APMTH 115 Final Project: Beluga's trip

WEI ZHANG

Harvard University, Cambridge MA, United States

Abstract

In this project, I am trying to analyze the traditional Travailing Salesman Problem (TSP) in a modern situation: the scale of trip is much larger with the guidance of (Geological Information System) GIS; the possible to relate the TSP with Hamiltonian Cycle Problem (HCP); finally I employ the classic Markov Chain and Markov Decision Process (MDP) to get the useful the conclusion concerning the steady state probability.

1. Introduction

The airports are not only to transport the passengers and cargo, they are also the place for the engineering site for aircraft; sometimes the oversize cargo (part of aircraft for example) need to be send to factory. Headquarter in Toulouse, Airbus company has the responsibility to transport all kinds of cargo among the airports. For the oversize cargo and the part of aircraft, the Airbus A300-600 Beluga takes this mission for years. The range of Beluga is quite short: 2779 km. But it can carry 40 ton with a capacity of 1700 m^3.

The 40 airports in Europe are selected for the trip of Beluga. All these airports are among the busiest airport in Europe geographically, including the airports of Britain, Turkey, and Russia.

Figure 1: Beluga: the transporter

.

1.1. structure of project

The project is realized by the files below:

Project files			
File name	Input	Output	function
Beluga_Trip.m	position_info.txt	airport_list.txt	main function
geolocation.py	airport_list.txt	position_info.txt	Geocoding for latitude and longitude of each airport
getdistance.m	location of two airports	distance	calculate the distance in earth
hamiltonian.m	Graph and nodes	the Hamiltonian path	Hamiltonian cycle problem
normalization.m	line of MDP Transition matrix	list	normalization the conditional probability for MDP matrix
airport_list.txt	N	N	generated by main function for list of airport
position_info.txt	N	N	generated by python for geographic info

2. Model

2.1. Geocoding

In the first part, the python Gocoding API is called from Matlab. The communication between Matlab and python is realized with the IO files. Matlab provides the list of European airports and to python; python will return the latitude and longitude for each airport.

This mechanism is time consuming and full of risk: during the debug step, the geographic info of 'Rome', 'Athens' are no correct. The precise information could be retrieved directly from the Google Maps or from the Google Maps API Javascript. To simplify this project, the Python Geocoding Toolbox geopy 1.11.0 is integrated in this program instead of Google API.

Another risk is the that version of python should be 64 bit to cooperate with Matlab. Both the command system ($'python **** .py'$) and the execution of $** .py$ file would cost much time, dependent on the number of cities in the list. To avoid the timeout problem and to make sure this step finish before next part. The parameter 'timeout' in python file and the command 'pause' in Matlab should be used.

In the condition that the Matlab do not have the $'try...catch'$ for IO error as in Python, the $'if...else'$ statement is employed to verify if the geographic information file *position_info.txt* is ready for next step.

To accelerate the program, the *position_info.txt* is prepared with all the information about latitude and longitude of the airports in the list. If the new airport is added, the *position_info.txt* and *airport_list.txt* should be deleted and the program will generate the updated ones.

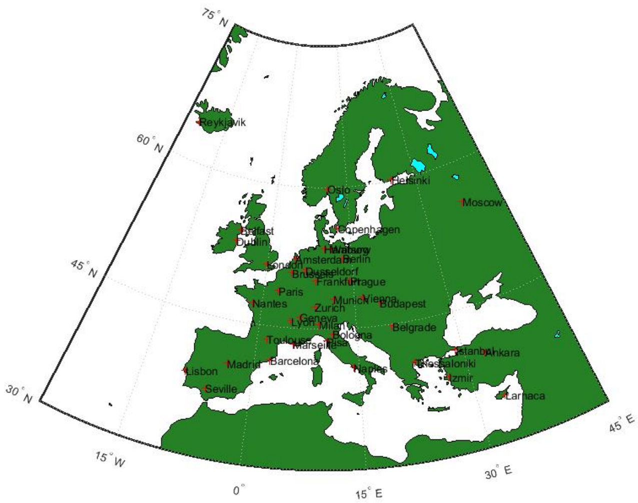

Figure 2: The 40 airports selected in Europe

.

2.2. Optimization: TSP

The Travailing Salesman Problem (TSP) has been well studied from last century. In this project, the dynamic programming is employed and this TSP is considered as one MIP optimization problem.

As the first step, the solver *intlinprog* deals the LP relaxation of TSP: without the subtour elimination constraints. In the second step, these subtours are eliminated gradually via a while loop.

The visualization plays an important role in the optimization. Several graphs are created to illustrate the Beluga's trip. The solution of this TSP MIP is a circle of 40 airports. At the end of this part, the Depth-first search (DFS) algorithm is employed to transform the the un-directed graph to a list.

2.3. Hamiltonian cycle problem

The first step of this part is to form a Markov transition matrix from the shortest iteration found in the previous optimization part. Different from the classic stochastic model, the element in this TSP transition matrix is 1 or 0. In Markov Chain, the action is deterministic from one state (airport) to another state (airport). In other way, the $'0-1'$ matrix can be regarded as the Hamiltonian path for the given problem. This fact is confirmed quickly with the help of $'hamiltonian'$ function.

Hamiltonian cycle problem is problem of Hamiltonian cycle exists in a given graph: a cycle in an un-directed or directed graph that visits each vertex exactly once. It is NP-complete problem. The Hamiltonian cycle problem is also a special case of the traveling salesman problem, by setting the distance between two cities to one if they are adjacent, with an addition weight matrix. This treatment is quite similar as the separation of decision variable in LP relaxation or MIP.

Therefore, the MDP matrix of TSP is then a Hamiltonian graph. Besides the given shortest iteration, the $'1'$ in other places of matrix mean the alternative way from one state (airport) to another state (airport).

Then it's time to take other scenario into account. As the giant transporter, Beluga could frequently arrive at the factory to fetch the special cargo. One list of engineering sites and factories is offered for this second condition. Firstly these additional iterations are added directly in the MDP transition matrix to form the Hamiltonian cycle problem. These new added air-ways provide the new possibility for the shortest iteration for Beluga. The Hamiltonian Path is found as well in this part.

2.4. Markov Chain and MDP

In previous part, the Markov Chain has already been mentioned. After the Hamiltonian cycle problem, the frequencies of the routine round trip and of the special mission are now considered: 0.75 for routine round trip and 0.25 for special mission. The new matrix is a MDP transition matrix, after the normalization of each line of the new matrix.

After about the iteration of 150 missions, the position frequencies of each airport is gained. This position frequencies is calculated theoretically as well to verify the calculation of the ergodic Markov chain.

3. Analysis of Results

As showed in the figure 3, the shortest iteration is a circle with the distance as weigh of arc. Obviously, the Beluga could start its trip from any airport including the Toulouse and return to the same airport.

Figure 4 : the same shortest iteration could be represented in the map of Europe as well.

Figure 5: to further analysis, the representation of iteration is simplified as a circle. The ID of nodes is stored in the cell structure $'airport_name'$ in main function file. Node 1 represents the first element in this data structure : Toulouse.

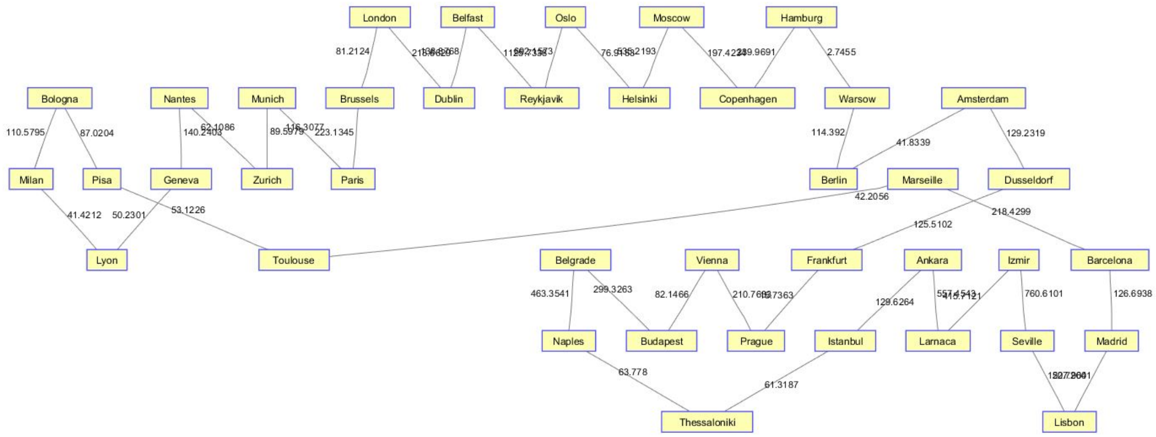

Figure 3: Graph of iteration for 40 airports

.

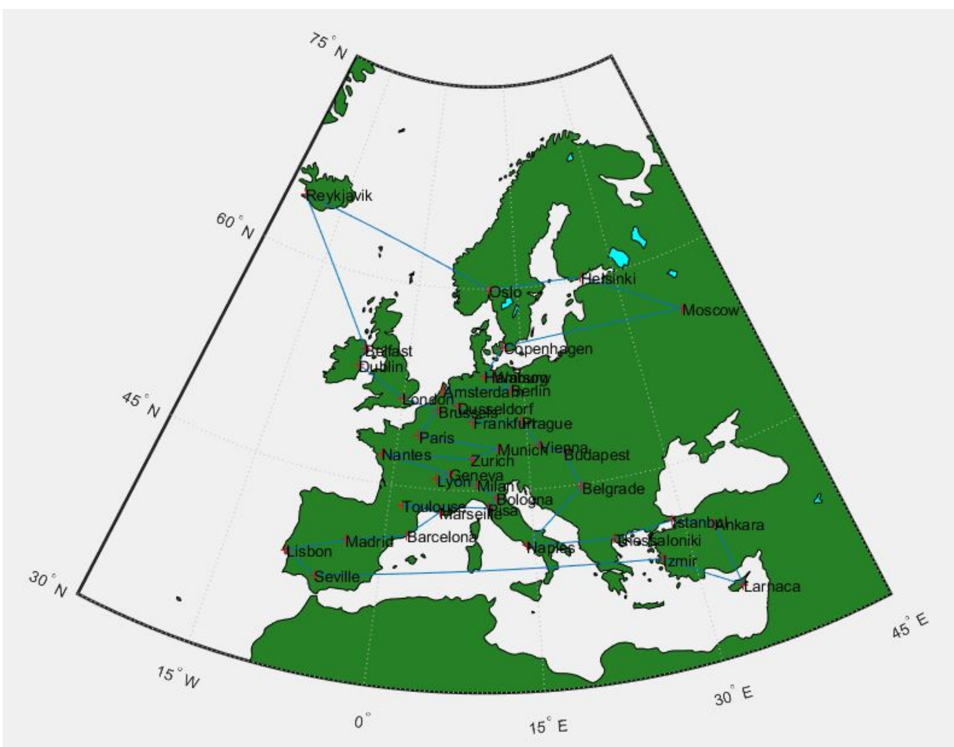

Figure 4: Graph of iteration for 40 airports

.

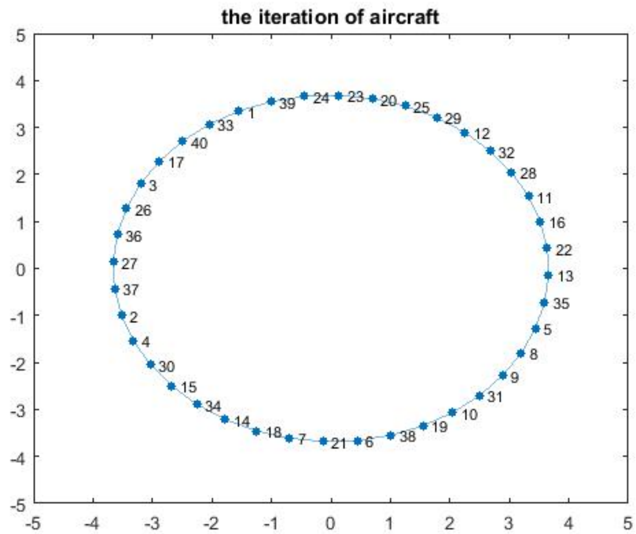

Figure 5: Graph simplified of iteration for 40 airports

.

Another fact is that Beluga has two option: clockwise and anti-clockwise.

As showed by Figure 6: verified by the Hamiltonian circle problem algorithm, the shortest iteration gained by the MIP optimization is one Hamiltonian circle as well.

The figure 7 shows that the Hamiltonian circle is still valid after the add of new way in the graph(matrix)

Figure 8 : the position frequencies for all the 40 airports. The number means the probability visited by Beluga. This figure is the iteration of 150 times as MDP.

Figure 9 : The probability distribution represents the same thing as in figure 8. But the result is calculated directly from the MDP transition matrix.

4. Discussion and Summary

As showed in the previous part, the shortest iteration of a TSP can be gained by the classic MIP optimization. The result is also the Hamiltonian cycle/Path, if we separate the weight of the arc between two nodes. Then if the perturbation or alternative possibilities are added to the original iteration. The MDP can also be used to find the probability of distribution of all the nodes in TSP.

In this way, the TSP connects the stochastic model - MDP and deterministic model - HCP. During this process, the various representation of data plays a quit important role. For example: Depth-first search produces the list from the un-direct graph; the list is the

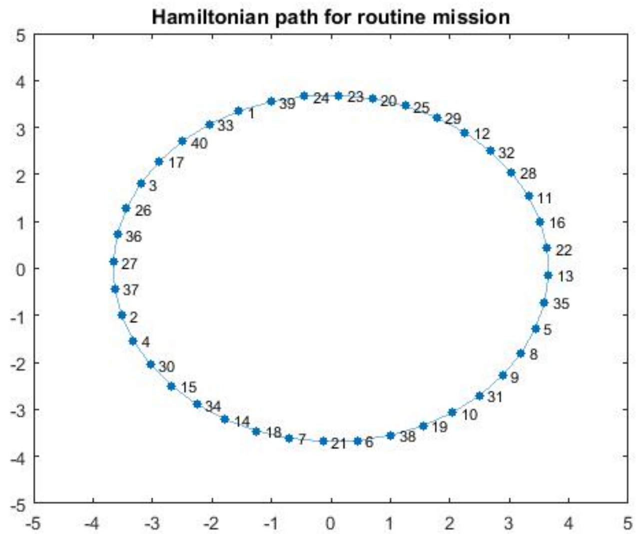

Figure 6: Hamiltonian circle/path for 40 airports

.

key point to form the Hamiltonian graph. In other words, the graph theory reveals the characteristic of TSP.

The conclusion of this project is obvious:
1. the shortest iteration from Toulouse
2. the important airports for Beluga are the engineering sits in condition of specially mission.

The second sentence is somehow too obvious: the range of Beluga is more than 2000 km in full of charge; considering the distance between the airports, it can change easily the destination if necessary. In other words: the decision-making is not quit difficult and the deviation from the shortest iteration would not be too much.

The interesting part of this project is to use lots of graph function including the python API. This practice enlarge the vision of math modeling. Surely the theory part is independent from the tools, but the tools can provide new possibility to regard the existent subject: the Matlab and Python manifests an good example to use the online GIS information to think the tradition problem in a larger scale.

5. References

The information about Airbus and the Beluga are available:
http://www.airbus.com/
https://en.wikipedia.org/wiki/Airbus_Beluga

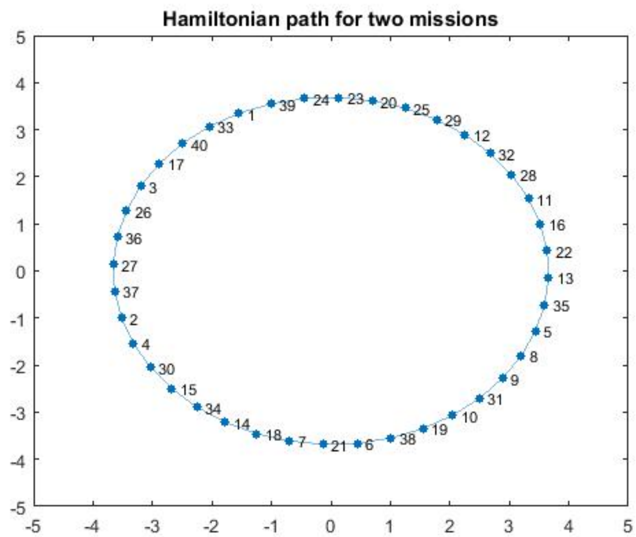

Figure 7: Hamiltonian circle/path for 40 airports

.

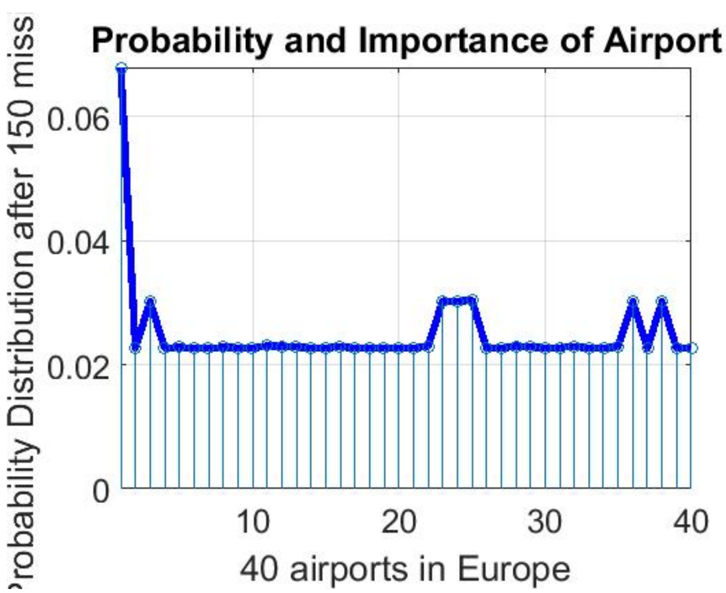

Figure 8: probability of 40 airports

.

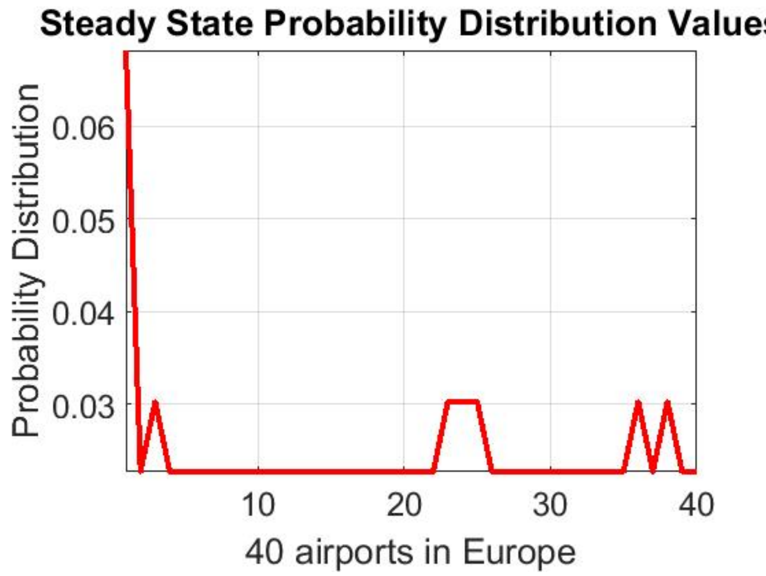

Figure 9: probability of 40 airports

.

6. postscript

The reason to choose the Beluga is personal: before studying the Architecture and Urban planning in Harvard, I studied mechanical engineering in aeronautic industry in Toulouse. Some Fridays, when the class was over, a Beluga flied over our engineering school from Toulouse–Blagnac Airport to factories somewhere in Europe. Sometimes it carried the main-body of another Airbus aircraft. But each time, it flied very slow and made very noisy sound; as a result, everybody stopped their own business and watched it passing by, as a huge white whale. I wondered always where it went for each mission.

All the files are available from the author's site :
https://github.com/zwei2016/B_TSP

The information in project is only to illustrate the mathematical models. It can not represent the real iteration of Airbus Beluga.