COMPUTER SIMULATIONS
OF
VARIOUS BLACKJACK STRATEGIES

BY

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ABSTRACT

A generalized blackjack strategy evaluator has been programmed. It has shown that the game of blackjack can be beaten. It also tells what the expectations are for a given strategy.

In addition to providing support for the expectations previously suggested for imitating the dealer and for Thorp's basic strategy, the evaluator has shown the true expectation for a never bust strategy.

The evaluator has shown the high-low index to be a more accurate partial deck evaluation function than the tens ratio method. The high-low index not only had a higher expectation than the tens ratio method, but it did so with a smaller average bet size.

Now that the strategy evaluator exists, it may be modified so that it copies the rules of any given casino (or group of casinos using the same rules, as in Puerto Rico). Then different strategies may be simulated until an acceptable strategy has been found.

ACKNOWLEDGEMENTS

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INTRODUCTION

In the past few years there has been quite a bit of controversy concerning the casino card game of blackjack. There have been many claims of winning strategies (3,4,5,6,7) as well as counterclaims from casino owners that system players are always welcome in their clubs (6,p.65). The public generally agreed with the casino owners. After all, if someone could consistantly win sizeable sums gambling, then why would he not do so to the exclusion of any other job?

This skepticism bothered me because of the structure of blackjack. In most games, the cards are shuffled after each hand. In blackjack, the cards are shuffled at the end of the deck. It is quite possible that someone with the knowledge of which cards remained to be played might be able to devise a winning strategy for the game. In order to resolve the question, I have written a blackjack strategy evaluator program which may be used to test given strategies, as well as to help devise newer, more powerful ones.

Before explaining the program and evaluating strategies, I think that an understanding of the rules of black jack is important. If the reader is not already familiar with those rules, he may refer to Appendix A

where they are listed.

As it may be seen from the rules, at various times the player is called upon to make such strategic decisions as how large his bet should be; whether to insure his hand when the dealer shows an ace; whether he should split a pair; whether he should double down; and whether he should stand on a given total. These decisions may be expressed in the form of a flow chart as in figure 1.

In general, the choice made will depend upon the cards that have been played and the dealer's up card. These decisions concerning bet size, insurance size, splitting, doubling down, and drawing comprise a player's strategy. The following questions arise. What comprises a good overall strategy? How can the player determine his chance of winning in a particular round? How can the player's expectation be changed by variations in his betting strategy? And finally, how can the expectation for a particular set of player strategies be determined? These are some of the questions which will be answered in this paper.

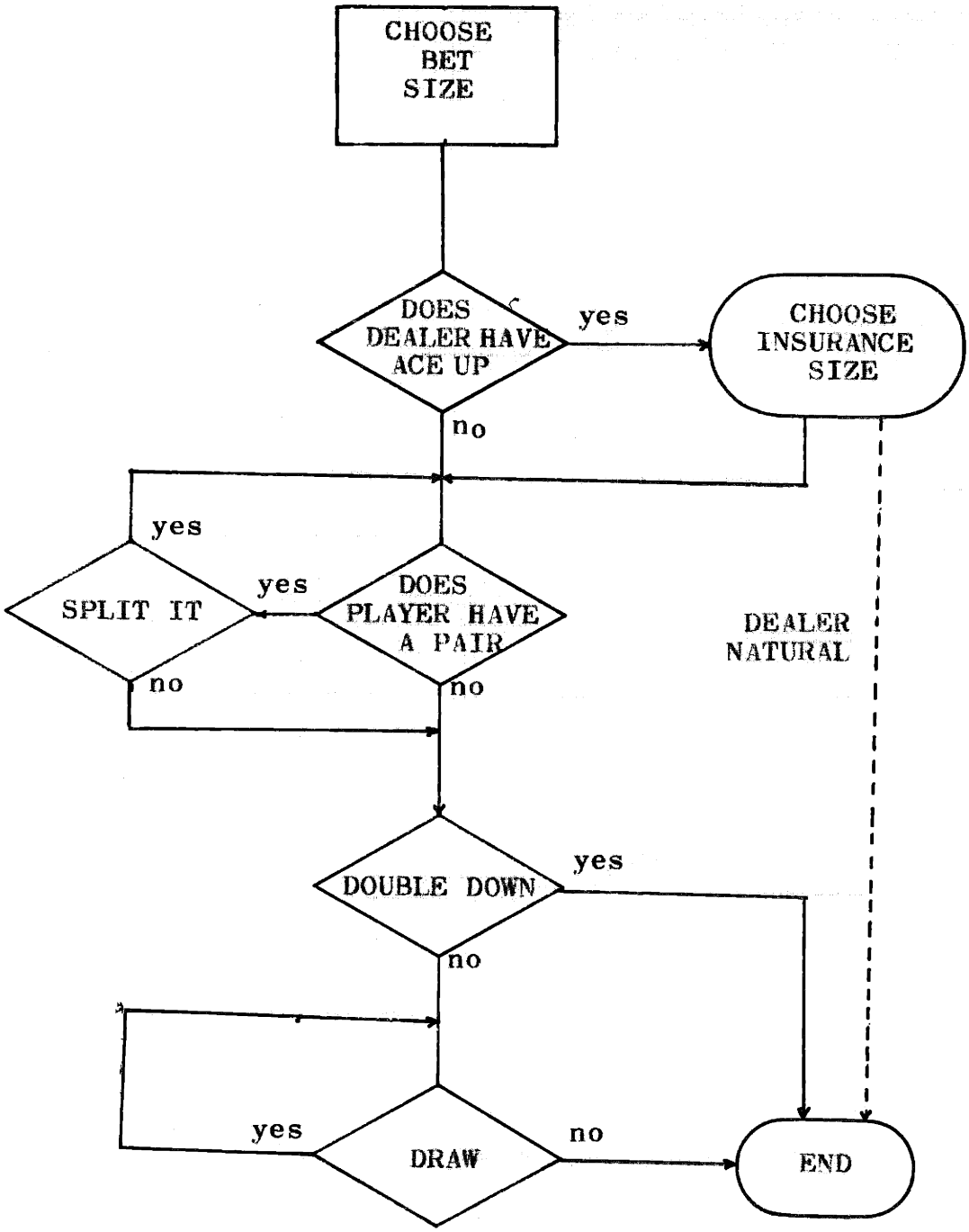


FIGURE 1 - - PLAYER DECISIONS

IMPORTANCE OF MEMORY IN BLACKJACK

Before continuing, we should understand exactly what we are looking for and why we expect to find it. As we have already seen, blackjack differs from other casino games in that the hands are not independent of one another. Since the cards are only shuffled at the end of the deck, we know that a card that has been exposed in a particular round cannot be dealt in a succeeding round. In particular, if we have observed all four aces dealt in previous rounds, then we know there will be no more naturals until the cards have been shuffled. This dependency upon the cards which have been played comprises the "memory" of the game. Thus, blackjack, unlike other casino games, disobeys the old gambler's adage that says "the cards have no memory." Once this has been observed, it is quite reasonable to expect that with a perfect memory one might be able to devise a strategy with which he could "beat the game."

Unfortunately, few of us are able to sit down amidst the chaotic surroundings of a casino gambling table and calmly memorize every card which has been dealt since the last shuffle. Even if this was a simple matter, however, there is still a question as

to just how useful this information would be. For example, if one had a list of all of the cards remaining to be played, how could this information be used in determining strategies? Clearly, if there are no tens left in the deck, then any offer to insure would be refused. In this case the insurance strategy would be changed. If the player knew the deck consisted of only five cards, namely three eights and two sevens, he would alter his strategy in that he would now bet the house limit and stand on whatever two cards he drew. The dealer would then have a total of either fourteen (7,7), fifteen (7,8), or sixteen (8,8). In any case, he would have to draw and bust. Generally, however, it is a difficult task to tell how to react to a particular partial deck composition.

PARTIAL DECK EVALUATION

We now see that a method of evaluating partial decks is at least as important as knowing what the partial decks are. If we could find a reliable method of partial deck evaluation with respect to our probability of winning a hand dealt from that partial deck we can obtain a winning strategy. This would be done by betting small amounts when our partial deck evaluation formula told us that we had a small probability of winning and betting large amounts when our partial deck evaluation formula told us that we had a large probability of winning.

Now that we know what is necessary for a winning strategy we must return from the theoretical world to reality. As we know, it is a difficult enough task to know what comprises the partial deck at any time. Yet, for a winning strategy, we must also be able to evaluate our probability of winning, given the partial deck composition. This is certainly not the easiest of tasks, and the only rational solution, short of bringing a computer along on trips to the casino, is to develop some approximate method of remembering and evaluating partial decks. Here a problem arises as to whether the particular approximate method actually

works and also whether it is better than other methods which are known to exist. Until now there has been no definitive way of answering this question.

COMPUTER SOLUTION

In order to determine how "good" a particular strategy is, I have programmed an IBM 360 computer to play the game of blackjack with the rules of Appendix A. Knowing that these rules are not strictly adhered to in all casinos, I provided enough generality in the program so that rule changes, such as not dealing to the bottom of the deck or using more than one pack of cards (both now used in Puerto Rico to make card counting more difficult) could easily be incorporated into the original program with a minimum amount of reprogramming.

The operation of the program may be understood by referring to the diagram of figure 2. This flow chart describes the game of blackjack. It is purposely left in a coarse form (i.e. settling of naturals is left out) in order to make it more understandable. There are five blocks which contain asterisks. These refer to the fact that player decisions are called for before the proper action can be taken. For example, the player must decide how much to bet before he or the dealer is dealt the initial hand. Since these decisions are dependent upon the player's strategy and not upon the evaluator program, they have been separated from the main program in the form of subprograms, or subrou-

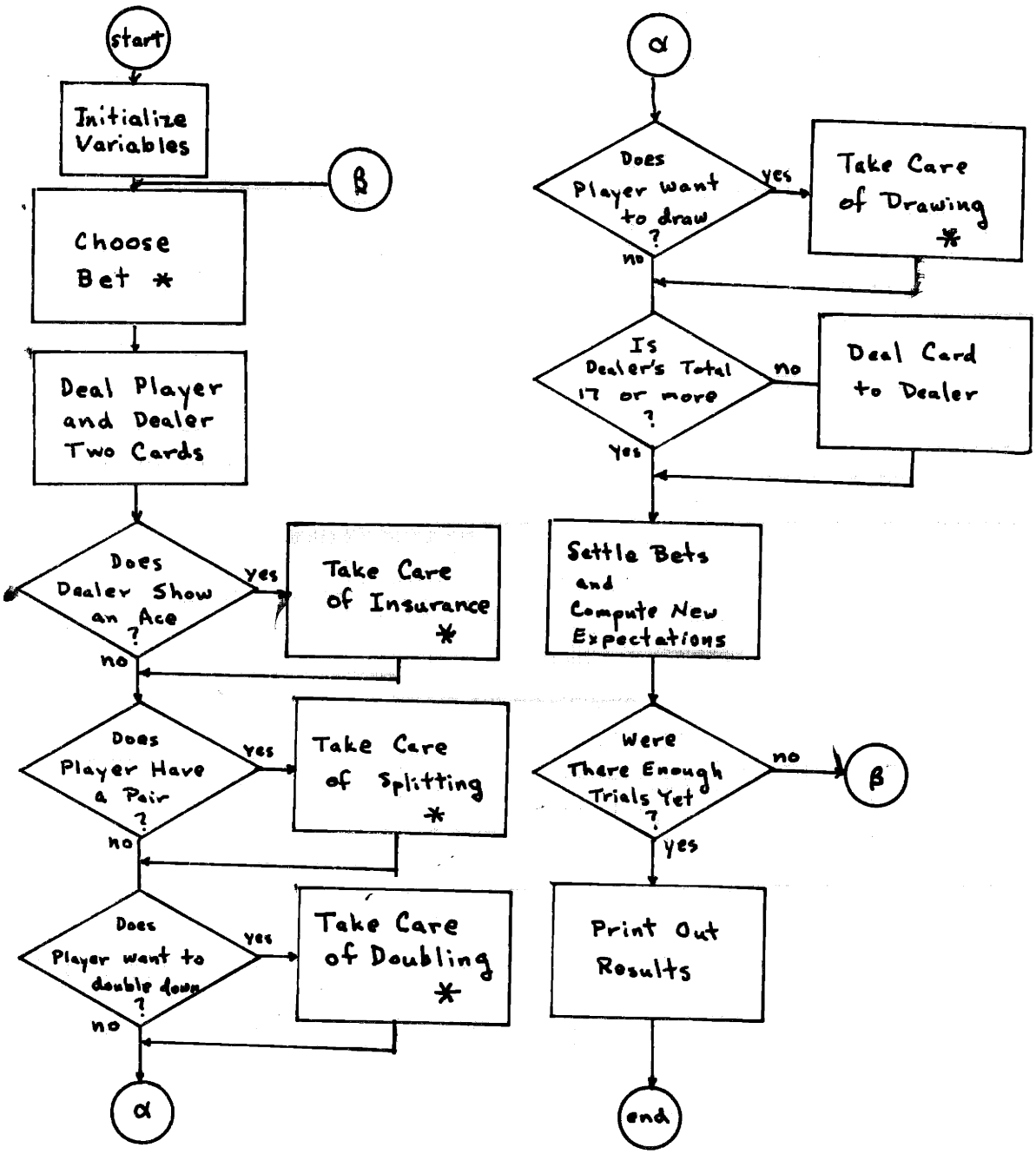


FIGURE 2 -- PROGRAM OPERATION

tines as they are called. Thus, changing the strategy (or strategies) being evaluated entails only changing the subroutine(s) associated with the respective strategy (or strategies). This provides numerous advantages. The complexity of the evaluator is absorbed in the main program, so the subroutines are very easy to write. They are also quite short. This means that a given strategy may usually be implemented as a subroutine in a time commensurate with the time it takes to describe the strategy.

Referring to figure 2, we see that there is a loop which is circled a number of times corresponding to the number of rounds of play we wish to simulate. This does not correspond to the number of hands we are simulating, as that number may be much larger due to splitting pairs. Obviously, the more rounds we simulate, the more valid our final answer will be. However, we will reach a point where additional simulations do us relatively little good. This may be seen by letting our average bet be B, the number of games simulated be G, our winnings be W, and our observed expectation E(G) be:

$$E(G) = \frac{W}{G}$$

If we are to simulate one additional round, our new

expectation will be:

$$E(G+1) = \frac{W \pm B}{G+1} = \frac{W}{G+1} \pm \frac{B}{G+1}$$

Assuming that G is much greater than 1 this becomes:

$$E(G+1) \approx \frac{W}{G} \pm \frac{B}{G} = E(G) \pm \frac{B}{G}$$

$$|\Delta E| = |E(G+1) - E(G)| = \frac{B}{G}$$

Now, if we arbitrarily decide that we want $|\Delta E|$ to be less than some small number, say 0.0005, we can determine the number of rounds we need to simulate from the inequality:

$$|\Delta E| < 0.0005$$

$$\frac{B}{G} < 5 \times 10^{-4}$$

$$G > 2 \times 10^{3} B$$

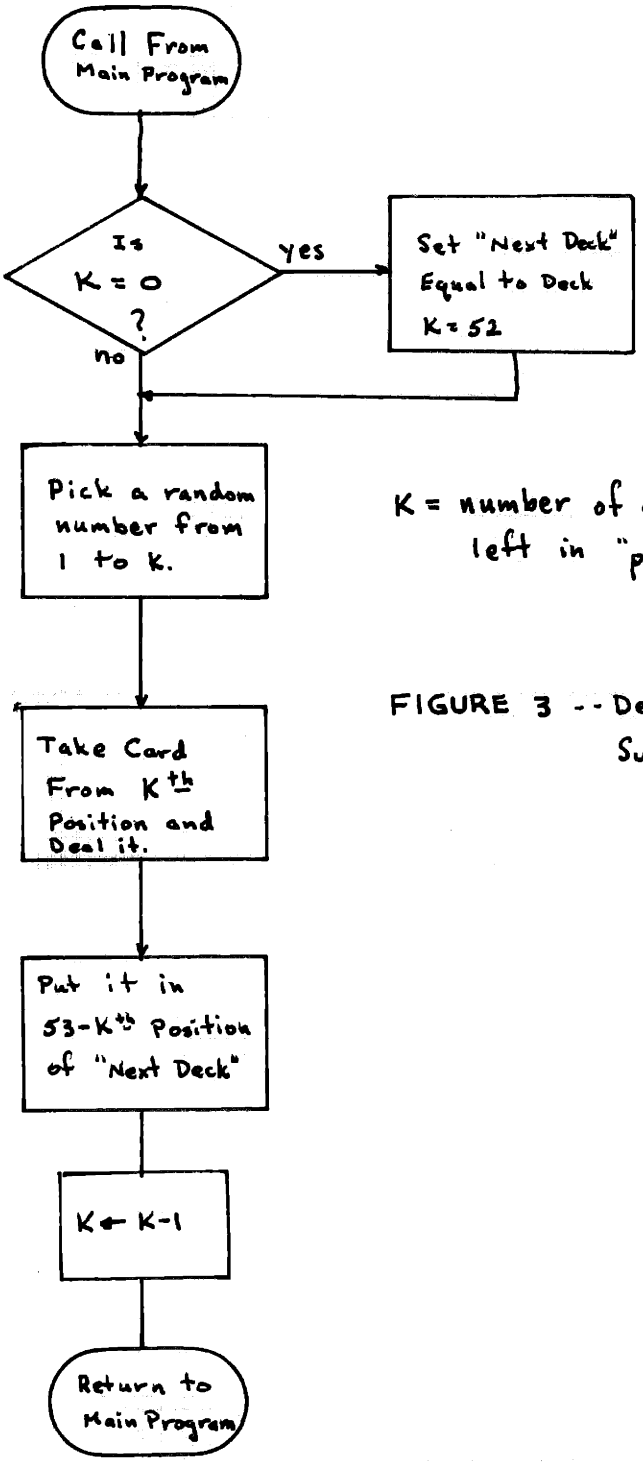
If we assume that the expectation is converging to the correct value and we use a constant bet size, B, of 2 units, the inequality tells us we should simulate at least $2 \times 10^{3} \times 2$ or 4000 hands. Since we want a high probability of obtaining the expectation to within 0.0005, we will simulate 5000 rounds (which may be more than 5000 hands). When the strategy calls for variable bet sizes of from 2 to 10 units, 50,000 rounds will

be simulated.

Another point of departure from the coarse flow chart is that the portion of the evaluator which is called upon to deal and shuffle cards is also maintained as a subroutine. This results from the fact that the algorithm used for dealing and shuffling is fairly complex and is called from various points in the main program. By writing it in this manner, each call may be written as only one statement.

Basically, the algorithm for dealing may be represented by the flow chart in figure 3. No initialization is shown because this occurs at the beginning of the main program as shown in figure 2. Each time this subroutine is called it "deals" a card, adds it to the "next deck", and eliminates it from the "present deck." As shown, when the "present deck" has been depleted of cards the "next deck" which now contains all fifty-two cards randomly ordered with respect to the "present deck" becomes the new deck.

In order to change the program to play under different rules, such as using more than one pack of cards or not dealing to the bottom of the deck, we need only change the portions of the main program corresponding to initialization and portions of this subroutine.



K = number of cards left in "present deck"

FIGURE 3 -- Dealing Subroutine

EVALUATED STRATEGIES

I. Imitate the Dealer

The first strategy which was simulated was that of imitating the dealer. This means that the player will always bet the same amount (2 units), never insure, never split pairs, never double down, and will always draw to totals of less than 17. Many players have chosen this to be their strategy assuming that it would cancel any advantage which the house has. Unfortunately for them, the game is not entirely symmetrical in two respects. First of all, they wrongly assume that any time there is a tie they neither win or lose their bet. This is called a "push". The fallacy of this argument occurs when the player busts. As this occurs before the dealer has had an opportunity to draw, he cannot also bust and "tie" for a push. Thus, the dealer has the advantage any time the player busts in a round in which the dealer would have subsequently busted. Since the probability of a bust using the dealer's strategy is approximately equal to 0.30, the probability of both the player and the dealer busting in the same round is $(0.30)^2$ or 0.09. This represents a 9% advantage for the house.

There is one other asymmetry in the game. When

the player is dealt a natural he is paid back at the rate of 1.5 to 1, whereas the dealer only receives a payment of 1 to 1 for receiving a natural. As naturals occur approximately 4.8% of the time, the house expectation (the probability of occurance times the payoff) is reduced by one-half this amount to 0.066 (i.e. 0.090 minus 0.024).

The above reasoning yields a figure which is in very close agreement to my derived expectation for the player using this strategy (see Appendix B1) of -0.072 (i.e. a house advantage of 0.072). The 0.006 discrepancy between the two figures may be attributed to the fact that the first figure is derived by assuming that all cards are being dealt from a full deck in order to simplify the calculations (5,p.328 and 6,p.38). This is not expected to yield a true value because conditional probabilities are neglected. The 0.072 house expectation is attributed to simulations involving full and partial decks, so I assume that it is the more accurate figure.

II. Never Bust

The second strategy which I simulated was that of never busting. With this strategy, the player has a "hard standing number" of 12 (i.e. He will stand on

hard totals of 12 or more). Since the "soft standing number" (the total which counts an ace as 11) is not clearly defined, I chose to let it be 17 as a high soft standing number is to the player's advantage. The rest of the strategy is that the player always bets the same amount and he never insures, splits, or doubles down. Using this conservative strategy, the player's simulated expectation is -0.039 (see Appendix B2).

Upon finding the above figure to be in sharp contrast with the -0.215 expectation derived by John Scarne (5,p.329), I followed through his derivation and found an error in his reasoning. He assumed that the dealer would only bust when the player had a total of less than 17, which is not true. After correcting the logical errors in Scarne's argument, his figures yield an expectation for this strategy which is quite close to the one which I have proposed.

III. Thorp's Basic Strategy

This strategy and the two which follow, rely upon the basic strategy presented by Edward Thorp (6,7). It is shown in figure 4. In the first version simulated we do not keep a record of the cards which have fallen. Also, we still maintain a constant bet

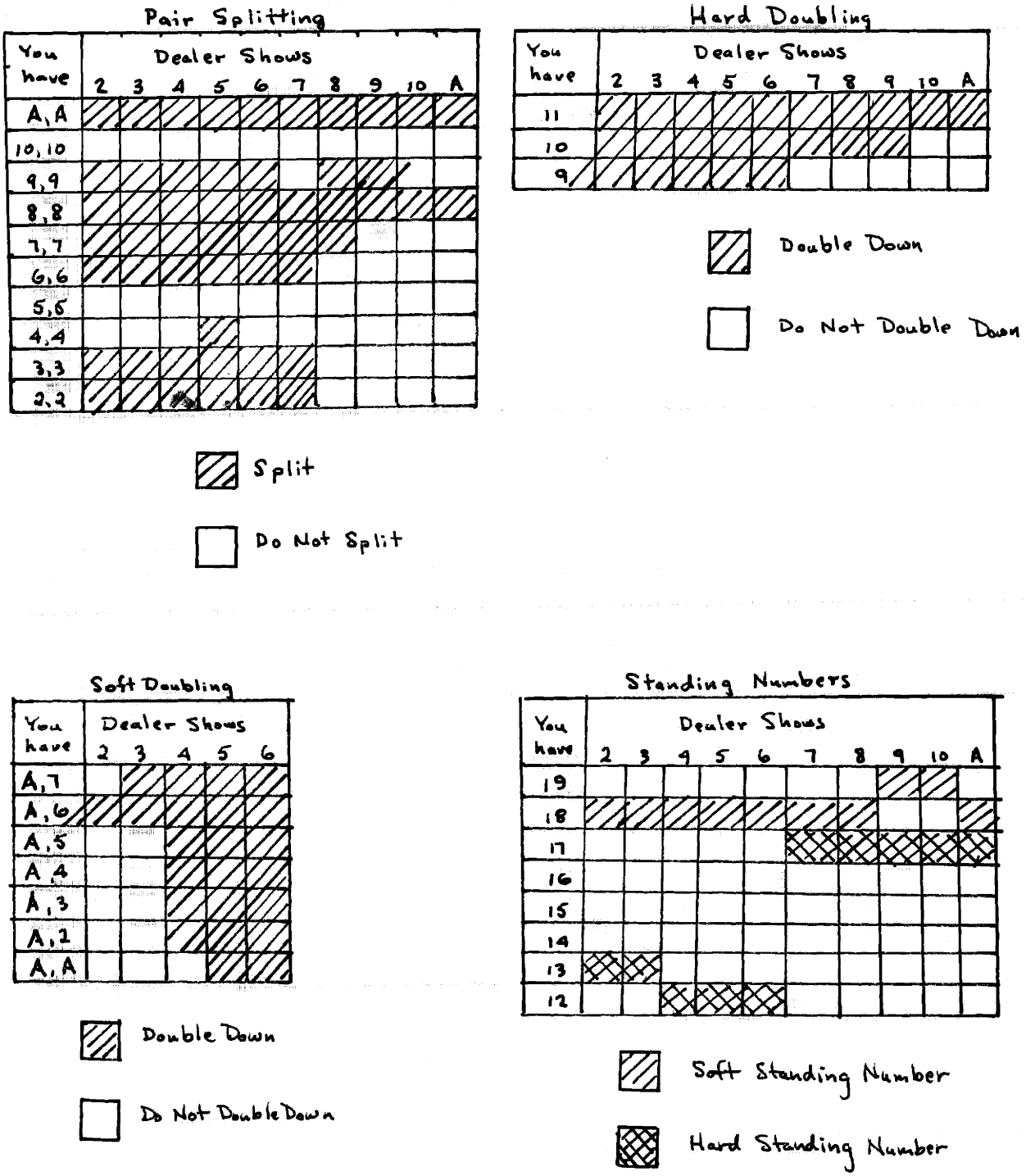


FIGURE 4 -- Thorp's Basic Strategy

size and do not insure. Yet, this strategy produces a positive player expectation of 0.001 (see Appendix B3). This is the approximate expectation which was claimed for the strategy (6, p. 31).

The relevence of a positive expectation without memorizing cards should be pointed out. Essentially this means that casino countermeasures such as adding packs or not dealing to the bottom of the deck cannot adversely affect the strategy. Furthermore, now that it has been shown that a positive expectation can be derived, we can explore those strategies pertaining to betting which may increase that expectation.

IV. Basic Strategy with Tens Ratio

By now we should realize that we can increase our expectation by betting large amounts when we expect to win and betting small amounts when we expect to lose. Obviously, the more tens which remain in the deck the poorer are the chances of the dealer of not busting a hand which is in the 12 to 16 range. Also, tens are an essential component of naturals (which have the only asymmetric expectation favorable to the player). This implies that the ratio of non-tens to tens (or the tens ratio as it will be called) in the undealt portion of the deck may be used to compute an

approximate probability of a player win. When the deck is "fresh" this ratio is 36/16 or 2.25. If more tens than non-tens are dealt this ratio decreases below 2.25. The lower it falls, the more favorable the deck becomes, and consequently, the larger the bet size should be. The bets suggested by Thorp (6, p. 106) are shown in figure 5. Also, this is the first strategy which suggests insuring. The insurance option should be accepted whenever the tens ratio is below 2.00. As it may be seen from the output of this strategy (see Appendix B4), the insurance expectation was negative. This figure is probably inaccurate though, because there were not enough trials simulated to get an accurate idea of the insurance expectation. The bet expectation (all bets except insurance) was seen to rise to 0.004 from its former value (without bet variations) of 0.001. In addition, the average bet size was seen to rise from 2 units to 7.6 units. This shows that the tens ratio does provide a somewhat accurate method of predetermining the probability of winning a given round. Thus, the tens ratio may be used as a partial deck evaluation formula.

V. Basic Strategy with High Low Index

For reasons similar to those which said that

BETTING SCHEME FOR THE TEN RATIO STRATEGY

RATIO	BET
above 2.00	2
2.00 - 1.75	4
1.75 - 1.65	8
below 1.65	10

FIGURE 5

tens in the deck are good for the player, it can be argued that low cards in the deck are bad for the player. The high-low point count index (high-low index) developed by Harvey Dubner (6, p.93) evaluates the probability of a win in the next hand by keeping track of the high cards (aces and tens) and the low cards (twos through sixes). Initially there are an equal number of high and low cards in the deck. Since the high cards in the deck are "good" and the low cards in the deck are "bad", we keep track of the high-low index by adding +1 each time we see a low card fall (The deck has become "better".). Similarly, we add -1 to the high-low index each time we see a high card fall (The deck has become "worse".). This method of counting allows us to remember a single total which gives us an idea of the relative "goodness" of the partial deck (The higher the high-low index, the better the deck is.). It should be clear, however, that a high-low index of +5 is more meaningful toward the end of the deck than it would be near the beginning of the deck. The high-low index is used in the following manner to determine the bet size. If the high-low index is not positive, then the bet is 2 units. Otherwise the bet size is:

$$\text{BET} = \left[\!\left[\frac{100 \times \text{high-low index}}{\text{number of cards remaining in deck}} \right]\!\right]$$

Using this strategy for betting and insuring whenever the high-low index is above 8, a bet expectation of 0.012 was obtained (see Appendix B5). The insurance expectation obtained was negative. However, once again, there were not enough trials to determine the accuracy of the insurance expectation figure. The significant advantages of this strategy are that it has the highest expectation of the strategies mentioned, it is easier to use than the tens ratio, and the average bet size is 6.6 units as opposed to 7.6 units for the tens ratio partial deck evaluation function. For these reasons, the high-low index provides a powerful method of predetermining the approximate probability of winning a given hand.

CONCLUSIONS

A generalized blackjack strategy evaluator has been programmed. It has shown that the game of blackjack can be beaten. It also tells what the expectations are for a given strategy.

In addition to providing support for the expectations previously suggested for imitating the dealer and for Thorp's basic strategy, the evaluator has shown the true expectation for a never bust strategy.

The evaluator has shown the high-low index to be a more accurate partial deck evaluation function than the tens ratio method. The high-low index not only had a higher expectation than the tens ratio method, but it did so with a smaller average bet size. This explains Thorp's comment that the tens ratio method repeatedly showed "...moderately heavy losing streaks, mixed with 'lucky streaks'..."(6, p. 67).

Now that the strategy evaluator exists, it may be modified so that it copies the rules of any given casino (or group of casinos using the same rules, as in Puerto Rico). Then different strategies may be simulated until an acceptable strategy has been found.

APPENDIX A

THE RULES OF BLACKJACK

Number of Players

Blackjack has a dealer and from one to seven players.

The Deck

One ordinary 52-card pack of playing cards is used. However, most casinos now use from two to four packs shuffled together in order to make card counting more difficult.

The Deal

Before play begins, the cards are shuffled by the dealer and cut by a player. A card is then "burned" (placed face up on the bottom of the deck). This card may or may not be shown. The dealer then deals two cards to himself and to each of the players. The dealer receives one card face up and one face deown.

Bets

The players place all bets other than insurance

before any cards are dealt. The house establishes a minimum and a maximum bet size.

Numerical Value of Cards

The player can choose either 1 or 11 to be the value of an ace. The numerical value of picture cards is 10, and the numerical value of all other cards is their face value. A hand is called "soft" if it contains an ace and that ace is counted as 11. All other hands are called "hard".

Object of the Player

The player tries to obtain a total which is greater than that of the dealer but which does not exceed 21. Hands which have exceeded 21 are said to have "busted".

Naturals

If the first two cards dealt either to the player or to the dealer consist of an ace and a 10-value card, they constitute a "natural" or "blackjack". If a player has a natural and the dealer does not, the player receives 1.5 times his original bet from the dealer. If a player does not have a natural and the dealer does, the player loses his original bet. If both

player and dealer have naturals, no money changes hands.

Drawing

The draw starts at the left of the dealer and proceeds in a clockwise fashion. A player looks at his cards and may decide to "stand"; otherwise, he can request additional cards from the dealer, which are dealt face up, one at a time. If a player busts, he immediately turns up his cards and pays his bet to the dealer.

The Dealer's Strategy

After each player has drawn, the dealer turns up his hole card. If his total is 16 or less, he must draw a card and continue to draw cards until his total is 17 or more, at which point he must stand. If the dealer receives an ace, and if counting it as 11 would bring his total to 17 or more without exceeding 21, then he must count it as 11 and stand.

Splitting Pairs

If the player's original cards are numerically identical, they are called a pair. He may choose to treat them as the initial cards in two separate hands. This is known as "splitting a pair". The original bet

goes on one of the split cars and an equal amount is bet on the other card. The player automatically receives a second card on each of the split cards. He then plays his two hands, one at a time, as though they were ordinary hands, with the following exceptions. In the case of split aces, the player receives only one card on each ace. Further, if a 10-value card falls on one of the split aces, the hand is not counted as a natural but only as ordinary 21. Similarly, if a player splits a pair of 10-value cards and then draws an ace, it counts only as an ordinary 21.

Doubling Down

After looking at his fist two cards a player may elect to double his bet and draw one, and only one more card. This strategy is known as "doubling down". A player who splits any pair except aces may, after receiving an additional card on each of the split cards, double down on one or both of his hands.

Insurance

If the dealer's up card is an ace, an additional wager is allowed before the draw. After checking his cards, a player may put up an additional side bet equal at most to half his original bet. After the player

has decided whether or not to do this, the dealer checks his down card. If the dealer has a natural, the side bet wins twice its amount. If the dealer does not have a natural, the side bet is lost and the play continues.

The Settlement

If the player does not go over 21 and the dealer does, the player wins an amount equal to his original bet. If neither player nor dealer busts, the person with the higher total wins an amount equal to the original bet of the player. If the player and the dealer have the same total, not exceeding 21, no money changes hands (This is called a "push".).

APPENDIX B

RESULTS OF COMPUTER SIMULATIONS

This Appendix contains copies of the computer outputs obtained for each of the strategies mentioned in the text. The format of each of the outputs is the same. The results for the last 50 rounds of the simulation are shown, so the reader is able to see how the expectations mentioned were obtained.

The ROUND column refers to the number of times the loop of figure 2 was circled. The FIRST through FOURTH PLAYER and the DEALER columns refer to the final totals which they each received (There will be more than one player only when the first player has split a pair.).

The columns for TOTAL BETS, TOTAL WINNINGS, and BET EXPECTATION do not take insurance into account. This was done because the insurance bet would have introduced inaccuracies with the number of rounds simulated.

The INSURANCE EXPECTATION is found by dividing the total amount won on insurance by the total amount bet on insurance.

GAMES PLAYED tells how many games were simulated.

The number of games simulated must be greater than, or equal to the number of rounds simulated. It will be a larger number only if the strategy simulated says to split pairs.

IMITATE THE DEALER

APPENDIX B1

ROUND *****	FIRST PLAYER ******	SECOND PLAYER ******	THIRD PLAYER ******	FOURTH PLAYER ******	DEALER ******	TOTAL BETS ****	TOTAL WINNINGS ********	INSURANCE EXPECTATION **********	BET EXPECTATION **********	GAMES PLAYED ******
4950	22	0	0	0	18	9900	-689	0.0	-0.069596	4950
4951	17	0	0	0	19	9902	-691	0.0	-0.069704	4951
4952	17	0	0	0	20	9904	-693	0.0	-0.069972	4952
4953	19	0	0	0	19	9906	-693	0.0	-0.069958	4953
4954	20	0	0	0	21	9908	-695	0.0	-0.070145	4954
4955	24	0	0	0	12	9910	-697	0.0	-0.070333	4955
4956	10	0	0	0	21	9912	-699	0.0	-0.070521	4956
4957	21	0	0	0	11	9914	-696	0.0	-0.070284	4957
4958	17	0	0	0	21	9916	-698	0.0	-0.070391	4958
4959	24	0	0	0	20	9918	-700	0.0	-0.070579	4959
4960	23	0	0	0	11	9920	-702	0.0	-0.070766	4960
4961	19	0	0	0	17	9922	-700	0.0	-0.070550	4961
4962	20	0	0	0	24	9924	-698	0.0	-0.070334	4962
4963	19	0	0	0	17	9926	-696	0.0	-0.070119	4963
4964	24	0	0	0	18	9928	-698	0.0	-0.070306	4964
4965	22	0	0	0	7	9930	-700	0.0	-0.070493	4965
4966	18	0	0	0	18	9932	-700	0.0	-0.070479	4966
4967	20	0	0	0	20	9934	-700	0.0	-0.070465	4967
4968	22	0	0	0	9	9936	-702	0.0	-0.070652	4968
4969	24	0	0	0	12	9938	-704	0.0	-0.070839	4969
4970	24	0	0	0	18	9940	-706	0.0	-0.071026	4970
4971	20	0	0	0	19	9942	-704	0.0	-0.070811	4971
4972	20	0	0	0	17	9944	-702	0.0	-0.070595	4972
4973	19	0	0	0	20	9946	-704	0.0	-0.070782	4973
4974	19	0	0	0	17	9948	-702	0.0	-0.070567	4974
4975	22	0	0	0	16	9950	-704	0.0	-0.070754	4975
4976	23	0	0	0	13	9952	-706	0.0	-0.070940	4976
4977	18	0	0	0	24	9954	-704	0.0	-0.070725	4977
4978	20	0	0	0	20	9956	-704	0.0	-0.070711	4978
4979	25	0	0	0	17	9958	-706	0.0	-0.070898	4979
4980	20	0	0	0	21	9960	-708	0.0	-0.071084	4980
4981	19	0	0	0	18	9962	-706	0.0	-0.070869	4981
4982	20	0	0	0	18	9964	-704	0.0	-0.070654	4982
4983	22	0	0	0	15	9966	-706	0.0	-0.070841	4983
4984	20	0	0	0	20	9968	-706	0.0	-0.070827	4984
4985	17	0	0	0	21	9970	-708	0.0	-0.071013	4985
4986	19	0	0	0	24	9972	-706	0.0	-0.070798	4986
4987	24	0	0	0	4	9974	-708	0.0	-0.070985	4987
4988	22	0	0	0	15	9976	-710	0.0	-0.071171	4988
4989	19	0	0	0	20	9978	-712	0.0	-0.071357	4989
4990	22	0	0	0	19	9980	-714	0.0	-0.071543	4990
4991	21	0	0	0	20	9982	-712	0.0	-0.071328	4991
4992	18	0	0	0	19	9984	-714	0.0	-0.071514	4992
4993	21	0	0	0	17	9986	-712	0.0	-0.071300	4993
4994	19	0	0	0	22	9988	-710	0.0	-0.071085	4994
4995	17	0	0	0	18	9990	-712	0.0	-0.071271	4995
4996	26	0	0	0	20	9992	-714	0.0	-0.071457	4996
4997	25	0	0	0	12	9994	-716	0.0	-0.071643	4997
4998	22	0	0	0	7	9996	-718	0.0	-0.071829	4998
4999	20	0	0	0	21	9998	-720	0.0	-0.072014	4999
5000	20	0	0	0	21	10000	-722	0.0	-0.072200	5000

NEVER BUST STRATEGY

APPENDIX B2

ROUND *****	FIRST PLAYER ******	SECOND PLAYER ******	THIRD PLAYER ******	FOURTH PLAYER ******	DEALER ******	TOTAL BETS ****	TOTAL WINNINGS ********	INSURANCE EXPECTATION **********	BET EXPECTATION **********	GAMES PLAYED ******
4950	17	0	0	0	21	9900	-393	0.0	-0.039697	4950
4951	20	0	0	0	21	9902	-395	0.0	-0.039891	4951
4952	17	0	0	0	17	9904	-395	0.0	-0.039883	4952
4953	14	0	0	0	17	9906	-397	0.0	-0.040077	4953
4954	19	0	0	0	18	9908	-395	0.0	-0.039867	4954
4955	12	0	0	0	17	9910	-397	0.0	-0.040061	4955
4956	16	0	0	0	17	9912	-399	0.0	-0.040254	4956
4957	15	0	0	0	22	9914	-397	0.0	-0.040044	4957
4958	14	0	0	0	25	9916	-395	0.0	-0.039835	4958
4959	17	0	0	0	24	9918	-393	0.0	-0.039625	4959
4960	13	0	0	0	20	9920	-395	0.0	-0.039819	4960
4961	21	0	0	0	21	9922	-395	0.0	-0.039811	4961
4962	20	0	0	0	21	9924	-397	0.0	-0.040004	4962
4963	17	0	0	0	17	9926	-397	0.0	-0.039996	4963
4964	14	0	0	0	26	9928	-395	0.0	-0.039786	4964
4965	12	0	0	0	25	9930	-393	0.0	-0.039577	4965
4966	20	0	0	0	26	9932	-391	0.0	-0.039368	4966
4967	16	0	0	0	20	9934	-393	0.0	-0.039561	4967
4968	17	0	0	0	18	9936	-395	0.0	-0.039754	4968
4969	12	0	0	0	18	9938	-397	0.0	-0.039948	4969
4970	14	0	0	0	20	9940	-399	0.0	-0.040141	4970
4971	12	0	0	0	20	9942	-401	0.0	-0.040334	4971
4972	19	0	0	0	19	9944	-401	0.0	-0.040326	4972
4973	20	0	0	0	17	9946	-399	0.0	-0.040117	4973
4974	21	0	0	0	17	9948	-396	0.0	-0.039807	4974
4975	14	0	0	0	22	9950	-394	0.0	-0.039598	4975
4976	18	0	0	0	23	9952	-392	0.0	-0.039389	4976
4977	13	0	0	0	22	9954	-390	0.0	-0.039180	4977
4978	14	0	0	0	18	9956	-392	0.0	-0.039373	4978
4979	14	0	0	0	17	9958	-394	0.0	-0.039566	4979
4980	19	0	0	0	22	9960	-392	0.0	-0.039357	4980
4981	14	0	0	0	18	9962	-394	0.0	-0.039550	4981
4982	20	0	0	0	17	9964	-392	0.0	-0.039342	4982
4983	21	0	0	0	25	9966	-390	0.0	-0.039133	4983
4984	17	0	0	0	21	9968	-392	0.0	-0.039326	4984
4985	18	0	0	0	17	9970	-390	0.0	-0.039117	4985
4986	15	0	0	0	25	9972	-388	0.0	-0.038909	4986
4987	13	0	0	0	26	9974	-386	0.0	-0.038701	4987
4988	21	0	0	0	15	9976	-383	0.0	-0.038392	4988
4989	15	0	0	0	22	9978	-381	0.0	-0.038184	4989
4990	16	0	0	0	17	9980	-383	0.0	-0.038377	4990
4991	18	0	0	0	20	9982	-385	0.0	-0.038569	4991
4992	15	0	0	0	20	9984	-387	0.0	-0.038762	4992
4993	19	0	0	0	17	9986	-385	0.0	-0.038554	4993
4994	14	0	0	0	19	9988	-387	0.0	-0.038746	4994
4995	16	0	0	0	18	9990	-389	0.0	-0.038939	4995
4996	21	0	0	0	15	9992	-386	0.0	-0.038631	4996
4997	12	0	0	0	22	9994	-384	0.0	-0.038423	4997
4998	14	0	0	0	20	9996	-386	0.0	-0.038615	4998
4999	13	0	0	0	18	9998	-388	0.0	-0.038808	4999
5000	12	0	0	0	23	10000	-386	0.0	-0.038600	5000

BASIC STRATEGY

APPENDIX B3

ROUND *****	FIRST PLAYER ******	SECOND PLAYER ******	THIRD PLAYER ******	FOURTH PLAYER ******	DEALER ******	TOTAL BETS ****	TOTAL WINNINGS ********	INSURANCE EXPECTATION **********	BET EXPECTATION **********	GAMES PLAYED ******
49950	20	0	0	0	17	113516	99	0.0	0.000872	51111
49951	20	0	0	0	23	113518	101	0.0	0.000890	51112
49952	19	0	0	0	23	113520	103	0.0	0.000907	51113
49953	18	0	0	0	19	113522	101	0.0	0.000890	51114
49954	20	0	0	0	20	113524	101	0.0	0.000890	51115
49955	16	0	0	0	19	113526	99	0.0	0.000872	51116
49956	19	0	0	0	26	113528	101	0.0	0.000890	51117
49957	20	0	0	0	21	113530	99	0.0	0.000872	51118
49958	20	0	0	0	20	113532	99	0.0	0.000872	51119
49959	17	0	0	0	18	113534	97	0.0	0.000854	51120
49960	18	0	0	0	23	113536	99	0.0	0.000872	51121
49961	19	0	0	0	20	113538	97	0.0	0.000854	51122
49962	17	0	0	0	17	113540	97	0.0	0.000854	51123
49963	19	0	0	0	20	113542	95	0.0	0.000837	51124
49964	19	0	0	0	24	113544	97	0.0	0.000854	51125
49965	18	0	0	0	17	113546	99	0.0	0.000872	51126
49966	19	0	0	0	19	113548	99	0.0	0.000872	51127
49967	20	0	0	0	20	113550	99	0.0	0.000872	51128
49968	21	0	0	0	21	113552	99	0.0	0.000872	51129
49969	18	0	0	0	20	113554	97	0.0	0.000854	51130
49970	20	0	0	0	18	113556	99	0.0	0.000872	51131
49971	21	0	0	0	7	113558	102	0.0	0.000898	51132
49972	25	0	0	0	18	113560	100	0.0	0.000881	51133
49973	21	0	0	0	23	113562	102	0.0	0.000898	51134
49974	21	0	0	0	17	113564	104	0.0	0.000916	51135
49975	17	0	0	0	24	113566	106	0.0	0.000933	51136
49976	19	0	0	0	24	113568	108	0.0	0.000951	51137
49977	20	0	0	0	21	113570	106	0.0	0.000933	51138
49978	17	0	0	0	20	113574	102	0.0	0.000898	51139
49979	20	0	0	0	23	113576	104	0.0	0.000916	51140
49980	17	20	0	0	19	113582	102	0.0	0.000898	51142
49981	20	0	0	0	22	113584	104	0.0	0.000916	51143
49982	21	0	0	0	19	113588	108	0.0	0.000951	51144
49983	13	0	0	0	21	113590	106	0.0	0.000933	51145
49984	14	0	0	0	24	113594	110	0.0	0.000968	51146
49985	25	0	0	0	20	113596	108	0.0	0.000951	51147
49986	17	0	0	0	18	113598	106	0.0	0.000933	51148
49987	22	0	0	0	13	113600	104	0.0	0.000915	51149
49988	20	0	0	0	17	113602	106	0.0	0.000933	51150
49989	17	0	0	0	17	113604	106	0.0	0.000933	51151
49990	24	0	0	0	12	113606	104	0.0	0.000915	51152
49991	18	0	0	0	22	113608	106	0.0	0.000933	51153
49992	15	0	0	0	23	113610	108	0.0	0.000951	51154
49993	13	0	0	0	19	113612	106	0.0	0.000933	51155
49994	21	0	0	0	19	113614	108	0.0	0.000951	51156
49995	18	0	0	0	17	113616	110	0.0	0.000968	51157
49996	26	0	0	0	11	113618	108	0.0	0.000951	51158
49997	21	0	0	0	18	113620	111	0.0	0.000977	51159
49998	18	0	0	0	21	113622	109	0.0	0.000959	51160
49999	19	0	0	0	23	113624	111	0.0	0.000977	51161
50000	20	0	0	0	20	113626	111	0.0	0.000977	51162

BASIC STRATEGY USING TENS-RATIO FOR BETS

APPENDIX B4

ROUND *****	FIRST PLAYER ******	SECOND PLAYER ******	THIRD PLAYER ******	FOURTH PLAYER ******	DEALER ******	TOTAL BETS ****	TOTAL WINNINGS ********	INSURANCE EXPECTATION **********	BET EXPECTATION **********	GAMES PLAYED ******
49950	20	0	0	0	17	387078	1408	-0.171867	0.003638	51111
49951	20	0	0	0	23	387088	1418	-0.171867	0.003663	51112
49952	19	0	0	0	23	387090	1420	-0.171867	0.003668	51113
49953	18	0	0	0	19	387092	1418	-0.171867	0.003663	51114
49954	20	0	0	0	20	387094	1418	-0.171867	0.003663	51115
49955	16	0	0	0	19	387096	1416	-0.171867	0.003658	51116
49956	19	0	0	0	26	387098	1418	-0.171867	0.003663	51117
49957	20	0	0	0	21	387108	1408	-0.172214	0.003637	51118
49958	20	0	0	0	20	387118	1408	-0.172214	0.003637	51119
49959	17	0	0	0	18	387128	1398	-0.172561	0.003611	51120
49960	18	0	0	0	23	387138	1408	-0.172561	0.003637	51121
49961	19	0	0	0	20	387148	1398	-0.172561	0.003611	51122
49962	17	0	0	0	17	387158	1398	-0.172561	0.003611	51123
49963	19	0	0	0	20	387168	1388	-0.172561	0.003585	51124
49964	19	0	0	0	24	387178	1398	-0.172561	0.003611	51125
49965	18	0	0	0	17	387188	1408	-0.172561	0.003636	51126
49966	19	0	0	0	19	387198	1408	-0.172561	0.003636	51127
49967	20	0	0	0	20	387208	1408	-0.172561	0.003636	51128
49968	21	0	0	0	21	387218	1408	-0.172561	0.003636	51129
49969	18	0	0	0	20	387220	1406	-0.172561	0.003631	51130
49970	20	0	0	0	18	387230	1416	-0.172561	0.003657	51131
49971	21	0	0	0	7	387232	1419	-0.172561	0.003664	51132
49972	25	0	0	0	18	387242	1409	-0.172561	0.003639	51133
49973	21	0	0	0	23	387244	1411	-0.172561	0.003644	51134
49974	21	0	0	0	17	387246	1413	-0.172630	0.003649	51135
49975	17	0	0	0	24	387256	1423	-0.172630	0.003675	51136
49976	19	0	0	0	24	387266	1433	-0.172630	0.003700	51137
49977	20	0	0	0	21	387270	1429	-0.172630	0.003690	51138
49978	17	0	0	0	20	387290	1409	-0.172630	0.003638	51139
49979	20	0	0	0	23	387300	1419	-0.172630	0.003664	51140
49980	17	20	0	0	19	387306	1417	-0.172630	0.003659	51142
49981	20	0	0	0	22	387308	1419	-0.172700	0.003664	51143
49982	21	0	0	0	19	387328	1439	-0.172700	0.003715	51144
49983	13	0	0	0	21	387338	1429	-0.171790	0.003689	51145
49984	14	0	0	0	24	387358	1449	-0.171790	0.003741	51146
49985	25	0	0	0	20	387368	1439	-0.171790	0.003715	51147
49986	17	0	0	0	18	387378	1429	-0.171790	0.003689	51148
49987	22	0	0	0	13	387388	1419	-0.171790	0.003663	51149
49988	20	0	0	0	17	387390	1421	-0.171790	0.003668	51150
49989	17	0	0	0	17	387400	1421	-0.171790	0.003668	51151
49990	24	0	0	0	12	387410	1411	-0.171790	0.003642	51152
49991	18	0	0	0	22	387412	1413	-0.171790	0.003647	51153
49992	15	0	0	0	23	387414	1415	-0.171790	0.003652	51154
49993	13	0	0	0	19	387416	1413	-0.171790	0.003647	51155
49994	21	0	0	0	19	387418	1415	-0.171859	0.003652	51156
49995	18	0	0	0	17	387428	1425	-0.171859	0.003678	51157
49996	26	0	0	0	11	387438	1415	-0.171859	0.003652	51158
49997	21	0	0	0	18	387440	1418	-0.171859	0.003660	51159
49998	18	0	0	0	21	387442	1416	-0.171859	0.003655	51160
49999	19	0	0	0	23	387444	1418	-0.171859	0.003660	51161
50000	20	0	0	0	20	387446	1418	-0.171859	0.003660	51162

BASIC STRATEGY USING HIGH-LOW INDEX FOR BETS

APPENDIX B5

ROUND *****	FIRST PLAYER ******	SECOND PLAYER ******	THIRD PLAYER ******	FOURTH PLAYER ******	DEALER ******	TOTAL BETS ****	TOTAL WINNINGS ********	INSURANCE EXPECTATION **********	BET EXPECTATION **********	GAMES PLAYED ******
49950	20	0	0	0	17	338490	3966	-0.021927	0.011717	51111
49951	20	0	0	0	23	338492	3968	-0.021927	0.011723	51112
49952	19	0	0	0	23	338494	3970	-0.021927	0.011728	51113
49953	18	0	0	0	19	338496	3968	-0.021927	0.011722	51114
49954	20	0	0	0	20	338498	3968	-0.021927	0.011722	51115
49955	16	0	0	0	19	338500	3966	-0.021927	0.011716	51116
49956	19	0	0	0	26	338510	3976	-0.021927	0.011746	51117
49957	20	0	0	0	21	338520	3966	-0.022412	0.011716	51118
49958	20	0	0	0	20	338530	3966	-0.022412	0.011715	51119
49959	17	0	0	0	18	338534	3962	-0.022412	0.011703	51120
49960	18	0	0	0	23	338536	3964	-0.022412	0.011709	51121
49961	19	0	0	0	20	338542	3958	-0.022412	0.011691	51122
49962	17	0	0	0	17	338552	3958	-0.022412	0.011691	51123
49963	19	0	0	0	20	338562	3948	-0.022412	0.011661	51124
49964	19	0	0	0	24	338572	3958	-0.022412	0.011690	51125
49965	18	0	0	0	17	338582	3968	-0.022412	0.011719	51126
49966	19	0	0	0	19	338592	3968	-0.022412	0.011719	51127
49967	20	0	0	0	20	338602	3968	-0.022412	0.011719	51128
49968	21	0	0	0	21	338612	3968	-0.022412	0.011718	51129
49969	18	0	0	0	20	338614	3966	-0.022412	0.011712	51130
49970	20	0	0	0	18	338616	3968	-0.022412	0.011718	51131
49971	21	0	0	0	7	338618	3971	-0.022412	0.011727	51132
49972	25	0	0	0	18	338620	3969	-0.022412	0.011721	51133
49973	21	0	0	0	23	338622	3971	-0.022412	0.011727	51134
49974	21	0	0	0	17	338624	3973	-0.022412	0.011733	51135
49975	17	0	0	0	24	338628	3977	-0.022412	0.011744	51136
49976	19	0	0	0	24	338636	3985	-0.022412	0.011768	51137
49977	20	0	0	0	21	338646	3975	-0.022412	0.011738	51138
49978	17	0	0	0	20	338650	3971	-0.022412	0.011726	51139
49979	20	0	0	0	23	338656	3977	-0.022412	0.011743	51140
49980	17	20	0	0	19	338662	3975	-0.022412	0.011737	51142
49981	20	0	0	0	22	338664	3977	-0.022412	0.011743	51143
49982	21	0	0	0	19	338672	3985	-0.022412	0.011767	51144
49983	13	0	0	0	21	338676	3981	-0.022412	0.011755	51145
49984	14	0	0	0	24	338680	3985	-0.022412	0.011766	51146
49985	25	0	0	0	20	338690	3975	-0.022412	0.011736	51147
49986	17	0	0	0	18	338700	3965	-0.022412	0.011707	51148
49987	22	0	0	0	13	338702	3963	-0.022412	0.011701	51149
49988	20	0	0	0	17	338704	3965	-0.022412	0.011706	51150
49989	17	0	0	0	17	338706	3965	-0.022412	0.011706	51151
49990	24	0	0	0	12	338708	3963	-0.022412	0.011700	51152
49991	18	0	0	0	22	338710	3965	-0.022412	0.011706	51153
49992	15	0	0	0	23	338712	3967	-0.022412	0.011712	51154
49993	13	0	0	0	19	338714	3965	-0.022412	0.011706	51155
49994	21	0	0	0	19	338720	3971	-0.022412	0.011724	51156
49995	18	0	0	0	17	338730	3981	-0.022412	0.011753	51157
49996	26	0	0	0	11	338740	3971	-0.022412	0.011723	51158
49997	21	0	0	0	18	338742	3974	-0.022412	0.011732	51159
49998	18	0	0	0	21	338744	3972	-0.022412	0.011726	51160
49999	19	0	0	0	23	338746	3974	-0.022412	0.011732	51161
50000	20	0	0	0	20	338748	3974	-0.022412	0.011731	51162

APPENDIX C

STRATEGY EVALUATOR PROGRAM

This Appendix contains a copy of the strategy evaluator program and the subroutine used for dealing.

The variables, arrays, and subroutines used and their purposes are presented in Table C1.

The operation of both routines is explained in the text. The comments on the listings should help the reader to distinguish portions of the program which correspond to different situations which occur in the game.

TABLE C1

LIST OF VARIABLES AND ARRAYS

BETEX - bet expectation

EXINS - insurance expectation

IACE - flag set to 1 if an ace is dealt

IBET - total winnings

IBTOT - total amount bet

ICARD(52) - present deck

IDECK(52) - "next" deck

IDUBL - flag set to 1 if player wants to double down

INS(4) - amount of insurance bet on hand

INSTOT - total amount of all insurance bets placed

INSWON - total amount of insurance winnings

IPLAY(4,11) - cards received by player(s)

ITOT(4) - each player's total

ISPLIT - flag set to 1 if player wants to split a pair

ISTRAT - flag set to 1 if player wants to draw

IVAL - numerical value of card dealt

IWON - sum of bet and insurance winnings

IX - input to random number generator

JBET(4) - amount bet by each player on hand

K- number of cards left in present deck

KDLR(11) - dealer's cards

KTR - face value of card dealt

LTOT - dealer's total

MACE(4) - number of aces in each player's hand

NGAME - total number of games played

NP - number of player actually playing

NPLAY - total number of players in that round

LIST OF SUBROUTINES

BETTR - sets the bet value by putting it into JBET(NP)

DBLDN - tell whether or not player wants to double down by setting the IDUBL flag

DEALA - deals cards by setting the IACE, IVAL, and KTR variables

INSUR - sets the value of the insurance bet in INS(NP)

SPLIT - tells whether player wants to split a pair by setting the ISPLIT flag

STRAT - tells whether player wants to draw by setting the ISTRAT flag

```
C*************************************************************************
C
C
C     THIS IS THE MAIN BODY OF THE PROGRAM.  IT ALLOWS THE GAME OF BLACKJACK TO
C        BE PLAYED ACCORDING TO THE RULES SEEN UNDER NORMAL CASINO SITUATIONS.
C
C
C*************************************************************************
C
C
C     INITIALIZE VARIABLES
C
C
C*************************************************************************
0001        COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KDLR(11),ITOT(4),JBET(4
           2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
           3NP,K,IACE
0002       1600 FORMAT(   10X,'FIRST',4X,'SECOND',3X,'THIRD',4X,'FOURTH',13X,'TOTA
           2L',4X,'TOTAL',6X,'INSURANCE',8X,'BET',7X,'GAMES')
0003       1601 FORMAT(2X,'ROUND',3X,4('PLAYER',3X),'DEALER',4X,'BETS',4X,'WINNING
           2S',3X,2('EXPECTATION',3X),'PLAYED')
0004       1602 FORMAT(1X,I6,5X,4(I2,7X),I2,5X,2(I6,4X),2(F11.6,3X),I6)
0005       1603 FORMAT(2X,5('*'),3X,5(6('*'),3X),1X,4('*'),4X,8('*'),2(3X,11('*'))
           2,3X,6('*'),//)
0006       1604 FORMAT(1H1,'      BASIC STRATEGY',//)
0007        JKLM=50000
0008        WRITE(6,1604)JKLM
0009        WRITE(6,1600)
0010        WRITE(6,1601)
0011        WRITE(6,1603)
0012        IX=913
0013        IWON=0
0014        IBTOT=0
0015        IBET=0
0016        INSTOT=0
0017        INSWON=0
0018        NGAME=0
0019        DO 10 IC=1,13
0020        IDECK(IC)=IC
0021    10  CONTINUE
0022        DO 20 IC=1,39
0023        IDECK(IC+13)=IDECK(IC)
0024    20  CONTINUE
0025        K=0
C*************************************************************************
C
C
C     THE MAIN LOOP STARTS HERE
```

```
                C
                C
                C*************************************************************************
0026                  DO 100 IXYZ=1,JKLM
0027                  DO 105 I=1,4
0028                  ITOT(I)=0
0029                  IBUST(I)=0
0030                  MACE(I)=0
0031                  IBJ(I)=0
0032                  INS(I)=0
0033                  JBET(I)=0
0034                  DO 105 J=1,11
0035                  IPLAY(I,J)=0
0036           105    CONTINUE
0037                  NPLAY=1
0038                  NP=NPLAY
0039                  LTOT=0
0040                  LACE=0
0041                  LBJ=0
                C*************************************************************************
                C
                C
                C     CHOOSE PLAYER'S BET
                C
                C
                C*************************************************************************
0042                  CALL BETTR
0043                  IBTOT=IBTOT+JBET(NP)
                C*************************************************************************
                C
                C
                C     DEAL PLAYER TWO CARDS
                C
                C
                C*************************************************************************
0044                  DO 110 I=1,2
0045                  CALL DEALA
0046                  ITOT(NP)=ITOT(NP)+IVAL
0047                  IPLAY(NP,I)=KTR
0048                  IF(IVAL-11)110,120,110
0049           120    MACE(NP)=MACE(NP)+1
0050           110    CONTINUE
0051                  IF(ITOT(NP)-21)130,140,150
0052           140    IBJ(NP)=999
0053                  GO TO 130
0054           150    ITOT(NP)=ITOT(NP)-10
0055                  MACE(NP)=MACE(NP)-1
                C*************************************************************************
```

```
                C
                C
                C      DEAL DEALER TWO CARDS
                C
                C
                C*********************************************************************
0056             130   DO 170 J=1,2
0057                   CALL DEALA
0058                   LTOT=LTOT+IVAL
0059                   KDLR(J)=KTR
0060                   IF(IVAL-11)170,180,170
0061             180   LACE=LACE+1
0062             170   CONTINUE
                C*********************************************************************
                C
                C
                C      CHECK FOR INSURANCE AND NATURALS
                C
                C
                C*********************************************************************
0063                   IF (LTOT-21)190,200,210
0064             200   LBJ=999
0065                   GO TO 190
0066             210   LTOT=LTOT-10
0067                   LACE=LACE-1
0068             190   IF (KDLR(1)-1)230,220,230
0069             220   CALL INSUR
0070                   INSTOT=INSTOT+INS(NP)
0071             230   IF(LBJ)240,240,250
0072             240   INSWON=INSWON-INS(NP)
0073                   IF(IBJ(NP))260,260,270
0074             270   IBET=IBET+(3*JBET(NP))/2
0075                   GO TO 680
0076             250   INSWON=INSWON+2*INS(NP)
0077                   IF(LBJ-IBJ(NP))680,680,280
0078             280   IBET=IBET-JBET(NP)
0079                   GO TO 680
                C*********************************************************************
                C
                C
                C      CHECK FOR SPLITTING PAIRS
                C
                C
                C*********************************************************************
0080             260   NP=1
0081             290   IF(IPLAY(NP,1)-IPLAY(NP,2))300,310,300
0082             310   CALL SPLIT
                C*********************************************************************
```

```
C
C
C     SPLIT IF ISPLIT IS SET TO 1
C
C*************************************************************************
0083        IF(ISPLIT-1)300,320,300
0084   320  NPLAY=NPLAY+1
0085        NPLAYP=NPLAY+1
0086        JPL=NPLAY-NP
0087        DO 330 KAS=1,JPL
0088        I=NPLAYP-KAS
0089        IPLAY(I,1)=IPLAY(I-1,1)
0090        IPLAY(I,2)=IPLAY(I-1,2)
0091        JBET(I)=JBET(I-1)
0092        IBTOT=IBTOT + JBET(I)
0093   330  CONTINUE
0094        CALL DEALA
0095        IPLAY(I-1,2)=KTR
0096        IF (IPLAY(I-1,1)-1)331,332,331
0097   331  IF (IPLAY(I-1,1)-10) 334,335,335
0098   334  ITOT(I-1)=IPLAY(I-1,1)
0099        GO TO 333
0100   335  ITOT(I-1)=10
0101        GO TO 333
0102   332  ITOT(I-1)=11
0103        MACE(I-1)=1
0104   333  ITOT(I-1)=IVAL*ITOT(I-1)
0105        IF(IVAL-11)340,350,340
0106   350  MACE(I-1)=MACE(I-1)+1
0107   340  CALL DEALA
0108        IPLAY(I,2)=KTR
0109        IF (IPLAY(I,1)-1)341,342,341
0110   341  IF (IPLAY(I,1)-10) 344,345,345
0111   344  ITOT(I)=IPLAY(I,1)
0112        GO TO 343
0113   345  ITOT(I)=10
0114        GO TO 343
0115   342  ITOT(I)=11
0116        MACE(I)=1
0117   343  ITOT(I)=IVAL*ITOT(I)
0118        IF (IVAL-11)290,370,290
0119   370  MACE(NP)=MACE(NP)+1
0120        GO TO 290
0121   300  IF (NP-NPLAY)380,390,390
0122   380  NP=NP+1
0123        GO TO 290
C*************************************************************************
C
```

```
                C
                C       SEE IF A PLAYER WANTS TO DOUBLE DOWN
                C
                C
                C*************************************************************************
0124             390    DO 531 NP=1,NPLAY
0125                    CALL DBLDN
0126                    IF (IDUBL-1)400,410,400
0127             410    IBTOT=IPTOT + JBET(NP)
0128                    JBET(NP)=2*JBET(NP)
0129                    CALL DEALA
0130                    IPLAY(NP,3)=KTR
0131                    ITOT(NP)=ITOT(NP)+IVAL
0132                    IF (IACE) 441,441,440
0133             440    MACE(NP)=MACE(NP)+1
0134             441    IF (ITOT(NP)-21) 531,531,442
0135             442    IF (MACE(NP)) 490,490,443
0136             443    ITOT(NP)=ITOT(NP)-10
0137                    MACE(NP)=MACE(NP)-1
0138                    GO TO 531
                C*************************************************************************
                C
                C
                C       CHECK IF PLAYER WOULD LIKE TO DRAW
                C
                C
                C*************************************************************************
0139             400    KNP=2
0140             460    CALL STRAT
0141                    IF (ISTRAT-1)531,450,531
0142             450    CALL DEALA
0143                    KNP=KNP+1
0144                    ITOT(NP)=ITOT(NP)+IVAL
0145                    IPLAY(NP,KNP)=KTR
0146                    IF(IACE)470,470,480
0147             480    MACE(NP)=MACE(NP)+1
0148             470    IF (ITOT(NP)-21) 460,460,500
0149             500    IF (MACE(NP))490,490,510
0150             510    MACE(NP)=MACE(NP)-1
0151                    ITOT(NP)=ITOT(NP)-10
0152                    GO TO 460
0153             490    IBUST(NP)=1
0154             531    CONTINUE
                C*************************************************************************
                C
                C
                C       SEE IF DEALER MUST DRAW
                C
```

```
                C
                C*************************************************************************
0155                    KBUST=0
0156                    DO 591 NP=1,NPLAY
0157                    KBUST=KBUST+IBUST(NP)
0158             591    CONTINUE
0159                    IF (KBUST-NPLAY)592,600,600
0160             592    KD=2
0161             590    IF (LTOT-16)580,580,570
0162             580    CALL DEALA
0163                    IF (IACE)593,593,594
0164             594    LACE=LACE+1
0165             593    LTOT=LTOT+IVAL
0166                    KD=KD+1
0167                    KOLR(KD)=KTR
0168                    GO TO 590
0169             570    IF (LTOT-21)620,620,610
0170             610    IF (LACE)630,630,640
0171             640    LACE=LACE-1
0172                    LTOT=LTOT-10
0173                    GO TO 590
                C*************************************************************************
                C
                C
                C       ALL PLAYER(S) HAVE BUSTED
                C
                C
                C*************************************************************************
0174             600    DO 601 NP=1,NPLAY
0175                    IBET=IBET-JBET(NP)
0176             601    CONTINUE
0177                    GO TO 680
                C*************************************************************************
                C
                C
                C       DEALER HAS BUSTED
                C
                C
                C*************************************************************************
0178             630    DO 631 NP=1,NPLAY
0179                    IF (IBUST(NP)) 632,632,633
0180             632    IBET=IBET+JBET(NP)
0181                    GO TO 631
0182             633    IBET=IBET-JBET(NP)
0183             631    CONTINUE
0184                    GO TO 680
                C*************************************************************************
                C
```

```
                C
                C      NEITHER HAS BUSTED
                C
                C
                C*************************************************************************
0185             620   DO 680 NP=1,NPLAY
0186                   IF (IBUST(NP))625,625,624
0187             625   IF (ITOT(NP)-LTOT) 624,680,622
0188             622   IBET=IBET+JBET(NP)
0189                   GO TO 680
0190             624   IBET=IBET-JBET(NP)
0191             680   CONTINUE
0192                   NGAME=NGAME+NPLAY
0193                   BET=IBET
0194                   BTOT=IBTOT
0195                   BETEX=BET/BTOT
0196                   BINS=INSTOT
0197                   BNS=INSWON
0198                   IF (INSTOT) 681,681,682
0199             682   EXINS = BNS / BINS
0200                   GO TO 683
0201             681   EXINS=0.
0202                   IF (200-JKLM+IXYZ) 100,683,683
0203             683   WRITE(6,1602)IXYZ,(ITOT(NP),NP=1,4),LTOT,IBTOT,IBET,EXINS,BETEX,NG
                      2AME
0204             100   CONTINUE
0205                   END
```

```
C*********************************************************************
C
C
C     THIS SUBROUTINE WILL DEAL A CARD WHENEVER ONE IS REQUESTED.  IT WILL
C        CONTINUALLY REMOVE ONE CARD AT A TIME FROM A SHUFFLED DECK UNTIL THAT
C        DECK HAS BECOME DEPLETED.  AT THAT TIME IT WILL START DEALING FROM A
C        FRESHLY SHUFFLED DECK.
C
C
C*********************************************************************
0001          SUBROUTINE DEALA
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KCLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          IACE=0
0004          IF(K)910,910,920
0005      910 DO 900 IK=1,52
0006          ICARD(IK)=IDECK(IK)
0007      900 CONTINUE
0008          K=52
0009          NUM=51
0010      920 CALL RANDU(IX,IX,YFL)
0011          ANUM=NUM
0012          N=ANUM*YFL
0013          N=N+1
0014          KTR=ICARD(N)
0015          IDECK(K)=ICARD(N)
0016          ICARD(N)=ICARD(K)
0017          GO TO(930,940,940,940,940,940,940,940,940,940,950,960,970),KTR
0018      930 TR=1
0019          IVAL=11
0020          IACE=1
0021          GO TO 990
0022      950 TR=2
0023          GO TO 980
0024      960 TR=3
0025          GO TO 980
0026      970 TR=4
0027      980 IVAL=10
0028          INDEX=2
0029          GO TO 990
0030      940 IVAL=KTR
0031          INDEX=1
0032      990 NUM=NUM-1
0033          K=K-1
0034          RETURN
0035          END
```

APPENDIX D

STRATEGY SUBROUTINES

This Appendix contains the listings of all the strategy subroutines used in the simulations broken up as follows:

D1. All of the strategies for imitating the dealer and never busting except for the drawing strategy.

D2. Drawing strategy for imitating the dealer.

D3. Drawing strategy for never busting.

D4. All the strategies for Thorp's basic strategy, tens ratio, and high-low except for betting and insuring.

D5. Betting and insuring for Thorp's basic strategy.

D6. Betting and insuring for tens ratio.

D7. Betting and insuring for high-low.

APPENDIX D1

```
C*************************************************************************
C
C
C      STRATEGY SUBROUTINES
C
C
C
C*************************************************************************
C
C
C      BET WILL ALWAYS BE 2 UNITS
C
C
C*************************************************************************
0001          SUBROUTINE BETTR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          JBET(NP)=2
0004          RETURN
0005          END

C*************************************************************************
C
C
C      NEVER INSURE
C
C
C*************************************************************************
0001          SUBROUTINE INSUR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          INS(NP)=0
0004          RETURN
0005          END

C*************************************************************************
C
C
C      NEVER DOUBLE DOWN
C
C
C*************************************************************************
0001          SUBROUTINE DBLDN
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          IDUBL=0
0004          RETURN
0005          END

C*************************************************************************
C
C
C      NEVER SPLIT
C
C
C*************************************************************************
0001          SUBROUTINE SPLIT
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          ISPLIT=0
0004          RETURN
0005          END
```

APPENDIX D2

```
C*****************************************************************************
C
C
C      IMITATE THE DEALER
C
C
C*****************************************************************************
0001        SUBROUTINE STRAT
0002        COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
           2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
           3NP,K,IACE
0003        IF (ITOT(NP)-16) 1020,1020,1030
0004   1020 ISTRAT=1
0005        RETURN
0006   1030 ISTRAT=0
0007        RETURN
0008        END
```

APPENDIX D3

```
C*************************************************************************
C
C
C      NEVER BUST
C
C
C*************************************************************************
0001          SUBROUTINE STRAT
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KDLR(11),ITOT(4),JRET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          IF (MACE(NP)) 1020,1020,1030
C      STOP DRAWING AT HARD 12
0004       1020 IF (ITOT(NP)-12) 1040,1050,1050
C      STOP DRAWING AT SOFT 17
0005       1030 IF (ITOT(NP)-18) 1040,1050,1050
0006       1040 ISTRAT=1
0007          RETURN
0008       1050 ISTRAT=0
0009          RETURN
0010          END
```

APPENDIX D4

```
C*************************************************************************
C
C
C     BASIC DOUBLING DOWN STRATEGY
C
C
C*************************************************************************
0001        SUBROUTINE DBLDN
0002        COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
           2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
           3NP,K,IACE
0003        IF (MACE(NP)) 2000,2000,2011
C     HARD DOUBLING
0004    2000 IF (ITOT(NP)-11) 2010,2050,2060
0005    2010 IF (ITOT(NP)-10) 2030,2020,2060
0006    2020 IF (KOLR(1) -10) 2051,2060,2060
0007    2030 IF (ITOT(NP)- 9) 2060,2040,2060
0008    2040 IF (KOLR(1)-7)   2051,2060,2060
C     SOFT DOUBLING
0009    2011 IF (KOLR(1)-7)   2070,2060,2060
0010    2070 IDUM=KOLR(1)
0011        GO TO  (2060,2072,2073,2074,2075,2075),IDUM
0012    2072 IF (ITOT(NP)-17) 2060,2050,2060
0013    2073 IF (ITOT(NP)-17) 2060,2050,2052
0014    2074 IF (ITOT(NP)-13) 2060,2050,2052
0015    2075 IF (ITOT(NP)-12) 2060,2050,2052
0016    2051 IF (KOLR(1)-1)   2060,2060,2050
0017    2052 IF (ITOT(NP)-19) 2050,2060,2060
C     DOUBLE DOWN
0018    2050 IDUBL=1
0019        RETURN
C     DON'T DOUBLE DOWN
0020    2060 IDUBL=0
0021        RETURN
0022        END
```

```
C*************************************************************************
C
C
C     BASIC STRATEGY FOR SPLITTING PAIRS
C
C
C*************************************************************************
0001        SUBROUTINE SPLIT
0002        COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
           2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
           3NP,K,IACE
0003        IDM=IPLAY(NP,1)
0004        GO TO (1070,1080,1080,1081,1090,1080,1082,1070,1083,1090,1090,1090
           2,1090),IDM
0005    1080 IF (KOLR(1)-8) 1071,1090,1090
0006    1081 IF (KOLR(1)-5) 1090,1071,1090
0007    1082 IF (KOLR(1)-9) 1071,1090,1071
0008    1083 IF (KOLR(1)-9) 1084,1071,1090
0009    1084 IF (KOLR(1)-7) 1071,1090,1071
0010    1071 IF (KOLR(1)-1) 1070,1090,1070
C     SPLIT
0011    1070 ISPLIT=1
0012        RETURN
C     DON'T SPLIT
0013    1090 ISPLIT=0
0014        RETURN
0015        END
```

```
C*************************************************************************
C
C
C     BASIC STRATEGY FOR DRAWING
C
C
C*************************************************************************
0001        SUBROUTINE STRAT
0002        COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KDLR(11),ITOT(4),JBET(4
           2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
           3NP,K,IACE
0003        IF (MACE(NP)) 1020,1020,1030
C     HARD STANDING
0004    1020 JDUM=KDLR(1)
0005        GO TO (1021,1022,1022,1023,1023,1023,1021,1021,1021,1021,1021,1021
           2,1021),JDUM
0006    1021 IF (ITOT(NP)-17) 1040,1050,1050
0007    1022 IF (ITOT(NP)-13) 1040,1050,1050
0008    1023 IF (ITOT(NP)-12) 1040,1050,1050
C     SOFT STANDING
0009    1030 IF (KDLR(1)-9) 1031,1032,1032
0010    1031 IF (ITOT(NP)-18) 1040,1050,1050
0011    1032 IF (ITOT(NP)-19) 1040,1050,1050
C     DRAW A CARD
0012    1040 ISTRAT=1
0013        RETURN
C     DON'T DRAW ANY MORE CARDS
0014    1050 ISTRAT=0
0015        RETURN
0016        END
```

APPENDIX D 5

```
C*************************************************************************
C
C
C      STRATEGY SUBROUTINES
C
C
C
C*************************************************************************
C
C
C      BET WILL ALWAYS BE 2 UNITS
C
C
C*************************************************************************
0001          SUBROUTINE BETTR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,ICUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          JBET(NP)=2
0004          RETURN
0005          END


C*************************************************************************
C
C
C      NEVER INSURE
C
C
C*************************************************************************
0001          SUBROUTINE INSUR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IOUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE
0003          INS(NP)=0
0004          RETURN
0005          END
```

APPENDIX D6

```
C*************************************************************************
C
C
C      STRATEGY SUBROUTINES
C
C
C
C*************************************************************************
C
C
C      TENS-RATIO USED TO DETERMINE BET SIZE
C
C
C*************************************************************************
0001          SUBROUTINE BETTR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IRUST(4),IBJ(4),
             3NP,K,IACE,TR
0003          IF (TR-2.00) 3003,3003,3000
0004         3003 IF (TR-1.65) 3005,3005,3006
0005         3006 IF (TR-1.75) 3004,3004,3002
0006         3000 JBET(NP)=2
0007          RETURN
0008         3002 JBET(NP)=4
0009          RETURN
0010         3004 JBET(NP)=8
0011          RETURN
0012         3005 JBET(NP)=10
0013          RETURN
0014          END


C*************************************************************************
C
C
C      TENS-RATIO USED TO DETERMINE INSURANCE SIZE
C
C
C*************************************************************************
0001          SUBROUTINE INSUR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KOLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IRUST(4),IBJ(4),
             3NP,K,IACE,TR
0003          IF (TR-2.00) 4000,4000,4010
0004         4000 INS(NP)=JBET(NP)/2
0005          RETURN
0006         4010 INS(NP)=0
0007          RETURN
0008          END
```

APPENDIX D7

```
C*************************************************************************
C
C
C      STRATEGY SUBROUTINES
C
C
C
C*************************************************************************
C
C
C      HIGH-LOW INDEX DETERMINES BET SIZE
C
C
C*************************************************************************
0001          SUBROUTINE BETTR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KDLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE,IHL,JHL
0003          IF (IHL) 3000,3000,3010
0004         3010 IF (K) 3000,3000,3020
0005         3020 HL=IHL
0006          HK=K
0007          BETS= 50. * HL / HK
0008          JHL=BETS
0009          IF (JHL-5) 3030,3030,3040
0010         3030 JBET(NP)=2 * JHL
0011          RETURN
0012         3040 JBET(NP)=10
0013          RETURN
0014         3000 JBET(NP)=2
0015          RETURN
0016          END
```

```
C*************************************************************************
C
C
C      HIGH-LOW INDEX DETERMINES INSURANCE SIZE
C
C
C*************************************************************************
0001          SUBROUTINE INSUR
0002          COMMON ICARD(52),IDECK(52),NUM,IX,KTR,IVAL,KDLR(11),ITOT(4),JBET(4
             2),INS(4),IPLAY(4,11),MACE(4),ISPLIT,IDUBL,ISTRAT,IBUST(4),IBJ(4),
             3NP,K,IACE,IHL,JHL
0003          IF (JHL*2-8) 4000,4000,4010
0004         4000 INS(NP)=0
0005          RETURN
0006         4010 INS(NP)=JBET(NP) /2
0007          RETURN
0008          END
```

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