1. $[x^{23}](1-2x^3)^{-7} + [x^{21}](1-2x^3)^{-7} = \binom{7+7-1}{7} 2^7$

2. First we choose A, since A is a k-subset of $[n]$, there are $\binom{n}{k}$ ways to pick A.
We then choose B, since B - - - - - - -
- - $\binom{n}{k}$ ways to pick B.
$\therefore \binom{n}{k}^2$
First we consider $\binom{n}{i}$ as $|A \cap B|$, $\binom{n-i}{k-i}$ as $A \setminus B$, & $\binom{n-i-(k-i)}{k-i}$ as $B \setminus k$.
For all possible values of i, we choose the appropriate $\binom{n}{k}$.
$\therefore \binom{n}{k}^2$

3. $A = \{4, 7, 10, 13, \ldots\}$
$\Phi_A = x^4 + x^7 + \ldots$
$= \frac{x^4}{1-x^3}$
$\Phi(x) = \frac{1}{1-\frac{x^4}{1-x^3}} = \frac{1-x^3}{1-x^3-x^4}$

4. $\Phi_S = \frac{1}{1-x^2} \frac{1}{1 - x\frac{1}{1-x^2} x\frac{1}{1-x^2}} = \frac{1}{1-x^2}$
$= \frac{x}{(1-x^2)^2} \frac{1}{1-\frac{x^2}{(1-x^2)^2}}$
$= \frac{x}{1-2x^2+x^4-x^2} = \frac{x}{1-3x^2+x^4}$

b) $1-x^3 = (a_0 + a_1 x + a_2 x^2) - (a_0 x^3 - a_1 x^4 \ldots)$
$(-a_0 x^4 - a_1 x^5 \ldots)$

$a_0 = 1$
$a_1 = 0$
$a_2 = 0$

$a_3 - a_0 = -1$
$a_3 = 0$
$a_4 - a_1 - a_0 = 0$
$a_4 = 1$
$a_5 - a_2 - a_1 = 0$
$a_5 = 0$

$\therefore a_i - a_{i-3} - a_{i-4} = 0$

5. $L \cup M = \{\varepsilon\} \cup L\{0,1\}$

$L = M\{11000\}$

$\{0\}^* \{\{1\}\{0\}\{0\}\{0\}^* \cup \{1\}\{1\}\{0,00,000\}\}^*$

$$\mathcal{D}_L + \mathcal{D}_M = 1 + \mathcal{D}_L(2x)$$

$$\mathcal{D}_L x^6 = \mathcal{D}_M$$

$$\mathcal{D}_L + x^6\mathcal{D}_L = 1 + \mathcal{D}_L(2x)$$

$$\mathcal{D}_L(1 + x^6 - 2x) = 1$$

$$\mathcal{D}_L = \frac{1}{1 - 2x + x^6}$$

6. $a_n - 3a_{n-1} = 0$

$r - 3 = 0$

$r = 3$

$\therefore f_n = \alpha_1 (3)^n$

$2 = \alpha_1$

$\therefore f_n = 2(3)^n$

7. G has an Euler Tour.

8. a)

b) a & b are even

c)

9. a)

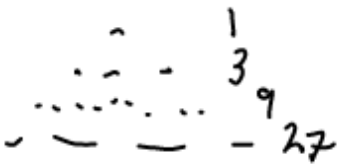
b) 42 leaves

$\frac{1-x^4}{1-x}$

$3^3 = 27$

$3^0 + 3^1 + 3^2 + 3^3 = 40$

$\therefore$ 40 vertices

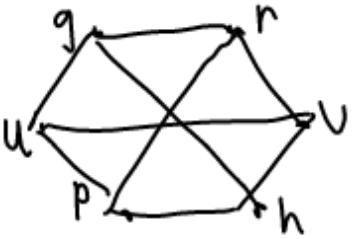


10. $|V| - |E| + |F| = 2$

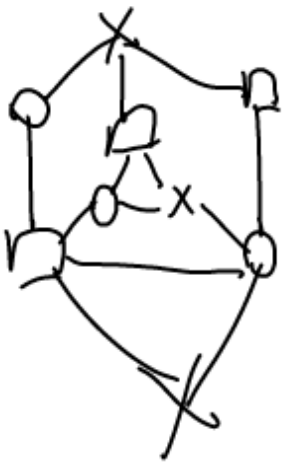
11.

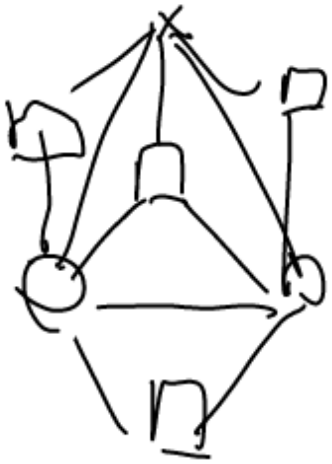
non-planar

planar



12.





13. a)

$\hat{x} = \{c, f\}$ $\hat{y} = \emptyset$

$\hat{y} = \{1,2,3,4,5\}$

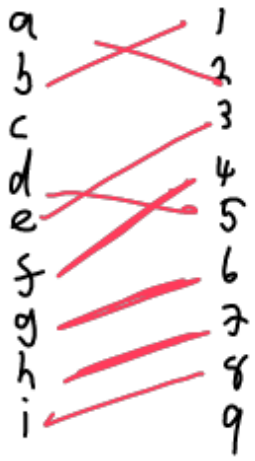
pr: 1 → c, 2 → c, 3 → g, 4 → f, 5 → f

$\hat{x} = \{c, f, a, b, e, g, d\}$ $\hat{y} = \{6\}$

a → 2
b → 1
e → 3
g → 4
d → 5

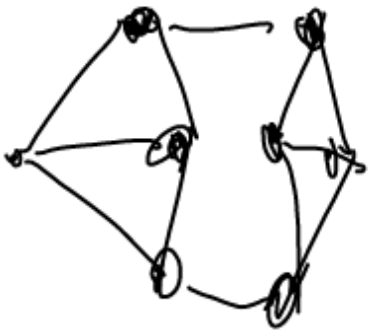
b → g
h → 6

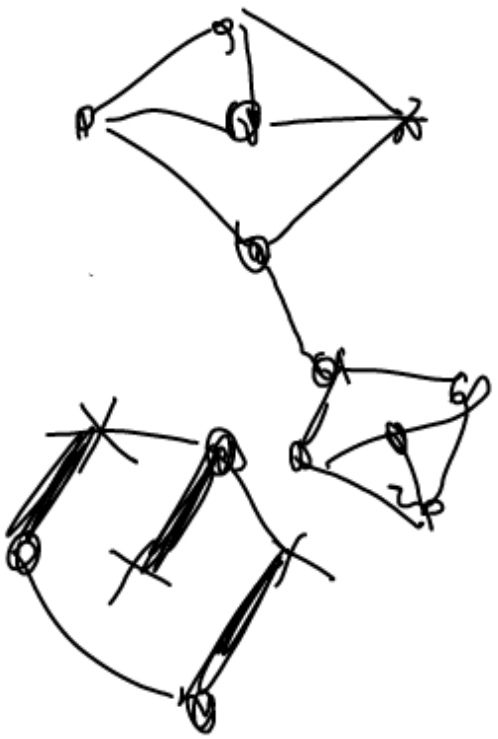
$\hat{x} = \{h\}$



7, h, 6, g, 4, f

14. a)





$x = \{7, 9\}$ $y = \{0\}$

$y = \{h, i\}$

$x = \{7, 9, 6, 8\}$ $y = \{h, i, 9\}$

$x = 4$ $y = f$

$h \to 7$
$i \to 9$
$6 \to h$
$8 \to i$
$9 \to 6$
$4 \to 9$
$f \to 4$

f 4 9 6 h 7