January 24 2014 Uniqueness 1. Unique 1865 To prove that an object x is unique, 2. Division Algorithm O Suppose & and y satisfy the conditions and x + y. Keach a contradiction. 3 Suppose x and y satisfy these conditions, prove x=y. Example: R, Additive identity] unique x & R such that YyER, yta=y Existence: Let x=0, then for any y & R, y +0=y. Uniqueness: Suppose $x_1, x_2 \in \mathbb{R}$ are both additive identities $\int S_0 x_1 + x_2 = x_1 (y = x_1, x = x_2)$ in descripting ctrl-c

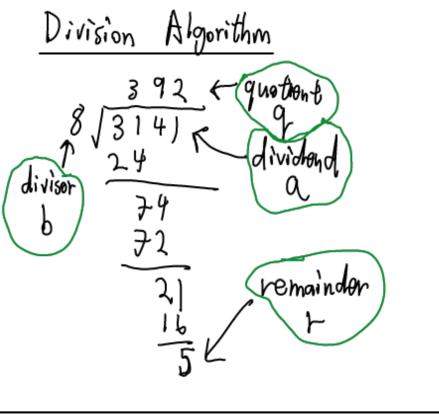
Since R is commutative under addition,

X, +22 = 22+2, So 20, = 22.

Another proof: Suppose $x_{1,2} \in \mathbb{R}$ are additive identities $x_1 + x_2$.

ctrl-V

Since x1=22 and 2+22, contradiction.



$$3141 = 8.392 + 5$$
 $A = 9.6 + r$
 f
 EZ
 EIN
 $0 \le r < 6$

13 divided by 3 equals 2 with rem 7.X

False

-13 = 3(-5)+(2)

Division Algorithm: Let a & Z, b & W. Then there exists unique q, r & Z such that azqb+r and 0 \le r < b.

Proof: Existence proof to done in A3

Uniqueness: Suppose both q, r, and q_2 , r_2 satisfy the conclusion of the div-alg. So $\alpha=q_1btr_1$ and $\alpha=q_2btr_2$ where $0\leq r$, < b and $0\leq r_2 < b$.

Subtract the two equations to get $q_1b+r_1-q_2b-r_2=0$. Then $b(q_1-q_2)=r_2-r_1$. Since $q_1-q_2\in \mathbb{Z}$, $b|(r_2-r_1)$. Since $0\leq r_1\leq b$ and $0\leq r_2\leq b$. $-(b-1)\leq r_2-r_1\leq b-1$

Since the only integer in [-(b-1), b-1] that is divisible by b is 0, $k_1-k_2=0$. So $k_1=k_2$. Then $b(q_1-q_2)=0$ Since $b\neq 0$, $q_1-q_2=0$, so $q_1=q_2$.

Example: Let A be an invertible matrix. Then A'is unique.

Suppose B,C are both invokes of A.

Then
$$B = BI = B(AC) = (BA)C = IC = C$$

Since C associative B is an inverse