

# Chapter 8

## Taxes

---

MSCI 261

SECTION 1 (CHE/GEOE) AND SECTION 2 (SOFTWARE)

INSTRUCTOR: TIFFANY BAYLEY  
SPRING 2015

# Overview

---

- ❑ Importance of Taxes
- ❑ Impact on MARR
- ❑ Canadian Corporate Taxation
  - ❑ Capital Cost Allowance
  - ❑ Undepreciated Capital Cost
  - ❑ Tax Benefit Factor
  - ❑ Capital Tax Factor
  - ❑ Half-year Rule
- ❑ After-tax IRR Analysis

## Why we need to consider taxes

---

- Estimate changes to tax payments, due to an investment
  - Gives estimates of after-tax cash flows
  - Then do PW, AW, or IRR analysis
  
- Similar to personal tax effects, e.g.:
  - Prof takes extra course, for \$7000 extra pay
  - The prof earns \$90,000/year without the course
    - But \$27,000 is paid in income tax, leaving \$63,000 after-tax
  - The prof pays 40% tax rate on extra income
  - Calculate the after-tax pay for the course.

tax paid on \$7000 =  $\$7000 / .4 = \$2800$   
after-tax pay from the course =  $\$7000 / .4 = \$4200$

## MARR with After-Tax Cash Flows

MARR is minimum acceptable profit, per dollar invested

- profit before-tax > profit after-tax
- therefore,  $MARR_{\text{before-tax}} > MARR_{\text{after-tax}}$
- $MARR_{\text{after-tax}} \approx (1-t) * MARR_{\text{before-tax}}$ , where  $t = \text{tax rate}$ .

Personal tax example for a Guaranteed Investment Certificate (GIC)

- extra income is taxed at 40%, personal  $MARR_{\text{after-tax}} = 3.5\%$
- a GIC would pay interest rate (before-tax) = 5%
- Is the GIC good enough ?

① GIC Interest after-tax =  $5\% (1 - .4) = 3\% < MARR \text{ after tax}$   
 $\therefore \text{not good}$

②  $MARR_{\text{before tax}} = \frac{MARR_{\text{after-tax}}}{1 - 0.4} = 5.83\% > \text{GIC rate before tax}$   
 $\therefore \text{not good}$

# Estimation of Corporate Tax Effects

- Invest P now; new net revenue of R for 10 years
- What is the future tax effect? – e.g., next year

| Income Statement, Next Year (forecast) |                     |   |
|--|---------------------|---|
| (without investment)                   | (in millions of \$) |   |
| <u>Revenues</u>                        |                     | <u>Changes</u>  |
| Sales                                  | 2010.0              |   |
| -Cost of Goods Sold                    | <u>1304.0</u>       |   |
| Net Revenue from Sales                 | 706.0               | ← + R   |
| <u>Expenses</u>                        |                     |   |
| Operating Expenses                     | 590.0               |   |
| Depreciation Expenses                  | 30.0                | ← + depreciation on P   |
| Interest Expenses                      | 11.0                |   |
| Total Expenses                         | 631.0               |   |
| <u>Profit Before Taxes</u>             | 75.0                | ← + R – (depreciation on P)   |
| -Income tax @ 30%                      | 30.0                | ← + 0.3(R – depreciation on P)  |
| <u>Profit After Taxes</u>              | 45.0                | $\therefore \text{After-tax cash flow}$<br>$= R - (\text{extra tax})$<br>$= R(1-0.3) + 0.3(\text{depreciation on P})$ |

# An Important Assumption: “Flow Through”

---

What if profit in a future year would be negative?

- does company “pay” a negative tax ?  
(meaning, government pays the company)
- NO !!!! The company pays zero tax when before-tax profit < 0 .

This complicates after-tax analysis of investments

- e.g. , government allows depreciation deductions to be delayed to a future, profitable year, reducing taxes in the future.

Assumption for this course:

**All future years are profitable. Therefore, all deductions  
(depreciation, other expenses) are taken in the years that they  
occur, and not delayed.**

## Example 8-1: After-tax PW Evaluation

- \$105,000 for equipment for new product
- life = 10 years; salvage value = \$20,000
- depreciation for tax purposes:  $\$105,000/10 = \$10,500/\text{year}$   
{N.B.: straight-line is NOT NORMAL for Canada ! }
- new product revenue = \$25,000 per year for 10 years
- other costs of new product = \$6000 per year for 10 years
- MARR<sub>after-tax</sub> = 10% ; tax rate t = 30%

rev:  $\$25,000/\text{yr} \rightarrow \text{after tax}, 25000(1-t)$

costs:  $\$6000/\text{yr} \rightarrow \text{after tax}, 6000(1-t)$

$P=105000 \Rightarrow$  no adjustment at time purchase, but depreciation will lower taxes.

In each year, taxes lowered by  $0.3(10,500) \rightarrow$  depreciation expense

Salvage value =  $\$20,000 @ n=10; \text{after tax } 20000(1-t)$

$$\begin{aligned} \text{PW}_{\text{after tax}} &= -105000 + 0.3(10500)(P/A, 10\%, 10) + 20000(1-0.3)(P/F, 10\%, 10) \\ &\quad + (25000 - 6000)(1-0.3)(P/A, 10\%, 10) \\ &= 1476 > 0 \end{aligned}$$

∴ good investment

6000 → reduces NIBT, so reduced income Tax  
before-tax  
cost

6000 - <sup>less</sup> (tax caused by extra deduction)

$$\hookrightarrow 6000(1 - .3)$$

Naive!

$$MARR_{\text{before tax}} = \frac{10\%}{1 - .3} = 14.3\%$$

$$\begin{aligned} PW_{\text{before-tax}} &= -105000 + 20000(P/B, 14.3\%, 10) + \\ &\quad (25000 - 6000)(P/A, 14.3\%, 10) \\ &= -1788 < 0 \end{aligned}$$

∴ not good

# Depreciation in Canadian Corporate Taxation System

---

- declining balance depreciation is required
  - book value is called “Undepreciated Capital Cost” (UCC)
  - depreciation in a year is called “Capital Cost Allowance” (CCA)
  - declining balance rate  $d$  is called “CCA rate”
- CCA rate varies by asset class, e.g.
  - Class 6: frame buildings,  $d=10\%$
  - Class 8: machinery, office furniture, ...,  $d=20\%$
  - Class 30: passenger vehicles, trucks, computers, ...,  $d=30\%$
  - UCC is tracked by Class: all things in a Class are “pooled”
- Half-Year Rule:
  - depreciable first cost is treated as occurring  $\frac{1}{2}$  way through the year, and “prorated” across first two years
  - i.e.  $\frac{1}{2}$  of first cost depreciates on first tax return after purchase and other  $\frac{1}{2}$  depreciates starting on second tax return after purchase

Tax benefits of 1<sup>st</sup> \$52500:

$$\text{First tax return: } 52500(0.2)(0.3)$$

$$\text{Second tax return: } \underbrace{52500(1-0.3)(0.2)(0.3)}_{\text{BV}}$$

$$\text{Third tax return: } 52500(1-0.2)^2(0.2)(0.3)$$

:

forever

Example 8-2: same as 8-1 but Canadian rules

- \$105,000 for equipment for new product
- life = 10 years; salvage value = \$20,000
- depreciation for tax purposes: CCA rate d = 20%
- new product revenue = \$25,000 per year for 10 years
- other costs of new product = \$6000 per year for 10 years
- MARR<sub>after-tax</sub> = 10% ; tax rate t = 30%

$$\begin{aligned} \text{PW}_{\text{after-tax}}(\text{tax benefits}) &= 52500(0.2)(0.3)(P/F, 10\%, 1) \\ &\quad + 52500(1-0.2)(0.2)(0.3)(P/F, 10\%, 2) \\ &\quad + \dots \\ &\quad \text{infinite series!} \end{aligned}$$

First  $\frac{1}{2}$  of first cost =  $\frac{105000}{2} = 52500$  available for depreciation (n) end of 1st year + later

Second  $\frac{1}{2}$  of first cost =  $\frac{105000}{2} = 52500$  @ end of 2nd year + later

$$CTF = 1 - \frac{td(1+i/2)}{(i+td)(1+i)}$$

$$CSF = 1 - \frac{td}{(i+td)}$$

## Present Worth of Tax Benefit on First $\frac{1}{2}$ of First Cost

P = entire first cost, t = tax rate, d = CCA rate, i = MARR<sub>after-tax</sub>

$$PW(\text{tax benefit}) = \frac{(P/2)td}{(1+i)} + \frac{(P/2)t(1-d)d}{(1+i)^2} + \dots + \frac{(P/2)t(1-d)^{N-1}d}{(1+i)^N} + \dots$$

$$\begin{aligned} PW(\text{tax benefit}) &= \frac{(P/2)td}{(1+i)} \left( 1 + \frac{(1-d)}{(1+i)} + \frac{(1-d)^2}{(1+i)^2} + \frac{(1-d)^3}{(1+i)^3} + \dots + \frac{(1-d)^N}{(1+i)^N} + \dots \right) \\ &= \frac{(P/2)td}{(1+i)} \left( \frac{1}{1 - \frac{(1-d)}{(1+i)}} \right) = \frac{(P/2)td}{i+d} \end{aligned}$$

Tax Benefit Factor, TBF =  $td/(i+d)$

Note: "TBF" is an acronym in text's 4<sup>th</sup> edition  
but not in the 5<sup>th</sup> edition.

Ex 8-2

$$\begin{aligned} PW_{\text{after-tax}} &= -P * CTF + S * CSF(P/F, 10\%, 10) + (25000 - 6000)(1 - .3)(P/A, 10\%, 10) \\ &= -105000 \left( 1 - \frac{(.3)(.2)(1 + \frac{1}{2})}{(.1 + .2)(1 + .1)} \right) + 20000 \left( 1 - \frac{(.3)(.2)}{(.1 + .2)} \right) (P/F, 10\%, 10) \\ &\quad + (25000 - 6000)(1 - .3)(P/A, 10\%, 10) \\ &= \$2936.89 > 0, \therefore \text{good investment.} \end{aligned}$$

## Present Worth of Tax Benefit on Second ½ of First Cost

---

Second P/2 starts depreciating on second tax return, 1 year later:

$$\begin{aligned} \text{PW(tax benefit, 2}^{\text{nd}} \text{ P/2)} &= (\text{P}/2) * \text{TBF} * (\text{P/F}, i, 1) \\ &= (\text{P}/2) * [\text{td}/(\text{i}+\text{d})]/(1+\text{i}) \end{aligned}$$

### After-tax First Cost

$$\begin{aligned} &= \text{P} - \text{PW(tax benefit, 1}^{\text{st}} \text{ P/2)} - \text{PW(tax benefit, 2}^{\text{nd}} \text{ P/2)} \\ &= \text{P} * \{1 - \text{TBF}/2 - (\text{TBF}/2) * (\text{P/F}, i, 1)\} \\ &= \text{P} * \text{CTF}, \text{ where Capital Tax Factor } \text{CTF} = 1 - \frac{\text{td}(1 + \text{i}/2)}{(\text{i} + \text{d})(1 + \text{i})} \end{aligned}$$

# After-tax First Cost and Salvage Values

---

## After-tax First Cost

$$= P - PW(\text{tax benefit, 1}^{\text{st}} \text{ P}/2) - PW(\text{tax benefit, 2}^{\text{nd}} \text{ P}/2)$$

$$= P * \{1 - TBF/2 - (TBF/2) * (P/F, i, 1)\}$$

$$= P * CTF, \text{ where Capital Tax Factor } CTF = 1 - \frac{td(1+i/2)}{(i+d)(1+i)}$$

## After-tax Salvage Value

- salvage revenue  $S$ , at time  $N$ , reduces depreciation later
- after-tax value =  $S - \{PW \text{ of lost depreciation, at time } N\}$
- rules do not distinguish first half of  $S$  from second half

After-tax Salvage =  $S * CSF$ , where Capital Salvage Factor

$$CSF = 1 - \frac{td}{(i+d)} = 1 - TBF$$

- Note:  $S * CSF$  is after-tax salvage at time  $N$ ; not at time 0.

## Back to Example 8-2

---

- \$105,000 for equipment for new product
- life = 10 years; salvage value = \$20,000
- depreciation for tax purposes: CCA rate  $d = 20\%$
- new product revenue = \$25,000 per year for 10 years
- other costs of new product = \$6000 per year for 10 years
- $MARR_{\text{after-tax}} = 10\% ; \text{tax rate } t = 30\%$

## Effect of Accelerated Depreciation (faster “writeoff”)

---

- P.W. of Tax Benefit on first cost  $P$   
$$= (P/2)[td/(i+d)] + (P/2)[td/(i+d)]/(1+i)$$
- What happens if government increases  $d$  ?
  - e.g. from  $d = 20\%$  to  $d = 30\%$
- $td/(i+d) = t/(i/d + 1)$ , so
  - increase  $d \rightarrow$  decrease  $i/d + 1 \rightarrow$  increase  $t/(i/d + 1)$
- ∴ P.W. of Tax Benefit on first cost  $P$  increases when  $d$  increases
- e.g., for example 8-2, PW Tax Benefit is
  - for  $d=20\%$ , \$20,045.45
  - for  $d=30\%$ , \$22,551.14

# Effect of Accelerated Depreciation

---

(Intuitive reason why larger d is better for company)

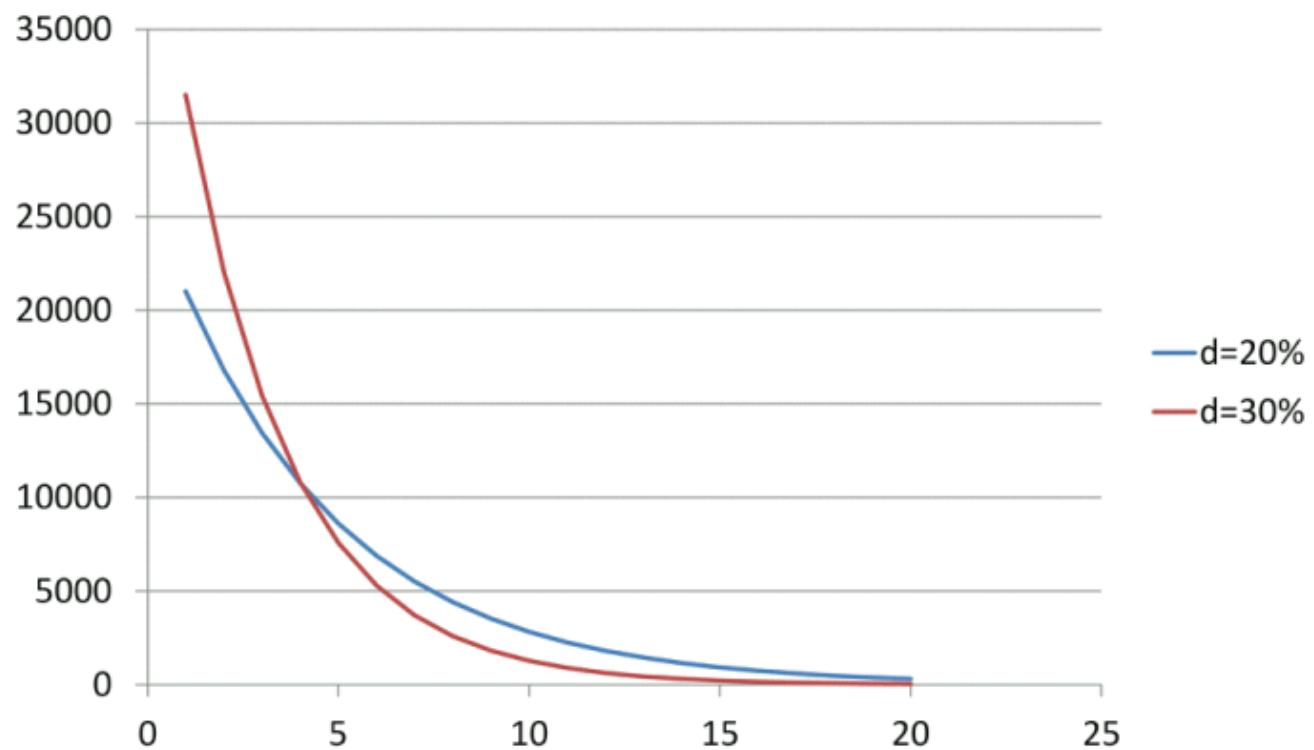
- Total depreciation deduction on first P/2 (same for second)

$$\begin{aligned} &= \sum_{n=1}^{\infty} (P/2)d(1-d)^{n-1} = (P/2)d \sum_{n=1}^{\infty} (1-d)^{n-1} \\ &= \frac{(P/2)d}{(1-(1-d))} = (P/2) \end{aligned}$$

- All of it is depreciated eventually.
- Larger d means benefit is sooner, so more valuable in P.W.
  - think of first year's depreciation,  $Pd/2$

**\$ of Depreciation  $Pd(1-d)^n$ , by Year n, P=\$105,000**

Note: total over 100 years is \$105,000 for both



## Example 8-3

---

A company is about to build a new warehouse costing \$400,000. It is not clear, however, whether the Canadian tax laws classify the building as class 3 (CCA rate = 5%) or as class 6 (CCA rate = 10%), since both classes include “buildings”.

The company’s accountant is going to the tax office to argue that the building should fall under one class or the other. Which class — 3 or 6 — do you think the accountant will advocate for the company? Justify your answer.

$$CTF = 1 - \frac{td(1+i/2)}{(i+d)(1+i)} = 1 - \frac{(.25)(.2)(1+.08/2)}{(0.08+.2)(1+.08)} \approx .828042$$

$$CSF = 1 - \frac{td}{(i+d)} = 1 - \frac{(.25)(.2)}{(.08+.2)} = 0.821429$$

$\underbrace{(i+d)}$   
TBP

## Example 8-4

An electric pallet truck costs \$12,000. It would save \$4000 per year over its 5 year life and would have a \$2000 salvage value. The after-tax MARR is 8%, the tax rate is 25% and the CCA rate for this type of equipment is 20%. Use the present worth method to evaluate the investment.

$$\begin{aligned}
 PW_{\text{after-tax}} &= -P_{\text{initial}}CTF + S_{\text{initial}}CSF(P/F, 8\%, 5) + 4000(1 - .25)(P/A, 8\%, 5) \\
 &= -12000(.828042) + 2000(.821429)(P/F, 8\%, 5) \\
 &\quad + 4000(1 - .25)(P/A, 8\%, 5) \\
 &= 3159.72 > 0 \quad \therefore \text{good.}
 \end{aligned}$$

$$AW_{after-tax} = -P \times CTF(A/P, 8\%, 5) + S \times CSF(A/P, 8\%, 5) \\ + 4000(1-t) = 791.37/\text{yr.}$$

## Example 8-5

---

Redo Example 8-4 using the annual worth method

$AW = PW^*(A/P, i, n) = \$791.37$  per year > 0 so good.

“Direct” method for AW: at board.

Find  $i$  that solves  $PW(i) = 0$

$$PW(i) = -12000 \left[ 1 - \frac{(0.25)(0.2)(1+i/2)}{(i+0.2)(1+i)} \right] + 2000 \left[ 1 - \frac{(0.25)(0.2)}{i+0.2} \right] (P/F, i, 5) + 4000 (1 - 0.25) (P/A, i, 5) = 0$$

$$IRR_{after-tax} = 16.6\%$$

## Example 8-6

---

Redo Example 8-4 using the IRR method

- solve for unknown interest rate
- CTF, CSF depend on unknown interest rate (not MARR) !!

# Approximate After-Tax IRR Calculation

---

- To ease the calculation, an approximate after-tax IRR can be estimated from the before-tax IRR as:

$$\text{IRR}_{\text{after-tax}} \cong \text{IRR}_{\text{before-tax}} \times (1 - t)$$

- Using the numbers from Example 8-6:

$$-12000 + 4000(P/A, i, 5) + 2000(P/F, i, 5) = 0 \text{ for } \text{IRR}_{\text{before-tax}}$$

$$\text{IRR}_{\text{before-tax}} \cong 22.7\%$$

$$\text{IRR}_{\text{after-tax}} \cong 22.7\%(1 - 0.25) = 17.0\% \text{ (vs. exact value of 16.6\%)}$$

- Compare  $\text{IRR}_{\text{before-tax}}$  to  $\text{MARR}_{\text{before-tax}}$
- or  $\text{IRR}_{\text{after-tax}}$  to  $\text{MARR}_{\text{after-tax}} = (1 - t) * \text{MARR}_{\text{before-tax}}$
- if approximate IRR<sub>after-tax</sub> is close to  $\text{MARR}_{\text{after-tax}}$ ,  
then exact IRR<sub>after-tax</sub> is required

$$\begin{aligned}
 CTF &= 0.695906 & EAC_{\text{after-tax}}(n) &= 6300(0.695906)(A/p, 8\%, n) \\
 (d = 30\%) & & & - 6300(1-0.684211)(A/f, 8\%, n) \\
 CSF &= 0.684211 & & + 1220(1-0.4)(P/A, g=10\%, i=8\%, n) \\
 & & & (A/p, 8\%, n)
 \end{aligned}$$

minimum EAC occurs when  $n=6$ .

## Example 8-7

A new ATV (all terrain vehicle) costs \$6300. It depreciates at 20% per year over its 9 year service life. Operating costs are \$1220 for the first year increasing by 10% per year. The CCA rate for ATVs is 30%, the after-tax MARR is 8%, and taxes are at 40%. Compute the economic life of ATV.

[Note: management thinks that 20% is more accurate to estimate salvage values; government requires 30% for this broad class of assets]

## Read for next class

---

Chapter 9 from textbook