One-story

$20(26k-4k)-250k=190k$

Avg. profit: $\frac{190k}{250k}=9500/yr$

profit rate: $\frac{9500}{250k}=3.8\%$

Two-story

$20(40k-6k)-400k=280k$

$\frac{280k}{400k}=14k/yr$

$\frac{14k}{400k}=3.5\%$

Chapter 2
Time Value of Money

MSCI 261
SECTION 1 (CHE/GEOE) AND SECTION 2 (SOFTWARE)

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Overview

- Time Value of Money (Interest)
- Future Worth Method
- Present Worth Method
- Cash Flow Diagrams
- Using Spreadsheets
- Nominal vs Effective Interest Rates

Today $1000

After 1 yr. $1000(1+.03) = \$1030$

2 yr. $1030(1+.03) = \$1060.90$

3 yr. $1060.90(1+.03) = \$1092.73$

4 yr. $1092.73(1+.03) = \$1125.51$

$= 1000(1+.03)^4$

In general, if I deposit P now, wait N periods w/ interest rate i per period, then after N periods, I'll have the future amt $F = P(1+i)^N$

compound interest: interest on top of interest

Example 2-1

(a) On September 1, you deposit $1000 into a bank account that earns interest at the rate of 3% per year. The bank calculates interest and adds it to your account once per year, on September 1, based on the balance in the account. If you make no withdrawals for four years, what is the balance in the account, just after the fourth interest payment is added?

(b) Suppose instead that you borrow $1000, and you must repay it in 4 years, with interest at 3% per year. How much do you owe in 4 years?

Interest: 12%/yr (%/unit time)

$\$0.12/\$1/yr$ ($/$/unit time)

Four Problem Types, based on $F = P(1 + i)^N$

Where P = principal, or present amount (in $)
F = future amount, with interest (in $)
i = interest rate (per period)
N = number of periods

1. Find F, given P, i, N (used in "Future Worth" method)
2. Find i, given F, P, N (used in "rate of return" methods)
3. Find N, given F, P, i
4. Find P, given F, i, N (used in "Present Worth" method)

More examples

Example 2-2:

- With i = 8% per year, how much is owed on a loan of $500 at the end of 3 years?

$$F = P(1+i)^N = 500(1+0.08)^3 = \$629.86$$

Example 2-3:

- I borrow $100 now and pay you $1000 four years from now. What is our implied interest rate?

$$1000 = 100(1+i)^4$$
$$i = \sqrt[4]{10} - 1$$
$$= 77.8\%$$

Example 2-4:

- How long will it take for a bank account balance to reach $2000, if $1500 is deposited now, and interest is at 3% per year ?

$$2000 = 1500(1+0.03)^N$$
$$N = \ln(4/3)/\ln(1.03)$$
$$= 9.733 \text{ yrs} \approx 10 \text{ yrs}$$

a) $900k(1+0.04)^5 + 200k(1+0.04)^4 + 200k(1+0.04)^3 + \ldots + 200k(1+0.04)^0$
$= 2178.^{25} k$

b) $5000k(1+0.04)^5 + 300k(1+0.04)^4 + \ldots + 300k(1+0.04)^0$
$= 2233.^{22} k$

c) automated is cheaper.

Including Interest in Automated versus Manual Decision: **Future Worth Method**

Example 2-5:

a) If the Automated line costs $900,000 now and $200,000 per year for 5 years, and if all of these amounts must be borrowed with interest at 4% per year, what is the total cost including interest after 5 years?

b) If the Manual line costs $500,000 now and $300,000 per year for 5 years, all borrowed at 4% per year, what is the total cost including interest after 5 years?

c) Which is cheaper?

Example 2-6:

- Redo Example 2-5, but this time assume that all money is taken from the company's bank account that earns interest at 4% per year.

same calc as above

2.7) $P = \frac{F}{(1+i)^N} = \frac{6500}{(1+0.03)^4} = \$5775.17 = P_1$

2.8) $\frac{3000}{(1+0.03)^5} = \$2587.83 = P_2$ $\quad P_1 + P_2 = \$8363$

2.9)a) $PW = 900k + \frac{200k}{1+0.4} + \frac{200k}{(1+0.4)^2} + \ldots + \frac{200k}{(1+0.4)^5} = 1790.4k$

Find P, given F, i, N: relevant for **Present Worth Method**

Example 2-7:

- If I need \$6500 in 4 years, how much should I deposit now in an account that earns interest at 3% per year?

Example 2-8:

- If I need \$6500 in 4 years, and also \$3000 in 5 years, how much should I deposit now in an account that earns interest at 3% per year?

Example 2-9:

a) If the Automated line costs \$900,000 now and \$200,000 per year for 5 years, how much should be deposited now, to cover all these costs, in the company's account that earns interest at 4% per year?

b) Do the same for the Manual line that costs \$500,000 now and \$300,000 per year for 5 years, and make a choice.

Example 2-10:

- Interpret the present worth calculation, if the company borrows all costs, at 4%.

2.9)b) $PW = 500k + \frac{300k}{(1+0.04)^1} + \ldots + \frac{300k}{(1+0.04)^5} = \$1835.4k$

automated is cheaper

2.10) Let PW be the amt I borrow today to pay all costs

$F = PW(1+0.4)^5 \Rightarrow PW = \frac{F}{(1+0.4)^5}$ ← calculated in 2.5a

2-11) FW @ end of yr 20

Revenue: $26k(1+.04)^{19} + 26k(1+.04)^{18} + \dots + 26k(1+0.04)^{0} = 734k$

Annual: $4k(1+.04)^{19} + 4k(1+.04)^{18} + \dots + 4k(1+.04)^{0} = 119k$

Init: $250k(1+.04)^{20} = 547k$ $\quad 774 - 547 - 119 = 107k$

Including Interest in 1-Storey versus 2-Storey Decision: Future Worth and Present Worth Methods

Total Future Worth = future worth of revenue, including interest, minus future worth of costs, including interest

Example 2-11:

- For the one-storey building, rental revenue is $26,000 per year, each year for 20 years, annual cost is $4,000 per year, and construction cost is $250,000. Suppose that the company earns interest at 4% per year, and that all revenues are deposited in this account, and all costs are withdrawn from this account. Calculate and interpret the future worth of investing in the 1-storey building.

Example 2-12:

- The 2-storey has rent = $40,000 per year, annual cost = $6,000 per year, and construction cost of $400,000. Calculate the future worth of the 2-storey building, and choose the better investment.

Example 2-13:

- Calculate and interpret the present worth of each alternative, assuming 4% is earned, and choose the better one.

Example 2-14:

- Redo Ex. 2-11,-12,-13, assuming all costs are borrowed at 4%, and revenues are counted as credits towards repaying the loans.

2-12: $FW = -400k(1+.04)^{20} + (40k-6k)[(1+.04)^{19} + (1+.04)^{18} + \dots + (1+.04)^{0}]$

$= 136.01k \Rightarrow$ choose 2 story

2-13: $P = \frac{F}{(1+i)^N}$ 1-story: $-250k + (26k-4k)\left[\frac{1}{(1+.04)} + \dots + \frac{1}{(1+.04)^{20}}\right] = 48.99k$

2-story: $62.07 \Rightarrow$ 2-story is better.

2-14

For costs, costs + interest accumulate in the same way
Calculate FW(revenues) the same way:

$26k[(1+0.04)^{19} + \dots + (1+0.04)^{0}]$

→ FW(revenues) − FW(costs) = amount I overpaid my loan

Comments on the Present Worth Method

Present Worth method is far more common than Future Worth method

Present Worth is essentially:

sum of revenues, minus sum of costs, over all periods

except that all future cash flows are “discounted”

e.g., 1-storey building

$$PW = \frac{26-4}{(1+.04)^1} + \frac{26-4}{(1+.04)^2} + \cdots + \frac{26-4}{(1+.04)^{20}} - 250$$

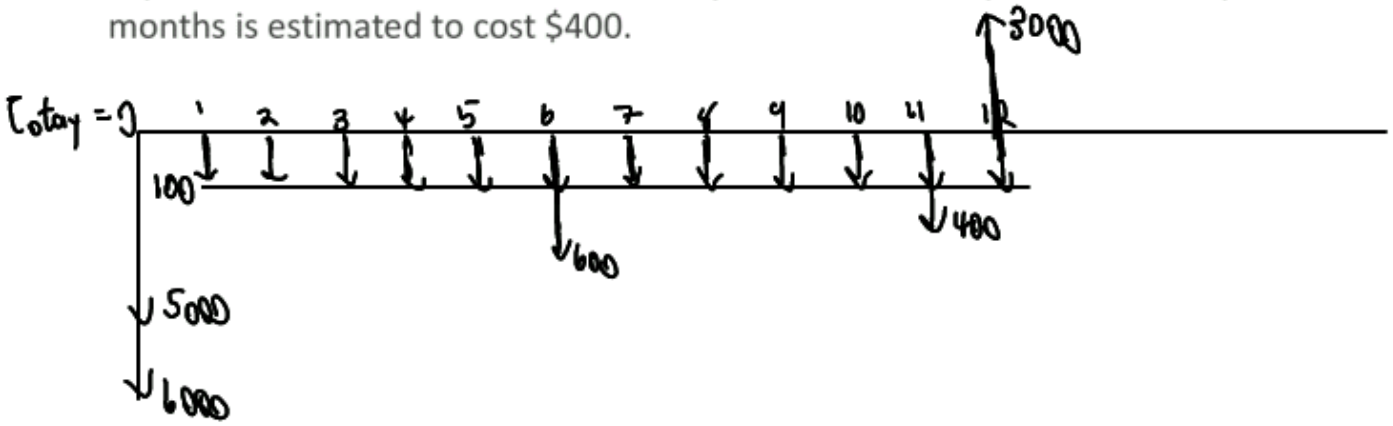
Each future cash flow shrinks with discounting.

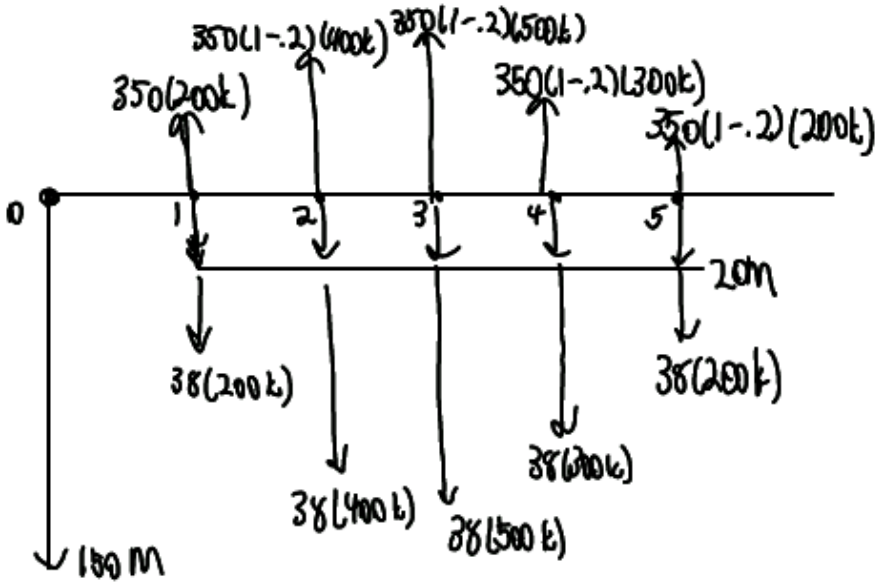
Cash Flow Diagrams

Useful to avoid errors in timing and revenue versus cost

Example 2-15:

- Draw a cash flow diagram for the following situation. You have just paid $5000 in cash for a car. Insurance costs for the year are $600 now and again at the end of 6 months. Gas costs will be $100 at the end of each month, starting at the end of month 1. You expect to sell the car at the end of the year for $3000. A pre-sale tune-up in 11 months is estimated to cost $400.





Spreadsheets for Present Worth Calculations

Example 2-16:

- R.I.N. Corporation is considering a proposal to produce a new model of cellphone. After much study, the following data have been estimated:
  - Product life = 5 years
  - Sales rate ('000 phones per year): 200, 400, 500, 300, 200
  - Unit selling price: $350 in first year, decreasing by 20% per year
  - First costs of production facility: $150,000,000
  - Annual operating costs (independent of output): $20,000,000 /year
  - Per-unit production costs: $38
  - Interest rate for discounting = 12% per year

a) Sketch a rough cash flow diagram.

b) Construct a spreadsheet to calculate the Present Worth of the proposal. Your spreadsheet should show the above data, the net cash flow for each year, and the Present Worth. Document the spreadsheet to make it understandable to others.

An Important Wrinkle in Interest Calculations: **Effective Interest Rates**

Example 2-17:

- You deposit $1000 at the end of each year for 4 years, into an account earning 1% per month, compounded monthly. How much accumulates, with interest, just after the fourth deposit ?

Generalizing, suppose interest is compounded each "small" period; let

- i_s = interest rate per small period,
- m = number of small compounding periods per "large" period
- i_e = effective interest rate per large period

Then $i_e = (1 + i_s)^m - 1$

Example 2-18:

- Find the present worth of house repainting costs of $8000 per decade, for 3 decades, starting in decade one, with interest at 5% per year, compounded annually.

Common Application of Effective Interest Rate: **Conversion of Nominal Annual Rate to Effective Annual**

Let r = the "nominal" interest rate per year

Let m = number of compounding periods per year

means r/m is the interest rate per small compounding period

i_e = the effective annual interest rate

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Example 2-19:

- For a daily interest bank account paying 3% per year (nominal), if we deposit $1000, how much will we have after one year? What is the effective interest rate?

Example 2-20:

- In Example 2-19, what would we have earned if there was only one compounding period during the year? What is the effective interest rate?

Compounding More Often

Example 2-21:

- Find the effective interest rate if r =12%/year and
  a) m = 2 (semi-annual compounding)
  b) m =12 (monthly compounding)
  c) m = 365 (daily compounding)
  d) let m go to infinity (continuous compounding)

Read for next week

Chapter 3 from textbook