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1. Introductions
2. Statements
3. Implications
MC 6492

Proofs

What is a proof?
- Verifying a statement of fact
- Involves logical steps
- Communication

Statement

A sentence that is either true or false.

e.g. $1+1=2$ T
         $1>2$ F

A sentence that has a variable is an open source.

   x is even   T for $x=2$
                  F for $x=\pi$

Combining Statements

① A and B   e.g. "x is even and $x \geq 314$"
② A or B   T when $x=314$
                      F when $x=315$
③ not A   "x is even or $x \geq 314$"
                      F when $x=3$
                      T when $x=1000$

Implications

if A, then B "Whenever A is true, B is true"

e.g.   ① if $x \geq 2$, then $x^2 \geq 4 \to T$ $A \to$ hypothesis

     ② if $x^2 \geq 4$, then $x \geq 2 \to F$ $B \to$ conclusion

For an implication with vars, it is true if it works for all values of the vars.

① $x = 3$ if $3 \geq 2$, then $9 \geq 4$ T

    $x = 0$ if $0 \geq 2$, then $0 \geq 4$ T   hyp. is false.

           if $1 = 2$, then blue is yellow   T

           if today is Friday, then tomorrow is saturday   T

② $x = -100$, $(-100)^2 \geq 4$ but $-100 \not\geq 2$ counterexample

   $x = 2$, $2^2 = 4$ $2 \geq 2$

To prove that an implication is fake, we need a counter example.

e.g. ① if $x \geq 2$, then $x^2 \geq 5$

     ② if $x \geq 2$, then $x^2 \geq 3$

     ③ if $xy > 0$, then $x > 0$ or $x + y < 0$

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① False.   let $x = 2$: $2 \geq 2$ hyp is true.

                  $2^2 \not\geq 5$ conc is false.

② True.   if $x \geq 2$, then $x^2 = x \cdot x \geq 2 \cdot 2 = 4 \geq 3$

③ True.   if $xy > 0$, then $\{ x > 0$ and $y < 0\}$

                       or $\{ x < 0$ and $y < 0\}$

    when $x > 0$ and $y > 0$, $x > 0$ So conclusion

    when $x < 0$ and $y < 0$, $x + y < 0$ holds.

Def^n: The <u>converse</u> of "if A, then B" is "if B, then A"

They are not equivalent. If both are true, then A and B are equivalent.

e.g. For integers x, "if x is even, then $x+1$ is odd"

  Converse. "if $x+1$ is odd, then x is even."

  both are true.