1. Topology as a Dietary choice

1.1. What is Topology?

When talking about topology, people draw cups with handles turning donuts. When I think of topology, I see nutritious food. In mathematics, topology is defined as a family of subsets of some space. We call theses subsets *open*. Open sets are like meaty, Figure 1 fruits.

Figure 1: Skinless fruits, are open set

For instance, in standard topology, the inside of a ball in 3-d is considered meaty. Contrast this with an empty sphere, a curve, or a point-these are skinny when embedded in 3-d-they have no nutritional value.

In one dimension (on a line), the inside of a segment is meaty, but a segment with endpoints is not open, because it has a rind (the endpoints).

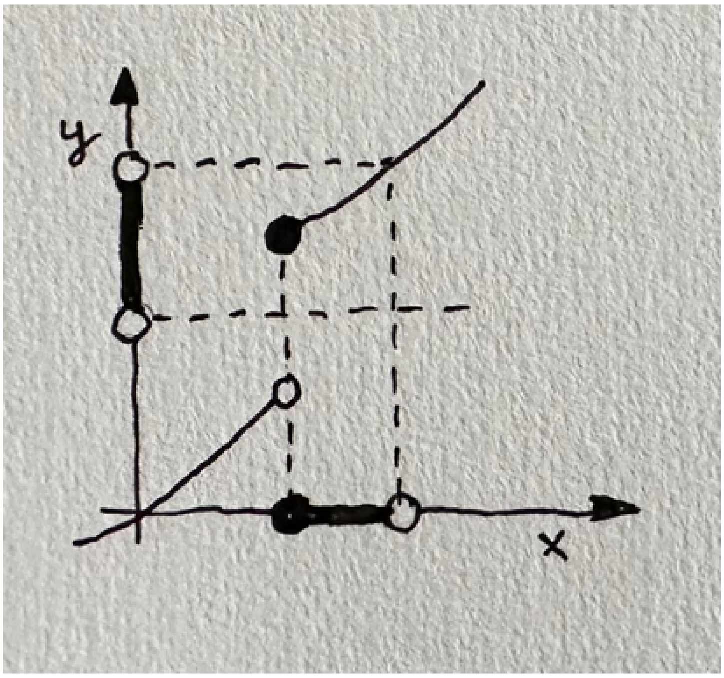
These four conditions define a topology.

1. The intersection of any two open sets is again an open set. This is what I mean by skinlessness. If you included skins, the intersection could end up skiny.
2. A union of open sets is again open. It's even more juicy, and no skin can be produced by a union. There is subtlety there: You can take a union of an arbitrary number of open sets and it's still open. But you have to be carful wiith intersections–only finite intersections are allowed. That's because by intersecting an infinite number of open sets you could end up with something very skinny-like a single point.
3. The whole space X is open. In a sense, it defines what it means to be juicy and it doesn't have a skin because it has no contact with outisde-it is its own Universe.
4. As usual, an empty set is an add item. Even though it's emppty, it's considered open. You may think of it as a union of zero open sets.

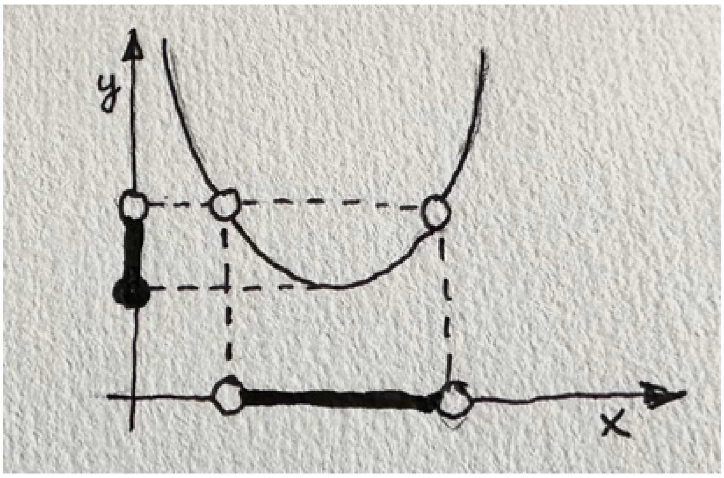
There are some extreme topologies, like the discrete topology in which all subsets are open (even individual points)a and a trivial (indiscrete) topology where only the whole space and the empty set are open. But most topologies are reasonable and adhere to our intuitions. So let's not worry about pathologies.

1.2. Continuity

Consider a function from one topological space X to another topological space Y. Intuitively, a function is continuous if it doesn't make sudden jumps. So naively you might think that a continuous function maps any open set to a point which, in most topologies, is not open.



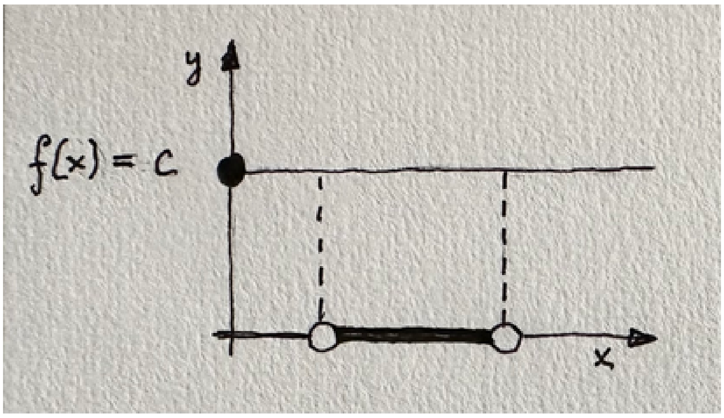
In fact any time a function stalls, or makes a turnaround (like the function $y = x^2$ at $x = 0$) you get a skinny point in its image.



The correct definition foes in the opposite direction: a function is continous if and only if the pre-image of every open set is open.

First of all, a function cannot stall or turn around in the x direction, since that would mean mapping one point to many.

Secondly, if a function makes a jump at some point x, it's possible to surround $f(x)$ with a samll open set whose counter-image contains x as its boundary.



It's also possible to define a continous function as a pair of functions. One function f is the usual mapping of points from X to Y. The other function g maps open sets in Y to open sets in X. The pair (f, g) defines a continuous function if for all points $x \in X$ and open sets O in Y we have the following equivalence:

$$f(x) \in O \Leftrightarrow x \in g(O)$$

The left-hand side tells us that x is the pre-image of O under f. The right-hand side tells us that g maps O to thus preimage. This formula looks a bit like an adjunction between f and g. It's an example of a more general notion of Chu constructions.

Finally, the cups and donuts magic trick uses invertible continous functions called homeomorphisms to deform shapes without tearing or gluing them.