Due: November 18, 2016

MATH 320: HOMEWORK 7

Please read through chapters 14, 15.1-15.3, and 17 in the textbook. Answer the following questions. Please submit all code and output with brief descriptions of what you are doing.

(1) Problem 14.5
The data is below:

x	0	2	4	6	9	11	12	15	17	19
y	5	6	7	6	9	8	8	10	12	12

The following code is used to compute the regression line, the standard error of the estimate, and the correlation coefficient – all for the regression line of y versus x.

```
X = [0 2 4 6 9 11 12 15 17 19;
    5 6 7 6 9 8 8 10 12 12]

%x-regression

A = [ones(10,1) X(1,:)']
b = [X(2,:)']
beta = A'*A \ A'*b

%Plotting the data & Regression line
plot(X(1,:),X(2,:),'.')
axis([-2 20 3 15])
hold on
plot([-2,20],[beta(1) + beta(2)*-2,
    beta(1) + beta(2)*20]);

%Std Error & Correlation computation
sr = sum((beta(1) + beta(2)*X(1,:) - X(2,:)).^2);
syx = sqrt(sr/(10-2));
st = sum((X(2,:) - mean(X(2,:))).^2);
r = ((st - sr)/st)^(1/2)
```

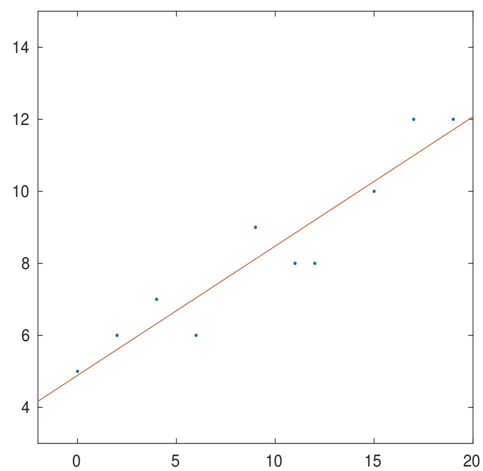
Regression line	$Y = 4.8881 + .3591X$
Standard error of the estimate	.8511
Correlation coefficient	.9449

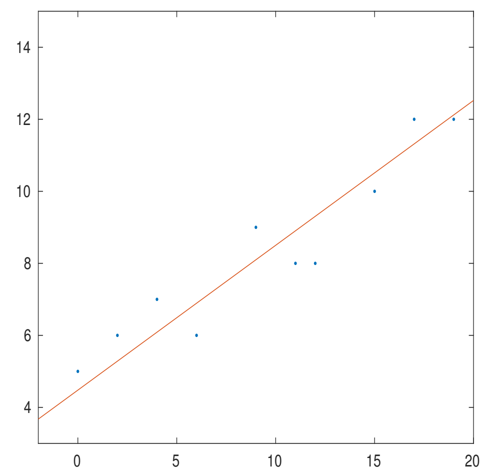
The following code is used to compute the regression line, the standard error of the estimate, and the correlation coefficient – all for the regression line of x versus y.

```
%y-regression

A = [ones(10,1) X(2,:)']
b = [X(1,:)']
beta = A'*A \ A'*b

plot(X(1,:),X(2,:),'.')
axis([-2 20 3 15])
hold on
plot([beta(1) + beta(2)*3,
    beta(1) + beta(2)*15],[3,15])

sr = sum((beta(1) + beta(2)*X(2,:) - X(1,:)).^2);
sxy = sqrt(sr/(10-2))
st = sum((X(1,:) - mean(X(1,:))).^2);
r = ((st - sr)/st)^(1/2)
```

Regression line	$X = -11.1349 + 2.4861Y$
Standard error of the estimate	2.2393
Correlation coefficient	.9449

The plot at left is for the regression of y versus x; the plot at right is the regression of x versus y.



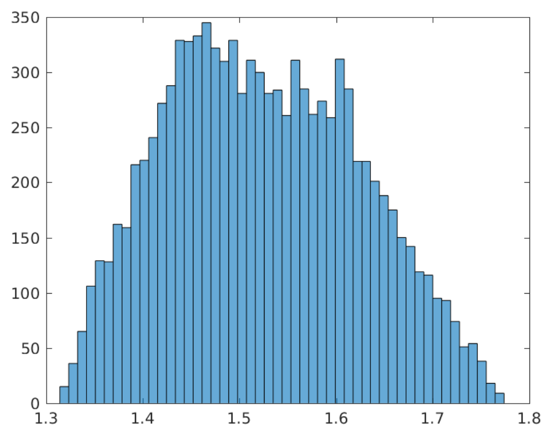


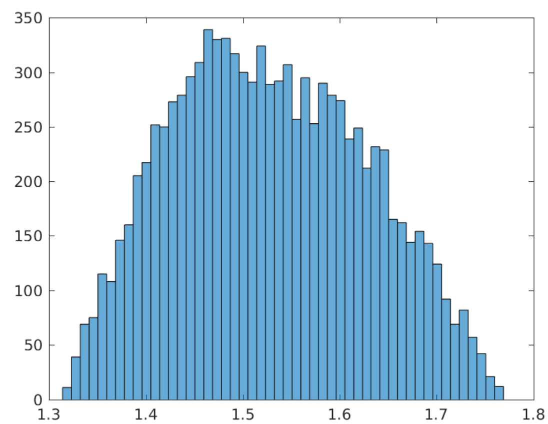
(2) Problem 14.34

We carry out the Monte Carlo simulation using the following MATLAB code:

```
Q = @(nm,B,H,S) (1/nm)*((B*H)^(5/3)/(B+2*H)^(2/3))*sqrt(S)
% Note B = 20, H = 0.3
% nm in range (0.027,0.033)
% S  in range (0.00027,0.00033)

in = rand(2,10000);
in(1,:) = .027 + .006*in(1,:);
in(2,:) = .00027 + .00006*in(2,:);
Qdata = zeros(1,10000);
for i=1:10000
    Qdata(i) = Q(in(1,i),20,0.3,in(2,i));
end
histogram(Qdata, 50)
```

The following histograms were obtained using two trials:





(3) Problem 15.12

x	1	2	3	4	5
y	2.2	2.8	3.6	4.5	5.5

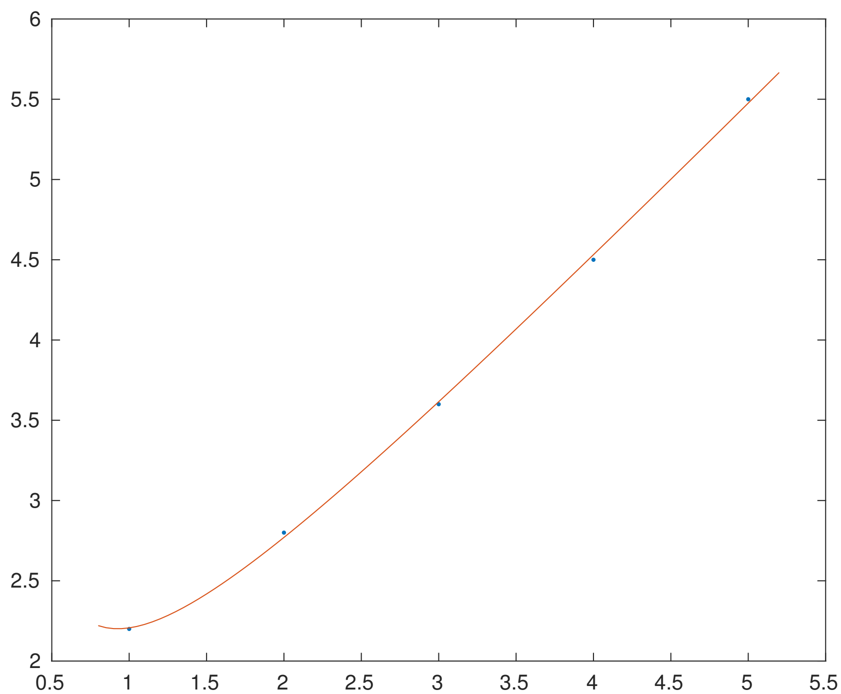
The MATLAB code we use to analyze this data and fit it to the model

$$y = a + bx + \frac{c}{x}$$

is below:

```
X = [1 2 3 4 5]';
Y = [2.2 2.8 3.6 4.5 5.5]';

A = [ones(5,1) X X.^(-1)];
b = A'*A \ A'*Y

f = @(x) b(1) + b(2)*x + b(3)*(1/x);
X2 = .8:0.05:5.2; Y2 = arrayfun(f,X2);
plot(X,Y,'.',X2,Y2)
```

The regression yields $Y = 0.3745 + 0.9864X + \dfrac{0.8456}{X}$, which is depicted below.



(4) Let $z = f(x, y)$ be a function of two variables.
Suppose we have the following 6 points:

x	1	1	2	2	3	3
y	2	3	1	3	1	2
z	3	2	3	1	2	1

Find a degree-5 homogeneous polynomial in x and y that includes these six points. Plot the corresponding surface in MATLAB.

We can treat this like a standard linear algebra problem– a homogeneous degree-5 polynomial has six terms:

$$F(X, Y) = \beta_5 X^5 + \beta_4 X^4 Y + \beta_3 X^3 Y^2 + \beta_2 X^2 Y^3 + \beta_1 X Y^4 + \beta_0 Y^5.$$

In matrix form:

$$\begin{pmatrix} | & | & | & | & | & | \\ X^5 & X^4Y & X^3Y^2 & X^2Y^3 & XY^4 & Y^5 \\ | & | & | & | & | & | \end{pmatrix} \begin{pmatrix} | \\ \beta \\ | \end{pmatrix} = \begin{pmatrix} | \\ Z \\ | \end{pmatrix}$$

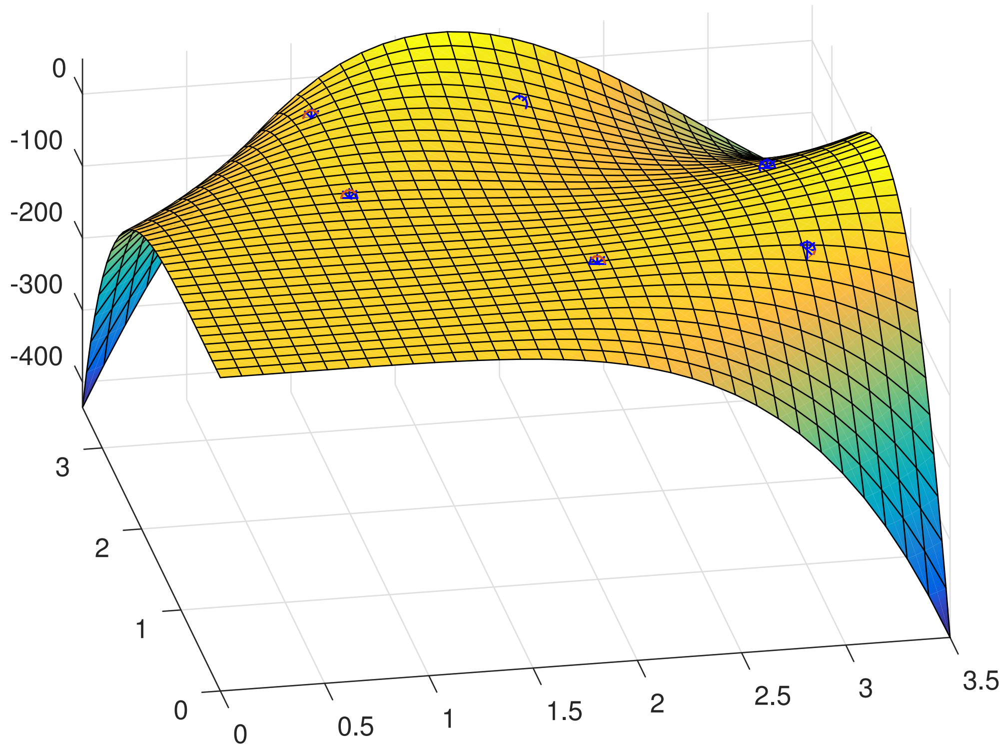
Row i of the XY matrix corresponds to data point (x_i, y_i) and coordinate i of the Z vector gives the value of z_i.

After evaluating everything, the equation is:

$$\begin{pmatrix} 1 & 2 & 4 & 8 & 16 & 32 \\ 1 & 3 & 9 & 27 & 81 & 243 \\ 32 & 16 & 8 & 4 & 2 & 1 \\ 32 & 48 & 72 & 108 & 162 & 243 \\ 243 & 81 & 27 & 9 & 3 & 1 \\ 243 & 162 & 108 & 72 & 48 & 32 \end{pmatrix} \begin{pmatrix} \beta_5 \\ \beta_4 \\ \beta_3 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \\ 1 \\ 2 \\ 1 \end{pmatrix}.$$

The MATLAB code to reach this equation and solve it, as well as to plot the corresponding surface is below:

```
X = [1 1 2 2 3 3]';
Y = [2 3 1 3 1 2]';
Z = [3 2 3 1 2 1]';

A = [X.^5 X.^4.*Y X.^3.*Y.^2 X.^2.*Y.^3 X.*Y.^4 Y.^5];
beta = A \ Z

func = @(x,y) ([x.^5 x.^4.*y x.^3.*y.^2 x.^2.*y.^3 x.*y.^4 y.^5]*beta)(1)

fsurf(func,[0,3.5])
hold on
plot3(X,Y,Z,'b*')
```



The blue stars signify the data points that we were trying to fit.