Kuratowski's Theorem:

G is planar iff G contains no subdivisions of K_5 or $K_{3,3}$ as a subgraph.

(minor version):

G is planar iff neither K_5 nor $K_{3,3}$ can be obtained from G by contracting/deleting edges, removing vertices.

Contraction: If e is an edge of G then G/e is the graph contained by 'contracting' e.

G $\quad\quad$ G/e

• if G is planar then so is G/e

Let G be a connected planar embedding of a graph. The planar dual of G is the graph G^* such that the set of vertices of G^* is the set of faces of G, and two vertices of G^* are joined by an edge iff the corresponding faces are adjacent in G.

Properties:

1) G^* has a drawing 'on top of' G so that each edge of G^* crosses exactly one edge of G, and each vertex of G^* is drawn inside its corresponding face.

2) each edge of G^* 'corresponds naturally' to a unique edge of G in particular, G and G^* have the same number of edges.

3) The faces of G^* 'corresponds naturally' to vertices of G.

4) $G^{**} = G$ (requires connectness of G)

5) $(G \backslash e)^* = G^* \backslash e$ & $(G/e)^* = G^*/e$

6) G^* may have multiple edges &/ loops when G does not.
7) G^* may depend on the particular embedding of G.
i.e. different embeddings of G may have nonisomorphic duals

8) Platonic graphs come in dual pairs

Matchings

Given a graph $G=(V,E)$, a <u>matching</u> of G is a set $M \subseteq E$ so that no two edges in M are incident with a common vertex