gcd-erasing ● EN

Erasing with GCD (gcd-erasing)

Dóra, a mathematics teacher, has just taught her students how to compute the greatest common divisor (GCD) of multiple numbers. Now, she wants to challenge them with an interesting game.

She writes the numbers 2, 3, ..., N on the board in some order of her choice. The students must then erase the numbers following these rules:

- In each step, they can remove one or more consecutive numbers from either the left or the right end of the sequence.
- The group of numbers selected for removal can only be erased if their GCD is greater than 1.



Figure 1: Dóra teaching her class.

Suppose N = 6 and Dóra writes the following sequence on the board:

2, 4, 6, 3, 5

In the first step, the students could erase the following:

- $2 \text{ (since } \gcd(2) = 2),$
- 2, 4 (since gcd(2, 4) = 2),
- 2, 4, 6 (since gcd(2, 4, 6) = 2), or
- $5 \text{ (since } \gcd(5) = 5).$

They cannot erase 3 and 5 together, since gcd(3,5) = 1. They cannot erase 6 and 3 together, as they are not at the left or right end of the sequence.

One possible way to erase the entire sequence in three steps is as follows:

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- 1. Erase 2,4 from the left end of the sequence.
- 2. Erase 5 from the right end of the sequence.
- 3. Erase 6, 3, which is the remaining sequence at this point.

It can be observed that for this sequence, and any reordering of these numbers, the minimum number of steps required to clear the board is three.

Dóra wants to finish the class as soon as possible, so she will arrange the numbers in an order that allows the students to erase them in the **fewest number of steps**. The students, being smart, will always erase the numbers optimally.

Given N, determine the minimum number of steps required to erase all numbers from the board.

Among the attachments of this task you may find a template file gcd-erasing.* with a sample incomplete implementation.

Input

The input consists of a single integer N.

Output

You need to write a single line with an integer: the minimum number of steps if both Dóra and the students act optimally.

Constraints

• $2 \le N \le 100000$.

Scoring

Your program will be tested against several test cases grouped in subtasks. In order to obtain the score of a subtask, your program needs to correctly solve all of its test cases.

- Subtask 1 (0 points) Examples.

- Subtask 2 (6 points) $N \le 6$.

- Subtask 3 (42 points) $N \le 1000$.

- Subtask 4 (52 points) No additional limitations.

Examples

8|8|8|**8**|

input	output
6	3
10	4

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Explanation

The first sample case is explained in the statement.

In the **second sample case**, a possible optimal order is

and the students can erase the numbers in the following 4 groups: (8,2,4), (10,5), (7), (3,6,9).

It can be proven that it is impossible to complete the process in 3 steps, no matter how the numbers $2, 3, \ldots, 10$ are ordered.

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