Due: September 16, 2016

MATH 320: HOMEWORK 2

Please read through sections 4.1 - 4.3 in the textbook. Then answer the following questions. Please submit all code and output with brief descriptions of what you are doing.

(1) Prove that a finite base-n representation is unique. In particular, show that if a number has a finite base-n representation, any other finite base-n representation must be the same one.

(2) (Problem 4.4) For computers, the machine epsilon ϵ can also be thought of as the smallest number that when added to one gives a number greater than 1. An algorithm based on this idea can be developed as

Step 1: Set $\epsilon = 1$.
Step 2: If $1 + \epsilon$ is less than or equal to 1, then go to Step 5. Otherwise go to Step 3.
Step 3: $\epsilon = \epsilon/2$.
Step 4: Return to Step 2.
Step 5: $\epsilon = 2 \times \epsilon$.

Write your own M-file based on this algorithm to determine the machine epsilon. Validate the result by comparing it with the value computed with the built-in function `eps`.

(3) (Problem 4.11) The Maclaurin series expansion for $\cos x$ is:

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

Starting with the simplest version, $\cos x = 1$, add terms one at a time to estimate $\cos(\pi/4)$. After each new term is added, compute the true and approximate percent relative errors. Determine the true value. Add terms until the absolute value of the approximate error estimate falls below an error criterion conforming to two significant figures.