Selectk(L, k) // k^{th} element

1. if $|L| < 10$, then return sort select ← $O(n \log n)$

2. select pivot x

$A \leftarrow \{L[i] \geq x\}$

$B \in \{L[i] < x\}$

3. if $|A| \geq k$, return selectk(A, k)

else return selectk(B, k − |A|)

$T(n) \leq n + T(\max(|A|, |B|))$

want: $\max(|A|, |B|) \leq (1-\varepsilon)\cdot n$

$$T(n) \leq n + T((1-\varepsilon)\cdot n)$$
$$= n + (1-\varepsilon)\cdot n + T((1-\varepsilon)^2\cdot n)$$
$$= n + (1-\varepsilon)n + (1-\varepsilon)^2 n + \ldots$$
$$= \frac{1}{\varepsilon} n = O(n) \text{ for constant } \varepsilon$$

SelectPivot(L)

1. divide L to $n/5$ groups — $O(n)$
2. Let x_i be medium of group i — $O(n)$
3. $x \leftarrow$ selectk($\{x_1, \ldots, x_{\frac{n}{5}}\}, \frac{n}{10}$) — $T(n/5)$
4. return x.

Lemma: x is such that $\max(|A|, |B|) \leq \frac{7}{10}\cdot n$

Proof: For each group G_i, $i=1, \dots, \frac{n}{5}$

there are 3 elements $\geq x_i$

there are $\frac{n}{10}$ $x_i \geq x$

$\therefore$ $\frac{3n}{10}$ elements are $\geq x$

$$T(n) = \underbrace{n + T(\tfrac{n}{5})}_{\text{pivot selection}} + T(\max(|A|,|B|))$$

$$\leq n + T(n/5) + T(\tfrac{7n}{10})$$

Claim: $T(n) \leq c \cdot n$

Proof: By induction

Base case: ✓ $T(0) \leq 0$

$$T(n) \leq n + T(\tfrac{n}{5}) + T(\tfrac{7n}{10})$$

$$\leq n + \frac{cn}{5} + \frac{7 \cdot c \cdot n}{10}$$

$$= (1 + \tfrac{9}{10} \cdot c) \cdot n$$

choose $c = 10$

$$= c \cdot n$$