

Name: SAMPLE

MATH 320: QUIZ 7

- (1) (4 points) Compute a least squares regression line fitting the three points (0,0), (1,0), and (1,1).

- (a) First, frame the regression as a linear algebra problem: $X^T X \beta = X^T Y$.
 (b) Then, solve for β using your preferred technique.

a) $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$, $Y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\min_{\beta} (X\beta - Y)$ is found at the solution of $X^T X \beta = X^T Y$

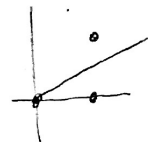
$$X^T X = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \quad X^T Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) Solve $\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Augm. Matrix: $\left[\begin{array}{cc|c} 3 & 2 & 1 \\ 2 & 2 & 1 \end{array} \right] \xrightarrow{R_2 + (-\frac{2}{3})R_1} \left[\begin{array}{cc|c} 3 & 2 & 1 \\ 0 & 2/3 & 1/3 \end{array} \right]$

$\Rightarrow \frac{2}{3}\beta_1 = \frac{1}{3} \Rightarrow \boxed{\beta_1 = \frac{1}{2}} \Rightarrow 3\beta_0 + 1 = 1 \Rightarrow \boxed{\beta_0 = 0}$

$y = \frac{1}{2}x$



- (2) (3 points) Compute the sum-of-squares residual error, and the correlation coefficient for this linear regression.

$$S_r = \left(1 - \frac{1}{2}\right)^2 + \left(0 - \frac{1}{2}\right)^2 = \frac{1}{2}$$

$$S_t = \sum (y_i - \bar{y})^2 \quad \bar{y} = \frac{0+0+1}{3} = \frac{1}{3}$$

$$= \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{2}{3}$$

$$r^2 = \frac{S_t - S_r}{S_t} = \frac{1/6}{2/3} = \frac{1}{4} \Rightarrow r = .5$$

- (3) (3 points) Find the Lagrange form of the interpolating polynomial for the points $(0, 0), (1, 1), (2, 4)$.

$$f(x) = \sum L_i(x) f(x_i)$$

$$= \cancel{L_1(x) \cdot 0} + L_2(x) \cdot 1 + L_3(x) \cdot 4$$

$$= \frac{(x-0)(x-2)}{(1-0)(1-2)} + \frac{(x-0)(x-1)}{(2-0)(2-1)} \cdot 4$$

$$= -x(x-2) + 2x(x-1)$$