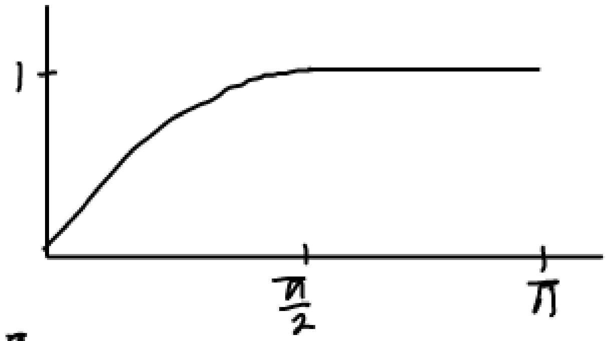
Seun

1. $\sin x, \quad 0 \le x \le \frac{\pi}{2}$

$1, \quad \frac{\pi}{2} \le x \le \pi$



period $= 2l = \pi$, so $l = \frac{\pi}{2}$

$$a_0 = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f\,dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin x\,dx + \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\pi} 1\,dx = \frac{1}{2} + \frac{1}{\pi}$$

$$a_n = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\cos 2nx\,dx = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin x\cos 2nx\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \cos 2nx\,dx$$

$$= \frac{2}{\pi}\left(\frac{1-2n\sin(\pi n)}{1-4n^2} + \frac{\sin(2\pi n) - \sin(\pi n)}{2n}\right)$$

$$= \frac{2}{\pi}\left(\frac{1}{1-4n^2}\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin x \sin 2nx\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \sin 2nx\,dx$$

$$= \frac{2}{\pi}\left(\frac{2n\cos(\pi n)}{1-4n^2} + \frac{\cos(2\pi n) - \cos(\pi n)}{2n}\right)$$

$$= \begin{cases} 2/\pi\left(\frac{-2n}{1-4n^2} + \frac{1}{n}\right) & \text{odd} \\ 2/\pi\left(\frac{2n}{1-4n^2}\right) & \text{even} \end{cases}$$

$$f(x) = \frac{1}{2} + \frac{1}{\pi} + \frac{2}{\pi}\sum \frac{1}{1-4n^2}\cos 2nx + \frac{2}{\pi}\left(\sum_{n=\text{odd}}\left(\frac{-2n}{1-4n^2} + \frac{1}{n}\right)\sin 2nx + \sum_{n=\text{even}}\left(\frac{2n}{1-4n^2}\right)\sin 2nx\right.$$

at $x = k\pi$ where $k \in \mathbb{Z}$

it fails to converge to $f(x)$

$$2.\ A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos(wv)\,dv$$

$$= \frac{1}{\pi}\int_{0}^{2} 3v^2\cos(wv)\,dv$$

$$= \frac{1}{\pi}\left(\frac{3v^2\sin(wv)}{w}\Big|_0^2 - \frac{6}{w}\int_0^2 v\sin(wv)\,dv\right)$$

$$= \frac{1}{\pi}\left(12\frac{\sin(2w)}{w} + \frac{12\cos(2w)}{w^2} - \frac{6\sin(2w)}{w^3}\right)$$

$$B(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin(wv)\,dv$$

$$= \frac{1}{\pi}\int_0^2 3v^2\sin(wv)\,dv$$

$$= \frac{1}{\pi}\left(\frac{-3v^2\cos(wv)}{w}\Big|_0^2 + \frac{6}{w}\int_0^2 v\cos(wv)\,dv\right)$$

$$= \frac{1}{\pi}\left(\frac{-12\cos(2w)}{w} + \frac{12\sin(2w)}{w^2} + \frac{6\cos(2w)}{w^3} - \frac{6}{w^3}\right)$$

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left(\left(\frac{12\sin(2w)}{w} + \frac{12\cos(2w)}{w^2} - \frac{6\sin(2w)}{w^3}\right)\cos(wx) + \left(\frac{-12\cos(2w)}{w} + \frac{12\sin(2w)}{w^2} + \frac{6\cos(2w)}{w^3} - \frac{6}{w^3}\right)\sin(wx)\right)dw$$

This fails to converge when $x=2$.