Solution for Problem Set 4

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October 28, 2015

Problem 1

A

$$\mathrm{P}(D=T) = \sum_S \sum_T \mathrm{P}(D=T|S,T)\mathrm{P}(S,T)$$
$$= 0.95 \times 0.75 \times 0.6 + 0.1 \times 0.75 \times 0.4 + 0.2 \times 0.6 \times 0.25$$
$$= \frac{39}{80}$$
$$\mathrm{P}(D=F) = 1 - \mathrm{P}(D=T) = \frac{41}{80}$$

B

$$\mathrm{P}(L=T|D=F) = \frac{\mathrm{P}(L=T, D=F)}{\mathrm{P}(D=F)}$$
$$= \frac{\mathrm{P}(L=T, D=F, S=T) + \mathrm{P}(L=T, D=F, S=F)}{\mathrm{P}(D=F)}$$
$$= \frac{0.05 \times 0.6 \times 0.75 + 0.9 \times 0.4 \times 0.75}{\frac{41}{80}}$$
$$= \frac{117}{205}$$

C

$$\mathrm{P}(D2=T|D1=F) = \frac{\mathrm{P}(D2=T, D1=F)}{\mathrm{P}(D1)}$$
$$= \frac{\sum_{S2,L} \mathrm{P}(D2, D1|S2 \cap L)\mathrm{P}(S2 \cap L)}{\mathrm{P}(D1)}$$
$$= \frac{\sum_{S2,L} \mathrm{P}(D2|S2 \cap L)\mathrm{P}(D1|S2 \cap L)\mathrm{P}(S2)\mathrm{P}(L)}{\mathrm{P}(D1)}$$
$$= \frac{\sum_{S2,L} \mathrm{P}(D2|S2 \cap L)\mathrm{P}(S2) \sum_{S1} \mathrm{P}(D1|L, S1)\mathrm{P}(L, S1)}{\mathrm{P}(D1)}$$
$$= 0.399659$$

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D

no ask   0

11.2615

accept
0.4875

Love
0.938462   19

no Love
0.0615385   -7

5.41458

ask

5.41458

no accept
0.5125

no ask   -4

-0.147171

ask

-0.147171

accept
0.399659

11.6488

love
0.871109   15
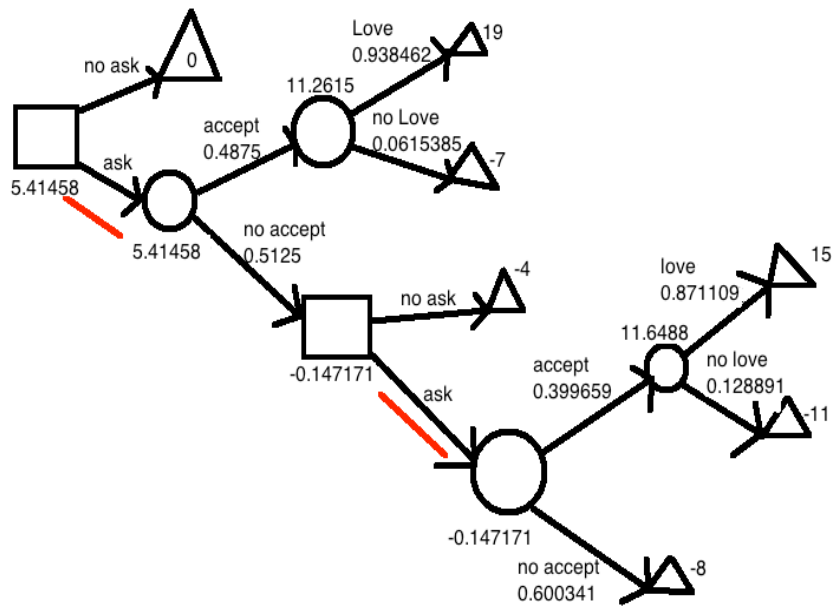
no love
0.128891   -11

no accept
0.600341   -8

Figure 1: Diagram for **Problem 1**

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Problem 2

A

Let X_i be the variable of the outcome of i-th clause, 0 for false and 1 for truth. Then the sum of them is K. Then

$$E(K) = \sum_{i=1}^{N} E(X_i) = \frac{7N}{8}$$

B

When enumerate all possible value, the result is:

- $K = 5$ has 8 possible values.

- $K = 4$ has 6 possible values.

- $K = 3$ has 2 possible values.

- $K = 2, 1, 0$ has no value.

So the distribution of K is

$$\mathrm{P}(K=5) = \frac{1}{2}, \mathrm{P}(K=4) = \frac{3}{8}, \mathrm{P}(K=3) = \frac{1}{8}$$