Due: November 9, 2016

MATH 320: HOMEWORK 6

Please read through chapters 11, 12.1, and 13 in the textbook. Answer the following questions. Please submit all code and output with brief descriptions of what you are doing.

(1) Section 11.2 discusses various norms on matrices. In class, we reviewed three properties of any matrix norm ρ:
  (a) $\rho(aM) = a\rho(M)$ for scalar a and matrix M.
  (b) $\rho(M + N) \leq \rho(M) + \rho(N)$ for M, N matrices.
  (c) $\rho(M) = 0$ only if M is the zero matrix.
  Prove that these properties are satisfied for 1) the column-sum norm, and 2) the spectral norm.
  Hint: for the spectral norm, use the induced norm definition: $||M||_2 = \sup(||Mx||_2/||x||_2)$, where the norm on the right-hand side is the Euclidean norm on vectors.

(2) Problem 11.9.

(3) Problem 12.2. You may use the textbook implementation of Gauss-Seidel, but add a subroutine that at each iteration, plots the first two coordinates of the approximation. Display the plots for part (a) and part (b).

(4) Consider a matrix F which acts on a vector in $\mathbb{R}^2$ by mapping (x_1, x_2) to (x_2, x_1+x_2).
  (a) Write F down explicitly.
  (b) Let $v = [0, 1]$. In a 2×10 matrix, display $F^k v$ for $k = 0, 10$.
  (c) What are the eigenvalues and eigenvectors of F? Use this to write an explicit formula for F^k.
  (d) Use everything you have done so far to write a formula for the k-th Fibonacci number, where $F_0 = 0$ and $F_1 = 1$.