1. Strong Induction
2. GCD

<u>Example</u>: ≥ 8 candies, divided into piles of 3's and 5's.

Base Cases: 8 candies $\rightarrow$ 3, 5

9 candies $\rightarrow$ 3, 3, 3

10 candies $\rightarrow$ 5, 5

Ind. Hyp: Assume only set of i candies $8 \leq i \leq k$ can be divided into 3's and 5's, for some $k \geq 10$

Ind. Step: K+1 candies put aside 3 k-2 candies left.

By Ind. Hyp, k-2 candies $\rightarrow$ 3's and 5's

Ind step is valid when $k-2 \geq 8$, or $k \geq 10$.

Example: Prove that every integer at least 2 is a product of (at least one) primes.

[$12 = 2 \cdot 2 \cdot 3$ $\quad 42 = 2 \cdot 3 \cdot 7$ $\quad 10^{100} = 2^{100} \cdot 5^{100}$]

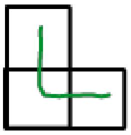
proof: Base case: When $n=2$, 2 is a prime.

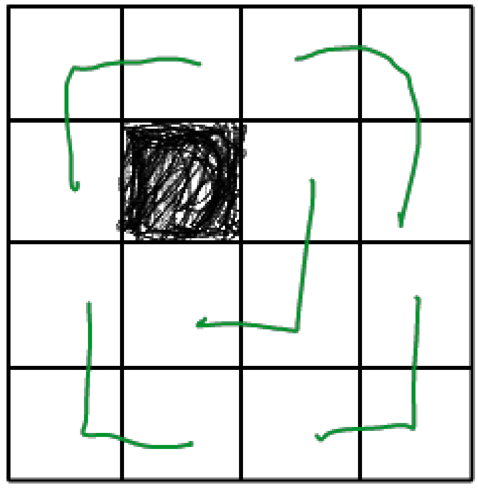
Ind. Hyp: For some $k \in \mathbb{N}$, $k \geq 3$, every integer between 2 and k is a product of primes.

Ind. Step: Consider $k+1$, if $k+1$ is prime, then it is alldeady a product of primes.

If $k+1$ is not prime, then $k+1 = ab$ where $a, b \in \mathbb{N}$, $2 \leq a, b$ and $a, b < k+1$. By ind. hyp., a and b are products of primes. Since $k+1$ is a product of a, b, $k+1$ is a product of primes.

Ind. Step applies to all int ≥ 3 since each int is either prime or composite so only one base case is needed.

Example: A unit square has been removed from a $2^n \times 2^n$ board. Prove that the board can be tiled using L-shaped triminoes. 

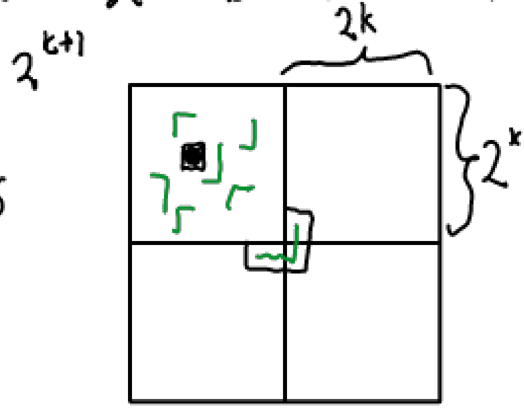


Proof: By induction on n.

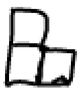
Base Case: When $n=1$, any 2×2 board with a square removed is already a trimino.

Ind. hyp: Assume true for some $k \in \mathbb{N}$.

Ind. Step: Suppose we have a $2^{k+1}\times 2^{k+1}$ board with a square removed.



Square A is a $2^k \times 2^k$ board minus a square. So A can be tiled by ind. hyp.

Put a  in the centre, the remaining 3 boards can be tiled by ind. hyp.