Preface

This volume emphasizes studies related to classical Stefan problems. The term 'Stefan problem' is generally used for heat transfer problems with phase-changes such as from the liquid to the solid. Stefan problems have some characteristics that are typical of them, but certain problems arising in fields such as mathematical physics and engineering also exhibit characteristics similar to them. The term 'classical' distinguishes the formulation of these problems from their weak formulation, in which the solution need not possess classical derivatives. Under suitable assumptions, a weak solution could be as good as a classical solution. In hyperbolic Stefan problems, the characteristic features of Stefan problems are present but unlike in Stefan problems, discontinuous solutions are allowed because of the hyperbolic nature of the heat equation. The numerical solutions of inverse Stefan problems, and the analysis of direct Stefan problems are so integrated that it is difficult to discuss one without referring to the other. So no strict line of demarcation can be identified between a classical Stefan problem and other similar problems. On the other hand, including every related problem in the domain of classical Stefan problem would require several volumes for their description. A suitable compromise has to be made.

The basic concepts, modelling, and analysis of the classical Stefan problems have been extensively investigated and there seems to be a need to report the results at one place. This book attempts to answer that need. Within the framework of the classical Stefan problem with the emphasis on the basic concepts, modelling and analysis, I have tried to include some weak solutions and analytical and numerical solutions also. The main considerations behind this are the continuity and the clarity of exposition. For example, the description of some phase-field models in Chapter 4 arose out of this need for a smooth transition between topics. In the mathematical formulation of Stefan problems, the curvature effects and the kinetic condition are incorporated with the help of the modified Gibbs–Thomson relation. On the basis of some thermodynamical and metallurgical considerations, the modified Gibbs–Thomson relation can be derived, as has been done in the text, but the rigorous mathematical justification comes from the fact that this relation can be obtained by taking appropriate limits of phase-field models. Because of the unacceptability of some phase-field models due to their so-called thermodynamical inconsistency, some consistent models have also been described. This completes the discussion of phase-field models in the present context.

Making this volume self-contained would require reporting and deriving several results from tensor analysis, differential geometry, nonequilibrium thermodynamics, physics and functional analysis. I have chosen to enrich the text with appropriate references so as not to enlarge the scope of the book. The proofs of propositions and theorems are often lengthy and different from one another. Presenting them in a condensed way may not be of much help to the reader. Therefore only the main features of proofs and a few results have been presented to suggest the essential flavour of the theme of investigation. However at each place, appropriate references have been cited so that inquisitive readers can follow them on their own.

Each chapter begins with basic concepts, objectives and the directions in which the subject matter has grown. This is followed by reviews—in some cases quite detailed—of published works. In a work of this type, the author has to make a suitable compromise between length restrictions and understandability. I have followed my best judgement in this regard. I hope the readers will appreciate my efforts.

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