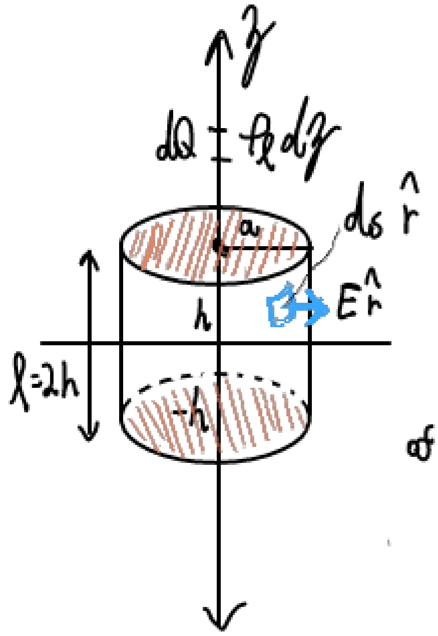
Gauss Law

$$\oint_{\text{closed surface}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \longleftarrow \text{flux}$$



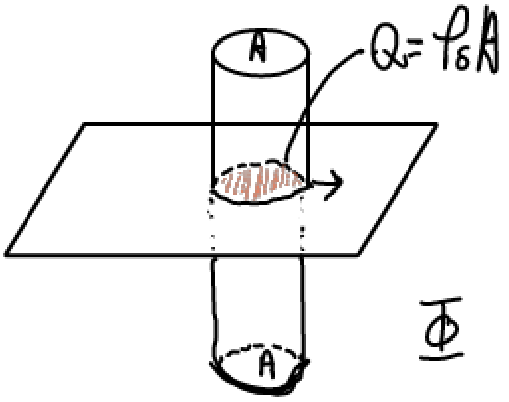
$$\oint_{\text{surface of the cyl.}} \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} = \frac{\rho_L l}{\varepsilon_0}$$

$$= E\int ds = E(2\pi a)\,l$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 a}$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 a}\hat{r}$$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r}$$



$$\oint_{\text{closed S of Cyl.}} \vec{E}\cdot d\vec{S} = \rho_s A \qquad \vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{z}$$

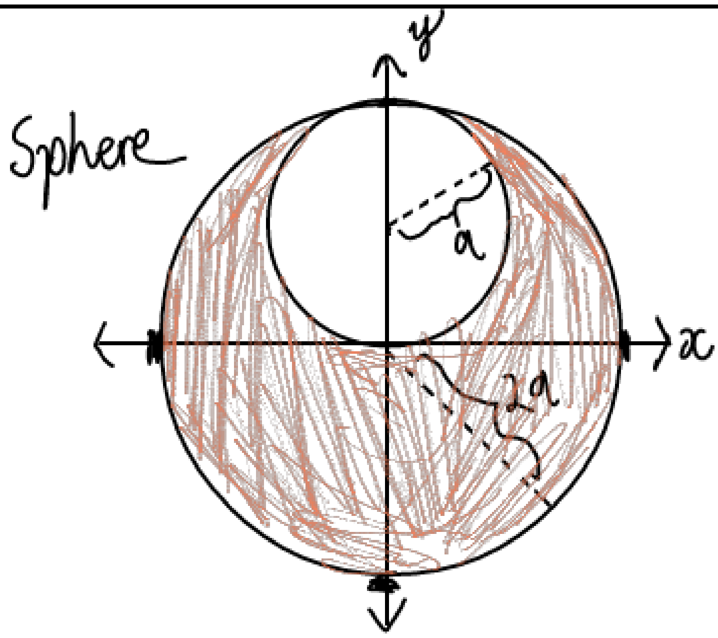
$Q = \rho_s A$

$$\Phi \Rightarrow \int_{A_1} \vec{E}\cdot d\vec{s} + \int_{A_2} \vec{E}\cdot d\vec{s}$$

$$\frac{Q}{\varepsilon_0} = \int E_{top}\,ds + \int E_{bot}\,ds$$

$$\frac{\rho_s A}{\epsilon_0} = 2EA$$

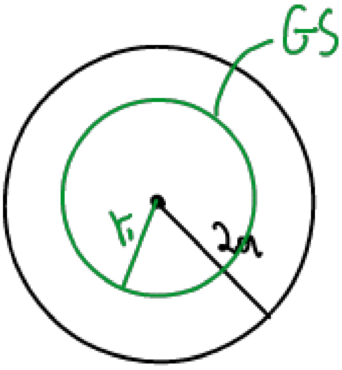
$$\Rightarrow \vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{y}$$



$$\rho_v = \frac{Q}{V}$$

Show $E_x = 0$, $E_y = \frac{\rho a}{3\epsilon_0}$ inside cavity.

①

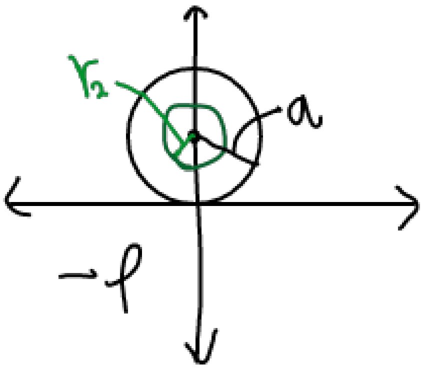


$$\Phi = \frac{Q_{in}}{\epsilon_0}$$

$$E_1 4\pi r_1^2 = \frac{\frac{4}{3}\pi r_1^3 \rho}{\epsilon_0}$$

$$\vec{E}_1 = \frac{r_1 \rho}{3\epsilon_0}\hat{r}$$

②



$$\Phi = \frac{Q_{in}}{\epsilon_0}$$

$$E_2 4\pi r_2^2 = \frac{\frac{4}{3}\pi r^3 (-\rho)}{\epsilon_0}$$

$$\vec{E}_2 = \frac{r_2 \rho}{3\epsilon_0}(-\hat{r})$$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\vec{r}_1 \rho}{3\epsilon_0} - \frac{\vec{r}_2 \rho}{3\epsilon_0}$$

$$= \frac{\rho}{3\epsilon} \left(\vec{r}_1 - \vec{r}_2 \right)$$

$$= \frac{\rho}{3\epsilon_0} \left(a\vec{y} \right)$$