Another Multilevel Circuit Example

Consider implementing the following 4 fcns. Using AND/OR/INV

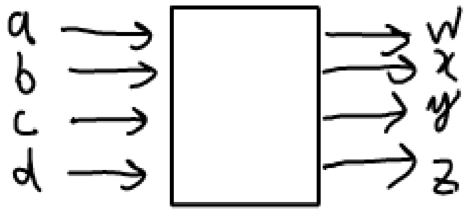
10 $w = a + bc + bd$

14 $x = \bar{b}c + \bar{b}d + b\underbrace{\bar{c}\bar{d}}_{(\overline{c+d})}$

9 $y = cd + \bar{c}\bar{d}$

0 $z = \bar{d}$

$\overline{33}$



factoring gives:

$w = a + b(c+d)$

$x = \bar{b}(c+d) + b(\overline{c+d}) = b \oplus (c+d)$

$y = cd + (\overline{c+d}) = c \odot d$

$z = \bar{d}$

$abc + (a+b+c)(\overline{ab+ac+bc})$

$= \qquad (\overline{ab} \cdot \overline{ac} \cdot \overline{bc})$

$= abc + \underbrace{(a+b+c)(\bar{a}+\bar{b})}(\bar{a}+\bar{c})(\bar{b}+\bar{c})$

$= abc + (a\bar{b} + \bar{a}b + \bar{a}c + \bar{b}c)(\bar{a}\bar{b} + \underbrace{\bar{a}\bar{c} + \bar{c}\bar{b} + \bar{c}}_{\bar{c}})$

$= abc + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c$

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Number Representation

* We will consider signed and unsigned integers.

Positional Number Notation

$$N = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \dots + a_1 r^1 + a_0 r^0$$

Value coefficients base

$$= (a_{n-1}\, a_{n-2} \cdots a_1\, a_2)_r$$ ← representation of value N in base r.

Coefficients are always $(0, 1, \dots, r-1)$

Fixed Point Notation

$$N = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \dots + a_1 r^1 + a_0 r^0 + a_{-1}r^{-1} + a_{-2}r^{-2} + \dots$$

fixed point

$$= (a_{n-2}\, a_{n-1} \cdots a_1\, a_0 \,.\, a_{-1}\, a_{-2} \cdots)_r$$

e.g.,

$$213.78 = (213.78)_{10}$$
$$= 2\times10^2 + 1\times10^1 + 3\times10^0 + 2\times10^{-1} + 8\times10^{-2}$$

$$(101.11)_2 = 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 0\times2^{-3}$$
$$= 4 + 0 + 1 + \frac{1}{2} + \frac{1}{4}$$
$$= 5.75 = (5.75)_{10}$$

To convert from base-r to base 10, just expand the representation.

To convert from base 10 to base-r...

→ Do integer part + fractional part separately.

$$\frac{N}{r} = \underbrace{a_{n-1} \times \frac{r^{n-1}}{r} + a_{n-2} \times \frac{r^{n-2}}{r} + \dots + \frac{a_1 r^1}{r}}_{\text{quotient}} + \underbrace{\frac{a_0 r^0}{r}}_{\substack{\text{remainder} \\ = a_0}}$$

$$\frac{\text{quotient}}{r} = \underbrace{\qquad\qquad\qquad}_{\text{another quotient}} + \underbrace{\quad}_{\text{another remainder}}$$

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e.g.

$(62.37)_{10}$ to base-2

2 | 62		
2 | 31	0	$\leftarrow a_0$
2 | 15	1	$\leftarrow a_1$
2 | 7	1	$\leftarrow a_2$
2 | 3	1	
2 | 1	1	
1	1	

$0.37 \times 2 = \overset{a_{-1}}{\boxed{0}}.74$

$0.74 \times 2 = \overset{a_{-2}}{\boxed{1}}.48$

$0.48 \times 2 = \overset{a_{-3}}{\boxed{0}}.96$

$0.96 \times 2 = \overset{a_{-4}}{\boxed{1}}.92$

$\therefore (111110.0101)_2$

$(62.37)_{10}$ to base-7

$$\begin{array}{l|ll} 7 & 62 & \\ 7 & 8 & 6 \\ 7 & 1 & 1 \\ & 0 & 1 \end{array}$$

$0.37 \times 7 = \overset{a_{-1}}{\boxed{2}}.59$

$0.59 \times 7 = \overset{a_{-2}}{\boxed{4}}.13$

$0.13 \times 7 = \overset{a_{-3}}{\boxed{0}}.91$

$0.91 \times 7 = \overset{a_{-4}}{\boxed{6}}.37$

$0.37 = \cdots$

repeats

$$\therefore (116.\overline{2406})_7$$