

Quantitative Macroeconomics - Homework 2

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Question 1. Function Approximation: Univariate.

- 1 Approximation of $f(x) = x^{0.321}$ with a Taylor series around $\bar{x} = 1$.

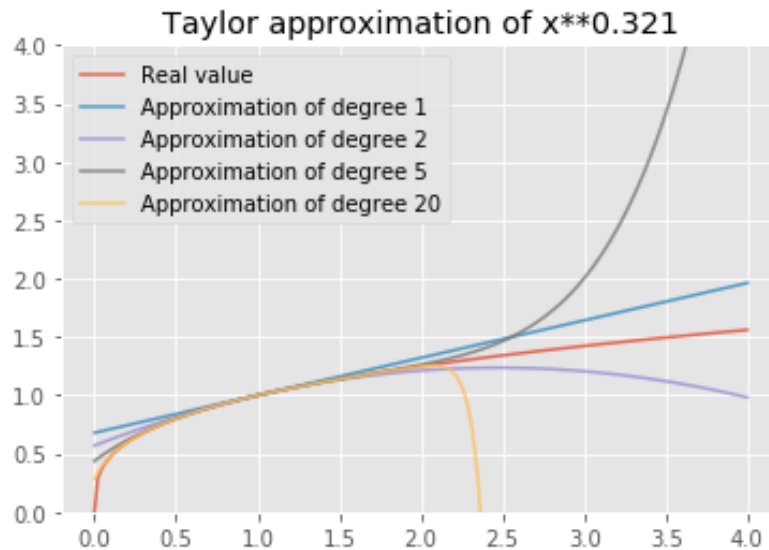


Figure 1

The further from \bar{x} we get, the bigger error of approximation becomes. What more, paradoxally, approximations of of higher degrees are less accurate.

2 Approximation of $f(x) = \text{frac}x + |x|/2$ with a Taylor series around $\bar{x} = 2$.

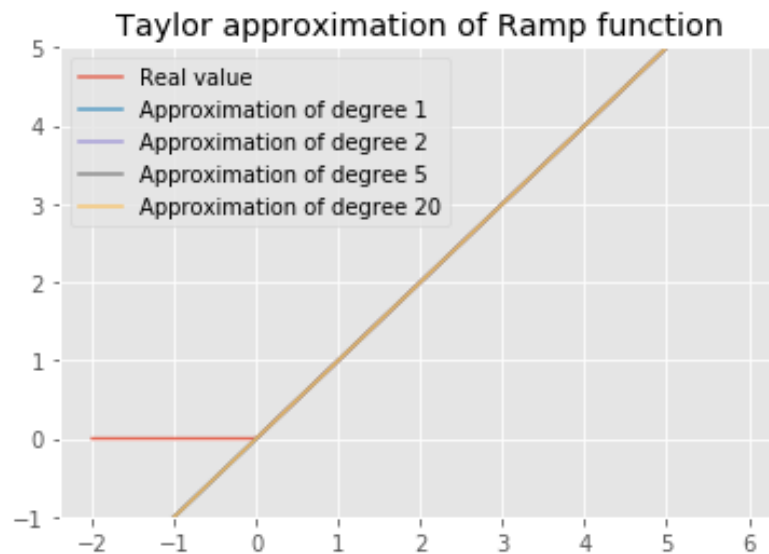


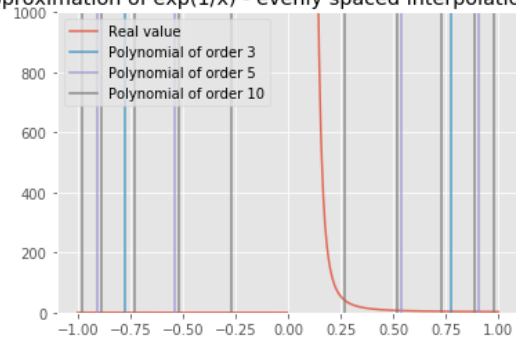
Figure 2

As the ramp function is affine, its derivative in $\bar{x} = 2$ is constant, and vanishes in higher orders. Thus Taylor series is constant and the same for all orders of approximations. What more, as we are approximation around $\bar{x} = 2$, the series 'cannot see' discontinuity in point 0.

3 Monomial interpolations

- Evenly spaced interpolation nodes

Approximation of $\exp(1/x)$ - evenly spaced interpolation nodes



Error of approximation of $\exp(1/x)$ - evenly spaced interpolation nodes

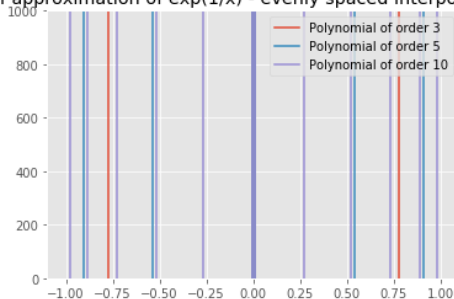
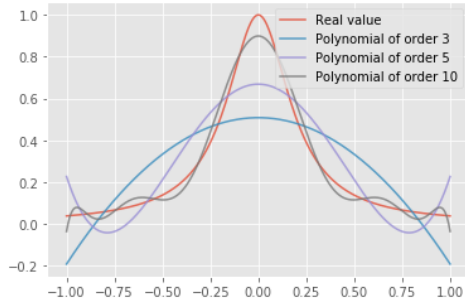


Figure 3

Approximation of runge function - evenly spaced interpolation nodes



Error of approximation of runge function - evenly spaced interpolation nodes

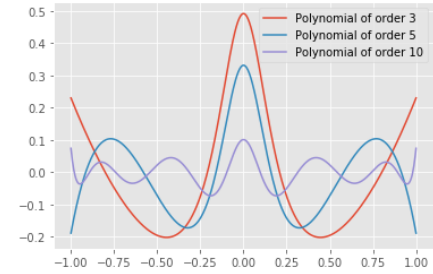
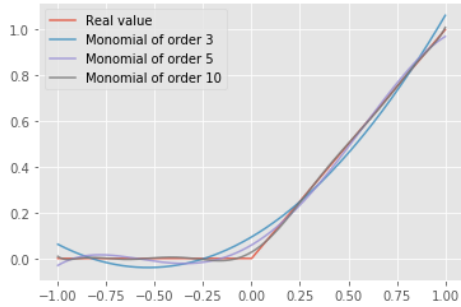


Figure 4

Approximation of ramp function - evenly spaced interpolation nodes



Error of approximation of ramp function - evenly spaced interpolation nodes

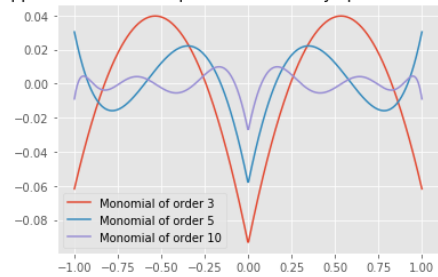
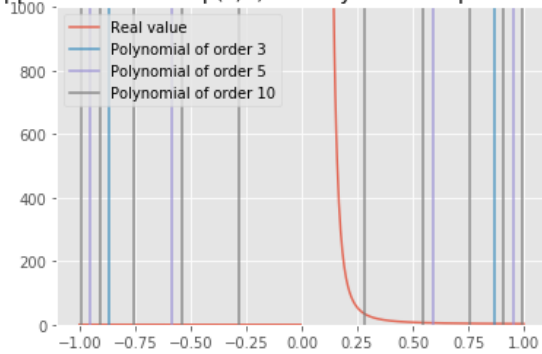


Figure 5

- Chebyshev interpolation nodes

Approximation of $\exp(1/x)$ - Chebyshev interpolation nodes



Error of approximation of $\exp(1/x)$ - Chebyshev interpolation nodes

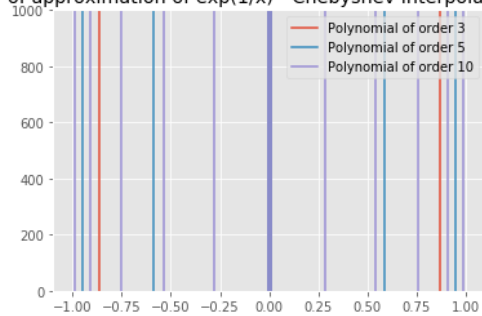
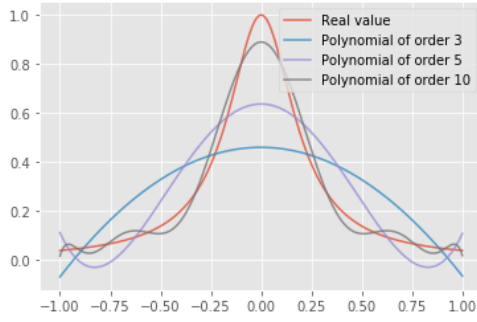


Figure 6

Approximation of runge function - Chebyshev interpolation nodes



Error of approximation of runge function - Chebyshev interpolation nodes

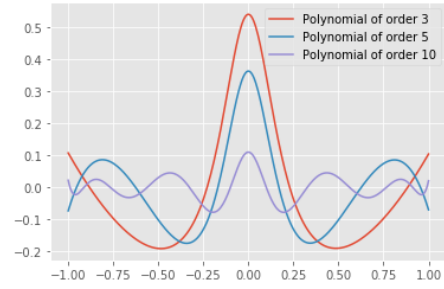
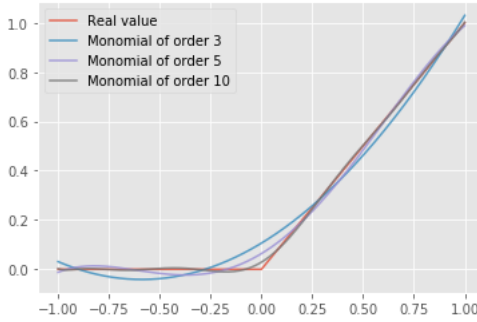


Figure 7

Approximation of ramp function - Chebyshev interpolation nodes



Error of approximation of ramp function - Chebyshev interpolation nodes

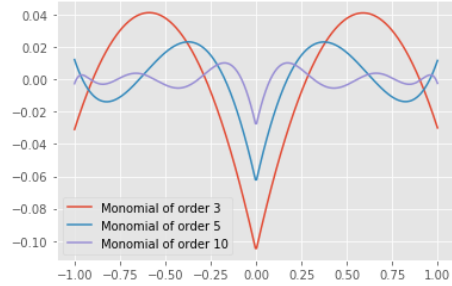
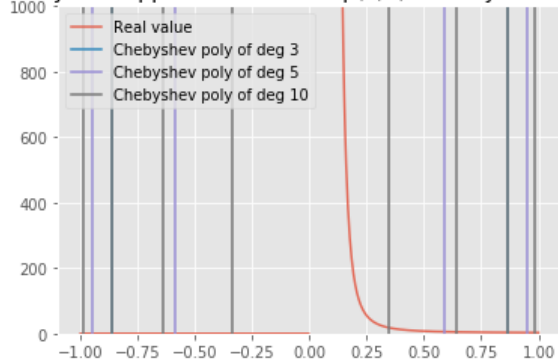


Figure 8

- Chebyshev interpolation nodes and Chebyshev polynomial

Chebyshev approximation of $\exp(1/x)$ - Chebyshev nodes



Chebyshev error of approximation of $\exp(1/x)$

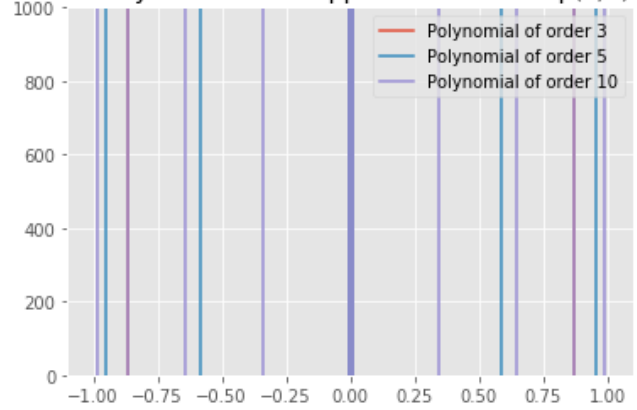
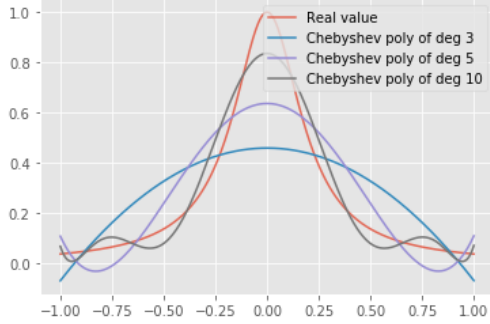


Figure 9

Chebyshev approximation of runge function - Chebyshev nodes



Chebyshev error of approximation of runge function

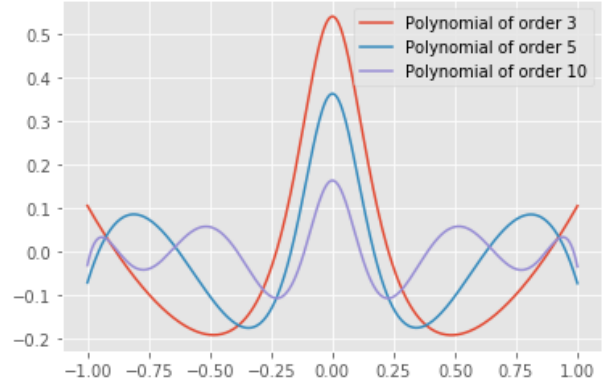
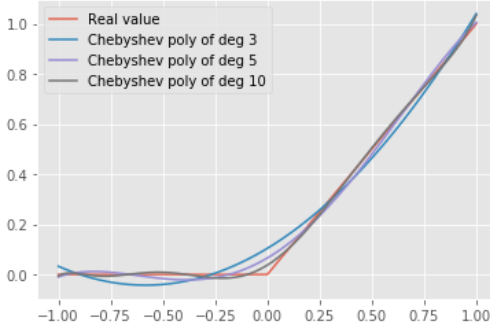


Figure 10

Chebyshev approximation of ramp function - Chebyshev nodes



Chebyshev error of approximation of ramp function

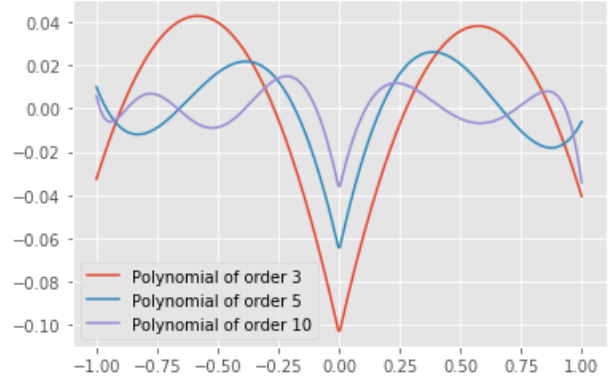


Figure 11

What one can easily see, the approximation of $f(x) = e^{\frac{1}{x}}$ is inaccurate for every way of approximation, because of its convergence to ∞ at point 0.

For runge function using Chebyshev interpolation nodes, decreases error at the minium and maximum of the scale. However using additional Chebyshev polynomials doesn't really improve the accuracy.

For ramp function using Chebyshev interpolation nodes, decreases error at the minium and maximum of the scale. However at the same time it increasaes the error in the middle of scale, especially for low degrees of polynomials.

Question 2. Function Approximation: Multivariate.

1 Show that σ is the ES

From definition:

$$\sigma = \frac{\partial \ln(\frac{h}{k})}{\partial \ln(MRS)}, \quad \text{where} \quad MRS = \frac{\frac{\partial f}{\partial k}}{\frac{\partial f}{\partial h}} \quad (1)$$

Then:

$$\frac{\partial f}{\partial k} = \frac{\sigma}{\sigma - 1} ((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}-1} (1 - \alpha) \frac{\sigma - 1}{\sigma} k^{\frac{\sigma-1}{\sigma}-1} \quad (2)$$

$$\frac{\partial f}{\partial k} = \frac{\sigma}{\sigma - 1} ((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} (1 - \alpha) \frac{\sigma - 1}{\sigma} k^{\frac{-1}{\sigma}} \quad (3)$$

Analogically:

$$\frac{\partial f}{\partial h} = \frac{\sigma}{\sigma - 1} ((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \alpha \frac{\sigma - 1}{\sigma} h^{-\frac{1}{\sigma}} \quad (4)$$

After crossing out:

$$MRS = \frac{\frac{\partial f}{\partial k}}{\frac{\partial f}{\partial h}} = \frac{1 - \alpha}{\alpha} \left(\frac{k}{h}\right)^{-\frac{1}{\sigma}} \quad (5)$$

And:

$$\sigma = \frac{\partial \ln\left(\frac{h}{k}\right)}{\partial \ln\left(\frac{1-\alpha}{\alpha} \left(\frac{k}{h}\right)^{-\frac{1}{\sigma}}\right)} \quad (6)$$

Let's assume:

$$x = \ln\left(\frac{1 - \alpha}{\alpha} \left(\frac{k}{h}\right)^{-\frac{1}{\sigma}}\right) \quad (7)$$

Then:

$$\ln\left(\frac{h}{k}\right) = \sigma(\ln(\alpha) - \ln(1 - \alpha) + x) \quad (8)$$

And:

$$\sigma = \frac{\partial \sigma(\ln \alpha - \ln(1 - \alpha) + x)}{\partial x} = \sigma \quad (9)$$

2 Compute labor share for this economy From definition Labor Share is:

$$LS = \frac{hw}{f}, \quad \text{where } w - \text{wages} \quad (10)$$

From assumption of fully competitive markets, wages are:

$$w = \frac{\partial f}{\partial h} = \frac{\sigma}{\sigma - 1} ((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \alpha \frac{\sigma - 1}{\sigma} h^{-\frac{1}{\sigma}} \quad (11)$$

Substituting:

$$LS = \frac{\alpha h^{\frac{\sigma-1}{\sigma}}}{((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})} \quad (12)$$

3-5 Approximation $f(k; h)$ with a 2-dimensional Chebyshev regression algorithm

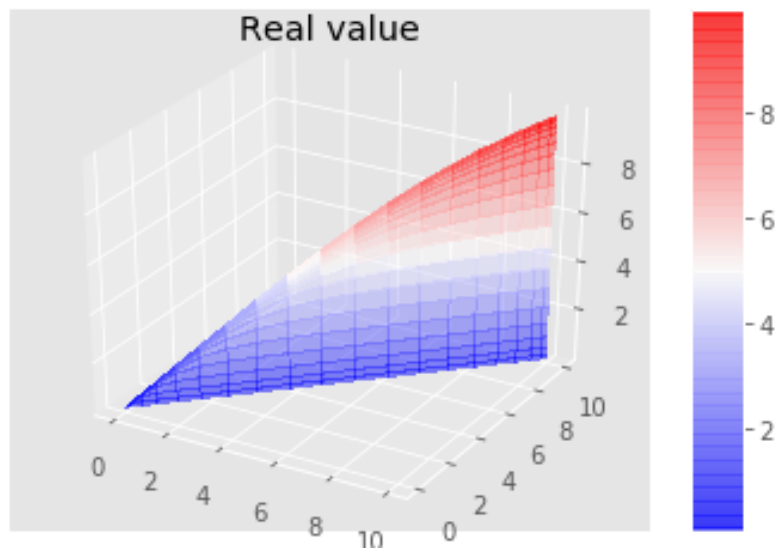


Figure 12

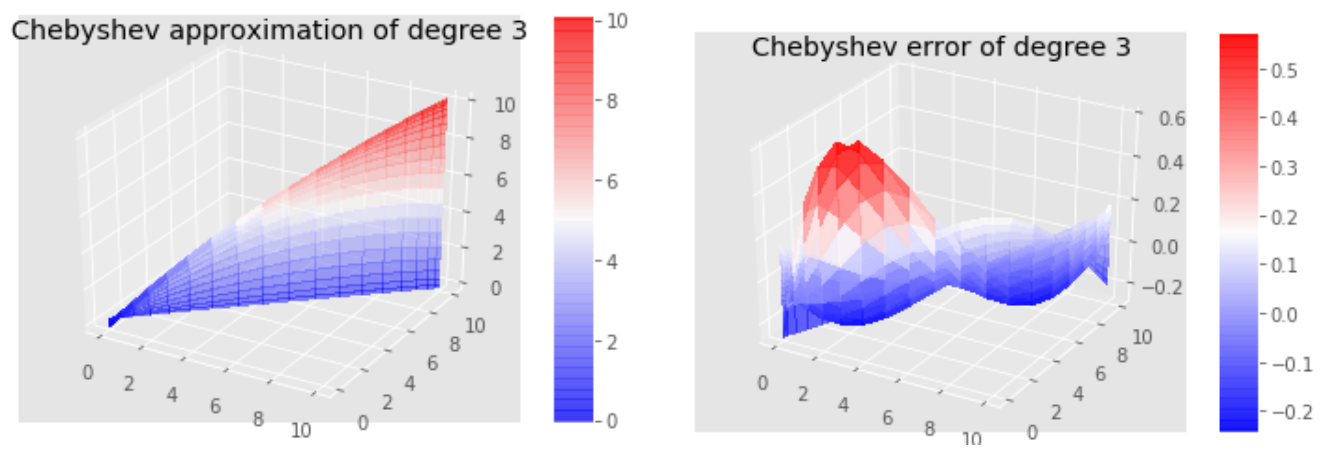


Figure 13

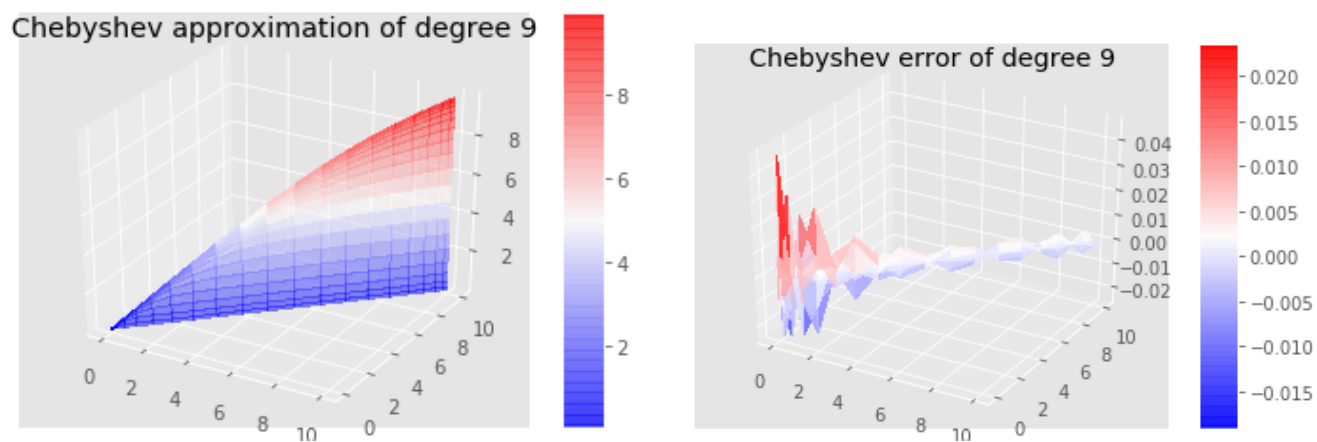


Figure 14

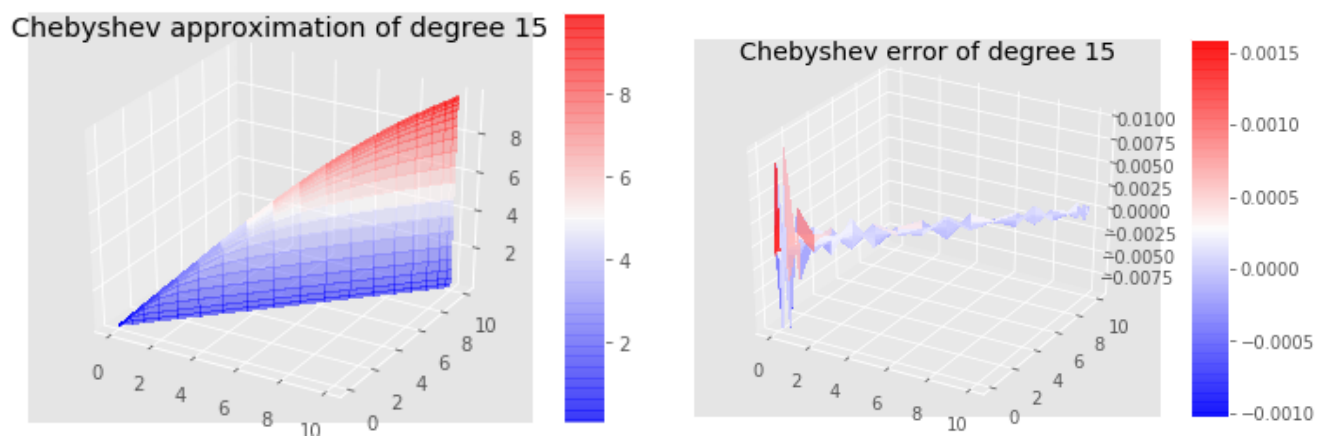


Figure 15