Math 2250 Lab 3 Name/Unid: ______________________________
Due Date: 02/05/2015

1. **Paratrooper with Linear Drag**
   A paratrooper bails out of an airplane at altitude 15,000 ft, then falls freely for 30 seconds, then opens a parachute. Assume linear air resistance kv ft/s^2 with drag coefficient ρ=0.4 before the parachute opens and ρ= 4 after the parachute opens.

   Note: A technology check is expected, as in the previous problem.

   (a) Write down the ODE for velocity v of the paratrooper before the parachute opens.

   (b) Find the velocity at time 5 seconds and the velocity at time 30 seconds, when the paratrooper just opens the parachute.

   (c) Find the height when the paratrooper opens the parachute.

   (d) Explain why $w' = -32 - 4w, w(0) = v(30) \approx -80$ is the model for the paratrooper's descent after the parachute opens.

   (e) Explain why the total time of descent for the paratrooper is $t = 30 + 1598 = 1628$s.

   Reference: Edwards & Penney Section 2.3. This traditional parachute problem can be modified to be more realistic. Read Meade & Struthers (1999) *Differential Equations in the New Millennium: the Parachute Problem,* Int. J. Engng Ed. 15(6), 417-424. Retrieved on January 14, 2014. See also Meade, D (1998), *ODE Models for the Parachute Problem,* SIAM Review. 40(2). 327-332. Retrieved on January 14, 2014 at

2. The growth of a certain bacteria in a reactor is assumed to be governed by the logistic equation:

$$\frac{dP}{dt} = k \cdot P(M - P)$$

where P is the population in millions and t is the time in days. Recall that M is the carrying capacity of the reactor and k is a constant that depends on the growth rate.

   (a) Suppose that the carrying capacity of the reactor is 10 million bacteria, and that the peak growth rate is 3 million bacteria per day. Determine the constants k and M in the above equation.

   (b) Supposing the bioreactor has 250,000 bacteria in it to begin with, find the number of bacteria in the tank. how long will it take for the population to reach 50% of the carrying capacity?

   (c) Suppose that bacteria are being harvested from the tank continuously. Let h be the rate at which the bacteria are harvested in millions per day. Write down the new differential equation governing the bacteria population. What is the maximum rate of harvesting h that will not cause the population of bacteria to go extinct? (Below this rate there will always be a stable equilibrium point where P is positive).

Reference: Edwards & Penney Section 2.1.