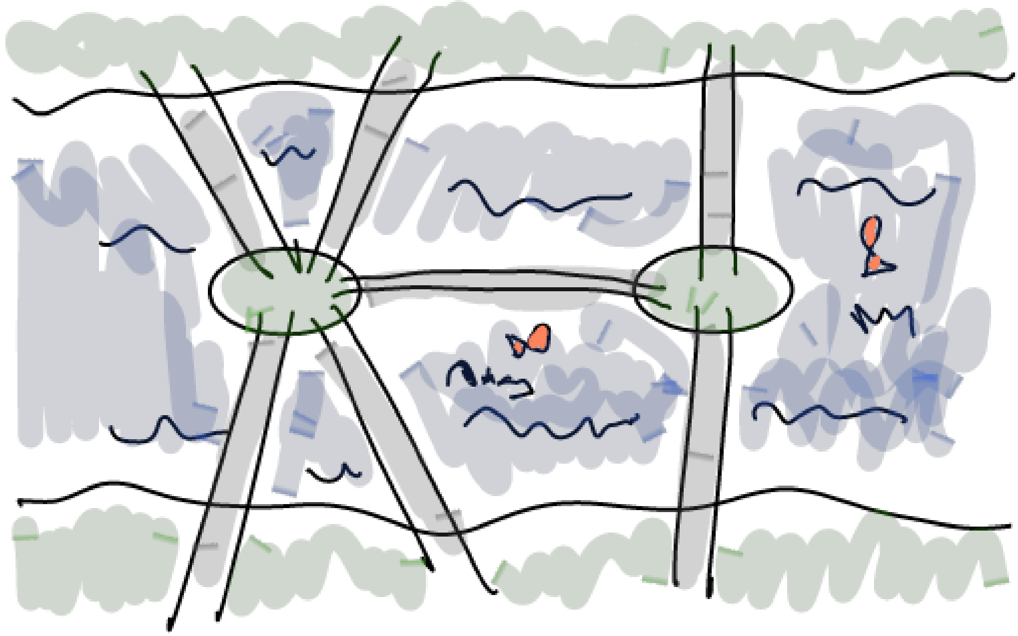
From last time

<u>Prop:</u> The following are equivalent for a graph $G=(V,E)$:

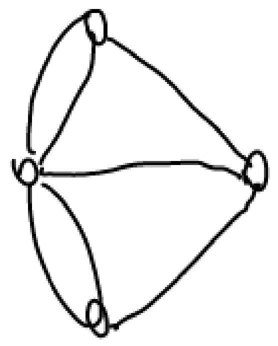
- G is connected
- There is no partition (A,B) of V such that $A\neq\emptyset$, $B\neq\emptyset$ and (A,B) reduces an empty cut.

<u>pf:</u> If G is connected, then no such (A,B) exists (last time). If G is disconnected, let C be a component of G. Let V_C be the vertex set of C. Since C is connected and G is not, we know that $V_C \subsetneq V$ and $V_C \neq \emptyset$ so $(V_C, V\setminus V_C)$ is a partition of V into nonempty parts. Since C is a maximal connected subgraph, there is no edge from a vertex in V_C to one in $V\setminus V_C$.



Konigsberg (1736)

Can we walk around Konigsberg, crossing each bridge once, and returning to the start?



An Euler Tour in a graph is a closed walk containing each edge exactly once. A graph with an Euler Tour is Eulerian.

Prop. If G has an Euler tour, then every vertex of G has even degrees.

pf. Let $v_0, e_1, v_1, e_2, \dots, v_{k-1}, e_k, v_k = v_0$ be an Euler Tour. let v be a vertex to G. Each occurence of v in the sequence $v_0, v_1, \dots, v_{k-1}$ has an edge both before and after it in the tour (where we consider e_k to be before v_0). Since the tour includes each edge exactly once, this means that every such v has even degree.

Thm. If G is a connected graph in which every vertex has even degree, then G is Eulerian.