

15.6 注释

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说明 1. 三角不等式的推论:

$$|z - w| \geq ||z| - |w||$$

证明:

$$\begin{aligned} |z| &= |z - w + w| \leq |z - w| + |w| \\ &\implies |z - w| \geq |z| - |w| \end{aligned}$$

同理可得,

$$|w - z| \geq |w| - |z|$$

由于 $|z - w| = |w - z|$, 于是

$$|z - w| \geq ||z| - |w||$$

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说明 2. 任意实数 $\epsilon > 0$,

$$\begin{aligned} |z - w| &\leq \epsilon \\ \implies \\ |w| - \epsilon &\leq |z| \leq \epsilon + |w| \end{aligned}$$

证明:

$$\begin{aligned} |z| - |w| &\leq |z - w| \leq \epsilon \\ \implies \\ |z| &\leq \epsilon + |w| \end{aligned}$$

又

$$\begin{aligned} |w| - |z| &\leq |z - w| \leq \epsilon \\ \implies \\ |w| - \epsilon &\leq |z| \end{aligned}$$

综上,

$$|w| - \epsilon \leq |z| \leq \epsilon + |w|$$