2009.

1. a) $[x^n]\frac{(1+2x)(1+x^2)^m}{(1-x)^3} = [x^n]\frac{(1+x^2)^m}{(1-x)^3} + \frac{2x(1+x^2)^m}{(1-x)^3}$

$= [x^n]$

b) $[x^n]\frac{1}{1-x-x^m+x^{m+1}} = [x^n]\frac{1}{1-(x+x^m+x^{m+1})}$

$= [x^n]\sum^{\infty}(x+x^m+x^{m+1})$

2. $\Phi_{20}(x) = 1 + x^{20} + x^{40} + \ldots$ $\quad w(a) = 20a$

$\Phi_{50}(x) = 1 + x^{50} + x^{100} + \ldots$ $\quad w(b) = 50b$

$\Phi_{100}(x) = 1 + x^{100} + x^{200} + \ldots$ $\quad w(c) = 100c$

$$[x^{1000}]\Phi(x) = [x^{1000}] \frac{1-x^{20\times(50+1)}}{1-x^{20}} \frac{1-x^{50\times(40+1)}}{1-x^{50}} \frac{1-x^{100\times(90+1)}}{1-x^{100}}$$ $w(tot) = 20a + 50b + 100c$

3\. a) any zero followed by odd 1's followed by even zeroes followed by any 1s.

b) because it is unambiguous.

c) $$\Phi = \frac{1}{1-x} \frac{1}{1-\left(x\frac{1}{1-x} x\frac{1}{1-x} - x^2\frac{1}{1-x^2} x\frac{1}{1-x^2}\right)} \frac{1}{1-x}$$

d)

4. if $2n_3+2$

Then there are n_3+2 vertices of degree 1.

In a tree $p-1=e$

$$\sum \deg v = 3n_3 + n_3+2$$

$$= 4n_3+2$$

∴ tree

$$= 2(2n_3+1)$$

$$= 2(p-1)$$

If tree, then, $e=p-1$

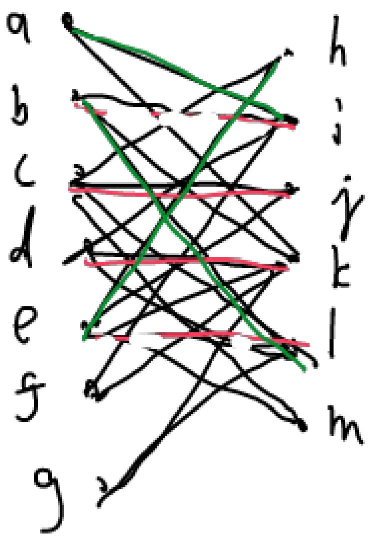
$$3n_3+n_1 = 2(n_3+n_1-1)$$

$$3n_3+n_1 = 2n_3+2n_1-2$$

$n_1 = n_3+2$

$$n_3 = n_1-2$$

∴ $p = 2n_3+2$

6. 

$x = \{a\}$ $y = \{i, k\}$

$x \to \{b, d\}$ $y \to \{l\}$

$x \to \{e\}$ $y \to \{h\}$

helbia

$j \to g$
$k \to g$

$b \to i$
$d \to k$
$l \to b$
$e \to l$

7. a) $v - e + f = 2$

b) $v - e + f = 2$

$\sum \deg f = 2e$

$\sum \deg v = 2e$

$5v = 2e$

$\frac{2e}{5} - e + f = 2$

$f = \frac{3e}{5} + 2$

$f = \frac{3e + 10}{5}$

$xf = 2e$

$e = \frac{5(f-2)}{3}$

$xf = 2\left(\frac{5(f-2)}{3}\right)$

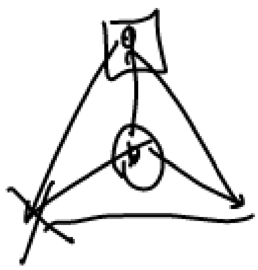
$\therefore f =$

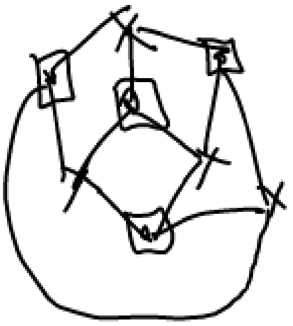
$\frac{3}{10} xf = f - 2$

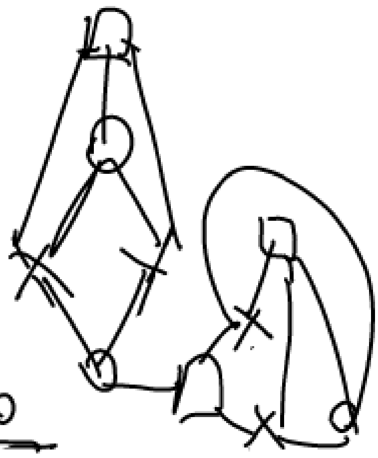
$2 = f\left(1 - \frac{3}{10} x\right)$

$f = \frac{2}{1 + \frac{3}{10} x} = \frac{20}{10 + 3x}$

$= x = 3$ $f = 20$







2008

1. since we don't contain n, we remove that from the list, hence $n-1$.

Since we already included 1, therefore $n-2$ and $k-1$

$\therefore \binom{n-2}{k-1}$

$\Phi_1 = 1 + x^{10} + x^{20} + \dots$

$\Phi_2 = 1 + x^{20} + x^{40} + \dots$

$\Phi_3 = 1 + x^{50} + x^{100} + \dots$

$\Phi = \Phi_1 \Phi_2 \Phi_3$ $\qquad w(x) = 10a + 20b + 50c$

$= \frac{1}{1-x^{10}} \frac{1}{1-x^{20}} \frac{1}{1-x^{50}}$ $\qquad x^{20} = (x^{10})^2$

$= \frac{1}{1-x^{10}} \frac{1}{(1+x^{10})(1-x^{10})} \frac{1}{1-x^{50}}$

$= \frac{1}{(1-x^{10})^2} \frac{1}{(1+x^{10})} \frac{1}{(1-x^{50})}$

3. a) $\{0\}^* \{\{1\}\{1\}^* \{0\}\{0\}^* \setminus \{1\}\{11\}^* \{0\}\{00\}^*\}^* \{0\}^*$

b) $(a_0+a_1x+a_2x^2) - (a_0x+a_1x^2+a_2x^3)$ $= 1+x$

$- (a_0x^2+a_1x^3+\dots)$

$a_0=1$

$a_1-a_0=1 \qquad a_1=2$

$a_2-a_1-a_0=0$

$a_2=3$

4、 $x^3-5x^2+8x-4=0$

$$
\begin{array}{r}
x^2-4x+4 \\
x-1 \,\overline{)\, x^3-5x^2+8x-4} \\
x^3-x^2 \\
\hline
-4x^2+8x \\
-4x^2+4x \\
\hline
4x-4
\end{array}
$$

$(x-1)(x-2)^2=0$

$\therefore \alpha\cdot 1^n + (\beta n+\gamma)2^n = a_n \qquad \therefore a_n=(2-n)2^n$

$2=\alpha+\gamma \qquad 2=\alpha+2(\beta+\gamma) \qquad 0=\alpha+8\beta+4\gamma$

$2=\alpha+2\beta+2\gamma$

$0=2\beta+\gamma \qquad -2=8\beta+3\gamma$

$0=6\beta+3\gamma$

$\alpha=0 \quad \gamma=2 \qquad -2=2\beta$

$\beta=-1$

$\sum \deg(f) = 2e$ $\quad V - e + f = 2$

$V = 2e$

$3f = 2e$ $\quad \frac{2e}{5} - e + \frac{2e}{3} = 2$

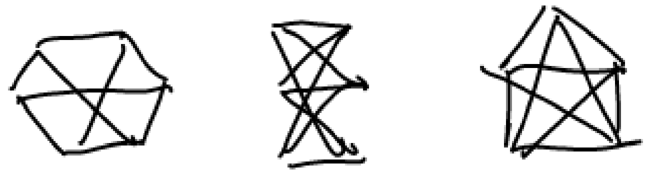
$f = \frac{2e}{3}$ $\quad 6e - 15e + 10e = 30$

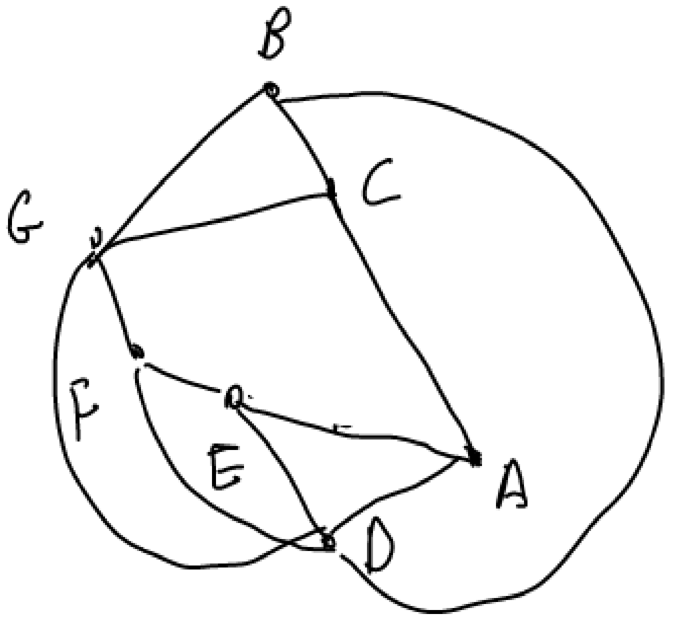
$e = 30$

$5V = 2e$

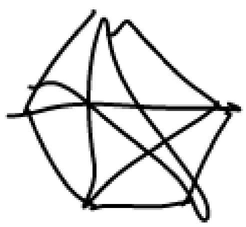
$V = \frac{2e}{5}$

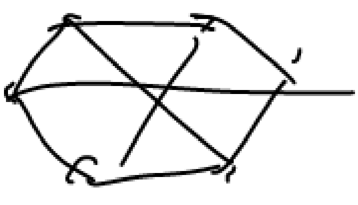
G,

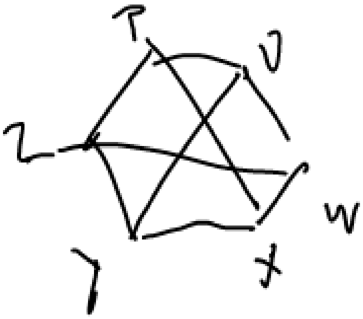




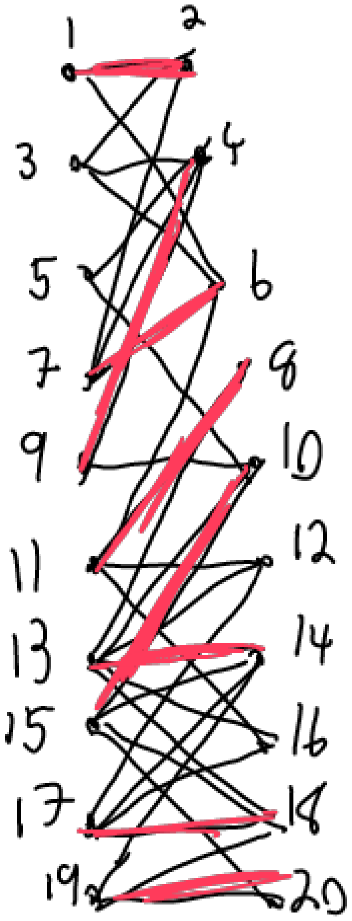
planar







9、



$x=\{3,5\}$ $y=\{2,4,6,10\}$

$x=\{3,5,1,9,7,15\}$

$y=\{2,4,6,10,14,18,20\}$

$x=\{13,17,19\}$

$y=\{12\}$

12, 13, 14, 15, 10, 5,

$p(2)=3$
$p(4)=5$
$p(6)=3$
$p(10)=5$
$p(1)=2$
$p(9)=4$
$p(7)=6$
$p(15)=10$
$p(14)=15$
$p(18)=15$
$p(20)=15$
$p(14)=13$
$p(18)=17$
$p(20)=19$