MSc Econometrics course
"Time Series Econometrics"
Assignments

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Assignments

The optional assignments are :

(A) "Quasi-Maximum Likelihood for Dynamic Factor Model" (QML)

(B) "Collapsed Kalman filter for Dynamic Nelson-Siegel Model" (CKFS)

(C) "Financial Cycles in the Euro Area and US" (FINC)

(D) "Stochastic Volatility with Realized Measures" (SVR)

Please choose the one that you like most !

Assignment A : Quasi-MLE for Dynamic Factor Model

Dynamic factor models are used oftentimes for macroeconomic forecasting. A key example is forecasting GDP growth.

Many contributions

- Stock & Watson (JASA, JBES 2002)
- Doz, Giannone & Reichlin (JEct 2011, RESTAT 2013)
- Bańbura & Rünstler (IJF 2011), Bańbura & Modugno (JAE 2014)
- Jungbacker, Koopman & van der Wel (JEDC 2014)
- Bräuning & Koopman (IJF 2014)

RESTAT = review of economics & statistics, JASA = journal of the american statistical association, JBES = journal of business economics statistics, JEct = journal of econometrics, IJF = international journal of forecasting, JEDC = journal of economic dynamic control

Important for Assignment A are Stock & Watson (JBES 2002), Doz, Giannone & Reichlin (JEct 2011) and Bräuning & Koopman (IJF 2014).

Principal Components Analysis (PCA)

Introductions to Principal Components Analysis, and the related Factor Analysis, are presented in most "Multivariate Statistics" study books: the one of T.W. Anderson is a good reference, while a short classic book on the topic is Lawley & Maxwell (1963).

The Wikipedia pages on PCA and Factor Analysis are rather good.

The PCA in our context is closely related to a Singular Value Decomposition (SVD) applied to a sample variance (or correlation) matrix of a dataset, for typically many variables.

The first principal component (PC) relates to the highest eigenvalue and the corresponding eigenvector provides the weights for the linear relationship of the variables. The resulting first PC represents the most commonly variation in the data set. The second highest eigenvalue is associated with the second PC, etc.

Use of PCs in macroeconomic forecasting

Let y_t be the time series of interest, the key variable, and let x_t be a very large column vector representing the many "instrumental" variables that are used to improve the forecasting of y_t.

Stock and Watson (2002) advocate to construct principal components series F_t from large data base of x_t variables. Then a parsimonious way to use x_t for the h-steps ahead forecasting of y_t is via the dynamic regression

$$y_{t+h} = \phi(L)y_t + \beta(L)F_t + \epsilon_t, \qquad t = 1, \ldots, n,$$

where $\phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + \ldots$, $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots$.

Many contributions in the literature have focussed on the appropriate choice of dimension for x_t and, most notably, for F_t, see Bai & Ng (2002).

Ox code for computing Principal Components

```
  PrincipalCmp(const mX, const NrCmp)
{
    decl mc = correlation(mX);
    decl eigval, eigvec;

    eigensym(mc, &eigval, &eigvec);

    decl percvar = 100 * eigval / sumr(eigval);
    decl mpc = standardize(mX) * eigvec;

    println("eigen values", eigval);
    println("percentage = ", percvar);
    println("cum perc = ", cumulate(percvar'));

    return standardize(mpc[][:NrCmp-1]);
}
```

Dynamic factor model

The dynamic factor model for the joint analysis of y_t and x_t is given by

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{bmatrix} \Lambda_y \\ \Lambda_x \end{bmatrix} f_t + u_t, \qquad u_t \sim \mathrm{NID}(0, \Sigma_u)$$

where u_t is assumed to be NID noise and where Σ_u is typically a diagonal variance matrix.

The underlying, unobserved vector of dynamic factors f_t can be modelled by the vector autoregressive process of order p, VAR(p),

$$f_t = \Phi_1 f_{t-1} + \ldots \Phi_p f_{t-p} + \eta_t,$$

where η_t is typically IID noise, mutually independent of u_t.

We assume standardized factors and imply specific initial conditions, see Slides Week 3. We have a linear Gaussian state space model.

Maximum likelihood estimation, full MLE

The number of unknown parameters in the DFM

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{bmatrix} \Lambda_y \\ \Lambda_x \end{bmatrix} f_t + u_t, \qquad f_t = \Phi_1 f_{t-1} + \ldots \Phi_p f_{t-p} + \eta_t,$$

is increasing quickly when dimension x_t becomes larger and larger. Some options for maximum likelihood estimation (MLE) :

- Standard MLE, direct maximization of the loglikelihood function wrt all unknown parameters, including those in Λ, is challenging;
    - Jungbacker and Koopman (2015) propose data transformation and collapse...
    - model dimension can be reduced considerably;
    - loglikelihood evaluation is fast via Kalman filter;
    - parameter dimension remains, but estimation task is challenging: we consider two other approaches next.

Maximum likelihood estimation, quasi-MLE

The number of unknown parameters in the DFM

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{bmatrix} \Lambda_y \\ \Lambda_x \end{bmatrix} f_t + u_t, \qquad f_t = \Phi_1 f_{t-1} + \ldots \Phi_p f_{t-p} + \eta_t,$$

is increasing quickly when dimension x_t becomes larger and larger.
Some options for maximum likelihood estimation (MLE) :

- Doz, Giannone and Reichlin (2011), two-step approach :
    1. replace unobservable f_t by principal component F_t and apply regression to both equations to obtain estimates for the parameters;
    2. replace parameters by these estimates and continue analysis based on state space representation above to analyze and obtain forecasts using the Kalman filter.
- The parameter and factor estimates are consistent and have good asymptotic properties.

Maximum likelihood estimation, quasi-MLE

The number of unknown parameters in the DFM

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{bmatrix} \Lambda_y \\ \Lambda_x \end{bmatrix} f_t + u_t, \qquad f_t = \Phi_1 f_{t-1} + \ldots \Phi_p f_{t-p} + \eta_t,$$

is increasing quickly when dimension x_t becomes larger and larger. Some options for maximum likelihood estimation (MLE) :

- Bräuning & Koopman (2014) propose a collapsing approach :
  - replace x_t by F_t and set $\Lambda_x = I$, that is

$$y_t = \Lambda_y f_t + u_{y,t}, \qquad F_t = f_t + u_{f,t}.$$

  - standard MLE for remaining parameters;
  - use this model to estimate parameters, analysis and forecasting.
- Extension is to let y_t have a factor of its own, say θ_t, we have

$$y_t = \theta_t + \Lambda_y f_t + u_{y,t},$$

  where θ_t can be an AR or cyclical dynamic process.

Assignment A, questions (a) – (b)

(a) Consider the data set "DFMtestdata.xlsx" (on Canvas), or a similar dataset of your own, with the single y_t variable in the second column and all the x_t variables in the remain columns (first column is time stamp). Here y_t is the target variable that we want to forecast. Compute the first four principal components for x_t and present them in a graphic.

(b) Carry out the Stock & Watson (SW) analysis for one-step ahead forecasting of y_t, $h = 1$. Take $\phi(L) = \phi_0 + \phi_1 L$, $\beta(L) = \beta_0 + \beta_1 L$ and consider 2 PCs. The forecast evaluation period is $3\frac{1}{2}$ years (last 41 observations). For each forecast, recompute the PCs and re-estimate the unknown coefficients by OLS. The forecast for y_{t+1} is computed from the values of y_t, y_{t-1} and F_t, F_{t-1}. Compute the MSE, MAE and MAPE for the forecast evaluation period.

Assignment A, questions (c) – (e)

(c) Implement the Kalman filter and smoother for the dynamic factor model with standardized factors and VAR(1) dynamics, make sure the initialization is correct. Allow flexibility in choosing the number of factors. You can randomly choose values for the unknown loading matrices Λ_y, Λ_x, the diagonal variance matrix Σ_u and the autoregressive coefficient matrix Φ_1.

(d) Carry out the Doz et.al (DGR) analysis of quasi-MLE for the unknown matrices, for the purpose of one-step ahead forecasting, $h = 1$. Consider a VAR(1) process for the two dynamic factors. Present your estimates of the unknown matrices in a Table.

(e) Based on your quasi-ML estimates, estimate the two dynamic factors using the Kalman filter smoother and present them in a graph. How do they compare to the PCs ? Comment.

(f) Carry out the Doz et.al. analysis for one-step ahead forecasting, $h = 1$. Consider the model as estimated in (d). The forecast evaluation period is last $3\frac{1}{2}$ years (41 observations). For each one-step ahead forecast, re-estimate the unknown coefficients by quasi-ML. The forecast for y_{t+1} is computed by the Kalman filter. Compute the MSE, MAE and MAPE for the forecast evaluation period.

(g) Consider the Bräuning & Koopman (BK) model and implement the Kalman filter smoother for their multivariate UC model with standardized factors and VAR(1) dynamics, with correct initialization. Allow flexibility in choosing the number of factors. You can randomly choose values for the loading vector Λ_y, the diagonal variance matrix Σ_u (for y_t and F_t) and the autoregressive coefficient matrix Φ_1 (for f_t).

(h) Redo the items (d), (e) and (f) for the BK analysis. You only need to estimate Λ_y and Σ_u by MLE; you can estimate Φ_1 as it is done for the DGR method. For each item, comment on differences in the results obtained from DGR and BK.

(i) For all three approaches to forecasting y_t (SW, DGR and BK), you can consider variations such as increasing the number of factors and/or adding more lags to the model. How different or sensitive are the forecasting results for these variations ? Consider some variations and report the corresponding MSE, MAE and MAPE values in a Table. Discuss your findings. Which methods and which variations produce the best forecast results in your study ?

Assignment B : Collapsed Kalman filter and smoother (CKFS)

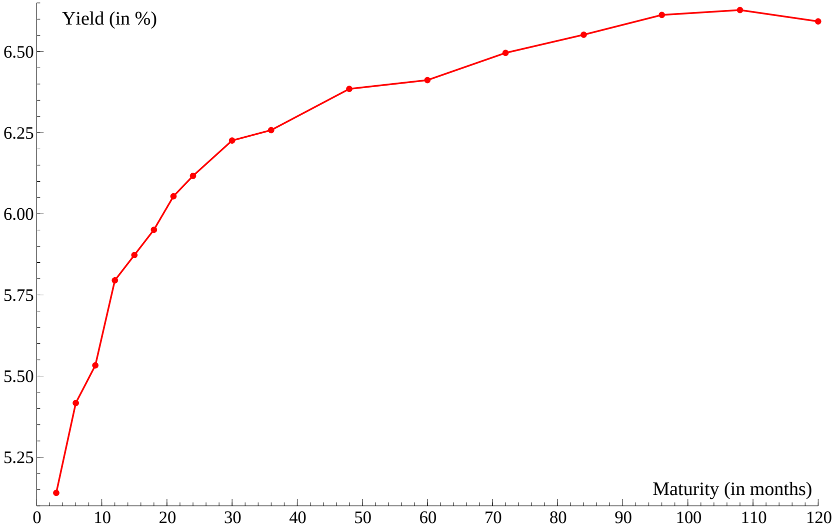
This assignment allows you to study the results of Jungbacker and Koopman (2015) and to apply them to the modelling of US interest rates (yield curve) by means of the dynamic Nelson-Siegel model.

Good references are :

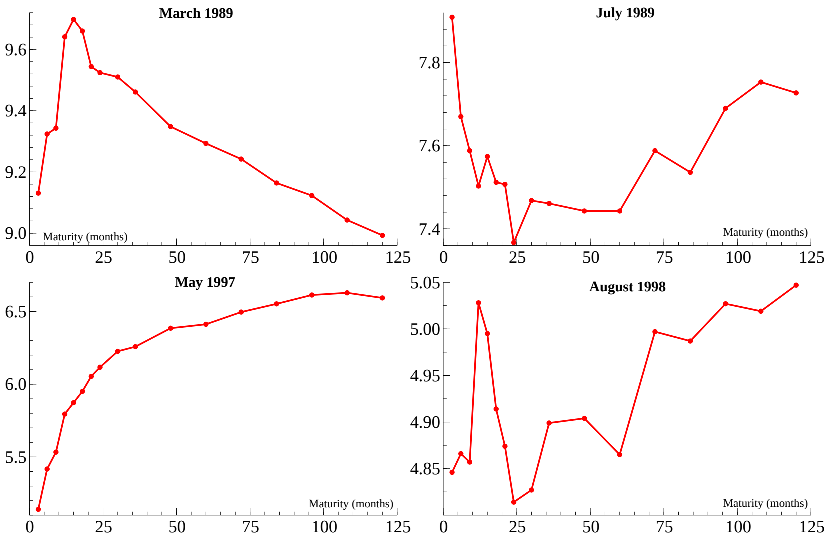
- Nelson and Siegel (J of Business, 1987)
- Diebold and Li (J of Econometrics, 2006)
- Diebold, Rudebusch and Aruoba (J of Econometrics, 2006)
- Koopman, Mallee and van der Wel (KMW, J Business Economics Statistics, 2010)
- Jungbacker and Koopman (Econometrics J, 2015)

The dataset of KMW is made available to you via Canvas (Fama-Bliss data)

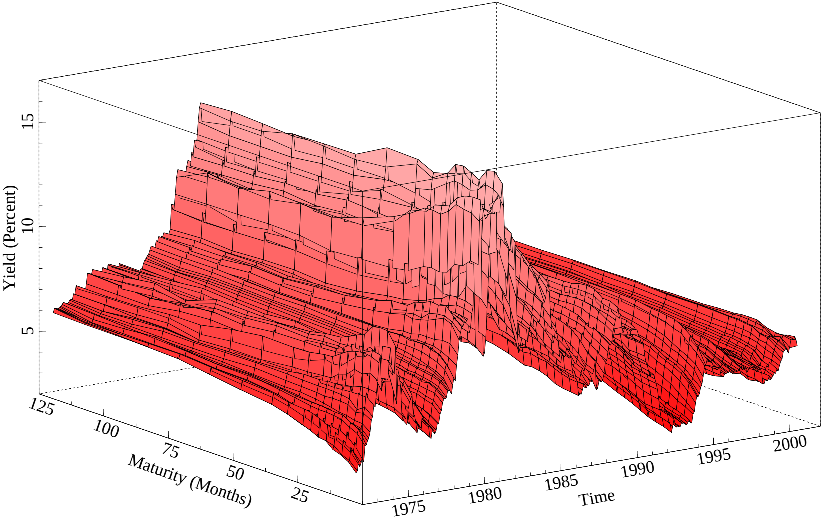
U.S. Yield Curve May 1997



U.S. Yield Curves for Four Months



U.S. Yield Curves over Time



Nelson-Siegel model for Interest Rates

- Nelson and Siegel (1987) provide a statistical model for the Yield curve with some economic interpretation.
- At month t, the yield is a function of maturity:

$$\theta(\tau;\lambda,\beta_t)=\beta_{1t}+\beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)+\beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}-e^{-\lambda\tau}\right),$$

  with τ time to maturity (in months), λ a coefficient, and β_{1t}, β_{2t} and β_{3t} (time-varying) factors.
- At each month the yield curve is given by three underlying factors and their associated factor loadings.
- Each of the factors carries an interpretation:
  - First Factor (β_{1t}) – *Level*,
  - Second Factor (β_{2t}) – *Slope*,
  - Third Factor (β_{3t}) – *Curvature*.

Dynamic Nelson-Siegel model for Forecasting

- When observing a series of interest rates $y_t(\tau_i)$ for a set of N maturities, $\tau_1 < \ldots < \tau_N$, at times $t = 1, \ldots, n$, for some λ value, we can use OLS to estimate the factors:

$$\begin{aligned} y_t(\tau_i) &= \theta_t(\tau_i; \lambda, \beta_t) + \varepsilon_{it} \\ &= \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \varepsilon_{it}, \end{aligned}$$

  with $\varepsilon_{1t}, \ldots, \varepsilon_{Nt}$ assumed independent, mean zero and constant variance σ^2 for a given t.
- Diebold and Li (2006) recognize that the factors β_{1t}, β_{2t} and β_{3t} are serially correlated and thus forecastable.
- The forecasts of these factors can be used to obtain yield curve forecasts, which outperform other methods such as the random walk and univariate autoregressive models.

State Space representation of DNS model

- Diebold, Rudebusch and Aruoba (2006) go a step further and treat the Nelson-Siegel framework as a state space model or a dynamic factor model.
- The general state space model is given by:

$$
\begin{aligned}
y_t &= Z_t \alpha_t + \varepsilon_t, & \varepsilon_t &\sim NID(0, H_t), \\
\alpha_{t+1} &= T_t \alpha_t + R_t \eta_t, & \eta_t &\sim NID(0, Q_t),
\end{aligned}
$$

  with α_t the unobserved state, initial condition $\alpha_1 \sim \mathrm{N}(a_1, P_1)$ and with system matrices Z_t, H_t, T_t, R_t and Q_t.
- The model is linear Gaussian. From the multivariate normal distribution properties, a filtering algorithm (Kalman Filter) can be derived to compute the likelihood function.
- Standard references include Anderson and Moore (1979) and Durbin and Koopman (2012).

DNS model in state space form

Dynamic Nelson-Siegel model as a state space model is given by:

$$
\begin{aligned}
y_t &= \Gamma(\lambda)\beta_t + \varepsilon_t, & \varepsilon_t &\sim NID(0, \Sigma_\varepsilon), \\
\beta_{t+1} &= (I - \Phi)\mu + \Phi\beta_t + \eta_t, & \eta_t &\sim NID(0, \Sigma_\eta),
\end{aligned}
$$

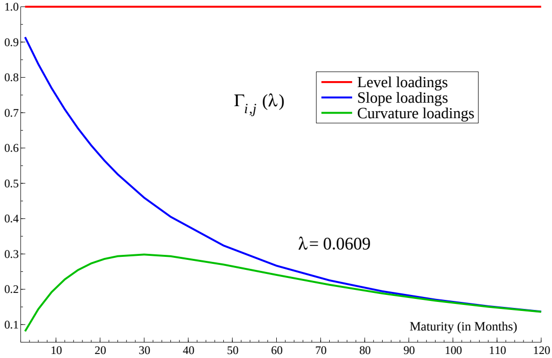
with

$$
\begin{aligned}
y_t &= (y_t(\tau_1), \ldots, y_t(\tau_N))', \\
\Gamma_{ij}(\lambda) &= \begin{cases} 1, & j = 1, \\ \left(1 - e^{-\lambda \cdot \tau_i}\right) \ / \ \lambda \cdot \tau_i, & j = 2, \\ \left(1 - e^{-\lambda \cdot \tau_i} - \lambda \cdot \tau_i e^{-\lambda \cdot \tau_i}\right) \ / \ \lambda \cdot \tau_i, & j = 3, \end{cases} \\
\beta_t &= (\beta_{1t}, \beta_{2t}, \beta_{3t})', \\
\varepsilon_t &= (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})', \\
\eta_t &= (\eta_{1t}, \eta_{2t}, \eta_{3t})'.
\end{aligned}
$$

Here τ_i is the maturity of the i-th interest rate series.
Typical choices for λ are 0.0609 and 0.077.

The λ coefficient and factor loadings

- The parameter λ determines the shape of the factor loadings



- In most studies assumptions are made about λ:
  - Diebold and Li (2006) assume λ is fixed at 0.0609
  - Diebold, Rudebusch and Aruoba (2006) estimate λ

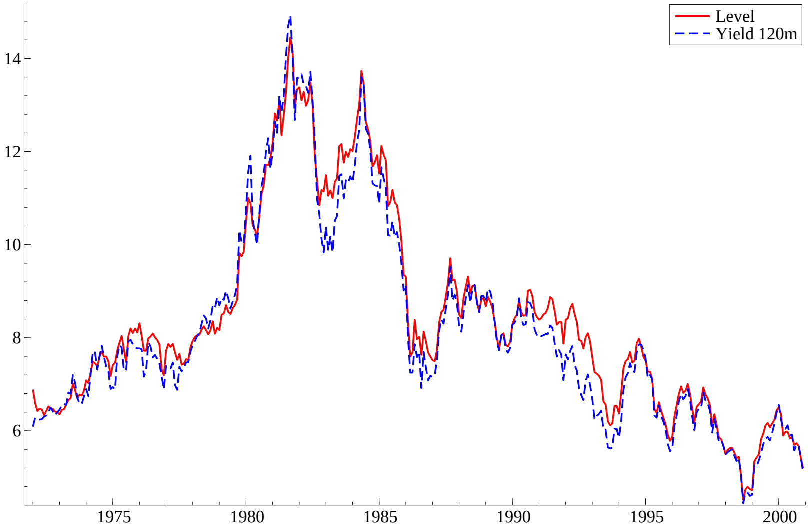
MLE estimates

DNS Model : Φ and μ				
	Level_{t-1}	Slope_{t-1}	Curvature_{t-1}	Constant (μ)
Level_t	0.997^{**} 0.00811	0.0271^{**} 0.00889	-0.0216^{*} 0.0105	8.03^{**} 1.27
Slope_t	-0.0236 0.0167	0.942^{**} 0.0176	0.0392 0.0212	-1.46^{**} 0.527
Curvature_t	0.0255 0.023	0.0241 0.0257	0.847^{**} 0.0312	-0.425 0.537

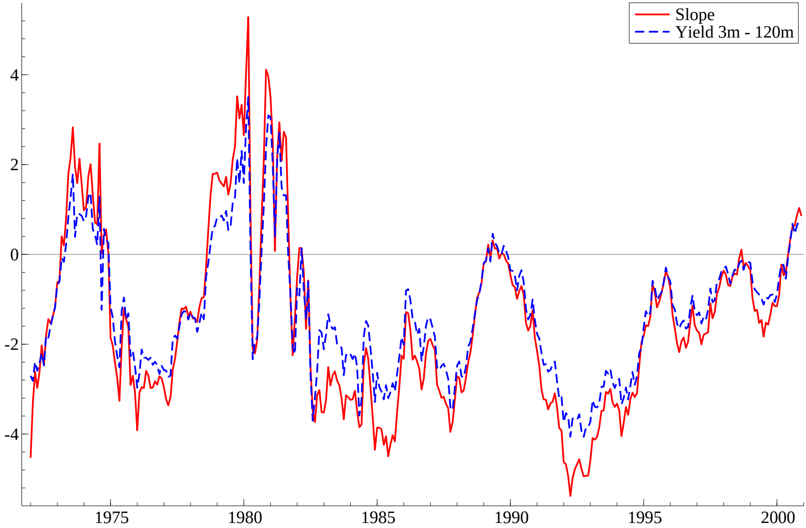
DNS Model : Σ_η			
	Level_t	Slope_t	Curvature_t
Level_t	0.0949^{**} 0.00841	-0.014 0.0113	0.0439^{*} 0.0186
Slope_t		0.384^{**} 0.0306	0.00927 0.0344
Curvature_t			0.801^{**} 0.0812

Choice for λ is 0.0609.

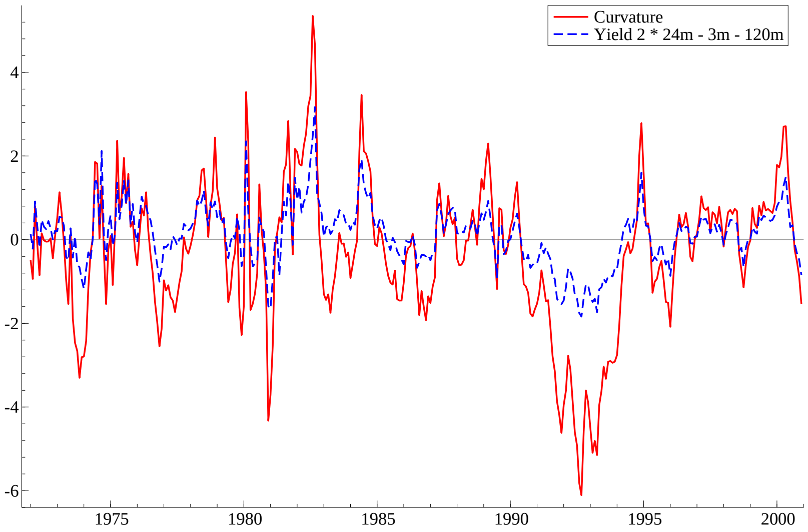
U.S. Yield Curve : estimated "Level" factor



U.S. Yield Curve : estimated "Slope" factor (spread)



U.S. Yield Curve : estimated "Curvature" factor



Assignment B, questions (a) – (c)

(a) Provide the details of state space model formulation of the Dynamic Nelson-Siegel (DNS) model.

(b) Develop computer code that computes the factor loadings $\Lambda_{i,j}(\lambda)$ as a function of λ. This procedure enables you to plot the factor loadings for level, slope and curvature, as above. How different are these plots when different values of λ are used ?

(c) The factors are modelled as a VAR(1) process. However, for this assignment, you can consider a trivariate random walk (RW) for the factors :

$$\beta_{t+1} = \beta_t + \eta_t, \qquad \eta_t \sim \mathrm{NID}(0, \Sigma_\eta),$$

where Σ_η is a 3×3 variance matrix. Implement the Kalman filter for the DNS model with $\beta_t \sim$ RW and $\Sigma_\eta = 0.05 \times I_3$.

Assignment B, questions (d) – (g)

(d) Develop computer code that calculates the Gaussian loglikelihood function for the DNS model with $\beta_t \sim$ RW and for the parameters in Σ_η. Initially, you can keep λ at a fixed value of your choice.

(e) Estimate Σ_η by MLE and report the results. How do your estimates compare with those reported above ?

(f) Given the ML estimate of Σ_η, produce plots of the smoothed estimated of β_t. How do they compare with the plots presented above ?

(g) Can you repeat (d)..(f), by treating λ as an unknown coefficient and estimate it together with Σ_η ? What are your findings ?

Assignment B, questions (h) – (m)

(h) Please return to (c) and implement the collapsed method of Jungbacker and Koopman (2015) for the Kalman filter and smoother. Do you obtain the same results for the Kalman filter values a_t and P_t, for $t = 1, \ldots, n$.

(i) Please repeat part (d) but now for the collapsed method. You need to obtain the same loglikelihood values for the two implementations, do you ?

(j) Please repeat part (e) using the collapsed method. Is this method much faster doing MLE ?

(k) Repeat part (f) using the collapsed method.

(m) When there is still time, you can consider a VAR(1) model rather than a RW model for β_t. Do you obtain similar results with the collapsed method as those reported by Diebold, Rudebusch and Aruoba (2006) and Koopman, Mallee and van der Wel (2010) ?

Assignment C : Financial Cycles in Euro Area

Background : Consider recent article of Koopman, Lit and Lucas (2016) : "Model-based business cycle and financial cycle decomposition for Europe and the U.S." which you can download from Canvas : Finc.pdf.

This assignment is relatively simple : replicate the empirical results of this article.

Are you able to replicate the analysis and estimation results, or at least are your results similar ?

Assignment C : Financial Cycles in Euro Area and U.S.

The only request is to reproduce the Tables and Figures in the article.

You need to collect the most recent data from the sources as provided in the article.

If your numerical results are different, they can still be very interesting. Hence a good documentation of your work is useful.

A part of the delivery of your Assignment, is the computer code. You are allowed to use pre-programmed packages for the state space methods.

Assignment D : Stochastic Volatility model with Realized Measures (SVR)

Background : denote closing price at trading day t by P_t with its return

$$r_t = \log(P_t \,/\, P_{t-1}) = \Delta \log P_t = \Delta p_t, \qquad t = 1, \ldots, n.$$

The price p_t can be regarded as a discretisized realisation from a continuous-time log-price process $\log P(t)$, that is

$$\mathrm{d} \log P(t) = \mu \,\mathrm{d}\, t + \sigma(t) \,\mathrm{d}\, W(t),$$

where μ is the mean-return, $\sigma(t)$ is a continuous volatility process and $W(t)$ is standardised Brownian motion. We concentrate on the volatility process and we let $\log \sigma(t)^2$ follow a so-called Ornstein-Uhlenbeck process

$$\log \sigma(t)^2 = \xi + H(t), \qquad \mathrm{d}\, H(t) = -\lambda H(t) \,\mathrm{d}\, t + \sigma_\eta \,\mathrm{d}\, B(t),$$

where ξ is constant, $0 < \lambda < 1$, σ_η is the "volatility-of-volatility" coefficient (strictly positive) and $B(t)$ is standardised Brownian motion, independent of $W(t)$.

Stochastic volatility model

The general framework can lead to a statistical model for the daily returns y_t. By applying the Euler-Maruyama discretisation method, we obtain

$$y_t = \mu + \sigma_t \varepsilon_t, \qquad \log \sigma_t^2 = \xi + H_t, \qquad H_{t+1} = \phi H_t + \sigma_\eta \eta_t,$$

where $\phi = 1 - \lambda$ so that $0 < \phi < 1$. Since both σ_t and u_t are stochastic processes, we have a nonlinear time series model.

However, after data transformation $x_t = \log(y_t - \mu)^2$ and some redefinitions, we obtain

$$x_t = h_t + u_t, \qquad h_{t+1} = \omega + \phi h_t + \sigma_\eta \eta_t,$$

where $u_t = \log \varepsilon_t^2$, $\omega = (1 - \phi)\xi$ and $h_t = H_t + \xi$.

We obtain the linear AR(1) plus noise model, but the disturbance u_t is not necessarily Gaussian...

This is the basis of QML for the stochastic volatility (SV) model.

Stochastic volatility model

The SV model for $x_t = \log(y_t - \mu)^2$ is

$$x_t = h_t + u_t, \qquad h_{t+1} = \omega + \phi h_t + \sigma_\eta \eta_t,$$

where $u_t = \log \varepsilon_t^2$. When we assume ε_t is Gaussian, u_t is generated from a $\log \chi^2$ distribution from which the mean and variance are well-defined.

The quasi-Maximum Likelihood (QML) method adopts the Kalman filter to compute the likelihood; do as if ε_t is Gaussian with mean and variance corresponding to those of the $\log \chi^2$ distribution.

The analysis above can be regarded as an approximate analysis.
In Weeks 4-6 we have treated methods for an exact analysis.
This is the main focus of Assignment D.

Stochastic Volatility: model and analysis

For a time series of financial returns y_t, the Stochastic Volatility (SV) model, with $\mu = 0$, is given by

$$y_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \exp(h_t), \quad \varepsilon_t \sim \mathrm{NID}(0,1),$$

where the log-volatility h_t follows a stationary autoregressive process given by

$$h_{t+1} = \omega + \phi h_t + \sigma_\eta \eta_t, \qquad \eta_t \sim \mathrm{NID}(0,1),$$

for $t = 1, \ldots, n$ with $h_1 \sim \mathrm{N}(\omega \,/\, 1 - \phi, \sigma_\eta^2 \,/\, 1 - \phi^2)$ and with the disturbances ε_t and η_t.

For Assignment D, you adopt the SV model to analyze a daily financial return series from `Yahoo Finance` or any other source. The return series can be from daily stock prices (eg IBM), daily stock index (eg S&P100), daily exchange rate (eg Dollar/Euro), commodity prices (eg corn), etc. You also need to collect a corresponding daily time series of a *realized measure*. Clearly state in your report which series are analyzed.

Assignment D, questions part (a) – (c)

(a) The financial series is in actual levels. Transform the series to returns, present graphs and descriptive statistics.

(b) The SV model can be made linear by transforming the returns data to $x_t = \log y_t^2$. This is the basis of the QML method. Compute x_t and present a graph. Hint : avoid taking logs of zeros, you can do so by demeaning y_t.

(c) The disturbances in the model for x_t will not be normally distributed. But we can **assume** that they are normal with mean and variance corresponding to those of the $\log \chi^2$ distribution. Estimate the unknown coefficients by the QML method using the Kalman filter and present the results in a Table.

Assignment D, questions part (d) – (g)

(d) Take the QML estimates as your final estimates. Compute the smoothed mean of h_t based on the approximate model for x_t by using the Kalman filter and smoother.

(e) By adopting the QML estimates for the unknown coefficients in the model for x_t, consider the SV model for y_t and compute the smoothed mode of h_t using Kalman filter smoothing methods.

(f) Compute the smoothed mean of h_t based on the SV model for y_t by using SPDK **and** the numerically accelerating importance sampling (NAIS) method.

(g) Compare the three different estimates for h_t (display them in graphs) and discuss your findings. Also compare these with your collected realized measure.

Assignment D, questions part (h) – (k)

Now consider the SV model with Student's t disturbances as given by

$$y_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \exp(h_t), \quad \varepsilon_t \sim \mathrm{t}(\nu),$$

where $\mathrm{t}(\nu)$ refers to the **standardized** Student's t distribution with ν degrees of freedom. The autoregressive specification for h_t remains the same as above.

(h) How would you modify the approximate maximum likelihood method for estimating the unknown parameters including ν ?

(i) Estimate the parameters by maximizing the simulated likelihood function for the Student's t SV model using an importance sampling method.

(j) Redo questions (e) – (f) but now for the Student's t SV model and the ML estimates of (i).

(k) Compare your estimates and analysis between the Gaussian and Student's t SV models.

Assignment D, questions part (m) – (o)

Finally we consider the SV model with Gaussian or Student's t disturbances where σ_t^2 is also function of the daily realized measure RM_t as given by

$$y_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \exp[h_t + \gamma \log(\mathrm{RM}_t)],$$

where disturbance $\varepsilon_t \sim \mathrm{N}(0,1)$ or $\varepsilon_t \sim \mathrm{t}(\nu)$ with $\mathrm{t}(\nu)$ refers to the standardized Student's t distribution with ν degrees of freedom. The autoregressive specification for h_t remains the same as above.

(m) Can you provide an estimate of γ via the method of maximum likelihood ? How does this model extension affect your results reported earlier ?

(n) How would you measure the goodness-of-fit improvement of including RM_t in the SV model ?

(o) Present the filtered estimates of h_t using the particle filter for both SV models (normal and t) and compare them with an earlier estimate of h_t. Comment on your findings.