Chapter 9
Inflation

MSCI 261
SECTION 1 (CHE/GEOE) AND SECTION 2 (SOFTWARE)

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Overview

- Measuring inflation
- Current vs Real
- Economic Evaluations with Inflation
  - Present Worth method
  - IRR method

Why we consider inflation

Prices of goods and services change over time

An increase in average prices over time is called **inflation**
- average over <u>all</u> goods and services

A decrease in average prices over time is called **deflation**
- doesn't happen much – in bad recessions, usually

Measuring Inflation: Consumer Price Index (CPI)

Statistics Canada observes the per-household average amounts of about 600 goods & services -- *average bundle*, updated every 4 years

Statistics Canada observes over 500,000 prices in each year

- Using prices in *base year* (now 2002) Statistics Canada computes how many dollars are needed to buy the average bundle
  - *in the base year*
  - *in the next year*
  - *in the year after that, etc.*
- Statistics Canada computes the CPI for each year as
  100 x (number of $ that year)/(number of $ in base year)

The inflation *rate* is the rate of change of the average price level: the % increase in the CPI from the previous year.

Historical CPI (2002=100) and its year over year change.

From Statistics Canada
http://www.statcan.gc.ca/tables-tableaux/sum-som/l01/cst01/econ46a-eng.htm

	All-items (2002 = 100)	Change from previous year (%)
1995	87.6	2.2
1996	88.9	1.5
1997	90.4	1.7
1998	91.3	1.0
1999	92.9	1.8
2000	95.4	2.7
2001	97.8	2.5
2002	100.0	2.2
2003	102.8	2.8
2004	104.7	1.8
2005	107.0	2.2
2006	109.1	2.0
2007	111.5	2.2
2008	114.1	2.3
2009	114.4	0.3
2010	116.5	1.8
2011	119.9	2.9
2012	121.7	1.5
2013	122.8	0.9
2014	125.2	2.0

Current and Real Dollars

Current Dollars:

- The dollars in our pockets, and recorded in our bank books, cheque books, accounting records
- Also known as *actual* or *nominal* dollars
  - **Note: old review questions use "actual" dollars**

Real Dollars:

- A hypothetical unit of measure, based on assuming zero inflation since the base year (or before the base year)
- Also known as *constant* dollars

a) $\left[\frac{21000}{0.476}\right] \times 1.19 = \$52500$ in 2002 dollars.

$\downarrow$ converts to "1992" dollars

$\downarrow$ bring to "2002" dollars

The 2002 jr. profs make more than the 1979 jr. profs.

Example 9-1

At LW University, the current dollar salaries for a new junior professor and for the highest paid senior professor are shown in the table, for the years 1979 and 2002, along with the CPI.

Year	CPI (1992=100)	Current Dollar Salaries	
		Junior	Senior
1979	47.6	$21,000	$60,000
2002	119.0	$58,000	$115,000

a) Express the 1979 junior salary in real 2002 dollars. Compare the standard of living of junior professors in 2002 and 1979.
b) Express the 2002 senior salary in real 1979 dollars. Compare the standard of living of senior professors in 2002 and 1979.
c) Calculate the geometric average inflation rate between 1979 and 2002, using the CPI in only these two years.

b) $\left[\frac{115000}{1.19}\right] 0.476 = \$46000$ in 1979 dollars

The 1979 sr. profs make more.

c) Geometric AVG $= \left(\frac{119}{47.6}\right)^{\frac{1}{2002-1979}} - 1$

$= 4.064\% / \text{yr.}$

Impact of Various Inflation Rates

Yr. (n)	$(1+f)^n$ for three f values: 2%	10%	170%
0	1	1	1
1	1.020	1.100	2.7
2	1.040	1.210	7.3
3	1.061	1.331	19.7
4	1.082	1.464	53.1
5	1.104	1.611	143.5
6	1.126	1.772	387.4
7	1.149	1.949	1046.0
8	1.172	2.144	2824.3
9	1.195	2.358	7625.6
10	1.219	2.594	20589.1

Some Theories on Causes of Inflation

- Amount of money grows more quickly than output of real goods and services ("too many dollars chasing too few goods"); since governments control money supply, it's their fault

- "wage-price spiral" – employees demand higher wages because they face higher prices; employers must raise prices of goods because wages are higher

- For more, take a course in Macroeconomics

Conversion Between Current and Real Dollar Forecasts

C_N = a current dollar cash flow forecast in year N

$R_{0,N}$ = same cash flow in year N, in real dollars relative to the base year, 0 (typically now, but not always)

f = the inflation rate forecast

- Assumed to be constant from year 0 to year N

Conversion (continued)

Then the conversion from current dollars in year N to real dollars in year N relative to the base year 0 is:

$$R_{0,N} = \frac{C_N}{(1+f)^N}$$

The base year (0) is usually omitted from the notation:

$$R_N = \frac{C_N}{(1+f)^N}$$

Mathematically, $R_N = C_N(P/F, f, N)$, but

- R_N is in real dollars <u>at time N</u> and <u>not</u> a present worth

$C_{2017} = \$1025 \rightsquigarrow R_{2017} = \frac{1025}{(1+0.015)^2}$

$= \$994.93$

Example 9-2

This is Spring 2015. The residence food plan in Spring 2017 is *expected to cost, in current dollars,* $1025. Inflation is expected to be 1.5% per year for the next 2 years. What is the real dollar cost of the 2017 food plan?

Rear to current: $(1+i')(1+f)-1=i$

Current and Real Interest Rates

Consider the current value of \$M invested for one year at a current interest rate i: \$M(1 + i)

- Note: because \$M is spent now, it is both "current" and "real"
- But \$M(1 + i) is only "current"

If the inflation rate over the next year is f, the real value of our cash flow is:

$$\$M\left(\frac{1+i}{1+f}\right)$$

Definition of real interest rate, i'

- Interest rate that yields the same real dollar amount <u>without</u> inflation as the current interest rate yields <u>with</u> inflation.

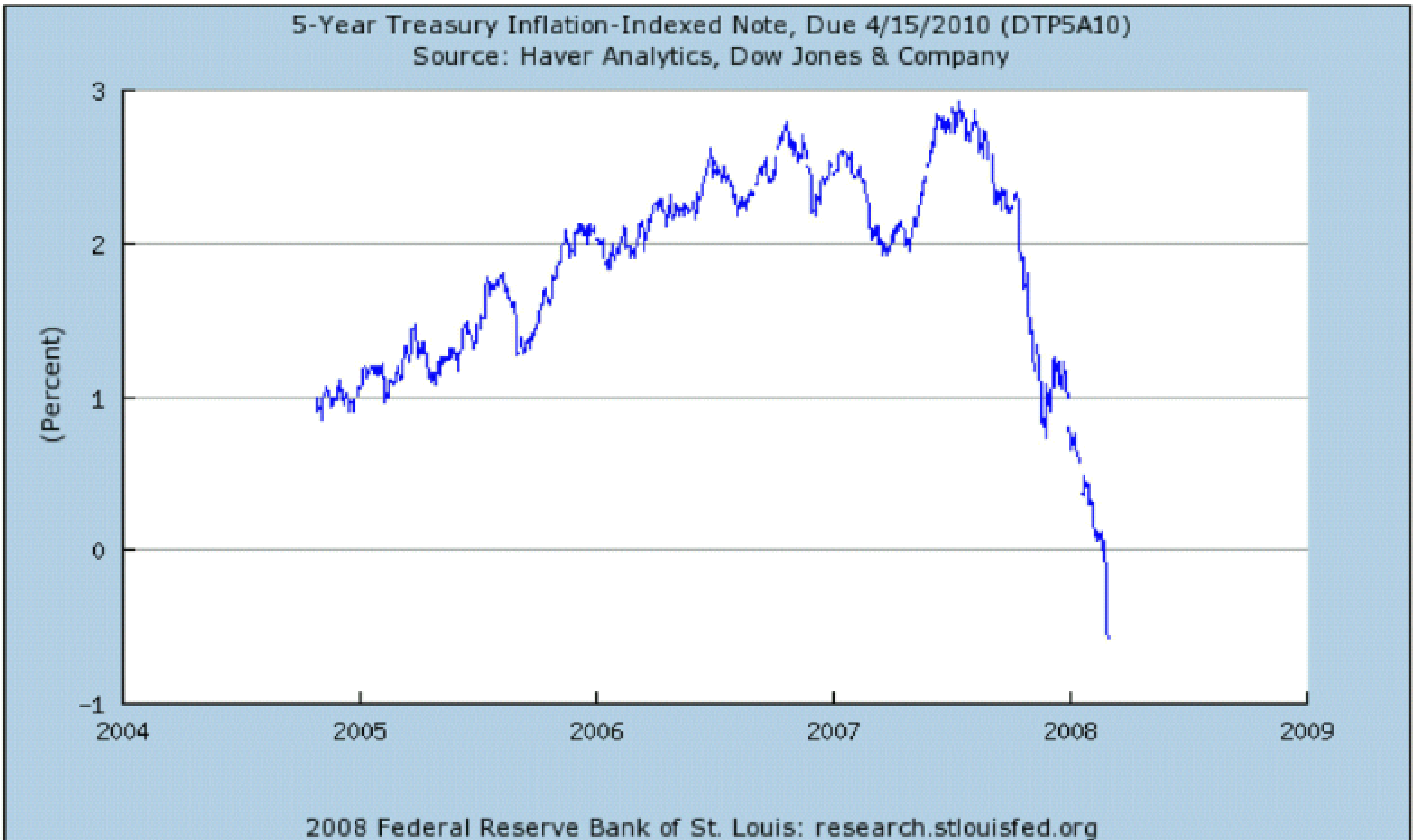
$$M(1+i') = M\left(\frac{1+i}{1+f}\right)$$

From current to real: $\boxed{i' = \frac{1+i}{1+f} - 1}$

From real to current: $\boxed{i = i' + f + i'f}$

- Note: for small i' and f, $i \approx i' + f$

Example of Real Interest Rates



Current and Real MARRs

- If investors expect inflation, they will require higher current rates of return on their investments than if no inflation was expected.
- i.e., analogous to $i = i' + f + i'f$,

From current to real: $$\boxed{MARR_{real} = \frac{1 + MARR_{current}}{1 + f} - 1}$$

From real to current: $$\boxed{MARR_{current} = MARR_{real} + f + MARR_{real}\, f}$$

① $MARR_{current} = (1+.03)(1+.015)-1 = 4.545\%/yr$

GIC offers $4\% < MARR_{current}$

$\therefore$ not gud

② real rate of return for GIC $= \left(\frac{1+0.4}{1+0.015}\right)-1 = 2.46\%/yr < MARR_{real}$

$\therefore$ not gud

Example 9-3

Suppose that Canada Trust offers a 4% interest rate for a 1-year GIC (Guaranteed Investment Certificate). If you require a real rate of return of 3% on your investments and you expect inflation to be 1.5% in the coming year, is a GIC a good investment for you?

Current and Real IRRs

Suppose a first cost C_0 , current net cash flows C_1, C_2, …, C_T
Then real first cost = C_0 , real net cash flows are:

$$R_1 = \frac{C_1}{(1+f)^1},\ R_2 = \frac{C_2}{(1+f)^2},\ \ldots,\ R_T = \frac{C_T}{(1+f)^T}$$

Then IRR$_{current}$ can be found by solving for i in:

$$-C_0 + \frac{C_1}{(1+i)} + \frac{C_2}{(1+i)^2} + \cdots + \frac{C_T}{(1+i)^T} = 0$$

and IRR$_{real}$ can be found by solving for i' in:

$$-C_0 + \frac{R_1}{(1+i')} + \frac{R_2}{(1+i')^2} + \ldots + \frac{R_T}{(1+i')^T} = 0$$

i.e. $$-C_0 + \frac{C_1}{(1+f)^1(1+i')} + \frac{C_2}{(1+f)^2(1+i')^2} + \ldots + \frac{C_T}{(1+f)^T(1+i')^T} = 0$$

Thefore: $(1+i) = (1+f)(1+i')$

To summarize

From current to real: $$\boxed{IRR_{real} = \frac{1+IRR_{current}}{1+f} - 1}$$

From real to current: $$\boxed{IRR_{current} = IRR_{real} + f + IRR_{real}\, f}$$

- Same as relationship between $MARR_{current}$ and $MARR_{real}$
- Therefore, correctly anticipated inflation has no effect on project evaluations by IRR method.

① Current cash flows: $-3000, 1700, 1800+500$

$$PW(i) = -3000 + \frac{1700}{(1+i)} + \frac{2300}{(1+i)^2} = 0$$

$MARR_{current} = (1+f)(1+MARR_{real}) - 1$

solve for $i = 20.36\%/yr > 17.6\%/yr$

IRR current

$\therefore$ gud

Example 9-4

A computer has a first cost of \$3000 and is expected to bring about current dollar savings of \$1700 in year 1 and \$1800 in year 2. Expected current dollar salvage value at the end of 2 years is \$500. Inflation is 5% per year and the real MARR is 12%. Based on an IRR analysis, should the project be undertaken?

② Real cash flows: $-3000, \frac{1700}{(1+0.05)}, \frac{2300}{(1+0.05)^2}$

$$PW(i') = -3000 + \frac{1619.05}{(1+i')} + \frac{2086.17}{(1+i')^2} = 0$$

$i' = IRR_{real} = 14.65\%/yr > MARR_{real}$

$\therefore$ gud

Current and Real Present Worth Calculations

Again, first cost C_0, current net cash flows C_1, C_2, …, C_T , real first cost C_0, real net cash flows

$$R_1 = \frac{C_1}{(1+f)^1}, R_2 = \frac{C_2}{(1+f)^2}, \ldots, R_T = \frac{C_T}{(1+f)^T}$$

Let i = MARR$_{current}$, and i' = MARR$_{real}$

$$\begin{aligned} PW_{real} &= -C_0 + \frac{R_1}{(1+i')^1} + \frac{R_2}{(1+i')^2} + \ldots + \frac{R_T}{(1+i')^T} \\ &= -C_0 + \frac{C_1}{(1+f)^1(1+i')^1} + \frac{C_2}{(1+f)^2(1+i')^2} + \ldots + \frac{C_T}{(1+f)^T(1+i')^T} \\ &= -C_0 + \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \ldots + \frac{C_T}{(1+i)^T} \\ &= PW_{current} \end{aligned}$$

Conclusion: if inflation base year <u>is</u> year zero, then

- $PW_{real} = PW_{current}$

① Find $MARR_{real} = (1+.08)(1+.04)-1 = 3.85\%/yr.$

$$PW_{real} = -97000 + 15000(P/A, 3.85\%, 10)$$
$$= 25600.59 > 0, \text{ Accept}$$

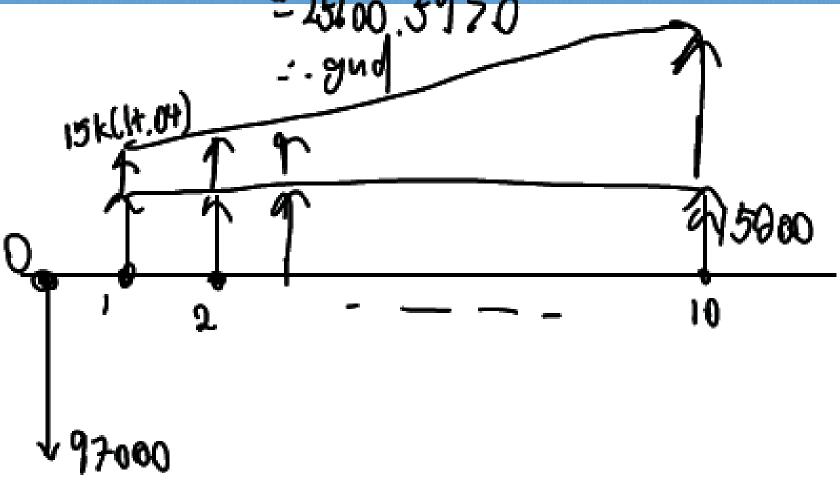
Example 9-5

Ken may insulate his company's building, at a cost of \$97,000. The energy savings are estimated at \$15,000 annually, in real "today's" dollars, starting one year from now, for 10 years altogether. Inflation is expected to average 4% per year and Ken's current MARR is 8%. Evaluate the investment by the present worth method.

② Convert cash flow to current dollars, with base annuity in yr 1. equal to 15000(1+.04), then increases by 4%/yr thereafter.

$$PW_{current} = -97000 + 15000(1+0.04)(P/A, g=4\%, i=8\%, 10)$$
$$= -97000 + \frac{15000\cancel{(1+.04)}(P/A, i', 10)}{\cancel{(1+.04)}}$$
$$= -97000 + 15000(P/A, 3.85\%, 10)$$
$$= 25600.59 > 0$$
$\therefore$ good

— Real \$



$$i' = \frac{1+i}{1+g} - 1 = \frac{1+.08}{1+.04} - 1 = MARR_{real}$$

Summary of Economic Evaluation with Inflation

- The engineer typically has been given an observed, i.e., current, MARR

- Projected cash flows are often stated in real (constant) dollars, i.e., they do not include adjustments for inflation.

- We must *match the cash flow type with the MARR type*
  - either convert real cash flows to current, or current MARR to real
  - both for IRR and PW methods

Example 9-6 (Taxes return!)

The CCA rate for sonar equipment is 30%. The current after-tax MARR is 9%. The tax rate is 50%. Inflation will be at 6% per year.

First cost	= $220,000
Real savings per year	= $80,000 (before-tax, year-0 dollars)
Real scrap at end of 4 years	= $20,000 (before-tax, year-0 dollars)

a) What is the real after-tax MARR?

b) What is the real after-tax IRR? Make a recommendation.

Always use $i_{current,\ after\text{-}tax}$ in CTF & CSF factors !

Why? Because taxes are paid in current \$; all formulas were developed with current \$ thinking.

-- e.g. salvage value, $S_{current} = S_{real}(1+f)^n$

After-tax PW of salvage value term

$$= S_{current}\,(1 - td/(i_{current}+d))/(1+i_{current})^n$$

$$= S_{real}(1+f)^n\,\{1 - td/(i_{current}+d)\}/((1+i_{real})^n\,(1+f)^n)$$

$$= S_{real}\{1 - td/(i_{current}+d)\}/(1+i_{real})^n$$

$$= S_{real}\{1 - td/([\,i_{real} + f + i_{real}\cdot f\,] + d)\}/(1+i_{real})^n$$

Interest Rate Conversions: Before & After Tax, and Current & Real

Given: before-tax, current i

Find: after-tax, real i'

Which procedure is valid:

(a) convert i to after-tax, then to real, or

(b) convert i to real, then to after-tax ?

Principle to remember: taxes are paid in current dollars

Example 9-7

Suppose that you earn interest on your savings account at the current, before-tax rate of 3.5% per year.

a) Suppose that any current dollars of interest earned are taxed (in the personal income tax system) at 25%. Convert the current, before-tax rate to a current, after-tax rate; then convert the current, after-tax interest rate to a real, after-tax interest rate, using an inflation forecast of 2% per year.

b) Find the value of the tax rate that makes the real, after-tax interest rate equal to zero.

Read for next class

Chapter 12 from textbook