$$u = XT$$

$$U_{xx} = X''T$$

$$U_{tt} = XT''$$

$$X''T = XT'' \implies \frac{X''}{X} = \frac{T''}{T}$$

$$\therefore \; X'' - X\lambda = 0$$

$$T'' - T\lambda = 0$$

$L = \pi$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_m \cos\frac{n\pi x}{L} + b_m \sin\frac{n\pi x}{L}$$

$$a_0 = \frac{1}{2L}\int_{-\pi}^{\pi} f(x)\,dx \qquad \text{avg} = \frac{\pi^2 + \frac{\pi^2}{2}}{2\pi} = 3\pi/4$$

$$a_m = \frac{1}{L}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{m\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\left(\int_0^{\pi} x\cos(mx)\,dx + \int_{-\pi}^{0} \pi\cos(\cancel{mx})\,dx\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{m^2}\cos(mx) + \frac{x}{m}\sin(mx)\Big|_0^{\pi} + \pi\frac{\sin(mx)}{m}\Big|_{-\pi}^{0}\right)$$

$$= \frac{1}{\pi}\left(\frac{(-1)^m - 1}{m^2}\right)$$

$$= \frac{1}{\pi}\frac{-2}{m^2} \qquad \text{when } m \text{ is od}$$