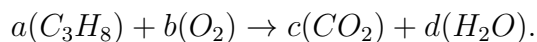


Due Date: 2/19/2015

**1. Balancing Chemical Equations**

Chemical balance means that matching elements on each side of a chemical equation have the same total amount of atoms. The balancing of a chemical reaction always leads to a system of linear equations. The equations are often “sparse” enough that the method of substitution will work well, but Gaussian elimination is guaranteed to solve the system.

The complete combustion of propane  $C_3H_8$  with oxygen  $O_2$  results in carbon dioxide  $CO_2$  and water  $H_2O$ , in such a way that the total number of atoms of each element are the same before and after the reaction:



The coefficients  $a, b, c, d$  for the numbers of each molecule need to be found from the three equations equating the number of  $C, H, O$  atoms on each side. For example, equating the number of oxygen atoms yields the equation

$$2b = 2c + d \Rightarrow 2b - 2c - d = 0.$$

Find the other two equations (i.e. find equation for carbon atoms and equation for hydrogen atoms). Then, use Gaussian elimination to find the solutions to this homogeneous system of three equations in four unknowns. Pick the free parameter in the solution so that  $a, b, c, d$  are each positive integers having no common factors. This will give you the usual balancing equation for the complete combustion of methane.

**References:** Edwards-Penney Sections 3.1-3.4

## 2. Mixing Fertilizers

When you buy fertilizer to put on your lawn or garden, one of the many things to consider is the relative amounts of nitrogen (N), phosphorus (P), and potassium (K) that it contains. A rating of the form N-P-K appears on the label, and means that the fertilizer is N% nitrogen, P% phosphorus, and K% potassium.

Suppose you intend to make your own fertilizer, and you want the mix to contain 5 lbs each of nitrogen, phosphorus, and potassium. The ingredients available to you are unlimited amounts of eggshells, feathers, and duck manure. The N-P-K ratings of the separate ingredients are

Eggshells:  $1.19 - 0.38 - 0.14$

Feathers:  $15.3 - 0 - 0$

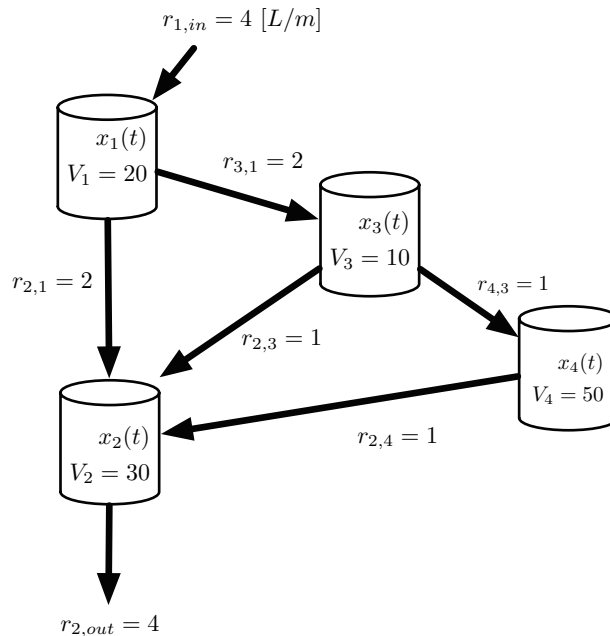
Duck Manure:  $1.22 - 1.44 - 0.6$

- (a) Set up a system of equations that describes the problem.
- (b) Solve the system. Is it possible to create the fertilizer you want?
- (c) What if, instead of feathers, you use ash from potato skins ( $0 - 5.18 - 27.5$ )?
- (d) In part c), how many pounds of fertilizer have you made? What is the N-P-K rating of your fertilizer?

**References:** Edwards-Penney Sections 3.1-3.4

3. (30 points) **Brine Tank Cascade**

This question exhibits a relationship between a system of linear equations and differential equations. Consider the cascade of four tanks depicted in the following figure with the volumes (in L) as  $V_1 = 20$ ,  $V_2 = 30$ ,  $V_3 = 10$  and  $V_4 = 40$ . The flow rates (in L/m) are indicated in the figure (e.g.  $r_{2,1}$  is the flow rate from tank 1 to tank 2). The functions  $x_1(t)$ ,  $x_2(t)$ ,  $x_3(t)$  and  $x_4(t)$  (in gal) represent the amount of salt in the tanks 1, 2, 3, and 4 at time  $t$  respectively. Suppose the concentration of the solute flowing into tank 1 is 1 gal/L.



- (a) (15 points) Express  $x'_1(t)$ ,  $x'_2(t)$ ,  $x'_3(t)$  and  $x'_4(t)$  in terms of  $x_1(t)$ ,  $x_2(t)$ ,  $x_3(t)$  and  $x_4(t)$ .
- (b) (15 points) Use the expressions in (a) to find a  $4 \times 4$  matrix  $\mathbf{A}$  and a vector  $\mathbf{v}$  such that the following relation is satisfied:

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \\ x'_4 \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \mathbf{v}.$$

**References:** Edwards-Penney Sections 1.5, 3.1-3.4, and 7.3. Course WEB notes for cascades and compartment analysis: *System Examples and Theory*.



## Some Maple, MuPad and Matlab Examples

### Creating a Matrix

The matrix  $A = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$  will be used as an example.

`A:=Matrix([[2,2,3],[4,5,6],[7,8,9]]);`      Create matrix  $A$  in Maple.

`A:=matrix([[2,2,3],[4,5,6],[7,8,9]]);`      Create matrix  $A$  in MuPad.

`A=[2 2 3; 4 5 6; 7 8 9]`      Create matrix  $A$  in Matlab.

Variable assignment uses `:=` in Maple and MuPad, whereas `=` is used in Matlab. The assignment creates a new symbol which is shorthand for the assignment, useful when you plan on using the matrix again. Uses are illustrated below.

The examples assume that two  $3 \times 3$  matrices are stored into variables  $A$  and  $B$ .

### Reduced Row Echelon Form

Maple: `linalg[rref](A);`

MuPad: `linalg::gaussJordan(A);`

Matlab: `rref(A)`

### Multiply two matrices A and B

Maple: `A.B`

MuPad: `A*B`

Matlab: `A*B`

### Inverse Matrix

Maple: `A^(-1);` or `1/A;`

Maple: `A^(-1);` or `1/A;`

MuPad: `A^(-1)` or `inv(A)`

### Augment Matrices A and B

Maple: `<A | B>`

MuPad: `A.B`

Matlab: `[A B]`

### Evaluate a Determinant

Maple: `linalg[det](A);`

MuPad: `linalg::det(A);`

Matlab: `det(A)`

### Create an $n \times n$ identity matrix

Maple: `Matrix(n,n,shape=identity);`

MuPad: `matrix::identity(n);`

Matlab: `eye(n)`

The Maple command `with(linalg):` issued just once in your Maple document allows `linalg[rref](A)` to be shortened to `rref(A);`. Maple package `LinearAlgebra` is a richer package replacement for `linalg`.

### Maple Package Equivalents

`linalg[det]` and `LinearAlgebra[Determinant]`

`linalg[rref]` and `LinearAlgebra[ReducedRowEchelonForm]`