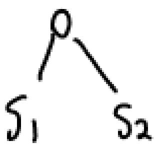
A <u>binary tree</u> is either

- The empty tree ε
- a triple $(\bullet, S_1, S_2)$, where S_1 & S_2 are (possibly empty) binary trees

We recursively define $w(\varepsilon) = 0$ $\quad w(S_1, S_2) = 1 + w(S_1) + w(S_2)$

N.B. $w(S)$ is the "# of vertices" of S,



$(\varepsilon, \varepsilon) = o$

$(S_1, \varepsilon) = o$

???

Let $\mathcal{T} = \{\text{binary trees}\}$

We have defined a weight function: Let $T(x) = \Phi_{\mathcal{T}}(x)$

So $[x^n]T(x) = $ # of binary trees with n vertices.

$\mathcal{T} = \{\varepsilon\} \cup \{\bullet\} \times \mathcal{T} \times \mathcal{T}$ 'unambiguously'

$\Phi_{\mathcal{T}}(x) = \Phi_{\{\varepsilon\}}(x) + \Phi_{\{\bullet\}}(x)\Phi_{\mathcal{T}}(x)^2$ by sum/product Lemma

$T(x) = 1 + xT(x)^2$

$xT(x)^2 - T(x) + 1 = 0 \Rightarrow 4x^2T(x)^2 - 4xT(x) + 4x = 0$

$\Rightarrow (2xT(x) - 1)^2 - 1 + 4x = 0$

$\Rightarrow (2xT(x) - 1)^2 = 1 - 4x$

By Assignment 3,

$$(1-4x)^{1/2} = 1-2\sum_{n\geq 0}\frac{1}{n+1}\binom{2n}{n}x^{n+1} \quad \cdots \text{ from A3}$$

$$\therefore\ 1-2xT(x) = \pm\left(1-2\sum_{n\geq 0}\frac{1}{n+1}\binom{2n}{n}x^{n+1}\right)$$

Can't be negative since the constant terms are different. (1 and -1)

$$\therefore\ 1-2xT(x) = 1-2\sum_{n\geq 0}\frac{1}{n+1}\binom{2n}{n}x^{n+1}$$

$$\therefore\ T(x) = \sum_{n\geq 0}\frac{1}{n+1}\binom{2n}{n}x^{n}$$

i.e. there are $\frac{1}{n+1}\binom{2n}{n}$ binary trees w/ n vertices.

GRAPH THEORY

- Given a circuit diagram, can we make a flat circuitboard to its specification without edges crossing? (planerity)
- How many colours are needed to colour each point in the plane so that no two points at distance 1 get the same colour?
- How many ways are there to get between two given intersectors in Manhattan's one way system?

- Given some SE students & co-op positions, where each position is compatible with only some students, can we give everyone a job?
- What is the {cheapest way / quickest way} to get between two cities?
- How many Fullvenes are there?

A *graph* is a pair (V, E), where V is a finite set, and E is a set of unordered pairs of distinct elements of V (i.e. two-element subsets of V).

We call the elements of V the *vetices* and the elements of E the *edges*.

Let $V = \{$ 5-letter english words in the SOWPODS Dictionary $\}$

$E = \{\{x, y\}$ s.t. x and y differ in one letter$\}$