ECE 124

How do we represent an integer?

decimal: $(219)_{10} = 2\times10^2 + 1\times10^1 + 9\times10^0 = 219$

$(\underbrace{\overset{\uparrow}{\text{digits}}\ \ \overset{\uparrow}{\text{base}}}_{\text{representation}})$ → placeholders; base; base; base; value

$D = d_{n-1}\ d_{n-2} \cdots d_2\ d_1\ d_0$

d_2 (100s), d_1 (10s), d_0 (units)

binary:

b_2 (4s), b_1 (2s), b_0 (units)

$B = \underbrace{b_{n-1}\ b_{n-2} \cdots b_2\ b_1\ b_0}_{\text{digits / bits}}$

0 or 1

$219 = 1\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$\underset{\text{value}}{\uparrow} = \underbrace{(11011011)_2}_{\text{representation}}$

Q. How do we represent a value to binary rep.?

$V(B) = b_{n-1}\times2^{n-1} + b_{n-2}\times2^{n-2} + \ldots + b_2\times2^2 + b_1\times2^1 + b_0\times2^0$

$\frac{V(B)}{2} = \underbrace{b_{n-1}\times2^{n-2} + b_{n-2}\times2^{n-3} + \ldots + b_2\times2^1 + b_1\times2^0}_{\text{quotient}} + \frac{b_0}{2}$ remainder

$\frac{\quad}{2} = \ldots$ quotient

eg. turn 53 into binaries

$$\begin{array}{r|l} 2 & 53 \\ 2 & 26 \rightarrow 1 \in b_0 \\ 2 & 13 \rightarrow 0 \in b_1 \\ 2 & 6 \rightarrow 1 \in b_2 \\ 2 & 3 \rightarrow 0 \\ 2 & 1 \rightarrow 1 \\ & 0 \rightarrow 1 \end{array}$$

$53 = (110101)_2$

Consider the integers

0	0000	0000
1	0001	+ 1
2	0010	0001
3	0011	+ 1
4	0100	0010
5	0101	⋮
6	0110	
7	0111	
8	1000	
⋮		

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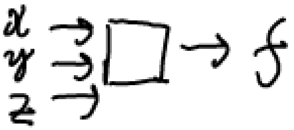
binary variables + binary functions (logic functions)

* variable + functions that are always 0 or 1

0 ↑ false, 1 ↑ true

How can we describe a logic function?
$\Rightarrow$ use a truth table

$f = f(x, y, z)$



\# rows $= 2^n$

n-inputs			outputs
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Also beneficial to be able to write binary functions as equations

Need logic operators.
- 3 of them: AND | OR | NOT

Must define how they work; must give them a symbol:

AND (AND2)

"$\bullet$" $f = x \bullet y = xy$

x	y	f
0	0	0
0	1	0
1	0	0
1	1	1

OR (OR2)

"+" $f = x + y$

x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

NOT

$f = \tilde{x}$
$= !x$
$= x'$
$= \neg x$

x	f
0	1
1	0

Aside:

AND can be expanded to n-inputs

$f = x_1 \cdot x_2 \cdot x_3 \cdot \cdots \cdot x_n$

x_1	x_2	---	x_n	f
0	0	----	0	0
		$\vdots$		$\downarrow$
1	1	----	1 0	0
1	1	----	1 1	1

OR can be expanded

$f = x_1 + x_2 + x_3 + \ldots + x_n$

x_1	x_2	---	x_n	f
0	0	----	0 0	0
0	0	----	0 1	1
		$\vdots$		$\downarrow$
1	1	----	1 1	1

x	y	f
0	0	1
0	1	0
1	0	0
1	1	1

$$\Longleftrightarrow$$

$$\underbrace{f = \bar{x}\bar{y} + xy}_{\text{f described as a logic eqn}}$$

f described via a truth table