Matrix Worksheet

(1) (a) How can we check if a matrix A is singular?

(b) Is the following matrix singular?

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

(c) For what value of α is the following matrix singular?

$$B = \begin{pmatrix} \alpha & 0 & 2 \\ 1 & 2 & 1 \\ -1 & 3 & 0 \end{pmatrix}$$

(2) We are trying to find values of x_1, x_2, x_3 satisfying $Ax = b$, where A is a 3×3 matrix and b is a 3×1 vector.

(a) If A is NOT singular, how many solutions are there?

(b) If A is NOT singular, and $b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, what is x?

(c) If A is singular, how many solutions will there be?

(3) Solve the system of equations:

$$-x + 2y + z = 4$$
$$x - z = 6$$
$$-2x + 3y + z = 0$$

(4) Find the eigenvalues and eigenvectors of the following matrix:

$$C = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$$

(5) (Challenge Question) Suppose x is an eigenvector of the matrix A. Explain why x is also an eigenvector of A^2.