Postulates/ Theorems of Boolean Algebra

P2	$x+0=x$	$x\cdot 1=x$
P3	$x+y=y+x$	$xy=yx$
P4	$x(y+z)=xy+xz$	$x+(y\cdot z)=(x+y)(x+z)$
P5	$x+\bar{x}=1$	$x\cdot\bar{x}=0$
T1	$x+x=x$	$x\cdot x=x$
T2	$x+1=1$	$x\cdot 0=0$
T3	$\bar{\bar{x}}=x$	
T4	$x+(y+z)=(x+y)+z$	$x\cdot(y\cdot z)=(x\cdot y)\cdot z$
T5	$\overline{(x+y)}=\bar{x}\cdot\bar{y}$	$\bar{x}\cdot\bar{y}=\bar{x}+\bar{y}$
T6	$x+xy=x$	$x(x+y)=x$

We can now manipulate and/or simplify eqns.

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$f = m_3 + m_5 + m_6 + m_7$

$= \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

"cost" => 4 AND gates
1 OR gate
12 AND INPUTS
4 OR INPUTS
21

* inverter at input is free.

minterm

$f = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

$= \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz + xyz + xyz$

$= (\bar{x}+x)yz + x(\bar{y}+y)z + xy(\bar{z}+z)$

$= 1\cdot yz + x\cdot 1\cdot z + xy\cdot 1$

product term

$= \boxed{yz + xz + xy}$

"cost" => 3 AND
1 OR
6 AND inputs
3 OR inputs
13

This is called a sum of products (SOP) impl. of f.

$f = M_0 M_2 M_3 M_5 M_6$

$= (x+y+z)(x+\bar{y}+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)$ ← Canonical POS.

maxterm

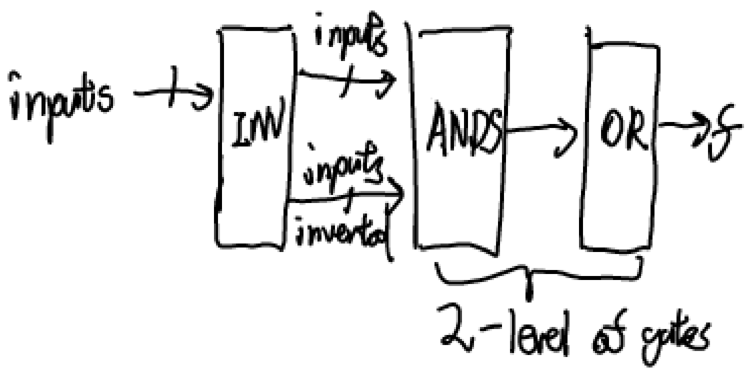
⋮

$= (x+z)(\bar{y}+z)(x+\bar{y})(\bar{x}+y+\bar{z})$ ← still a POS

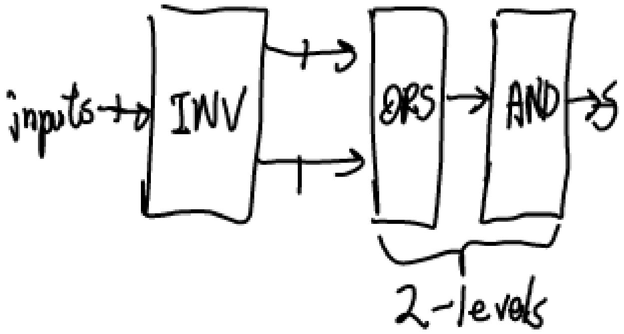
sum term

POS + SOP are called "2-level" function implimentations.

SOP



POS



Other Logic Gates

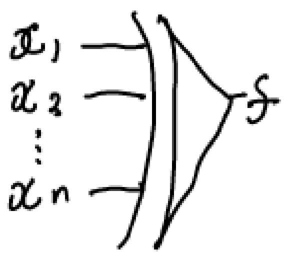
XOR

2-inputs

x	y	$f = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

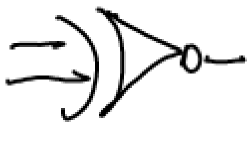
n-inputs "odd"

x	y	z	$f = x \oplus y \oplus z$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

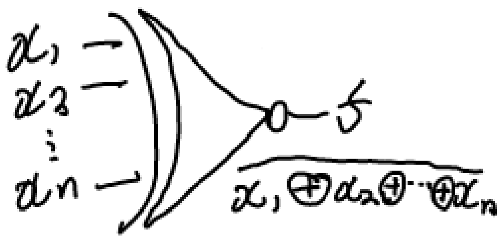


NXOR

x	y	$f = \overline{x \oplus y}$
0	0	1
0	1	0
1	0	0
1	1	1

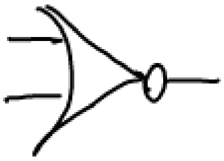


"even"



NOR "not or"

x	y	$f=\overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0



NAND "not and"

x	y	$f=\overline{xy}$
0	0	1
0	1	1
1	0	1
1	1	0

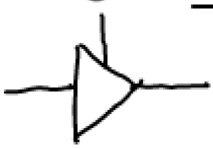
buffer

x  f

x	f
0	0
1	1

tri-state buffer

e

x  f

x	e	f
0	1	0
1	1	1
0	0	"Z"
1	0	

$\left.\begin{matrix}0\\1\end{matrix}\right\}$ buffer

if $e=0$

x —o o— f

disconnected