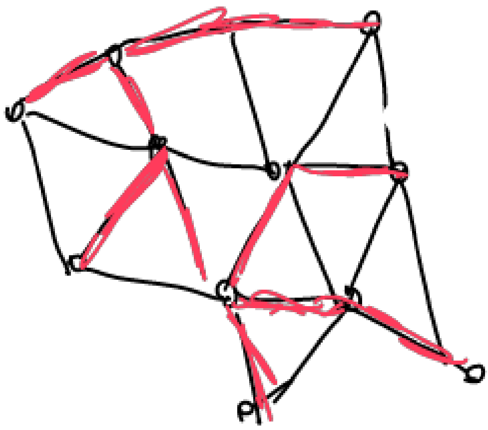
A connected graph is a tree iff every edge is a bridge.

Trees are bipartite.

Spanning Tree

A spanning tree of a connected graph G is a subgraph of G that is a tree with the same vertex set as G.



Prop: Every connected graph has a spanning tree

pf: let $G=(V,E)$ be a connected graph. Let F be a minimal subset of E so that the graph $H=(V,F)$ is connected.

Since F is minimal, the graph $H-e$ is disconnected for every $e \in F$, so every edge of H is a bridge. Thus, H is a tree, so it is a spanning tree.

Prop. A graph G is bipartite iff it contains no odd cycle.

pf. We may assume that G is connected, since we could otherwise just apply the result to each component. Let T be a spanning tree of G.

Suppose G has no odd cycles. We know that trees are bipartite.
Let (A,B) be a bipartition of T.
We show that (A,B) is also a bipartition of G.
Suppose otherwise.

Let x, y be adjacent vertices of G that are both in A or both in B. Let $x = u_0, \dots, u_k = y$ be a path from x to y in T. Since each edge of T has an end in A and an end in B, the vertices in this path alternates between A & B. The ends are in the same set, so the length is even (k). Now $x = v_0 v_1 \dots v_k = y$, x is an odd cycle of G, a contradiction.

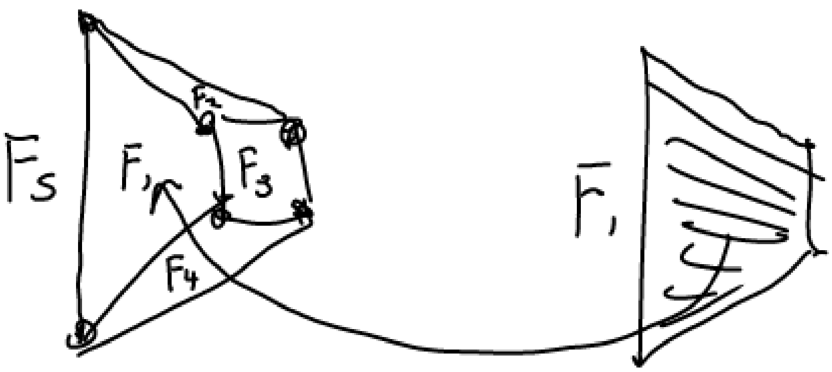
No odd cycle $\Rightarrow$ bipartite.
The inverse is true because all odd cycles are not bipartites.

A drawing of a graph G is a subset of the plane such that: every vertex corresponds to a distinct point,
every edge corresponds to an open arc
the closure of every edge is exactly its endpoints.

Fary's Theorem: If G is planar, then G can be embedded in the plane using only staright lines

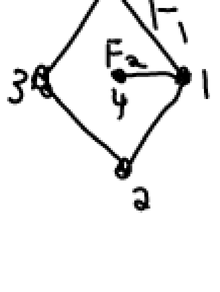
If G is disconnected, then G is planar iff every component of G is planar.

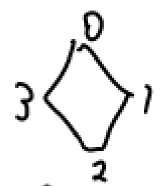
If G is embedded in the plane P, the closures of the connected components of $P \setminus G$ are the faces of the embedding.

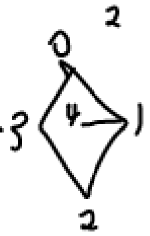


<u>Defn:</u> The unbounded face of an embedding is called the outer face.

The subgraph of G formed by the vertices and edges in the boundary of F is the boundary of F

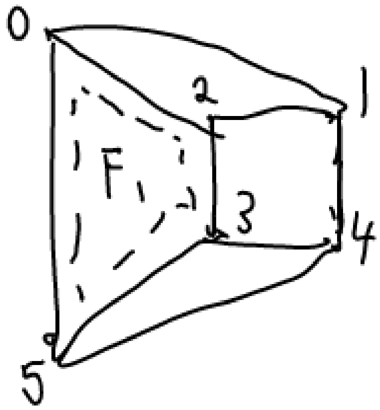


$bd(F_1) =$ 

$bd(F_2) =$ 

A vertex or edge of G in the boundary of F is incident w/ F.

As we walk along the boundary of F, we get a closed walk in G.



$W_{F_1} = \{0, 2, 3, 5, 0\}$