Name:____________________

MATH 320: QUIZ 6

(1) (4 points) Compute the largest eigenvalue and its corresponding eigenvector using three iterations of the power method for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix}.$$

Let $x_0 = [1, 1/2, 1/4]$ and iterate $x_{i+1} = Ax_i/\lambda_i$, where λ_i = largest coordinate of Ax_i. Please compute x_3 and λ_3.

(2) (3 points) Write down a matrix whose characteristic polynomial is $x^3 + 5x^2 - x + 1$.

(3) (3 points) Compute the following vector norms for the vector $[3, -2, 1, 4]$:

(a) 1-norm,

(b) 2-norm,

(c) ∞-norm.

1) $x_0 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

$$x_1 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \\ 1/4 \end{bmatrix} / \lambda_0 = \begin{bmatrix} 11/4 \\ 9/4 \\ 5/4 \end{bmatrix} / \frac{11}{4} = \begin{bmatrix} 1 \\ 9/11 \\ 5/11 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 9/11 \\ 5/11 \end{bmatrix} / \lambda_1 = \begin{bmatrix} 4 \\ 28/11 \\ 16/11 \end{bmatrix} / 4 = \begin{bmatrix} 1 \\ 7/11 \\ 4/11 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7/11 \\ 4/11 \end{bmatrix} / \lambda_2 = \begin{bmatrix} 37/11 \\ 27/11 \\ 15/11 \end{bmatrix} / \frac{37}{11} = \begin{bmatrix} 1 \\ 27/37 \\ 15/37 \end{bmatrix} \approx \begin{bmatrix} 1 \\ .73 \\ .40 \end{bmatrix}$$

$$\lambda_3 = \max(A\lambda_3) = \max \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 27/37 \\ 15/37 \end{bmatrix} = \max \begin{bmatrix} 136/37 \\ 92/37 \\ 52/37 \end{bmatrix}$$

$$= 136/37 \approx 3.68$$

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2) Let M be our matrix, which must be 3×3.

Suppose λ_i is an eigenvalue of M, with corresponding eigenvector $(\lambda_i^2, \lambda_i, 1)^T$, denoted v_i.

Then, $Mv_i = \lambda_i v_i \Rightarrow M \cdot \begin{bmatrix} \lambda_i^2 \\ \lambda_i \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_i^3 \\ \lambda_i^2 \\ \lambda_i \end{bmatrix} = \begin{bmatrix} -5\lambda_i^2 + \lambda_i - 1 \\ \lambda_i^2 \\ \lambda_i \end{bmatrix}$

(using the characteristic polynomial to rewrite λ_i^3.)

$$\Rightarrow M = \begin{bmatrix} -5 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Check: $\det(xI - M) = \begin{vmatrix} x+5 & -1 & 1 \\ -1 & x & 0 \\ 0 & -1 & x \end{vmatrix} = x^2(x+5) + (-x+1)$

$= x^3 + 5x^2 - x + 1.$ ✓

3) Let $v = [3, -2, 1, 4]$

a) 1-norm : $\sum_{i=1}^{4} |v_i| = 3 + 2 + 1 + 4 = 10$

b) 2-norm : $\sqrt{\sum_{i=1}^{4} v_i^2} = \left[9 + 4 + 1 + 16\right]^{1/2} = \sqrt{30} \approx 5.5$

c) ∞-norm : $\max_i |v_i| = 4.$