Proof: Let $x = v_0, v_1, \dots, v_n = y$ be a path from x to y in G.

Let $k \in \{0, 1, 2, \dots, n\}$ be maximal so that G contains two egde-disjoint paths from x to v_n. If $v_k = v_n$, the theorem holds, so suppose $k < n$.

Let P, P' be edge-disjoint paths from x to v_k.
The edge $v_k v_{k+1}$ is not a bridge, so there is some path Q' from v_{k+1} to k that does not contain the edge $v_k v_{k+1}$.
Let w be the first vertex of Q' that is contained in $P \cup P'$, and let Q be the subpath of Q' from x_{k+1} to w,
Now, the edges in P, P', Q and $\{v_k, v_{k+1}\}$ contain edge-disjonit paths from x to v_{k+1}, contradicting the maximality of k.

↑

Theorem: If G is a connected graph with no bridges and x and y are vertices of G, then there exist at least two edge-disjoint paths from x to y in G.

Trees

A tree is a connected graph with no cycle. (Acyclic)

Prop. A connected graph G is a tree iff every edge is a bridge
pf. We saw that an edge is a bridge iff it is contained in no cycle. The result follows

A leaf of a tree is a degree-1 vertex.

Prop. Every tree on ≥ 2 vertices has ≥ 2 leaves.

pf: Let $v_0, v_1, \dots, v_k$ be a longest path. By maximality, every neighbour of v_0 or v_k is in the path. By Acyclicity, v_0 and v_k have only neighbours v_1, v_{k-1} respectively.
So $\deg(v_0) = \deg(v_k) = 1$.

Prop: If T is a tree with n vertices, then T has $n-1$ edges.

Pf: Trivial if $n=1$. Suppose that the statement holds for every tree on k vertices for some $k \geq 1$. Let T be a tree on $k+1$ vertices. Let v be a leaf of T, and let T' be the graph obtained by removing v and its single incident edge from T. Clearly T is acyclic. If x, y are vertices of T', then by connectedness of T, there is a path of T from x to y, since $\deg(v)=1$, this path does not contain v, so it's also a path of T', so T' is connected and is a tree.
T' has k vertices, $k-1$ edges, T has k edges.

Prop Trees are bipartite

pf reverse leaf, use induction