(1)

1) If the dog is barking, then the dog is in the house.

2) If the dog is in the house then someone is at the front door unless the dog is not barking.

3) The dog is barking.

4) Someone is at the front door.

b = the dog is barking
h = the dog is in the house
d = someone is at the front door.

1) $b \Rightarrow h$

2) $h \Rightarrow (d \vee \neg b)$ } equivalent
$(h \Rightarrow d) \vee \neg b$ }

3) b

4) d

b	d	h	b⇒h	d∨¬b	h⇒(d∨¬b)
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

← BV

This BV shows this set of formulas is consistent.

A set of formulas is consistent if there is at least one BV in which all the formula are T.

A formula is satisfiable if there is at least one BV in which it is T.

(2) a) $(a \Rightarrow \neg b) \wedge (c \Rightarrow \neg d)$

Is this formula satisfiable?

Yes.

$v(a) = F$
$v(b) = T$
$v(c) = F$
$v(d) = T$

$v((a \Rightarrow \neg b) \wedge (c \Rightarrow \neg d))$
$= v(a \Rightarrow \neg b)$ AND $v(c \Rightarrow \neg d)$
$= (v(a)$ IMP $(\text{NOT } v(b)))$ AND $(v(c)$ IMP $(\text{NOT } v(d)))$
$= (F$ IMP NOT $(T))$ AND $(F$ IMP $(\text{NOT } T))$
$= T$

b) $(\overset{F}{a} \Rightarrow \overset{F}{\neg} b) \wedge (\neg a \Rightarrow \neg c)$

$(\overset{F}{a} \Rightarrow \overset{F}{\neg} b)$: $\neg$ is T, so T

$(\neg a \Rightarrow \neg c)$: a is F, c is F, so T

$T \wedge T = T$

Satisfiable?

Yes.

$v(a) = F$
$v(b) = T$
$v(c) = F$

$v((a \Rightarrow \neg b) \wedge (\neg a \Rightarrow \neg c))$
$= (v(a)\,\text{Imp}\,(\text{NOT } v(b)))$ AND $((\text{NOT } v(a))$ IMP $(\text{NOT } v(c)))$
$= (F$ Imp $(\text{NOT } T))$ AND $((\text{NOT } F)$ IMP $(\text{NOT } F))$
$= T$

$\vdash p$: defined to be true.

$\models p$: proved to be true.

$\vdash p$ $\overset{\text{soundness}}{\Leftarrow}$ $\overset{\text{completeness}}{\Rightarrow}$ $\models p$