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2. $y'' + y' + y = x^2 + 2e^x + 4\sin x$

Homog case:

$$y'' + y' + y = 0 \Rightarrow \lambda^2 + \lambda + 1 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j$$

$$\therefore y_h = C_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + C_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right)$$

Particular case:

$$y_p = k_1 x^2 + k_2 x + k_3 + Ce^x + A\cos x + B\sin x$$

$$y_p' = 2k_1 x + k_2 + Ce^x - A\sin x + B\cos x$$

$$y_p'' = 2k_1 + Ce^x - A\cos x - B\sin x$$

Sub back into original equation

$$(2k_1 + Ce^x - A\cos x - B\sin x) + (2k_1 x + k_2 + Ce^x - A\sin x + B\cos x)$$
$$+ k_1 x^2 + k_2 x + k_3 + Ce^x + A\cos x + B\sin x = x^2 + 2e^x + 4\sin x$$

$$k_1 x^2 + (k_2 + 2k_1)x + (2k_1 + k_2 + k_3) + 3Ce^x - A\sin x + B\cos x = x^2 + 2e^x + 4\sin x$$

$$k_1 = 1,\ k_2 = -2,\ k_3 = 0,\ C = \frac{2}{3},\ A = -4,\ B = 0$$

$$\therefore y_p = x^2 - 2x + \frac{2}{3}e^x - 4\sin x$$

$$\because y = y_h + y_p$$

$$\therefore y = C_1 e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{3}}{2}x\right) + C_2 e^{-\frac{1}{2}x}\sin\left(\frac{\sqrt{3}}{2}x\right) + x^2 - 2x + \frac{2}{3}e^x - 4\cos$$

Marko & Sameer

1. $y'' - 20y' + 100y = xe^{10x}$

homog. case:

$$y'' - 20y' + 100y = 0$$

$$\lambda^2 - 20\lambda + 100 = 0$$

$$(\lambda - 10)^2 = 0$$

$$\lambda = 10$$

$$\therefore y_h = C_1 e^{10x} + C_2 x e^{10x}$$

particular case:

$$y_p = (k_1 x + k_2)(Ce^{10x}) \quad \text{part of } y_h$$

$$= (k_1 x + k_2)(C x e^{10x}) \quad \text{still part of } y_h$$

$$= (k_1 x + k_2)(C x^2 e^{10x})$$

$$= k_1 C x^3 e^{10x} + k_2 C x^2 e^{10x}$$

$$y_p' = k_1 C x^2 e^{10x}(10x + 3) + k_2 C x e^{10x}(10x + 2)$$

$$y_p'' = 2k_1 C x e^{10x}(50x^2 + 30x + 3) + 2k_2 C e^{10x}(50x^2 + 20x + 1)$$

sub in to original

$$2k_1 C x e^{10x}(50x^2 + 30x + 3) + 2k_2 C e^{10x}(50x^2 + 20x + 1)$$
$$- 20(k_1 C x^2 e^{10x}(10x + 3) + k_2 C x e^{10x}(10x + 2)) + 100(k_1 C x^3 e^{10x} + k_2 C x^2 e^{10x}) = x e^{10x}$$

$(100-200+100)k_1Cx^3e^{10x} + (60-60)k_1Cx^2e^{10x} + 6k_1Cxe^{10x}$

$(100-200+100)k_2Cx^2e^{10x} + (40-40)k_2Cxe^{10x} + 2k_2Ce^{10x} = xe^{10x}$

$$6k_1Cxe^{10x} + 2k_2Ce^{10x} = xe^{10x}$$

$$\therefore k_1C = \frac{1}{6},\ k_2C = 0$$

$$\therefore y_p = \frac{1}{6}x^3e^{10x}$$

$$\therefore y = y_h + y_p$$

$$\therefore y = C_1e^{10x} + C_2xe^{10x} + \frac{1}{6}x^3e^{10x}$$