Due: September 30, 2016

MATH 320: HOMEWORK 3

Please read through chapters 5 and 6 in the textbook. Answer the following questions. Please submit all code and output with brief descriptions of what you are doing.

(1) *Solution:*

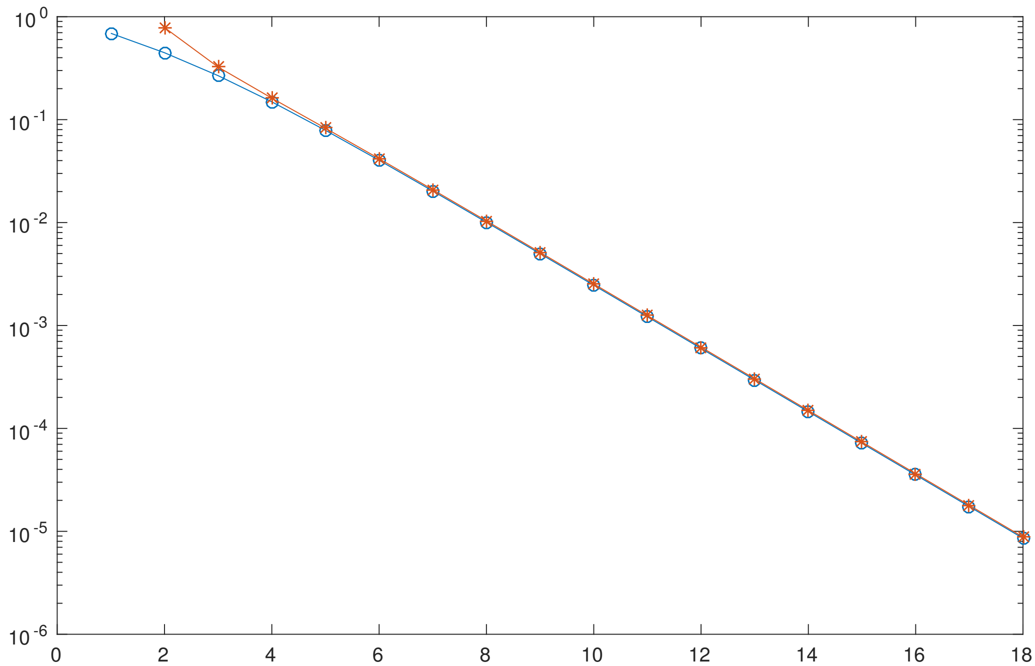
(a) The following code works as a `falsi` MATLAB function:

```
function approx = falsi(l,r,func,epsilon,maxSteps)
%INPUT: a function func, with bracket [l,r], a.r.e. bound epsilon,
% and maximum step number maxSteps.
%OUTPUT: a point approx giving the x-value of the most
%recent approximation to the root.
new = l;
old = r;
iterCount = 0;
while (abs(r - l) > 2*epsilon) & ...
        (iterCount <= maxSteps)
    %Compute y-values for endpoints
    old = new;
    fl= func(l); fr = func(r);
    %Compute linear interpolation point and y-value
    new = l - fl*(r-l)/(fr- fl);
    val = func(new);
    %Redefine your bracket based on the sign.
    if sign(val) == sign(fl)
        l = new;
    elseif sign(val) == sign(fr)
        r = new;
    end
    %exit the loop if the a.r.e. is small enough.
    if ((new - old)/new <= epsilon)
        break
    end
    %Step up the count of iterations.
    iterCount = iterCount + 1;
end
approx = new;
end
```

(b) The value output by MATLAB after 18 iterations is 2.030525784672327. (Use `format long` to obtain this level of precision.)

(c) There are a few ways to go about this. I added an optional argument "root" to the `falsi` function, and included these lines of code before the while loop:

```
if nargin == 6
    truerr = [];
    apperr = [];
end
```
Then, inside of the while loop, I included these lines of code:
```
if nargin == 6
    truerr = [truerr [abs(new - rt)/rt]];
    apperr = [apperr [abs(new - old)/old]];
end
```
After obtaining the sequences of errors. I plotted them using the following code:
`semilogy(1:18,t,'o-',2:18,a(2:18),'-*')`.



(2) *Solution:*

(a) The following code works for implementing the secant method.
```
function approx = secant(x0,x1,func,epsilon,maxSteps)
%INPUT: a function func, with two approximations,
%a.r.e. bound epsilon, and maximum step number maxSteps.
%OUTPUT: a point approx giving the x-value of the most
%recent approximation to the root.
new = x1;
old = x0;
iterCount = 0;

while (iterCount <= maxSteps)
    %Step up the count of iterations.
```

```
    iterCount = iterCount + 1;
    %Compute y-values for two points
    f1= func(new); f2 = func(old);
    %Compute linear interpolation point and y-value
    mid = new - f1*(new-old)/(f1 - f2);
    old = new;
    new = mid;
    %exit the loop if the a.r.e. is small enough.
    if (abs(new - old)/new <= epsilon)
        break
    end
end
approx = new;
end
```

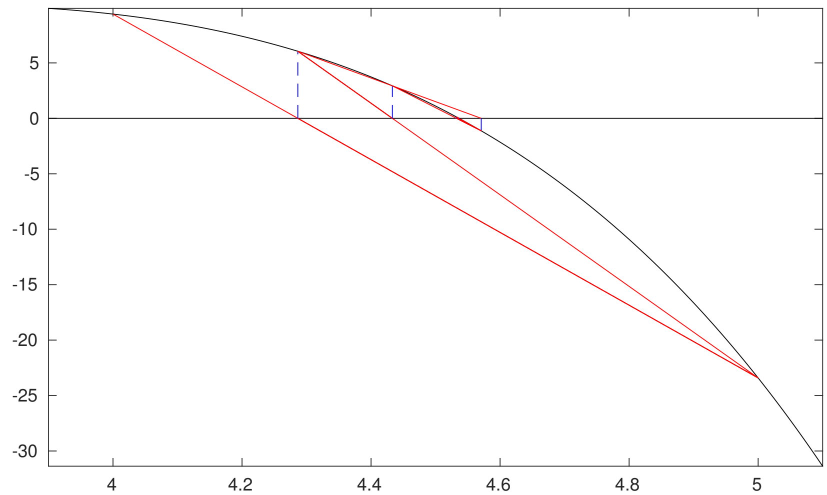
(b) The value output by MATLAB after 7 iterations is 4.536403654894378.

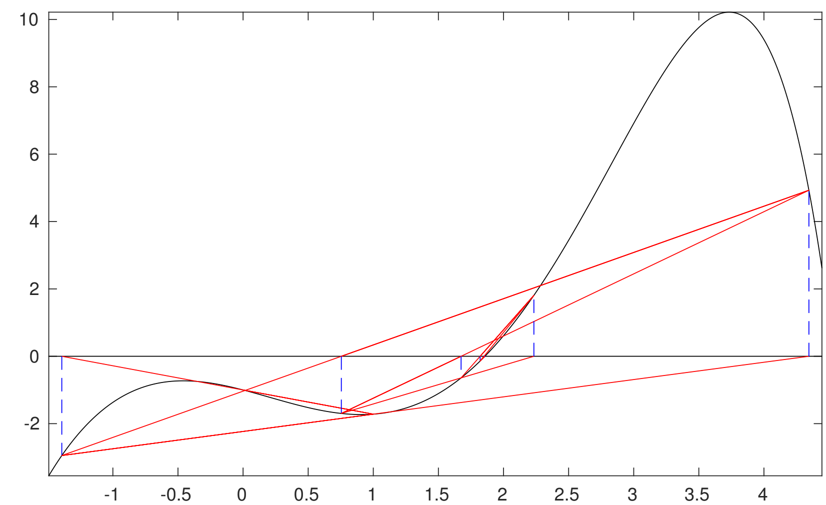
(c) The value output by MATLAB after 7 iterations is 1.857183860152164.

(d) There are also a number of ways to do this. I used the following command to initialize a collection of points: `pts = [old new];` and then the following code inside the loop to collect points: `pts = [pts new];`
After the loop is complete we carry out the plots using the following sequence of commands:

```
X = linspace(min(pts)-.1,max(pts)+.1,500);
Y = arrayfun(func, X);
plot(X,Y,'-k');
hold on
plot(X,zeros(1,500),'-k');
fpts = arrayfun(func, pts);
for i=3:length(pts)
    plot(pts(i-2:i),[fpts(i-2),fpts(i-1),0],'r')
    plot([pts(i),pts(i)],[0,fpts(i)],'--b')
end
axis([min(X),max(X),min(Y),max(Y)]);
```

The output for our two plots is here:





(3) Let $E_n = x_n - r$. We claim that $\lim_{n\to\infty} E_n/E_{n-1}^2 = \mu$.

The formula for $x_n = x_{n-1} - f(x_{n-1})/f'(x_{n-1})$. Because the function is smooth, and the root is simple, we can use Taylor's theorem to write $f(x_n) = f(r) + f'(r)(x_n - r) + \frac{1}{2}f''(\xi)(x_n - r)^2$ where ξ is between r and x_n and $f'(r)$ is nonzero.

Using this expansion, we have:

$$E_n = E_{n-1} - \frac{f'(r)E_{n-1} + \frac{1}{2}f''(\xi)E_{n-1}^2}{f'(r) + f''(\xi)E_{n-1}}$$

$$E_n = \frac{E_{n-1}(f'(r) + f''(\xi)E_{n-1})}{f'(r) + f''(\xi)E_{n-1}} - \frac{f'(r)E_{n-1} + \frac{1}{2}f''(\xi)E_{n-1}^2}{f'(r) + f''(\xi)E_{n-1}}$$

$$E_n = \frac{\frac{1}{2}f''(\xi)E_{n-1}^2}{f'(r) + f''(\xi)E_{n-1}}$$

$$E_n/E_{n-1}^2 = \frac{1}{2}f''(\xi)\frac{1}{f'(r) + f''(\xi)E_{n-1}}$$

The limit of the expression on the right as $n \to \infty$ is finite since $f'(r) \neq 0$.

(4)

- Bisection Method for $f(x)$. This method will not find the root, since the values on either side of it are nonnegative.
- Incremental Search Method for $f'(x)$. This method can find a collection of intervals where the root might be. Since $f(l)$ and $f(r)$ are presumably greater than 0 (if not, then we found the root), the Mean Value Theorem implies that the derivative is negative at some point before the root, and positive at some point after the root. The incremental search will find an interval where it's zero.
- Newton's Method. This method should converge to the root, but slower than if it were a simple root. Consider the example of the parabola x^2. Any point x_n nearby points to $x_{n+1} = x_n - x_n/2$. This clearly points towards zero.
- Secant Method. This method should also converge to the root; like Newton's method, it will converge more slowly than usual. Here, too, consider the parabola $f(x) = x^2$ and begin with points within 1/2 of zero. Then $x_{n+1} = x_n - x_n^2 \dfrac{x_n^2 - x_{n-1}^2}{x_n - x_{n-1}} = x_n - x_n^2(x_n + x_{n-1})$. Assuming $0 < x_n, x_{n-1} < 1/2$, the subtracted term will be less than x_n meaning that it moves towards zero without overshooting.