Second Partial Derivatives

For $f: \mathbb{R}^2 \to \mathbb{R}$, the partials $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ are also functions of x & y, so we can calculate partials with respect to either variable.

<u>Notation:</u>

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

$$\left.\begin{aligned} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) &= \frac{\partial^2 f}{\partial y \partial x} = f_{xy} \\ \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) &= \frac{\partial^2 f}{\partial x \partial y} = f_{yx} \end{aligned}\right\} \text{usually equal}$$

Called mixed partials.

Exercise: Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ for

$f(x,y) = x^3 + x^2 y^3 - 2y^3$ & verify $f_{xy} = f_{yx}$

Read about 1) tangent plane.

2) Taylor polynomial/series.

Midterm Info

Wed. Feb. 26th 2:30-4:15

A-Liu EIT 1015

- No calculators
- up to and including lectures 16/17 in notes & assign 5.
- polynomial interpolation not on midterm.
- convergence tests given.

Things not on the sheet that are important

- geometric & p-series
- the alternating series estimation theorem.

· Know the Maclaurin series for (and where they converge)

- e^x
- $\sin x$
- $\cos x$

converge for all x

· $\frac{1}{1-x}$ ← $|x|<1$

· $\arctan x$ ← converge at endpoints

· $(1+x)^k$ ← $|x|<1$

· Know how to obtain related series starting from

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad |x|<1$$

Testing for convergence

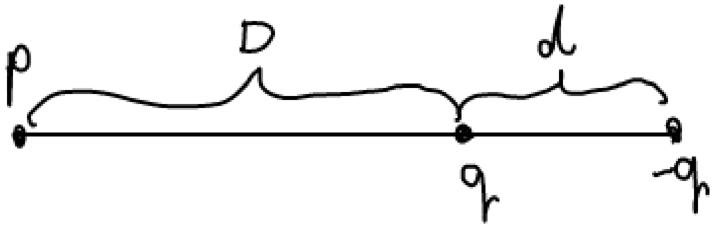
1) test for divergence

- is it geometric or p-series?
  - is it similar to geometric or p-series?
    - comparison/limit comparison test.
  - Can you integrate the associated function?
    - integral test
  - alternating?
    - AST
  - factorials or powers of k ($k!$, 2^k)
    - ratio test
  - are some terms negative?
    - check absolute convergence.

Consider the electric dipole shown below, with charges q & $-q$ located a distance of d.

The electric field E at p below is

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$$



when P is far from the dipole, show that E is proportional to $1/D^3$.

<u>Sol^n</u>: D is large compared with d. ($D \gg d$ or $\frac{d}{D} \ll 1$)

Expand something in powers of $\frac{d}{D}$.

$\rightarrow$ Make $\frac{d}{D}$ appear.

$$E = \frac{q}{D^2} - \frac{q}{D^2(1+\frac{d}{D})^2} = \frac{q}{D^2}\left(1 - \frac{1}{(1+\frac{d}{D})^2}\right)$$

Expand $\frac{1}{(1+\frac{d}{D})^2} = (1+\frac{d}{D})^{-2}$ as a binomial ($k=-2$, $x = \frac{d}{D}$

Note $|\frac{d}{D}| < 1$)

$$\left(1+\frac{d}{D}\right)^{-2} = 1 - 2\left(\frac{d}{D}\right) + O\left(\left(\frac{d}{D}\right)^2\right) \text{ as } \frac{d}{D} \to 0$$

$$E = \frac{q}{D^2}\left[1 - \left(1 - 2\left(\frac{d}{D}\right) + O\left(\left(\frac{d}{D}\right)^2\right)\right)\right]$$

$$= \frac{q}{D^2}\left(2\left(\frac{d}{D}\right) + O\left(\left(\frac{d}{D}\right)^2\right)\right)$$

when $D \gg d$, $E \approx \frac{2qd}{D^3}$