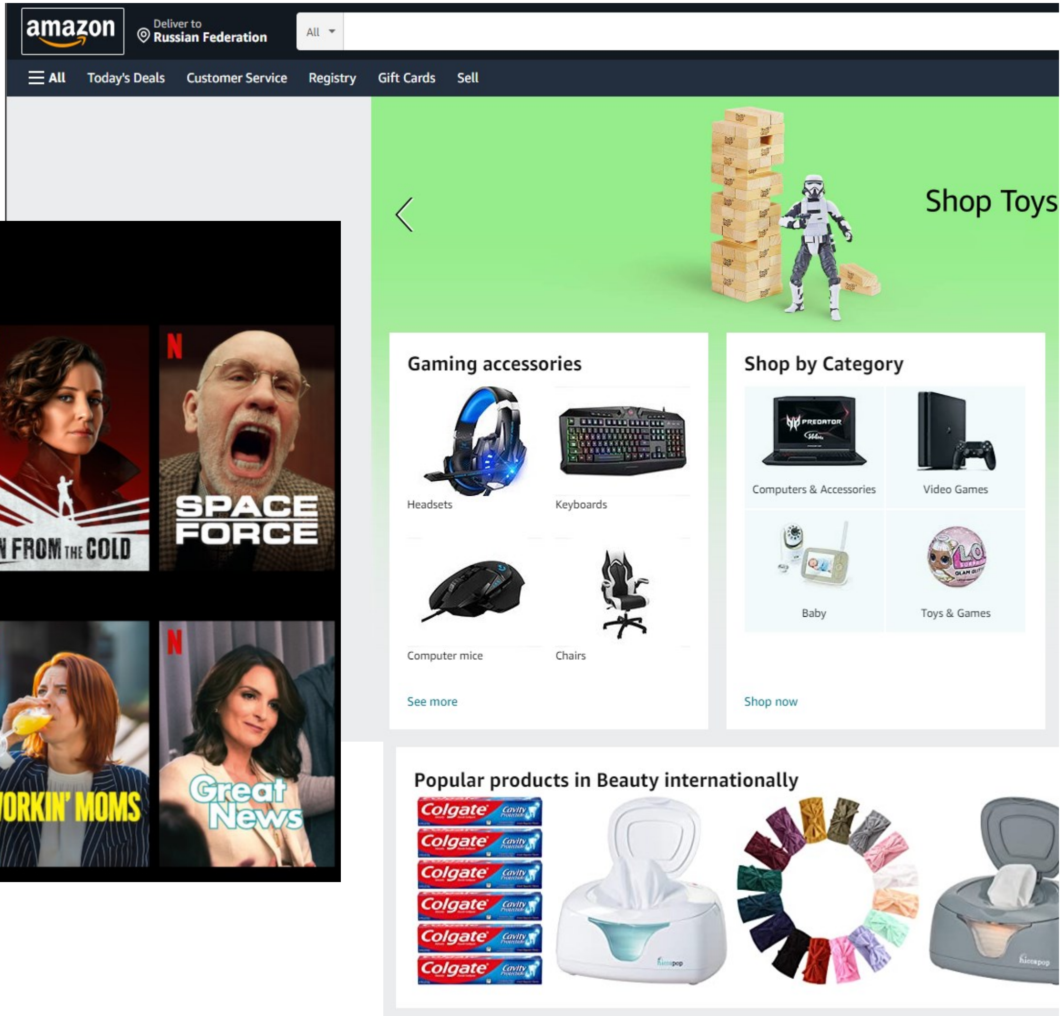
Recommender Systems

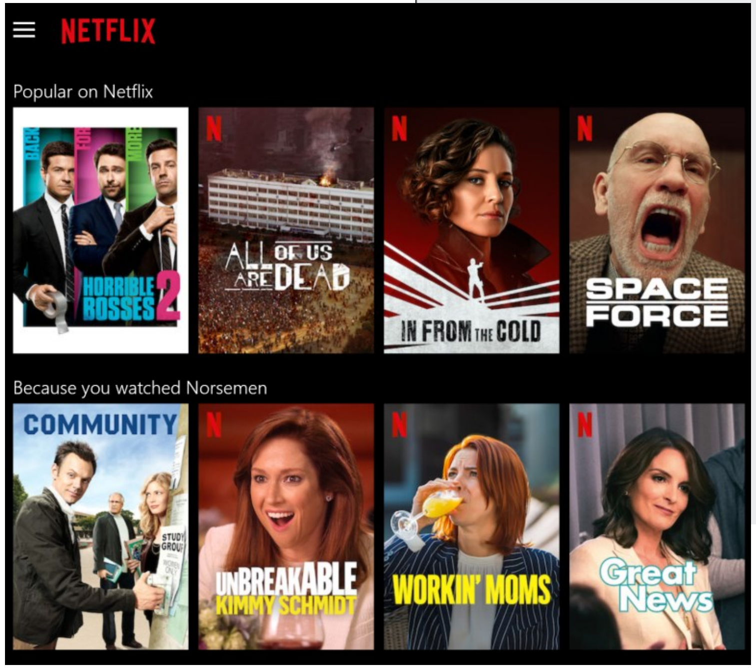
Lecture 2

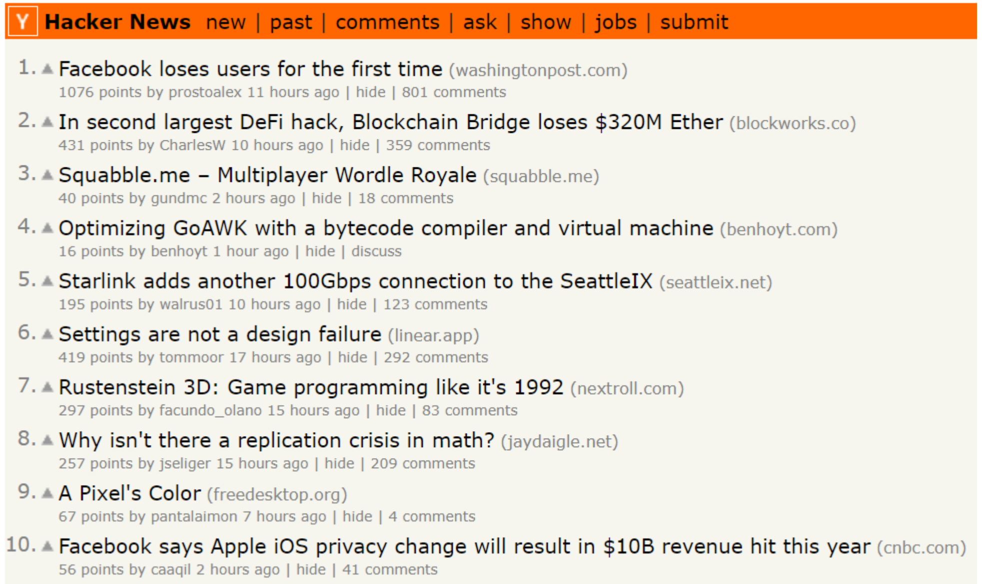
Today's Lecture

- Popularity-based models
- Content-based recommendations
- Baseline Predictors

Usecases for popularity-based predictions







Popularity-based scoring

Task:

- assign a popularity score based on accumulated feedback from users

- mostly based on heuristics
  - e.g., content “interestingness” ↔ consumption frequency

- typical challenges / issues
  - dynamics and trends
    - e.g., seasonal purchases
  - non-homogeneous distribution
    - e.g., experts vs non-experts
  - attacks and fraud

Examples of user feedback

1. Ratings
2. Upvotes/Downvotes
3. Likes (w/o dislikes)

Таблица сравнения

Grow Food	Performance Food
Официальный сайт:growfood.pro	Официальный сайт:p-food.ru
Рейтинг: 8.77	Рейтинг: 9.59
Просмотров: 127 518	Просмотров: 36 058
Отзывов: 14	Отзывов: 8

Hacker News new | past | comments | ask | show | jobs | submit

1. ▲ Seawater electrolysis by adjusting the local reaction environment of a catalyst (nature.com)
   99 points by CharlesW 2 hours ago | hide | 46 comments
2. ▲ Show HN: I trained an AI model on 120M+ songs from iTunes (maroofy.com)
   91 points by subtech 2 hours ago | hide | 60 comments
3. ▲ Play Counter Strike 1.6, with full multiplayer, in the browser (play-cs.com)
   951 points by philosopher1234 10 hours ago | hide | 372 comments
4. ▲ I'm Now a Full-Time Professional Open Source Maintainer (filippo.io)
   191 points by chmaynard 4 hours ago | hide | 29 comments
5. ▲ An obituary for the man who saved North Carolina from Nuclear Disaster (ncrabbithole.com)
   75 points by joeatwork 2 hours ago | hide | 3 comments

Rating-based scoring

Popularity-based recommendations

naïve estimation of item j popularity:

$$\mathrm{score}_{\mathrm{POP}}(j) = \frac{1}{|U_j|} \sum_{i \in U_j} r_{ij}$$

- U_j - set of users who rated item j
- r_{ij} - rating of item j provided by user i

Popularity-based recommendations

$$\mu_j = \quad \text{score}_{\text{POP}}(j) = \frac{1}{|U_j|} \sum_{i \in U_j} r_{ij}$$

What are potential flaws?

- insufficient amount of data
  - more data means higher reliability /certainty
  - an item with 100 ratings and average score 4.95 vs item with two 5-star ratings
- different dynamics
  - trends
  - seasonality

In the early days

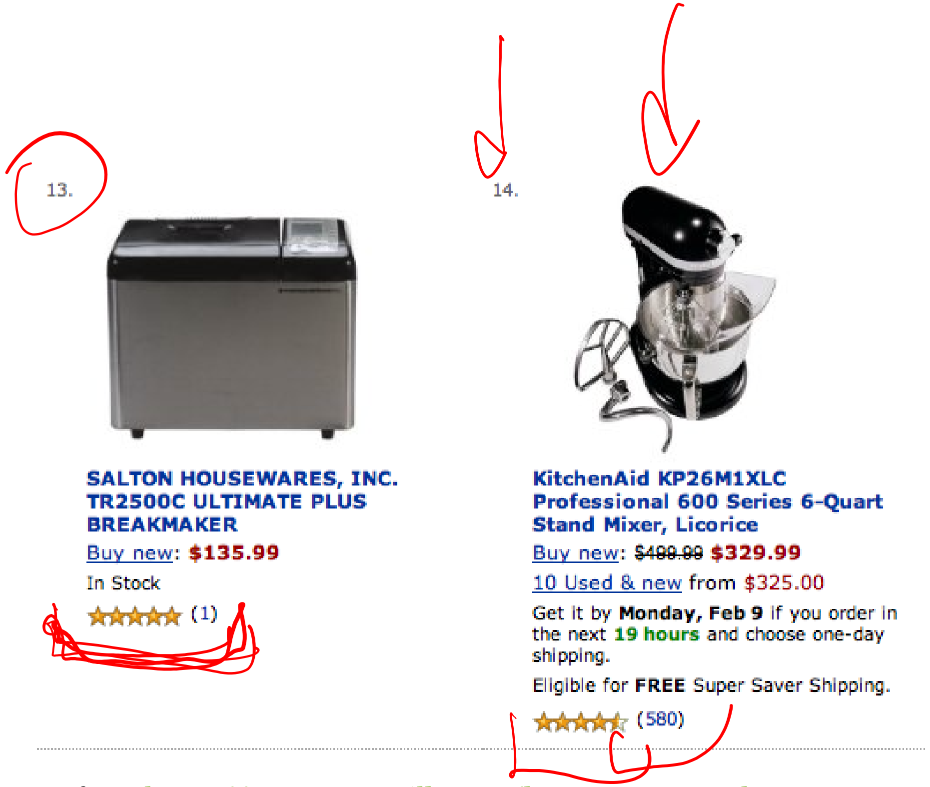
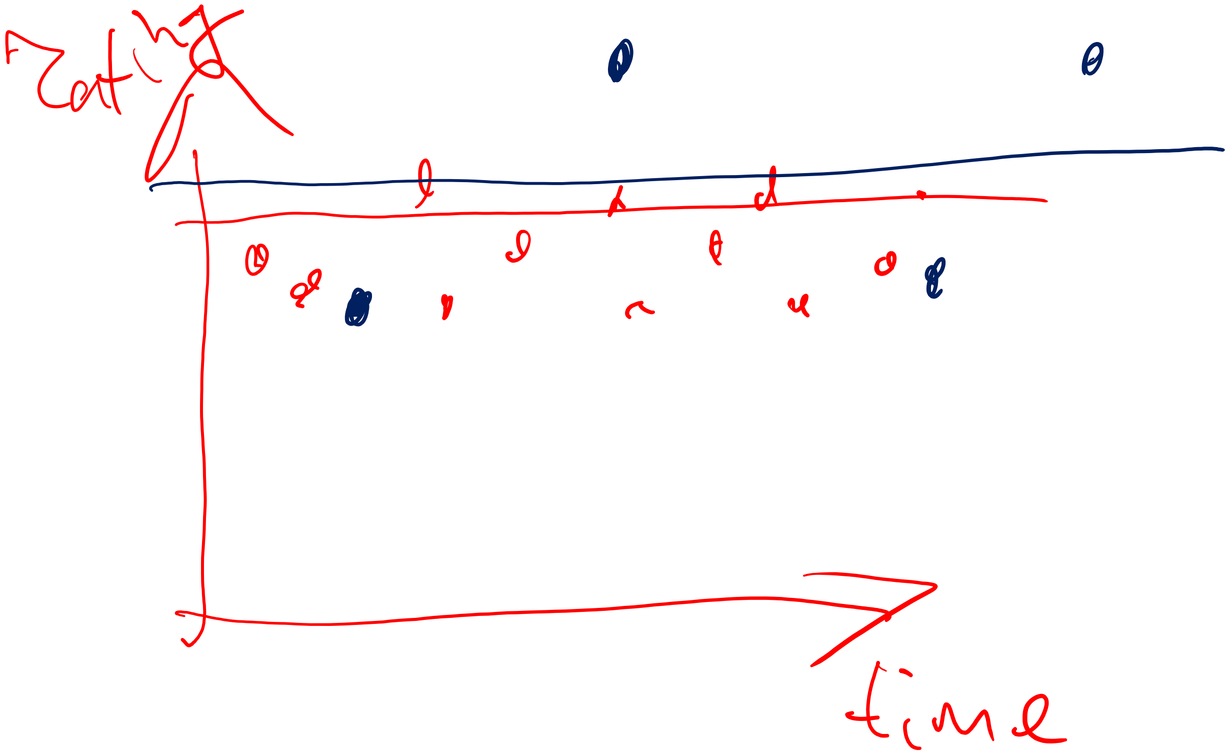
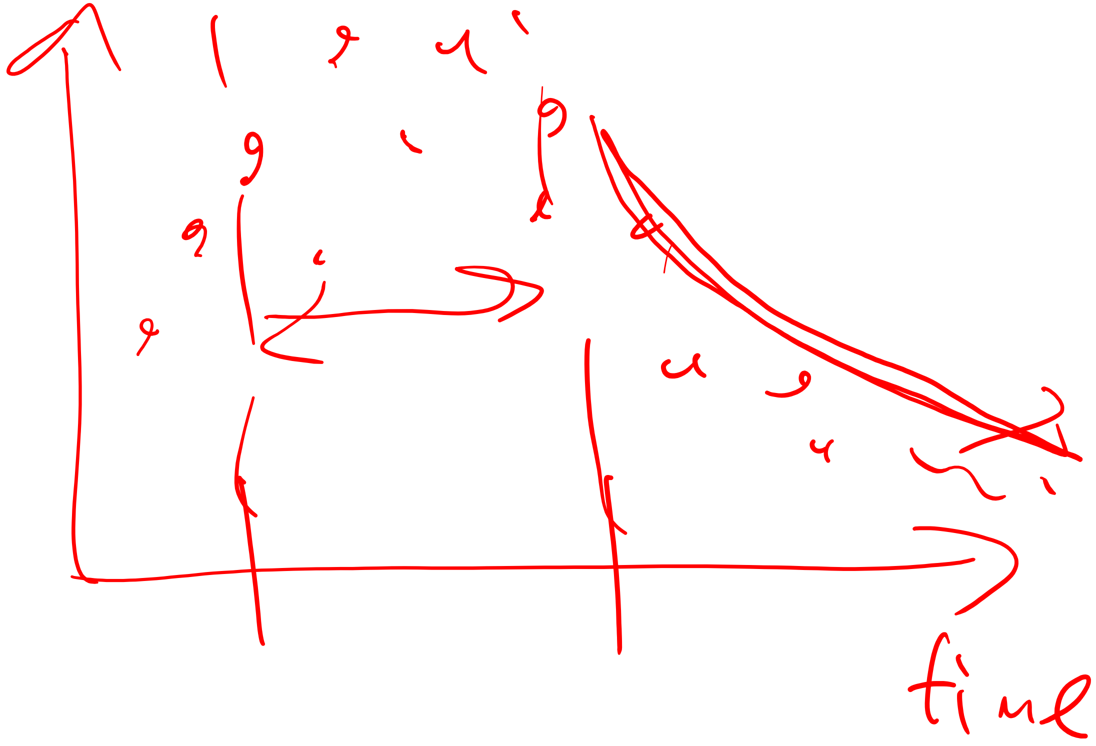


Image from https://www.evanmiller.org/how-not-to-sort-by-average-rating.html

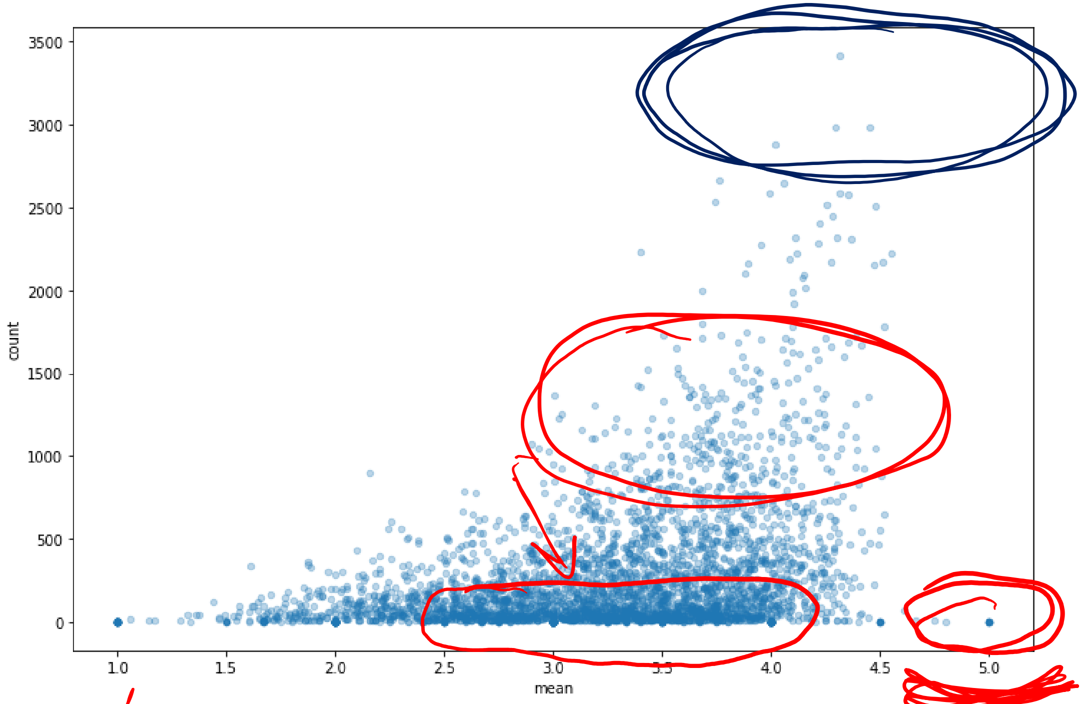
Popular vs niche products





Simple adjustment of popularity

Average rating distribution on Movielens-1M



#

1 2 3 4 5

mean

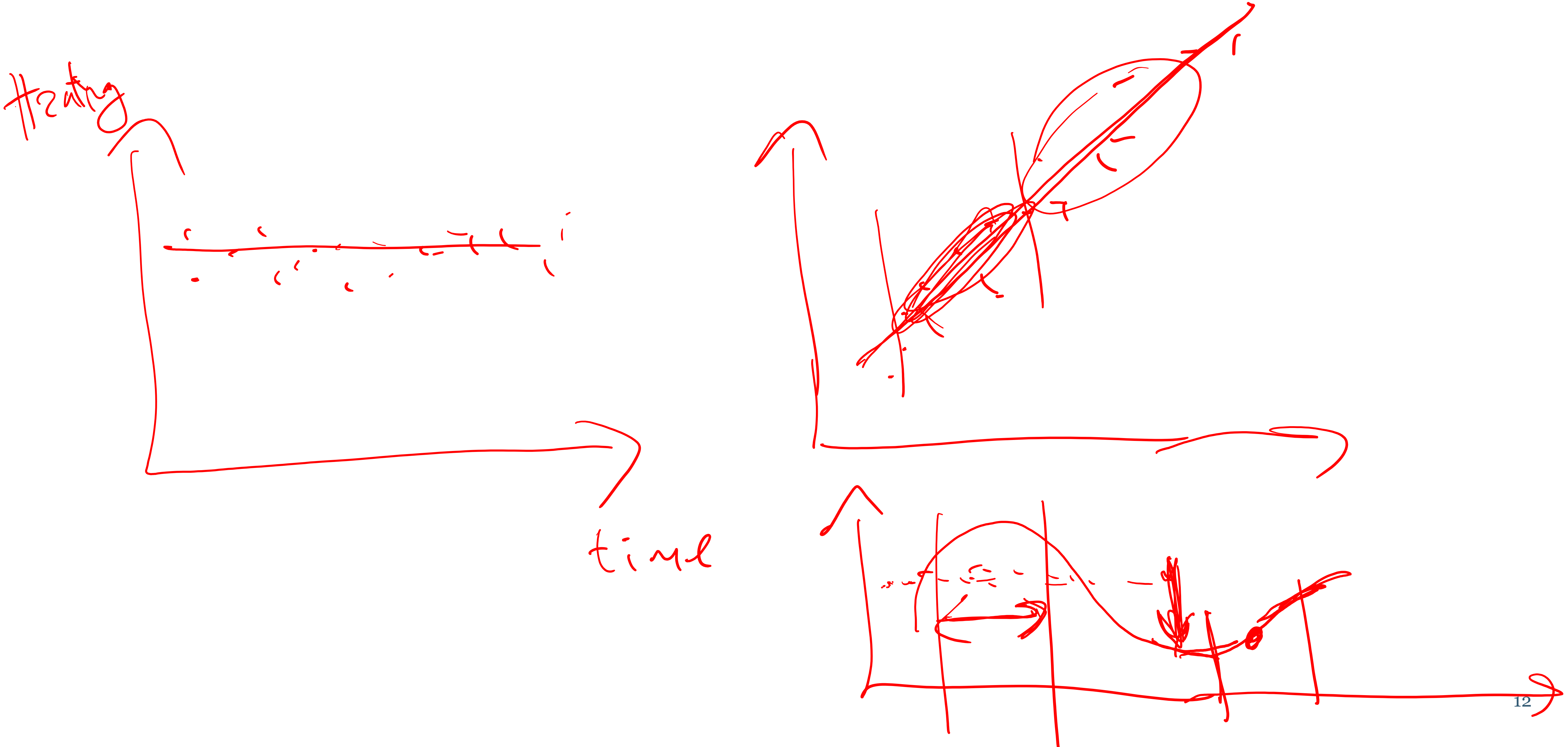
minimization problem:

$$\sum_{i \in U_j} (\mu_j - r_{ij})^2 + \lambda \mu_j^2 \to \min$$

$$\mu_j = \frac{1}{|U_j| + \lambda} \sum_{i \in U_j} r_{ij}$$

≥ 0

Popularity vs trends in data



Bayesian averaging

combination of prior belief and observed sample average:

$$\mu_{BA} = \underbrace{\alpha \cdot \mu_{PB}} + \underbrace{(1-\alpha) \cdot \mu_{SA}}, \qquad 0 \le \alpha \le 1$$

assume we're in the future and already have enough data:

$$\frac{1}{C+n}\sum_{k=1}^{C+n} x_k = \underbrace{\frac{C}{C+n}}_{\alpha} \boxed{\frac{1}{C}\sum_{k=1}^{C} x_k} + \underbrace{\underbrace{\frac{n}{C+n}}_{1-\alpha} \frac{1}{n}\sum_{k=C+1}^{n} x_k}$$

- x_k – sample
- C – amount of prior samples
- n – amount of newly added samples

Bayesian averaging

$$\mu_{BA} = \frac{C\bar{\mu} + \sum_{k=1}^{n} x_k}{C + n}, \qquad \alpha = \frac{C}{C+n}$$

- For item rating data:

μ_j^t

$t+1$

$$\mu_j = \frac{C\overline{r_j} + \sum_{i \in U_j} r_{ij}}{C + |U_j|}$$

U_j - set of users, who rated items j

- too high C → need more observations
- too low C → unreliable estimate

Other possible adjustments

current

$$\sum_i z_{ij} \cdot f_{20}(t - t_0)$$

$e^{-|t-t_0|}$

init

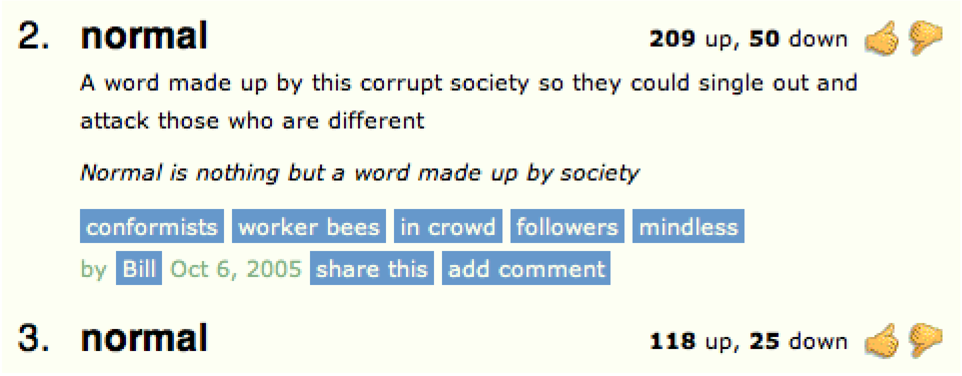
- dampening weights based on recency
- activity of users (bots/trolls or not invested users)
- side information (features, attributes)
  - typically rules-based

Scoring for binary feedback

Possible scoring strategies

- Amount of positive feedback
- # positive feedback – # negative feedback
- proportion of positive feedback

In the early days



Urban Dictionary, image credit: https://www.evanmiller.org/how-not-to-sort-by-average-rating.html

Estimating popularity on binary feedback

- lower bound of Wilson score confidence interval*
  - estimate for a Bernoulli parameter
  - estimates the bound with 95% probability given current observations

$$\left(\hat{p} + \frac{z_{\alpha/2}^2}{2n} - z_{\alpha/2}\sqrt{\left[\hat{p}(1-\hat{p}) + z_{\alpha/2}^2/4n\right]/n}\right) / \left(1 + z_{\alpha/2}^2/n\right)$$

- $\hat{p}$ is the observed fraction of positive ratings
- $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard normal distribution
- n is the total number of ratings

Assumptions: data follows a binomial distribution with

- fixed probability of a success,
- statistically independent trials.

*Example of calculation in python
https://gist.github.com/aditya00kumar/ab81a77258c70f9c3a811f3763a6fb62

Laplace (add-one) smoothing

- $r_{ij} \in \{0, 1\}$ – binary feedback
  $\text{upvotes}_j = \{r_{ij} > 0, i \in U_j\}$

$$\frac{\sum_{i \in U_j} \mathbb{I}(r_{ij} > 0) + \cancel{1}\alpha}{|U_j| + 2}$$

observ.

classes

- $r_{ij} \in \{-1, 0, 1\}$

$$\frac{\sum_{i \in U_j} \ldots + 1}{\ldots + 3}$$

- $r_{ij} \in \{1, 2, 3, 4, 5\}$

Some other schemes: Valcarce, D., Parapar, J. and Barreiro, Á., 2016, June. Additive smoothing for relevance-based language modelling of recommender systems. In *Proceedings of the 4th Spanish Conference on Information Retrieval* (pp. 1-8).

Note on popularity / average ratings

- Horror movies ratings are typically lower, even if a user actually likes it.

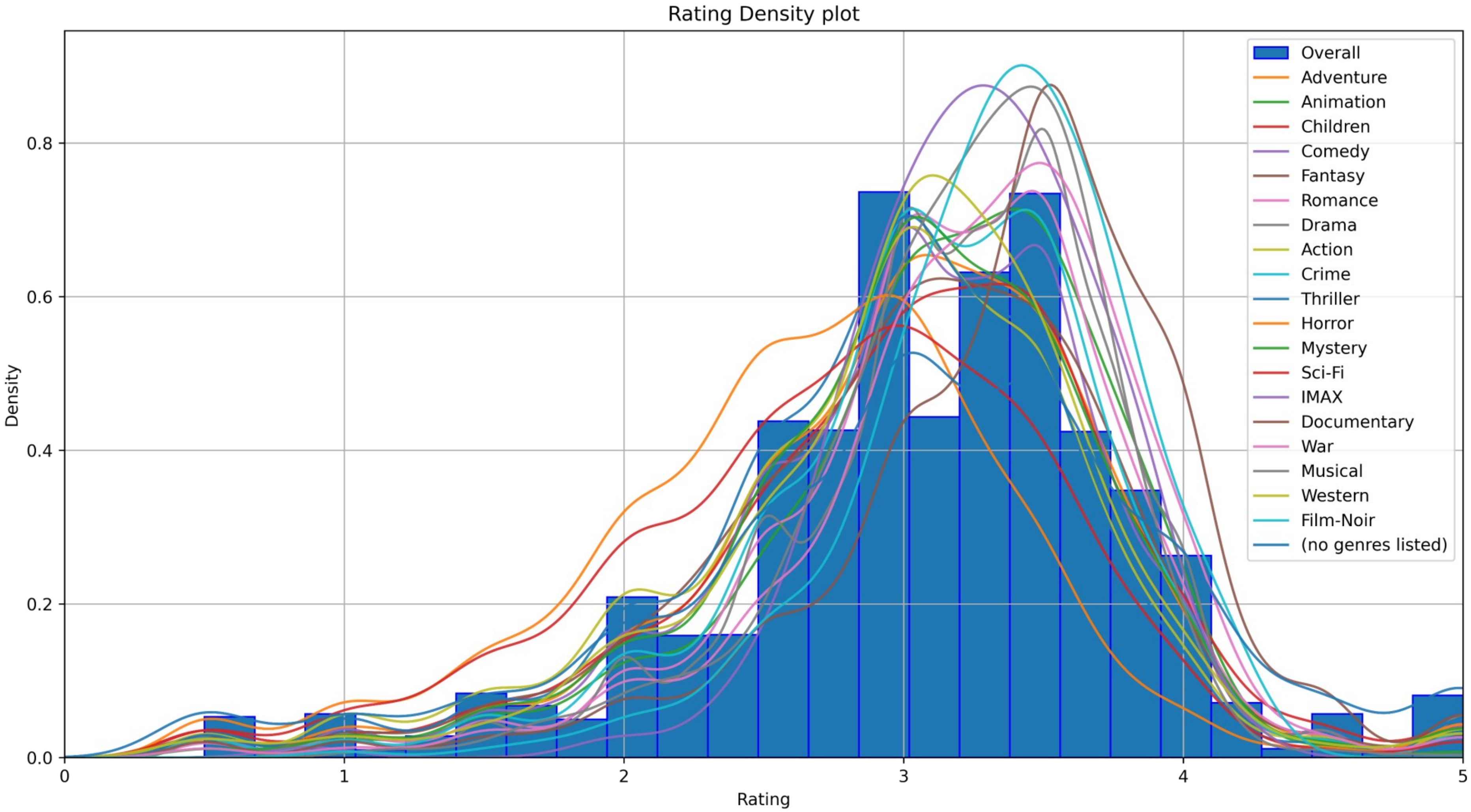
- "Ghostbusters" Is A Perfect Example Of How Internet Movie Ratings Are Broken.

- IMDb average user rating: 4.1 out of 10, of 12,921 reviewers
- IMDb average user rating among *men*: 3.6 out of 10, of 7,547 reviewers
- IMDb average user rating among *women*: 7.7 out of 10, of 1,564 reviewers

Source: http://fivethirtyeight.com/features/ghostbusters-is-a-perfect-example-of-how-internet-ratings-are-broken/

Content-based Recommendations

Ratings distribution over movie genres (ML-25M)



Using content for matching

			
Good for gaming	+		+
Good for movies	+	+	+
Good for TV shows	+		
Blue-ray support		+	+

Gamer

Cinema fan

“Family guy”

generating recommendations require:

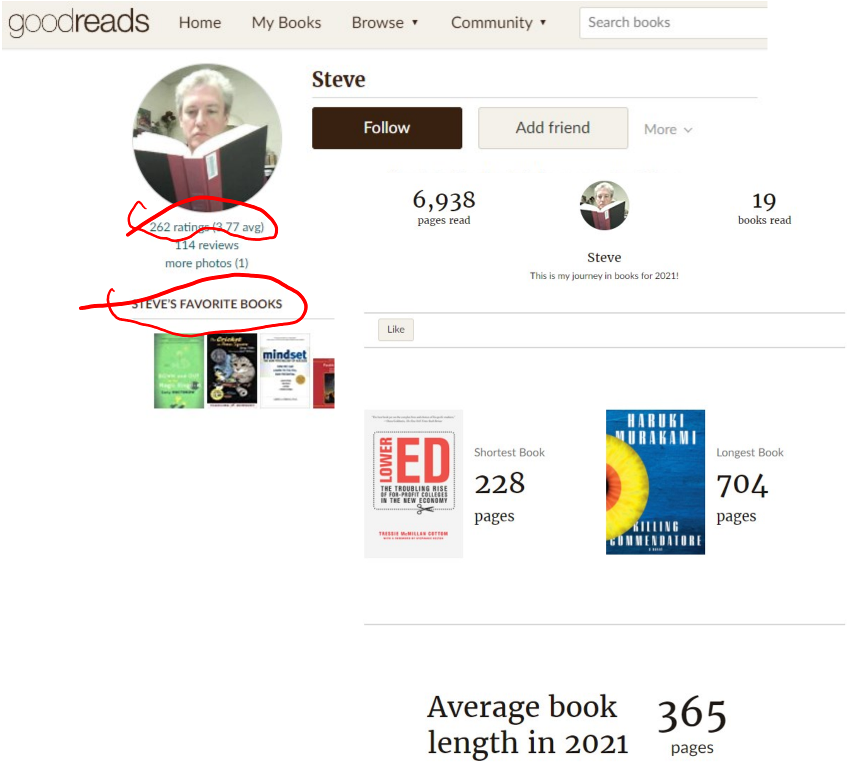
- item features to compute items relevance
- user profile information to match items
- previous user actions to match items
- user attributes for tailoring predictions
- criteria specified by user / expert

- knowledge-based
- case-based
- constraint-based

Typical use case: “cold” items

Example of content features

Users



More attributes:
- demographics
- location
- occupation
- ...

Items

Other features:
- price
- format/style
- language
- ...

Simple rating prediction

Task: find a utility function f:

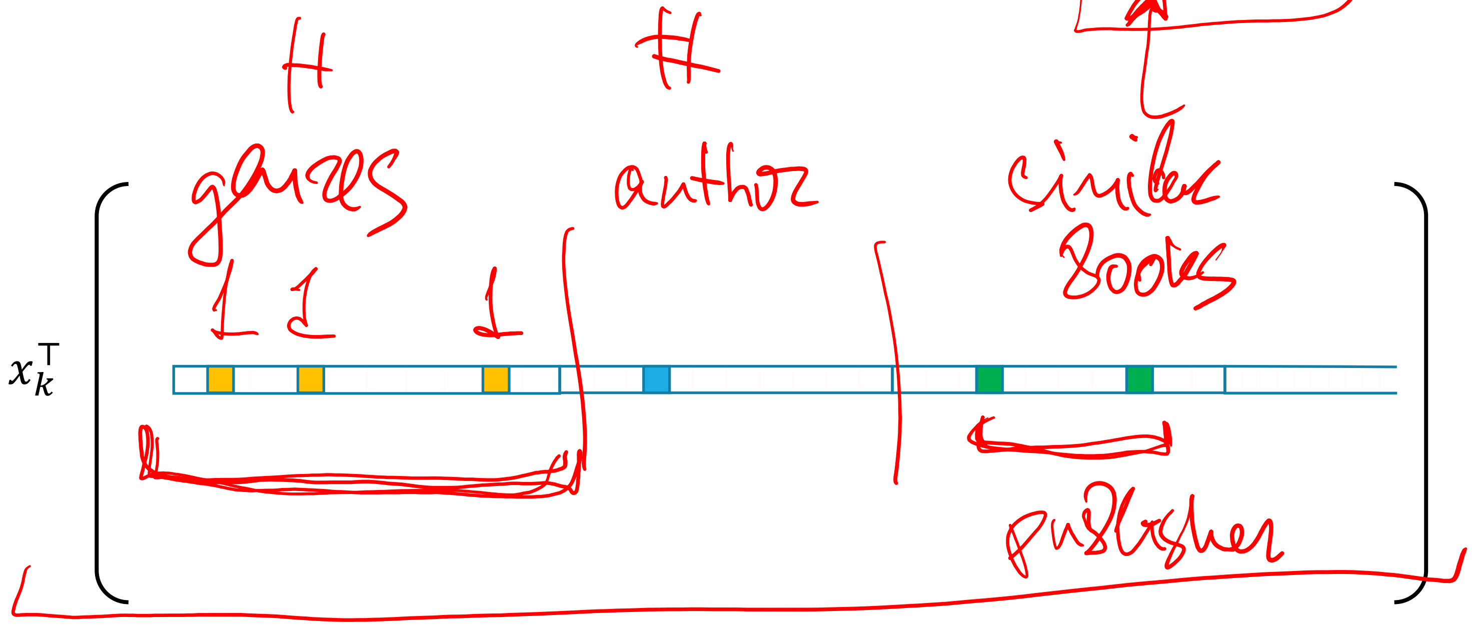
$$f(\text{item features}) \to \text{feedback}$$

weight

Linear regression model: $X\boldsymbol{w} = \boldsymbol{y}$

Linear w.r.t. what?

X - design matrix



Linear Regression

$$\boldsymbol{y} = X\boldsymbol{w} + \boldsymbol{\epsilon}$$

$$y_k = \boldsymbol{x}_k^\top \boldsymbol{w} + \epsilon_k$$

Under assumption of normally distributed noise, standard linear least squares problem:

$$||\boldsymbol{y} - X\boldsymbol{w}||_2^2 \to \min$$

Multicollinearity problem

$$\lambda \|w\|^2 + \|y - Xw\|_2^2 \to \min$$

$$X = U \Sigma V^T = \sum_i \sigma_i u_i v_i^T$$

$$\underbrace{w^*}_{\Lambda} = (X^T X)^{-1} X^T y = X^+ y \qquad \hat{y} = U U^T y$$

$$\underline{\underline{y}} \approx \hat{y} = X X^+ y \qquad P_X = X X^+$$

$$w^* = X^+ y = V \Sigma^{-2} \cancel{V^T V} \Sigma U^T y = V \Sigma^{-1} U^T y = \sum_i \frac{(u_i^T y)}{\sigma_i} v_i$$

$$\hat{y} = \sum_i (u_i^T y) u_i$$

Multicollinearity problem

Typical ways to combat the ill-posedness:

- add regularization term
  - e.g., l2-penalty $\lambda||\boldsymbol{w}||^2$
- feature selection
  - hard problem
- feature transformation
  - e.g., dimensionality reduction

A. Nikitin Are quantum computer practical yet

Ridge regression

How to select features?

Types of features:

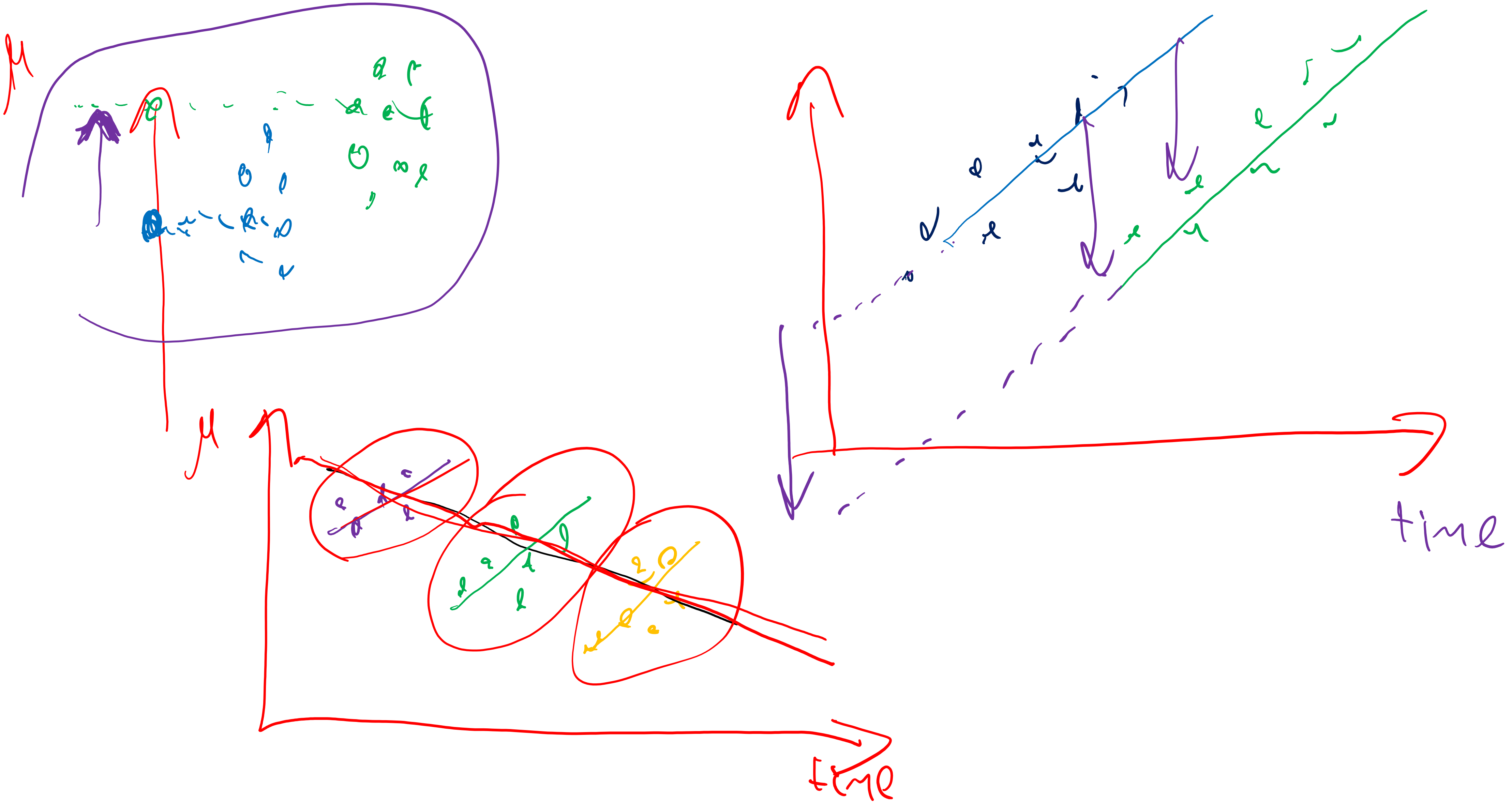
- Known knowns
- Known unknowns
- Unknown unknowns

Adding/omitting regressors:

- Missing important variables → bias
- Adding meaningless variables → overfitting

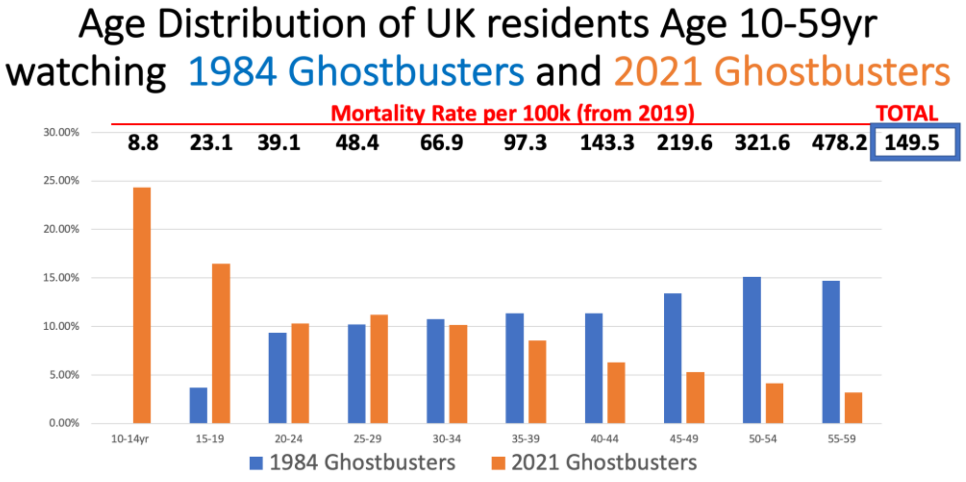
- No standard “silver bullet” recipe exists.
- Choose the simplest possible model, but not simpler.

Simpson Paradox



Correlation vs causation

Is watching the 1984 Ghostbusters movie killing people?



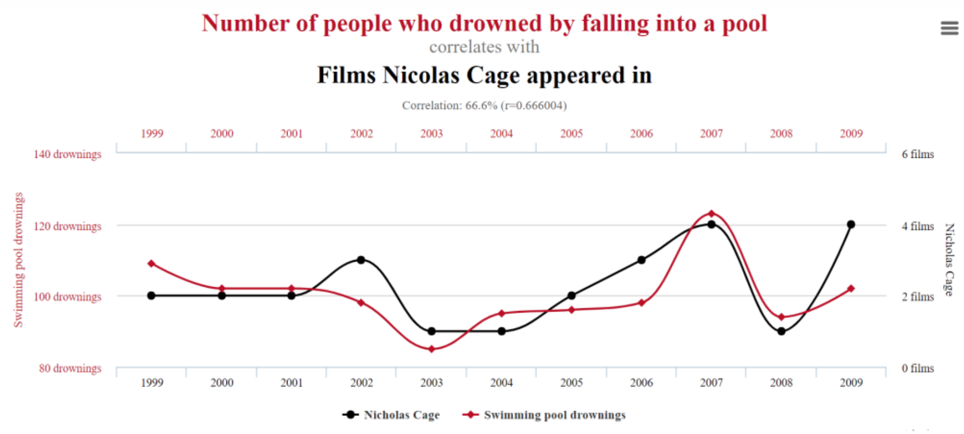


Image from: https://tylervigen.com/spurious-correlations

Images from https://www.covid-datascience.com/post/is-watching-the-1984-ghostbusters-movie-killing-people-a-statistician-s-perspective

Other methods used for content-based filtering

- SVM
- Decision Tree-based models
  - CatBoost
  - LightGBM
  - XGBoost
  - Automated feature selection, e.g. as in LightAutoML
- Neural Networks

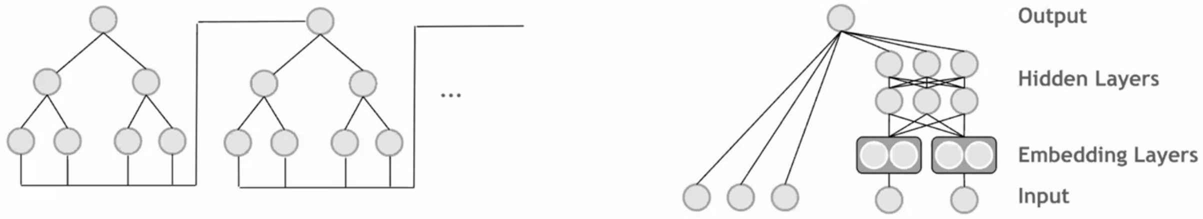
Feature engineering usecase

Feature Type	Example	Feature Engineering
Categorical	User ID / Item ID Brand Main Category	Target Encoding Count Encoding Categorify + Combining Categories
Unstructured list	Keywords Subcategories Colors	Target Encoding Count Encoding Categorify
Numeric	Price Deliver time Avg. reviews	Binning Normalization Gauss Rank
Timestamp	Timestamp	Extract month, weekday, weekend, hour
Timeseries	Events in order Time since last event	# of events in past X Difference in time (lag)
Image	Product image	Extract latent representation with deep learning
Text	Description	Extract latent representation with deep learning
Social graph	Follower/Following graph	Link analysis
Geo location	Addresses	Distances to point of interest

- Massive experiments in parallel
- Trying, not guessing

Review Recommender Model

Tree Based	Deep Learning



Tree Based

Types:
- CatBoost
- XGBoost
- LightGBM

Components:
- Defines split by information gain per feature
- Does not require normalization of input features

XGBoost cannot handle raw categorical features

Deep Learning

Types:
- Wide And Deep
- DeepFM
- DLRM

Components:
- Embedding Layers
- Feed-Forward Layers
- Requires normalization of input features

How to Build a Winning Deep Learning Recommender System
https://medium.com/rapids-ai/winning-solution-of-recsys2020-challenge-gpu-accelerated-feature-engineering-and-training-for-cd67c5a87b1f
https://www.youtube.com/watch?v=bHuww-l_Sq0

Classification

Task: predict whether user will like an item or not.

Example:

$$y_k = \begin{cases} 1, & \text{rating above threshold,} \\ 0, & \text{otherwise.} \end{cases}$$

In linear regression: $y_k \approx \boldsymbol{x}_k^\top \boldsymbol{w}$.

How to translate $\boldsymbol{x}_k^\top \mathbf{w} \in \mathbb{R}$ into binary domain?

Classification – probabilistic formulation

Logistic Regression for classification task

The model:

$$p_{\boldsymbol{w}}(y_k = 1 \mid \boldsymbol{x}_k) \sim \frac{1}{1 + e^{-\boldsymbol{x}_k^\top \boldsymbol{w}}}$$

Target:

$$y_k = \begin{cases} 1, & \text{if } \boldsymbol{x}_k^\top \mathbf{w} > 0, \\ 0, & \text{otherwise.} \end{cases}$$

- Our observations data can be viewed as a set of random trials.
- Outcome of each trial is binary (coin flip).

$$y \sim \text{Bernulli}(\mu)$$

$$E[y] = \mu, \qquad 0 \le \mu \le 1$$

A bit of probability theory

- We aim to maximize the joint probability distribution (likelihood)

$$p_w(y, X) = \prod_{k=1}^{n} p_{\mathbf{w}}(y_k, \boldsymbol{x}_k), \qquad p_{\mathbf{w}}(y_k, \boldsymbol{x}_k) = p_{\mathbf{w}}(\, y_k \mid \boldsymbol{x}_k \,) \cdot p_{\mathbf{w}}(\boldsymbol{x}_k)$$

$$\operatorname{argmax}_{\mathbf{w}} \prod_{k=1}^{n} p_{\mathbf{w}}(y_k, \boldsymbol{x}_k) = \operatorname{argmax}_{\mathbf{w}} \log \prod_{k=1}^{n} p_{\mathbf{w}}(y_k, \boldsymbol{x}_k)$$

$$= \operatorname{argmax}_{\mathbf{w}} \left(\log \prod_{k=1}^{n} p_{\mathbf{w}}(\, y_k \mid \boldsymbol{x}_k \,) + \log \prod_{k=1}^{n} p_{\mathbf{w}}(\boldsymbol{x}_k) \right)$$

$$= \operatorname{argmax}_{\mathbf{w}} \sum_{k=1}^{n} \log p_{\mathbf{w}}(\, y_k \mid \boldsymbol{x}_k \,)$$

Maximum likelihood derivation

$$\mathcal{L}(w) = \sum_{k=1}^{n} \log p_{\boldsymbol{w}}(\, y_k \mid \boldsymbol{x_k} \,)$$

- The probability of each outcome is parametrized via:

$$p_w(\, y_k \mid \boldsymbol{x}_k \,) = p_{\boldsymbol{w}}(\, y = 1 \mid \boldsymbol{x_k} \,)^{y_k} \cdot \left(1 - p_{\boldsymbol{w}}(\, y = 1 \mid \boldsymbol{x}_k \,)\right)^{(1-y_k)}$$

Logistic Regression Objective

More concise/practical form with $y_k \in \{-1, +1\}$ instead of $\{0, 1\}$:

$$\mathcal{L}(\boldsymbol{w}) = -\sum_{k=1}^{n} \log\left(1 + e^{-y_k \cdot \boldsymbol{x}_k^\top \boldsymbol{w}}\right)$$

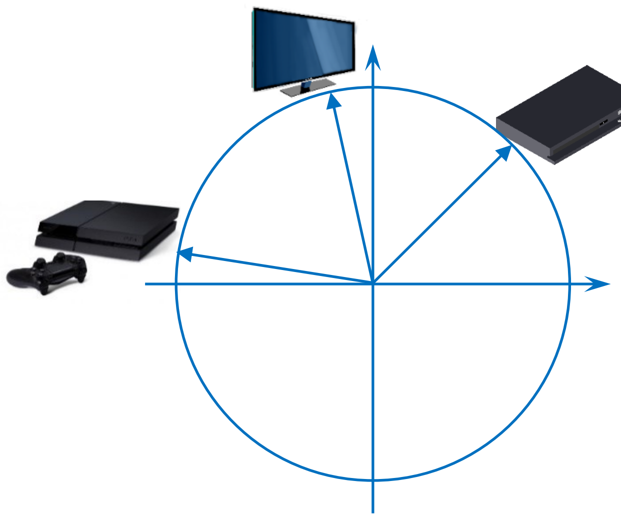
- Turns into standard loss minimization:

$$\boldsymbol{w}^* = \operatorname{argmin}_{\mathbf{w}} \mathcal{Q}(\boldsymbol{w}), \qquad \mathcal{Q}(\boldsymbol{w}) = -\mathcal{L}(\boldsymbol{w}) + \lambda \|\boldsymbol{w}\|^2$$

- Can be solved with e.g.:
  - SGD
  - Newton-Raphson (Iteratively Reweighted Least Squares)

Similarity-based models

Each item as a vector in d-dimensional feature space.



user profile

item fact

Given two item feature vectors $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$, we can estimate their proximity in on a unit sphere:

$$\cos(\alpha) = \frac{\boldsymbol{x}_i^\top \boldsymbol{x}_j}{\|\boldsymbol{x}_i\| \cdot \|\boldsymbol{x}_j\|}$$

What happens if some features are missing?

How to match with a user?

$\boldsymbol{x}_k^\top$

User profile:

$$\overline{\boldsymbol{x}}_u = \sum_{j \in I_u} w_{uj} \boldsymbol{x}_j \,, \qquad I_u - \text{set of items of user } u$$

The simplest case:

$$w_{uj} = \frac{1}{|I_u|} \quad \text{or} \quad w_{uj} = \frac{r_{uj}}{\sum_k r_{uk}},$$

Relevance estimation:

$$r_{ui} = \frac{\overline{\boldsymbol{x}}_u^\top \boldsymbol{x}_i}{\|\overline{\boldsymbol{x}}_u^\top\| \cdot \|\boldsymbol{x}_i\|}$$

How to evaluate

Intuitive heuristic:

- Based on proximity to user preferences:
  - must be close to what user likes
  - must be distant from what user dislikes

- Construction of evaluation dataset:
  - select k pairs of likes/dislikes for each user
    - must be excluded from training data
  - rank other items (e.g., user favorites) based on their proximity to test pairs