

$$3. a) \underbrace{\begin{bmatrix} -6 & 24 & 30 & 3 \\ -12 & 24 & 12 & -6 \\ -4 & 4 & -1 & -2 \\ -3 & 3 & 6 & -6 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 42 \\ 36 \\ 5 \\ -12 \end{bmatrix}}_b$$

b) swap ① & ②

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ -6 & 24 & 30 & 3 \\ -4 & 4 & -1 & -2 \\ -3 & 3 & 6 & -6 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ 1/2 & 12 & 24 & 6 \\ 1/3 & -4 & -5 & 0 \\ 1/4 & -3 & 3 & -9/2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ 1/2 & 12 & 24 & 6 \\ 1/3 & -1/3 & 3 & 2 \\ 1/4 & -1/4 & 9 & -3 \end{bmatrix}$$

↓ swap ③ & ④

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ 1/2 & 12 & 24 & 6 \\ 1/4 & -1/4 & 9 & -3 \\ 1/3 & -1/3 & 3 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ 1/2 & 12 & 24 & 6 \\ 1/4 & -1/4 & 9 & -3 \\ 1/3 & -1/3 & 1/3 & 3 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 1/4 & -1/4 & 1 & 0 \\ 1/3 & -1/3 & 1/3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} -12 & 24 & 12 & -6 \\ 0 & 12 & 24 & 6 \\ 0 & 0 & 9 & -3 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

c) 2 steps to compute  $x$

1)  $Lz = pb$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 1/4 & -1/4 & 1 & 0 \\ 1/3 & -1/3 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 42 \\ 36 \\ 5 \\ -12 \end{bmatrix}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 36 \\ 1/2 & 1 & 0 & 0 & 42 \\ 1/4 & -1/4 & 1 & 0 & -12 \\ 1/3 & -1/3 & 1/3 & 1 & 5 \end{array} \right] \Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 36 \\ 0 & 1 & 0 & 0 & 24 \\ 0 & -1/4 & 1 & 0 & -21 \\ 0 & -1/3 & 1/3 & 1 & -7 \end{array} \right]$$

$$\Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 36 \\ 0 & 1 & 0 & 0 & 24 \\ 0 & 0 & 1 & 0 & -15 \\ 0 & 0 & 1/3 & 1 & 1 \end{array} \right] \Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 36 \\ 0 & 1 & 0 & 0 & 24 \\ 0 & 0 & 1 & 0 & -15 \\ 0 & 0 & 0 & 1 & 6 \end{array} \right]$$

2)  $Ux = z$

$$\begin{bmatrix} -12 & 24 & 12 & -6 \\ 0 & 12 & 24 & 6 \\ 0 & 0 & 9 & -3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 36 \\ 24 \\ -15 \\ 6 \end{bmatrix}$$

$$\left[ \begin{array}{cccc|c} -12 & 24 & 12 & -6 & 36 \\ 0 & 12 & 24 & 6 & 24 \\ 0 & 0 & 9 & -3 & -15 \\ 0 & 0 & 0 & 3 & 6 \end{array} \right] \Rightarrow \left[ \begin{array}{cccc|c} 1 & -2 & -1 & 1/2 & -3 \\ 0 & 1 & 2 & 1/2 & 2 \\ 0 & 0 & 1 & -1/3 & -5/3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\Rightarrow \left[ \begin{array}{cccc|c} 1 & -2 & -1 & 0 & -4 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \Rightarrow \left[ \begin{array}{cccc|c} 1 & -2 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\therefore x = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}$$

4.  $x A = b$

$$\therefore A^T x^T = b^T \quad (1)$$

From  $LU = A$ ,  $U^T L^T = A^T \quad (2)$

$$(2) \rightarrow (1)$$

$$U^T L^T x^T = b^T$$

$$\therefore U^T z = b^T$$

$$L^T x^T = z$$

Algorithm:

$$z = \text{forward\_sub}(U^T, b^T);$$

$$y = \text{back\_sub}(L^T, z);$$

$$x = y^T;$$

5. a)  $x = A \setminus B;$

$$x = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$b) \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ -3 & 2.099 & 6 & | & 3.901 \\ 5 & -1 & 5 & | & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & -0.001 & 6 & | & 6.001 \\ 0 & 2.5 & 5 & | & 2.5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & -0.001 & 6 & | & 6.001 \\ 0 & 0 & 15005 & | & 15006 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & -0.001 & 0 & | & 0.0006 \\ 0 & 0 & 15005 & | & 15006 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 10 & 0 & 0 & | & 2.8 \\ 0 & -0.001 & 0 & | & 0.0006 \\ 0 & 0 & 15005 & | & 15006 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} 0.28 \\ -0.6 \\ 1.0001 \end{bmatrix}$$

$$c) \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & -0.001 & 6 & | & 6.001 \\ 0 & 2.5 & 5 & | & 2.5 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & 2.5 & 5 & | & 2.5 \\ 0 & -0.001 & 6 & | & 6.001 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & 2.5 & 5 & | & 2.5 \\ 0 & 0 & 6.002 & | & 6.002 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & -7 & 0 & | & 7 \\ 0 & 2.5 & 0 & | & -2.5 \\ 0 & 0 & 6.002 & | & 6.002 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 0 & | & 0 \\ 0 & 2.5 & 0 & | & -2.5 \\ 0 & 0 & 6.002 & | & 6.002 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

d) c is more accurate.