

Score: /10

Math 1321 Week 7 Lab Worksheet Due Thursday 10/23

1. **(3 points) Surfaces & Level Sets:** Investigate the shape of the surface S given by the following parametric equation $x = \sqrt{\nu} \cos u$, $y = \sqrt{\nu} \sin u$, and $z = 2\nu$
 - (a) Write the equation of S in standard form, i.e. find the function of two variables $z = f(x, y)$, such that S is the graph of this function.
 - (b) Find three level curves of $z = f(x, y)$, i.e. the curve with equation $f(x, y) = k$, where k is a constant. Can you figure out what all the level curves look like?
 - (c) Is the surface S a surface of revolution? Try to illustrate your conclusion.

2. Suppose that $f(x, y) = \frac{x^2 y}{x^4 + y^2}$.

(a) (**1 point**) Show that $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ along any line $y = mx$.

(b) (**1 point**) Show that $f(x, y) \rightarrow \frac{1}{2}$ as $(x, y) \rightarrow (0, 0)$ along the parabola $y = x^2$.

(c) (**1 point**) What conclusions can you draw? Explain.

3. Find the Limits Using Known Limits

(a) (1 point) $\lim_{(x,y) \rightarrow (0,0)} \left(1 + \frac{3x^2y}{x^2 + y^2}\right)^{\frac{x^2+y^2}{3x^2y}}.$

(b) (1 point) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin[(y+1)\sqrt{x^2+y^2}]}{\sqrt{x^2+y^2}}.$

4. **Polar Coordinates and Continuity**

- (a) **(1 point)** Suppose that $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$, use polar coordinates to verify that f is not continuous at the origin.

- (b) **(1 point)** Use the following plot of several contours of f to argue that f is not continuous at the origin.

