Math Test

1.1 $\frac{x^{n+2}}{x^{n-2}} = x^{n+2-n+2} = \boxed{x^4}$

1.2 $x^{-1} \cdot 8 = 2$

$x^{-1} = \frac{2}{8}$

$x^{-1} = \frac{1}{4}$

$\boxed{x = 4}$

1.3 $a=5$ $b=10$ $(a^b)^0 = (5^{10})^0 = \boxed{1}$

1.4 $\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} = \boxed{2}$

1.5 $x^2+(x+1)^2=(x+2)^2$

$x^2+x^2+x+x+1 = x^2+2x+2x+4$

$2x^2+2x+1 = x^2+4x+4$

$2x^2-x^2+2x-4x+1-4=0$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$\boxed{x_1=-1}$

$\boxed{x_2=3}$

1.6 $2^x > 1024$

$2^x > 2^{10}$

$x > 10$

2.1 $F = a+b\cdot c$ if $C=0$ then $F=32$ if $C=100$ then $F=212$

$32 = a + b\cdot 0$

$\boxed{a=32}$

$212 = 32 + b\cdot 100$

$b = 1.80$

$F = 32 + 1.8C$

$-32 = 0.8\cdot C$

$\boxed{C = -40}$

2.2 $f(x) = 5x+4$ if $f(3)=y$

$f(3) = 5\cdot 3+4$

$f(3) = 15+4$

$\boxed{f(3)=19}$

2.3 $x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\boxed{x_1 = 1}$

$\boxed{x_2 = 3}$

2.4 $10 \cdot 1.02^{90} = \boxed{59.4313}$

2.5 $e^{\ln 5} = \boxed{5}$

3.1 $\sum_{i=1}^{\infty} \frac{12}{6^i} \Rightarrow a_n = 12 \cdot \frac{1}{6^i}$ $\quad a = 12 \quad b = \frac{1}{6} \quad \sum_{i=1}^{\infty} a_i = \frac{ab}{1-b}$

$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{2}{\frac{5}{6}} = \boxed{\frac{12}{5}}$

3.2 $\lim_{x \to 1} \frac{6^{1-x}}{x} = \frac{6^{1-1}}{1} = \frac{6^0}{1} = \frac{1}{1} = \boxed{1}$

3.3 $f(x) = x^5 - 8$ at $x = -3$

$f'(x) = 5x^4$

$f'(-3) = 5(-3)^4 = 81 \cdot 5 = \boxed{405}$

3.4 $\frac{d}{dx} \frac{x^3+2x-1}{x-2} = \frac{(3x^2+2)(x-2) - (x^3+2x-1) \cdot 1}{(x-2)^2} =$

$\left(\frac{f}{g}\right)' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{(g(x))^2}$ $= \frac{3x^3 - 6x^2 + 2x - 4 - x^3 - 2x + 1}{x^2 - 4x + 4} =$

$= \frac{2x^3 - 6x^2 - 3}{x^2 - 4x + 4}$

3.5 $\frac{d^2}{dx^2} 4x^4 + 4x^2$

$d_1 = 16x^3 + 8x$

$d_2 = 48x^2 + 8$

3.6 $\frac{d}{dx}\ \frac{\ln x}{e^x} = \frac{\frac{1}{x}\cdot e^x - \ln x \cdot e^x}{(e^x)^2} = \frac{\frac{1}{x} - \ln x}{e^x}$

3.7

$f(x) = 3x^2 - 5x + 2$ $\quad 6x - 5 = 0$ $\quad (3x-2)(x-1)$

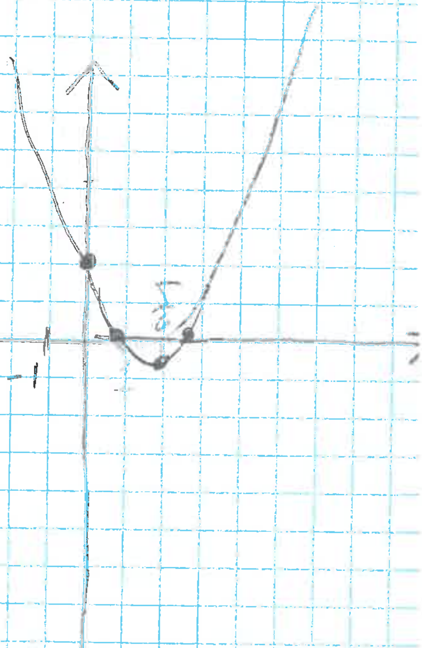
$3x = 2$ $\quad \boxed{x = \frac{2}{3}}$ $\quad \boxed{x = 1}$

$d_1 = 6x - 5$ $\quad 6x = 5$

$x = \frac{5}{6}$ ↗ stationary point

$d_2 = 6$ ↘ local min

x	$x < \frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$> \frac{5}{6} < x < 1$	1	> 1
$f(x)$	+	0	−	−	−	0	+
$f'(x)$	−	−	−	0	+	+	+
slope	↘	↘	↘	local min	↗	↗	↗
$f''(x)$	+	+	+	+	+	+	+
convexity	∪	∪	∪	∪	∪	∪	∪



3.8 $f(x,y) = x^2 + y^3$

$f(2,3) = 2^2 + 3^3 = 4 + 27 = 31$

3.9 $f(x,y) = \ln(x - y)$

$x - y > 0$

$\boxed{x > y}$

3.10 $\frac{\partial}{\partial x}\ x^5 + xy^3 = \boxed{5x^4 + y^3}$

3.11 $f(x,y) = x^2 y^2 + 10$

$f'_x(x,y) = 2xy^2$ $\quad f'_y(x,y) = x^2 2y$

$\boxed{x = y}$

$2x^3 = 0$

$\boxed{x = 0 \quad y = 0} \Rightarrow$ local minimum

3.12 max x^2y^2 $\quad x+y=10$

$x+y-10=0$

$\mathcal{L} = x^2y^2 - \lambda(x+y-10)$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= y^2 \cdot 2x - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= x^2 \cdot 2y - \lambda = 0 \end{aligned}\right\} \quad 2xy^2 = 2yx^2$$

$y = x$

$\frac{\partial \mathcal{L}}{\partial \lambda} = x+y-10 = 0$

$\boxed{\begin{aligned} &x+y=10 \\ &x=y=5 \end{aligned}}$

$x+x=0$

$2x=10$

$x=\frac{10}{2}$

$x=5$

4.1

$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 \\ 1 & 8 \\ 7 & 2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 1\cdot2+6\cdot2 & 2\cdot1+6\cdot8 & 2\cdot7+6\cdot2 \\ 1\cdot5+2\cdot1 & 5\cdot1+8\cdot1 & 5\cdot7+1\cdot2 \\ 1\cdot1+9\cdot2 & 1\cdot1+9\cdot8 & 1\cdot7+9\cdot2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$

4.2 $A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$B \cdot A =$

	2 2 4 6 1 3
1 5 1	$2\cdot1+9\cdot4+1\cdot1$ $\quad 2\cdot1+9\cdot6+3\cdot1$
2 1 2	$5\cdot2+1\cdot4+2\cdot1$ $\quad 2\cdot2+6\cdot1+2\cdot3$

$= \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$

4.3 $A^T = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix} = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.3 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$

4.4 $\begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix} = 1\cdot8 - 9\cdot2 = \boxed{-10}$

5.1

$\#\Omega = n^k$ $\quad n = 6$

$k = 2$

$\#\Omega = 6^2 = \boxed{36}$

5.2

Drug User 1% B_1

Non-drug 99% B_2

$P(A) \rightarrow$ positive on a random drug test

$P(A|B_1) = 99\%$ $\quad P(A) = \sum_{i=1}^{2} P(B_i) \cdot P(A|B_1) + P(B_2)\, P_{(A|B_2)}$

$= 1\% \cdot 99\% + 99\% \cdot 0.5\% = \boxed{1.49\%}$

$P(A|B_2) = (100 - 99.5) = 0.05$

5.3 $$\mathbb{P}(B_j|A) = \frac{\mathbb{P}(A|B_j)\,\mathbb{P}(B_j)}{\sum_{i=1}^{n} \mathbb{P}(B_i)\,\mathbb{P}(A|B_i)} =$$

$$= \frac{99\% \cdot 1\%}{1.49\%} = \boxed{66.44\%}$$