

# Monte Carlo Method & Simpson's Rule

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## 1 Introduction

First of all to calculate the approximate value of a polynomial, using Monte Carlo Method, on a given interval, we have to calculate the approximate value of the integral, of the given polynomial.

## 2 Labels

Interval:

$$[a, b]$$

Polynomial:

$$f(x)$$

Number of trials:

$$n$$

Formula:

$$\int_a^b f(x)dx = \left(\frac{b-a}{n}\right) * \sum_{i=0}^n f(x_i)$$

## 3 Main

In main() to compile our code, we include all headers and all typical libraries we need. First we ask user to input

- degree - which is our polynomial degree,
- \*arrayCoefficients - which is our pointer to array with all coefficients,
- a, b, n - which are our lower, upper limits, and number of samples.

Then we create four arrays (TabX, TabY), (tabX, tabY) - which are our arrays for Monte Carlo, and Simpsons, with random x, and f(x) for both in range of [a, b].

Thanks to having these arrays we can calculate approximate values of integral using the Monte Carlo Method and Simpson's Rule. Accordingly monte\_carlo(a, b, TabY, n), simpsons(a, b, tabY, n, tablica, st).

Once we have calculated our values, we extract the entire specifications into a separate file.

And in yet another files we write out our x and f(x) for both methods. writeDataToFile\_monte(TabX, TabY, n) and writeDataToFile\_simpson(tabX, TabY, n)

Once we have performed all these steps we draw the graphs of both methods using drawPlot\_monte() and drawPlot\_simpson().

```

1 // MONTE CARLO PROJECT
2
3 #include <stdio.h>
4 #include <stdlib.h>
5 #include <time.h>
6 #include "plot.h"
7 #include "input.h"
8 #include "monte_carlo.h"
9 #include "simpson.h"
10 #include "func.h"
11 #include "data.h"
12 #include "calculation_error.h"
13
14
15 int main()
16 {
17     printf("\nMonte Carlo and Simpson Integration, based on Riemann Sum and Thomas Simpson\n\n");
18     printf("Ten program wylicza wartosc calek numerycznych wielomianow za pomoca metody Monte Carlo oraz metody Simpsona.\nProsze wpisac nastepujace dane.\n\n");
19
20
21     double* TabX;
22     double* TabY;
23     double* tabX;
24     double* tabY;
25
26     int degree = degreeOfPolynomial();
27     int *arrayCoefficients = arrayWithCoefficients(degree);
28
29     int a = lowerLimit();
30     int b = upperLimit();
31     int n = numOfSamples();
32
33
34     createTabXandTabYMonte(&TabX, &TabY, a, b, n, arrayCoefficients, degree);
35     createTabXandTabYSimpson(&tabX, &tabY, a, b, n, arrayCoefficients, degree);
36
37
38     printf("Przyblizona wartosc calki numerycznej metoda Monte Carlo z wielomianu to: %f\n\n",
39     monteCarlo(a, b, TabY, n));
40     printf("Przyblizona wartosc calki numerycznej metoda Simpsona z wielomianu to: %f\n\n",
41     simpson(a, b, n, arrayCoefficients, degree));
42
43     double integrationMonte = monteCarlo(a, b, TabY, n);
44     double integrationSimpson = simpson(a, b, n, arrayCoefficients, degree);
45     double errorMonte = error(TabY, n, integrationMonte);
46
47     writePolynomialToFile(degree, arrayCoefficients, a, b, n, integrationMonte,
48     integrationSimpson, errorMonte);
49
50     writeDataToFileMonte(TabX, TabY, n);
51     drawPlotMonte();
52
53     writeDataToFileSimpson(tabX, tabY, n);
54     drawPlotSimpson();
55
56     printf("Bl d obliczeniowy wynosi:%f\n\n", error(TabY, n, integrationMonte));
57
58     free(TabX);
59     free(TabY);
60     free(tabX);
61     free(tabY);
62
63     return 0;
64 }

```

Listing 1: C main

## 4 Functions

Here we made a function which helps us calculate  $f(x)$  using Monte Carlo Method and Simpson's Rule. We are using  $x$ ,  $tab$  - our coefficients,  $stopien$  - degree of polynomial. To calculate  $f(x)$  we use Horner's diagram.

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include "func.h"
4
5 double f(double x, int tab[], int stopien) {
6     double wynik = tab[0];
7
8     for (int i = 1; i <= stopien; i++)
9         wynik = wynik * x + tab[i];
10
11     return wynik;
12 }
```

Listing 2: C functions

## 5 Input

Here are some usefull functions we made to interact with the user.

- `degreeOfPolynomial()` - takes the input of the degree of a polynomial from the user.
- `*arrayWithCoefficients()` - takes the input of the rates of change of a polynomial from the user
- `lowerLimit()`, `upperlimit()` - takes the input of the down and top limit of the interval on which the integration is being calculated from the user
- `numOfSamples()` - takes the input of the number of values for which the integration is being calculated from the user

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include "input.h"
4 #include <stdbool.h>
5 //wciecia
6 int degreeOfPolynomial(){
7     int degree;
8     while (true) {
9         printf("Podaj stopien (liczbe do ktorej podnoszona jest najwieksza potega) wielomianu:
10         ");
11         if (scanf("%d", &degree) == 1) {
12             printf("\n");
13             break;
14         }
15         else {
16             printf("Nieprawidlowe dane. Podaj poprawny stopien wielomianu (liczba).\n");
17             while (getchar() != '\n');
18         }
19     }
20     return degree;
21 }
22
23 int* arrayWithCoefficients(int degree){
24     int* array_coefficients = (int*)malloc(degree*sizeof(int));
25     printf("Podaj prosze wspolczynniki liczonego wielomianu zaczynajac od tego przy najwyzszej
26     potedze.\n\n");
27     while(1){
28         for (int i = 0; i < degree+1; i++){
29             printf("Podaj %d wspolczynnik:", i);
30             if(scanf("%d", &array_coefficients[i])!=1){
31                 continue;
32             }
33             else{
34                 printf("Nieprawidlowe dane. Podaj poprawny stopien wielomianu (liczba).\n");
35                 while (getchar() != '\n');
36                 i--;
37             }
38         }
39         printf("\n");
40         break;
41     }
42     int n = degree;
```

```

41     printf("Liczony wielomian to:");
42     for(int i=0; i<n; i++){
43         printf(" %dx^%d +",array_coefficients[i],degree);
44         degree--;
45     }
46     printf(" %d\n",array_coefficients[n]);
47
48     printf("\n");
49     return array_coefficients;
50 }
51
52 int lowerLimit(){
53     int a;
54     printf("Calka liczona jest dla danego przedzialu <a,b>\n");
55     while(1){
56         printf("Podaj dolna granice (a) zasiegu liczenia calki:");
57         if(scanf("%d",&a)==1){
58             break;
59         }
60         else{
61             printf("Nieprawidlowe dane. Podaj poprawny stopien wielomianu (liczba).\n");
62             while (getchar() != '\n');
63         }
64     }
65     return a;
66 }
67
68
69 int upperLimit() {
70     int b;
71     while(1){
72         printf("Podaj gorna granice (b) zasiegu liczenia calki:");
73         if(scanf("%d",&b)==1){
74             printf("\n");
75             break;
76         }
77         else {
78             printf("Nieprawidlowe dane. Podaj poprawna granice dolna(liczba).\n");
79             while (getchar() != '\n');
80         }
81     }
82     return b;
83 }
84
85 int numOfSamples(){
86     int n;
87     printf("Obie metody liczenia opieraja sie na wyliczeniu wartosci dla wielomianu dla danej
88     ilosci probek.\nIm wyzsza liczba probek tym wyzsza dokladnosc calki.\n\n");
89     while(1){
90         printf("Podaj liczbe probek dla ktorej chcesz liczyc calke (dowolna dodatnia liczba):"
91 );
92         if(scanf("%d", &n)==1){
93             printf("\n");
94             break;
95         }
96         else{
97             printf("Nieprawidlowe dane. Podaj poprawna dolna granice (liczba).\n");
98             while (getchar() != '\n');
99         }
100    }
101    return n;
102 }

```

Listing 3: C Input

## 6 Monte Carlo Method

In createTabXandTabY\_monte as the name says we create two arrays which we will use to calculate our integral in function below named monte\_carlo we use the label from the beginning. To specify:

- szerokosc\_przedzialu - width where we calculate integral
- (\*xs), (\*ys) - our pointers to arrays where we record our x's and f(x's)
- xi - is a random generated float between a and b
- (double)rand()/RAND\_MAX is random value between (0, 1], after we add an a, we get number between [a, b]

In monte\_carlo() we simply use our labels to calculate value as we said before.

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <time.h>
4 #include "monte_carlo.h"
5 #include "func.h"
6
7 void createTabXandTabYMonte(double** xs, double** ys, int a, int b, int n, int tab[], int
  stopien) {
8
9     srand(time(NULL));
10    double szerokosc_przedzialu =(double) b - a;
11
12    *xs = (double*) malloc(n*sizeof(double));
13    *ys = (double*) malloc(n*sizeof(double));
14
15    if (*xs == NULL || *ys == NULL) {
16        fprintf(stderr, "Memory allocation failed\n");
17        exit(EXIT_FAILURE);}
18
19    for (int i = 0; i < n; i++) {
20        double xi = a + ((double)rand() / RAND_MAX) * szerokosc_przedzialu; // Losowa
21        warto z zakresu [a, b]
22        (*xs)[i] = xi;
23        (*ys)[i] = f(xi, tab, stopien);
24    }
25
26 double monteCarlo(int a, int b, double ys[], int n){
27     double suma = 0.0;
28     double szerokosc_przedzialu = (double) b - a;
29
30     for (int i = 0; i < n; i++) suma += ys[i];
31
32     double calka = (suma * szerokosc_przedzialu) / n;
33     return calka;
34 }
```

Listing 4: C monte carlo method

## 7 Simpson's rule

This function is very similar to the previous one. We change how we calculate the value in `simpsons()`. Instead of random numbers we take equal compartments. For even multiplies the results by two, for odd by four and then multiplies all the numbers added to themselves by width and divides by three.

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include "simpson.h"
4 #include "func.h"
5
6 void createTabXandTabYSimpson(double** xs, double** ys, int a, int b, int n, int tab[], int
  stopien){
7     double przedzial = (double)(b - a)/n;
8     (*xs) = (double*) malloc(n*sizeof(double*));
9     (*ys) = (double*) malloc(n*sizeof(double*));
10
11     if (*xs == NULL || *ys == NULL) {
12         fprintf(stderr, "Memory allocation failed\n");
13         exit(EXIT_FAILURE);}
14
15     for(int i=0; i<=n; i++){
16         double k = a + przedzial * i;
17         (*xs)[i] = k;
18         (*ys)[i] = f(k, tab, stopien);
19     }
20 }
21
22 double simpson(int a, int b, int n, int tab[], int stopien){
23     double przedzial = (double) (b - a) / n;
24     double suma = 0.0;
25
26     for (int i = 0; i <= n; i++) {
27         double k = a + przedzial * i;
28         if (i == 0 || i == n) suma += f(k, tab, stopien);
29         else if (i % 2 == 0) suma += 2 * f(k, tab, stopien);
30         else suma += 4 * f(k, tab, stopien);
31     }
32
33     double calka = (przedzial / 3) * suma;
34     return calka;
35 }
```

Listing 5: C simpson's rule

## 8 Calculation Error

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <math.h>
4 #include "calculation_error.h"
5
6 double error(double ys[], int n, double monte){
7     double y_squared = 0.0;
8     double monte_squared = 0.0;
9     double error;
10
11     double* ys_squared = (double*)malloc(n*sizeof(double));
12     for(int i = 0; i < n ; i++){
13         ys_squared[i] = ys[i]*ys[i];
14     }
15
16     for(int j = 0; j < n; j++){
17         y_squared += ys_squared[j];
18     }
19
20     y_squared = y_squared/n;
21     monte_squared = monte*monte;
22     error = sqrt((monte_squared-y_squared)/(n));
23
24     return error;
25 }
```

Listing 6: C Calculation Error

## 9 Plot

In this section we draw a plots.

- drawPlot\_monte() - using the data from the .txt data file for Monte Carlo
- drawPlot\_simpson - same but for Simpson's calculations
- writeDataToFile\_monte() - writing the data from the arrays with the points of a Monte Carlo integration, so then we can draw plots
- writeDataToFile\_simpson() - same but for Simpson's

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include "plot.h"
4
5 void drawPlotMonte(){
6     FILE *gnuplotPipeMonte = popen("gnuplot -persistent", "w");
7     if (gnuplotPipeMonte != NULL) {
8         fprintf(gnuplotPipeMonte, "set terminal pngcairo enhanced color\n");
9         fprintf(gnuplotPipeMonte, "set output 'calka_wykres_monte.png'\n");
10        fprintf(gnuplotPipeMonte, "set xlabel 'X'\n");
11        fprintf(gnuplotPipeMonte, "set ylabel 'Y'\n");
12        fprintf(gnuplotPipeMonte, "set title 'Calka Monte Carlo'\n");
13        fprintf(gnuplotPipeMonte, "plot 'plot_data_monte.txt' smooth unique title 'Data'\n");
14    }
15    else {
16        printf("Error: Gnuplot not found or unable to open a pipe.\n");
17        exit(EXIT_FAILURE);
18    }
19 }
20
21 void drawPlotSimpson(){
22     FILE *gnuplotPipeSimpson = popen("gnuplot -persistent", "w");
23     if (gnuplotPipeSimpson != NULL) {
24         fprintf(gnuplotPipeSimpson, "set terminal pngcairo enhanced color\n");
25         fprintf(gnuplotPipeSimpson, "set output 'calka_wykres_simpson.png'\n");
26         fprintf(gnuplotPipeSimpson, "set xlabel 'X'\n");
27         fprintf(gnuplotPipeSimpson, "set ylabel 'Y'\n");
28         fprintf(gnuplotPipeSimpson, "set title 'Calka Simpson'\n");
29         fprintf(gnuplotPipeSimpson, "plot 'plot_data_simpson.txt' smooth unique title 'Data'\n");
30     }
31     else {
32         printf("Error: Gnuplot not found or unable to open a pipe.\n");
33         exit(EXIT_FAILURE);
34     }
35 }
36
37 void writeDataToFile(char* name, double x[], double y[], int n) {
38     char filename [100];
39     snprintf(filename, sizeof(filename), "plot_data_%s.txt", name);
40     FILE *dataFile = fopen(filename, "w");
41     if (dataFile != NULL) {
42         for (int i = 0; i < n; i++) {
43             fprintf(dataFile, "%f %f\n", x[i], y[i]);
44         }
45         fclose(dataFile);
46     }
47     else {
48         printf("Error: Unable to open data file.\n");
49         exit(EXIT_FAILURE);
50     }
51 }
52
53 void writeDataToFileMonte(double x[], double y[], int n) {
54     writeDataToFile("monte", x, y, n);
55 }
56
57 void writeDataToFileSimpson(double x[], double y[], int n){
58     writeDataToFile("simpson", x, y, n);
59 }
```

Listing 7: C Plot

## 10 Data

In this section we use all data we got from the the code and simply write out it to file.

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include "data.h"
4
5 void writePolynomialToFile(int st, int tab[],int a, int b, int n, double cal1, double cal2,
6 double error){
7     FILE *dataFile = fopen("data.txt", "a");
8     if (dataFile != NULL) {
9         fprintf(dataFile, "Dla wielomianu:\n ");
10        int m = st;
11        for(int i=0; i<m; i++){
12            fprintf(dataFile, "%dx^%d + ",tab[i],st);
13            st--;
14        }
15        fprintf(dataFile, "%d ",tab[m]);
16        fprintf(dataFile, " w przedziale(%d,%d) ", a, b);
17        fprintf(dataFile, " dla %d ilosci probek:\n\n", n);
18        fprintf(dataFile, "Monte Carlo:%f\n", cal1);
19        fprintf(dataFile, "Simpson:%f\n\n", cal2);
20        fprintf(dataFile, "Blad obliczeniowy:%f\n\n", error);
21        fprintf(dataFile, "
22        -----\n");
23
24        fclose(dataFile);
25    } else {
26        printf("Error: Unable to open data file.\n");
27        exit(EXIT_FAILURE);
28    }
29 }
```

Listing 8: Data

## 11 Headers

Here are headers we made in our code.

```
1 #ifndef calculation_error_h
2 #define calculation_error_h
3
4 double error(double ys[], int n, double monte);
5
6 #endif
7
8
9 #ifndef data_h
10 #define data_h
11
12 void writePolynomialToFile(int st, int tab[],int a, int b, int n, double cal1, double cal2,
13 double error);
14 #endif
15
16
17 #ifndef func_h
18 #define func_h
19
20 double f(double x, int tab[], int stopien);
21
22 #endif
23
24
25 #ifndef input_h
26 #define input_h
27
28 int degreeOfPolynomial();
29 /*
30 ** takes the input of the degree of a polynomial from the user
31 */
32
33 int* arrayWithCoefficients(int degree);
34 /*
35 ** takes the input of the rates of change of a polynomial from the user
36 */
37
38 int lowerLimit();
39 
```



```

15  /*
16  ** takes the input of the down limit of the interval on which the integration is being
    calculated from the user
17  */
18
19
20  int upperLimit();
21  /*
22  ** takes the input of the top limit of the interval on which the integration is being
    calculated from the user
23  */
24
25  int numOfSamples();
26  /*
27  ** takes the input of the number of values for which the integration is being calculated from
    the user
28  */
29  #endif

```

```

1  #ifndef monte_carlo_h
2  #define monte_carlo_h
3
4  double f(double x, int tab[], int stopien);
5  void createTabXandTabYMonte(double** xs,double** ys,int a, int b, int n,int tab[],int stopien)
    ;
6  double monteCarlo(int a,int b, double ys[], int n);
7
8  #endif

```

```

1  #ifndef plot_h
2  #define plot_h
3
4  void writeDataToFileMonte(double x[],double y[],int n);
5
6  /*
7  ** writing the data from the arrays with the points of a monte carlo integration
8  */
9  void writeDataToFileSimpson(double x[],double y[],int n);
10
11  /*
12  ** writing the data from the arrays with the points of a simpson integration
13  */
14
15  void drawPlotMonte();
16  /*
17  ** drawing the plot using the data from the txt data file (monte carlo)
18  */
19
20  void drawPlotSimpson();
21
22  /*
23  ** drawing the plot using the data from the txt data file (monte carlo)
24  */
25
26  #endif

```

```

1  #ifndef simpson_h
2  #define simpson_h
3
4  double f(double x, int tab[], int stopien);
5  void createTabXandTabYSimpson(double** xs,double** ys,int a, int b, int n,int tab[],int
    stopien);
6  double simpson(int a,int b, int n,int tab[],int stopien);
7
8  #endif

```

## 12 Makefile

```
1 CC = gcc
2 CFLAGS = -Wall -Wextra
3
4 # lista plikow .o
5 OBJECTS = func.o input.o main.o monte_carlo.o plot.o simpson.o data.o calculation_error.o
6 PNG = calka_wykres_monte.png calka_wykres_simpson.png
7 TXT = plot_data_monte.txt plot_data_simpson.txt
8 all: Monte_carlo
9
10 # linkowanie
11 Monte_carlo: $(OBJECTS)
12     $(CC) $(CFLAGS) -o $@ $^
13
14 # kompilowanie
15 %.o: %.c
16     $(CC) $(CFLAGS) -c -o $@ $<
17
18 # czyszczenie
19 clean:
20     rm -f $(OBJECTS) Monte_carlo
21     rm -f $(PNG) Monte_carlo
22     rm -f $(TXT) Monte_carlo
23
24 run: Monte_carlo
25     ./Monte_carlo
```

Listing 9: Makefile