Index

Note: Page numbers followed by f indicate figures and t indicate tables.

| A | Bijective operator, 223 |
|--|--|
| Ablation problems, 59, 61, 414–415, 575, 582 | Bilinear form, 670 |
| Abstract space, 665 | Biot number, 405–406, 483 |
| Activation energy, 47 | Biot's technique, 611–612 |
| Adjoint operator, 671 | Blow-up, 77–78, 130–131 |
| Admissibility conditions, 194–195 | essential, 100 |
| Adomian decomposition method (ADM), | finite-time, 100, 102 |
| 460–466, 470 | nonessential, 100 |
| for inverse Stefan problems, 497–511 | in SSP, 101–102 |
| Adomian polynomials, 460–461, 464 | Boley's embedding technique, 424, 443–444, |
| AFS. See Approximate fundamental solution | 446–447 |
| (AFS) | Boltzman transformation, 318, 340 |
| Analytical-numerical solutions | Boundary conditions |
| ADM, 498–511 | of fifth type, 29–30 |
| approximate solutions in series form, | with multivalued functions, 30–31 |
| 490–494 | nonlocal, 29 |
| Green's functions, 470–486 | of radiative-convective type, 28–29 |
| of inverse Stefan problems, 497–511 | type I, 27–28 |
| Picard's iterative method, 486–490 | Boundary element method (BEM), 478 |
| VIM, 466–470 | Boundary norm, 200–201 |
| Anisotropic Holder space, 265 | Bounded linear operator, 667 |
| a-posteriori parameter-choice rule, 217 | norm, 667–668 |
| Approximate fundamental solution (AFS), 494 | norm, 007–008 |
| a-priori parameter-choice rule, 217 | |
| Arbitrary functions, 1–4, 465, 652 | C |
| Ascoli–Arzela theorem, 270, 677–678 | Caffarelli's criterion, 169 |
| Associative law, 665 | Canonical mapping, 669 |
| Asymptotic analysis | Caputo derivatives, 353, 355 |
| large-time solution, 360–361 | Cartesian coordinates, 448–449, 516, 556 |
| non-linear undercooling, 361–369 | problems in, 470–479 |
| small-time solution, 360 | Cauchy integral equation, 538–539 |
| α -th strong derivative, 679 | Cauchy integral method, 530–531 |
| α -th weak derivative, 679 | Cauchy principle value, 479–481 |
| | Cauchy problem, 448–449 |
| В | in heat conduction, 231–232 |
| Bäcklund transformation, 342, 344–345, 347 | Cauchy–Riemann conditions, 532–533 |
| Baiocchi transformation, 113–114, 168, 325 | Cauchy–Stefan problem, 266–267 |
| Banach contraction mapping theorem, 677 | Cauchy-type free boundary problems, |
| Banach fixed point theorem, 677 | 272–275 |
| Banach space, 242, 666 | C^{∞} -compatible, 136 |
| Bergman-type series expansions, 447–448 | CEF. See Classical enthalpy formulation (CEF |
| Bessel functions, 317–318, 456–457, 513, 657 | Characteristic function, 113–114, 169–170 |
| | CICE scheme, 496 |
| Best-approximate solution, 217 | CICE SCHEIRE, 470 |

| Claration 44.57 | Convey set 672 |
|---|---|
| Clapeyron's equation, 44, 57 Classical enthalpy formulation (CEF), | Convex set, 673 Convolution integral, 224, 234 |
| 121–124, 128–129 | Correctly-posed problem, 210 |
| Classical solution, 5–7, 32 | Cosmol Multiphysics Software, 420 |
| Classical Stefan problems | Crack-model, 545 |
| one-phase, 163–174 | Crack-model, 545 |
| quasi-variational inequality formulation, | 5 |
| 174–178 | D |
| two-phase, 178–182 | Darcy's law, 5–7, 541 |
| Classical Stefan solution (CSS), 128 | Degenerate free boundary problems, 4 |
| Class I problems, 64–65, 427 | Degenerate parabolic-elliptic problems, 5–7 |
| Class II problems, 64–66, 427–428 | Degenerate Stefan problems, 134–138 |
| Class III problems, 64–66, 427–428 | Dendrites, 333–334 |
| Clausius–Duhem inequality, 94–95 | equiaxed, 50, 52 <i>f</i> |
| Coercivity, 670 | formation of, 50, 50f |
| Coincidence set, 144–145 | growth, 50, 50 <i>f</i> |
| Columnar solidification, 49–50 | polycrystalline structure, 50, 51f |
| Combined integral balance method (CIM), 635 | Dense set, 667 |
| Commutative law, 665 | Diffuse interface model, 357 |
| Compact operators, 668 | Dirac delta function, 189, 306, 476 |
| Compact operators, 668 | Green's function, 316 |
| Compact support, 667 | regularization of, 494–497 |
| Compatibility conditions, 60, 64 | Directional solidification, 50, 119 |
| Complementarity problem, 143–146 | Direct metal layer siltering (DMLS), 580 Dirichlet problem, 196 |
| coincidence set, 144–145 | Dirichlet–Stefan problem, 209, 266–267 |
| noncoincidence set, 144–145 | Dirichlet type boundary conditions, 27–28 |
| quadratic programming, 145 | Discrepancy, 222 |
| Complete linear space, 666 | Distribution, 680 |
| Completely continuous operator, 668 | Distributional derivative, 679 |
| Complex-variable method, 541, 549 | Distributional sense, 30–31, 82–83, 119, |
| Confluent hypergeometric function, 509 | 124–126, 142, 169–170, 206, 298, 304, |
| Conformal mapping method, 529f, 530–531 | 402–403, 476 |
| for steady-state two-dimensional solutions | DMLS. See Direct metal layer siltering |
| of Stefan problems, 531–541 | (DMLS) |
| Conformal transformation method, 530–549, | Dual least-squares method, 225 |
| 532f, 533f, 534f, 535f | Dual space, 140–141, 669 |
| Conjugate gradient method, 463 | Duhamel's theorem, 425–426 |
| Conjugate operator, 214 | Duvait's transformation, 164–169 |
| Conservation | Dynamical boundary conditions, 30–31 |
| of energy, 32, 34–35, 54–57 | Dynamic energy balance condition, 18–19 |
| of forces, 53–54 | Dynamic linear stability analysis, 635–636 |
| of linear momentum, 33–34 | |
| of mass, 32–33 | E |
| of momentum, 32 | EHBIM, 577–580 |
| Constrained integral method, 582 | Eigenvalues, 456–457 |
| Constrained oxygen-diffusion problem | Elliptic variational inequality, 140 |
| (CODP), 77–78 | classical Stefan problems |
| Contact angle, 54–55 | one-phase, 163–174 |
| Continuous dependence, 211–213 | quasi-variational inequality formulation, |
| Contraction, 677 | 174–178 |
| Control volume, 33 | two-phase, 178-182 |
| Convective type boundary condition, 28 | complementarity problem, 143-146 |
| Convex function, 673 | definition, 140–142 |
| | |

| existence and uniqueness results | Fourier series, 449 |
|--|---|
| Lions-Stampacchia theorem, 150-152 | Fourier's law, 18, 184–185 |
| variational equation, 152–156 | Fourier transform method, 520 |
| minimization problem, 142–143 | Fractional diffusion equation (FDE), 352 |
| obstacle problem, string, 156–159 | Frechet derivative, 116–117, 681 |
| Elliptic variational inequality with obstacle, | Free boundary, 1–4 |
| 140 | Free boundary conditions, 1–4 |
| Embedding, 682 | of codimensional-two, 17 |
| Energy balance, 187, 299, 458, 476 | discontinuity in multidimensional problems. |
| Energy conservation principle, 202–203 | 7–15 |
| Enthalpy, 40, 82–83 | with frictional oscillator problem, 7–15 |
| Enthalpy formulation of Stefan problems, | Free boundary problems (FBPs), 311 |
| 577–580 | Cauchy-type, 272–275 |
| Entropy, 39 | Freezing front, 1–4 |
| Equiaxed solidification, 119 | |
| Equicontinuity, 673 | Freezing index, 163–164 |
| Equicontinuous functions, 673 | Freezing temperature, 17–18 |
| Equilibrium phase-change temperature, 17–18, | Frictional oscillator, 7–15, 10 <i>f</i> |
| 376f | Functionals, 669 |
| Equilibrium temperature, 43, 43f | Riesz representation theorem for, 670 |
| Essential blow-up, 100 | |
| Essential boundary conditions, 594 | G |
| Essentially bounded function, 676 | Galerkin approximation, 166–167 |
| Essential supremum, 676 | Galerkin approximation, 160–167 Galerkin method, 592–598 |
| Euler equation, 154–155, 214, 603–605, 608 | • |
| Eulerian coordinate system, 33 | Gamma function, 353 |
| Euler–Lagrange equation, 83–84 | Gâteaux derivative, 151–152, 681 |
| Euler transformation, 440–441, 646 | Generalized derivative, 679 |
| Eutectic temperature, 372 | Generalized discrepancy principle, 222 |
| Explicit free boundary condition, 31–32 | Generalized Fourier's law, 184–185 |
| Exponential Fourier transform, 211 | Generalized solution, 297–298 |
| Extensive property, 40 | Generalized Stefan condition, 31, 52, 55f |
| • • | Genuinely ill-posed problems, 215 |
| F | Gibbs free energy, 41–42 |
| | Gibbs-Thomson kinetic condition, 661 |
| Fickian diffusion systems, 352 | Gibbs-Thomson relation, 47 |
| Fickian flux vector, 352 | multidimensional SSPs, 113-118 |
| Fick's law, 7–15, 511, 567–568 | one-dimensional one-phase SSPs, 106-108 |
| FIDAP, 460 | one-dimensional two-phase SSPs, 108–113 |
| Fifth type boundary condition, 29–30 | Glassy solid, 49 |
| Finite positive integer, 465 | Graph of an operator, 676 |
| Finite-time blow-up, 100 | Grashof number, 661 |
| Finite-time extinction, 100 | Green's formulas, 304 |
| Finite volume method, 508 | application, 144–145, 173, 481 <i>f</i> , 529 <i>f</i> |
| First law of thermodynamics, 39 | Green's functions, 257, 429, 434, 452, |
| First-order phase transitions, 20–21 | 470–486, 493–494, 497 |
| First order relaxation process, 512–513 | in cylindrical polar coordinates, 317–318 |
| Fixed boundary condition, 1–4, 27–31 | Dirac delta function, 315–316 |
| Fixed grid enthalpy method, 389 | for linear parabolic heat equation, 315–316 |
| Fixed point of a mapping, 677 | * |
| Fixed point theorem | for parabolic and hyperbolic Stefan |
| Banach, 677 | problems, 520–530 |
| Schauder's, 677 | quasi-analytical solutions, 315–325 |
| Flux-prescribed boundary condition, 28 | source solution, 316 |
| Fourier heat conduction, 436 | Green's identity, 470–471 |

| Н | energy conservation equation for two-phase |
|--|---|
| HAM. See Homotopy analysis method (HAM) | problem, 202–207 |
| Hanzawa transformation, 116–117, 135 | Green's functions for, 520–530 |
| HBIM. See Heat balance integral method | relaxation models, 184-185 |
| (HBIM) | relaxation time, 184–185 |
| Heat balance equation, 555 | with temperature continuity at interface, |
| Heat balance integral | 185–193 |
| equation, 559 | with temperature discontinuity at interface, |
| in HBIM and RIM formulations, 575 | 193–201 |
| Heat balance integral method (HBIM), 361, | weak formulations, 201-207 |
| 550–592, 635 | Hyperboloid, 331 |
| enthalpy formulation of Stefan problems, 577–580 | Hypergeometric function, 354 |
| heat balance integrals in, 575 | _ |
| • | I |
| in multi-dimensional problems, 582–587 | Ideal equilibrium temperature, 43 |
| polynomial profile for temperature in, | Ill-posed problem, 79, 210 |
| 588–590 | Imbedding, 675 |
| polynomial representation of temperature in, | Implicit free boundary condition, 5–7, 31–32, |
| 571–577 | 72–76 |
| refinement and variations of, 558–570 | Incompatibility measure, 222 |
| Stefan-like multiphase problem using, | Incorrectly-posed problem, 210 |
| 580–582 | Independent variables, 437–438 |
| Heat conduction equation, 18 | Indicator function, 96–97 |
| Heat conduction problem, 445–446 | Initial approximation, 469 |
| Heat flux relaxation function, 184 | Inner product, 666 |
| Heat flux vector, 18, 184 | Inner product space, 666 |
| Heat polynomials, 244 | Integral equation formulations, 252–264 |
| Heat transfer, 527, 557–558 | Integral equation method, 319–320 |
| Heaviside functions, 416 | Integral heat balance, 71–72 |
| Heaviside shift theorem, 522–524 | solutions using integral heat-balance |
| Hele-Shaw problem (HSP), 78-79, 134, | method, 503–507 |
| 136–137, 168, 328 | Interface curvature, 47–48, 56 <i>f</i> |
| use of conformal transformation in solutions | Interface kinetic effect, 49 |
| of, 541–549 | Internal energy, 39 |
| Helmholtz free energy, 41–42, 43f | Inverse problem, 209 |
| Hermite interpolating polynomial expression, | Inverse Stefan problems, 497–498 |
| 590–591 | ADM, 498–511 |
| Heterogeneous nucleation, 44 | defect minimization problems, 239–250 |
| Hilbert-adjoint operator, 671 | homotopy analysis method for, 498–511 |
| Hilbert space, 460–461, 666 | input data, 209–210 |
| isomorphic, 669 | regularization |
| Holder continuous function, 674 | convolution integrals, 234–238 |
| Hölder stability, 212 | generalized discrepancy principle, |
| Homogeneous nucleation, 44 | 215–217 |
| Homotopy, 614–627 | generalized inverse, 217–218 |
| Homotopy analysis method (HAM) | heat conduction problems, 231–234 |
| for inverse Stefan problems, 501–502 | methods, 219–226 |
| Homotopy perturbation method (HPM), | regularizing operator principle, 215–217 |
| 614–620 | solutions obtained without |
| for inverse Stefan problems, 501–502 | regularization, 499 |
| Hyperbolic conductivity model, 517 | Tikhonov regularization approach, 507–51 |
| Hyperbolic heat equation, 517–518 | unknown parameters |
| Hyperbolic η -weighted interior norm, 200–201 | one-phase, 228–230 |
| Hyperbolic Stefan problem 183 511–530 | two-phase 230–231 |
| | |

| variational iteration method for, 500–502 | Lions–Stampacchia theorem, 150–152 |
|---|--|
| well-posedness of solution | Lipschitz continuous function, 673 |
| approximate, 213–215 | Liquid–mush boundary, 69, 71, 302–303, |
| continuous dependence, 211–213 | 306–307 |
| nonexistence, 210 | Lithium concentration, 464–465 |
| nonuniqueness, 210–211 | Locally compact space, 676 |
| Isomorphism, 669 | Locally Holder continuous function, 674 |
| Isotherm conditions, 18–19, 187, 454–456, 476 | Locally integrable function, 676 |
| Iterated error functions, 318 | Lower semi-continuous function, 96–97, 673 |
| Ivantsov solution, 333–334 | |
| | M |
| | |
| J | Macroscopic models, 37–38 |
| Joule heating, 121–123 | Mass transfer problem, 445, 557–558 |
| | Material surface, 52 |
| K | MATLAB, 458, 465–466 |
| | Maximal monotone graph, 676 |
| Keller box finite difference technique, 366, | Maxwell-Cattaneo model, 511 |
| 576–577 | Melting front, 1–4 |
| Kinetic condition, 31 | Melting problem, 19 |
| Kirchoff's transformation, 26–27, 394 | Melting temperature, 17–18, 405, 461–462 |
| Kronecker delta function, 26 | Metallurgical aspects |
| kth order compatibility conditions, 199–200 | glassy (amorphous) solids, 49 |
| | interface curvature effect, 47–48 |
| L | interface kinetics, 49 |
| | melting, nucleation of, 48 |
| Lagrange–Bürmann expansions, 452 | nucleation, 44–46 |
| Lagrange multipliers, 221, 466–468 | supercooling, 44–46 |
| Lagrangian coordinate system, 33 | Metastable state, 46 |
| Lanczos regularization technique, 495 | Method of fundamental solutions (MFS), 601 |
| Landau–Ginzburg free energy functional, | Method of invariant manifold/ method of |
| 82–87 | integral manifold (MIM), 625 |
| Landau transformation, 437–440, 445–446, | Method of over-specified boundary conditions |
| 630, 633–634 | 226–227 |
| Laplace–Beltrami operator, 294 | Methods of weighted residual (MWR) |
| Laplace transform, 368–369, 450–451, 576 | Galerkin method, 592–598 |
| of linear heat equation, 492 | orthogonal collocation method, 599-602 |
| method, 512–513, 520 | Metric, 666 |
| Latent heat, 42 | MFS. See Method of fundamental solutions |
| Least squares method, 601 | (MFS) |
| Least-squares solution, 217 | Microscopic models, 37–38 |
| Legendre polynomials, 599–600 | MIM. See Method of invariant manifold/ |
| Lever rule, 373 | method of integral manifold (MIM) |
| Lewis number, 378–379 | Minimization problem, 142 |
| Lightfoot's method, 477, 523–525 | Modified error function, 228 |
| Lightfoot's source and sink method, 489–490 | Modified Gibbs–Thomson relation, 81 |
| Linear heat equation, 442, 450–451 | Mole number, 46 |
| Laplace transform of, 492 | Moment integral method, 561 |
| Linearization, 4–16 | Moore–Penrose (MP) generalized inverse, |
| Linear operator, 667 | 217–218 |
| Linear parabolic equation, 447 | <i>m</i> -order compatible, 136 |
| Linear phase diagram, 375, 377 | Moser–Nash theorem, 135 |
| Linear regularization method, 217 | Moving boundary, 1–4 |
| Linear space, 665 | Moving least-square (MLS) approximation, |
| Linear stability analysis, 118, 129 | 598 |
| Linear statinty analysis, 110, 147 | 370 |

| Multi-dimensional perturbation solutions, | finite-difference schemes, 268–271 |
|--|---|
| 636–642 | integral equation formulations, 252–264 |
| Multidimensional Stefan problems, 64–66, | analyticity, 264–268 |
| 290–295 | Cauchy-type free boundary problems, |
| Multiplicative identity, 665 | 272–275 |
| Multivalued functions, 30–31 | density changes, 276 |
| Mushy region, 121, 304 | existence and uniqueness, 275–276 |
| characterization of, 302, 306 | infinite differentiability, 264–268 |
| disappearance, 302, 306 | One-dimensional three-phase problem, 497 |
| enthalpy of, 298 | One-dimensional two-phase Stefan problems, |
| one-phase multidimensional Stefan | 63–64 |
| problem, 307–308 | analyticity, 286–287 |
| | differentiability, 286 |
| N | existence, 276–286 |
| Natural boundary conditions, 166, 606 | <i>n</i> -phase problem, 287–290 |
| Natural convection, 315, 335 | uniqueness and stability results, 276–286 |
| Navier–Stokes equation, 34 | One-phase continuous casting model, 169–172 |
| Neumann problem, 196–201, 439–440, 442, | One-phase problems, 20–21, 163–174 |
| 452, 454 | continuous casting model, 169–172 |
| Neumann solution, 18–21, 110–111, 313–314 | Duvait's transformation, 164–169 |
| Newton potential, 484–485 | freezing index, 163 |
| Newton's second law of motion, 7–15 | multidimensional |
| Noisy data, 215 | Signorini-type boundary condition, 62–63 |
| Non-characteristic Cauchy problem, 227–228 | three-dimensional ablation problem, |
| Non-coincidence set, 144–145 | 61–63 |
| Non-degenerate problem, 206–207 | one-dimensional, 59–61 |
| Non-essential blow-up, 100 | oxygen-diffusion problem, 173–174 |
| Non-Fickian law, 511 | Operator(s) |
| | compact, 668 |
| Non-Fourier melting, 513–514 Non-Fourier's law, 184–185 | continuity, 668 |
| | graph, 676 |
| Nonlinear degenerate equation, 5–7 | restriction, 668 |
| Non-linear heat equation, 491–492 | Optimal homotomy analysis method |
| Non-linear operator, 460–461, 466 | (OHAM), 624 |
| Non-local boundary condition, 7–15, 29 | Optimal regularization method, 226 |
| Nonmaterial singular surface, 52–57 | Order parameter, 82–83 |
| Norm, 666 | Orthogonal collocation method, 599–602 |
| Normed space, 666 | Orthogonal complement, 668–669 |
| <i>n</i> -phase Stefan problem, 287–288 | Orthogonal projection, 669 |
| Nucleation, 44, 102 | Orthonormality condition, 494 |
| of melting, 48, 49 <i>f</i> | Oxyen-diffusion problem (ODP), 76–79, 99, |
| Nyström discretization method, 485–486 | 105–106, 123–124, 173–174, 324–325 |
| | constrained, 77–78 |
| 0 | one-phase Stefan problem, 174 |
| Obstacle, 140 | quasi-static two-dimensional, 78–79 |
| Obstacle problem, 7–15, 10 <i>f</i> , 140, 156–159, | in radially symmetric domain, 78 |
| 157 <i>f</i> | unconstrained, 77–78 |
| OHAM. See Optimal homotomy analysis | variational inequality formulation, 173 |
| method (OHAM) | |
| One-dimensional melting problem, 523 | P |
| One-dimensional one-phase solidification | Parabolic boundary, 159–160, 678 |
| problem, 489–490 | Parabolic distance, 163 |
| One dimensional one-phase Stefan problems | Parabolic duality, 597 |
| analysis, 251–252 | Parabolic–elliptic Stefan problems, 134–136 |

| Parabolic operator, 135 | <i>n</i> -phase Stefan problems with $n > 3$, |
|--|---|
| Parabolic regularity theory, 114–115 | 391–395 |
| Parabolic variational inequality, 159–163 | Neumann solution, 313–337 |
| Paraboloids, 332–333 | in Cartesian coordinates, 470–479 |
| PCM. See Phase change materials (PCM) | in cylindrical and spherical geometry, |
| Peclet number, 334–335 | 479–486 |
| Penalty method, 162–163, 168 | explicit solutions of <i>n</i> -phase problems, |
| Penetration depth method, 552 | 352–358 |
| Perturbations, 50–52, 51 <i>f</i> | phase-change boundary, 437–439 |
| methods and solutions, 627–630 | series solutions, 452–460 |
| parameter, 627–628 | short-time analytical solutions, 423–452 |
| series expansions, 643 | Stefan-like problems, 337–352, 358–359 |
| singular methods, 629–630, 643 | dilute binary alloys, solidification, |
| solutions, multi-dimensional, 636-642 | 374–382 |
| Petrov–Galerkin approach, 598 | with kinetic condition, 358–359 |
| Phase-change boundary, 1–4, 17–18, 437–438, | heat and mass transfer, 374–382 |
| 445 | heat transport in formulation, 370–374 |
| solution with temperature continuity across, | in porous media, 382–391 |
| 512–520 | in which thermo-physical parameters, |
| Phase change materials (PCM), 458, 507, | 337–352 |
| 632–633, 659 | VIM, 466–470 |
| Phase-change problem, 591–592 | Quasimonotonic, 221 |
| Phase-change temperature, 17–18 | Quasi-solution, 214 |
| Phase-field model, 81–82, 82f | Quasi-static hyperbolic equation, 516 |
| for solidification, 82–87, 88f, 89f | Quasi-static process, 38 |
| Phase function, 82–83, 83f | Quasi-static two-dimensional ODP, 78–79 |
| Phase relaxation, 94 | Quasi-steady conditions, 447, 458–459 |
| for supercooling, 93–98 | Quasi steady-state free boundary problems, |
| Picard iteration method, 257, 486 | Quasi steady-state Stefan problems, 134–135 |
| Planar interface, 50–52 | Quasi-steady-state Stefan problems, 134–133 Quasi-steady state two-dimensional one-phase |
| Poincare's inequality, 153 | problem, 489 |
| Polubarinova-Galin equation, 545–546 | Quasi-variational inequality, 174 |
| Polycrystalline structure, 50, 51 <i>f</i> | Quasi-variational inequality, 174 |
| Prandtl number, 418 | _ |
| Precompact set, 668 | R |
| Prior distribution, 223–224 | Radiative-convective type boundary condition, |
| Projection operator, 175, 669. See also | 28 |
| Orthogonal projection | Radon measure, 158 |
| Properly-posed problem, 210 | Rankine–Hugoniot conditions, 7–15, 9f, |
| Pseudo-steady-state solution (PSS), 562–563 | 194–196, 516 |
| | Rayleigh–Taylor instability, 549 |
| 0 | Refined HBIM (RHBIM), 558–570 |
| Q | Refinement RIM, 571–577 |
| Quadratic bilinear form, 140 | Reflexive space, 670 |
| Quadratic polynomials, 569 | Regularization, 101, 130–131 |
| Quasi-analytical solutions | Regularization by projection, 224 |
| ADM, 460–466 | Regularization methods, 216 |
| asymptotic analysis | convergence rate, 226–227 |
| large-time solution, 360–361 | convolution integral type, 224 |
| non-linear undercooling, 361–369 | of inverse heat conduction problems, |
| small-time solution, 360 | 231–234 |
| Green's function, 315–325 | of inverse Stefan problems, |
| of hyperbolic Stefan problems, 511–530 | 234–238 |
| hyperboloidal moving boundary, 332 | maximum entropy, 223 |

| Regularization methods (Continued) | Euler transformation, 646 |
|---|--|
| by projection, 224 | matched asymptotic expansions, 646–658 |
| Tikhonov, 219 | strained coordinates method, 643 |
| Regularization operator, 215–216 | Singular perturbed systems (SPS), 624–625 |
| Regularization (regularizing) operator, 216 | Singular state, 52 |
| Regularization parameter, 216 | Singular surface, 52 |
| Regular perturbation expansions, 630 | SLS. See Selective laser sintering (SLS) |
| one- and multidimensional problems, | Smoothing functional, 219 |
| 628–630 | Sobolev imbedding theorem, 682 |
| Regular perturbation method, 629 | Sobolev space, 680 |
| Relaxation models, 184–185 | Solid freedom fabrication (SFF), 568 |
| Relaxation time, 183–185 | Solidification, 61 |
| Representative elementary volume (REV), 379 | entropy functional, 90–93 |
| Reproductive toxic mass diffusion, 5 | at liquid–mush boundary, 71 |
| Restricted variational principle, 611 | in liquid region, 70 |
| Restriction of an operator, 668 | in mushy region, 70 |
| Reversible process, 38, 41 | phase-field model for, 82–87 |
| RHBIM. See Refined HBIM (RHBIM) | at solid–mush boundary, 70 |
| Riemann invariants, 190–191, 195–196 | • |
| Riemann–Liouville fractional derivative, | in solid region, 70 |
| 357–358 | supercooled liquids, 98–106, 358 |
| Riemann's mapping theorem, 547 | supercooling, 97–98 |
| Riesz representation theorem for | phase relaxation models for, 93–98 |
| functionals, 670 | superheating, 97–98 |
| RKM. See Runge–Kutta method (RKM) | with transition temperature range, 69–72 |
| Rule of solution expression (RSE), 624 | Solid–liquid interface, 49–52, 50 <i>f</i> , 51 <i>f</i> , 52 <i>f</i> , |
| Runge–Kutta method (RKM), 464, 581, | 170 <i>f</i> |
| 601, 626 | Solid–mush boundary, 69, 302–303, 306–307 |
| 001, 020 | Soret effect, 660 |
| _ | Source and sink method (SSM), 476, 478 |
| S | Specific enthalpy, 40 |
| Scalar multiplication axioms, 665 | Specific heat, 40 |
| Scalar product. See also Inner product | Specific heat capacity, 40 |
| definition, 148 | Splicing algorithm, 590–591 |
| Schatz transformations, 72–76 | SPS. See Singular perturbed systems (SPS) |
| Schauder's fixed point theorem, 115, 254, 677 | Stability, 211, 213 |
| Scheil's equation, 410–411 | Stability of the solution, 213 |
| Schwarz-Christoffel transformation, 533-541 | Stabilizers of pth order, 221–222 |
| Second law of thermodynamics, 39, 84 | Stabilizing functional, 220–221 |
| Selective laser sintering (SLS), 568–569, 580 | Standard phase-field (SPF) model, 87, 93 |
| Semi-norm, 666 | Steady-state continuous-casting |
| Sensitivity analysis, 247–248 | two-dimensional problem, 537–538 |
| Sensitivity coefficients, 247–248 | Steady-state free boundary problems, 133 |
| Separation constants, 456–457 | Steadystate temperature, 1–4 |
| Sequence(s) | Steady-state two-dimensional Stefan problem, |
| strongly convergent, 671 | 493, 531–541 |
| weakly convergent, 671 | Stefan–Boltzman constant, 396 |
| Series solutions, 423 | Stefan condition, 19, 441, 444, 457–458 |
| SFF. See Solid freedom fabrication (SFF) | Stefan problems, 1-4, 349, 436-438, 448, 493 |
| Shank's transformation, 362 | in Cartesian Coordinates, 470–479 |
| Sharp-interface model, 355–356, 370 | in cylindrical and spherical geometry, |
| Shock, 7–15 | 479–486 |
| Shrinking core model, 437–438 | degenerate, 134–138 |
| Signorini-type boundary condition, 62–63 | enthalpy formulation of, 577–580 |
| Singular perturbation method, 629–630, 643 | hyperbolic, 183 |
| | |

| with implicit free boundary conditions, | first law, 39 |
|---|--|
| 72–79 | macroscopic models, 37–38 |
| inverse, 497–498 | microscopic models, 37–38 |
| with linear kinetic undercooling, 360 | second law, 39 |
| multidimensional, 64–66 | variables and thermal parameters, 40-42 |
| one dimensional one-phase, 251–252 | Thermodynamical property, 40 |
| one-dimensional two-phase, 63–64 | Thermodynamic equilibrium, 38 |
| one-phase, 163–174 | Three-dimensional ablation problem, 61–63 |
| quasi steady-state, 134 | Three-dimensional heat equation, 493 |
| quasi-variational inequality formulation, | Tikhonov-regularization method, 213, 219, |
| 174–178 | 246–247, 507–511 |
| steady-state two-dimensional, 531–541 | Tikhonov stabilizer, 221–222 |
| Stefan and Stefan-like problems, 358–359, | Trace operator, 682 |
| 614–658 | Transient heat equation, 494 |
| two-phase, 178–182 | Transient nonlinear heat conduction |
| using conformal transformation method, | problem, 661 |
| 530–549 | Transient two-dimensional problem, 493 |
| Stefan-type problem, 75–76 | Two-phase Stefan problems, 20–21, 178–182 |
| Stoke's theorem, 127 | classical Stefan problems with <i>n</i> -phases, |
| Storm transformation, 394–395 | n > 2, 66-69 |
| Storm-type metals, 348–349 | multidimensional, 64–66 |
| Strained coordinates method, 643 | in multiple dimensions, 21–25, 22 <i>f</i> , 23 <i>f</i> , 25 <i>f</i> |
| Strictly convex function, 673 | one-dimensional, 63–64 |
| Strong derivative, 679 | solidification with transition temperature |
| Strongly convergent sequence, 671 | range, 69–72 |
| Strong maximum principle, 678 | Type I boundary condition, 27–28 |
| Sturm–Liouville problem, 457–458, 496 | ** |
| Subdifferential, 96–97 | Type II boundary condition, 28 |
| Suction problems, 136–137 | Type III boundary condition, 28 |
| Supercooled liquids, 43, 99–101, 358 | Type IV boundary condition, 28 |
| Supercooled state, 45–46 | |
| Supercooled Stefan problem (SSP), 61 | U |
| blow-up in, 101–102 | Unconstrained oxygen-diffusion problem |
| initial and boundary conditions in, 102–106 | (UODP), 77–78 |
| with modified Gibbs–Thomson relation, | Uniformly bounded function, 673 |
| 106–120 | Uniformly Holder continuous function, 674 |
| one-dimensional one-phase solidification, | Uniformly parabolic, 5–7, 162 |
| 99–101 | Unilateral boundary condition, 268–270 |
| Supercooling effects, 97–98 | Upper semi-continuous function, 673 |
| Superheated solid, 45–46, 45 <i>f</i> | oppor some communation, over |
| Superheating effects, 97–98 | • / |
| Surface divergence, 54 | V |
| Surface tension, 1–4, 53–54, 106, 117 | Variational equation, 152–156 |
| Surface tension, 1-4, 33-34, 100, 117 | Variational formulation, 16 |
| | Variational inequality, 139 |
| T | Variational iteration method (VIM), 466–470 |
| Taylor's series, 46 | for inverse Stefan problems, 500–502 |
| Telegrapher's equation, 185–186, 511, | Variational methods and principles, 602–613 |
| 522–523 | Vector space, 665 |
| Test function, 680 | completeness, 666 |
| Theodorsen's integral equation, 540 | Vertical gradient freeze (VGF) technique, |
| Thermal conductivity, 23, 26–27, 42 | 458–460 |
| Thermal diffusivity, 422, 476–477, 485 | Visco-plastic bar, 7–15 |
| Thermodynamic(s) | Viscosity solutions, 310 |
| equilibrium temperature, 43–44 | Volterra integral equation, 235, 237, 346–347 |

726 Index

W

Waiting time, 5–7 Weak derivative, 679 Weak formulation, 119–120 Weak free boundary, 307 Weakly compact set, 667 Weakly convergent sequence, 671 Weak maximum principle, 678 Weak solution (WS), 124–130, 297–298 Well-posed problem, 210