QUALIFYING EXAM SYLLABUS

ZVI ROSEN

Committee: Bernd Sturmfels (advisor), Mark Haiman (chair), Dan-Virgil Voiculescu, Satish Rao (Computer Science)

Major Topic: Commutative Algebra (Algebra)

- Noetherian, Artinian, Localization, Hom, Tensor Product, Spec.
- Associated Primes, Primary Decomposition, Prime Avoidance.
- Hilbert Basis Theorem, Noether Normalization, Nullstellensatz, Cayley-Hamilton Theorem, Nakayama's Lemma.
- **Integral Ring Extensions**: Integral Closure, Going-Up, Going-Down, Incomparable Property.
- **Gröbner Bases**: Monomial Orders, Ideal Membership, Elimination, Initial Ideals.
- **Combinatorial Commutative Algebra:** Stanley-Reisner Ideals, Alexander duality, Buchberger graphs, Hochster's formula, Resolutions (Eliahou-Kervaire, Cellular, Cocellular, Hull, Cohull, Taylor, Scarf).

Reference: Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Chapters 1-4, 12-15

Miller & Sturmfels, *Combinatorial Commutative Algebra*, Chapters 1-4, 5.1-5.4, 6.1-6.2

Major Topic: Algebraic Combinatorics (Algebra)

- **Enumeration**: Twelvefold Way, Permutation Statistics (inversions, descents, etc.), Ordinary and exponential generating functions, species, Matrix-Tree theorem.
- **Symmetric Function Theory**: Bases for the space of symmetric functions, transition matrices, Jacobi-Trudi Identity, Kostka numbers, Littlewood-Richardson coefficients, Littlewood-Richardson rule.
- **Representations of the symmetric group**: Young tableaux and tabloids, Specht modules, irreducible representations and characters of the symmetric group, Schur functions.

Reference: Stanley, *Enumerative Combinatorics*, chapters 1 and 7.

Fulton, *Young Tableaux*, chapters 1-7.

Sagan, *The Symmetric Group*, chapter 2.

Date: March 8, 2013, 10 AM - 1 PM.

Minor Topic: Complex Analysis (Classical Analysis)

- **Holomorphic functions**: Cauchy-Riemann equations, conformal maps, linear fractional transformations, Taylor and Laurent series, Liouville's Theorem
- **Complex Integration**: Cauchy's Theorem, Cauchy's Integral Formula, Taylor's Theorem, Morera's Theorem, Maximum Modulus Principle, Residue Theorem, Argument Principle, Rouché's Theorem.
- Fundamental Theorem of Algebra (statement and proof).

Reference: Ahlfors, *Complex Analysis*, Chapters 1-4.