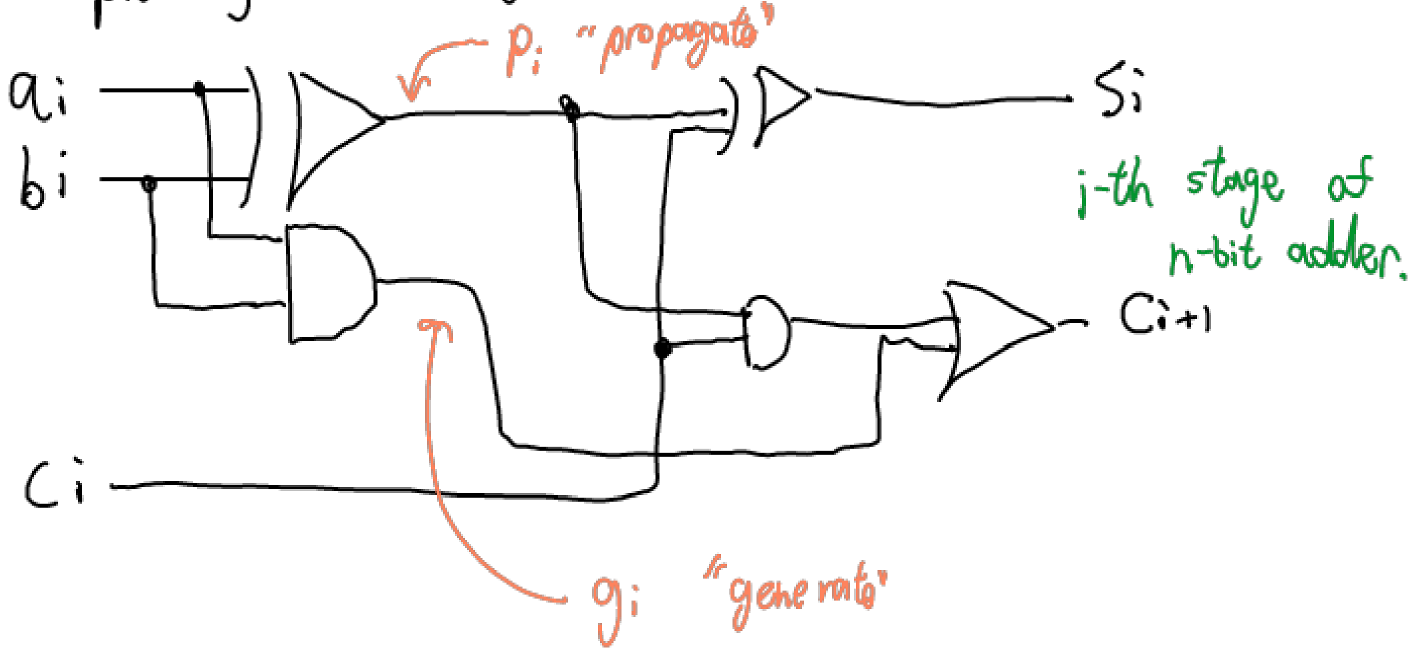
Last time:

* n-bit ripple adder.
  → follows logic of how we add.
  → slow for large n due to the worst path down the carry chain.

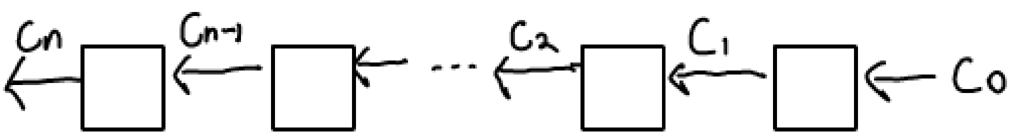
*Q: what can we do to improve performance?

A: Shorten critical path; in this circuit it means producing carries faster.



$$C_{i+1} = a_i b_i + c_i(a_i \oplus b_i)$$

$$= g_i + c_i p_i$$



$$C_1 = g_0 + c_0 p_0$$

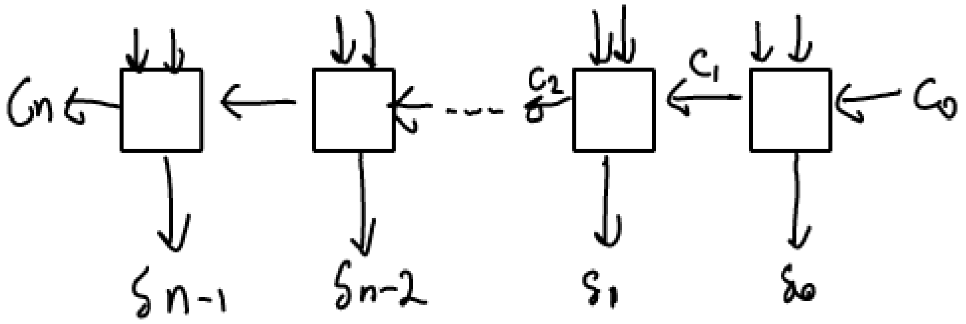
$$C_2 = g_1 + C_1 P_1 = g_1 + P_1 g_0 + P_1 P_0 C_0$$

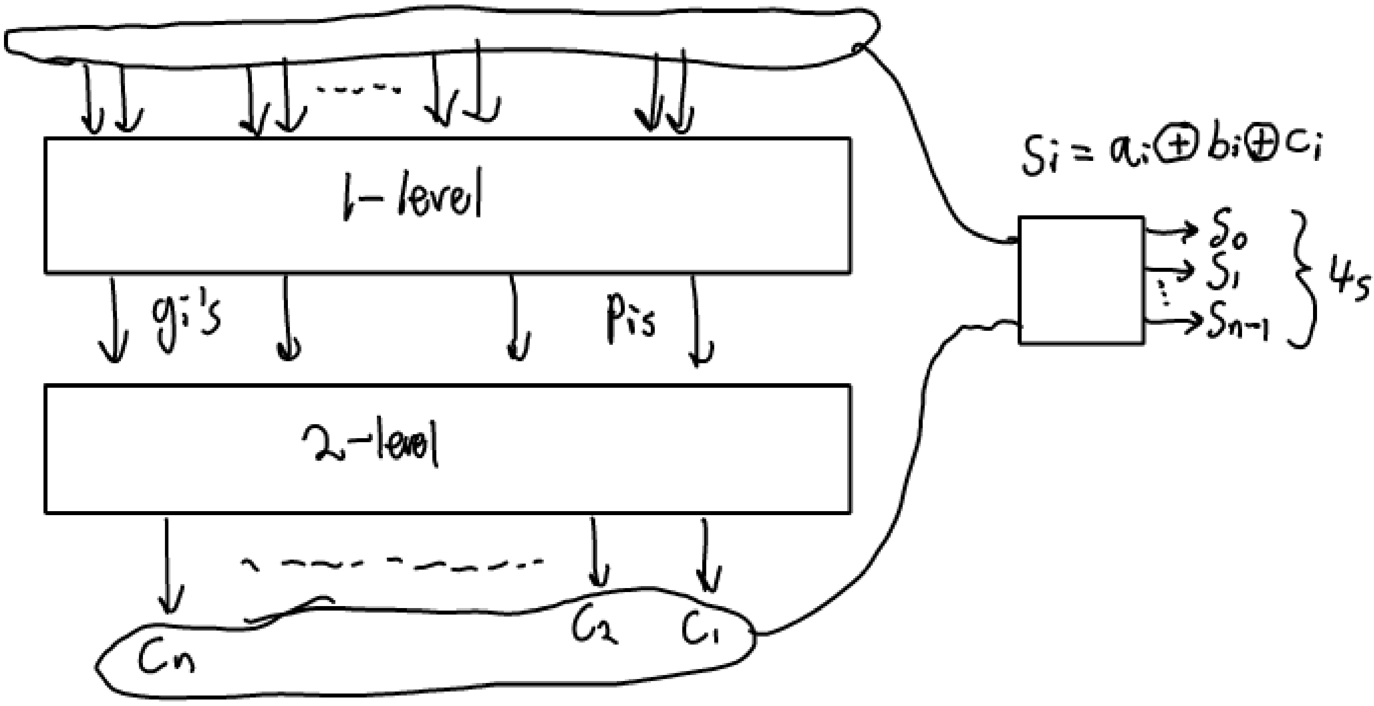
$$C_3 = g_2 + C_2 P_2 = g_2 + P_2 g_1 + P_2 P_1 g_0 + P_2 P_1 P_0 C_0$$

By substituting, all c_is are now 2 levels from the p_i's and g_i's.

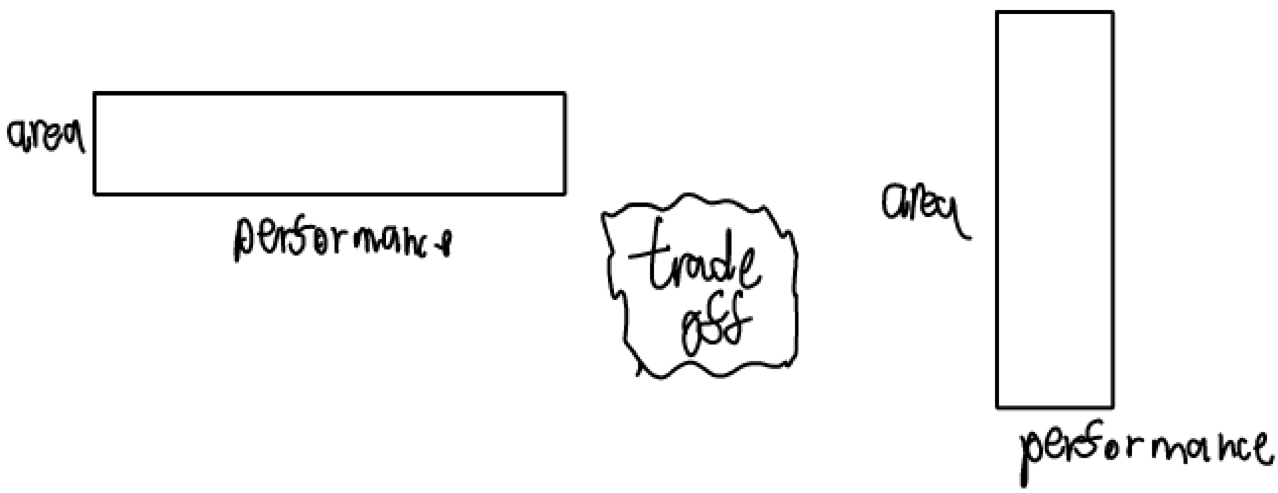
The p_i's & g_i's are 1 level from the inputs.

∴ all c_is are now 3-levels from the inputs.

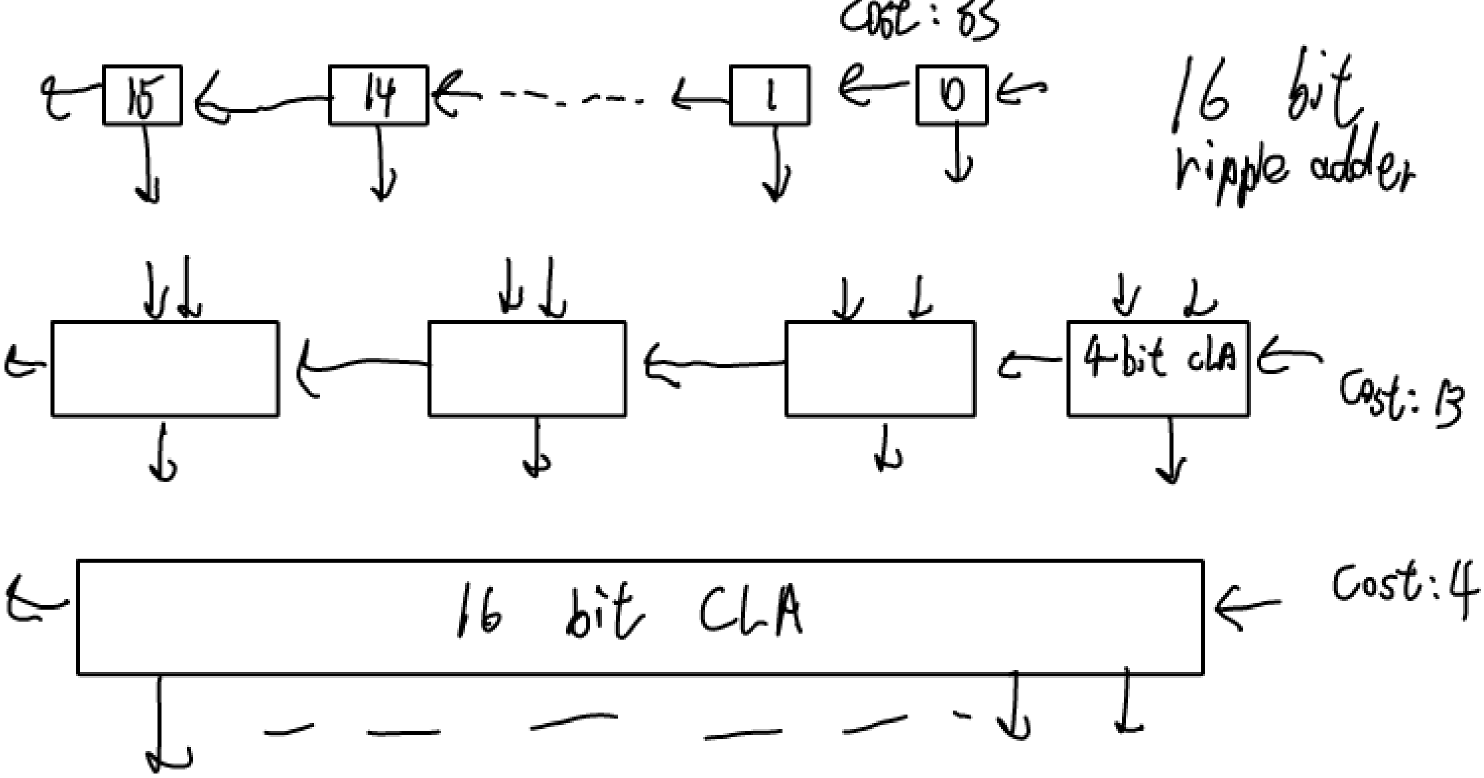




This is called a <u>carry-lookahead adder</u> (CLA)



* Solutions exist in between.



Radix Complements

* Or "r" complements.
* Given a value N in base-r using n digits. The r's complement is

$$r^n - N \quad \text{if } N \neq 0$$
$$0 \quad \text{otherwise}$$

e.g. $(\underbrace{546700}_{n=6})_{10}$, $r=10$ $\qquad$ $(\underbrace{1011000}_{n=7})_2$, $r=2$

$10^6 - 546700 = (453300)_{10}$ $\qquad$ $2^7 - 1011000$

$$\begin{array}{r} 10000000 \\ -\ 1011000 \\ \hline (0101000)_2 \end{array}$$

In base-2, the 2s complement is formed by flipping the bits and adding 1.

$$\begin{array}{rr} 1011000 \rightarrow & 0100111 \\ & +1 \\ \hline & 0101000 \end{array}$$

Subtraction of unsigned binary numbers.

Consider $M-N$.

Instead consider adding the r's complement of N.

$$M+(r^n-N) = M-N+r^n \quad (1)$$
$$= r^n-(N-M)$$

2 cases.

1) $M \geq N$ $\qquad \underbrace{1}_{r^n}\underbrace{(M-N)}_{n\text{-bits}}$

2) $M<N \rightarrow$ underflow

$\underbrace{0}\underbrace{(r^n-(N-M))}_{n\text{-bits}}$ 2s complement of $N-M$

So just add -ve sign

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \rightarrow \begin{array}{r} 101 \\ -011 \\ \hline 2 \end{array} \rightarrow \begin{array}{r} 101 \\ +101 \\ \hline \cancel{1}\,010 \end{array}$$

$M-N=2$

use 3-bits

$$\begin{array}{r} 3 \\ -5 \\ \hline -2 \end{array} \qquad \begin{array}{r} 011 \\ -101 \\ \hline -2 \end{array} \rightarrow \begin{array}{r} 011 \\ +011 \\ \hline \cancel{0}\,110 \end{array}$$

2s comp of 5-3

$\Downarrow$

$-010 = -2$