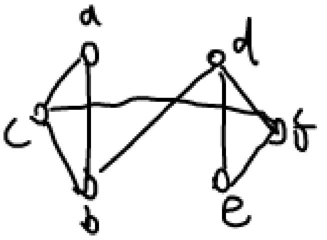
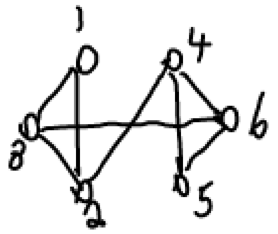
Graph:

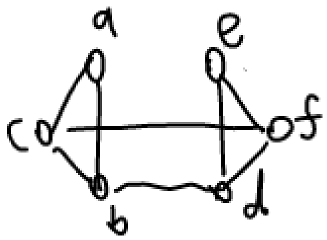
V is a finite set

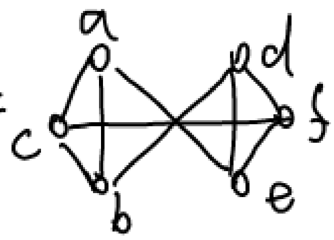
E is a set of unordered pairs of elements of V.

(i.e. 2-element subsets of V)

$G_1 =$ 

$G_2 =$ 

$G_3 =$ 

$G_4 =$ 

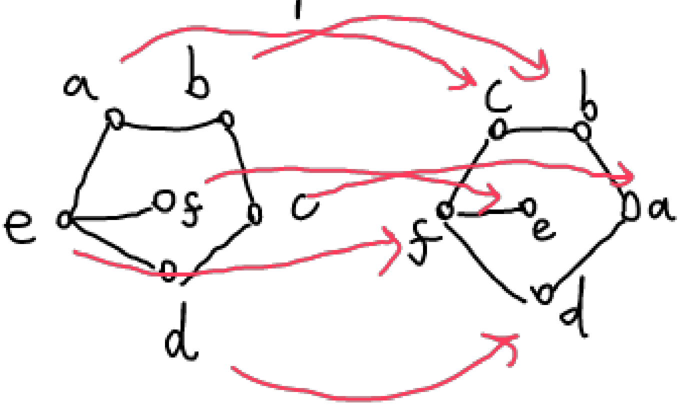
$G_1 == G_2$

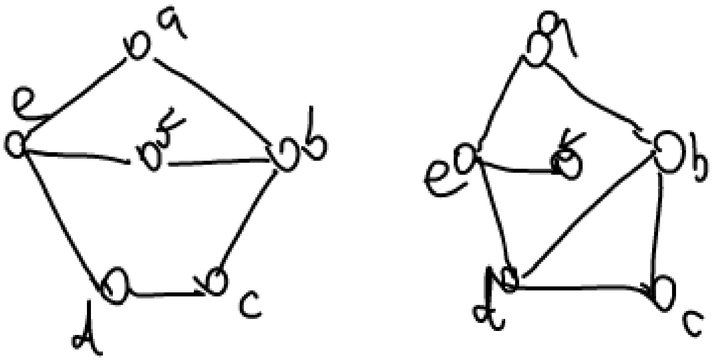
Let $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$

An isomorphism from G_1 to G_2 is a bijection $\psi : V_1 \to V_2$ so that:

- If $u, v \in V_1$ & $\{u, v\} \in E_1$, then $\{\psi(u), \psi(v)\} \in E_2$
- If $u, v \in V_1$ and $\{u, v\} \notin E_1$, then $\{\psi(u), \psi(v)\} \notin E_2$

If an isomorphism exists, G_1, G_2 are <u>isomorphic</u>.





are not isomorphic, because the second graph has a vertex in exactly on edge, and the first one does not.

(such a property would be preserved by isomorphism)

We abbreviate an edge $\{u,v\}$ by uv.

If $uv \in E$ then u and v are adjacent or neighbours.

The degree of a vertex is its number of neighbours, we write $\deg(v)$ for the degree of v.

<u>Handshake Theorem</u>: $\sum_{v\in V} \deg(v) = 2|E|$

The edge uv is <u>incident</u> with vertices u & v.

<u>pf</u>: Let $S = \{(v,e),\ v \text{ is incident with } e\}$.

$$|S| = \sum_{v\in V} (\#\text{ edges incident with } v) = \sum_{v\in V} \deg(v)$$

Also $$|S| = \sum_{e\in E} (\#\text{ vertices incident with } e) = 2|E|$$

So $\sum_{v\in V} \deg(v) = 2|E|$ $\blacksquare$

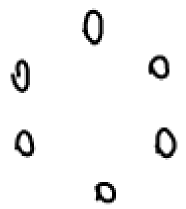
Corollary: Every graph has an even number of vertices of odd degree.

pf: $\sum_{v \in V} \deg(v) = 2|E|$ is even, so $\deg(v)$ is odd for an even number of $v \in V$.

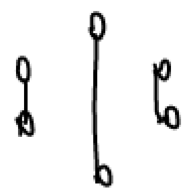
A graph is regular if every vertex has the same degree. If this degree is d, the graph is d-regular.

The nonisomorphic graphs on 6 vertices.

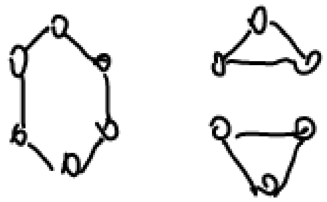
0-regular



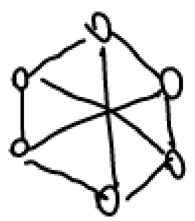
1-regular



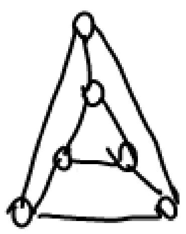
2-regular



3-regular

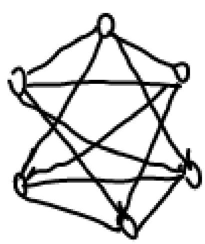


$K_{3,3}$



3-prism

4-regular



5-regular