

CS370 HW2

1. Refer to page 3

$$2. \quad S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 - \frac{1}{4}(x-1)^2(x-2) & 1 \leq x < 2 \\ e + f(x-2) + g(x-2)^2 + \frac{1}{4}(x-2)^2(x-3) & 2 \leq x \leq 3 \end{cases}$$

$$\frac{d^2 S}{dx^2}(1) = 0, \quad \frac{d^2 S}{dx^2}(3) = 0$$

$$\frac{dS}{dx}(x) = \begin{cases} b + 2c(x-1) - \frac{3x^2}{4} + 2x - \frac{5}{4} \\ f + 2g(x-2) - \frac{3x^2}{4} + \frac{7x}{2} - 4 \end{cases}$$

$$\frac{d^2 S}{dx^2}(x) = \begin{cases} 2c - \frac{3x}{2} + 2 \\ 2g - \frac{3x}{2} + \frac{7}{2} \end{cases}$$

$$a) \quad S(1) = a + b(0) + c(0) - 0$$

$$1 = a$$

$$S(2) = a + b + c + 0$$

$$1 = 1 + b + c$$

$$0 = b + c$$

$$S(3) = e + f + g + 0$$

$$0 = 1 + f + g$$

$$-1 = f + g$$

$$S(2) = e + f(0) + g(0) + 0$$

$$1 = e$$

$$b) \quad S'(2) = S'(2)$$

$$b + 2c(2) - 2c - \frac{3(2)^2}{4} + 2(2) - \frac{5}{4} = f + 2g(2) - 4g - \frac{3(2)^3}{4} + \frac{7(2)}{2} - 4$$

$$b + 2c - \frac{1}{4} = f$$

$$b + 2c - f = \frac{1}{4}$$

$$c) \frac{d^2 S}{dx^2}(3) = 2g - \frac{3(3)}{2} + \frac{7}{2}$$

$$0 = 2g - 1$$

$$g = -\frac{1}{2}$$

$$\frac{d^2 S}{dx^2}(1) = 2c - \frac{3(1)}{2} + 2$$

$$0 = 2c + \frac{1}{2}$$

$$c = -\frac{1}{4}$$

$$d) 0 = b + c$$

$$b = \frac{1}{4}$$

$$-1 = f + g$$

$$f = -\frac{1}{2}$$

e) We must check that the second derivative is also continuous by computing $S''_{1 \leq x \leq 2}(2) = S''_{2 \leq x \leq 3}(2)$.

3) refer to page 4 and after