1. Sets
2. Sets Operations
3. Proving with sets

Sets

A set is a collection of objects

$S=\{1,2,3,4,5\}$

$T=\{3,5,\pi,e,10^{100}\}$

Order does not matter.
No repeated elements.

$3\in S$ (3 is in S)

$6\notin S$ (6 is not in S)

Set-building notation. $S=\{\underbrace{\boxed{\quad\quad}}_{\text{objects in } S} \mid \underbrace{\boxed{\quad\quad}}_{\text{constraints these objects satisfy}}\}$

$\{1,2,3,4,5\}=\{x\in\mathbb{Z} \mid 1\le x\le 5\}$

set of even integers $\{2x \mid x\in\mathbb{Z}\}$

Some Common Sets:

$\mathbb{N}=\{1,2,3,\ \}$ $\quad \mathbb{Z}$ integers $\quad \mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z}, b\neq 0\}$

$\mathbb{R}$ reals $\quad \mathbb{C}$ complex $\quad \mathbb{C}=\{a+bi \mid a,b\in\mathbb{R}\}$

Cartesian product: if S and T are sets,

$S\times T=\{(a,b) \mid a\in S, b\in T\}$

$\mathbb{R}\times\mathbb{R}=\{(a,b) \mid a,b\in\mathbb{R}\}$

$\{1,2\}\times\{3,4,5\}=\{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$

$S=\{(x,y)\in\mathbb{Z}\times\mathbb{Z} \mid x+2y=0\}$ $\quad (1000,-500)\in S$

$1000\notin S$

Empty set $\emptyset$. The only set that has no elements.

Subsets: S is a <u>subset</u> of T, denoted $S \subseteq T$, if all elements of S are in T. "if $x \in S$, then $x \in T$".

$$\{2,3\} \subseteq \{1,2,3,4,5\}$$

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

Is $\emptyset \subseteq \{1,2,3\}$? Yes, it satisfies $x \in \emptyset \Rightarrow x \in \{1,2,3\}$

Is $\underbrace{\emptyset}_{\text{set}} \in \underbrace{\{1,2,3\}}_{\text{not an element}}$? No, but $\emptyset \in \{\emptyset\}$ yes

set $\longrightarrow$ not an element

<u>Set operations</u>

① Union $S \cup T = \{x \mid x \in S \text{ or } x \in T\}$

② Intersection $S \cap T = \{x \mid x \in S \text{ and } x \in T\}$

③ Set Difference: $S \setminus T = \{x \mid x \in S \text{ and } x \notin T\}$

$$S = \{1,2,3,4,5\} \quad T = \{3,5,\pi,e,10^{100}\}$$

$$S \cup T = \{1,2,3,4,5,\pi,e,10^{100}\}$$

$$S \cap T = \{3,5\}$$

$$S \setminus T = \{1,2,4\}$$

<u>Proving with Sets</u>

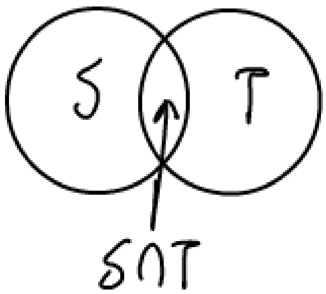
How to prove $S \subseteq T$? Prove "if $x \in S$, then $x \in T$".

e.g. $(S \cap T) \subseteq (S \cup T)$

"if $x \in S \cap T$, then $x \in S \cup T$"

if $x \in S \cap T$, then $x \in S$. So $x \in S$ or $x \in T$.

So $x \in S \cup T$.



$S = T$ if $S \subseteq T$ and $T \subseteq S$

e.g. $S = (S \cap T) \cup (S \setminus T)$

$(\subseteq)$ Let $x \in S$. We know $x \in T$ or $x \notin T$.

If $x \in T$, then $x \in S \cap T$

If $x \notin T$, then $x \in S \setminus T$

So $x \in (S \cap T) \cup (S \setminus T)$

$(\supseteq)$ Let $x \in (S \cap T) \cup (S \setminus T)$

So $x \in S \cap T$ or $x \in S \setminus T$.

If $x \in S \cap T$, then $x \in S$. If $x \in S \setminus T$, $x \in S$

So $x \in S$.

Since $S \subseteq (S \cap T) \cup (S \setminus T)$ and $(S \cap T) \cup (S \setminus T) \subseteq S$

So $S = (S \cap T) \cup (S \setminus T)$

e.g. Let $A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 2x + 3y = 0\}$

$B = \{x \in \mathbb{Z} \mid 2 \mid x\}$

$C = \{x \in \mathbb{Z} \mid 3 \mid x\}$

Prove $A \subseteq C \times B$.

Let $(x, y) \in A$. Then $2x + 3y = 0$. Then $x = \frac{-3y}{2}$

since $x \in \mathbb{Z}$, $\frac{y}{2} \in \mathbb{Z}$. So $x = 3\left(\frac{-y}{2}\right)$, and $3 \mid x$, so $x \in C$

$y = \frac{-2x}{3}$, since $y \in \mathbb{Z}$, $\frac{x}{3} \in \mathbb{Z}$, so $y = 2\left(-\frac{x}{3}\right)$

and $2 \mid y$, so $y \in B$.

So $(x, y) \in C \times B$.