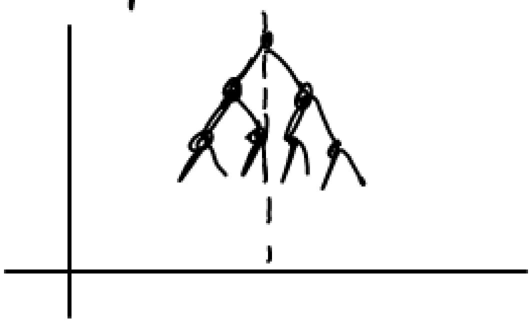
Prop: Given G embedded in the plane, the bridges of G are exactly the edges that appear twice in some face boundry walk.

Eg. Let T be a tree, is T planar?

Yes, trivial.



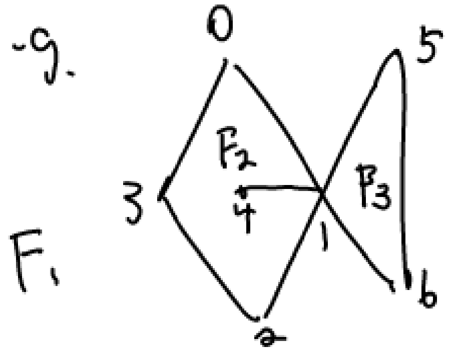
In an embedding of T in the plane, we have exactly one face.
And every edge of T is contained in the boundry walk twice. So, $\deg(F) = 2|E(T)| = 2|V(T)| - 2$

Theorem 7.1.2: If we have a planar embedding of a connected graph G with faces $F_1, \dots, F_k$, then

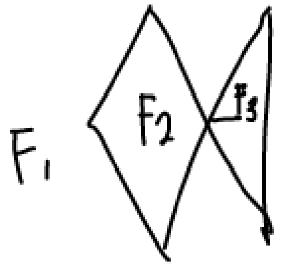
$$\sum_{i=1}^{k} \deg(F_i) = 2|E(G)|$$

Corollary 7.1.3: If we have an embedding of a connected graph G in the plane with f faces, then the avg face degree is $\dfrac{2|E(G)|}{f}$

e.g.



$$\left.\begin{array}{l} \deg(F_1) = 7 \\ \deg(F_2) = 6 \\ \deg(F_3) = 3 \end{array}\right\} 16 = 2|E(G)|$$



$$\left.\begin{array}{l} \deg(F_1) = 7 \\ \deg(F_2)\ 4 \\ \deg(F_3)\ 5 \end{array}\right\} 16 = 2|E(G)|$$

Euler's formula:

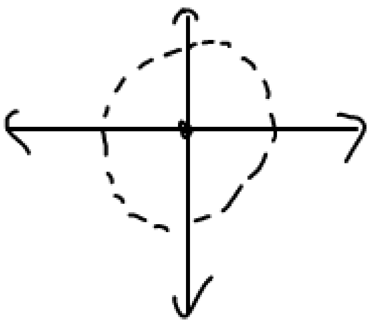
The # of faces in any embedding of a planar graph G is constant.

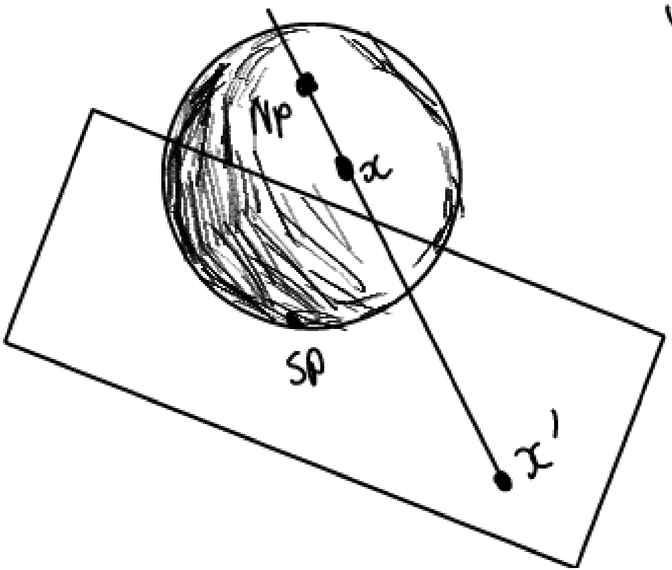
Theoren 7.2.1: Let G be a connected graph with V vertices and e edges, if G has an embedding in the plane with f faces, then

$$V - e + f = 2$$

pf: look @ notes

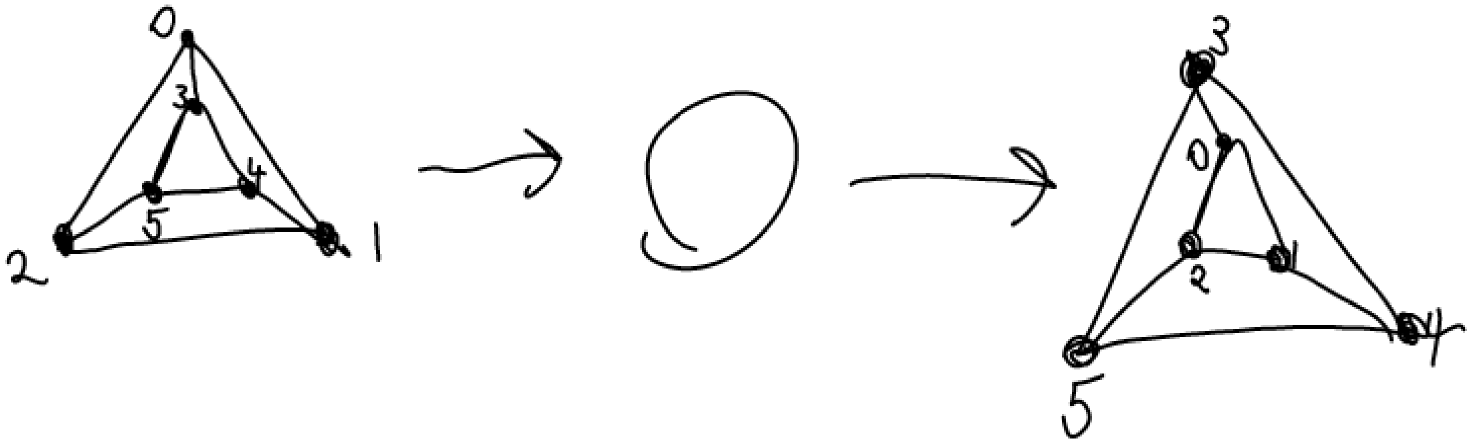
$$\mathbb{R}^2 \cong$$





$f : S \setminus \{NP\} \to \mathbb{R}^2$

$f(x) = x'$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