Complex Stuff

$$z = a + bi$$
$$i^2 = -1$$

Division

$$\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di} = \frac{\sim}{\underbrace{c^2+d^2}_{\text{real}}}$$

Properties of $\mathbb{C}$

Let $u, v, z \in \mathbb{C}$

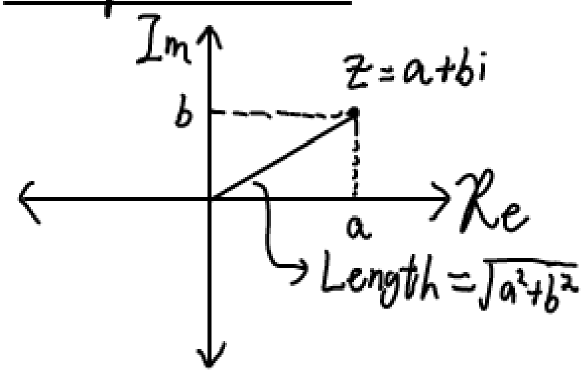
1. Associativity: $(u+v)+z = u+(v+z)$
   $(uv)z = u(vz)$
2. Commutativity: $u+v = v+u$
   $vu = uv$
3. Distributivity: $u(v+z) = uv+uz$

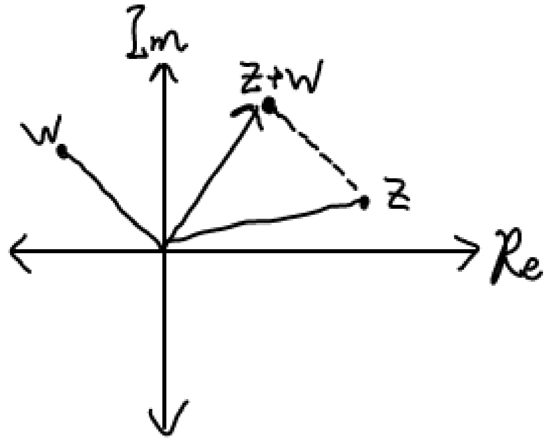
Conjugate of $z = a+bi$ is $\overline{z} = a-bi$

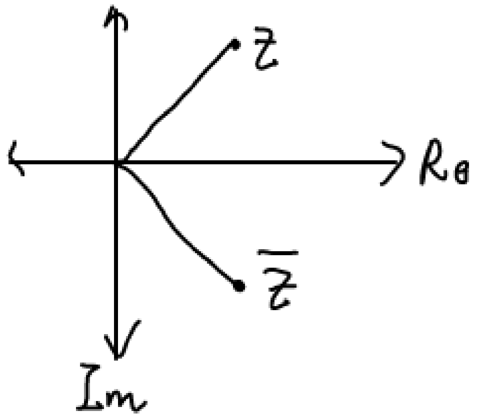
Properties of $\overline{z}$... Let $z, w \in \mathbb{C}$

1. $\overline{z+w} = \overline{z} + \overline{w}$
2. $\overline{zw} = \overline{z}\cdot\overline{w}$
3. $\overline{\overline{z}} = z$
4. $z + \overline{z} = 2Re(z)$
5. $z - \overline{z} = 2iIm(z)$
6. z is real iff $z = \overline{z}$
   z is purely imaginary iff $z = -\overline{z}$

Complex Plane





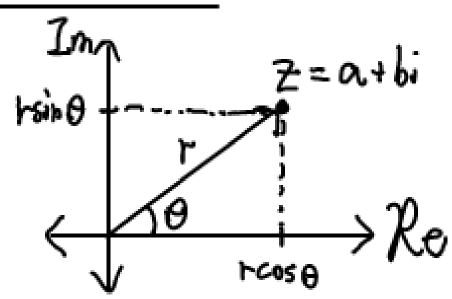


Modulus of $z = a+bi$ is $|z| = \sqrt{a^2+b^2} = \sqrt{z \cdot \bar{z}}$ $\quad |z|^2 = z \cdot \bar{z}$

Properties of $|z|$

1. $|z| \geq 0$
2. $|z| = |\bar{z}|$
3. $z^{-1} = \frac{1}{z} \cdot \frac{\bar{z}}{\bar{z}} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2}$
4. $|zw| = |z| \cdot |w|$
5. $|z+w| \leq |z| + |w|$

Polar Form



Associate z with r (length) and θ (angle with Re axis)

$z = r(\cos\theta + i\sin\theta)$

$r = |z| = \sqrt{a^2+b^2}$

$\theta = \arcsin\frac{b}{r} = \arccos\frac{a}{r}$] not unique

If $0 \leq \theta \leq 2\pi$, then the polar form is unique.

Multiplication: If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then $z_1 z_2 = r_1 r_2 (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

Example: $(-\sqrt{3}+i)(1-i) = (-\sqrt{3}+1) + (\sqrt{3}+1)i$

$$= \left[2\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)\right]\left[\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)\right]$$

$$= 2\sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$$

$$\cos\frac{7\pi}{12} = \frac{-\sqrt{3}+1}{2\sqrt{2}} \qquad \sin\frac{7\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$z = r(\cos\theta + i\sin\theta)$$

$$z^{-1} = \frac{1}{r}(\cos(-\theta) + i\sin(-\theta))$$