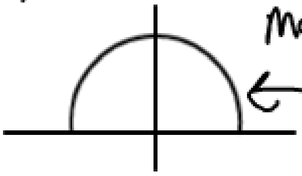
For one more example using Lagrange multipliers, see 'example' posted on Learn



Max/min of f in here.

1) Crit pts inside
2) boundary – a) $y=0$
   b) $x^2+y^2=1$ (Lagrange)

Partial Integration

Is there a fcn. $f(x,y)$ such that $\frac{\partial f}{\partial x} = xe^y + y\sin x$ & $\frac{\partial f}{\partial y} = \frac{1}{2}x^2e^y - \cos x$?

If so, find the most general form of f.

If f exists, $f(x,y) = \int f_x\,dx = \int f_y\,dy$

$$\int f_x\,dx = \frac{1}{2}x^2e^y - y\cos x + \underbrace{g(y)}_{\text{unknown fcn.}}$$

$$\int f_y\,dy = \frac{1}{2}x^2e^y - y\cos x + \underbrace{h(x)}_{\text{unknown fcn.}}$$

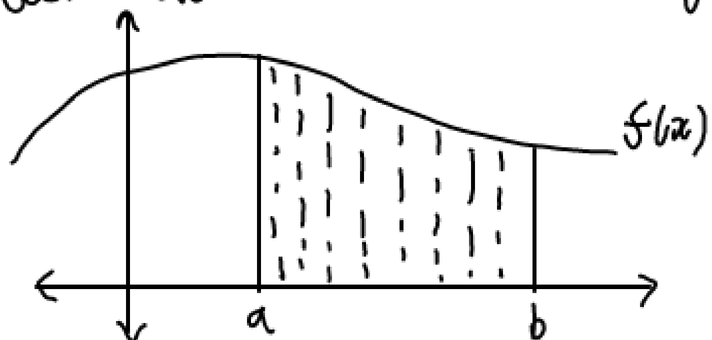
If f exists, the expressions are equal:

$g(y) = h(x)$ ← only possibility – both const.

$$\Rightarrow f(x,y) = \frac{1}{2}x^2e^y - y\cos x + C$$

Double integrals

Recall defn. of definite integral for $f: \mathbb{R} \to \mathbb{R}$.



$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

If $R = [a,b] \times [c,d]$, then

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx.$$

These are called <u>iterated integrals</u>

As long as the domain is rectangular, we can switch the order.

To evaluate iterated integrals, use partial integration.

<u>Ex.</u>

Evaluate $\iint_D (x-3y^2)\,dA$, where $D = \{(x,y) \mid 0 \le x \le 2, 1 \le y \le 2\}$

<u>Soln.</u>

$$\iint_D (x-3y^2)\,dA = \int_1^2 \int_0^2 x-3y^2\,dx\,dy$$

$$= \int_1^2 \left[\int_0^2 x-3y^2\,dx \right] dy$$

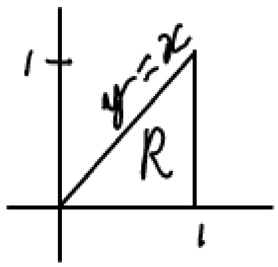
$$= \int_1^2 \left[\frac{1}{2}x^2 - 3y^2 x \Big|_0^2 \right] dy$$

$$= \int_1^2 (2-6y^2)\,dy$$

$$= 2y - 2y^3 \Big|_1^2$$

$$= -12$$

Excersise: Show that you get the same answer with the order switched.



$x \in [0,1]$

$y \in [0,x]$