Quantitative Macroeconomics - Homework 2

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Question 1. Function Approximation: Univariate.

1 Approximation of $f(x) = x^{.321}$ with a Taylor series around $\overline{x} = 1$.

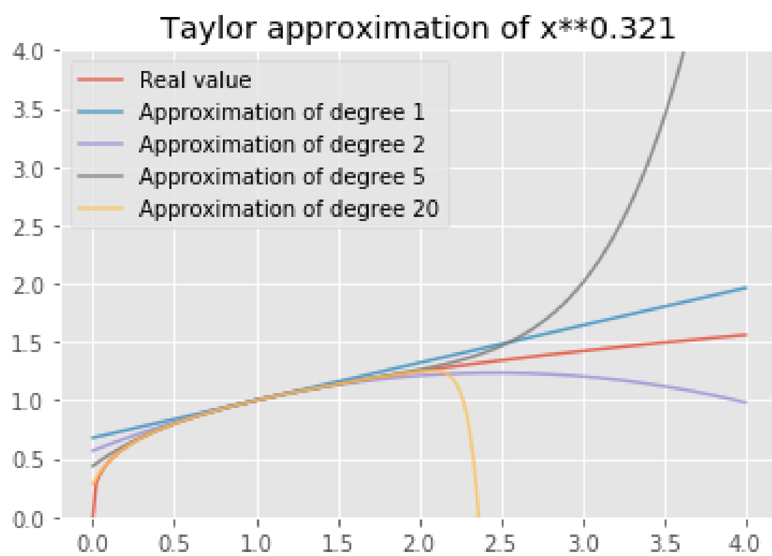

Figure 1

The further from $\overline{x}$ we get, the bigger error of approximation becomes. What more, paradoxally, approximations of of higher degrees are less accurate.

2 Approximation of $f(x) = fracx + |x|2$ with a Taylor series around $\overline{x} = 2$.

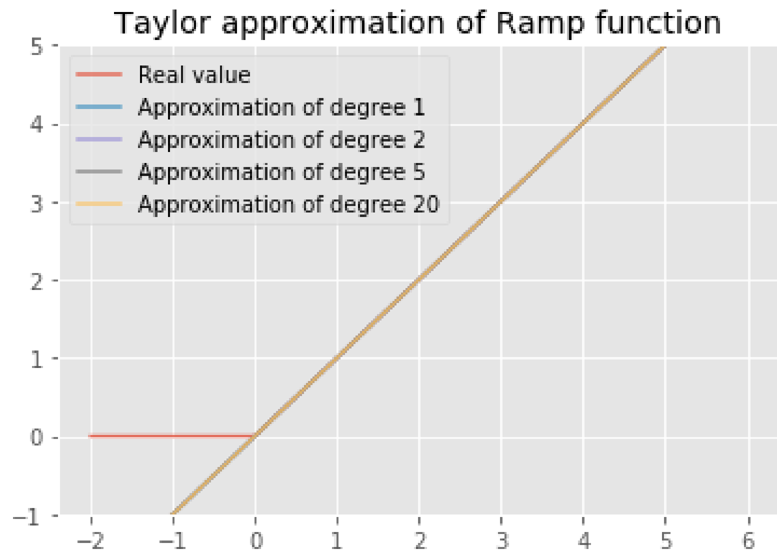

Figure 2

As the ramp function is affinne, its derivative in $\overline{x} = 2$ is constant, and vanishes in higher orders. Thus Taylor series is constant and the same for all orders of approximations. What more, as we are approximation around $\overline{x} = 2$, the series 'cannot see' discontinuity in point 0.

3 Monomial interpolations

- Evenly spaced interpolation nodes

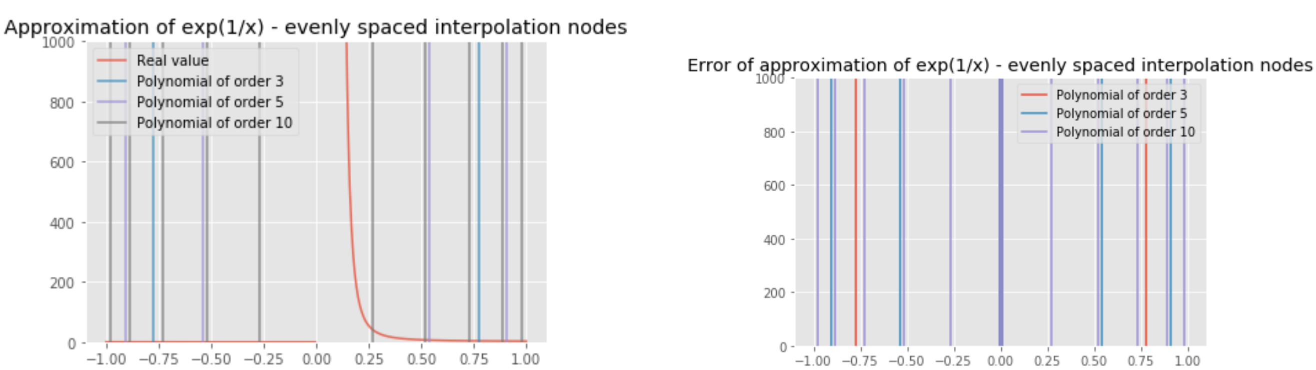

Figure 3

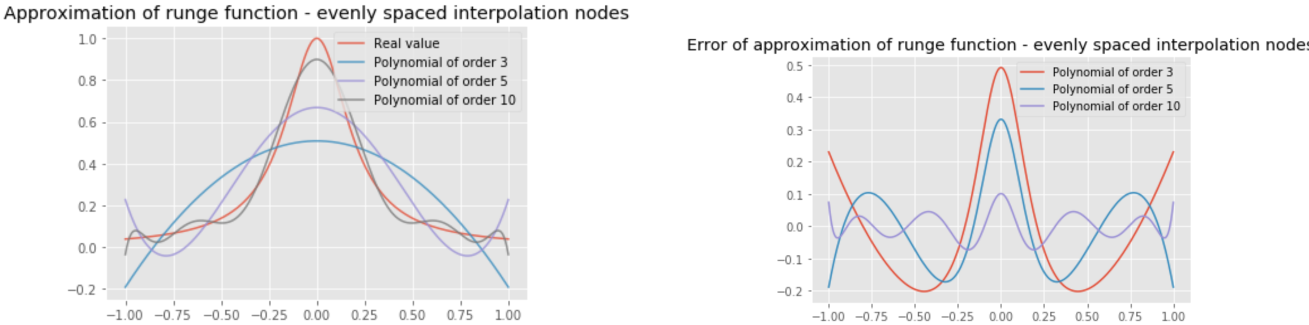

Figure 4

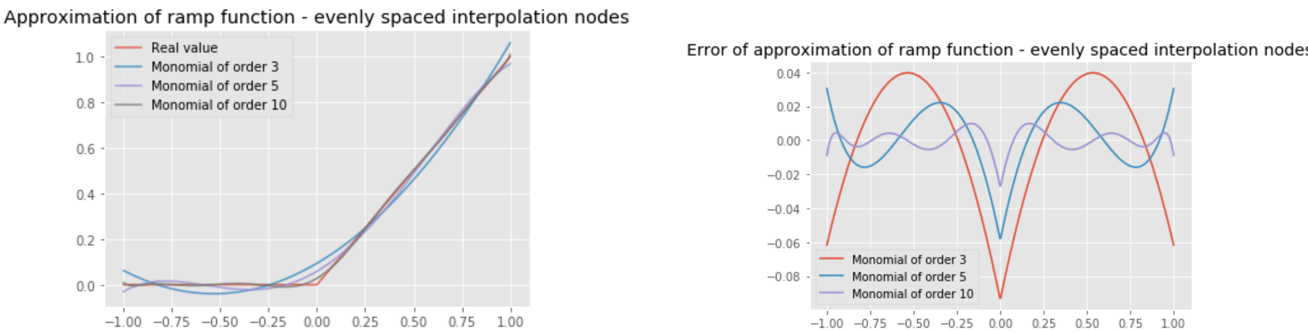

Figure 5

- Chebyshev interpolation nodes

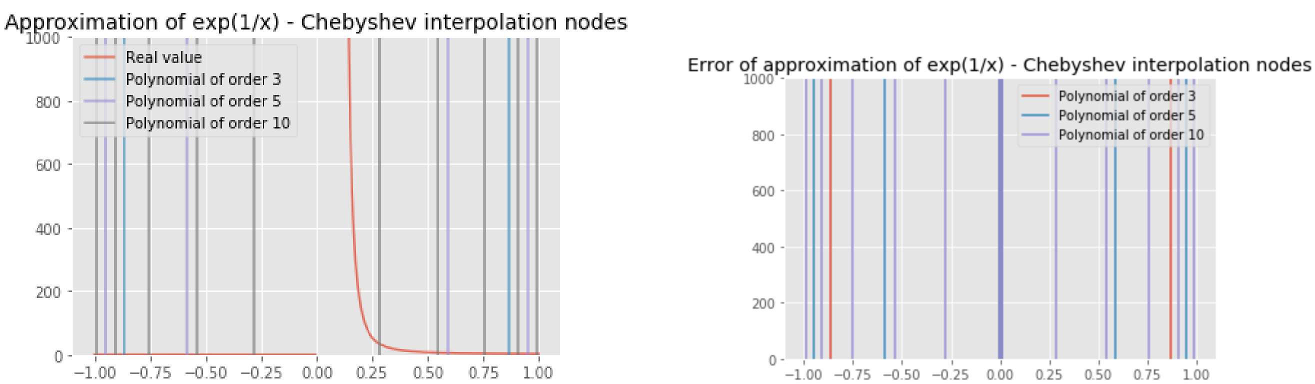

Figure 6

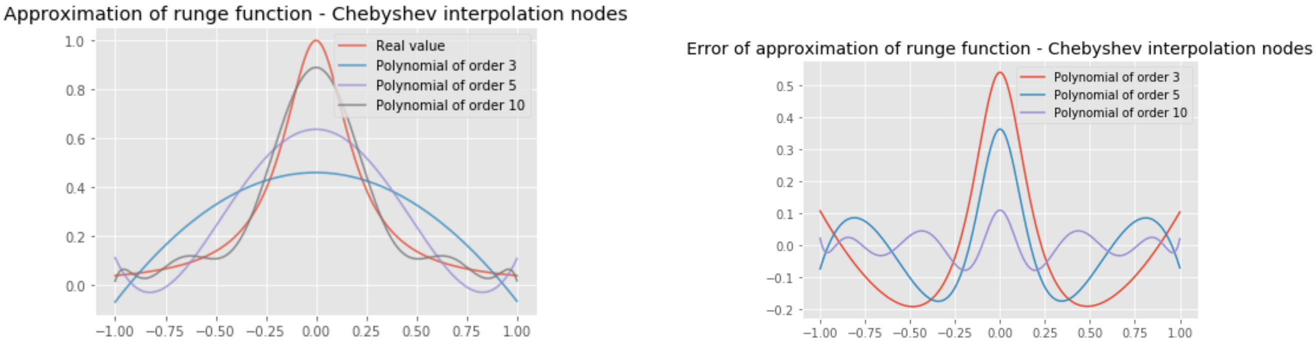

Figure 7

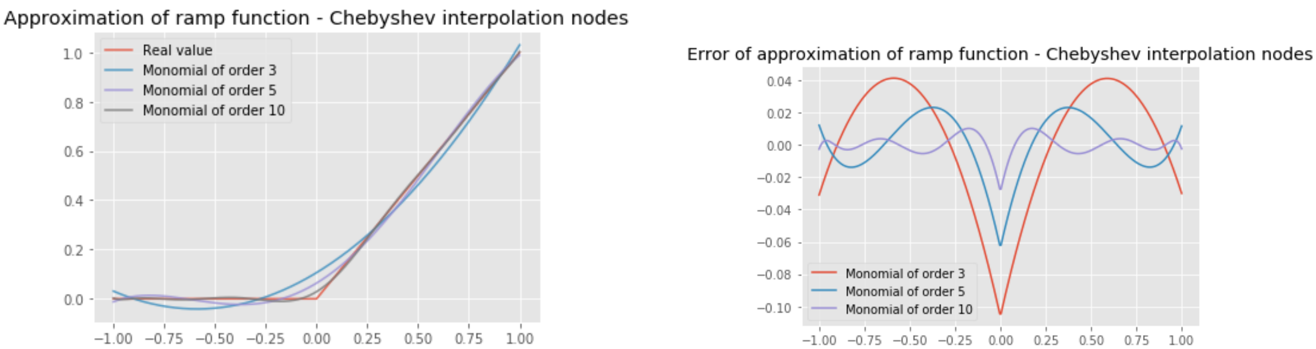

Figure 8

- Chebyshev interpolation nodes and Chebyshev polynomial

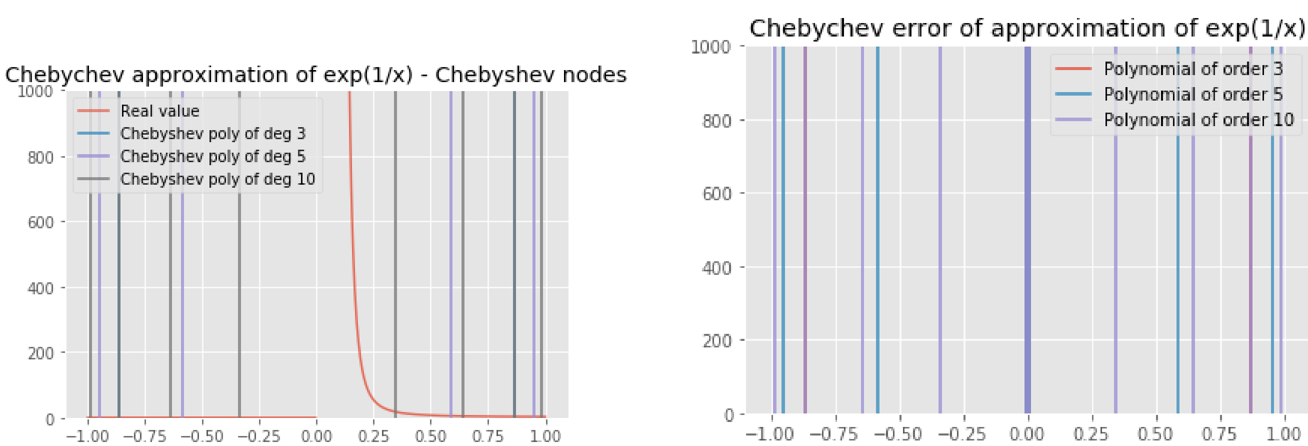

Figure 9

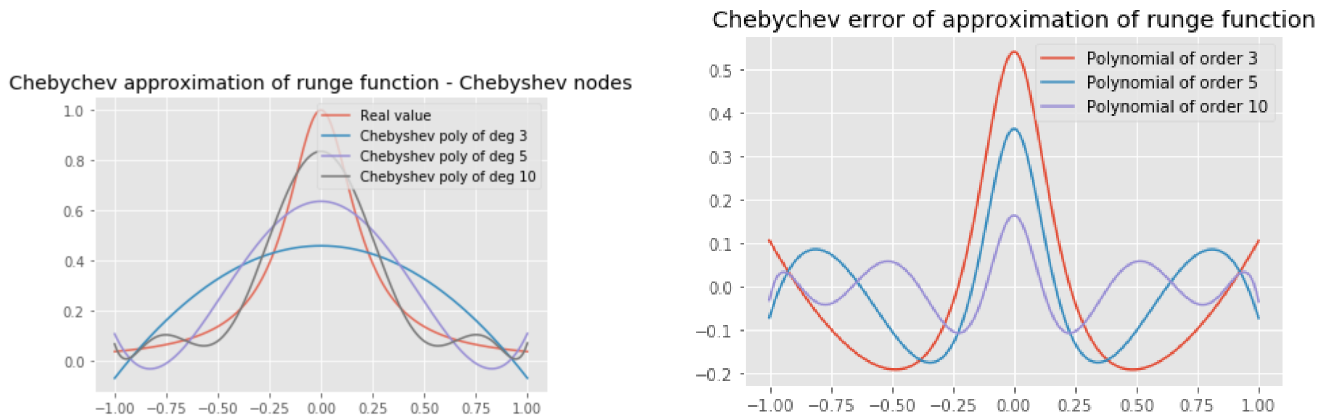

Figure 10

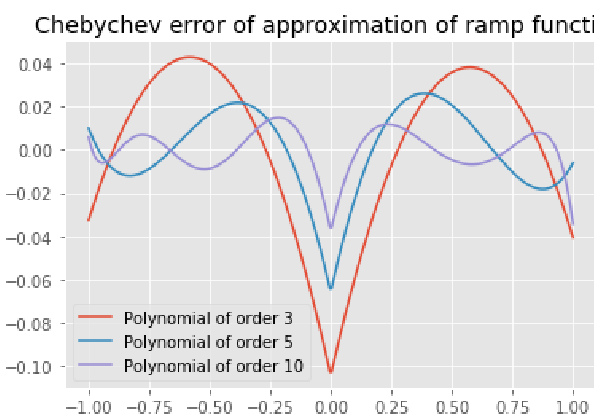

Figure 11

What one can easily see, the approximation of $f(x) = e^{\frac{1}{x}}$ is inaccurate for every way of approximation, because of its convergence to $\propto$ at point 0.
For runge function using Chebyshev interpolation nodes, decreases error at the minium and maximum of the scale. However using additional Chebyshev polynomials doesn't really improve the accuracy.
For ramp function using Chebyshev interpolation nodes, decreases error at the minium and maximum of the scale. However at the same time it incresaes the error in the miidle of scale, especially for low degrees of polynomials.

Question 2. Function Approximation: Multivariate.

1 Show that σ is the ES
From definition:

$$\sigma = \frac{\partial ln(\frac{h}{k})}{\partial ln(MRS)}, \quad where \quad MRS = \frac{\frac{\partial f}{\partial k}}{\frac{\partial f}{\partial h}} \tag{1}$$

Then:

$$\frac{\partial f}{\partial k} = \frac{\sigma}{\sigma - 1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}-1}(1-\alpha)\frac{\sigma-1}{\sigma}k^{\frac{\sigma-1}{\sigma}-1} \tag{2}$$

$$\frac{\partial f}{\partial k} = \frac{\sigma}{\sigma - 1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}(1-\alpha)\frac{\sigma-1}{\sigma}k^{\frac{-1}{\sigma}} \tag{3}$$

Analogically:

$$\frac{\partial f}{\partial h} = \frac{\sigma}{\sigma - 1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}\alpha\frac{\sigma-1}{\sigma}h^{-\frac{1}{\sigma}} \tag{4}$$

After crossing out:

$$MRS = \frac{\frac{\partial f}{\partial k}}{\frac{\partial f}{\partial h}} = \frac{1-\alpha}{\alpha}(\frac{k}{h})^{-\frac{1}{\sigma}} \tag{5}$$

And:

$$\sigma = \frac{\partial ln(\frac{h}{k})}{\partial ln(\frac{1-\alpha}{\alpha}(\frac{k}{h})^{-\frac{1}{\sigma}})} \tag{6}$$

Let's assume:

$$x = ln(\frac{1-\alpha}{\alpha}(\frac{k}{h})^{-\frac{1}{\sigma}}) \tag{7}$$

Then:

$$ln(\frac{h}{k}) = \sigma(ln(\alpha) - ln(1-\alpha) + x) \tag{8}$$

And:

$$\sigma = \frac{\partial \sigma(ln\alpha - ln(1-\alpha) + x)}{\partial x} = \sigma \tag{9}$$

2 Compute labor share for this economy From definition Labor Share is:

$$LS = \frac{hw}{f}, \quad where \quad w - wages \tag{10}$$

From assumption of fully competitive markets, wages are:

$$w = \frac{\partial f}{\partial h} = \frac{\sigma}{\sigma - 1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}\alpha\frac{\sigma-1}{\sigma}h^{-\frac{1}{\sigma}} \tag{11}$$

Substituting:

$$LS = \frac{\alpha h^{\frac{\sigma-1}{\sigma}}}{((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})} \tag{12}$$

3-4 Approximation f(k; h) with a 2-dimensional Chebyshev regression algorithm

Graphs below are presented for real value as well as approximations of degrees: 3, 9 and 15. There are available 3d projections of values and error in tadem with isoquants representations.

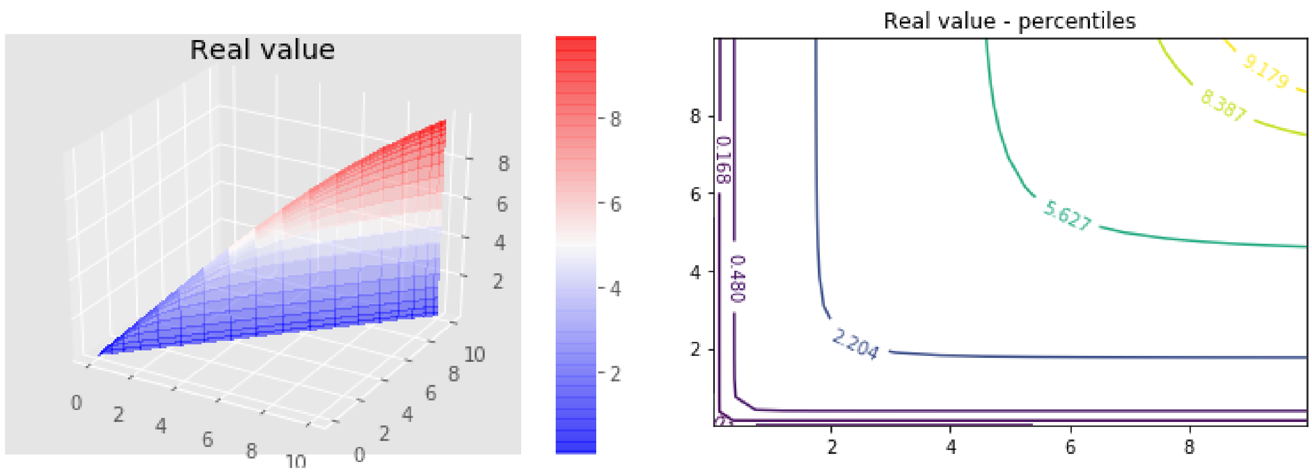

Figure 12

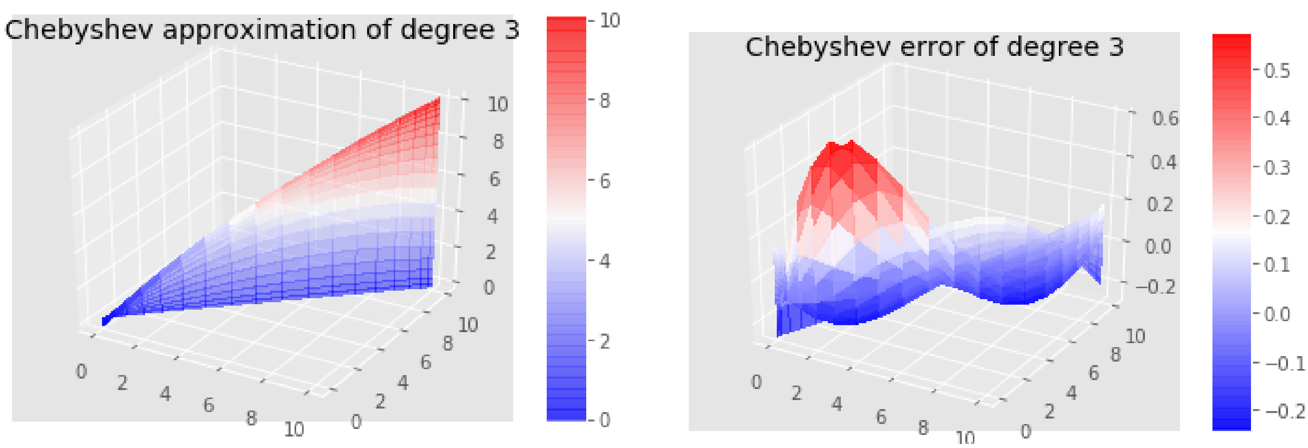

Figure 13

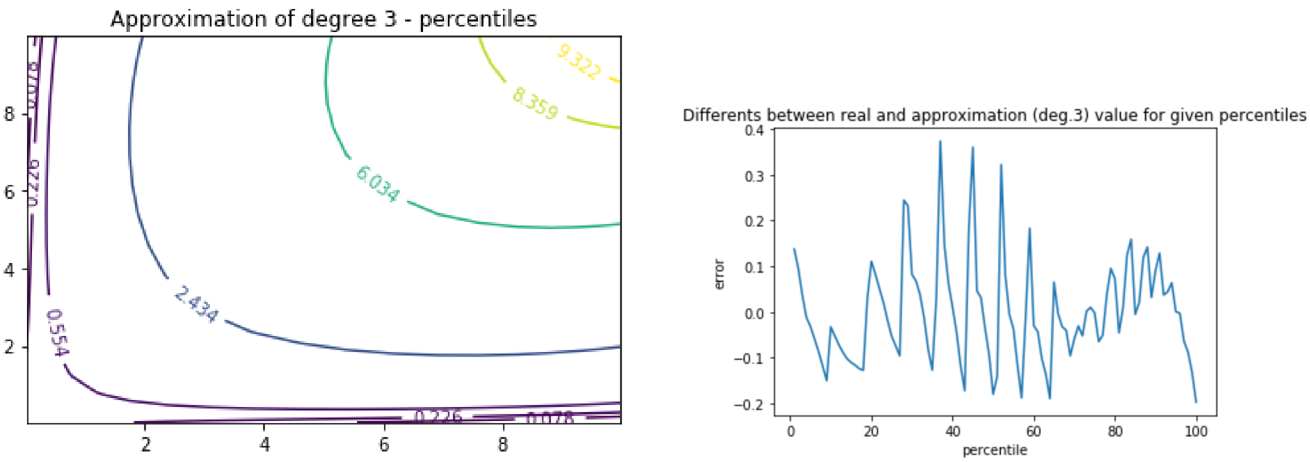

Figure 14

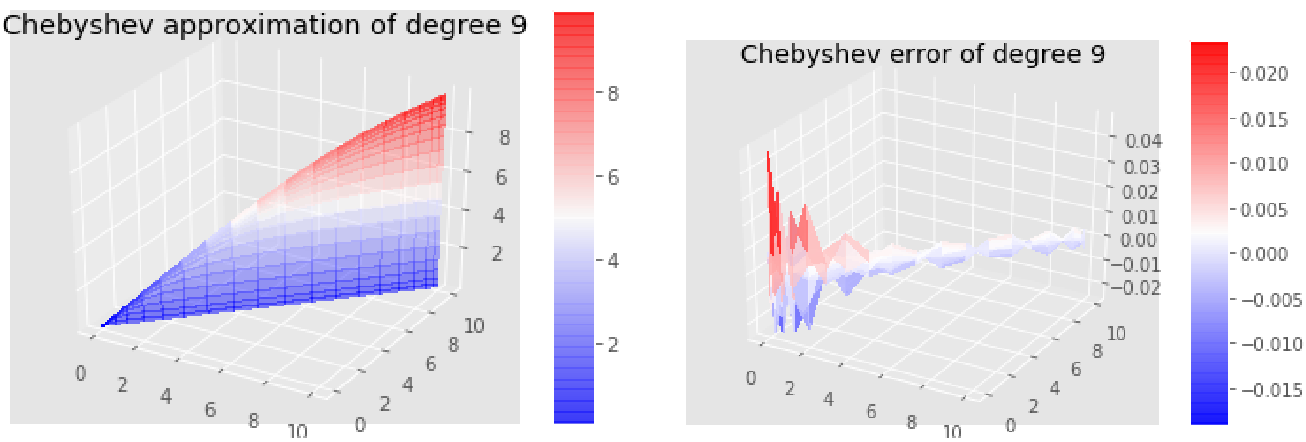

Figure 15

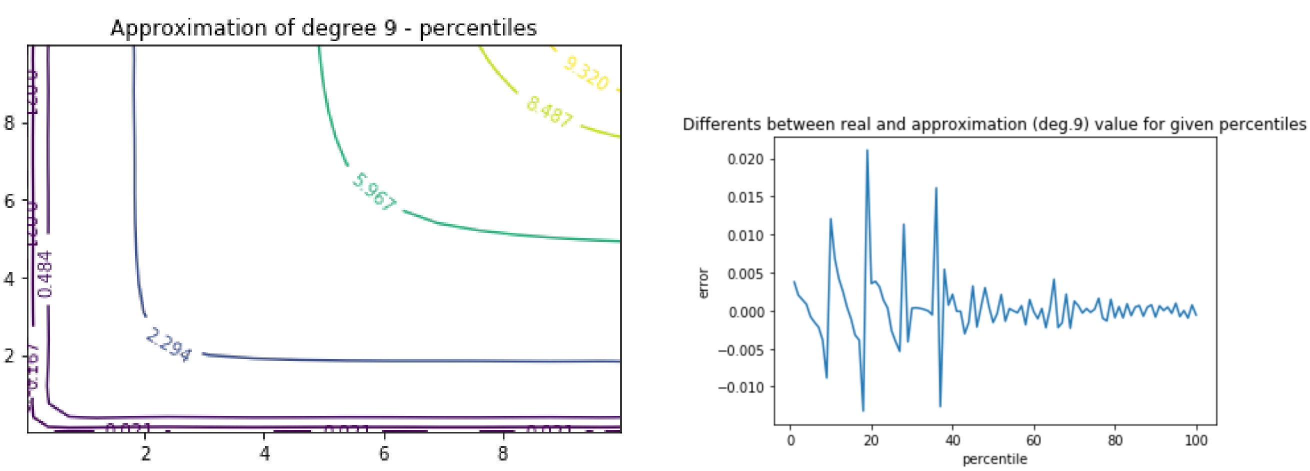

Figure 16

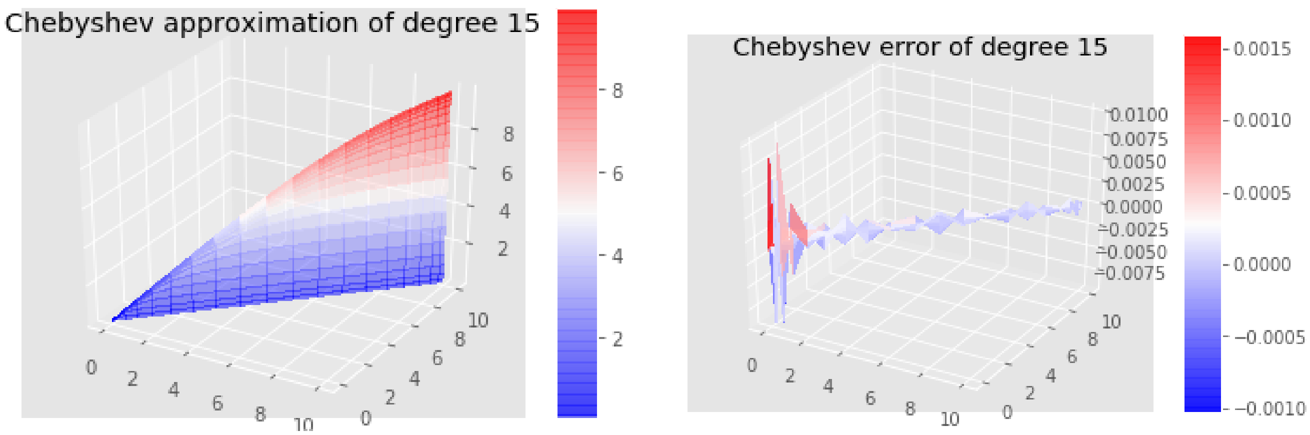

Figure 17

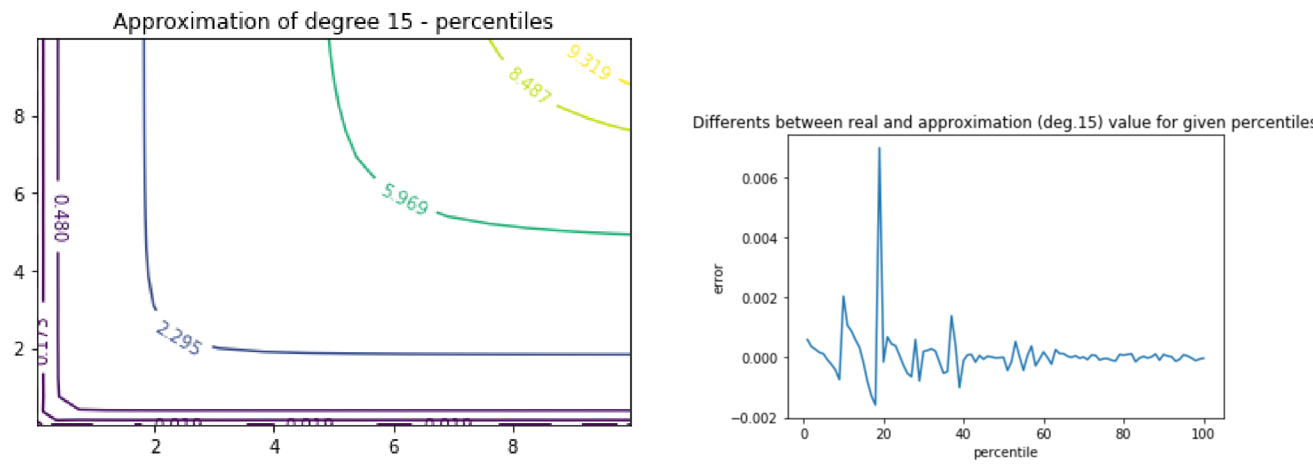

Figure 18

In the simple graphs one cannot easily see differences between real values and approximations. Only after looking specifically at error, one can see, if given approximations are accurate.
In general the error variates highly, but decreases with increasing degrees of approximations and level of percentile

5 Approximation f(k; h) with a 2-dimensional Chebyshev regression algorithm for $\sigma = 5$

Graphs presented below are analogical to ones from poin 3-4. Values are computer for σ =5

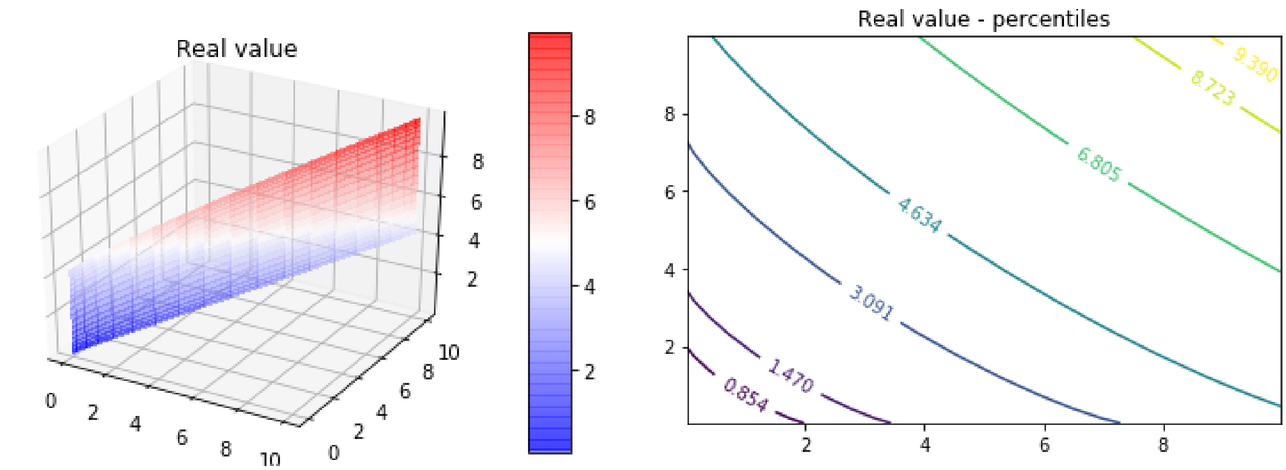

Figure 19

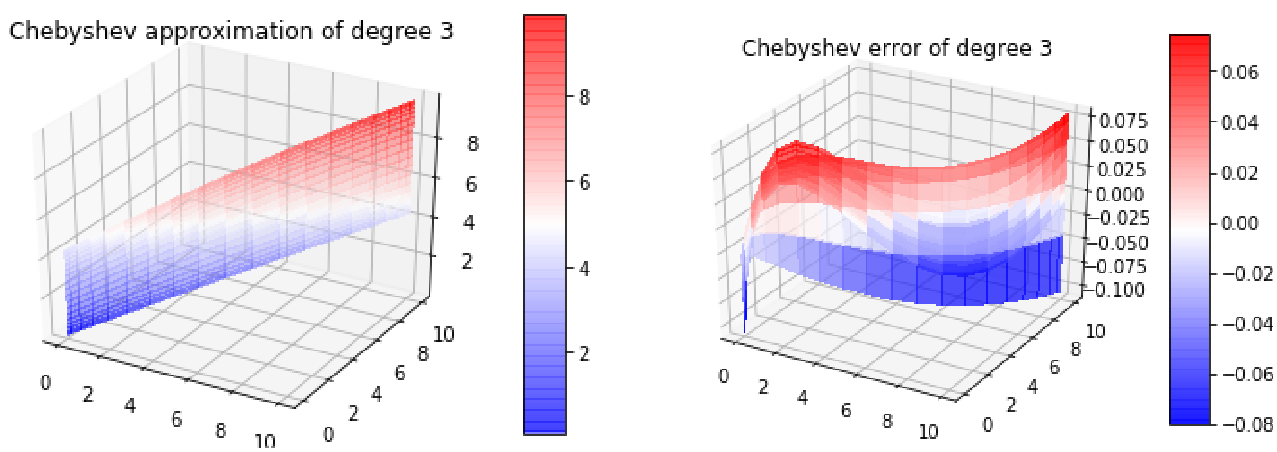

Figure 20

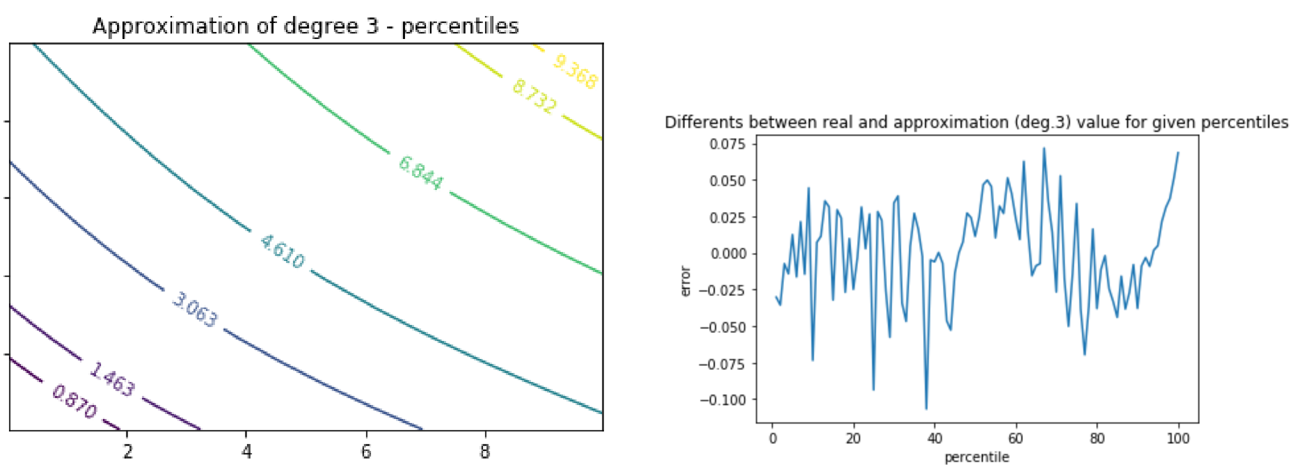

Figure 21

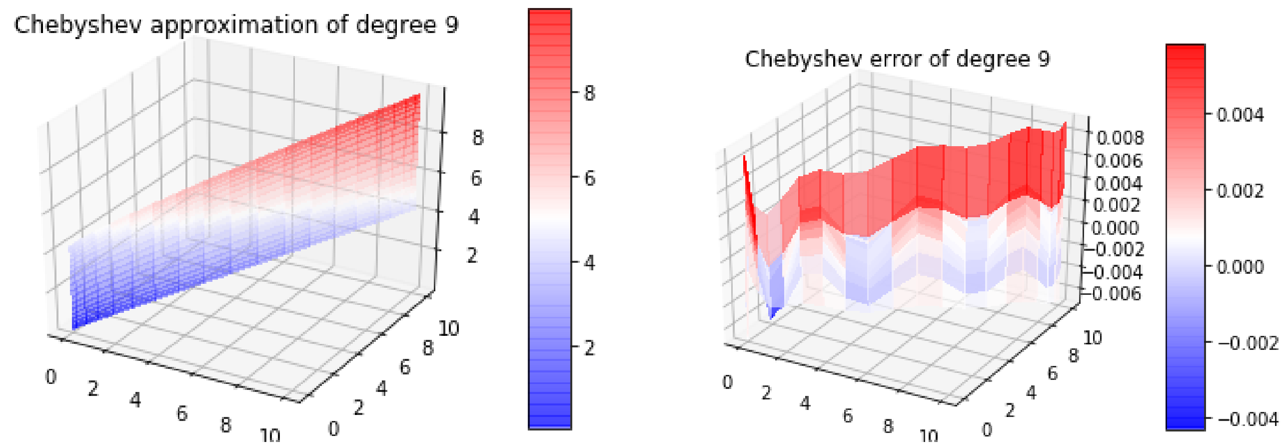

Figure 22

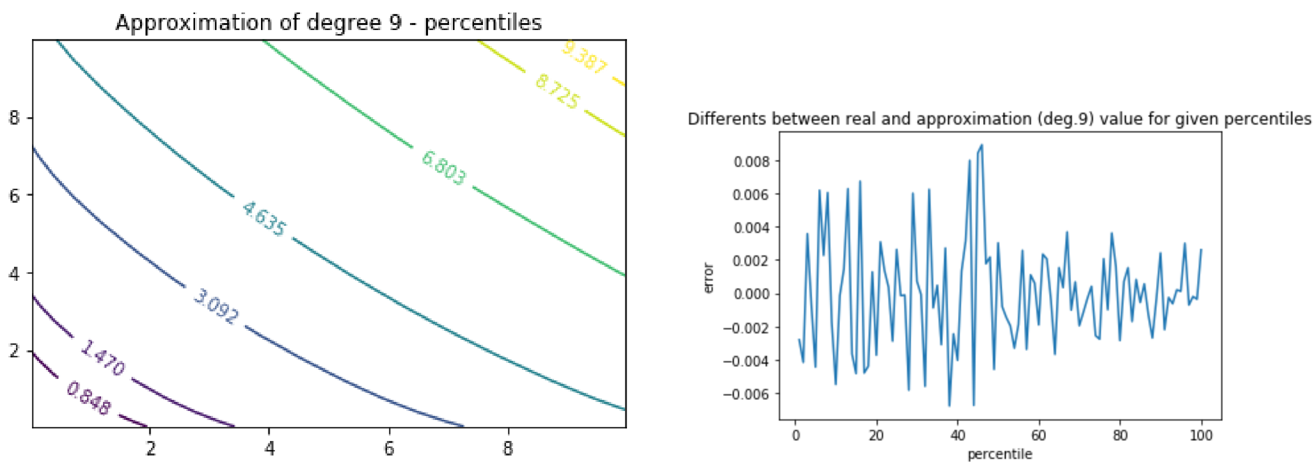

Figure 23

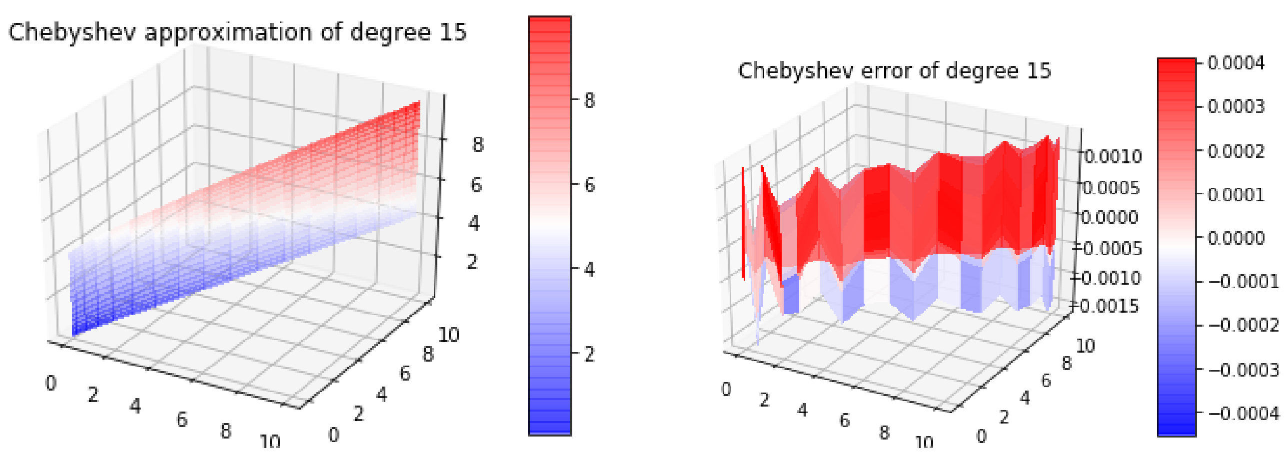

Figure 24

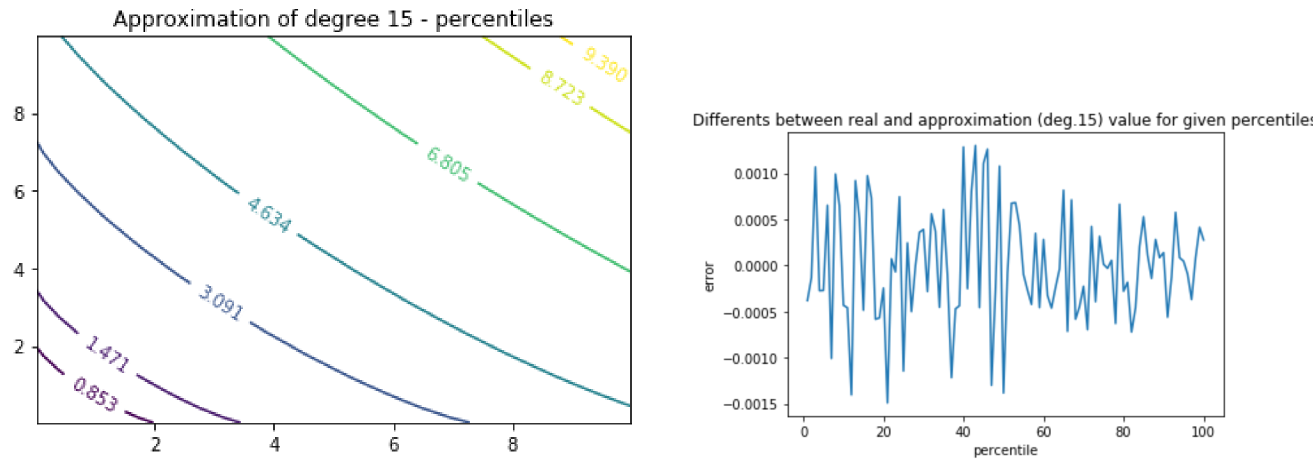

Figure 25

Though the function has different values for σ=5, the general comment on behavior of Chebyshev approximations is similar.