Electrostatic Fields

- Ring of Charges
- Disk of Charges

- Concept of Field lines
- Flux

$$K_e = 8.98\times10^{9}\ \frac{N\cdot m^2}{C^2}$$
$$K_e = \frac{1}{4\pi\epsilon_0}$$
$$\epsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}$$

$$dQ = (r\,d\phi)\,\rho_L$$
$$\rho_L = \frac{Q}{2\pi a}$$

∵ it is a uniform ring
∴ the $\hat{r}$ component all gets canceled out.

$$\vec{E}_{dQ}(z) = \frac{k\,dQ}{R^2}\left(\frac{r\hat{r} + z\hat{z}}{\underbrace{\sqrt{r^2+z^2}}_{R}}\right)$$

$$\vec{E}_Q(z) = \int_Q \left(\frac{k\,dQ}{R^2}\left(\frac{r\hat{r}+z\hat{z}}{R}\right)\right)$$

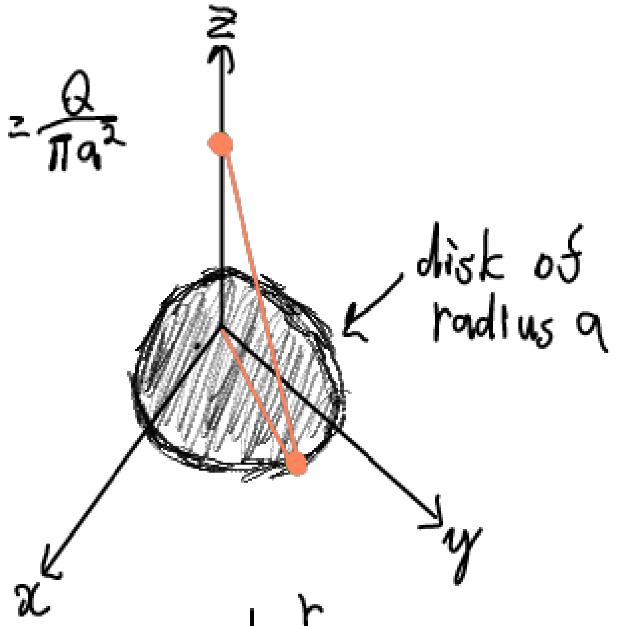
$$\vec{E} = \int_{\phi=0}^{2\pi} \frac{k\rho_L r\,d\phi}{(r^2+z^2)^{3/2}}\left(r\hat{r} + z\hat{z}\right)$$

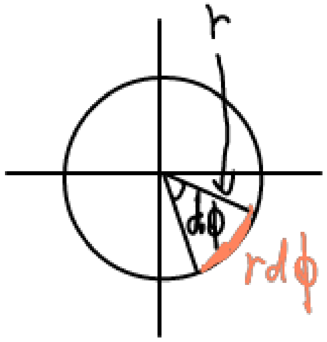
$$\vec{E} = k\rho_L a \int_{\phi=0}^{2\pi} \frac{z\,d\phi}{(a^2+z^2)^{3/2}}(\hat{z})$$

$$= \frac{k\rho_L a(2\pi)z}{(a^2+z^2)^{\frac{3}{2}}}(\hat{z})$$

$$= \frac{kQz}{(a^2+z^2)^{3/2}}(\hat{z})$$

$$\rho_s = \frac{Q}{\pi a^2}$$





$$dQ = dS\,\rho_s$$

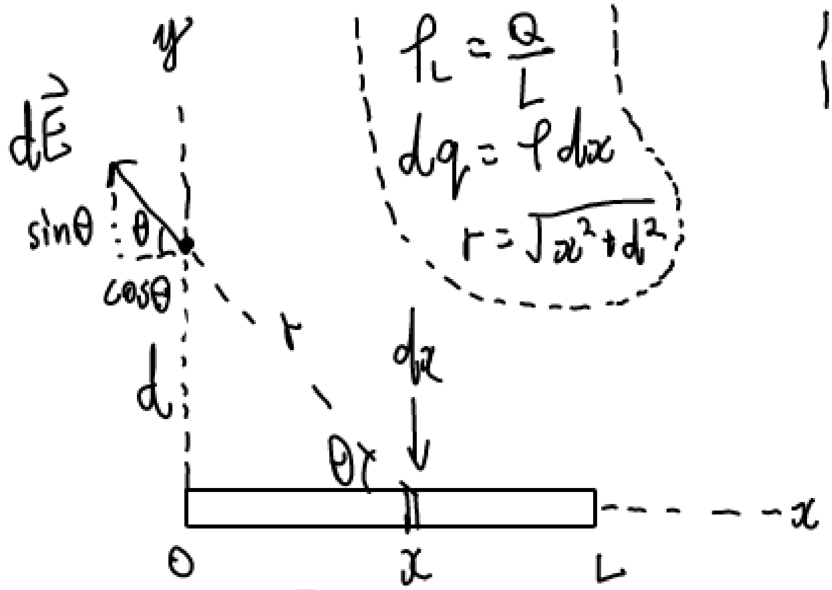
$$dS = dr\,(r\,d\phi)$$

$$\vec{E}_{dQ} = \int \frac{k\,dQ}{(z^2+r^2)^{3/2}}\left(r\hat{r} + z\hat{z}\right)$$

$$= k \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \frac{\rho_s\, z\, r\,dr\,d\phi}{(z^2+r^2)^{3/2}}\,\hat{z}$$

$$= k \int_{r=0}^{a} \frac{\rho_s\, z\, r\,dr\, 2\pi}{(z^2+r^2)^{3/2}}\,\hat{z}$$

$$= \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]\hat{z}$$



$$\rho_L = \frac{Q}{L}$$

$$dq = \rho\, dx$$

$$r = \sqrt{x^2 + d^2}$$

$$\boxed{\begin{aligned} \hat{r} &= \sin\theta\,\hat{y} - \cos\theta\,\hat{x} \\ \sin\theta &= \frac{d}{r} \\ \cos\theta &= \frac{x}{r} \end{aligned}}$$

$$\vec{E}_{\substack{\text{point}\\\text{charge}}} = \frac{kq}{r^2}\hat{r}$$

$$\vec{E} = \int_{\text{bar}} d\vec{E}_x$$

$$\vec{E} = \int_0^L \frac{k\,dq}{r^2}\left(\frac{d}{r}\hat{y} - \frac{x}{r}\hat{x}\right)$$

$$E = \int_0^L \frac{k\rho\, dx}{(x^2+d^2)^{3/2}}\left(d\hat{y} - x\hat{x}\right)$$

① ②

$$①: E_y = k\rho d\,\hat{y}\int_0^L \frac{dx}{(x^2+d^2)^{3/2}}$$

$$= k\rho d\,\hat{y}\left(\frac{x}{d^2\sqrt{x^2+d^2}}\right)\Bigg|_0^L$$

$$= k\rho d\left(\frac{L}{d^2\sqrt{L^2+d^2}}\right)\hat{y}$$

$$②: E_x = k\rho\int_0^L \frac{x\,dx}{(x^2+d^2)^{3/2}}\;(-\hat{x})$$

$$= k\rho\left(\frac{-1}{\sqrt{x^2+d^2}}\right)\Bigg|_0^L(-\hat{x})$$

$$= -k\rho\left(\frac{1}{d} - \frac{1}{\sqrt{L^2+d^2}}\right)\hat{x}$$

$$\therefore \vec{E} = \frac{kQ}{L}\left(\frac{dL}{d^2\sqrt{L^2+d^2}}\hat{y} - \left(\frac{1}{d} - \frac{1}{\sqrt{L^2+d^2}}\right)\hat{x}\right)$$

as $d >> L$ $\quad \vec{E} \to \dfrac{kQ}{d^2}\hat{y}$

alternate method:

$$\vec{E} = \int_{bar} d\vec{E}(\theta)$$

$$\rho = \frac{Q}{L}$$

$$S = r\theta$$

$$dx = r\,d\theta$$

$$dq = \rho_L\,dx$$

$$= \rho_L\, r\, d\theta$$

$$\sin\theta = \frac{d}{r} \to r = \frac{d}{\sin\theta}$$

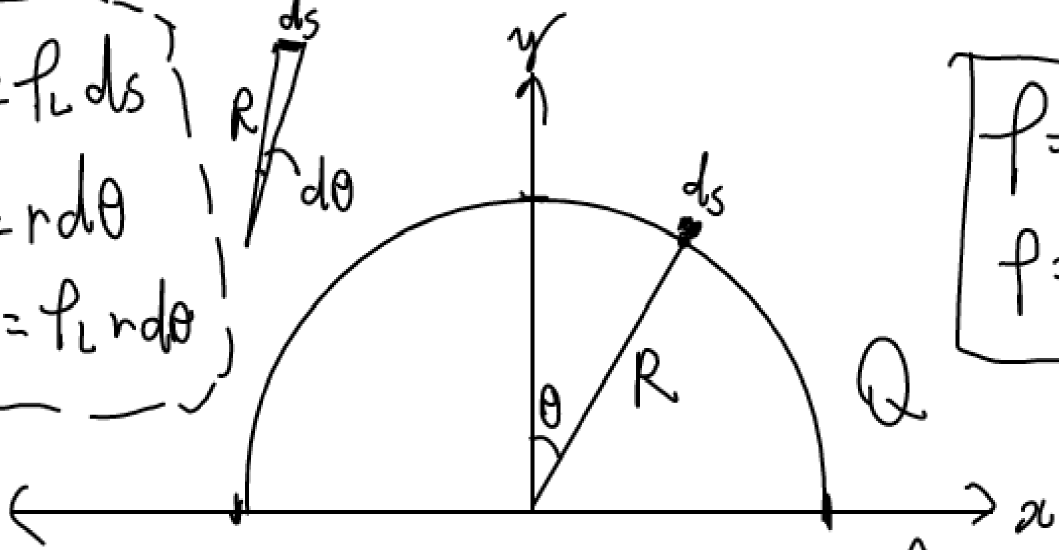
$$= \int_0^{\theta_L} \frac{k\,dq\,\hat{r}}{r^2}$$

$$= \int_0^{\theta_L} \frac{k\rho_L\,d\theta}{r}\hat{r}$$

$$= \int_0^{\theta_L} \frac{k\rho_L\,d\theta}{d/\sin\theta}\left(\sin\theta\,\hat{y} - \cos\theta\,\hat{x}\right)$$

$$= \frac{kQ}{dL}\int_0^{\theta_L}\left(\sin^3\theta\,\hat{y} - \sin\theta\cos\theta\,\hat{x}\right)$$

$$dq = \rho_L \, ds$$
$$ds = r \, d\theta$$
$$dq = \rho_L \, r \, d\theta$$



$$\rho = \frac{Q}{L}$$
$$\rho = \frac{Q}{\pi R}$$

By symmetry, only need to integrate $\hat{y}$.

$$\vec{E} = \int_{\text{half ring}} d\vec{E}_y = \int_{\text{half ring}} \frac{k\,dq}{R^2} \cos\theta \, (-\hat{y})$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{k \rho_L R \, d\theta}{R^2} \cos\theta \, (-\hat{y})$$

$$= \frac{k \rho_L}{R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, d\theta \, (-\hat{y})$$

$$= \frac{k \rho_L}{R} (\sin\theta) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\hat{y})$$

$$= \frac{2 k \rho_L}{R} (-\hat{y})$$

$$= \frac{2kQ}{\pi R^2} (-\hat{y})$$