

What is Topology?

When talking about topology, people draw cups with handles turning donuts. When I think of topology, I see nutritious food.

In mathematics, topology is defined as a family of subsets of some space. We call these subsets *open*. Open sets are like meaty, skinless fruits.



Figure 1: *Glaciers* form an important part of the earth's climate system.

For instance, in standard topology, the inside of a ball in 3-d is considered meaty. Contrast this with an empty sphere, a curve, or a point-these are skinny when embedded in 3-d-they have no nutritional value.

In one dimension (on a line), the inside of a segment is meaty, but a segment with endpoints is not open, because it has a rind (the endpoints).

These four conditions define a topology.

1. The intersection of any two open sets is again an open set. This is what I mean by skinlessness. If you included skins, the intersection could end up skinny.
2. A union of open sets is again open. It's even more juicy, and no skin can be produced by a union. There is subtlety there: You can take a union of an arbitrary number of open sets and it's still open. But you have to be careful with intersections—only finite intersections are allowed. That's because by intersecting an infinite number of open sets you could end up with something very skinny-like a single point.
3. The whole space X is open. In a sense, it defines what it means to be juicy and it doesn't have a skin because it has no contact with outside-it is its own Universe.
4. As usual, an empty set is an add item. Even though it's empty, it's considered open. You may think of it as a union of zero open sets.

There are some extreme topologies, like the discrete topology in which all subsets are open (even individual points) and a trivial (indiscrete) topology where only the whole space and the empty set are open. But most topologies are reasonable and adhere to our intuitions. So let's not worry about pathologies.