Due: September 30, 2016

MATH 320: HOMEWORK 3

Please read through chapters 5 and 6 in the textbook. Answer the following questions. Please submit all code and output with brief descriptions of what you are doing.

(1) Implement the False Position Method (regula falsi) in MATLAB.

(a) The method should be implemented as

```
falsi(l,r,func,epsilon,maxSteps)
```

taking a bracket $[l, r]$, a function `func`, an approximate relative error bound ϵ, and a positive integer `maxSteps`. The algorithm should terminate when the approximate relative error goes below ϵ or when the number of iterations exceeds `maxSteps`. The output should be the midpoint value most recently computed.

(b) Use your implementation to compute:

```
falsi(0, 3, @(x) x^4 - 17, 10^(-5), 200)
```

(c) Modify your `falsi` code to output the sequence of true relative errors and approximate relative errors at each iteration. Plot both of these sequences for the function as above.
Let the y-axis be on a log scale.

(2) Implement the Secant method in MATLAB.

(a) The method should be implemented as

```
secant(x0,x1,func,epsilon,maxSteps)
```

taking two initial guesses x_0 and x_1, a function `func`, an approximate relative error bound ϵ, and a positive integer `maxSteps`. The algorithm should terminate when the approximate relative error goes below ϵ or when the number of iterations exceeds `maxSteps`. The output should be the final approximation.

(b) Evaluate the function for

```
secant(4,5,@(x) x^3 - exp(x),10^(-6),100)
```

(c) Evaluate the function for

```
secant(0,1,@(x) x^3 - exp(x),10^(-6),100)
```

(d) Incorporate a graphical component to your code, so that the function `secant` plots the graph of `func`, as well as the secant line segment computed at each iteration. Repeat the computations above and display the associated graphs.

(3) Prove that Newton's method converges quadratically, given that the function being studied is smooth, and the algorithm is converging to a simple root. **Hint:** Estimate the error using Taylor's theorem with remainder term.

(4) You suspect that a bracket $[l, r] \subset \mathbb{R}$ contains a multiple root of a smooth function $f(x)$ which is everywhere nonnegative. For each of the following methods describe if it will work and provide reasons.
- Bisection Method for $f(x)$.
- Incremental Search Method for $f'(x)$.
- Newton's Method.
- Secant Method.