Reading Notes for
Abstract Algebra
by Dummit and Foote

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Contents

0 Preliminaries

Read the book

1 Introduction to Groups

1.1 Basic Axioms and Examples

if a is any element of G and for some $b \in G$, ab = e or ba = e, then $b = a^{-1}$, i.e., we do not have to show both equations hold.

the order of an element in a group is the same as the cardinality of the set of all its (distinct) powers so the two uses of the word "order" are naturally related.

1.2   Dihedral Groups

presentation = Generators + Relations

some "collapsing" may occur because the relations are intertwined in some unobvious way, i.e., there may be "hidden," or implicit, relations that are not explicitly given in the presentation but rather are consequences of the specified ones.

This collapsing makes it difficult in general to determine even a lower bound for the size of the group being presented.

While in general it is necessary to be extremely careful in prescribing groups by presentations, the use of presentations for known groups is a powerful conceptual and computational tool. Additional results about presentations, including more elaborate examples, appear in Section 6.3.

1.3   Symmetric Groups

For any $a \in S_n$, the cycle decomposition of a^{-1} is obtained by writing the numbers in each cycle of the cycle decomposition of a in reverse order.

1.4   MATRIX GROUPS