Math 2250 Lab 4 Name: ________________ Unid: ________________
Due: 2/12/2015

1. A mathematical model for the rate at which a drug disseminates into the bloodstream is given by

$$\frac{dx(t)}{dt} = r - kx(t),$$

where r and k are positive constants. The function $x(t)$ describes the concentration of the drug in the bloodstream at time t. Assuming $r = 0.3$, $k = 0.1$,

   (a) Sketch the phase diagram to find the limiting value of $x(t)$ as $t \to \infty$.

   (b) Find the particular solution of the initial value problem $x(0) = 0$. Compute the limiting value of x(t) as $t \to \infty$? Sketch the solution and verify your prediction in part(a).

2. Consider the same initial value problem as in problem 1,

$$\frac{dx(t)}{dt} = 0.3 - 0.1x(t),$$
$$x(0) = 0,$$

apply the following methods to approximate the solution function on the time interval $0 \leq t \leq 1$. Do these computations by hand (using a calculator or other technology to do each step, but not using programmed code). You may wish to use programmed code saved in Canvas to check your work before you hand it in.

(a) Euler Method: use step size $h = 0.5$, i.e. 2 time steps.

(b) Improved Euler Method: use step size $h = 0.5$ again.

(c) Runge Kutta Method: use step size $h = 1.0$, i.e. just 1 time step.

(d) Compare these approximations for $x(1)$ with the exact solution at $t = 1$ and comment about the relative accuracy of the three techniques, by computing the relative errors

$$\left|\frac{x_{approx} - x_{exact}}{x_{exact}}\right|$$

in each case.(The step sizes are so large that you don't expect any of the techniques to be extremely accurate.)

3. If we drop a package from a helicopter with a parachute attached, the wind resistance provided by the parachute is not really linear, assume for a certain type of parachute the acceleration equation is modeled by (Assume velocity in ft/s)

$$v'(t) = 32 - 0.17v - 0.13v^{1.8}$$
$$v(0) = 0$$

   (a) What is the terminal velocity in this model? Notice that the "slope function" $32 - 0.17v - 0.13v^{1.8}$ is decreasing function of v for $v > 0$ that its value is 32 when v=0, and that its limit as $v \to \infty$ is $-\infty$. Thus the slope function has exactly one root and you can find it with the Matlab (or other software)"solve" command.

   (b) This is a differential equation which does NOT have an elementary solution. Use the numerical code numericsWN.m (You may modify the Matlab code according your need) to estimate the solution values at t=2 and t=4 seconds with the methods Euler, Improved Euler and Runge Kutta with step sizes h=0.2 and h=0.02.
   • Write the approximate solution for each method with each h and each time t=2 and t=4.
   • Plot your results and comment about apparent accuracy of the three methods with these different choices of time step size.

   (c) Since we don't know the exact solution function in this problem, there is not a quick direct way to see how accurate our numerical approximate solutions are. There are error estimates that some of you will learn in later numerical analysis classes (analogous to error estimates you may have learned for Taylor series), but for the purposes of this lab undertake the following crude test: Compare the numerical approximations for velocity at $t = 2$ and $t = 4$, for Runge-Kutta with $h = 0.2$ and $h = 0.02$. If these approximations are close (and the approximate solution graphs appear to line up exactly), then it's likely that they are also close to the exact solution.

   (d) What does the asymptote on the plot mean? What happen if you change the initial condition to $v(0)$= 5 and $v(0)$= 50?