Terminology

* a product term is an implicant of a function if the function is a "1" for all minterms in the product term.

An implicant is called prime if removal of an input makes in not an implicant.

Fact: Any function can be implimented using any prime implicants.

A prime is essential if it includes a minterm which is <u>not</u> inside another prime.

Fact: if f is implemented with primes you must include the essentials.

$z \backslash xy$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

green: prime / essential

blue: prime

$yz \backslash wx$	00	01	11	10
00	1	0	1	1
01	0	1	0	0
11	1	1	1	0
10	0	0	1	1

$w\bar{z}$

$\bar{x}\bar{y}\bar{z}$

$\bar{w}xz$

wxy

$\bar{w}yz$

xyz

6 prime implicants

$$f = \underbrace{\bar{x}\bar{y}\bar{z} + w\bar{z} + \bar{w}yz + \bar{w}xz}_{\text{essential}}$$

$+ \begin{cases} xyz \\ wxz \end{cases}$ choose one to cover remaining minterm

Standard procedure:

① Enumerate Prime impl.

② Identify essential

③ choose least amount of primes to cover any minterms left over

XOR

* mostly an aside..., sometimes XORs can help minimize functions (if you can see them).

4 Variable XOR

yz \ wx	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$$f = x \oplus y \oplus z$$

So consider the following

yz \ wx	00	01	11	10
00	0	1	0	0
01	1	0	1	1
11	0	1	0	0
10	1	0	1	1

$$f = \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}yz + w\bar{y}z + wy\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z}$$

$$= \bar{w}x(\bar{y}\bar{z} + yz) + w(\bar{y}z + y\bar{z}) + \bar{x}(\bar{y}z + y\bar{z})$$

$$= \bar{w}x(\bar{y}\bar{z} + yz) + (w+\bar{x})(\bar{y}z + y\bar{z})$$

$$= \bar{w}w(\bar{y}\bar{z} + yz) + (\overline{\bar{w}x})(\bar{y}z + y\bar{z})$$

$$= \bar{w}x(\overline{y \oplus z}) + (\overline{\bar{w}x})(y \oplus z)$$

$$= a \cdot \bar{b} + \bar{a} \cdot b = a \oplus b$$