Problem 1:

To begin with my simulation, I set the initial price:

$P_0 = 100,\ r_t \sim N(0,\ 0.1) \Rightarrow \mu = 0,\ \sigma = 0.01$

Mathematical deduction of my expected values:

- Classical Brownian Motion

  According to property of the Normal Distribution that any linear function of a normal variable is also Normally Distributed

  $Y = A + BX$

  $Y \sim N(A + B\mu,\ B^2\sigma^2)$

  $Know\ that\ r \sim N(0,\ \sigma^2)\ is\ normally\ distributed$

  $P_1 = P_0 + r_1$

  $\Rightarrow A = P_0,\ B = 1$

  $\Rightarrow P_1 \sim N(P_0,\ \sigma^2)$

  $\Rightarrow P_1 = 100,\ \sigma = 0.01$

- Arithmetic Return:

  $P_1 = P_0(1 + r_1) = P_0 + P_0 r_1$

  $Y = A + BX \Rightarrow A = P_0,\ B = P_0$

  $Y \sim N(A + B\mu,\ B^2\sigma^2),\ r \sim N(0,\ \sigma^2)$

  $\Rightarrow P_1 \sim N(P_0 + P_0\mu,\ P_0^2\sigma^2) \Leftrightarrow P_1 \sim N(P_0,\ P_0^2\sigma^2)$

  $\Rightarrow P_1 = 100,\ \sigma = \sqrt{0.01^2(100^2)} = 1$

- Log Return:

  $P_1 = P_0 e^{r_1}$

  $\Rightarrow ln(P_1) = ln(P_0) + r_1$

  $Y = A + BX \Rightarrow A = ln(P_0),\ B = 1$

  $\Rightarrow ln(P_1) \sim N(ln(P_0) + \mu,\ \sigma^2) \Leftrightarrow P_1 \sim LN(ln(P_0),\ \sigma^2)$

  $\Rightarrow P_1 = e^{(ln(100) + 0.01^2/2)} \approx 100,\ \sigma = \sqrt{(e^{0.01^2} - 1)e^{(ln(P_0)\ + 0.01^2)}} \approx 0.1$

My actual results:

- Classical Brownian Motion

```
np.mean(pc[1:])
```

100.00001489756146

```
np.std(pc[1:])
```

0.010061015077766045

- Arithmetic Return

```
np.mean(pa[1:])
```

```
100.00148975614617
```

```
np.std(pa[1:])
```

```
1.0061015077766013
```

- Log Return

```
np.mean(pl[1:])
```

```
100.00655108430765
```

```
np.std(pl[1:])
```

```
1.0061660640831025
```

Conclusion:

For both classical brownian motion and arithmetic return, the mean and σ are approximately equal to my expectation. For the log return, mean was approximately equal to my expectation but $\sigma \approx 1.006$, about 10 times larger than my expectation.

Problem 2:

- Normal Distribution:

```
var_n
```

0.0342555180863239O5

- EWMA Normal Distribution:

```
var_exp
```

0.05072918921533632

- Maximum Likelihood assuming T-distribution:

```
var_mle
```

1.0025513513104038

- Historical

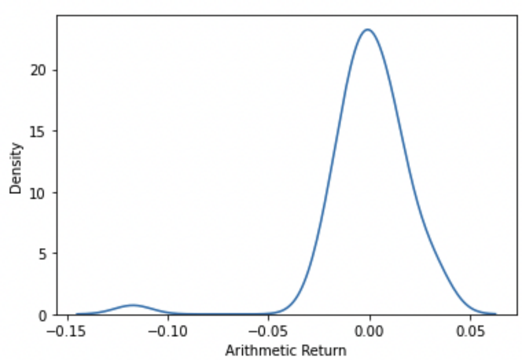
```
VaR(intc_demean)
```

0.11450355388382355

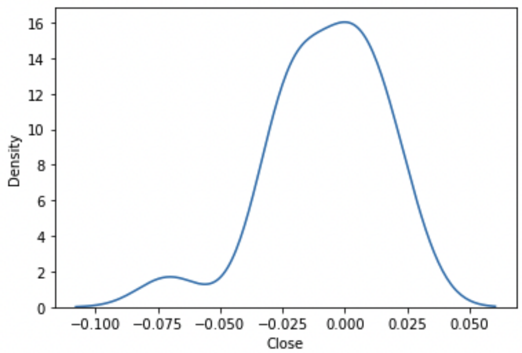
Conclusion:

As calculated above, the value at risk when assuming normal distribution is the smallest, VaR calculated assuming normal distribution (parametric, ewma, and historical) have similar result $VaR \in (0,\ 0.12)$ while VaR that was calculated using MLE assuming T-distribution is significantly larger than the rest.

- Return distribution generated from DailyPrices.CSV



- Most recent returns generated from Yahoo Finance



As shown in both graphs from earlier returns and most recent returns from Yahoo Finance, the returns for INTC are left skewed.

The risk for INTC is that investors may expect frequent small gains and a few large losses.

Problem 3:

```
varA
```

```
1.013918537073878
```

```
varB
```

```
1.0144714479417678
```

```
varC
```

```
1.0110980676399755
```

```
varT
```

```
1.0053690699731563
```

Conclusion:

I used parametric VaR because the only things I would need to calculate are the mean and standard deviation. It is precise to some extent and time-efficient.

At a = 0.05, the value at risk of portfolio A, B, C and total portfolio are very similar.