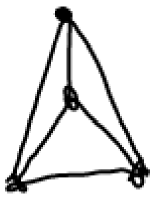
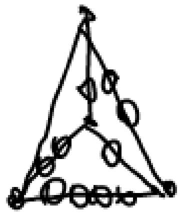
A subdivision of a graph G is a graph obtained by replacing each edge of G by a path of length ≥ 1.

eg.  K_4  subdivision of K_4

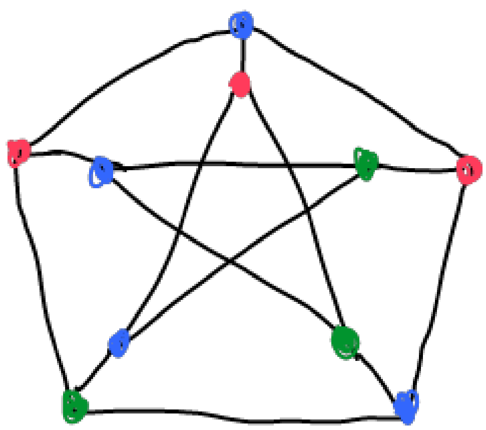
<u>Prop.</u> If H is a sub-division of a graph G then H is planar iff G is planar.

<u>Corollary.</u> If H is a non-planar graph & G is a graph containing a subdivision of H as a subgraph, then G is non-planar.

<u>Kuratowski's Theorem:</u> G is planar iff G contains no subdivision of K_5 or $K_{3,3}$ as a subgraph.

<u>Graph Coloring</u>

Let $k \in \mathbb{N}$, a <u>k-colouring</u> of a graph $G=(V,E)$ is a function from V to a set of size k (whose elements are called <u>colours</u>) so that adjacent vertices are mapped to different colours always.



A graph that has a k-colouring is <u>k-colourable.</u>

<u>Prop:</u> G is bipartite iff G is 2-colourable

<u>Prop:</u> The complete graph K_n is n-colourable but not $(n-1)$-colourable

<u>Prop:</u> The cycle C_n is 2-colourable iff n is even, and is 3-colourable if n is odd.

<u>Prop:</u> If G is k-colourable, it is also k'-colourable for all $k' \geq k$.

<u>Theorem:</u> Planar graphs are 4-colourable