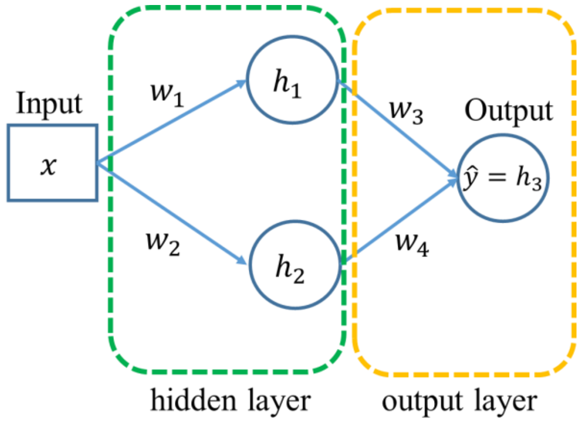
Homework Assignment 4

Part-1: Basic Concepts

1. Backpropagation in A Neural Network (14 points)



$h_1 = f_1(w_1 x + b_1)$

$h_2 = f_2(w_2 x + b_2)$

$\hat{y} = f_3(w_3 h_1 + w_4 h_2 + b_3)$

$f_n' = \frac{\partial f_n(v)}{\partial v}$, n = 1,2,3

$x, w_1, w_2, w_3, w_4, b_1, b_2, b_3, h_1, h_2, h_3$ are scalars

Compute the derivatives of the loss L with respect to parameters and input, assuming $\frac{\partial L}{\partial h_3}$ is known.

Example:

This is the complete solution: $\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial h_3}\frac{\partial h_3}{\partial h_1}\frac{\partial h_1}{\partial w_1} = \frac{\partial L}{\partial h_3} f_3' w_3 f_1' x$ and you get 2 points.

This is a partial solution: $\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial h_3}\frac{\partial h_3}{\partial h_1}\frac{\partial h_1}{\partial w_1}$, and you get 1 point.

Each of these derivatives is worth 2 points

$\frac{\partial L}{\partial w_2}$

$\frac{\partial L}{\partial w_3}$

$\frac{\partial L}{\partial w_4}$

$\frac{\partial L}{\partial b_1}$

$\frac{\partial L}{\partial b_2}$

$\frac{\partial L}{\partial b_3}$

$\frac{\partial L}{\partial x}$

2. Computational Graph (6 points)

$h = 2x + 1$

$z = x^2 + h^2$

$y = \frac{1}{1 + e^{-h}}$

(1: 1 point) Draw the computational graph based on the above three lines of code

(2: 5 points) What is the value of $\frac{\partial y}{\partial z}$ according to the computational graph ? (not pure math)

3. Why are Nonlinear Activation Functions Needed in Neural Networks ? (5 points)

(1: 1 point) Assume that we have developed an MLP. Inside the MLP, there are *N* linear layers (including hidden layers and output layer), and each of these layers is followed by ReLU activation. Now, we remove those ReLU activations from the MLP. Show that the MLP without ReLU activations is equivalent to a simple linear transform of the input, i.e., $y = Wx$

Hint:

Linear Layer-1: $y_1 = W_1 x$ (note: it can also be written as $y_1 = W_1^T x$)
Linear Layer-1 Followed by ReLU activation: $y_1 = ReLU(W_1 x)$

(2: 4 point) Assume that we have developed a 1D CNN. This CNN has *N* convolution layers (including hidden layers and output layer), and each of these layers is followed by ReLU activation. Now, we remove those ReLU activations from the CNN. Show that the CNN without ReLU activations is equivalent to a simple linear transform of the input, i.e., $y = Wx$

Hint: Can a convolution layer be written as a linear transform? If so, how? Read the lecture notes

4. Target (output) Normalization for a Neural Network (2 points)

Usually, we need to apply normalization/standardization to the inputs for classification and regression tasks, so that the input will be in the range of 0 to 1, or -1 to +1. For example, if the input is an image, then every pixel value is divided by 255, so that the pixel values of the normalized image are in the range of 0 to 1. Input normalization facilitates the convergence of training algorithms.

Do we need to apply normalization to the output ? To answer this question, let's consider the following example: assume the input is an image of a person, the output vector from a neural network has two components, $\hat{y}_{(1)}$ and $\hat{y}_{(2)}$: $\hat{y}_{(1)}$ is the monthly income (in the range of 0 to 10,000), and $\hat{y}_{(2)}$ is the age (in the range of 0 to 100). The MSE loss for a single data sample is defined as

$$L = (\hat{y}_{(1)} - y_{(1)})^2 + (\hat{y}_{(2)} - y_{(2)})^2$$

where $y_{(1)}$ and $y_{(2)}$ are ground truth values of an input data sample.

(2 points): is output (i.e., the output target $y_{(1)}, y_{(2)}$) normalization necessary for this task? Why? If it is necessary, what kind of normalization can be applied ?

5. Activation Functions for Regression (3 points)

Neural networks can be used for regression. To model nonlinear input-output relationship, a neural network needs nonlinear activation functions in the hidden layers. Usually, the output layer does not need nonlinear activation functions. However, sometimes, there are requirements for outputs. For example, if the output is the sale price of a house, then the output should be nonnegative.

Assume $\boldsymbol{z}$ is the scalar output of a network, and the network does not have nonlinear activation function in the output layer. Now, there is some requirement for output, and you decide to add a nonlinear activation function.

You design nonlinear activation functions for three different requirements:

(1: 1 point) the final output y should be nonnegative ($y \geq 0$), then what is the activation function $y = f(z)$?

(2: 1 point) the final output y should be nonpositive ($y \leq 0$), then what is the activation function $y = f(z)$?

(3: 1 point) the final output y should be $a \leq y \leq b$, then what is the activation function $y = f(z)$?

The above activation functions should be based on activation functions in PyTorch: choose appropriate activation functions or combine some activation functions or modify some activation functions available in PyTorch.
https://pytorch.org/docs/stable/nn.functional.html#non-linear-activation-functions
Do NOT use if statements, for example:

```
def activation(z):
    if z < a:     return a
    elif z > b:   return b
    else: return z
```

The above activation function is not an acceptable answer.

6. ReLU and Piecewise Linear (5 points)

(1: 4 points) Prove that an MLP with ReLU activations is a piecewise linear function of the input

(1: 1 points) Prove that a 1D CNN with ReLU activations is a piecewise linear function of the input

Note: this is not the answer "it is because ReLU is piecewise linear"

Note: it is also true that 2D/3D CNN with ReLU activations is a piecewise linear function of the input

Part-2 Programming

Programming tasks: H4P2T1.ipynb and H4P2T2.ipynb. Read H4P2_torchview.ipynb

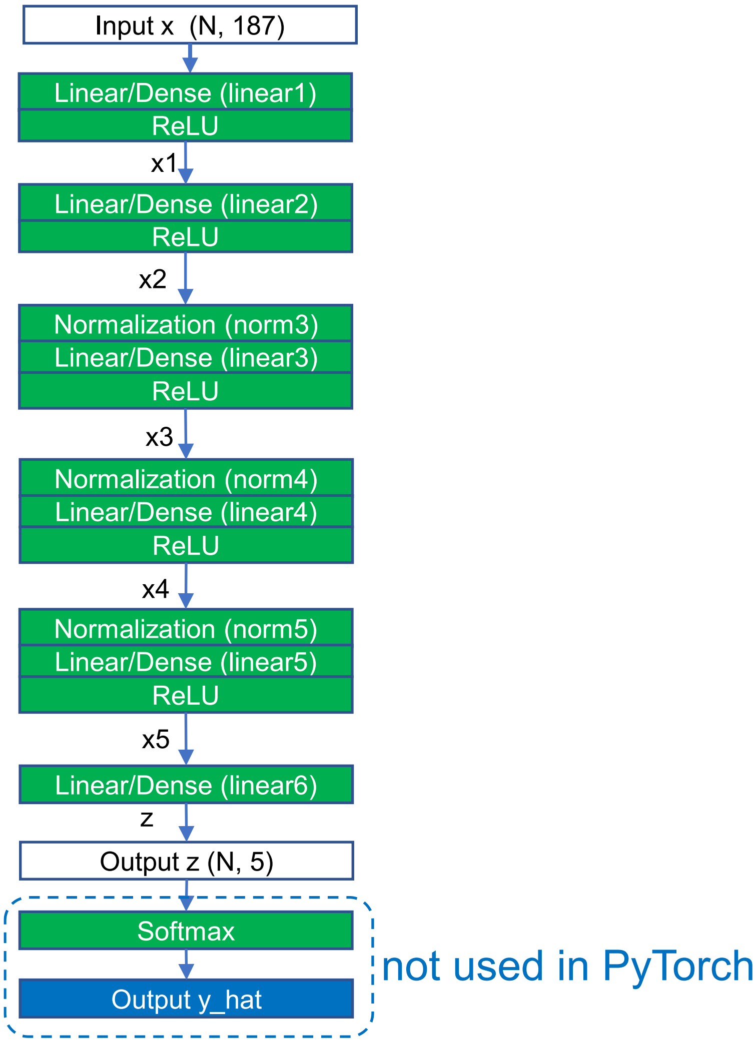
Grading : The number of points for each question/task

	Undergrad Student	Graduate Student
1. Backpropagation	14	14
2. Computational Graph	6	6
3. Nonlinear Activations in NN	extra 5 points (bonus)	5
4. Output Target Normalization	2	2
5. Activations for Regression	3	3
6. ReLU and piecewise linear	N.A.	extra 5 points (bonus)
H4P2T1 (MLP)	25	20
H4P2T2 (CNN)	50	50
Total	100 + 5	100 + 5

Read the instructions about H4P2T1 and H4P2T2 on the following pages.

In H4P2T1, you will implement an MLP for ECG signal classification according to the diagram below.

You will lose points if your network deviates from the diagram: **one deviation costs 10 points**. The following actions are deviations: miss a connection, add an extra connection, miss a layer, add an extra layer. Note: Softmax is optional in Pytorch when cross-entropy loss is used, so missing Softmax in Pytorch is not a deviation.



Please read more detailed instructions in the file H4P2T1.ipynb carefully

You will get **zero score** if the test accuracy < 80%

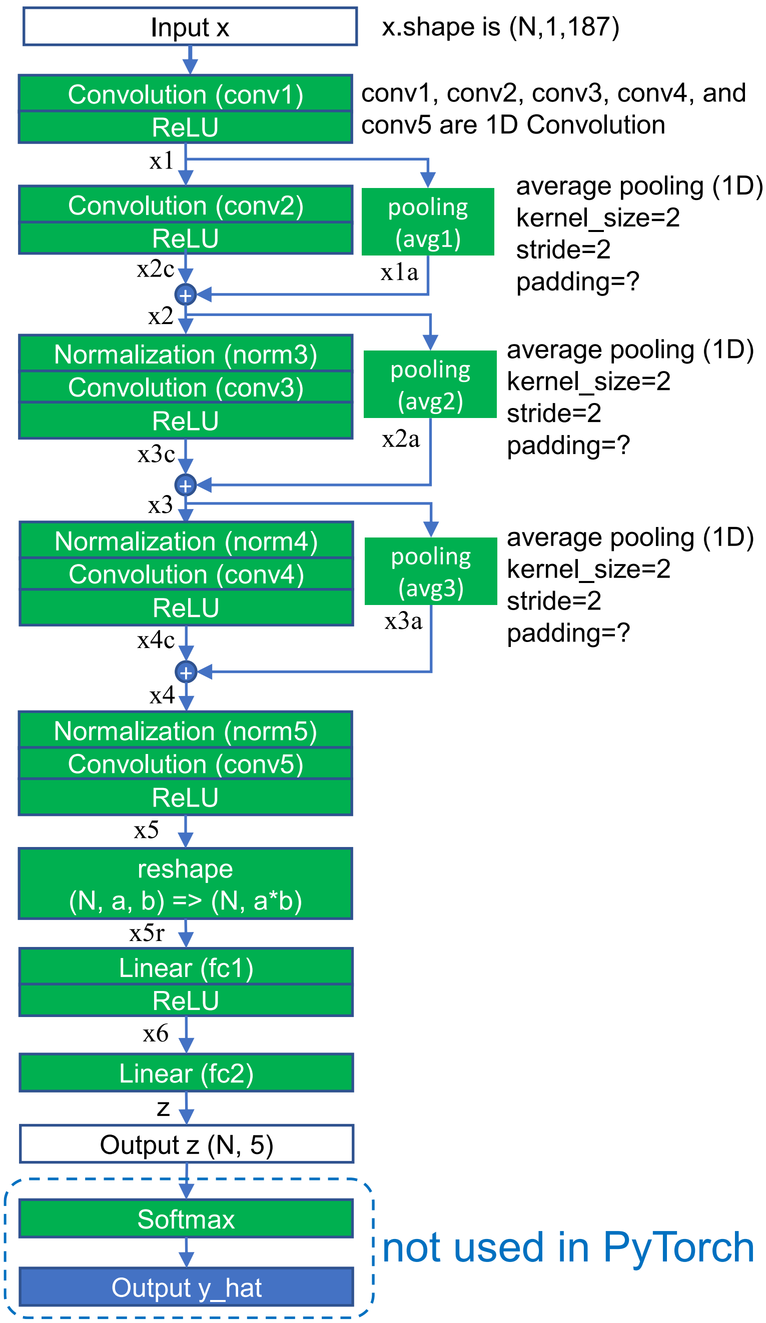
Submit H4P2T1.ipynb and H4P2T1_MLP_best.pt to Blackboard

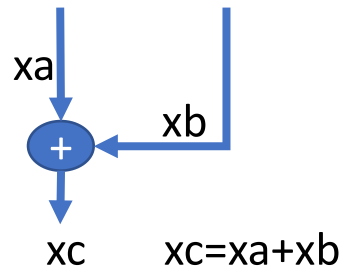
You will get zero score if the test accuracy < 80%

You will get zero score if H4P2T1.ipynb is missing

You will get zero score if H4P2T1_MLP_best.pt is missing

In H4P2T2, you will implement a CNN with residual connections for ECG signal classification according to the diagram below. You will lose points if your network deviates from the diagram: **one deviation costs 10 points**. The following actions are deviations: miss a connection, add an extra connection, miss a layer, add an extra layer, or change kernel_size /stride of the pooling layers. For example, you will lose 30 points if the three residual/skip connections are missing.





residual/skip connection

Please read more detailed instructions in the file H4P2T2.ipynb carefully

Submit H4P2T2.ipynb and H4P2T2_CNN_best.pt to Blackboard

You will get zero score if the test accuracy < 85%

You will get zero score if H4P2T2.ipynb is missing

You will get zero score if H4P2T2_CNN_best.pt is missing

Before you submit homework files, make sure you run each and every code cell of your program files.

If a code cell is supposed to generate some output (text or figure) and nothing shows up below the cell, you will lose the points of that cell.