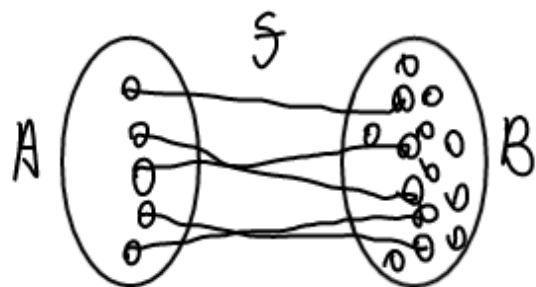


Countability

Use functions to count

Let A, B be sets, $f: A \rightarrow B$ is a function.

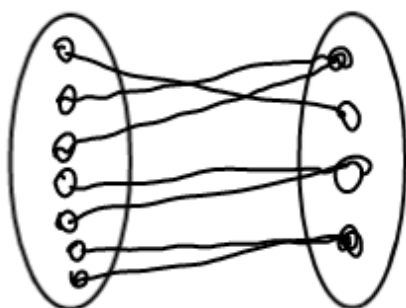
- ① If f is 1-1... each element in A is mapped to a diff element in B .



$$\text{So } |A| \leq |B|$$

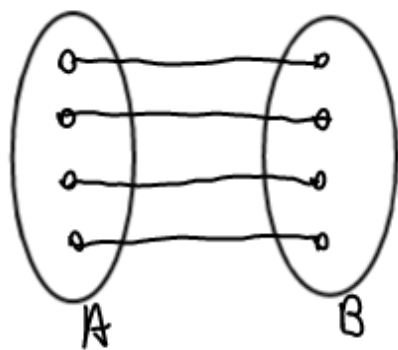
- ② If f is onto...

$$|A| \geq |B|$$



- ③ If f is a bijection...

$$|A| = |B|$$



What if A, B are infinite? $\mathbb{N} \quad \mathbb{Z} \quad \mathbb{Q}$

Defn: A set S is countable if there exists a fcn

$f: S \rightarrow \mathbb{N}$ that is 1-1.

Example: $S = \{a, b, c\}$ $f: S \rightarrow \mathbb{N}$

$$f(a) = 1, \quad f(b) = 2, \quad f(c) = 3$$

f is 1-1 $\Rightarrow S$ is countable

Ex $S = \{2k \mid k \in \mathbb{N}\} = \{2, 4, 6, 8, \dots\} \quad S \subseteq \mathbb{N}$

$f: S \rightarrow \mathbb{N} \quad f(x) = \frac{x}{2}$ This is a bijection.

S	2	4	6	8	10	...
\mathbb{N}	\updownarrow 1	\updownarrow 2	\updownarrow 3	\updownarrow 4	\updownarrow 5	...

$|S| = |\mathbb{N}|$