Preface to the New Edition

The first edition of *The Classical Stefan Problem: Basic Concepts, Modelling and Analysis with Quasi-Analytical Solutions and Methods* was published by Elsevier in 2003 as a volume in the North-Holland Applied Mathematics and Mechanics book series. The main objective was to discuss comprehensively, insofar as possible, the theoretical aspects of classical formulations and analysis of some of the topics of the study of the Stefan problem.

The Stefan problem, which has some characteristic features, forms only a small part of a bigger class of problems known as Free Boundary Problems. Even in 2003, the existing literature on Stefan problems was so vast that it seemed feasible to discuss only classical formulations of Stefan problems related to topics such as supercooling, variational inequality, hyperbolic Stefan problems, inverse problems, existence and uniqueness and other aspects of analysis. While discussing variational inequalities, inverse problems, analysis aspects, etc., the discussion of weak solutions was unavoidable, and they were discussed as needed and not comprehensively. There are weak solutions, which are as good as classical solutions. To bridge the gap between other basic sciences and mathematics, and to deepen the understanding of the book's contents, some definitions, theory and results from thermodynamics, metallurgy, physics, applied mathematics, etc., were included as separate chapters.

A 5-year effort by the sole author produced the earlier edition. The positive reviews and readers' indirect encouragement provided the author inspiration and courage to undertake this new project. This new edition features an extensive Chapter 12, which deals with quasianalytical solutions and methods of classical Stefan and Stefan-like problems. Because the class of Stefan-like problems is very large, only those problems whose formulations are similar to those of Stefan problems and whose physics and formulations can be easily explained have been included. Rather than publish the contents of Chapter 12 as a separate book, with the aim of bridging theoretical and solution aspects of Stefan problems, Chapter 12 has been introduced along with earlier chapters.

A solution method is a procedure, and it is not confined only to Stefan problems. It can be applied to any mathematical physics problem. However, for illustrative purposes, the discussion is focused only on formulations of classical Stefan problems and some Stefan-like problems. It is easier to explain the method with clarity in a concise way than the solution, as describing the solutions requires too much space.

Chapter 12 is divided into 10 sections and each section into several subsections. First, Section 12.1 begins with an overview of the aims, objectives, and contents of the chapter. Some preliminaries, such as Green's functions in various geometries, similarity variable, and similarity solution, are discussed.

A lengthy Section 12.2 is devoted to exact analytical solutions pertaining to various geometries, including ellipsoidal and paraboloidal. Sections are assigned to various geometries, different types of heat equations (such as with parameters depending on temperature and space variables), Stefan problems with kinetic conditions, equations with fractional derivatives, multiple-phase problems and dilute binary alloy problems.

Section 12.3 is about series solutions of various types, including short-time solutions.

Section 12.4 deals with the analytical-numerical solutions of Stefan problems. Here the term *analytical-numerical* is used for those solutions in which a complete analytical solution cannot be obtained, and after some analytical derivation, numerical solutions are attempted with the help of some suitable numerical schemes. The analytical derivation part should be dominant and should have some variety if possible. Terms, such as *semianalytical* solution and *quasianalytical* solution, are also used, but we prefer *analytical-numerical* solution. The Adomian decomposition method, variational iteration method, integral equation approach and regularization of Dirac-delta function are also discussed.

Section 12.5 is about analytical-numerical solutions of inverse Stefan problems. In addition to the methods discussed in Section 12.4, the homotopy analysis method and some regularization methods are also discussed.

The analytical-numerical solutions of hyperbolic Stefan problem are discussed in Section 12.6, with the background information provided in Chapter 8. A rigorous background of deriving Green's function in the planar case is also briefly discussed.

Section 12.7 is about the use of complex variable methods in solving solidification/melting problems and Hele-Shaw problems. The singularity development in suction problems, types of singularity and its possible removal are described.

Approximate solutions and methods are discussed in Section 12.8. A major portion of this section is devoted to the heat-balance integral method and its refinements and variations, such as RIM, ARIM and hybrid methods. Weighted residual methods, such as the Galerkin method and the orthogonal collocation method, are discussed briefly with only a few illustrative examples. This section also discusses the first variation, variational principles and the derivation of Euler's equation for a given functional using calculus of variations. Finally, the section describes the method of constructing a functional for a given problem whose first variation or Euler's equation will be the required differential equation.

A considerable amount of literature exists on perturbation solutions because of their easiness in application. Therefore the emphasis in Section 12.9 is on applications of the homotopy analysis method and the homotopy perturbation method. Regular perturbation and singular perturbation methods applied to solutions of Stefan problems in various geometries as well as a variety of formulations are discussed. Applications of the methods of strained coordinates and matched asymptotic expansions are also illustrated.

Section 12.10 offers brief reviews of some supplementary references. Chapter 12 concludes with an extensive bibliography of about 455 references.

The presentation of material in all 12 chapters is characterized by discussions based on the thorough study of full-length research papers. The discussion includes my own comments on many published works in Chapter 12. Reporting purely numerical solutions was never the objective of the chapter, as that would require several separate volumes. However, for analytical-numerical solutions, the highlights of numerical solutions and results are given very briefly along with names of the software used if given in the referenced paper.

Invariably, the author thinks conceptually in terms of obtaining the solution first and then devising a method to achieve it. This is why *solution* comes first in the book title, followed by *method*.