Boltzmann distribution

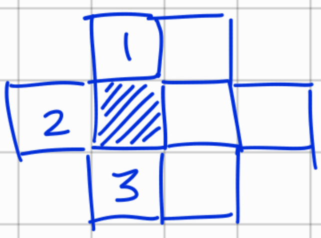
The probability of a state with energy E is

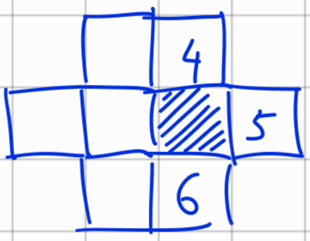
$$p_i \propto \exp\left(-\frac{E}{kT}\right)$$

We assume molecules with affinity, so that

$$E = -(\text{\# adjacent molecules})$$

In particular, in the case of moving a single mol:





Before

$E = m_1 + m_2 + m_3$

After

$E = m_4 + m_5 + m_6$

$$\Delta E = E_2 - E_1 = m_4 + m_5 + m_6 - m_1 - m_2 - m_3$$

$$p_2 \propto \exp\left(-\frac{E_2}{kT}\right), \quad p_1 \propto \exp\left(-\frac{E_1}{kT}\right)$$

$$\frac{p_2}{p_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right) = \exp\left(-\frac{\Delta E}{kT}\right)$$

If we only want to choose between the two options (1), (2), then $q \triangleq \frac{p_2}{p_1 + p_2}$ is the probability of (2) occuring.

$$\frac{1}{q} = \frac{p_1 + p_2}{p_2} = 1 + \frac{p_1}{p_2} = 1 + \exp\left(\frac{\Delta E}{kT}\right)$$

$$\Rightarrow q = \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)}.$$