

MATH 320: CLASS NOTES

Chapter 7: Optimization

1. SINGLE VARIABLE

Local optimum: there exists a neighborhood of x , such that for all a in the neighborhood, $f(x) > f(a)$.

Global optimum: For all a in the domain of the function, $f(x) > f(a)$.

1.1. **Gradient Method.** Compute $f'(x)$ analytically and use a root-finding algorithm.

Why is $f'(x) = 0$ at a local optimum? Suppose $f'(x_0) = a \neq 0$. Then by Taylor's theorem, $f(x) \approx f(x_0) + a(x - x_0) + O((x - x_0)^2)$. For sufficiently small $x - x_0$, the function is linear with nonzero slope, and you can follow the line up or down.

1.2. **Direct Method.** Golden Section.

Parabolic Interpolation.

2. MULTIVARIABLE

2.1. **Gradient Method.** Gradient Descent

2.2. **Direct Method.** Nelder-Mead