

1

① $y_n = 1, \hat{y}_n = 0.9$, two decisions

$$L = -[y_n \ln(\hat{y}_n) + (1 - y_n) \ln(1 - \hat{y}_n)] = -[\ln(0.9)]$$

② $y_n = 2$, one hot $= [0, 0, 1]$, $\hat{y} = [0.01, 0.09, 0.9]^T$

$$L = -\sum_k y_k \ln(\hat{y}_k) = -\ln(0.9)$$

③ $f(x) = -\ln\left(\frac{1}{1+e^{-x}}\right) = \ln(1+e^{-x})$

$$f'(x) = \frac{1}{1+e^{-x}}$$

$$f''(x) = \frac{e^{-x}}{(1+e^{-x})^2} \geq 0 \text{ so it's convex}$$

④ loss is a convex composition of logs/sums of linear fncs, these operations preserve convexity.

⑤ $L = -[y \ln(\hat{y}) + (1-y) \ln(1-\hat{y})]$
 $\hat{y} = \frac{1}{1+e^{-z}}$

$$\frac{\partial L}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

$$\frac{d\hat{y}}{dz} = \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial z} = \hat{y} - y$$

⑥

$$\hat{y} = \text{softmax}$$

$$L = -\sum_i y_i \ln(\hat{y}_i)$$

$$\frac{\partial \hat{y}_i}{\partial z_j} = \hat{y}_i (\delta_{ij} - \hat{y}_j)$$

$$\frac{\partial L}{\partial z_j} = \hat{y}_j - y_j$$

2

$$MSE = \frac{1}{2N} \sum_{n=1}^N [(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2]$$

$$MAE = \frac{1}{2N} \sum_{n=1}^N (|y_1 - \hat{y}_1| + |y_2 - \hat{y}_2|)$$

$$MAPE = \frac{1}{2N} \sum_{n=1}^N \frac{|y_1 - \hat{y}_1|}{|y_1|} + \frac{|y_2 - \hat{y}_2|}{|y_2|}$$

3

1. 100, 2. 3, 3. = 1.761 4. 2, 5. 2

4

Bagging will not work if the base learners highly correlated data. Random Forest, because they are trained on bootstrap samples, reducing correlation.

5

Bagging, reduces variance as we are taking averages, boosting reduces bias by minimizing error from previous rounds, may overfit noise.

6

Stacking of same degree doesn't have benefit, as its space is very correlated. Stacking different types/structures may help capture other patterns.

7

1-1
2-2

8

1. Parameters done at beginning, such as depth of tree, configurations.
2. Validation set helps choose hyperparameters, preventing overfitting.
3. Test set is not seen until we evaluate our model.

9

1. Maximizing the margin makes the decision boundary less sensitive to noise.
2. Nonlinear SVM maximizes margin in the dimensions it creates in the kernel.
3. The kernel function computes the inner products in an above dimension.

1. Standard can make most frequent seem very accurate
Weighted accuracy is more fair gives importance to all.

2. Synthetic data generation

Yes, when our targets are unevenly distributed, we can try and transform target

$$H(p) = - \sum_{k=1}^K p_k \log p_k$$

$$1. 0 \leq p_k \leq 1 \text{ \& } \ln(p_k) \geq 0$$

$$H(p) \geq 0$$

$$2. \sum p_k = 1$$

$$L = - \sum p_k \ln(p_k) + \lambda (\sum p_k - 1)$$

$$\frac{\partial L}{\partial p_k} = -\ln(p_k) - 1 + \lambda = 0$$

$$\sum p_k \quad p_k = e^{\lambda-1}$$

$$\sum p_k = 1 \rightarrow \frac{1}{K}, \text{ entropy occurs at uniform distribution}$$