CS370 HW3

1.

$$G_k = \sum_{n=0}^{N-1} g_n \overline{W}^{nk}$$

$$= \sum_{n=0}^{N-1} f_{N-n} \overline{W}^{nk}$$

let $m = N-n$
$n = N-m$

$\therefore$ when $n = N-1$, $m = 1$
when $n = 0$, $m = N$

$$= \sum_{m=1}^{N} f_m \overline{W}^{(N-m)k}$$

$$= \sum_{m=1}^{N} f_m \cancel{\overline{W}^{Nk}} \, \overline{W}^{-mk}$$

$$= \sum_{m=1}^{N} f_m W^{mk} + f_0 - f_N \qquad \because f_0 = f_N$$

$$= \sum_{m=0}^{N-1} f_m W^{mk}$$

$$= \overline{\sum_{m=0}^{N-1} f_m \overline{W}^{mk}}$$

$$= \overline{F_k} \qquad \square$$

2.

$$f = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \rightarrow \begin{matrix} (1+4) \\ (2+3) \\ (3+2) \\ (4+1) \\ (1-4)\overline{W}_8^0 \\ (2-3)\overline{W}_8^1 \\ (3-2)\overline{W}_8^2 \\ (4-1)\overline{W}_8^3 \end{matrix}$$

$$\begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \rightarrow \begin{matrix} (5+5) \\ (5+5) \\ (5-5)\overline{W}_4^0 \\ (5-5)\overline{W}_4^1 \end{matrix}$$

$$\begin{bmatrix} -3 \\ -\frac{1-i}{\sqrt{2}} \\ -i \\ -3\frac{1+i}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{matrix} -3-i \\ -\frac{1-i}{\sqrt{2}} - 3\frac{1+i}{\sqrt{2}} \\ (-3+i)\overline{W}_4^0 \\ \left(-\frac{1-i}{\sqrt{2}} + 3\frac{1+i}{\sqrt{2}}\right)\overline{W}_4^1 \end{matrix}$$

$$\begin{bmatrix} 10 \\ 10 \end{bmatrix} \rightarrow \begin{matrix} 10+10 \\ (10-10)\overline{W}_2^0 \end{matrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} 0+0 \\ (0-0)\overline{W}_2^0 \end{matrix}$$

$$\begin{bmatrix} -3-i \\ (-2-i)\sqrt{2} \end{bmatrix} \rightarrow \begin{matrix} -3-i-(2+i)\sqrt{2} \\ -3-i+(2+i)\sqrt{2} \end{matrix}$$

$$\begin{bmatrix} -3+i \\ (2-i)\sqrt{2} \end{bmatrix} \rightarrow \begin{matrix} -3+i+(2-i)\sqrt{2} \\ -3+i-(2-i)\sqrt{2} \end{matrix}$$

$$\begin{matrix} [20] \\ [0] \\ [0] \\ [0] \\ [-3-i-(2+i)\sqrt{2}] \\ [-3-i+(2+i)\sqrt{2}] \\ [-3+i+(2-i)\sqrt{2}] \\ [-3+i-(2-i)\sqrt{2}] \end{matrix} \rightarrow \begin{matrix} \begin{bmatrix} 20 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} -3-i-(2+i)\sqrt{2} \\ -3-i+(2+i)\sqrt{2} \end{bmatrix} \\ \begin{bmatrix} -3+i+(2-i)\sqrt{2} \\ -3+i-(2-i)\sqrt{2} \end{bmatrix} \end{matrix} \rightarrow \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ -3-i-(2+i)\sqrt{2} \\ -3+i+(2-i)\sqrt{2} \\ -3-i+(2+i)\sqrt{2} \\ -3+i-(2-i)\sqrt{2} \end{bmatrix} \rightarrow \begin{bmatrix} 20 \\ -3-i-(2+i)\sqrt{2} \\ 0 \\ -3+i+(2-i)\sqrt{2} \\ 0 \\ -3-i+(2+i)\sqrt{2} \\ 0 \\ -3+i-(2-i)\sqrt{2} \end{bmatrix} = F$$