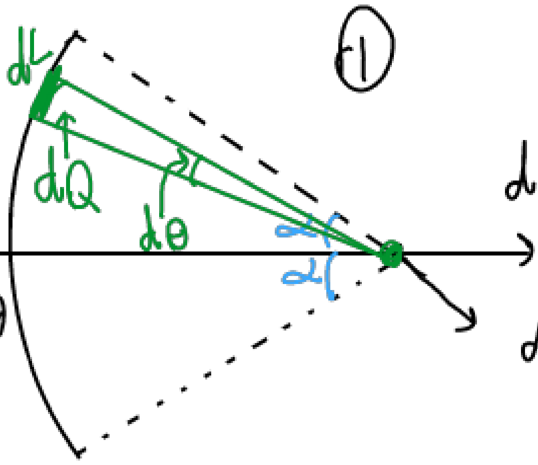
1

$\lambda = \frac{Q}{2\alpha R}$

$dL = R\,d\theta$

$dq = \lambda\, dL$

$dQ = \lambda R\, d\theta$



$dE_x = \frac{k\,dq}{R^2}\cos\theta$

$dE = \frac{k\,dq}{R_2}$

$$E_x = \int_{arc} \frac{k\,dQ}{R^2}\cos\theta$$

$$= \int_{-\alpha}^{\alpha} \frac{k\lambda R\,d\theta}{R^2}\cos\theta$$

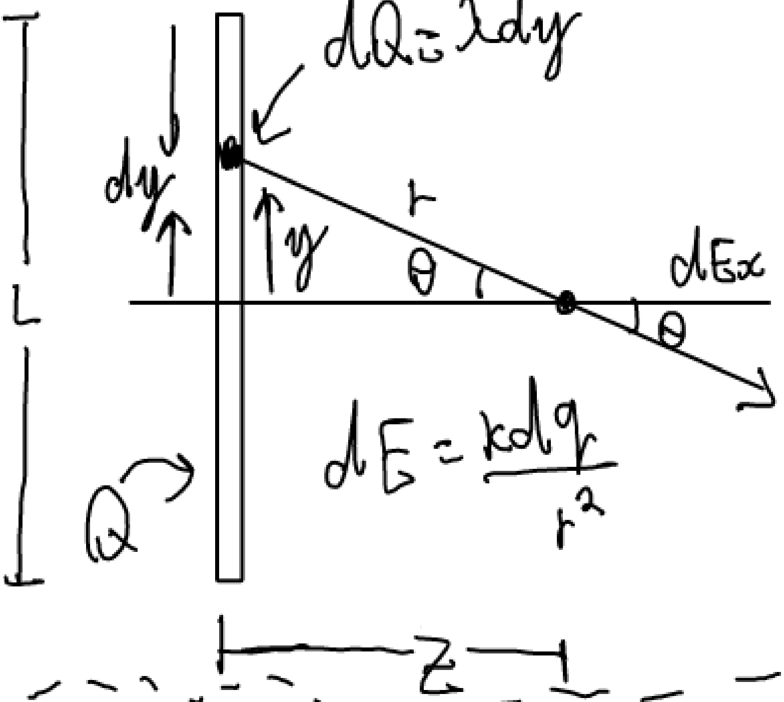
$$= \frac{k\lambda}{R}\int_{-\alpha}^{\alpha} \cos\theta\, d\theta$$

$$= \frac{k\lambda}{R}\sin\theta \Big|_{-\alpha}^{+\alpha}$$

$$= \frac{k\lambda}{R}\sin\alpha - \frac{k\lambda}{R}\sin(-\alpha)$$

$$E_x = \frac{2k\lambda}{R}\sin\alpha$$

2



$dQ = \lambda\, dy$

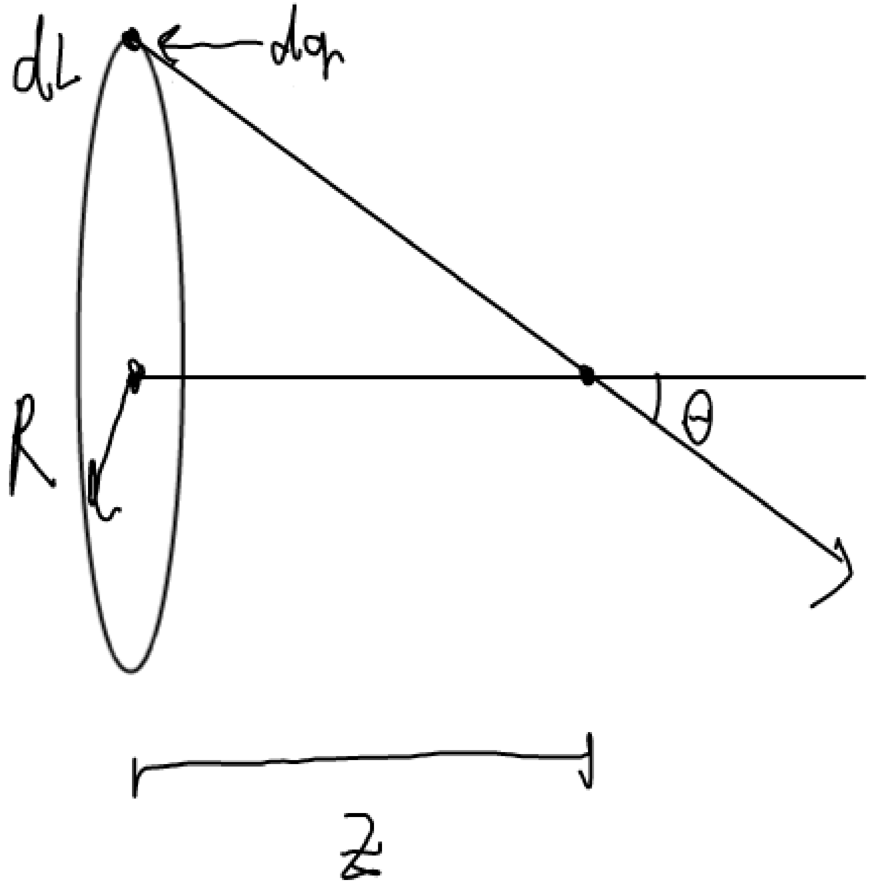
$dE = \frac{k\,dq}{r^2}$

$$E_z = \int_{bar} dE_z$$

$$= \int_{-\frac{L}{2}}^{L/2} \frac{kz\lambda\, dy}{(y^2+z^2)^{3/2}}$$

$$= kz\lambda \int_{-L/2}^{L/2} \frac{dy}{(y^2+z^2)^{3/2}}$$

$$dE_z = \frac{k\,dq}{r^2}\cos\theta = \frac{k\,dq}{r^2}\left(\frac{z}{r}\right) = \frac{kz\,dq}{r^3} = \frac{kz\,dq}{\left(\sqrt{z^2+r^2}\right)^{\frac{3}{2}}}$$



$$dE_z = \frac{k\,dq}{r^2}\cos\theta = \frac{k\,dq}{r^2}\left(\frac{z}{r}\right) = \frac{k\,dq\,z}{r^3}$$

$$dE = \frac{k\,dQ}{r^2} \qquad dE_z = \frac{k\,dq\,z}{(R^2+z^2)^{3/2}}$$

$$E_z = \int_{ring} \frac{k\,dq\,z}{(R^2+z^2)^{3/2}}$$

$$= \frac{kz}{(R^2+z^2)^{3/2}} \int_{ring} dq$$

$$= \frac{Qkz}{(R^2+z^2)^{3/2}}$$

② cont.

$$\lim_{L\to\infty} E_z$$ ← important

$y = z\tan\beta$

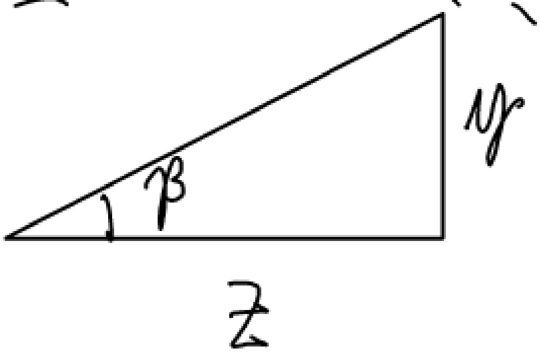
$dy = z\sec^2\beta$

$y^2+z^2 = z^2\tan^2\beta + z^2 = z^2(1+\tan^2\beta)$

$= z^2\sec^2\beta$

plug back to integral

$$E_z = Kz\lambda\int_{bar}\frac{z\sec^2\beta\, d\beta}{z^3\sec^3\beta} = \frac{k\lambda}{z}\int_{bar}\cos\beta\, d\beta$$



$\sin\beta = \dfrac{y}{\sqrt{y^2+z^2}}$

$$= \frac{k\lambda}{z}\sin\beta\Big|_{bar}$$

$$= \frac{k\lambda}{z}\left(\frac{y}{\sqrt{y^2+z^2}}\right)\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$E = \dfrac{kQ}{r^2} = \dfrac{Q}{4\pi\epsilon_0 r^2}$

$K = \dfrac{1}{4\pi\epsilon_0}$

$$= \frac{k\lambda}{z}\left(\frac{L/2}{\sqrt{\frac{L^2}{4}+z^2}} - \frac{-L/2}{\sqrt{\frac{L^2}{4}+z^2}}\right)$$

$$= \frac{k\lambda}{z}\left(\frac{L}{\sqrt{\frac{L^2}{4}+z^2}}\right) = \frac{\lambda}{4\pi\epsilon_0 z}\left(L/\sqrt{\tfrac{L^2}{4}+z^2}\right)$$