First Latex Assignment

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Consider two rad cars racing around the inner lane of a circular racetrack. Each car is 2.0 meters wide and 5.0 meters long. They travel at a speed of 50 m/s in a circle of radius 40 meters, with the rear car 1.0 meters behind the rear bumper of the front car. The rear car can perform constant acceleration from 50 m/s to 60 m/s in two seconds stay at 60 m/s for 2 seconds and then return (at a constant rate) to 50 m/s over the next two seconds. Describe a possible passing maneuver in physical detail. Discuss the physical significance of each of the four acceleration terms from the polar coordinate representation.

1 Explanation of Stage 1

Before passing the leading rad car, the trailing rad car (the leading and trailing rad cars will be referred to as R_1 and R_2, respectively) must accelerate into the outer lane. In order to avoid collision, the trailing rad car should shift to a concentric circle of a 43 meter radius, preserving a meter gap in the radial direction between the accelerating rad car and the first rad car. During its first two seconds of motion, R_2 will maintain a constant tangential velocity of $50\frac{m}{s}$ and a constant radial velocity of $1.5\frac{m}{s}$.

In order to determine the total acceleration of R_2 in its first two seconds of this passing maneuver, a number of additional variables must be determined. Among them are α, ω, r, V_{rad}, a_{rad} and V_{tan}:

$$V_{rad} = 1.5\frac{m}{s}$$

$$V_{tan} = 50\frac{m}{s}$$

$$a_{rad} = 0$$

$$r = 42$$

$$\omega = \frac{\Delta\theta}{\Delta T} = \frac{2\pi(\frac{50}{2\pi(42)})}{1} = 1.19\frac{rad}{sec}$$

$$\alpha = 0$$

After obtaining those values, finding total acceleration, when R_2 is racing around a circle with a 42 meter radius, is simply a matter of plugging these numbers into an equation:

$$\vec{a} = \underbrace{r\alpha\hat{\theta}}_{tangential} - \overbrace{r\omega^2\hat{r}}^{centripetal} + \underbrace{2V_r\omega\hat{\theta}}_{Coriolis} + \overbrace{a_r\hat{r}}^{radial}$$

$$\vec{a} = \cancel{42(0)\hat{\theta}} - (43)(1.19)^2\hat{r} + (2)(1.5)(1.19)\hat{\theta} + \cancel{(o)\hat{r}}$$

$$\boxed{\vec{a} = -60.9\hat{r} + 3.57\hat{\theta}}$$

The cancelling of the tangential and radial acceleration terms indicates that R_2 is not accelerating radially or tangentially. The size of centripetal acceleration is a clear indicator that R_2 is being drawn strongly toward the center of the track. Additionally, the Coriolis acceleration is small but significant.

The difference in angular displacement between R_1 and R_2 ought to be calculated at this point as well.

$$\Delta\theta = \bar{\omega}t$$

$$t = 2sec$$

$$\bar{\omega}_{R_2} = \frac{\omega_o + \omega f}{2} = \frac{2\pi(\frac{50}{2\pi(43)}) + 2\pi(\frac{50}{2\pi(40)})}{2} = \frac{1.19\frac{rad}{sec} + 1.25\frac{rad}{sec}}{2} = 1.22\frac{rad}{sec}$$

$$\Delta\theta_{R_2} = (1.22\frac{rad}{sec}) \times (2sec) = 2.44rad$$

$$\Delta\theta_{R_1} = (1.25\frac{rad}{sec}) \times (2sec) = 2.5rad$$

$$\Delta\theta_{R_1 and R_2} = 2.44rad - 2.5rad = .06rad$$

R_1 is .06 radians ahead of R_2 at the end of the first two seconds of this maneuver.

2 Explanation of Stage 2

Given that the trailing rad car, referred to as R_2 from now on, has the capability to accelerate at a constant rate to $60\frac{m}{s}$ while shifting to the outer lane, it would be expedient for R_2 to do so.

In order to determine the total acceleration of R_2 in its second two seconds of this passing a maneuver, a number of variables must be determined. Among them are α, ω_o, ω_f, r, V_{rad}, a_{rad} and V_{tan}:

$$r = 43meters$$

$$V_{rad} = 0$$

$$a_{rad} = 0$$

$$V_{tan} = r\omega = (43)(1.4) = 60.2\frac{m}{s}$$

$$\omega_{oR2} = \frac{\Delta\theta}{\Delta T} = \frac{2\pi(\frac{50}{2\pi(43)})}{1} = 1.16\frac{rad}{sec}$$

$$\omega_{R2,t=2} = \frac{\Delta\theta}{\Delta T} = \frac{2\pi(\frac{60}{2\pi(43)})}{1} = 1.4\frac{rad}{sec}$$

$$\alpha_{R2} = \frac{1.4\frac{rad}{s} - 1.16\frac{rad}{s}}{2} = .015\frac{rad}{s^2} = .24\frac{rad}{sec^2}$$

After obtaining those values, finding total acceleration, when R_2 is accelerating around a circle with a 43 meter radius, is simply a matter of plugging these numbers into an equation:

$$\vec{a} = \underbrace{r\alpha\hat{\theta}}_{tangential} - \overbrace{r\omega^2\hat{r}}^{centripetal} + \underbrace{2V_r\omega\hat{\theta}}_{Coriolis} + \overbrace{a_r\hat{r}}^{radial}$$

$$\vec{a} = (43)(.24)\hat{\theta} - (43)(1.4)^2\hat{r} + \cancel{2(0)(1.4)\hat{\theta}} + \cancel{(0)\hat{r}}$$

$$\boxed{\vec{a} = 10.32\hat{\theta} - 84.28\hat{r}}$$

It is not surprising that the magnitude of tangential acceleration increased from 3.57 to 10.32, because the speed of R_2 increased from $50\frac{m}{s}$ to $60\frac{m}{s}$. In addition, it is logical that centripetal acceleration would increase given that the radius of track has increased for R_2 as well. Lastly, there were no Coriolis or radial accelerations as radial velocity was equal to zero for the entire period.

The difference in angular displacement between R_1 and R_2 ought to be calculated at this point as well.

$$\Delta\theta = \bar{\omega}t$$

$$t = 2sec$$

$$\bar{\omega}_{R_2} = \frac{\omega_o + \omega f}{2} = \frac{2\pi(\frac{50}{2\pi(43)}) + 2\pi(\frac{60}{2\pi(43)})}{2} = \frac{1.19\frac{rad}{sec} + 1.4\frac{rad}{sec}}{2} = 1.295\frac{rad}{sec}$$

$$\Delta\theta_{R_2} = (1.295\frac{rad}{sec}) \times (2sec) + 2.44rad = 5.03rad$$

$$\Delta\theta_{R_1} = (1.25\frac{rad}{sec}) \times (2sec) + 2.5rad = 5rad$$

$$\Delta\theta_{R_1 and R_2} = 5.00rad - 5.03rad = -.03rad$$

R_1 is .03 radians behind R_2 at the end of the first four seconds of this maneuver.

3 Explanation of Stage 3

In this stage, R_2 has finished accelerating and should pass R_1 while staying in the outer lane. In order to do this, R_1 will maintain it's current speed of $60\frac{m}{s}$ for the next period of two seconds.

As in the prior two stages, the values of a handful of values will need to be identified in order to determine the total acceleration of R_2 after six seconds have elapsed in total. Among these variables are: r, α, ω, V_r, and a_r.

$$r = 43$$

$$\alpha = 0$$

$$\omega = \frac{\Delta\theta}{\Delta T} = \frac{2\pi(\frac{60}{2\pi(43)})}{1} = 1.4\frac{rad}{sec}$$

$$V_r = 0$$

$$a_r = 0$$

After obtaining those values, finding total acceleration, when R_2 is accelerating around a circle with a 43 meter radius, is simply a matter of plugging these numbers into an equation:

$$\vec{a} = \underbrace{r\alpha\hat{\theta}}_{tangential} - \overbrace{r\omega^2\hat{r}}^{centripetal} + \underbrace{2V_r\omega\hat{\theta}}_{Coriolis} + \overbrace{a_r\hat{r}}^{radial}$$

$$\vec{a} = \cancel{(43)(0)\hat{\theta}} - (43)(1.4)^2\hat{r} + \cancel{2(0)(1.4)\hat{\theta}} + \cancel{(0)\hat{r}}$$

$$\boxed{\vec{a} = -84.28\hat{r}}$$

As R_2 is maintaining a constant speed of $60\frac{m}{s}$, its total acceleration is equal to its centripetal acceleration, which is slightly greater with a radius of 43 meters than it was it with a radius of 40 meters.

The difference in angular displacement between R_1 and R_2 ought to be calculated at this point as well.

$$\Delta\theta = \bar{\omega}t$$

$$t = 2sec$$

$$\bar{\omega}_{R_2} = \frac{\omega_o + \omega f}{2} = \frac{2\pi(\frac{60}{2\pi(43)}) + 2\pi(\frac{60}{2\pi(43)})}{2} = \frac{1.4\frac{rad}{sec} + 1.4\frac{rad}{sec}}{2} = 1.4\frac{rad}{sec}$$

$$\Delta\theta_{R_2} = (1.4\frac{rad}{sec}) \times (2sec) + 5.03rad = 7.83rad$$

$$\Delta\theta_{R_1} = (1.25\frac{rad}{sec}) \times (2sec) + 5rad = 7.5rad$$

$$\Delta\theta_{R_1 and R_2} = 7.5rad - 7.83rad = -.33rad$$

R_1 is .33 radians behind R_2 at the end of the first six seconds of this maneuver.

4 Explanation of Stage 4

In this stage, R_2 should decelerate to $50\frac{m}{s}$ at a constant rate and maneuver back into the inner lane in front of R_1. At the end of this stage, the passing maneuver should be complete. In order to return to the inner lane, R_2 ought to have a radial velocity of $-1.5\frac{m}{s}$ and a radial acceleration of 0.

As in the prior two stages, the values of a handful of values will need to be identified in order to determine the total acceleration of R_2 after six seconds have elapsed in total. Among these variables are: r, α, ω, V_r, and a_r.

$$r = 40$$

$$\omega = \frac{\Delta\theta}{\Delta T} = \frac{2\pi(\frac{50}{2\pi(40)})}{1} = 1.25\frac{rad}{sec}$$

$$\alpha = \frac{1.25\frac{rad}{s} - 1.4\frac{rad}{s}}{2} = -.075\frac{rad}{s^2}$$

$$V_r = -1.5\frac{m}{s}$$

$$a_r = 0$$

After obtaining those values, finding total acceleration, when R_2 is accelerating around a circle with a 43 meter radius, is simply a matter of plugging these numbers into an equation:

$$\vec{a} = \underbrace{r\alpha\hat{\theta}}_{tangential} - \overbrace{r\omega^2\hat{r}}^{centripetal} + \underbrace{2V_r\omega\hat{\theta}}_{Coriolis} + \overbrace{a_r\hat{r}}^{radial}$$

$$\vec{a} = (40)(-.075)\hat{\theta} - (40)(1.25)^2\hat{r} + 2(1.5)(1.25)\hat{\theta} + \cancel{(0)\hat{r}}$$

$$\boxed{\vec{a} = .75\hat{\theta} - 62.5\hat{r}}$$

Given that R_2 is decelerating, it makes sense that there would be a significant and negative tangential acceleration. This negative tangential acceleration combines with a slightly larger in magnitude Coriolis acceleration, which brings the total acceleration in the direction of $\hat{\theta}$ to $.75\frac{m}{s^2}$. Centripetal acceleration will decrease to $62.5\frac{m}{s^2}$ from $84.28\frac{m}{s^2}$ in the previous stage, because radius has decreased from 43 meters to 40 meters. Finally, there will be no radial acceleration, as R_2 is not accelerating radially.

The difference in angular displacement between R_1 and R_2 ought to be calculated at this point as well.

$$\Delta\theta = \bar{\omega}t$$

$$t = 2sec$$

$$\bar{\omega}_{R_2} = \frac{\omega_o + \omega f}{2} = \frac{2\pi(\frac{60}{2\pi(43)}) + 2\pi(\frac{50}{2\pi(40)})}{2} = \frac{1.4\frac{rad}{sec} + 1.25\frac{rad}{sec}}{2} = 1.325\frac{rad}{sec}$$

$$\Delta\theta_{R_2} = (1.325\frac{rad}{sec}) \times (2sec) + 7.83rad = 10.48rad$$

$$\Delta\theta_{R_1} = (1.25\frac{rad}{sec}) \times (2sec) + 7.5rad = 10.0rad$$

$$\Delta\theta_{R_1 and R_2} = 10.00rad - 10.48rad = -.48rad$$

R_1 is .48 radians behind R_2 after the maneuver is complete.