Zero-Noise Extrapolation (ZNE) Interactive Demo

An end-to-end demonstrator for ZNE error mitigation.
Inspired by Temme, Bravyi & Gambetta (arXiv:1612.02058).
Built with Mathematica 14.3 and Wolfram QuantumFramework.

References

1. Temme, Bravyi, Gambetta – "Error Mitigation for Short-Depth Quantum Circuits" (arXiv:1612.02058)
   https://arxiv.org/pdf/1612.02058

2. Kandala et al. – "Error mitigation extends the computational reach..." (arXiv:1805.04492)
   https://arxiv.org/abs/1805.04492

Environment

In[1]:=
```
Print["Mathematica Version: ", $VersionNumber]
```
```
Mathematica Version: 14.3
```

In[2]:=
```
Print["System: ", $SystemID]
```
```
System: Windows-x86-64
```

In[3]:=
```
Print["QuantumFramework: ", PacletFind["Wolfram/QuantumFramework"]]
```
```
QuantumFramework: {PacletObject[ Name: Wolfram/QuantumFramework
                                 Version: 1.6.5 ]}
```

Load ZNE Package

In[4]:=
```
Get[FileNameJoin[{NotebookDirectory[], "..", "src", "ZNEDemo.wl"}]];
Print["ZNE package loaded."]
```
```
ZNE package loaded.
```

Interactive ZNE Demo

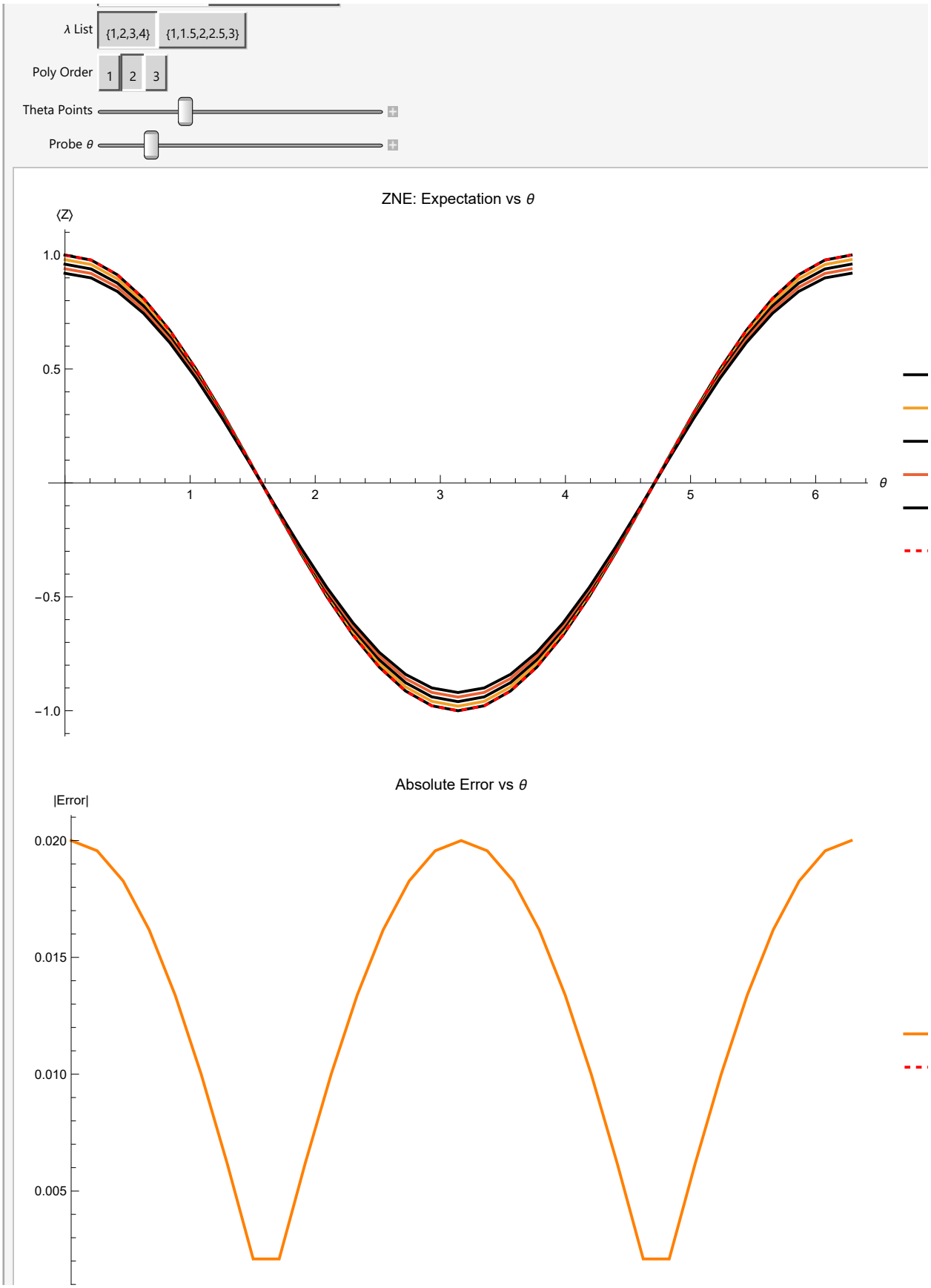
Use the controls below to explore how ZNE mitigates depolarizing noise on a Ry(θ) circuit.

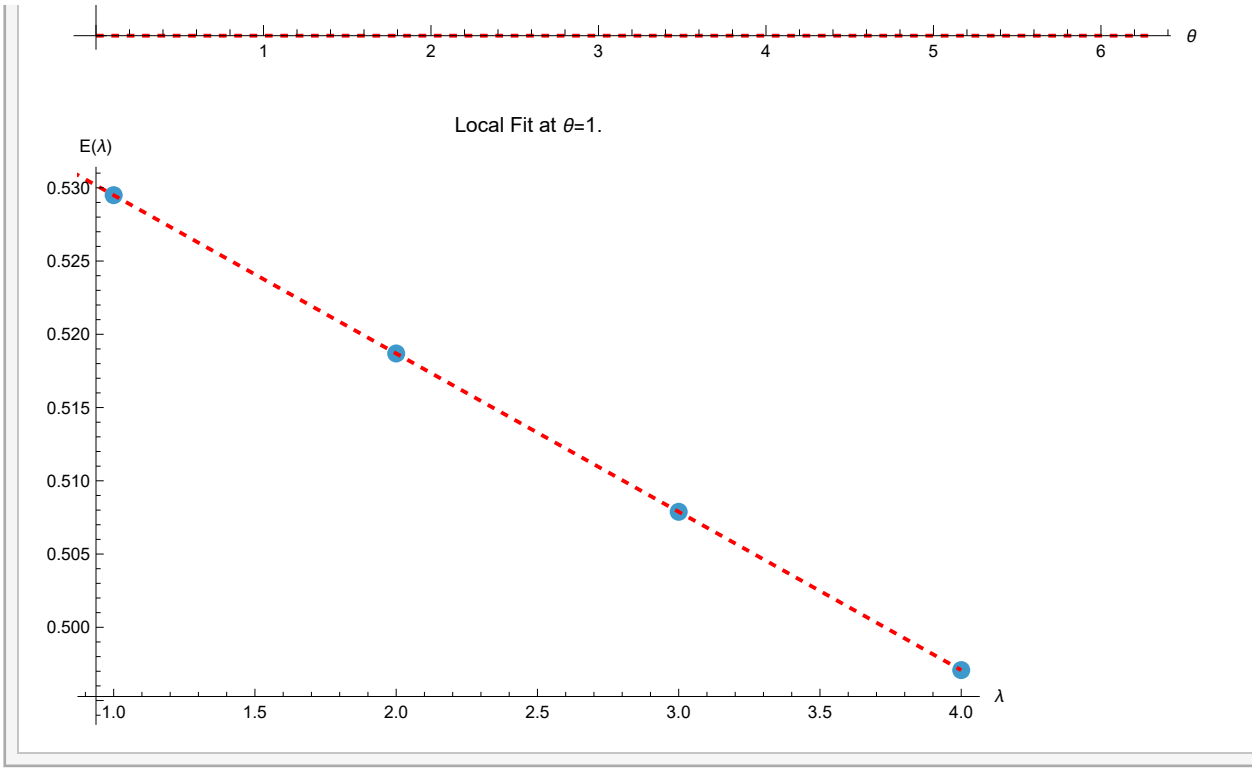
Three plots are shown:

1. Main: Ideal vs Noisy vs ZNE expectation curves
2. Error: Absolute error of Noisy and ZNE vs Ideal
3. Fit: Local polynomial extrapolation at the probe angle

In[5]:=
```
Manipulate[
Module[{sweepResult, idealPts, znePts, mainPlot, errorPlot, noisyErrors, zneErrors,
   fitPlot, fitData, fitLambdas, fitMeans, polyOrder2, fit, lam, extrapolated},
sweepResult = ZNE`GenerateSweep[
N[Range[0, 2 Pi, 2 Pi / nPts]], lambdaList, polyOrd,
"NoiseModel" → noiseModel, "BaseNoise" → p0, "Shots" → Infinity];
idealPts = sweepResult["IdealData"]; znePts = sweepResult["ZNEData"];
mainPlot = Show[
ListLinePlot[Prepend[Values[sweepResult["NoisyData"]], idealPts],
PlotLegends → Prepend[Table["λ=" <> ToString[l], {l, lambdaList}], "Ideal"],
PlotLabel → "ZNE: Expectation vs θ", AxesLabel → {"θ", "⟨Z⟩"},
     ImageSize → 600, PlotStyle → {{Thick, Black}, Automatic}],
ListLinePlot[znePts, PlotStyle → {Thick, Red, Dashed}, PlotLegends → {"ZNE"}]];
noisyErrors = MapThread[Function[{ideal, noisy}, {ideal[[1]], Abs[noisy[[2]] - ideal[[2]]]}],
{idealPts, sweepResult["NoisyData", First[lambdaList]]}];
  zneErrors = MapThread[
    Function[{ideal, zne}, {ideal[[1]], Abs[zne[[2]] - ideal[[2]]]}], {idealPts, znePts}];
errorPlot = ListLinePlot[{noisyErrors, zneErrors},
PlotLegends → {"Noisy (λ=1)", "ZNE"},
    PlotLabel → "Absolute Error vs θ", AxesLabel → {"θ", "|Error|"},
    ImageSize → 600, PlotStyle → {{Thick, Orange}, {Thick, Red, Dashed}}];
fitLambdas = N[lambdaList];
  fitMeans = Table[ZNE`NoisyExpectation[thetaProbe,
     l, "NoiseModel" → noiseModel, "BaseNoise" → p0], {l, fitLambdas}];
  fitData = Transpose[{fitLambdas, fitMeans}];
  polyOrder2 = Min[polyOrd, Length[fitLambdas] - 1];
  fit = Fit[fitData, Table[lam^k, {k, 0, polyOrder2}], lam];
  extrapolated = fit /. lam → 0;
fitPlot = Show[
ListPlot[fitData, PlotStyle → {PointSize[0.02]},
     PlotLabel → "Local Fit at θ=" <> ToString[NumberForm[thetaProbe, 3]],
     AxesLabel → {"λ", "E(λ)"}, ImageSize → 500],
Plot[fit, {lam, 0, Max[fitLambdas]}, PlotStyle → {Red, Dashed}],
ListPlot[{{0, extrapolated}}, PlotStyle → {Red, PointSize[0.03]}],
Epilog →
     {Text["ZNE = " <> ToString[NumberForm[extrapolated, 4]], {0.5, extrapolated}]}];
Column[{mainPlot, errorPlot, fitPlot}, Spacings → 2]],
{{p0, 0.02, "Base Noise"}, 0, 0.1, 0.005},
 {{noiseModel, "DepolarizingEnd", "Noise Model"},
  {"DepolarizingEnd", "DepolarizingPerGate"}}, {{lambdaList, {1, 2, 3, 4}, "λ List"},
  {{1, 2, 3, 4} → "{1,2,3,4}", {1, 1.5, 2, 2.5, 3} → "{1,1.5,2,2.5,3}"}},
 {{polyOrd, 2, "Poly Order"}, {1, 2, 3}}, {{nPts, 30, "Theta Points"}, 10, 80, 5},
 {{thetaProbe, 1.0, "Probe θ"}, 0.1, 6.0, 0.1}]
```

Out[5]=


λ List
{1,2,3,4}
{1,1.5,2,2.5,3}
Poly Order
1
2
3
Theta Points
Probe θ
ZNE: Expectation vs θ
⟨Z⟩
1.0
0.5
−0.5
−1.0
1
2
3
4
5
6
θ
Absolute Error vs θ
|Error|
0.020
0.015
0.010
0.005



Validation Checks

Automated pass/fail acceptance checks.

In[6]:=
```
Module[{thetaTest, lambdas},
 thetaTest = N[Range[0.1, 2 Pi, 0.3]]; lambdas = {1, 2, 3, 4};
 Module[{sweep, maxErr},
   sweep = ZNE`GenerateSweep[thetaTest, lambdas, 2, "BaseNoise" → 0.001];
   maxErr = Max[Abs[sweep["ZNEData", All, 2] - sweep["IdealData", All, 2]]];
   Print["Check 1 (low noise): max|ZNE-Ideal| = ",
    maxErr, If[maxErr < 0.01, " PASS", " FAIL"]]];
 Module[{sweep, noisyErr, zneErr, improvement},
   sweep = ZNE`GenerateSweep[thetaTest, lambdas, 2, "BaseNoise" → 0.05];
   noisyErr = Mean[Abs[sweep["NoisyData", 1, All, 2] - sweep["IdealData", All, 2]]];
   zneErr = Mean[Abs[sweep["ZNEData", All, 2] - sweep["IdealData", All, 2]]];
   improvement = (noisyErr - zneErr) / noisyErr;
   Print["Check 2: Noisy MAE=",
    NumberForm[noisyErr, 4], " | ZNE MAE=", NumberForm[zneErr, 4], " | ",
    NumberForm[100 improvement, 3], "%", If[zneErr < noisyErr, " PASS", " FAIL"]]];
 Module[{sweepE, sweepP, endErr, pgErr}, sweepE = ZNE`GenerateSweep[
     thetaTest, lambdas, 2, "NoiseModel" → "DepolarizingEnd", "BaseNoise" → 0.03];
   sweepP = ZNE`GenerateSweep[thetaTest, lambdas, 2,
     "NoiseModel" → "DepolarizingPerGate", "BaseNoise" → 0.03];
   endErr = Mean[Abs[sweepE["NoisyData", 1, All, 2] - sweepE["IdealData", All, 2]]];
   pgErr = Mean[Abs[sweepP["NoisyData", 1, All, 2] - sweepP["IdealData", All, 2]]];
   Print["Check 3: End MAE=", NumberForm[endErr, 4], " | PerGate MAE=",
    NumberForm[pgErr, 4], If[pgErr > endErr, " PASS", " Equal (single gate)"]]]]
```

```
Check 1 (low noise): max|ZNE-Ideal| =
 Abs[-<|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
         {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
         {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
         {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
         {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
       NoisyData → <|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
           {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
           {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
           {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
           {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
         2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
           {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
           {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
           {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
           {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
         3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
           {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
           {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
           {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
           {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
         4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078},
           {1.3, 0.246099}, {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421},
           {2.5, -0.737052}, {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454},
           {3.7, -0.780252}, {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
```

```
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[IdealData, All, 2] +
  ⟨|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    NoisyData → ⟨|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
        {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
        {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
        {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
        {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
      2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
        {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
        {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
        {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
        {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
      3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
        {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078},
        {1.3, 0.246099}, {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421},
        {2.5, -0.737052}, {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454},
        {3.7, -0.780252}, {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[ZNEData, All, 2]]
If[Abs[-⟨|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
        {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
        {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
        {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
        {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
      NoisyData → ⟨|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
          {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
          {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
          {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
          {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
        2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
```

```
    {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
    {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
    {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
    {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
  3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
    {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
    {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
    {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
    {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
  4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
    {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
    {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
    {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
    {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
 ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
   {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
   {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
   {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
   {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
 Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
   LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[IdealData, All, 2] +
⟨|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
   {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
   {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
   {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
   {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
 NoisyData → ⟨|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
    {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
    {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
    {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
    {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
  2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
    {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
    {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
    {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
    {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
  3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
    {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
    {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
    {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
    {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
  4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078},
    {1.3, 0.246099}, {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421},
    {2.5, -0.737052}, {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454},
    {3.7, -0.780252}, {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
    {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
 ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
   {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
   {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
   {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
   {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
```

```
    Meta → ⟨| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞, LambdaList → {1, 2, 3, 4},
      Order → 2, NumPoints → 21 |⟩ |⟩ [ZNEData, All, 2]] < 0.01,  PASS,  FAIL]

Check 2: Noisy MAE=
 Mean[Abs[- ⟨| IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
         {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011}, {2.8, -0.9422},
         {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6,
          -0.1122}, {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
       NoisyData → ⟨| 1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
           {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
           {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
            -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
         2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
           {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
           {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
            -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
         3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
           {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
           {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
            -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
         4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
           {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
           {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
            -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}} |⟩,
       ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
         {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
         {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
         {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
         {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
       Meta → ⟨| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
         LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21 |⟩ |⟩ [IdealData, All, 2] +
     ⟨| IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403},
         {1.3, 0.2675}, {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885},
         {2.5, -0.8011}, {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668},
         {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122},
         {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
       NoisyData → ⟨| 1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
           {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
           {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
            -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
         2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
           {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
           {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
            -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
         3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
           {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
           {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
            -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
         4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
           {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
           {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
```

```
        −0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|⟩,
    ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
      {1.6, −0.0292}, {1.9, −0.3233}, {2.2, −0.5885}, {2.5, −0.8011},
      {2.8, −0.9422}, {3.1, −0.9991}, {3.4, −0.9668}, {3.7, −0.8481},
      {4., −0.6536}, {4.3, −0.4008}, {4.6, −0.1122}, {4.9, 0.1865},
      {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[NoisyData, 1, All, 2]]]
| ZNE MAE=Mean[Abs[−⟨|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648},
      {1., 0.5403}, {1.3, 0.2675}, {1.6, −0.0292}, {1.9, −0.3233}, {2.2, −0.5885},
      {2.5, −0.8011}, {2.8, −0.9422}, {3.1, −0.9991}, {3.4, −0.9668},
      {3.7, −0.8481}, {4., −0.6536}, {4.3, −0.4008}, {4.6, −0.1122},
      {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
        {1.6, −0.02862}, {1.9, −0.3168}, {2.2, −0.5767}, {2.5, −0.7851}, {2.8, −0.9234},
        {3.1, −0.9792}, {3.4, −0.9475}, {3.7, −0.8311}, {4., −0.6406}, {4.3, −0.3928}, {4.6,
         −0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
      2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
        {1.6, −0.02803}, {1.9, −0.3104}, {2.2, −0.565}, {2.5, −0.7691}, {2.8, −0.9045},
        {3.1, −0.9592}, {3.4, −0.9281}, {3.7, −0.8142}, {4., −0.6275}, {4.3, −0.3848}, {4.6,
         −0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
      3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
        {1.6, −0.02745}, {1.9, −0.3039}, {2.2, −0.5532}, {2.5, −0.7531}, {2.8, −0.8857},
        {3.1, −0.9392}, {3.4, −0.9088}, {3.7, −0.7972}, {4., −0.6144}, {4.3, −0.3768}, {4.6,
         −0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
      4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
        {1.6, −0.02686}, {1.9, −0.2974}, {2.2, −0.5414}, {2.5, −0.7371}, {2.8, −0.8668},
        {3.1, −0.9192}, {3.4, −0.8895}, {3.7, −0.7803}, {4., −0.6014}, {4.3, −0.3687}, {4.6,
         −0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|⟩,
    ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
      {1.6, −0.0292}, {1.9, −0.3233}, {2.2, −0.5885}, {2.5, −0.8011},
      {2.8, −0.9422}, {3.1, −0.9991}, {3.4, −0.9668}, {3.7, −0.8481},
      {4., −0.6536}, {4.3, −0.4008}, {4.6, −0.1122}, {4.9, 0.1865},
      {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[IdealData, All, 2] +
  ⟨|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403},
      {1.3, 0.2675}, {1.6, −0.0292}, {1.9, −0.3233}, {2.2, −0.5885},
      {2.5, −0.8011}, {2.8, −0.9422}, {3.1, −0.9991}, {3.4, −0.9668},
      {3.7, −0.8481}, {4., −0.6536}, {4.3, −0.4008}, {4.6, −0.1122},
      {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
        {1.6, −0.02862}, {1.9, −0.3168}, {2.2, −0.5767}, {2.5, −0.7851}, {2.8, −0.9234},
        {3.1, −0.9792}, {3.4, −0.9475}, {3.7, −0.8311}, {4., −0.6406}, {4.3, −0.3928}, {4.6,
         −0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
      2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
        {1.6, −0.02803}, {1.9, −0.3104}, {2.2, −0.565}, {2.5, −0.7691}, {2.8, −0.9045},
        {3.1, −0.9592}, {3.4, −0.9281}, {3.7, −0.8142}, {4., −0.6275}, {4.3, −0.3848}, {4.6,
         −0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
      3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
        {1.6, −0.02745}, {1.9, −0.3039}, {2.2, −0.5532}, {2.5, −0.7531}, {2.8, −0.8857},
```

```
{3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
  -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
 4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
  {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
  {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
  -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|>,
ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
  {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
  {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
  {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
  {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
  LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[ZNEData, All, 2]]] |
(100 (-Mean[Abs[-⟨|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
    {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942}, {3.1,
     -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6,
     -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
   NoisyData → ⟨|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
     {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923},
     {3.1, -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393}, {4.6,
      -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
    2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
     {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905},
     {3.1, -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385}, {4.6,
      -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}}, 3 →
    {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
     {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886},
     {3.1, -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377}, {4.6,
      -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
    4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
     {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867},
     {3.1, -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369},
     {4.6, -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815},
     {6.1, 0.905}}|>, ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54},
    {1.3, 0.267}, {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801},
    {2.8, -0.942}, {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3,
     -0.401}, {4.6, -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886},
    {6.1, 0.983}}, Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots →
     ∞, LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[IdealData, All, 2] +
  ⟨|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
    {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
    {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6,
     -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
   NoisyData → ⟨|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
     {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923},
     {3.1, -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393}, {4.6,
      -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
    2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
     {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905},
     {3.1, -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385},
     {4.6, -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}},
```

```
        3 → {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
          {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886},
          {3.1, -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377}, {4.6,
           -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
        4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
          {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867},
          {3.1, -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369}, {4.6,
           -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815}, {6.1, 0.905}}|>,
      ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
        {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
        {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6,
         -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
      Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
        LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[ZNEData, All, 2]]] +
Mean[Abs[-<|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
        {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
        {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6,
         -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
      NoisyData → <|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
          {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923},
          {3.1, -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393}, {4.6,
           -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
        2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
          {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905},
          {3.1, -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385},
          {4.6, -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}},
        3 → {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
          {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886},
          {3.1, -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377}, {4.6,
           -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
        4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
          {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867},
          {3.1, -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369}, {4.6,
           -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815}, {6.1, 0.905}}|>,
      ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
        {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
        {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6,
         -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
      Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
        LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[IdealData, All, 2] +
  <|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
      {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
      {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401},
      {4.6, -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
    NoisyData → <|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
        {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923}, {3.1,
         -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393}, {4.6,
         -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
      2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
        {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905}, {3.1,
         -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385}, {4.6,
```

```
        -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}},
      3 → {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
        {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886}, {3.1,
        -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377}, {4.6,
        -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
      4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
        {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867}, {3.1,
        -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369}, {4.6,
        -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815}, {6.1, 0.905}}|>,
    ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
      {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801}, {2.8, -0.942},
      {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848}, {4., -0.654}, {4.3, -0.401},
      {4.6, -0.112}, {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
    Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[NoisyData, 1, All, 2]]]])) /
Mean[Abs[-<|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54},
      {1.3, 0.267}, {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589},
      {2.5, -0.801}, {2.8, -0.942}, {3.1, -0.999}, {3.4, -0.967},
      {3.7, -0.848}, {4., -0.654}, {4.3, -0.401}, {4.6, -0.112},
      {4.9, 0.187}, {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
    NoisyData → <|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
        {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923},
        {3.1, -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393},
        {4.6, -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
      2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
        {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905},
        {3.1, -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385},
        {4.6, -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}},
      3 → {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
        {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886},
        {3.1, -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377},
        {4.6, -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
      4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
        {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867},
        {3.1, -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369},
        {4.6, -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815}, {6.1, 0.905}}|>,
    ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
      {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801},
      {2.8, -0.942}, {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848},
      {4., -0.654}, {4.3, -0.401}, {4.6, -0.112}, {4.9, 0.187},
      {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
    Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[IdealData, All, 2] +
  <|IdealData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
      {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801},
      {2.8, -0.942}, {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848},
      {4., -0.654}, {4.3, -0.401}, {4.6, -0.112}, {4.9, 0.187},
      {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
    NoisyData → <|1 → {{0.1, 0.975}, {0.4, 0.903}, {0.7, 0.75}, {1., 0.529}, {1.3, 0.262},
        {1.6, -0.0286}, {1.9, -0.317}, {2.2, -0.577}, {2.5, -0.785}, {2.8, -0.923},
        {3.1, -0.979}, {3.4, -0.947}, {3.7, -0.831}, {4., -0.641}, {4.3, -0.393},
```

```
      {4.6, -0.11}, {4.9, 0.183}, {5.2, 0.459}, {5.5, 0.694}, {5.8, 0.868}, {6.1, 0.964}},
    2 → {{0.1, 0.955}, {0.4, 0.884}, {0.7, 0.734}, {1., 0.519}, {1.3, 0.257},
      {1.6, -0.028}, {1.9, -0.31}, {2.2, -0.565}, {2.5, -0.769}, {2.8, -0.905},
      {3.1, -0.959}, {3.4, -0.928}, {3.7, -0.814}, {4., -0.627}, {4.3, -0.385},
      {4.6, -0.108}, {4.9, 0.179}, {5.2, 0.45}, {5.5, 0.68}, {5.8, 0.85}, {6.1, 0.944}},
    3 → {{0.1, 0.935}, {0.4, 0.866}, {0.7, 0.719}, {1., 0.508}, {1.3, 0.251},
      {1.6, -0.0274}, {1.9, -0.304}, {2.2, -0.553}, {2.5, -0.753}, {2.8, -0.886},
      {3.1, -0.939}, {3.4, -0.909}, {3.7, -0.797}, {4., -0.614}, {4.3, -0.377},
      {4.6, -0.105}, {4.9, 0.175}, {5.2, 0.44}, {5.5, 0.666}, {5.8, 0.832}, {6.1, 0.924}},
    4 → {{0.1, 0.915}, {0.4, 0.847}, {0.7, 0.704}, {1., 0.497}, {1.3, 0.246},
      {1.6, -0.0269}, {1.9, -0.297}, {2.2, -0.541}, {2.5, -0.737}, {2.8, -0.867},
      {3.1, -0.919}, {3.4, -0.889}, {3.7, -0.78}, {4., -0.601}, {4.3, -0.369},
      {4.6, -0.103}, {4.9, 0.172}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.815}, {6.1, 0.905}}|>,
  ZNEData → {{0.1, 0.995}, {0.4, 0.921}, {0.7, 0.765}, {1., 0.54}, {1.3, 0.267},
    {1.6, -0.0292}, {1.9, -0.323}, {2.2, -0.589}, {2.5, -0.801},
    {2.8, -0.942}, {3.1, -0.999}, {3.4, -0.967}, {3.7, -0.848},
    {4., -0.654}, {4.3, -0.401}, {4.6, -0.112}, {4.9, 0.187},
    {5.2, 0.469}, {5.5, 0.709}, {5.8, 0.886}, {6.1, 0.983}},
  Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
    LambdaList → {1, 2, 3, 4}, Order → 2,
    NumPoints → 21|>|>[NoisyData, 1, All, 2]]]%
If[Mean[Abs[-<|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842},
      {1., 0.540302}, {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    NoisyData → <|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
        {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
        {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
        {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
        {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
      2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
        {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
        {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
        {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
        {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
      3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
        {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
        {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
        {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
        {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|>,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
```

```
    Meta → ‹| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
       LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21 |› |› [IdealData, All, 2] +
  ‹| IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    NoisyData → ‹| 1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
        {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
        {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
        {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
        {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
      2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
        {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
        {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
        {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
        {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
      3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
        {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
        {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
        {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
        {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}} |›,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → ‹| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21 |› |› [ZNEData, All, 2]]] <
Mean[Abs[- ‹| IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842},
       {1., 0.540302}, {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
       {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
       {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
       {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
     NoisyData → ‹| 1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
         {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
         {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
         {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
         {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
       2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
         {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
         {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
         {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
         {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
       3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
         {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
```

```
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
        {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
        {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
        {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|>,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[IdealData, All, 2] +
  <|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    NoisyData → <|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
        {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
        {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
        {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
        {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
      2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
        {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
        {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
        {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
        {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
      3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
        {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
        {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
        {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
        {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|>,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[
   NoisyData, 1, All, 2]]], PASS, FAIL]
Check 3: End MAE=
 Mean[Abs[-<|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
```

```
    {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011}, {2.8, -0.9422},
    {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6,
     -0.1122}, {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}}},
  NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
      {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
      {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
       -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
    2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
      {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
      {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
       -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
    3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
      {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
      {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
       -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
    4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
      {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
      {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
       -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|⟩,
  ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
    {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
    {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
    {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
    {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
  Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
    LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[IdealData, All, 2] +
⟨|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403},
    {1.3, 0.2675}, {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885},
    {2.5, -0.8011}, {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668},
    {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122},
    {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
  NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
      {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
      {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
       -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
    2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
      {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
      {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
       -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
    3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
      {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
      {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
       -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
    4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
      {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
      {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
       -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|⟩,
  ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
    {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
    {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
    {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
```

```
      {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[NoisyData, 1, All, 2]]]
| PerGate MAE=Mean[Abs[-⟨|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648},
      {1., 0.5403}, {1.3, 0.2675}, {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885},
      {2.5, -0.8011}, {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668},
      {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122},
      {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
        {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
        {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
         -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
      2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
        {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
        {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
         -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
      3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
        {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
        {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
         -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
      4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
        {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
        {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
         -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|⟩,
    ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
      {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
      {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
      {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
      {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[IdealData, All, 2] +
  ⟨|IdealData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403},
      {1.3, 0.2675}, {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885},
      {2.5, -0.8011}, {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668},
      {3.7, -0.8481}, {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122},
      {4.9, 0.1865}, {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    NoisyData → ⟨|1 → {{0.1, 0.9751}, {0.4, 0.9026}, {0.7, 0.7495}, {1., 0.5295}, {1.3, 0.2621},
        {1.6, -0.02862}, {1.9, -0.3168}, {2.2, -0.5767}, {2.5, -0.7851}, {2.8, -0.9234},
        {3.1, -0.9792}, {3.4, -0.9475}, {3.7, -0.8311}, {4., -0.6406}, {4.3, -0.3928}, {4.6,
         -0.1099}, {4.9, 0.1828}, {5.2, 0.4591}, {5.5, 0.6945}, {5.8, 0.8678}, {6.1, 0.9636}},
      2 → {{0.1, 0.9552}, {0.4, 0.8842}, {0.7, 0.7342}, {1., 0.5187}, {1.3, 0.2568},
        {1.6, -0.02803}, {1.9, -0.3104}, {2.2, -0.565}, {2.5, -0.7691}, {2.8, -0.9045},
        {3.1, -0.9592}, {3.4, -0.9281}, {3.7, -0.8142}, {4., -0.6275}, {4.3, -0.3848}, {4.6,
         -0.1077}, {4.9, 0.1791}, {5.2, 0.4498}, {5.5, 0.6803}, {5.8, 0.8501}, {6.1, 0.9439}},
      3 → {{0.1, 0.9353}, {0.4, 0.8658}, {0.7, 0.719}, {1., 0.5079}, {1.3, 0.2514},
        {1.6, -0.02745}, {1.9, -0.3039}, {2.2, -0.5532}, {2.5, -0.7531}, {2.8, -0.8857},
        {3.1, -0.9392}, {3.4, -0.9088}, {3.7, -0.7972}, {4., -0.6144}, {4.3, -0.3768}, {4.6,
         -0.1054}, {4.9, 0.1753}, {5.2, 0.4404}, {5.5, 0.6661}, {5.8, 0.8324}, {6.1, 0.9243}},
      4 → {{0.1, 0.9154}, {0.4, 0.8474}, {0.7, 0.7037}, {1., 0.4971}, {1.3, 0.2461},
        {1.6, -0.02686}, {1.9, -0.2974}, {2.2, -0.5414}, {2.5, -0.7371}, {2.8, -0.8668},
        {3.1, -0.9192}, {3.4, -0.8895}, {3.7, -0.7803}, {4., -0.6014}, {4.3, -0.3687}, {4.6,
```

```
        -0.1032}, {4.9, 0.1716}, {5.2, 0.431}, {5.5, 0.652}, {5.8, 0.8147}, {6.1, 0.9046}}|>,
    ZNEData → {{0.1, 0.995}, {0.4, 0.9211}, {0.7, 0.7648}, {1., 0.5403}, {1.3, 0.2675},
      {1.6, -0.0292}, {1.9, -0.3233}, {2.2, -0.5885}, {2.5, -0.8011},
      {2.8, -0.9422}, {3.1, -0.9991}, {3.4, -0.9668}, {3.7, -0.8481},
      {4., -0.6536}, {4.3, -0.4008}, {4.6, -0.1122}, {4.9, 0.1865},
      {5.2, 0.4685}, {5.5, 0.7087}, {5.8, 0.8855}, {6.1, 0.9833}},
    Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[NoisyData, 1, All, 2]]]
If[Mean[Abs[-<|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842},
        {1., 0.540302}, {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
        {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
        {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
        {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
      NoisyData → <|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
          {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
          {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
          {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
          {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
        2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
          {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
          {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
          {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
          {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
        3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
          {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
          {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
          {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
          {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
        4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
          {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
          {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
          {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
          {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|>,
      ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
        {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
        {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
        {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
        {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
      Meta → <|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
        LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|>|>[IdealData, All, 2] +
    <|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
        {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
        {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
        {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
        {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
      NoisyData → <|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
          {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
          {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
          {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
          {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
        2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
```

```
    {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
    {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
    {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
    {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
  3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
    {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
    {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
    {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
    {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
  4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
    {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
    {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
    {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
    {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
 ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
   {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
   {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
   {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
   {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
 Meta → ⟨|NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
   LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21|⟩|⟩[NoisyData, 1, All, 2]]] >
Mean[Abs[-⟨|IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842},
    {1., 0.540302}, {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
    {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
    {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
    {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
  NoisyData → ⟨|1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
     {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
     {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
     {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
     {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
   2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
     {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
     {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
     {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
     {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
   3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
     {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
     {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
     {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
     {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
   4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
     {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
     {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
     {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
     {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}}|⟩,
  ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
    {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
    {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
    {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
    {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
```

```
    Meta → ⟨| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
       LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21 |⟩ |⟩ [IdealData, All, 2] +
  ⟨| IdealData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    NoisyData → ⟨| 1 → {{0.1, 0.975104}, {0.4, 0.90264}, {0.7, 0.749545}, {1., 0.529496},
        {1.3, 0.262149}, {1.6, -0.0286155}, {1.9, -0.316824}, {2.2, -0.576731},
        {2.5, -0.785121}, {2.8, -0.923378}, {3.1, -0.979152}, {3.4, -0.947462},
        {3.7, -0.831138}, {4., -0.640571}, {4.3, -0.392783}, {4.6, -0.109909},
        {4.9, 0.182782}, {5.2, 0.459146}, {5.5, 0.694496}, {5.8, 0.867809}, {6.1, 0.963603}},
      2 → {{0.1, 0.955204}, {0.4, 0.884219}, {0.7, 0.734248}, {1., 0.51869},
        {1.3, 0.256799}, {1.6, -0.0280315}, {1.9, -0.310358}, {2.2, -0.564961},
        {2.5, -0.769098}, {2.8, -0.904533}, {3.1, -0.95917}, {3.4, -0.928126},
        {3.7, -0.814176}, {4., -0.627498}, {4.3, -0.384767}, {4.6, -0.107666},
        {4.9, 0.179052}, {5.2, 0.449776}, {5.5, 0.680323}, {5.8, 0.850099}, {6.1, 0.943938}},
      3 → {{0.1, 0.935304}, {0.4, 0.865797}, {0.7, 0.718952}, {1., 0.507884},
        {1.3, 0.251449}, {1.6, -0.0274476}, {1.9, -0.303892}, {2.2, -0.553191},
        {2.5, -0.753075}, {2.8, -0.885689}, {3.1, -0.939187}, {3.4, -0.90879},
        {3.7, -0.797214}, {4., -0.614425}, {4.3, -0.376751}, {4.6, -0.105423},
        {4.9, 0.175322}, {5.2, 0.440406}, {5.5, 0.66615}, {5.8, 0.832388}, {6.1, 0.924272}},
      4 → {{0.1, 0.915404}, {0.4, 0.847376}, {0.7, 0.703655}, {1., 0.497078}, {1.3, 0.246099},
        {1.6, -0.0268636}, {1.9, -0.297426}, {2.2, -0.541421}, {2.5, -0.737052},
        {2.8, -0.866845}, {3.1, -0.919204}, {3.4, -0.889454}, {3.7, -0.780252},
        {4., -0.601352}, {4.3, -0.368735}, {4.6, -0.10318}, {4.9, 0.171591},
        {5.2, 0.431035}, {5.5, 0.651976}, {5.8, 0.814678}, {6.1, 0.904607}} |⟩,
    ZNEData → {{0.1, 0.995004}, {0.4, 0.921061}, {0.7, 0.764842}, {1., 0.540302},
      {1.3, 0.267499}, {1.6, -0.0291995}, {1.9, -0.32329}, {2.2, -0.588501},
      {2.5, -0.801144}, {2.8, -0.942222}, {3.1, -0.999135}, {3.4, -0.966798},
      {3.7, -0.8481}, {4., -0.653644}, {4.3, -0.400799}, {4.6, -0.112153},
      {4.9, 0.186512}, {5.2, 0.468517}, {5.5, 0.70867}, {5.8, 0.88552}, {6.1, 0.983268}},
    Meta → ⟨| NoiseModel → DepolarizingEnd, BaseNoise → 0.02, Shots → ∞,
      LambdaList → {1, 2, 3, 4}, Order → 2, NumPoints → 21 |⟩ |⟩ [
  NoisyData, 1, All, 2]]], PASS, Equal (single gate)]
```

Export Artifacts

Run the cell below to export figures and PDF.

In[7]:=
```
Module[{exportDir, figDir, sweep, mainFig, errorFig},
exportDir = FileNameJoin[{NotebookDirectory[], "exports"}];
 figDir = FileNameJoin[{exportDir, "figures"}];
 If[! DirectoryQ[figDir], CreateDirectory[figDir, CreateIntermediateDirectories → True]];
sweep = ZNE`GenerateSweep[N[Range[0, 2 Pi, 2 Pi / 40]], {1, 2, 3, 4}, 2, "BaseNoise" → 0.03];
mainFig = Show[ListLinePlot[Prepend[Values[sweep["NoisyData"]], sweep["IdealData"]],
    PlotLegends → Prepend[Table["λ=" <> ToString[l], {l, {1, 2, 3, 4}}], "Ideal"],
    PlotLabel → "ZNE: Ideal vs Noisy vs Mitigated", ImageSize → 700,
    PlotStyle → {{Thick, Black}, Automatic}], ListLinePlot[sweep["ZNEData"],
    PlotStyle → {Thick, Red, Dashed}, PlotLegends → {"ZNE"}]];
errorFig =
  ListLinePlot[{MapThread[Function[{i, n}, {i[[1]], Abs[n[[2]] - i[[2]]]}], {sweep["IdealData"],
      sweep["NoisyData", 1]}], MapThread[Function[{i, z}, {i[[1]], Abs[z[[2]] - i[[2]]]}],
     {sweep["IdealData"], sweep["ZNEData"]}]}, PlotLegends → {"Noisy (λ=1)", "ZNE"},
   PlotLabel → "Absolute Error Comparison", ImageSize → 700];
Export[FileNameJoin[{figDir, "zne_main_plot.png"}], mainFig, ImageResolution → 150];
 Export[FileNameJoin[{figDir, "zne_error_plot.png"}], errorFig, ImageResolution → 150];
 Print["Figures exported to: ", figDir];
NotebookPrint[EvaluationNotebook[], FileNameJoin[{exportDir, "ZNE_Demo.pdf"}]];
 Print["PDF exported."]]
```

```
Figures exported to: C:\Amp_demos\Qedma\qedma-zne-mathematica-demo\notebook\exports\figures
```

Reproducibility Runbook

Follow these steps to reproduce results on another machine:

1. Install Mathematica 14.3 (or compatible version).
2. Install the QuantumFramework paclet:
   PacletInstall["Wolfram/QuantumFramework"]
3. Open this notebook (notebook/ZNE_Demo.nb) in Mathematica.
4. Evaluate All Cells (Evaluation menu).
5. Interactive demo appears in the ZNE Demo section.
6. Export artifacts by running the Export section.

Default parameters for exports:
- BaseNoise: 0.03
- NoiseModel: DepolarizingEnd
- LambdaList: {1, 2, 3, 4}
- Polynomial Order: 2
- Theta points: 40
- Shots: Infinity (analytic)