Solution for Problem Set 3

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Problem 1

I would like to set some functions on it.

- Intersec(W_i, W_j) returns TRUE if W_i and W_j have same element, otherwise it returns FALSE.
- In(x, W) returns TRUE if W contains x, otherwise it returns FALSE.

Then we have two constraints.

1. $\forall_{i \neq j} \text{Intersec}(W_i, W_j) \Rightarrow \neg W_i \vee \neg W_j$
2. $\bigwedge_{x \in U} \bigvee_{W:\text{In}(x,W)} W$

Supposed there is a simple example: $U = \{a, b, c, d, e, f\}$ and

$W_1 = \{a, b\}$

$W_2 = \{b, c\}$

$W_3 = \{c, d\}$

$W_4 = \{d, e\}$

$W_5 = \{e, f\}$

$W_6 = \{f, a\}$

Then we can have the following constraints:

1. $\forall_{i \neq j} \text{Intersec}(W_i, W_j) \Rightarrow \neg W_i \vee \neg W_j$
2. $W_1 \vee W_6$
3. $W_1 \vee W_2$
4. $W_2 \vee W_3$
5. $W_3 \vee W_4$
6. $W_4 \vee W_5$
7. $W_5 \vee W_6$

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Problem 2

a. $\forall_{p,u} W(p,u) \Rightarrow S(p,u)$

b. $\forall_{u} A(u,H) \wedge S(B,u) \Rightarrow L(B,u)$

c. $\forall_{p,u} F(B,p) \wedge W(p,u) \Rightarrow S(B,u)$

d. $\forall_{p} (F(T,p) \Rightarrow \exists_{u} A(u,H) \wedge W(p,u))$

e. $\exists_{p} F(T,p) \wedge F(B,p)$

f. $\exists_{p,u} F(T,p) \wedge W(p,u) \wedge L(B,u)$

g. $\forall_{u} W(G,u) \Rightarrow A(u,H)$

h. $\forall_{p} (F(B,p) \Rightarrow \exists_{u} W(p,u))$

i. $\forall_{u} W(G,u) \Rightarrow \neg L(B,u)$

j. $\neg F(B,G)$

Problem 3

Use counter example. Let (f) become (f*): $\neg(\exists_{p,u} F(T,p) \wedge W(p,u) \wedge L(B,u))$. Then we change b, c, d, e and f* to CNF:

1. $\neg A(u,H) \vee \neg S(B,u) \vee L(B,u)$
2. $\neg F(B,p) \vee \neg W(p,u) \vee S(B,u)$
3. $\neg F(T,p) \vee A(sk1(p),H)$
4. $\neg F(T,p) \vee W(p,sk1(p))$
5. $F(T,sk2)$
6. $F(B,sk2)$
7. $\neg F(T,p) \vee \neg W(p,u) \vee \neg L(B,u)$

Then we have the following steps.

Step 1. $5+3$, $p \leftarrow sk2$ we have 8. $A(sk1(sk2),H)$

Step 2. $5+4$, $p \leftarrow sk2$ we have 9. $W(sk2,sk1(sk2))$

Step 3. $5+7$, $p \leftarrow sk2$ we have 10. $\neg W(sk2,u) \vee \neg L(B,u)$

Step 4. $6+2$, $p \leftarrow sk2$ we have 11. $\neg W(sk2,u) \vee S(B,u)$

Step 5. $9+10$, $u \leftarrow sk1(sk2)$ we have 12. $\neg L(B,sk1(sk2))$

Step 6. $9+11$, $u \leftarrow sk1(sk2)$ we have 13. $S(B,sk1(sk2))$

Step 7. $8+1$, $u \leftarrow sk1(sk2)$ we have 14. $\neg S(B,sk1(sk2)) \vee L(B,sk1(sk2))$

Step 8. $14 + 12$, we have 15. $\neg S(B, sk1(sk2))$

Step 9. $15 + 13$, we have 16. NULL

Then we finish the proof.

Problem 4

Use counter example. Let (j) become (j*): $F(B, G)$. Then change (b), (c), (g), (h), (i) and (j*) to CNF:

1. $\neg A(u, H) \vee \neg S(B, u) \vee L(B, u)$

2. $\neg F(B, p) \vee \neg W(p, u) \vee S(B, u)$

3. $\neg W(G, u) \vee A(u, H)$

4. $\neg F(B, p) \vee W(p, sk1(p))$

5. $\neg W(G, u) \vee \neg L(B, u)$

6. $F(B, G)$

Follow the resolution step:

Step 1. $4 + 6$, $p \leftarrow G$ we have 7. $W(G, sk1(G))$

Step 2. $2 + 6$, $p \leftarrow G$ we have 8. $\neg W(G, u) \vee S(B, u)$

Step 3. $7 + 5$, $u \leftarrow sk1(G)$ we have 9. $\neg L(B, sk1(G))$

Step 4. $7 + 3$, $u \leftarrow sk1(G)$ we have 10. $A(sk1(G), H)$

Step 5. $7 + 8$, $u \leftarrow sk1(G)$ we have 11. $S(B, sk1(G))$

Step 6. $9 + 1$, $u \leftarrow sk1(G)$ we have 12. $\neg A(sk1(G), H) \vee \neg S(B, sk1(G))$

Step 7. $12 + 10$, we have 13. $\neg S(B, sk1(G))$

Step 8. $13 + 11$, we have 14. NULL

Then we finish the proof.