Solution for Problem Set 2

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Problem 1

Follow alpha-beta pruning algorithm, I finally get this diagram.

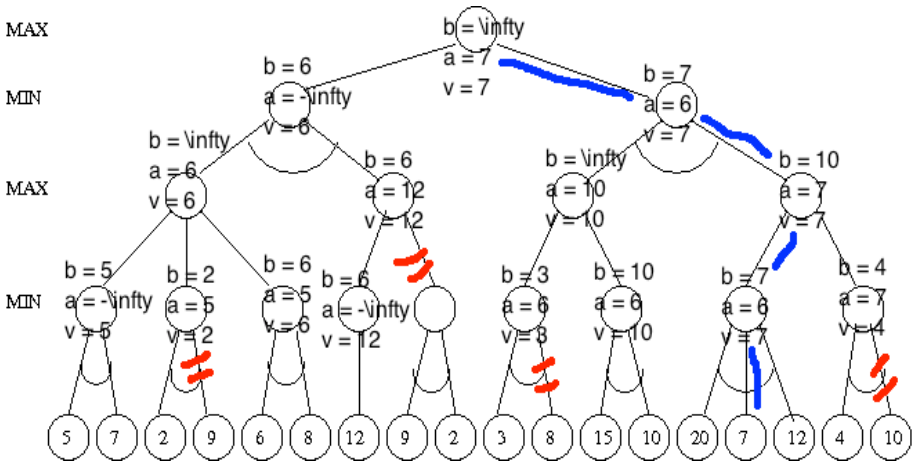

Figure 1: Diagram for **Problem 1**

In this diagram, 'b' means β value in this node, which means the upper bound of the best choice, and 'a' means α value in this node, which means the lower bound of the best choice (under this sub-tree). Also 'v' means the best value, the red slashing line means the pruning and the blue line means the best move.

Problem 2

1.

$$
\begin{aligned}
& C \Rightarrow (A \Leftrightarrow E) \\
\Leftrightarrow & \neg C \vee ((A \Rightarrow E) \wedge (E \Rightarrow A)) \\
\Leftrightarrow & \neg C \vee ((\neg A \vee E) \wedge (\neg E \vee A)) \\
\Leftrightarrow & (\neg C \vee \neg A \vee E) \wedge (\neg C \vee \neg E \vee A)
\end{aligned}
$$

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2.

$$\begin{aligned}
&(\neg C \vee E) \Rightarrow B \\
\Leftrightarrow\ &\neg(\neg C \vee E) \vee B \\
\Leftrightarrow\ &(C \wedge \neg E) \vee B \\
\Leftrightarrow\ &(C \vee B) \wedge (\neg E \vee B)
\end{aligned}$$

3.

$$\begin{aligned}
&D \Rightarrow \neg B \\
\Leftrightarrow\ &\neg D \vee \neg B
\end{aligned}$$

4.

$$\begin{aligned}
&(A \wedge D) \Rightarrow \neg E \\
\Leftrightarrow\ &\neg(A \wedge D) \vee \neg E \\
\Leftrightarrow\ &\neg A \vee \neg D \vee \neg E
\end{aligned}$$

5. This is clearly a clausal form.

6.

$$\begin{aligned}
&E \Rightarrow D \\
\Leftrightarrow\ &\neg E \vee D
\end{aligned}$$

Problem 3

Initial set of clauses S0:

1. $\neg C \vee \neg A \vee E$
2. $\neg C \vee \neg E \vee A$
3. $C \vee B$
4. $\neg E \vee B$
5. $\neg D \vee \neg B$
6. $\neg A \vee \neg D \vee \neg E$
7. $C \vee D \vee E$
8. $\neg E \vee D$

Initial valuation V0: All atoms unbound. Sequence of calls.

I. Call dp1(ATOMS,S0,V0)

No pure literals, no singleton clauses.

Try V[A] := TRUE; V1[A] = TRUE.

Call propagate(A,S0,V1): Delete clause 2, delete $\neg A$ from 1, 6

New set of clauses S1:

1. $\neg C \vee E$
3. $C \vee B$
4. $\neg E \vee B$
5. $\neg D \vee \neg B$
6. $\neg D \vee \neg E$
7. $C \vee D \vee E$
8. $\neg E \vee D$

II. Call dp1(ATOMS,S1,V1)

No pure literals, no singleton clauses.

Try V[B] := TRUE; V2 is the valuation V2[B] = TRUE, V2[A] = TRUE.

Call propagate(B,S1,V2): Delete clauses 3 and 4, delete $\neg B$ from 5.

New set of clauses S2:

1. $\neg C \vee E$
5. $\neg D$
6. $\neg D \vee \neg E$
7. $C \vee D \vee E$
8. $\neg E \vee D$

III. Call dp1(ATOMS, S2, V2)

5 is a singleton clause with literal $\neg D$;

V[D] = FALSE;

V3 is the valuation V3[D] = FALSE, V3[B] = TRUE, V3[A] = TRUE.

Call propagate(D,S2,V3): Delete clauses 5 and 6, delete D from 7 and 8.

New set of clauses S3:

1. $\neg C \vee E$
7. $C \vee E$
8. $\neg E$

8 is a singleton clause with literal $\neg E$;

V[E] = FALSE;

V4 is the valuation V4[E] = FALSE, V4[D] = FALSE, V4[B] = TRUE, V4[A] = TRUE.

Call propagate(E,S3,V4): Delete clause 8, delete E from 1 and 7.

New set of clause S4:

1. $\neg C$

7. C

1 is a singleton clause with literal $\neg C$;

V[C] = FALSE;

V5 is the valuation V5[C] = FALSE, V4[E] = FALSE, V4[D] = FALSE, V4[B] = TRUE, V4[A] = TRUE.

Call propagate(C,S4,V5): Delete clause 1, delete C from 7.

New set of clauses S5:

7. $\emptyset$

7 is the empty clause.

III returns NIL to II.

II. continuing.
Try V[B] := FALSE; V6 is the valuation V6[B] = FALSE, V6[A] = TRUE.
Call propagate(B,S1,V6): Delete clause 5, delete B from 3 and 4.
New set of clauses S6:
1. $\neg C \vee E$
3. C
4. $\neg E$
6. $\neg D \vee \neg E$
7. $C \vee D \vee E$
8. $\neg E \vee D$

IV. Call dp1(ATOMS, S6, V6)

3 is a singleton clause with literal C;

V[C] = TRUE; V7 is the valuation V7[C] = TRUE, V7[B] = FALSE, V7[A] = TRUE.

Call propagate(C,S6,V7): Delete clause 3 and 7, delete $\neg C$ from 1.

New set of clauses S7:

1. E

4. $\neg E$

6. $\neg D \vee \neg E$

8. $\neg E \vee D$

1 is a singleton clause with literal E;

V[E] = TRUE; V8 is the valuation V8[E] = TRUE, V8[C] = TRUE, V8[B] = FALSE, V7[A] = TRUE.

Call propagate(E,S7,V8): Delete clause 1, delete $\neg E$ from 4, 6 and 8.

New set of clauses S8:

4. $\emptyset$

6. $\neg D$

8. D

4 is the empty clause.

IV returns NIL to II.

II having failed with both TRUE and FALSE for B, return NIL to I.

I continuing

Try V[A] = FALSE; V9 is the valuation V9[A] = FALSE.
Call propagate(A,S0,V9); Delete clause 1 and 6, delete A from 2
New set of clauses S9:
2. $\neg C \vee \neg E$
3. $C \vee B$
4. $\neg E \vee B$
5. $\neg D \vee \neg B$
7. $C \vee D \vee E$
8. $\neg E \vee D$

V. Call dp1(ATOMS, S9, V9).

No pure literals, no singleton clauses.

Try V[B] := TRUE; V10 is the valuation V10[B] = TRUE, V10[A] = FALSE.

Call propagate(B,S9,V10): Delete clauses 3 and 4, delete $\neg B$ from 5

New set of clauses S10:

2. $\neg C \vee \neg E$

5. $\neg D$

7. $C \vee D \vee E$

8. $\neg E \vee D$

5 is a singleton with literal $\neg D$;

V[D] = FALSE; V11 is the valuation V11[D] = FALSE, V11[B] = TRUE, V11[A] = FALSE.

Call propagate(D,S10,V11): Delete clause 5, delete D from 7 and 8.

New set of S11:

2. $\neg C \vee \neg E$

7. $C \vee E$

8. $\neg E$

8 is a singleton with literal $\neg E$;

V[E] = FALSE; V12 is the valuation V12[E] = FALSE, V12[D] = FALSE, V12[B] = TRUE, V12[A] = FALSE.

Call propagate(E,S11,V12): Delete clauses 2 and 8, delete E from 7

New set of S12:

7. C

7 is a singleton with literal C;

V[C] = TRUE; V13 is the valuation V13[C] = TRUE, V13[E] = FALSE, V13[D] = FALSE, V13[B] = TRUE, V13[A] = FALSE.

Call propagate(C,S12,V13): Delete clause 7.

S13 is the empty set of clauses.

Return V13 to the top level.

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