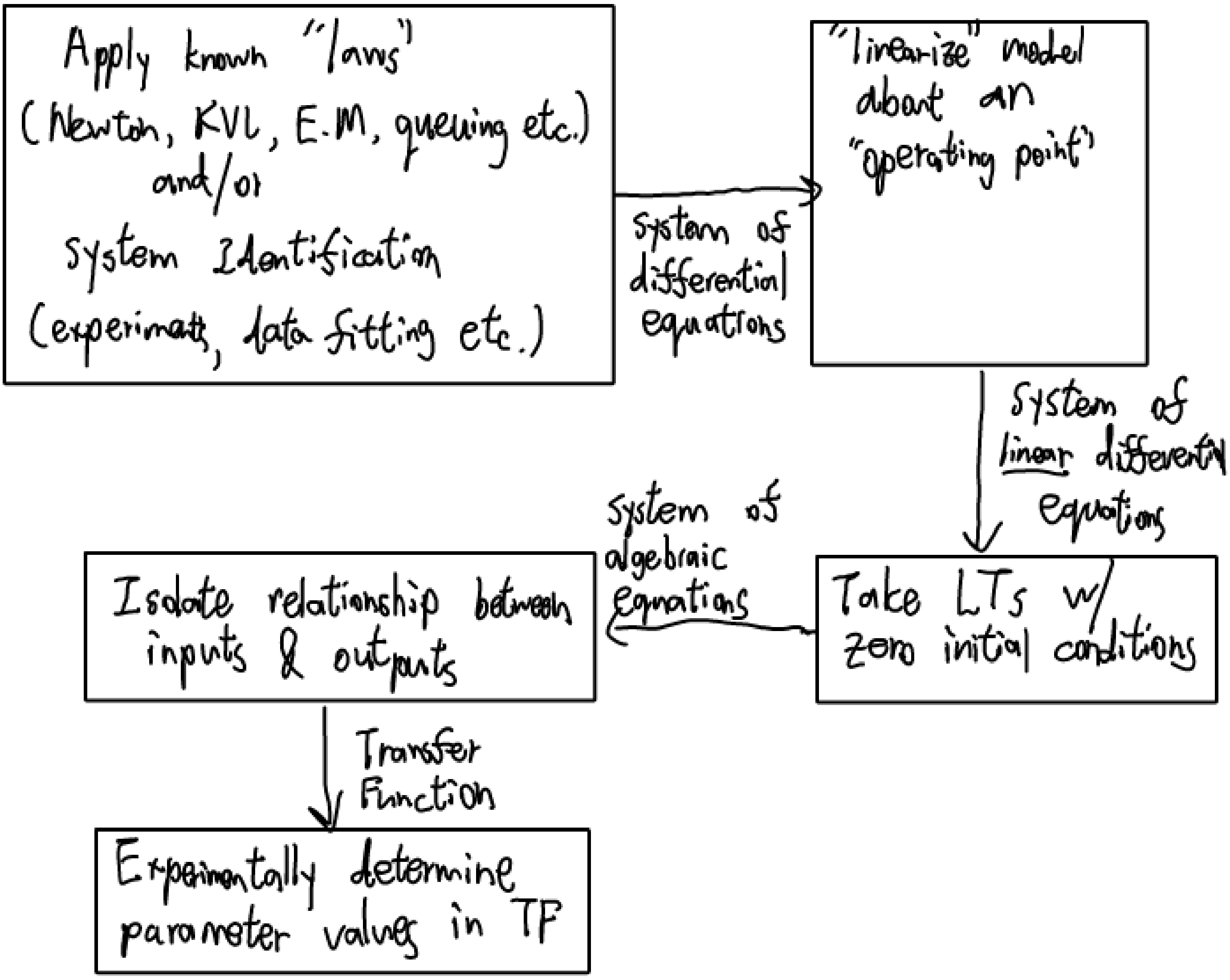
Ch. 2 Mathematical Models of System

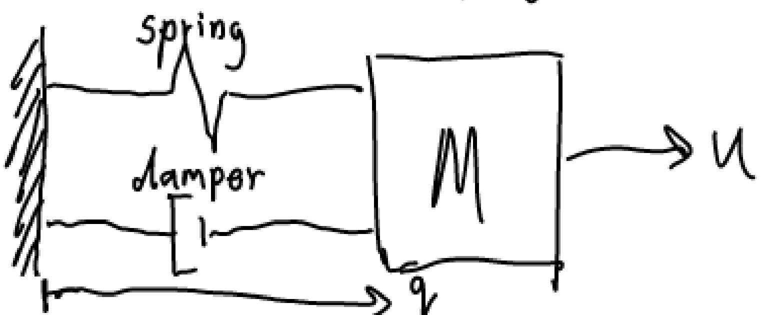
- for control design, we need a 'good' math model of the plant
  'good' := simple but accurate

2.1 General Comments on Modelling

- a model is a set of equations that represent a system.
  <u>NO</u> model is perfect.
- models allow us to simulate, test, and refine designs in a cost effective & safe way.



Ex. 2.1.1. (mass-spring-damper)



$q \in \mathbb{R}$, pos of mass
$M \in \mathbb{R}$, mass in kg
$u \in \mathbb{R}$, applied force
$\dot{q} = \frac{dq}{dt}$, velocity
$\ddot{q} = \frac{d^2q}{dt^2}$, acceleration

Take q_0 to be the position where spring is not compressed (stretched).

<u>Newton's 2^{nd} Law: $M\ddot{q} = \sum$ forces acting on M</u>

Force due to spring (assumed linear, Hooke's Law) $F_k q = kq$ (opposes motion)

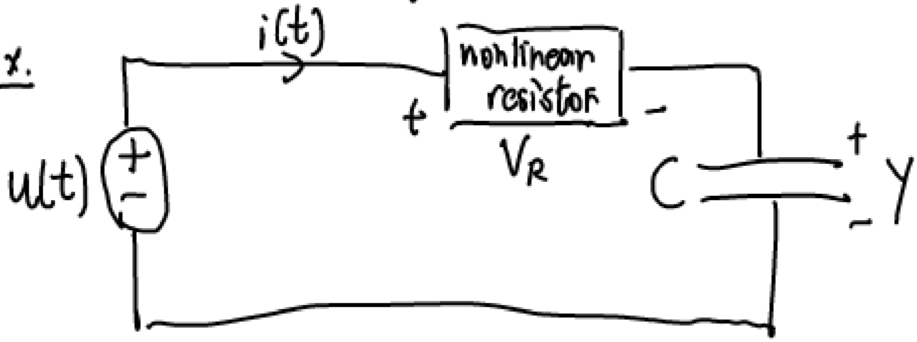
" " " damper (possibly non-linear, models friction) $c(\dot{q})$ $c: \mathbb{R} \to \mathbb{R}$ (opposes motion)

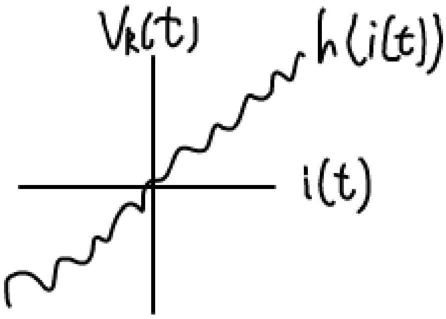
$$\boxed{M\ddot{q} = u - kq - c(\dot{q})}$$

2^{nd} order, non-linear, ODE

If the damper were linear, i.e., $c(\dot{q}) = b\dot{q}$, b constant then the overall system is linear.

Ex.





$V_R(t) = h(i(t))$, $h: \mathbb{R} \to \mathbb{R}$

$u(t)$ = applied voltage

$y(t)$ = voltage across capacitor

Apply KVL:

$$-u(t) + V_R(t) + y(t) = 0$$

$V_R(t) = h(i(t))$, $i(t) = C\frac{dy(t)}{dt}$ (capacitor device eqn)

$$\Rightarrow \boxed{-u(t) + h(C\dot{y}) + y(t) = 0}$$

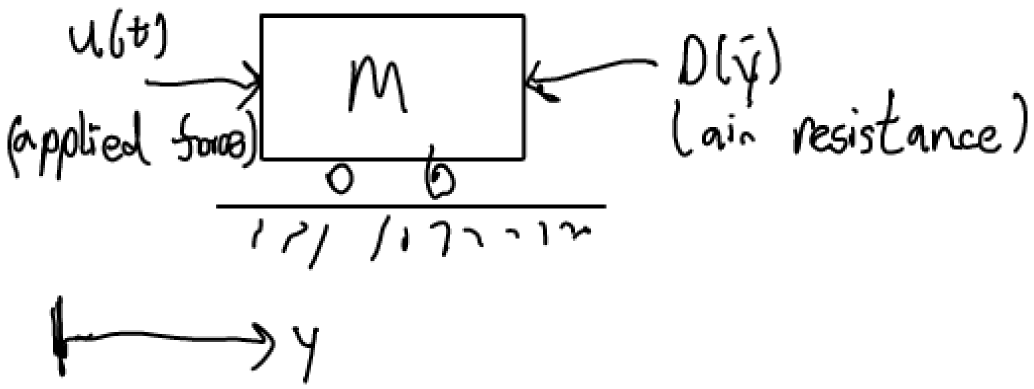
1^{st} order, non-linear ODE

If the resistor were linear, i.e. $h(i) = Ri$, R constant then the circuit is linear (see Ex. 2.3.4)

2.4 State-Space Models

- state-space models are a way of expressing math models in a standard form.

Ex. 2.4.1



$$M\ddot{y} = u - D(\dot{y})$$

We put this model into a standard form by defining two so-called "state variables"