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Modeling of ice heating and melting in approximation of the Stefan problem considering radiation*

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The problem of radiation-conductive heating and subsequent melting of ice in a climatic chamber in a single-phase approximation of the Stefan problem was posed and solved by mathematical modeling methods taking into account a thin water film formed on the irradiated surface. The fields of temperature and density of the resulting radiation flux are obtained, as well as the rate of melting and heating of the non-irradiated ice surface. Comparison of results with experimental data showed satisfactory agreement.

Key words: ice, melting, Stefan problem, radiation-conductive heat transfer, water film.

Introduction

Modeling of ice melting is necessary both for understanding the processes taking place in nature and ensuring the safety of building structures, equipment, and population in the northern latitudes. The problem of ice melting is a classical problem of phase transition. The history of Stefan problem statements is presented in [1]. Theoretically and experimentally, the two- and three-phase Stefan problems with radiation are well studied [2–7]. Mathematical modeling of the single-phase Stefan problem with radiation was considered in a series of works (see, for instance, [8, 9]), however, there are few experimental works, which can verify the statement and method of solution. The work [10] is computationally experimental. There, ice was placed on a vertical opaque substrate in the climatic chamber at the constant temperature of 0 °C illuminated by two kinds of lamps (halogen with filament temperature T_b = 3200 K and lamps with a nichrome filament with T_b = 800 K). Ice melting was considered under the conditions of short-wave and long-wave radiation. In the mathematical model of the process, the authors neglected the presence of a thawed water film on the surface and calculation was carried out in a single-phase statement of the Stefan problem. They compared the rates of melting and heating of the non-radiated ice side and obtained a satisfactory agreement, using in calculations the fitting parameters and direct integration of radiation transfer according to

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the Bouguer law. The effect of short-wave radiation on formation of highly rough surfaces in ice was shown. Among the computational and experimental works, there is also [11], which is devoted to the topical problem of preventing icing of various surfaces. The authors carried out an experiment on a vertical substrate of transparent and opaque for radiation materials at the room temperature. The results of experiments and calculations showed that ice is better removed from the opaque materials than from a transparent acrylic substrate. When comparing these calcu-lations with experiment, it was found that they are in good agreement for thin substrates.

The modern state of thermal physics of snow and ice was analyzed in detail in monograph [12], and a new class of problems, where snow and ice are considered as the light-scattering media with volume absorption and reflection, was posed. This approach makes it possible to explain a number of observed effects of metamorphism in the near-surface layers of glacial masses and describe the regularities of dynamics of their thermal state and interfacial exchange.

In this paper, we present a mathematical model of experiment [10] for verification of problem statement and method for solving the radiation part used in [9], with experimental data also obtained in [10].

Problem statement

The geometric scheme of the problem, assuming an opaque bakelite substrate of thickness L_1 with an adhered layer of pure ice, absorbing radiation, with thickness L_2, considered in approximation of the gray medium, is shown in Fig. 1. Radiation from a lamp with filament temperature T_b = 800 K falls on the right side of ice. This models approximately solar radiation on a cloudy day; the data obtained are used for subsequent comparison with the results of [10]. The ice boundaries diffusely absorb, reflect, and pass radiation in such a way that $A_i + R_i + D_i = 1$, where A_i, R_i, and D_i are absorptive, reflective, and transmissive hemispherical abilities of the ice surface, i = 1, 2. The left boundary of substrate $T_1(0, t)$ is maintained at a constant temperature $T_{\text{sub}} = 256.15$ K, ambient air is kept at T_∞ = 273.15 K.

The solution of the problem includes two stages. At the first stage, radiative-conductive heat transfer is considered, it continues until the right ice boundary (with initial temperature $T_2(L_2, t)$) reaches the phase transition temperature T_f At the second stage, we consider the Stefan problem with a fixed value of T_f at this stage, a thin film of water flows under the influence of gravitational forces causing an additional heat load in the form of convection and radiation. The position of interface $L_2(t)$ is determined from the solution to the boundary-value problem.

The equations of energy conservation of substrate with temperature $T_1(z, t)$ and ice with temperature $T_2(x, t)$ are written as follows:

$$c_{p1}\rho_1 \frac{\partial T_1(z,t)}{\partial t} = \lambda_1 \frac{\partial^2 T_1(z,t)}{\partial z^2}, \quad 0 < z < L_1, \tag{1}$$

$$c_{p2}\rho_2 \frac{\partial T_2(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_2 \frac{\partial T_2(x,t)}{\partial x} - E(x,t)\right), \quad 0 < x < L_2, \tag{2}$$

here, c_{pi} is the specific heat at constant pressure, ρ_i is the density, λ_i is the heat conductivity

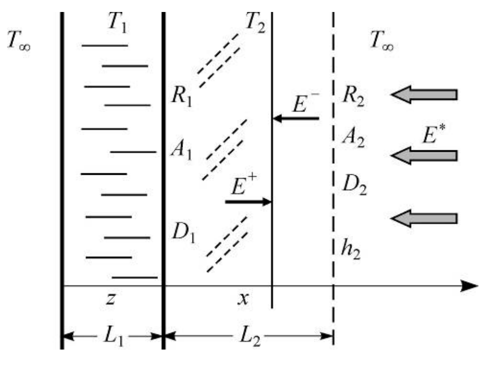

Fig. 1. Geometrical scheme of the problem.

coefficient (i = 1, 2), $E(x,t) = 2\pi \int_{-1}^{1} I(\tau, \mu)\mu d\mu$ is the density of resulting radiation flux written in terms of radiation intensities $I(\tau, \mu)$, where $\tau = \alpha \cdot x$ is the optic thickness, α is the coefficient of volumetric absorption, μ is the cosine of an angle between the direction of radiation propagation and coordinate axis x. Boundary conditions for the first stage are written as relationships

$$T_1 = T_{\text{sub}}, \text{ at } z = 0, \tag{3}$$

$$-\lambda_1 \frac{\partial T_1}{\partial z} + D_1 E^-(x, t) = \lambda_2 \frac{\partial T_2}{\partial x} + \left|E_{\text{res, 1}}\right| \text{ at } z = L_1 \text{ and } x = 0, \tag{4}$$

$$\lambda_2 \frac{\partial T_2}{\partial x} - h\left(T_2 - T_\infty\right) - \left|E_{\text{res, 2}}\right| = 0 \text{ at } x = L_2. \tag{5}$$

Radiation component $\left|E_{\text{res}, i}\right|$ in (4) and (5) is determined as

$$\left|E_{\text{res, 1}}\right| = A_1 \sigma_0 T_1^4(z, t) - \varepsilon_1 \sigma_0 T_2^4(x, t),$$
$$\left|E_{\text{res, 2}}\right| = A_2 E^* - \varepsilon_2 \sigma_0 T_2^4(x, t),$$

where h is the heat transfer coefficient, σ_0 is the Stefan–Boltzmann constant, ε_i is the emissivity, E^* is the constant falling flux, i = 1, 2. In (4), the second term in the left is responsible for radiation penetration through ice; the terms included into $\left|E_{\text{res, 1}}\right|$ are the flux densities determining absorption and radiation from the substrate and ice surfaces, respectively; the validity of Kirchhoff law is assumed: $A_i = \varepsilon_i$. The system of equations (1) and (2) is supplemented with initial condition $T_1(z, 0) = T_2(x, 0) = T_{\text{sub}}$. At the stage of ice melting, the temperature of the right boundary is fixed:

$$T(x, t) = T_{\text{f}}, \quad \text{at } x = L_2(t). \tag{6}$$

Boundary condition (4) is transformed into the Stefan condition taking into account a thin layer of ice water that appears on the surface. We assume that the film temperature is isothermal, there is no temperature gradient, and thickness of the film itself is much smaller than the ice thickness:

$$\lambda_2 \frac{\partial T_2}{\partial x} - h\left(T_{\text{fil}} - T_\infty\right) - \left|E_{\text{res, fil}}\right| = \rho_2 \gamma \frac{\partial L_2}{\partial t}, \tag{7}$$

where, $\left|E_{\text{res, fil}}\right|$ takes the following form:

$$\left|E_{\text{res, fil}}\right| = A_2 E^* + \varepsilon_2 \sigma_0 \left(T_{\text{fil}}^4 - T_2^4(x, t)\right), \; x = L_2(t). \tag{8}$$

Here, T_{f} = 273.15 K is the melting temperature for ice, T_{fil} = 277.15 K is the water film temperature. Condition (7) takes into account heat transfer from the outer surface of the water film, and in (8), intrinsic radiation of the film and right surface are taken into account.

The assumption that a thin water film exists on the ice surface does not contradict the one-phase approximation of the Stefan problem, since there is no energy transfer in the film and it acts only as an additional boundary condition on the interface with the constant values. The thermal problem is solved only in the thickness of ice on a vertical substrate.

The Stefan condition without consideration of the water film (as in [10]) takes the form:

$$\lambda_2 \frac{\partial T_2}{\partial x} - h_2\left(T_2 - T_\infty\right) - \left|E_{\text{res, 2}}\right| = \rho_2 \gamma \frac{\partial L_2}{\partial t}. \tag{9}$$

Dimensionless equations (1), (2) with boundary conditions (3) are written in the following form:

$$\frac{\partial\theta_1(\zeta,\eta)}{\partial\eta} = AX^2\frac{\partial^2\theta_1(\zeta,\eta)}{\partial\zeta^2}\text{, at } 0<\zeta<1, \quad (10)$$

$$\frac{\partial\theta_2(\xi,\eta)}{\partial\eta} = \frac{\partial^2\theta_2(\xi,\eta)}{\partial\xi^2} - \frac{1}{N}\cdot\frac{\partial\Phi}{\partial\xi}\text{, at } 0<\xi<1, \quad (11)$$

$$\theta_1 = \theta_{\mathrm{sub}} = \mathrm{const}\text{, at } \zeta = 0; \quad (12)$$

$$-\Lambda X\frac{\partial\theta_1(\zeta,\eta)}{\partial\zeta} = \frac{\partial\theta_2(\xi,\eta)}{\partial\xi} + \frac{1}{N}\left(\frac{\varepsilon_1}{4}\left(\theta_1^4-\theta_2^4\right) - D_1\Phi^-(\xi,\eta)\right)\text{, at } \zeta=1 \text{ и } \xi=0; \quad (13)$$

$$\frac{\partial\theta_2(\xi,\eta)}{\partial\xi} - \mathrm{Bi}\left(\theta_\infty-\theta_2\right) - \frac{\varepsilon_2}{N}\left[\Phi^+(\xi,\eta)+\Phi^* - \frac{\theta_2^4}{4}\right] = 0\text{, at } \xi=1. \quad (14)$$

Initial conditions for equations (7)–(8) take the form: $\theta_1(\zeta,0)=\theta_2(\xi,0)=\theta_{\mathrm{sub}}$. Here, $\zeta = z/L_1$ is the dimensionless coordinate of substrate, $\xi = x/L_2(0)$ is the dimensionless coordinate of ice; $\theta_i = T_i/T_{\mathrm{f}}$, where i = 1, 2, ∞, "sub", and "fil" are dimensionless temperatures of substrate, ice, ambient medium, left boundary of substrate, and film, respectively; $\eta = \left(a_2\cdot t\right)/L_2^2$ is the dimensionless time; $a_i = \lambda_i/c_{\mathrm{p}i}\rho_i$ is the thermal diffusivity, i = 1, 2; $\mathrm{Bi} = L_2h/\lambda_2$ is the Biot number; $\Lambda = \lambda_1/\lambda_2$, $X = L_2(0)/L_1$, and $A = a_1/a_2$ are the ratios of heat conductivity coefficients, initial lengths, and thermal diffusivities of substrate and ice, respectively; $N = \lambda_2\Big/\left(4\sigma_0T_{\mathrm{f}}^3L_2(0)\right)$ is radiation-conductive parameter, $\Phi^\pm = E^\pm\Big/\left(4\sigma_0T_{\mathrm{f}}^4\right)$ is the dimensionless density of resulting radiation flux, $\Phi^* = E^*\Big/\left(4\sigma_0T_{\mathrm{f}}^4\right)$ is the dimensionless density of the radiation flux.

The authors of [10] pointed out that long-wave radiation with T_{b} = 800 K smooths the unevenness of irradiated boundary, and the ice has a smooth surface without roughness. In this connection, at the stage of melting, it is convenient to use the transition to dimensionless coordinates in the ice thickness with the use of Lagrangian transformations $\xi = x/L_2(t)$ [8, 9]. Such a variable makes it possible to fix the coordinate of the phase transition front in boundaries $0\le\xi\le1$, while the front itself becomes plane-parallel (the method of front rectification).

Energy equation (2) and boundary conditions (4) are transformed to the form

$$\frac{\partial\theta_2(\xi,\eta)}{\partial\eta} = \xi\frac{\dot{s}}{s}\cdot\frac{\partial\theta_2(\xi,\eta)}{\partial\xi} + \frac{1}{s^2}\cdot\frac{\partial^2\theta_2(\xi,\eta)}{\partial\xi^2} - \frac{1}{sN}\cdot\frac{\partial\Phi_\nu(\xi,\eta)}{\partial\xi}\text{, at } 0<\xi<1, \quad (15)$$

$$\Lambda X\frac{\partial\theta_1}{\partial\zeta} = \frac{1}{s}\cdot\frac{\partial\theta_2}{\partial\xi} + \frac{1}{N}\left(\frac{A_1}{4}\left(\theta_1^4-\theta_2^4\right) + D_1\Phi(0,\eta)\right)\text{, at } \zeta=1 \text{ and } \xi=0, \quad (16)$$

and boundary condition (6) and Stefan equation (7) with consideration of (8) take the form

$$\theta_2(\xi,\eta)=1,\ \frac{1}{s}\cdot\frac{\partial\theta_2(\xi,\eta)}{\partial\xi} + \mathrm{Bi}\left(\theta_{\mathrm{fil}}-\theta_\infty\right) - \frac{A_2}{N}\left[\Phi^+(\xi,\eta)+\Phi^* + \frac{\theta_{\mathrm{fil}}^4}{4} - \frac{\theta_2^4}{4}\right] = \frac{\dot{s}}{\mathrm{St}}\text{ at } \xi=1. \quad (17)$$

Condition without a film (6) is written as

$$\frac{1}{s}\cdot\frac{\partial\theta_2(\xi,\eta)}{\partial\xi}-\mathrm{Bi}\left(\theta_\infty-\theta_2\right)-\frac{A_2}{N\theta_b^4}\left[\Phi^+(\xi,\eta)+\Phi^*-\frac{\theta_2^4}{4}\right]=\frac{\dot{s}}{\mathrm{St}}. \tag{18}$$

Here, $s(\eta)=L_2(t)/L_2(0)$, $\dot{s}=ds/d\eta$ is the velocity of melting front propagation, $\mathrm{St}= =T_\mathrm{f}c_{\mathrm{p}2}/\gamma$ is the Stefan number. Boundary conditions (15)–(18) are supplemented by initial conditions $\theta_2(\xi,0)=f(\xi)$, $s(0)=1$.

The modified method of mean flows [3, 7] represents a wide range of possibilities for the simplicity of solution and effectiveness of result obtaining. In the framework of this method, the radiation transfer equation is reduced to the system of two nonlinear differential equations for a plane layer of a semitransparent absorbing medium. The differential dimensionless analog of the radiation transfer equation for hemispherical flows $\Phi^\pm$, included as the source term $E(x,t)$ into equation (1), can be represented in the form [3]

$$\begin{aligned}&\frac{d}{d\tau}\Big(\Phi^+(\tau,\eta)-\Phi^-(\tau,\eta)\Big)+\Big(m^+(\tau)\Phi^+(\tau,\eta)-m^-(\tau)\Phi^-(\tau,\eta)\Big)=n^2\Phi_0,\\&\frac{d}{d\tau}\Big(m^+(\tau)l^+(\tau)\Phi^+(\tau,\eta)-m^-(\tau)l^-(\tau)\Phi^-(\tau,\eta)\Big)+\Big(\Phi^+(\tau,\eta)-\Phi^-(\tau,\eta)\Big)=0.\end{aligned} \tag{19}$$

The boundary conditions for equation system (19) in dimensionless variables are written as

$$\begin{aligned}&\Phi^+(0,\eta)=A_1n^2\frac{\theta_2^4(0,\eta)}{4}+D_1\frac{\theta_1^4(1,\eta)}{4}+\left(1-\frac{1-R_1}{n^2}\right)\Phi^-(0,\eta),\\&\Phi^-(1,\eta)=A_2n^2\frac{\theta_2^4(1,\eta)}{4}+D_2F^*+\left[1-\frac{1-R_2}{n^2}-A_2\left(\frac{1+n^2}{n^2}\right)\right]\Phi^+(1,\eta),\end{aligned} \tag{20}$$

here, $\Phi^\pm(\tau,\eta)=\dfrac{2\pi\int_{0(-1)}^{1(0)}I(\tau,\mu)\,\mu d\mu}{4\sigma_0T_r^4}$, $m^\pm(\tau)=\dfrac{\int_{0(-1)}^{1(0)}I(\tau,\mu)\,d\mu}{\int_{0(-1)}^{1(0)}I(\tau,\mu)\mu d\mu}$, $l^\pm(\tau)=\dfrac{\int_{0(-1)}^{1(0)}I(\tau,\mu)\mu^2d\mu}{\int_{0(-1)}^{1(0)}I(\tau,\mu)\mu d\mu}$,

$\Phi_0=B_\nu\Big/\left(4\sigma_0T_f^4\right)$ is the dimensionless density of equilibrium radiation, B_ν is the Plank function of blackbody radiation, n is the refractive index; the values of coefficients $m^\pm, l^\pm$ are determined from the recurrence relationship obtained by means of a formal solution to the equation of radiation transfer [3, 7].

The boundary-value problem solution is reduced to determination of temperatures $\theta_1(\zeta,\eta)$ and $\theta_2(\xi,\eta)$ and densities of resulting radiation fluxes $\Phi(\xi,\eta)$ in range $\{G=0\le\xi\le1,\ 0\le\eta\le\eta_1\}$, which is a flat layer of pure, absorbing, radiating, and non-dispersing ice. The position of the phase transition front varies from 1 to 0. The boundary-value problem (10)–(18) is solved by the finite-difference method, the nonlinear system of implicit differential equations is solved by the sweep and iteration method. When solving the radiation problem, we used iterations, when boundary-value problem (19), (20) is solved at each step by the matrix factorization method. Rapid convergence of such a solution method allows obtaining results with a high degree of accuracy.

Analysis

Below, we present the results of numerical modeling of ice melting on a substrate with the following physical parameters: substrate thickness L_1 = 0.015 m, initial ice thickness $L_2(0)$ = 0.045 m, temperature of the left boundary of substrate and initial temperature of substrate and ice T_{sub} = 256.15 K, temperature of the atmosphere inside the chamber is maintained at constant value T_∞ = 273.15 K, equal to ice melting temperature T_{f}, and constant density of the incident radiation flux E^* = 1162.22 W/m^2. Thermophysical properties of substrate and ice have the following values: thermal conductivity of bakelite and ice, respectively, λ_1 = 0.232 and λ_2 = 1.9 W/(m·K); $a_1 = 1.1\cdot10^{-7}$ m^2/s, $a_2 = 9.3\cdot10^{-7}$ m^2/s; latent heat of phase transition γ= = 335 kJ/kg. The optical parameters of ice are as follows: refractive index n = 1.31, coefficient of volumetric absorption taking into account the long-wavelength region a = 1000 m^{-1} [12], reflection coefficients $r_{1,2}$ = 0.063, and emissivity at the right boundary ε_2 = 0.97.

When solving the problem, two parameters (emissivity ε_1 on the left boundary and coefficient of heat transfer from the right boundary of ice) were varied. At the first stage, emissivity of the left boundary, conjugated with the substrate, is assumed to be ε_1 = 0.002. At the second stage, ε_1 = 0. Such an approach does not allow the temperature of the left boundary to reach the melting temperature ahead of time. The heat transfer coefficient at the first stage is assumed to be h = 7 W/(m^2·K), which equals heat transfer from the vertical wall. At the second stage, h = = 80 W/(m^2·K) is assumed, as in calculations of [10].

The temperature field in the ice layer at different instants of time is shown in Fig. 2. The curves between *1* and *2* refer to the heating stage. The presence of high absorption coefficients on the right boundary (ε_2) and coefficient of volumetric absorption (α) in the medium lead to deep ice heating. The effect of radiation is also observed on the left boundary. At the stage of melting, the temperature curves (lines between *2* and *3*) take on the character of curves, when heat transfer by thermal conductivity prevails. The constant values of the flux densities of resulting radiation $\Phi(\xi, \eta)$ (Fig. 3) throughout the volume at different times indicate that there is no flow gradient in the medium and the difference between intrinsic and absorbed radiations remains unchanged. This circumstance is caused by the large optical thickness of the gray medium $\tau = \alpha x(t)$.

Calculation of the rate of ice melting and its comparison with experimental data of [10] is shown in Fig. 4 (for the convenience of comparison with the experiment, the measurement units are given in accordance with those presented in [10]). It can be seen that calculation, which takes into account a thin water film on the surface,

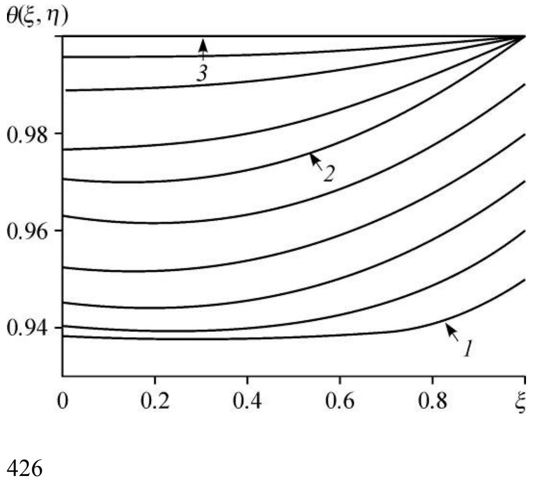

Fig. 2. Temperature field in the process of ice heating and melting.

1 — heating beginning, *2* — melting beginning, *3* — end of melting calculation.

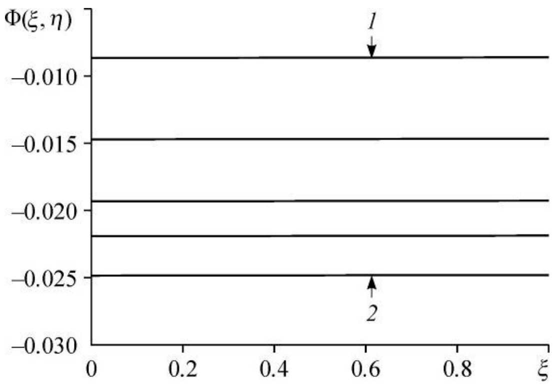

Fig. 3. Density of resulting radiation flux during heating.

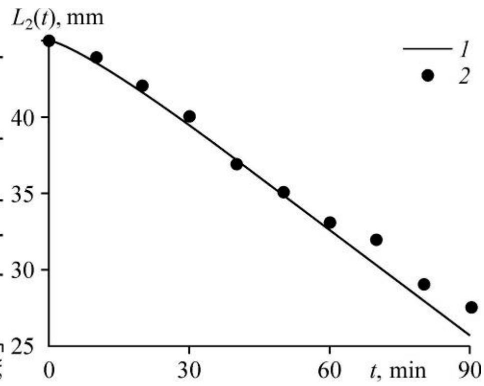

Fig. 4. Rate of ice melting.
1 — calculation of the current study, *2* — data of [10].

is in good agreement with experiment. Left boundary heating with time is shown in Fig. 5. Here, the calculation results diverge quantitatively with the experimental data, but there is a qualitative agreement. This divergence is related to the mathematical model used in this paper. In the model, the left boundary of ice does not absorb, but reflects the radiation. In turn, radiation reflected from the boundaries equalizes the temperature over the entire volume of the medium (curves between *2* and *3* in Fig. 2), including the left boundary.

The importance of taking into account the flowing water film is shown in Fig. 6, where the curves of the rates of virtual melting and heating of the left ice boundary without a film are shown. In this case, the melting stage is significantly stretched in time, and there is no agreement between the calculated and experimental data.

Conclusion

A mathematical model of the experiment on heating and subsequent melting of ice at its radiation by a long-wave source was constructed. To solve the radiation part of the problem, a gray medium model was used. Consideration of the presence of a thin film of ice water on the irradiated surface is in good agreement with the experimental data on the rate of ice melting.

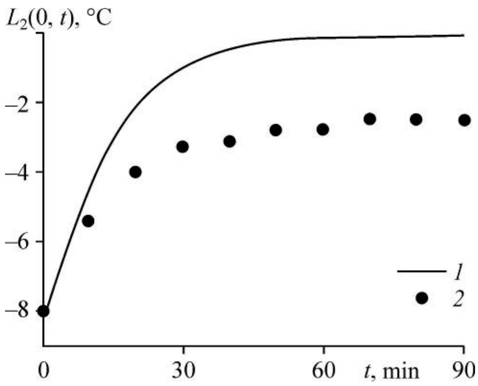

Fig. 5. Heating rate of the left ice boundary at melting.
See symbols in Fig. 4.

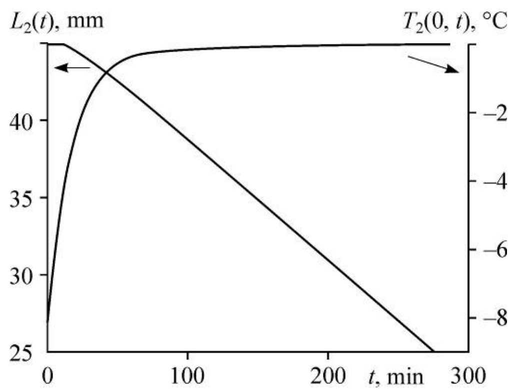

Fig. 6. Rates of melting (left ordinate) and heating (right ordinate) of the left ice boundary without consideration of water film.

Matching of calculated and experimental data makes it possible to consider verification of the single-phase Stefan problem for a semitransparent medium implemented. At the same time, the indicated problem with increasing temperature at the substrate-ice boundary requires further calculations and improvements of the proposed mathematical model.

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