Divisibility

Defⁿ

An integer m divides an integer n, denoted by $m|n$, if there is an integer k such that $n=mk$.

e.g. $3|21$ ✓ Yes, since $21=3\cdot 7$

$-3|21$ ✓ Yes, since $21=(-3)\cdot(-7)$

$3|7$ X No interger k satisfies $7=3k$.

$5|0$ ✓ Yes since $0=5\cdot 0$

$0|5$ X No $5=0\cdot k$, no int k

$0|0$ ✓ Yes $0=0\cdot 314$

Proposition (Transitivity of divisibility, TD):

Let a, b, c be integers.

If $a|b$ and $b|c$, then $a|c$

e.g. $3|21$ $21|42 \Rightarrow 3|42$

proof: since $a|b$, there is an integer k where $b=ak$

since $b|c$, there is an integer l where $c=bl$

substituting b for ak in the second equation to get $c=a(kl)$. since k,l are integers, kl is an integer so $a|c$.

Proposition (Divisibility of integer combinations, DIC):

Let a, b, c be integers. If $a|b$, $a|c$, and x, y are integers, then $a|(bx+cy)$.

e.g. $\left.\begin{matrix} 3|15 \\ 3|12 \end{matrix}\right\} 3|(15x+12y) \Longrightarrow 3|1512$

proof: Since $a|b$, there is an integer k where $b=ak$,
since $a|c$, there is an integer l where $c=al$

Then $bx+cy = akx + aly = a(kx+ly)$

since k, x, l, y are integers, kx and ly is also an integer.

So $a|(bx+cy)$

Proposition (bounds by divisibility, BBD).

Let a, b be integers. If $a|b$ and $b \neq 0$ then, $|a| \leq |b|$.

proof: since $a|b$, there is an integer k where $b=ak$

Then $|b| = |ak| = |a| \cdot |k|$

Since $b \neq 0$ and $b=ak$, $|k| \geq 1$

So $|b| = |a| \cdot |k| \geq |a|$