

Irreducible

$$\mathbb{C}[x] \quad f(x) = a(x-r_1)(x-r_2)\cdots(x-r_n)$$

$$\mathbb{R}[x] \quad f(x) = a(x-r_1)\cdots(x-r_k)(x^2+2\operatorname{Re}(r_{k+1})x+|r_{k+1}|^2)$$

$$\mathbb{Z}_2[x] \quad x^2+1 = (x+1)(x+1)$$

$$x^2+x+1$$

$$x^3+1 = (x+1)(x^2+x+1)$$

$$x^3+x+1 \quad \text{no lin. factor, irreducible.}$$

$$x^4+x+1$$

$$x^4+x^2+1 = (x^2+x+1)^2$$

Product and Sums of Roots

$$f(x) = (x-r_1)(x-r_2)\cdots(x-r_n) \quad \text{in } \mathbb{C}[x]$$

$$f(0) = (-1)^n r_1 r_2 \cdots r_n = \text{constant term.}$$

$$f(x) = x^3 + \frac{1}{3}x^2 + 2x + \frac{2}{3} \quad (x+\frac{1}{3})(x-\sqrt{2}i)(x+\sqrt{2}i)$$

$\left. \begin{array}{l} \text{---trace(A)} \\ \text{---}(-\sum r_i) \end{array} \right\} \rightarrow \frac{1}{3}$

$\left. \begin{array}{l} \text{---}(-1)^n \det \\ \text{---}(-1)^3(-\frac{2}{3}) \end{array} \right\} \rightarrow \frac{2}{3}$

$\left. \begin{array}{l} \text{---}(-\frac{1}{3}, \sqrt{2}i, -\sqrt{2}i) \end{array} \right\} \rightarrow -\frac{2}{3}$

Coefficient of x^{n-1} in $f(x)$

$$\boxed{-r_1 - r_2 - \cdots - r_n = -\sum r_i}$$