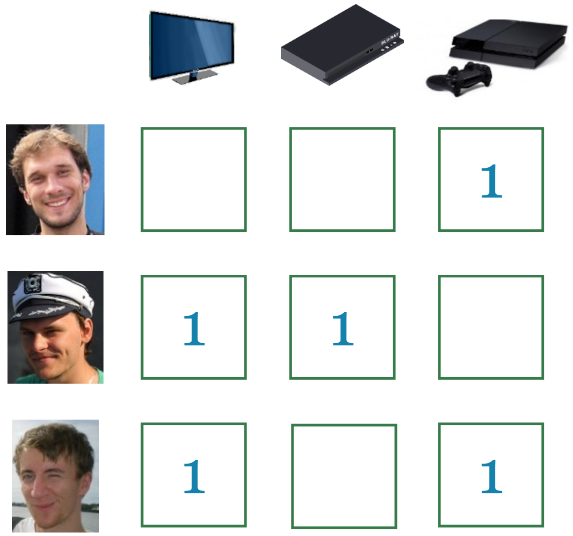
$$\mathrm{score}_{\mathrm{I2I}}(u,i) = \sum_{\substack{j \in I_u \\ j \neq i}} \mathrm{pairCount}(i,j)$$

Item-to-item is a strong baseline on very "sparse" datasets.

Simplest item-to-item approach

Convenient representation of logs– sparse matrix

user id	item id	transact.
0	575	view
0	1881	view
0	846	basket
1	1878	purchase
1	576	view



- Can be efficiently stored in CSR or CSC formats.
- Also enables efficient computations (especially useful for experiments).

Computing I2I scores

- How to compute item-to-item co-occurrence matrix in symmetric case?
- How to compute similarity scores in that case?

Computing I2I scores

$$C = A^\top A - \operatorname{diag}\left(\operatorname{diag}(A^\top A)\right)$$

If p is a vector of known user preferences,
then the vector of predicted relevance scores is:

$$r = Cp$$

Recommendations:

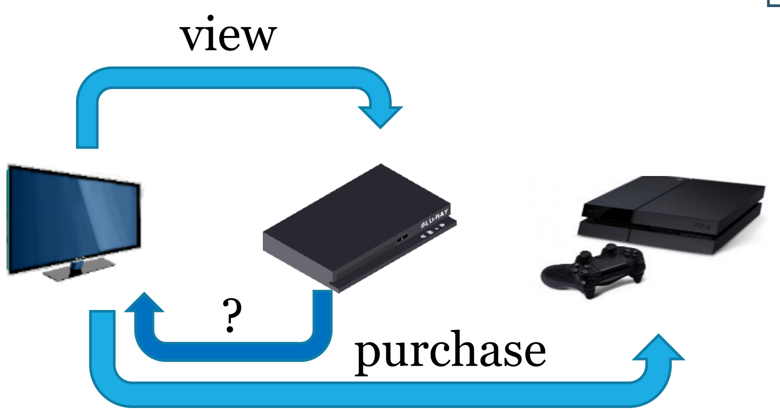
$$\operatorname{toprec}(n) \coloneqq \operatorname*{arg\,max}^{n}_{j} r_j$$

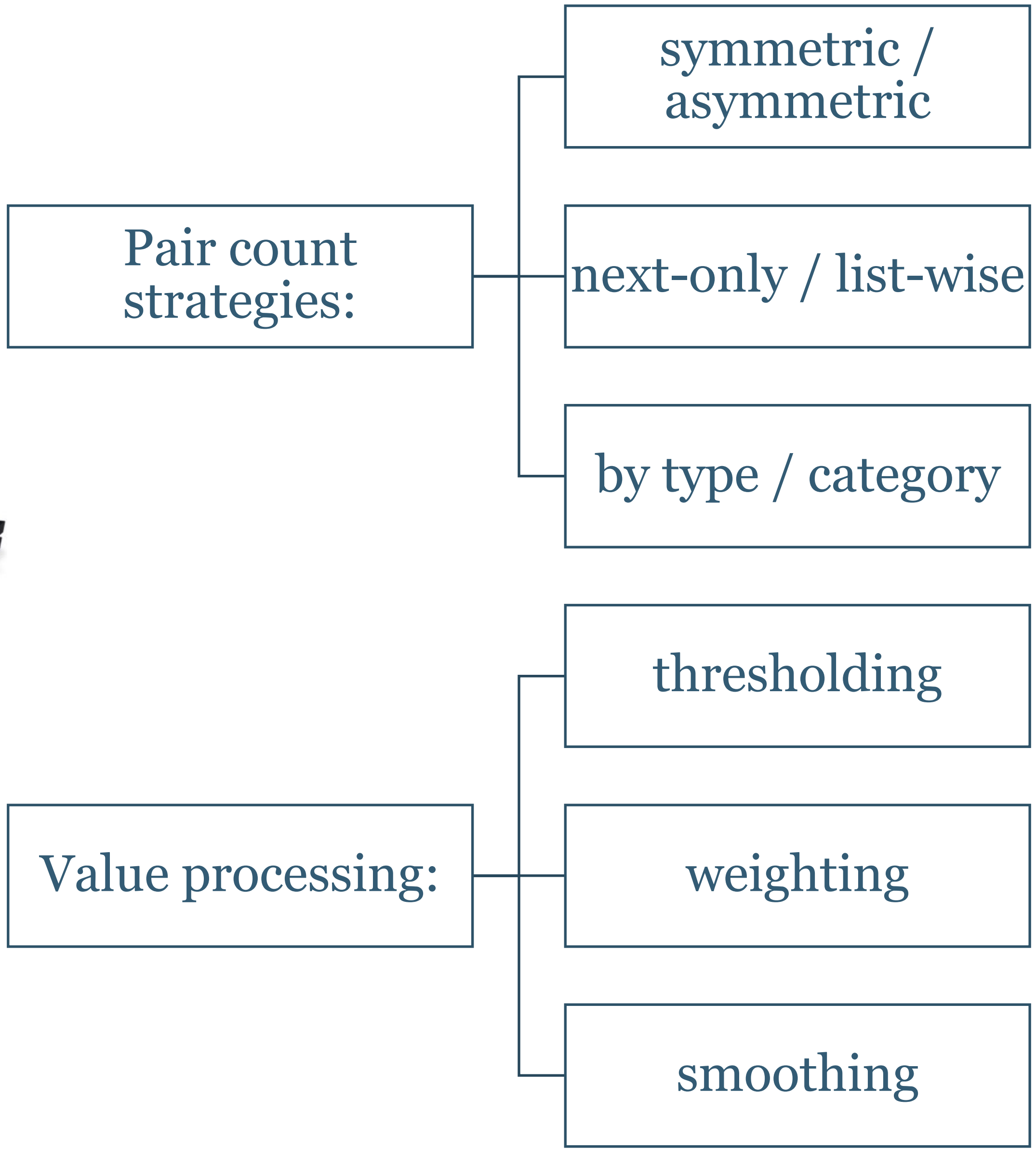
Complexity analysis

Item-to-item issues

- somewhat obvious recommendations
  - high influence of popular items
- i2i matrix can also become dense if there are too many interactions per user

Item-to-item variants

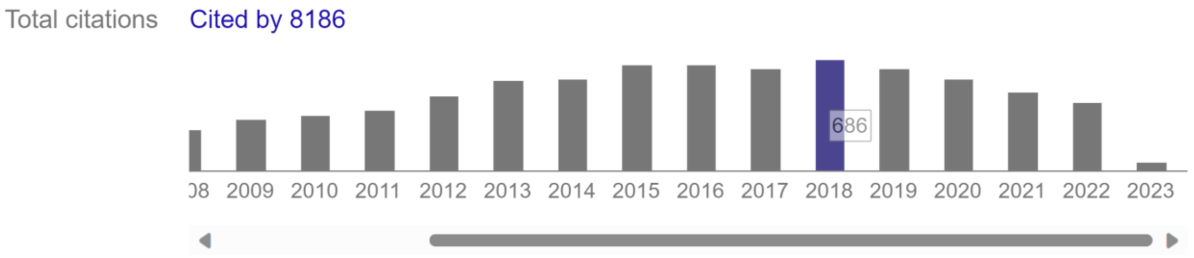




Case study: Amazon item-to-item

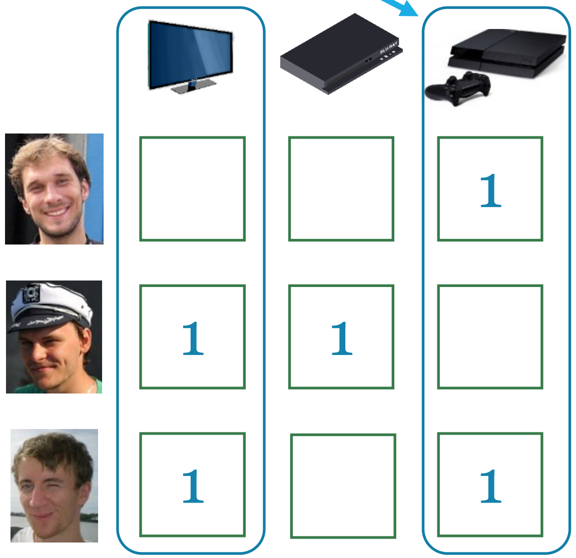
```
For each item in product catalog, I1
  For each customer C who purchased I1
    For each item I2 purchased by
      customer C
        Record that a customer purchased I1
        and I2
  For each item I2
    Compute the similarity between I1 and I2
```

G. Linden, B. Smith and J. York, "Amazon.com recommendations: item-to-item collaborative filtering," in IEEE Internet Computing, vol. 7, no. 1, pp. 76-80, Jan.-Feb. 2003.



Iterative algorithm

Computes similarity of items based on user purchases.



$\bar{\boldsymbol{a}}_1$ $\bar{\boldsymbol{a}}_2$

$$\mathrm{sim}(I_1, I_2) = \cos(\bar{\boldsymbol{a}}_1, \bar{\boldsymbol{a}}_2) = \frac{(\bar{\boldsymbol{a}}_1, \bar{\boldsymbol{a}}_2)}{\|\bar{\boldsymbol{a}}_1\|\|\bar{\boldsymbol{a}}_2\|}$$

$\bar{\boldsymbol{a}}_k$ - "one-hot" representation of item k

Scalability trick: incremental updates in binary case

$$\text{sim}(i,j) = \frac{\bar{\boldsymbol{a}}_i^\top \bar{\boldsymbol{a}}_j}{\|\bar{\boldsymbol{a}}_i\| \cdot \|\bar{\boldsymbol{a}}_j\|} = \frac{\text{pairCount}(i,j)}{\sqrt{\text{itemCount}(i)} \cdot \sqrt{\text{itemCount}(j)}}, \qquad j \neq i$$

$$\|\bar{\boldsymbol{a}}_i\|^2 = \sum_u a_{ui}^2 = \sum_u a_{ui} = \text{itemCount}(i) \qquad \bar{\boldsymbol{a}}_i^\top \bar{\boldsymbol{a}}_j = \sum_u a_{ui}\, a_{uj} = \text{pairCount}(i,j)$$

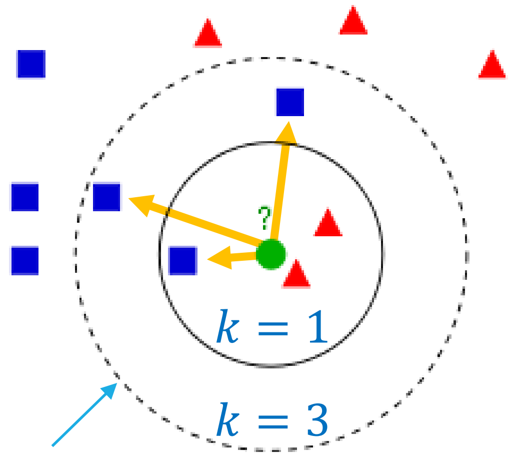
After observing ΔA new interactions s.t. $A' = A + \Delta A$, the updated similarity is:

$$\boxed{\text{sim}'(i,j) = \frac{\text{pairCount}(i,j) + \sum_u [\Delta A]_{ui}\, [\Delta A]_{uj}}{\sqrt{\text{itemCount}(i) + \sum_u [\Delta A]_{ui}} \cdot \sqrt{\text{itemCount}(j) + \sum_u [\Delta A]_{uj}}}}$$

Source: Huang, Yanxiang, Bin Cui, Wenyu Zhang, Jie Jiang, and Ying Xu. "Tencentrec: Real-time stream recommendation in practice." *In Proceedings of the 2015 ACM SIGMOD International Conference on Management of Data*, pp. 227-238. 2015.

Nearest neighbors models

kNN-based approach



neighborhood $\mathcal{N}_i(u)$

- user u
- neighbors of user u, who rated item i
- other users, who have not rated item i

User-based approach

- aggregated opinion of like-minded users:

$$\mathrm{score}_{\mathrm{uKNN}}(u, i) = \underset{v \in \mathcal{N}_i(u)}{\mathrm{agg}}\, a_{vi}$$

Item-based approach:

-

$$\mathrm{score}_{\mathrm{iKNN}}(u, i) =$$

Simple user-based kNN

$$\text{score}_{\text{uKNN}}(u, i) = \frac{1}{|\mathcal{N}_i(u)|} \sum_{v \in \mathcal{N}_i(u)} a_{vj}$$

Potential issues:

- users may have very different interests
- neighborhood size is unlimited

Improved user-based kNN

$$\text{score}_{\text{uKNN}}(u,i) = \frac{1}{z} \sum_{v \in \mathcal{N}_i(u)} \text{sim}(u,v) \cdot a_{vi}$$

$$\mathcal{N}_i(u) = U_i \setminus \{u\}, \qquad z = \sum_{v \in \mathcal{N}_i(u)} |\text{sim}(u,v)|$$

Potential issues:

- ~~other users may have very different interests~~
- large neighborhood size

Dealing with large neighborhood size

- Storing similarities or on-the-fly computations?
- Aggressive subsampling
- Approximate nearest neighbors
  - e.g., NMSLib, Faiss, Annoy
- Dimensionality reduction

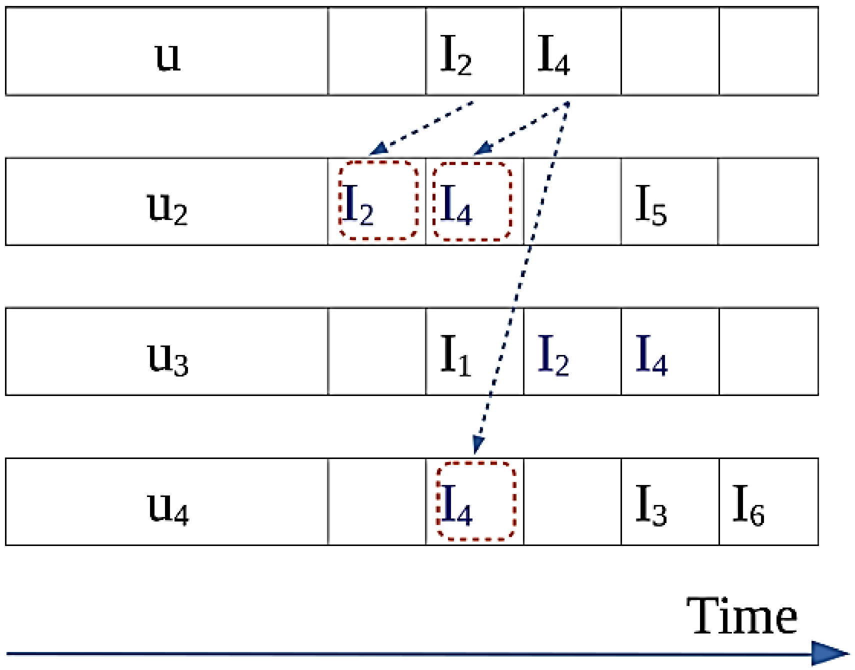
Reducing neighborhood

- from N total entities sample $n \ll N$
- select top-k most similar among n samples, $k \ll n$

Possible sampling strategies (must be fast):

- randomly
- most recent only
- most ratings in common (turns into fast MIPS problem)

Example: local time-aware sampling



Sampling strategy:

- select users that have items in common with a target user u
  - each item of a neighbour-user must precede the corresponding item in the target user profile
  - filter out neighbours with too few items in common

Additional weighting:

- users with no recent ratings → lower weights
- active neighbour-user but old rating on a target item → lower weights

	recent user ($t_0 \approx t_{u'l}$)	old user ($t_0 \gg t_{u'l}$)
recent item ($t_{u'l} \approx t_{u'i}$)	≈ 0	$t_0 - t_{u'l}$
old item ($t_{u'l} \gg t_{u'i}$)	$t_{u'l} - t_{u'i}$	$t_0 - t_{u'l}$

Images source and more details: Anelli, V. W., Di Noia, T., Di Sciascio, E., Ragone, A., & Trotta, J. (2019). Local popularity and time in top-N recommendation. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, *11437 LNCS*, 861–868

Centered kNN

- baseline estimators contain most of useful signal
- every user may have individual “rating scale”

$$\text{score}_{\text{uKNN}}(u,i) = \bar{a}_u + \frac{1}{z}\sum_{v\in\mathcal{N}_i(u)} \text{sim}(u,v)\cdot(a_{vi} - \bar{a}_v)$$

$\bar{a}_u$ - average rating of user u

item-based kNN:

$$\text{score}_{\text{iKNN}}(u,i) =$$

Centered kNN

- baseline estimators contain most of useful signal
- every user may have individual “rating scale”

user-based kNN:

$$\text{score}_{\text{uKNN}}(u,i) = \bar{a}_u + \frac{1}{z}\sum_{v\in\mathcal{N}_i(u)} \text{sim}(u,v)\cdot(a_{vi} - \bar{a}_v)$$

$\bar{a}_u$ - average rating of user u

item-based kNN:

$$\text{score}_{\text{iKNN}}(u,i) = \bar{a}_i + \frac{1}{z}\sum_{j\in\mathcal{N}_u(i)} \text{sim}(i,j)\cdot(a_{uj} - \bar{a}_j)$$

$\bar{a}_i$ - average rating of user i

Similarity measures

- Cosine Similarity
- Pearson Correlation
- Adjusted Cosine Similarity
- Jaccard Index
- Weighted Jaccard Index
- Asymmetric Similarities
- …
- Spearman's Rank Correlation
- Kendall Tau

insensitive to ranking of "bad" items vs "good " items

Baseline-adjusted similarity

- **Pearson correlation** (adopted for CF):

$$\text{score}_{\text{Pearson}}(u,v) = \frac{\sum_{i \in I_u \cap I_v} (a_{ui} - \bar{a}_u)(a_{vi} - \bar{a}_v)}{\sqrt{\sum_{i \in I_u \cap I_v} (a_{ui} - \bar{a}_u)^2} \cdot \sqrt{\sum_{i \in I_u \cap I_v} (a_{vi} - \bar{a}_v)^2}}$$

$\bar{a}_u$ - average rating of user u

- **Adjusted Cosine Similarity**:

$$\text{score}_{\text{AC}}(u,v) = \frac{\sum_{i \in I_u \cap I_v} (a_{ui} - \bar{a}_i)(a_{vi} - \bar{a}_i)}{\sqrt{\sum_{i \in I_u \cap I_v} (a_{ui} - \bar{a}_i)^2} \cdot \sqrt{\sum_{i \in I_u \cap I_v} (a_{vi} - \bar{a}_i)^2}}$$

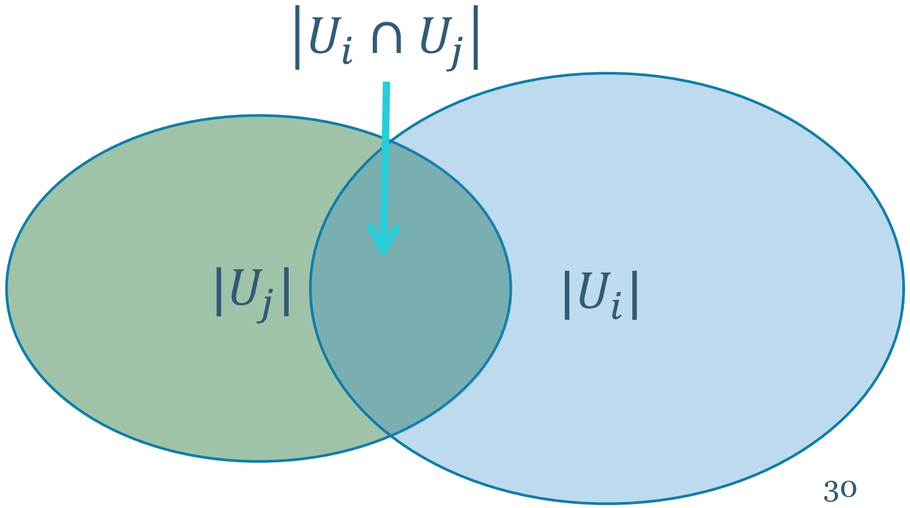
$\bar{a}_i$ - average rating of item i

Jaccard Index

Item-based similarity:

$$\mathrm{sim}_{\mathrm{JI}}(i,j) = \frac{\left|U_i \cap U_j\right|}{\left|U_i \cup U_j\right|}$$

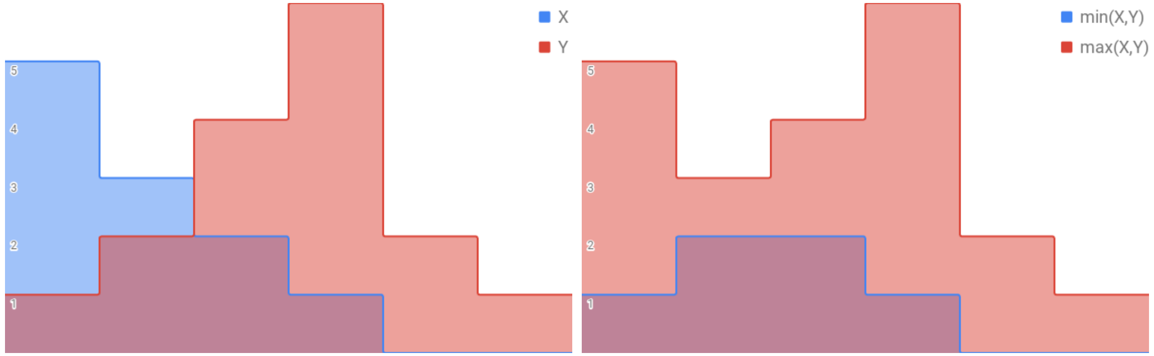
$$\left|U_i \cup U_j\right| = |U_i| + \left|U_j\right| - \left|U_i \cap U_j\right|$$

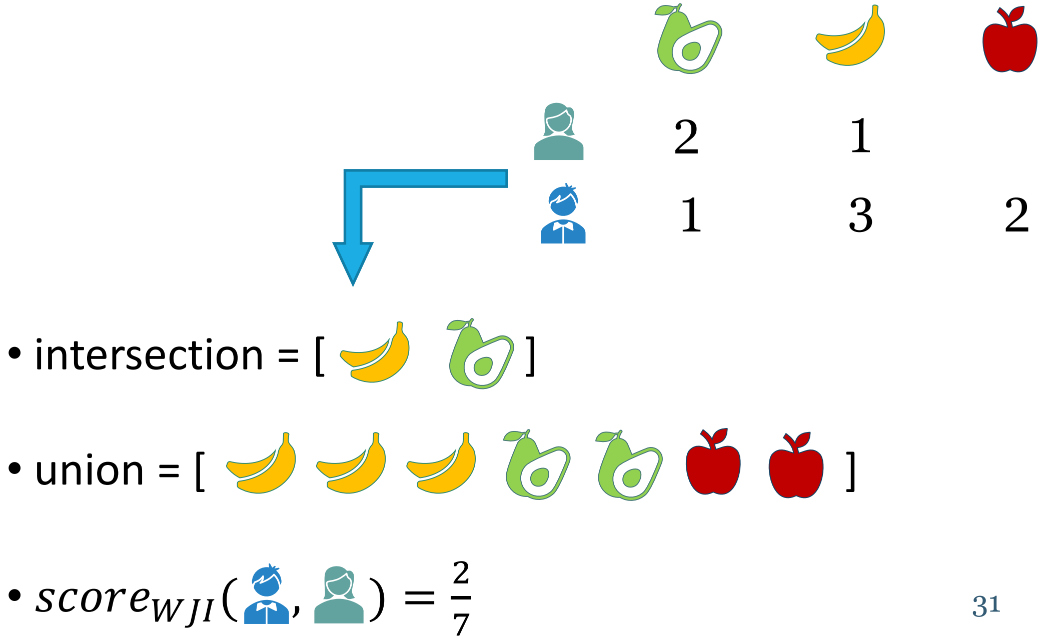


Weighted Jaccard Index

- Jaccard Index only operates on sets
- often some values are associated with interactions (e.g., ratings, frequencies)

$$\mathrm{sim}_{\mathrm{WJI}}(u,v) = \frac{\sum_{i=1}^{N} \min\{w_i(a_u), w_i(a_v)\}}{\sum_{i=1}^{N} \max\{w_i(a_u), w_i(a_v)\}}, \qquad w_i(a_u) = f(a_{ui})$$





Images from: https://moultano.wordpress.com/2018/11/08/minhashing-3kbzhsxyg4467-6/

kNN in matrix form

Element-wise weighting for user-based KNN:

$$r_{ui} = \frac{1}{z_{ui}} \cdot \sum_{v \in \mathcal{N}_i(u)} \mathrm{sim}(u, v) \cdot a_{vi}\,, \qquad \mathcal{N}_i(u) = U_i \backslash \{u\}$$

we impute A with 0's

kNN in matrix form

Row-wise weighting for user-based KNN:

$$r_{ui} = \frac{1}{z_{ui}} \cdot \sum_{v \in \mathcal{N}_i(u)} \mathrm{sim}(u, v) \cdot a_{vi}\,, \qquad \mathcal{N}_i(u) = U \backslash \{u\}$$

i.e., “explicit” 0’s

kNN weighting schemes

K – user similarity, $k_{ii} = 0, k_{ij} \geq 0, i \neq j$; S –item similarity matrix, $s_{ii} = 0, \;\; s_{ij} \geq 0, i \neq j$.

element-wise weighting:

- User-based:

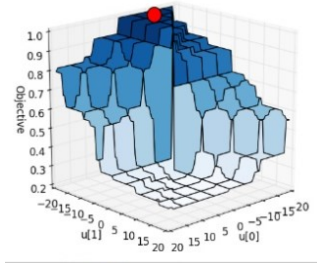
$$R = KA \oslash (KB)$$

$$b_{ui} = \begin{cases} 1, & \text{if } a_{ui} \text{ is known} \\ 0 & \text{otherwise} \end{cases}$$

- Item-based:

$$R = AS^\top \oslash (BS^\top)$$

- filters known ratings only
- better for rating prediction



row-wise / no weighting:

- User-based:

$$R = D_K^{-1} KA$$

$$D_K = \mathrm{diag}(K\boldsymbol{e}) \;\text{ or }\; D_K = I$$

$$\boldsymbol{e} = [1, 1 \dots, 1]^\top$$

- Item-based

$$R = AS^\top D_S^{-1}$$

$$D_S = \mathrm{diag}(S\boldsymbol{e}) \;\text{ or }\; D_S = I$$

- assumes 0-imputation of unknowns
- better for top-n recommendations

Let’s implement simple KNN models

User-based kNN for top-n recommendations

- Row-wise weighting:

$$R = D^{-1}KA$$

$$r_{ui} = w_u \cdot \sum_{v \in \mathcal{N}(u)} \mathrm{sim}(u, v) \cdot a_{vi}$$

- Alternative (column-wise) weighting:

$$R = KD^{-1}A$$

$$r_{ui} = \sum_{v \in \mathcal{N}(u)} \mathrm{sim}(u, v) \cdot w_v \cdot a_{vi}$$

- Is it different from the unweighted case?

kNN with asymmetric similarity

kNN similarity (e.g., item-based):

row-wise weighted symmetric $\rightarrow$ unweighted asymmetric

$$S_{\text{asym}} = D^{-\alpha} S$$

$$R = A S_{\text{asym}}^{\top}$$

Example: cosine similarity, assuming $d_{ii} = \|\bar{\boldsymbol{a}}_i\|$:

$$\text{sim}(i, j) = \left[S_{\text{asym}}\right]_{ij} = \frac{\bar{\boldsymbol{a}}_i^{\top} \bar{\boldsymbol{a}}_j}{\|\bar{\boldsymbol{a}}_i\|^{1+\alpha} \cdot \|\bar{\boldsymbol{a}}_j\|}$$

For binary data, $\alpha = -1$ gives a simple conditional probability $p(i|j)$

Popularity effect in asymmetric similarity

$$\mathrm{sim}(i,j) = [D^{-\alpha}S]_{ij} = \frac{\overline{\boldsymbol{a}}_i^\top \overline{\boldsymbol{a}}_j}{\|\overline{\boldsymbol{a}}_i\|^{1+\alpha} \cdot \|\overline{\boldsymbol{a}}_j\|}$$

What do we recommend?

popular item $\xrightarrow{\text{sim}}$ unpopular vs. unpopular $\xrightarrow{\text{sim}}$ popular item

i j i j

Popularity effect in asymmetric similarity

$$\text{popular item} \xrightarrow{\text{sim}} \text{unpopular} \qquad \text{vs.} \qquad \text{unpopular} \xrightarrow{\text{sim}} \text{popular item}$$

$\alpha < 0$

- Popular products → too trivial recommendations.
- Easy to guess but low value for users + low diversity.

$\alpha > 0$

- Recommending niche products increases diversity.
- For users with generic tastes may not fit well.

- <u>Observation</u>: popular items are not very descriptive of users interests.
- Suggest a normalization that would improve item-KNN recommendations.

New scheme – emphasizing contribution of specific user tastes:

$$S_{\text{asym}} = SD^{-\beta}, \qquad \text{sim}(i,j) = \frac{\bar{\boldsymbol{a}}_i^\top \bar{\boldsymbol{a}}_j}{\|\bar{\boldsymbol{a}}_i\| \cdot \|\bar{\boldsymbol{a}}_j\|^{1+\beta}}$$