Name: SAMPLE

MATH 320: QUIZ 3

(1) (3 points) Let $f(x) = x^3 - 7$. Suppose we would like to find a root of $f(x)$ in the interval $(0, 2)$ using the two endpoints as our "bracket."

(a) What are the first two points selected inside the interval using the bisection method? Justify your response.

(b) How many iterations of the bisection method must be performed to guarantee error less than or equal to 2^{-16}?

(c) What is the first point inside the interval selected by the false position (regula falsi) method?

a) Note that $f(0) = -7$ and $f(2) = 1$.
The first iteration takes $\boxed{m_1 = \frac{2+0}{2} = 1.}$
$f(1) = -6$ which has the same sign as $f(0)$,
so our new bracket is $[1, 2]$.
The next point we choose is $\boxed{m_2 = \frac{1+2}{2} = 1.5.}$

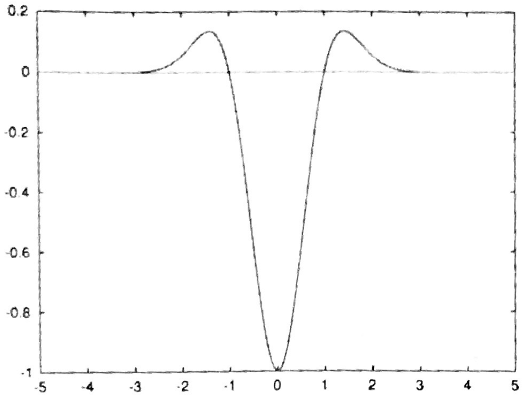
b) Error here refers to half the final bracket. At iteration 1, the estimate 1 has error 1. Each subsequent iteration shrinks the error by $\frac{1}{2}$ $\Rightarrow$ after 17 iterations the error is 2^{-16}.

c) In the false position method,

$$c = b - \frac{f(b)(b-a)}{f(b) - f(a)}.$$

Here, $c = 2 - \frac{1 \cdot 2}{8} = 1.75$.

(2) (4 points) Let $f(x) = (x^2 - 1)e^{-x^2}$ be our function of interest, with graph shown below.



Suppose we try to find a root using Newton's method.

(a) Compute $f'(x)$.

(b) Describe (without actually computing) our approximations by Newton's method if our initial guess is $x = -2$.

(c) Describe (without actually computing) our approximations by Newton's method if our initial guess is $x = -1/2$.

a) $f'(x)$ (by the product rule)

$$= 2x \cdot e^{-x^2} + (x^2-1)\cdot(-2x)e^{-x^2} = (4x - 2x^3)e^{-x^2}.$$

b) Approximations will head down the slope to the left. As $n \to \infty$, $\lim_{n \to \infty} x_n = -\infty$, since the slope stays positive while the function stays above zero.

c) Approximations will approach the left root while staying above it, because the curve is concave down in this interval.

(3) (3 points) For the image below, draw and label secant lines and values for x_2, x_3, and x_4 given x_0 and x_1 shown here.