A7 - Giliam

1. a) $\mathcal{L}\left\{\int_0^t e^{t-\tau}\cos(3\tau)\,d\tau\right\} = \mathcal{L}\{e^t * \cos(3t)\}$

$$= \mathcal{L}\{e^t\}\mathcal{L}\{\cos 3t\}$$

$$= \frac{1}{s-1}\frac{s}{s^2+9}$$

b) $\mathcal{L}\left\{\int_0^t \tau^3 \sin(t-\tau)\,d\tau\right\} = \mathcal{L}\{\sin t * t^3\}$

$$= \mathcal{L}\{\sin t\}\mathcal{L}\{t^3\}$$

$$= \frac{1}{s^2+1}\frac{3!}{s^4}$$

$$= \frac{6}{s^4(s^2+1)}$$

2. $x''(t) + x(t) = 3\delta(t-1)$

$$X(s)(s^2+1) = 1$$

$$X(s) = \frac{1}{s^2+1}$$

$\therefore\ x(t) = \sin t \leftarrow$ impulse respone

Now, $x(t) = \int_0^t g(t-\tau)\,u(\tau)\,d\tau$

$$= \int_0^t 3\delta(\tau-1)[\sin(t-\tau)]\,d\tau$$

gives 0 if $t<1$ & $3\sin(t-1)$ if $t>1$

$\therefore\ x(t) = 3H(t-1)\sin(t-1)$