

A7 - Giliam

$$\begin{aligned} 1. a) \mathcal{L} \left\{ \int_0^t e^{t-\tau} \cos(3\tau) d\tau \right\} &= \mathcal{L} \{ e^t * \cos(3t) \} \\ &= \mathcal{L} \{ e^t \} \mathcal{L} \{ \cos 3t \} \\ &= \frac{1}{s-1} \frac{s}{s^2+9} \end{aligned}$$

$$\begin{aligned} b) \mathcal{L} \left\{ \int_0^t \tau^3 \sin(t-\tau) d\tau \right\} &= \mathcal{L} \{ \sin t * t^3 \} \\ &= \mathcal{L} \{ \sin t \} \mathcal{L} \{ t^3 \} \\ &= \frac{1}{s^2+1} \frac{3!}{s^4} \\ &= \frac{6}{s^4(s^2+1)} \end{aligned}$$

$$2. x''(t) + x(t) = 3\delta(t-1)$$

$$\underline{X}(s)(s^2+1) = 1$$

$$\underline{X}(s) = \frac{1}{s^2+1}$$

$$\therefore x(t) = \sin t \leftarrow \text{impulse response}$$

$$\begin{aligned} \text{Now, } x(t) &= \int_0^t g(t-\tau) u(\tau) d\tau \\ &= \int_0^t 3\delta(\tau-1) [\sin(t-\tau)] d\tau \end{aligned}$$

gives 0 if $t < 1$ & $3\sin(t-1)$ if $t > 1$

$$\therefore x(t) = 3H(t-1) \sin(t-1)$$