Name: SAMPLE

MATH 320: QUIZ 4

(1) (3 points) Let $f(x) = x^2 e^{-x}$.

(a) Compute the first and second derivatives $f'(x)$ and $f''(x)$

$$f'(x) = 2x \cdot e^{-x} + (-1)e^{-x} \cdot x^2 = (2x - x^2)e^{-x} \quad \text{(product rule)}$$

$$f''(x) = (2-2x)e^{-x} + (-1)e^{-x}(2x - x^2) = (x^2 - 4x + 2)e^{-x}.$$

(b) Based on this computation, list the local optima of $f(x)$ and whether each point is a maximum or minimum.

$f'(x) = 0$ at all local optima.

$\Leftrightarrow (2x - x^2)e^{-x} = 0 \Leftrightarrow x = 0$ or 2 since $e^{-x} > 0$ for all x.

At $x = 0$, $f''(x) > 0 \Rightarrow x = 0$ is a local minimum.

$x = 2$, $f''(x) < 0 \Rightarrow x = 2$ is a local maximum.

(c) Are these points global optima?

$x = 2$ is not a global max, since as $x \to -\infty$, $x^2 e^{-x} \to +\infty$.

$x = 0$ is a global min, since $f(x) = 0$ there, and is strictly positive everywhere else.

(2) Let $g(x) = x^3 - x^2 - 3x - 1$.

(a) We would like to minimize the value of $g(x)$ between 0 and 2. Suppose our initial root estimate is $x = 1$. What is the equation (in the form $y = ax^2 + bx + c$) for the parabola P that intersects the graph of $g(x)$ at each of these x-values.

x	0	1	2
y	-1	-4	-3

We use the following to derive our equation:

$$y = (x-1)(x-2) \cdot \frac{-1}{2} + x(x-2) \cdot \frac{-4}{-1} + x(x-1)\frac{-3}{2}.$$

$$\Rightarrow y = (x^2 - 3x + 2) \cdot -\frac{1}{2} + (x^2 - 2x)4 + (x^2 - x) \cdot -\frac{3}{2}$$

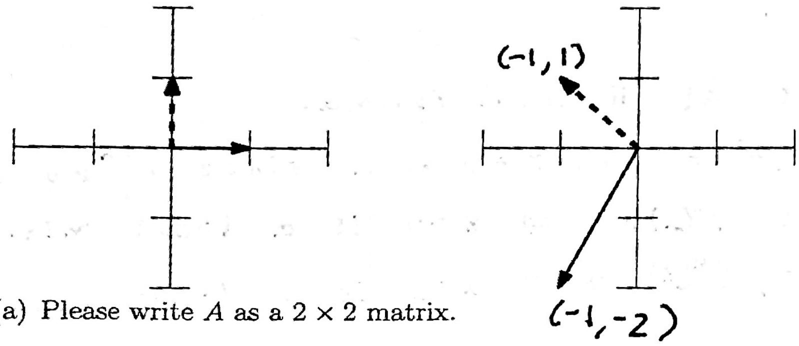
$$= 2x^2 - 5x - 1.$$

(b) Where does P attain its minimum?

$y = 2x^2 - 5x - 1$ has a minimum when $\frac{dy}{dx} = 0$

$\Rightarrow 4x - 5 = 0 \Rightarrow x = \frac{5}{4} = 1.25$.

(3) Suppose A is a matrix describing a map from $\mathbb{R}^2$ to $\mathbb{R}^2$ sending the solid vector $(1, 0)$ and the dashed vector $(0, 1)$ to the corresponding vectors in the picture at right.



(a) Please write A as a 2×2 matrix.

$$A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

(b) Evaluate the determinant of A.

$$\det \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix} = -1 \cdot 1 - (-1) \cdot (-2) = -3.$$