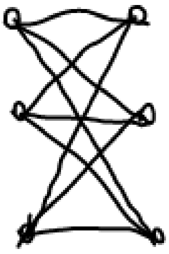
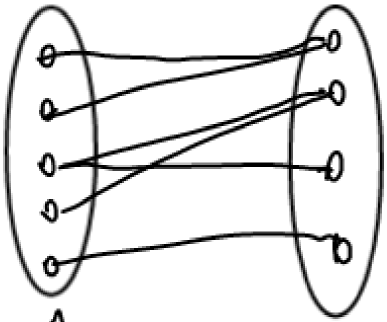
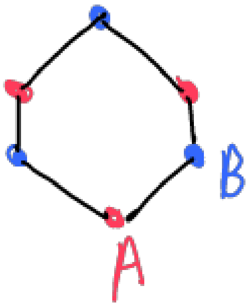


$K_{3,3}$

$K_{3,3}$ is a bipartile graph.



A bipartile graph is a graph $G=(V,E)$ for which there exists sets A, B such that $A \cup B = V$, $A \cap B = 0$, and every edge is incident with a vertex in A and a vertex in B. (A,B) is a bipartition of G.

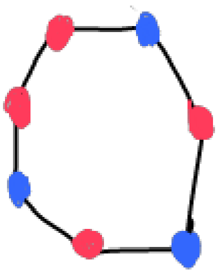


C_6

A k-cycle is a graph $G=(V,E)$ so that V has an ordering $v_1, v_2, \dots, v_k$ so that $E = \{v_1v_2, v_2v_3, \dots, v_{k-1}v_k, v_kv_1\}$.

Prop: A k-cycle is bipartile iff k is even.

pf: If k is even, then $(\{v_1, v_3, v_5, \dots, v_{k-1}\}, \{v_2, v_4, \dots, v_k\})$ is a bipartition. So C_k is bipartile.



If k is odd, suppose (A,B) is a bipartition with $v_1 \in A$. (otherwise, switch A & B so that $v_1 \in A$).

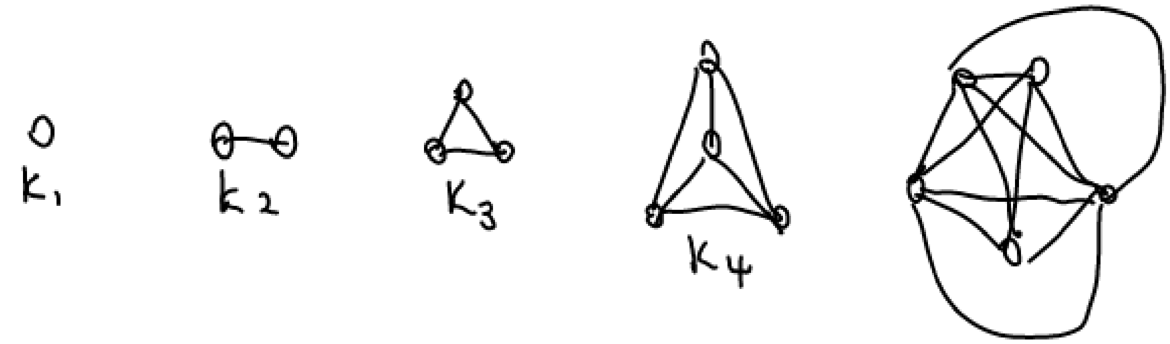
We show recursively that $v_i \in A$ whenever i is odd. True for $i=1$. If it is true for some V_i, then since $V_iV_{i+1} \in E$ and $V_{i+1}V_{i+2} \in E$, we have $V_{i+1} \in B$ and $V_{i+2} \in A$. By induction, $V_i \in A$ for all i. Thus $V_k \in A$ and $V_1 \in A$, since $V_kV_1 \in E$, (A, B) is not a bipartition.

A complete graph K_n is a graph $G=(V,E)$ so that $|V| = n$ and every pair of vertices is adjacent.

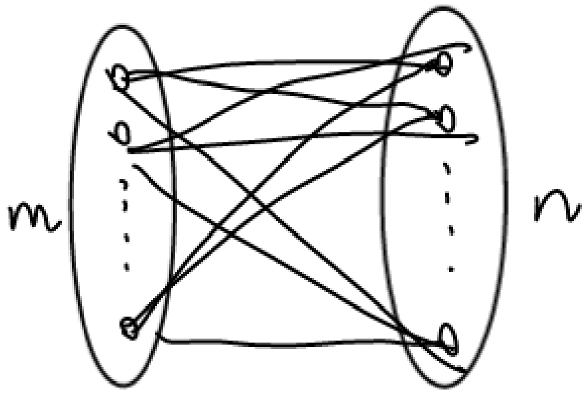
A complete graph has $\binom{n}{2}$ edges. $\left(= \frac{n(n-1)}{2}\right)$

Q. Which complete graphs are bipartite? K_1 and K_2.

Q. Which complete graphs are planar? (can be drawn in the plane without edges crossing)



A complete biparte graph $K_{m,n}$ is a biparte graph with a bipartition (A, B) so that every vertex in A is adjacent to every vertex in B, and $|A| = m$, $|B| = n$



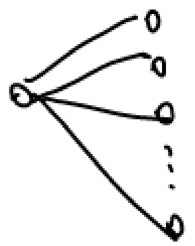
N.B. Complete bipartite graphs are not (usually) complete

$K_{m,n}$ has mn edges.

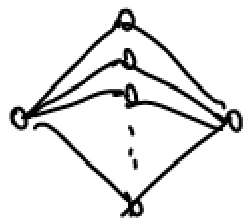
Q: Which complete bipartite graphs are regular?

When $n=m$, because the vertices in A have degree n, and those in B have degree m.

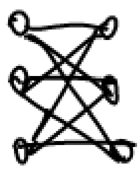
Q. Which complete bipartite graphs are planar?

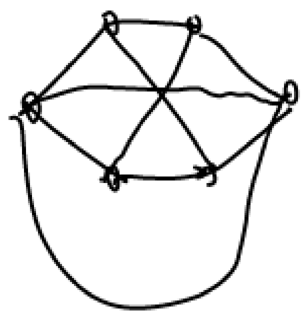


$K_{1,n}$



$K_{2,n}$

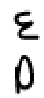




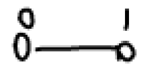
For $n \geq 0$, an n-cube is a graph with

$V = \{$binary strings of length $n\}$

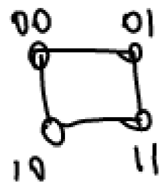
in which 2 vertices are adjacent if they differ in exactly one position.



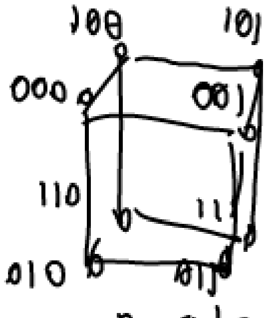
0-cube



1-cube



2-cube



3-cube