1. Negations
2. Contrapositive

Negations

Not P is true if and only if P is false.

Not (A and B) is equivalent to (Not A) or (Not B).

Not (A or B) = (Not A) and (Not B)

Not ($A \underset{\uparrow}{\Rightarrow} B$) = A and (Not B)

implication

Not ($\forall x \in S$, $P(x)$) = $\exists x \in S$, not $P(x)$

Not ($\exists x \in S$, $P(x)$) = $\forall x \in S$, not $P(x)$

Example: $P(x)$: "x is even or $x \geq 314$" $x \in \mathbb{Z}$

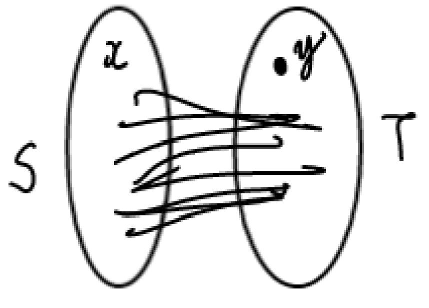
not $P(x)$: "x is odd and $x < 314$".

$P(300)$ T, not $P(300)$ F

Example: $f : S \to T$

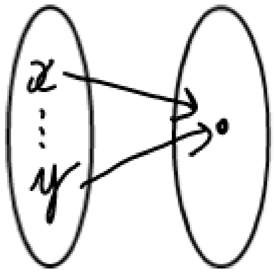
f is onto if for all $y \in T$, there exists $x \in S$ such that $f(x) = y$

f is not onto if there exists $y \in T$, for all $x \in S$, $f(x) \neq y$.



f is 1-1 if for all $x, y \in S$, if $f(x)=f(y)$, then $x=y$.

f is not 1-1 if there exists $x, y \in S$, s.t. $f(x)=f(y)$ and $x \neq y$.



Example: $f: \mathbb{R} \to \mathbb{R}$ $f(x)=x^2$

Not onto: Let $y=-1$, Let $x \in \mathbb{R}$, since $x^2 \geq 0$, $f(x) \neq -1$

Not 1-1: Let $x=1, y=-1$.

$f(x)=1=f(y)$

Also, $x \neq y$.

To disprove a statement, prove its negation.

Contrapositive

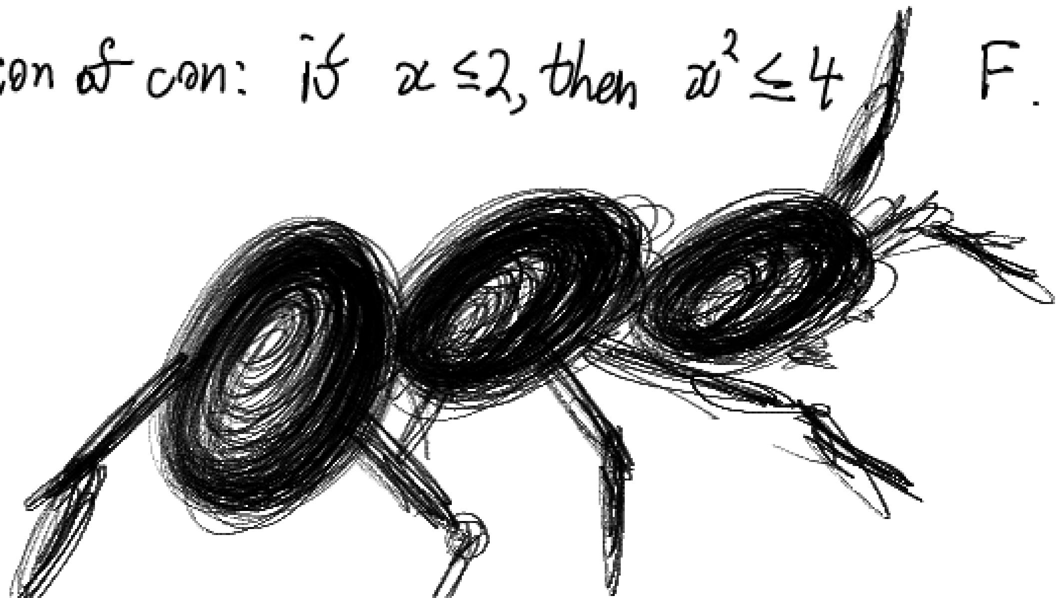
Def^n: The contrapositive of "if P, then Q" is "if not Q, then not P."

Example: if $x>2$, then $x^2>4$. $x \in \mathbb{R}$ T

contrapositive: if $x^2 \leq 4$, then $x \leq 2$. T

Converse: if $x^2>4$, then $x>2$. F. $x=-3000$

con of con: if $x \leq 2$, then $x^2 \leq 4$ F. $x=-3000$



Fact: "If P, then Q" is equivalent to "if not Q, then not P"

P	Q	$P \Rightarrow Q$		not $Q \Rightarrow$ not P
1	1	1	$\longleftrightarrow$	1
1	0	0	$\longleftrightarrow$	0
0	1	1	$\longleftrightarrow$	1
0	0	1	$\longleftrightarrow$	1

Proof by contrapositive: prove $P \Rightarrow Q$ by proving not $Q \Rightarrow$ not P.

Assume conclusion is false. Conclude the hyp is false.

Example: let $n \in \mathbb{Z}$. If n^2 is even, then n is even.

proof: (if n is odd, then n^2 is odd.)

suppose n is odd, then $n = 2k+1$ for some $k \in \mathbb{Z}$.

Then $n^2 = (2k+1)^2 = 4k^2+4k+1$

$$= 2(2k^2+2k)+1$$

Since $2k^2+2k \in \mathbb{Z}$, n^2 is odd.

Example: Let $x, y \in \mathbb{R}$. If $xy > 0$, then $x > 0$ or $x+y < 0$.

proof: suppose $x \leq 0$ and $x+y \geq 0$. (Goal $xy \leq 0$)

Then $y \geq 0$. Since $x \leq 0$ and $y \geq 0$, $xy \leq 0$.

When to use contrapositives?

→ When direct proof is hard

→ When 'not Q' gives more information to work with than "P".