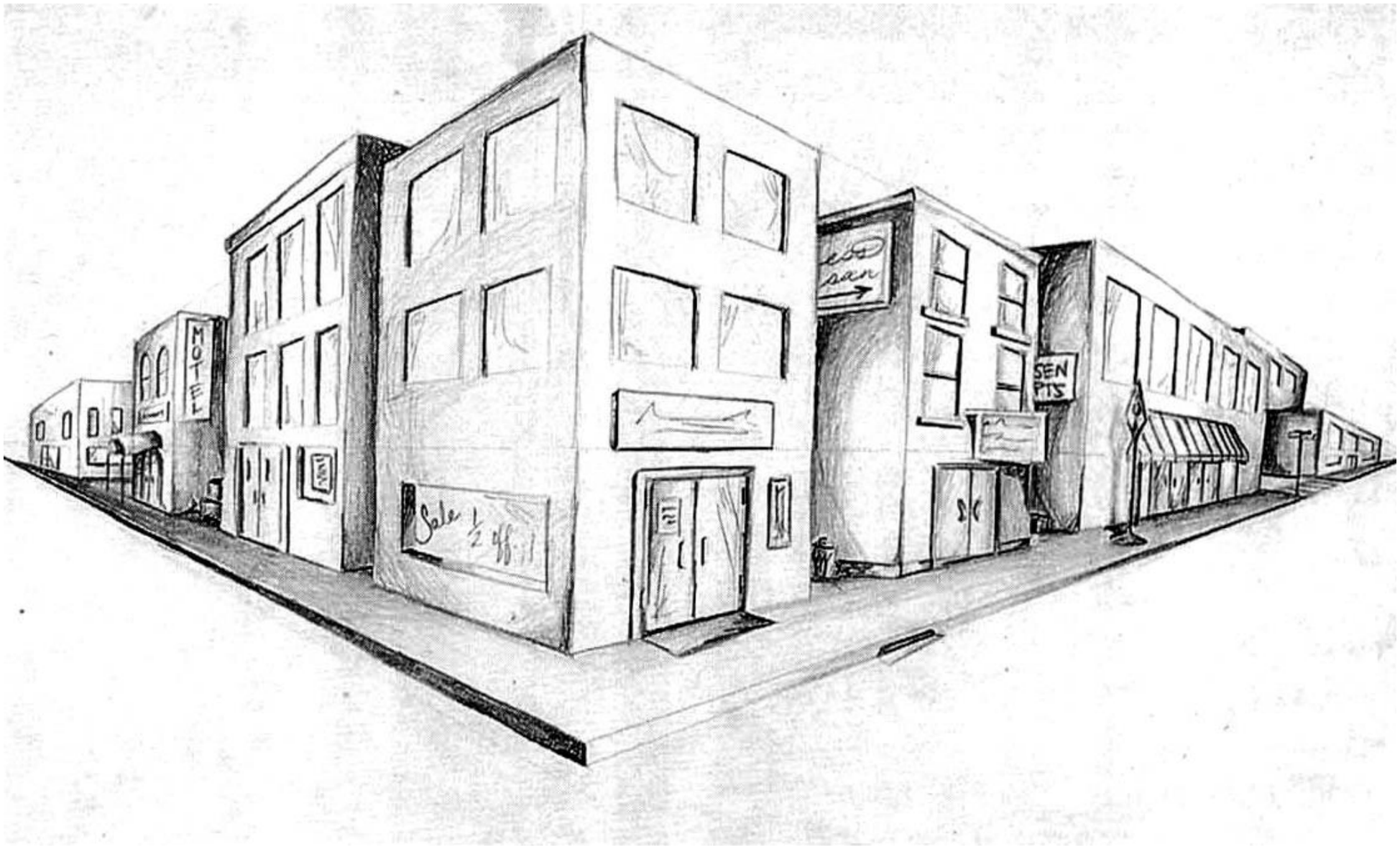
Detecting corners



Recap

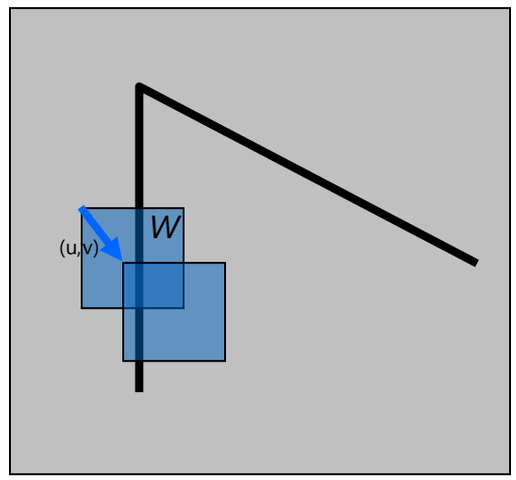
- **Why detect corners?**

- Visualizing quadratics.
- Harris corner detector.
- Multi-scale detection.
- Multi-scale blob detection.

Corner detection: the math

Consider shifting the window *W* by (*u,v*)

- define an SSD "error" *E(u,v)*:

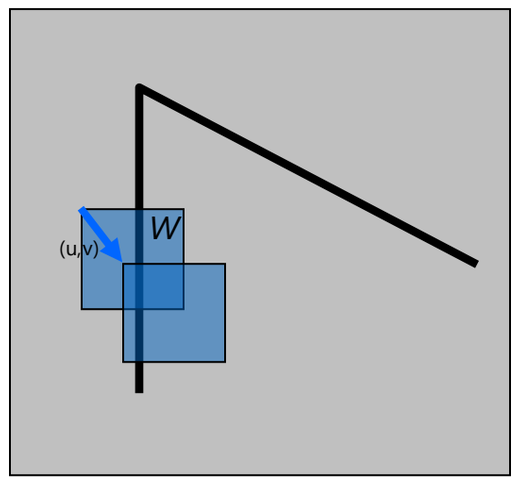


$$
\begin{aligned}
E(u,v) &= \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2 \\
&\approx \sum_{(x,y)\in W} [I(x,y) + I_x u + I_y v - I(x,y)]^2 \\
&\approx \sum_{(x,y)\in W} [I_x u + I_y v]^2
\end{aligned}
$$

Corner detection: the math

Consider shifting the window *W* by (*u*,*v*)

- define an SSD "error" *E(u,v)*:



$$E(u,v) \approx \sum_{(x,y)\in W} [I_x u + I_y v]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

- Thus, *E*(*u*,*v*) is locally approximated as a quadratic error function

The second moment / covariance matrix

The surface $E(u,v)$ is locally approximated by a quadratic form.

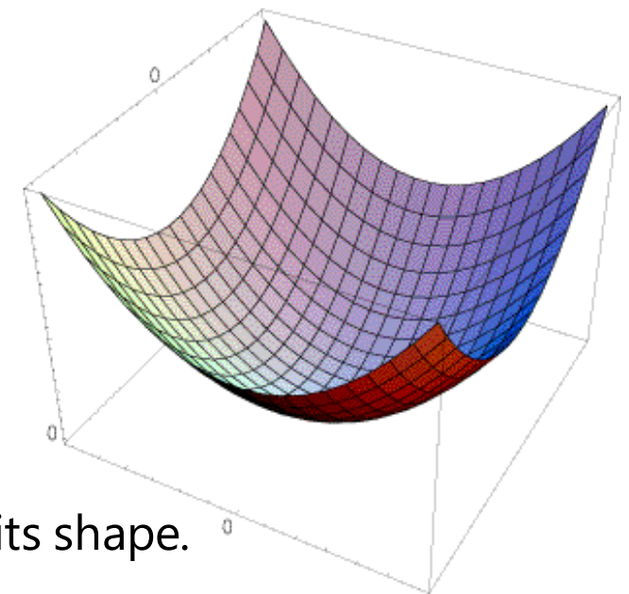
$$E(u,v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

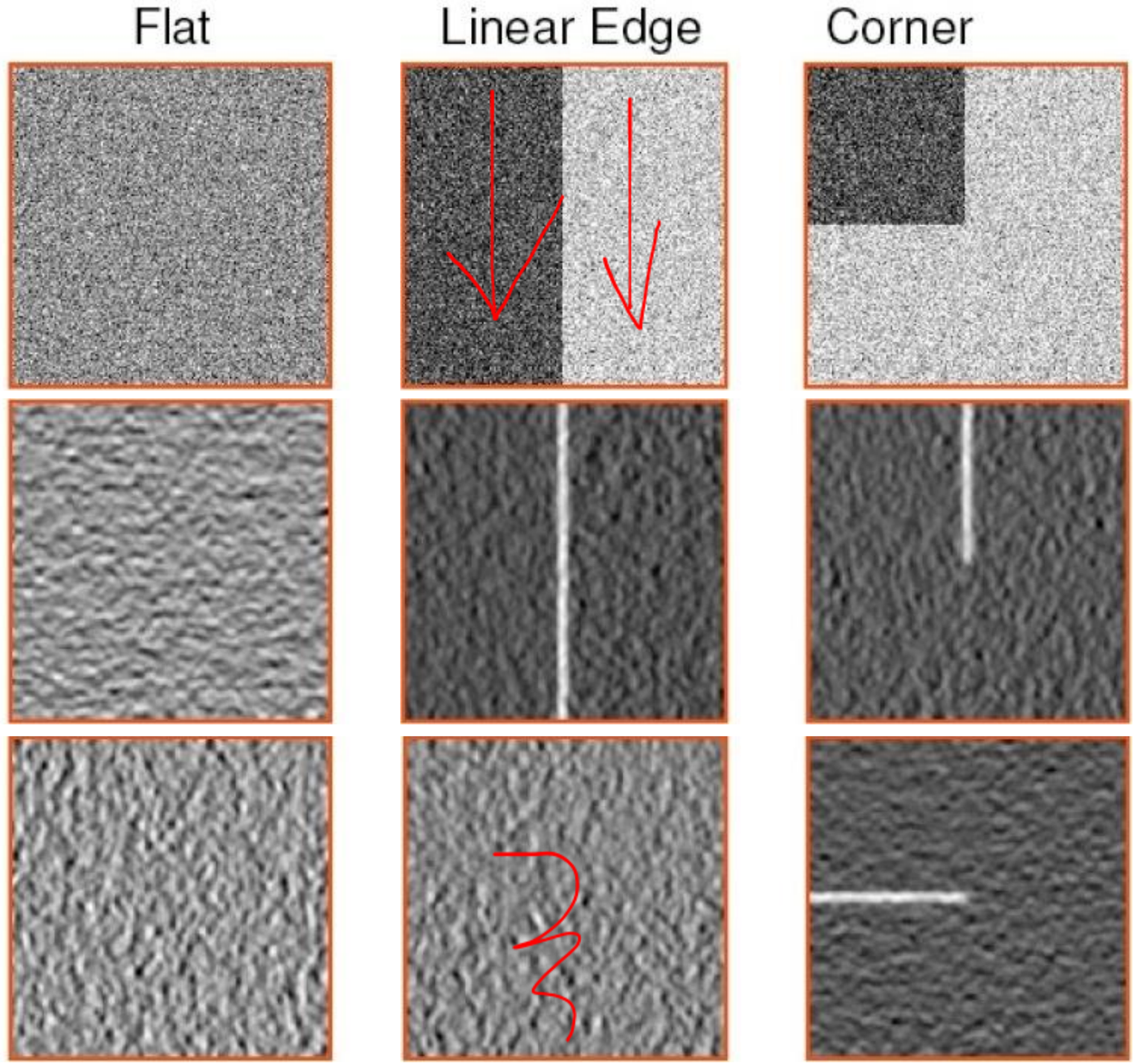
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Let's try to understand its shape.

visualization of gradients

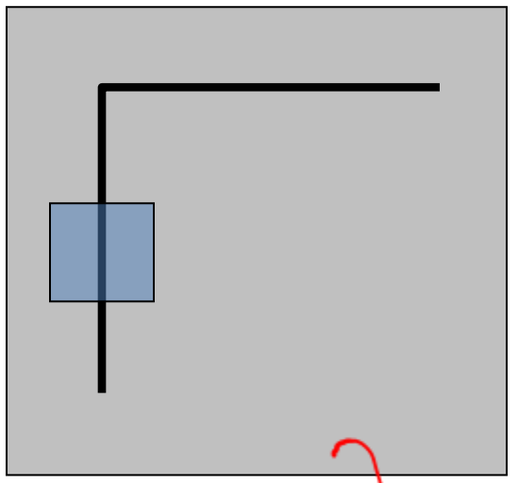


$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

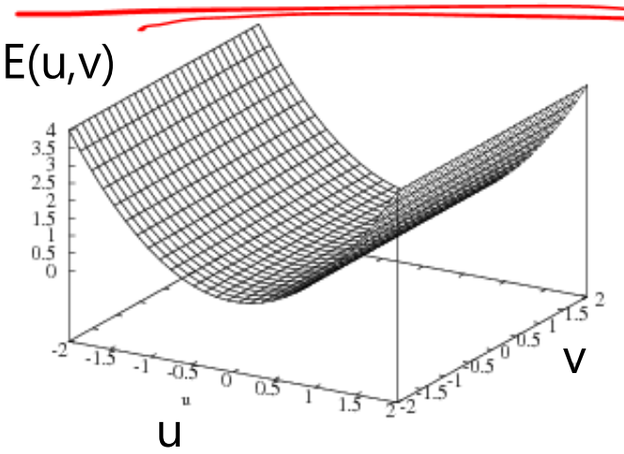
$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge: $I_y = 0$

$$H = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$

E(u,v)



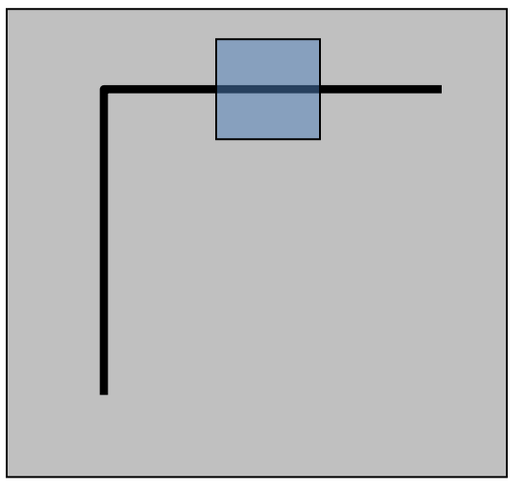
u

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

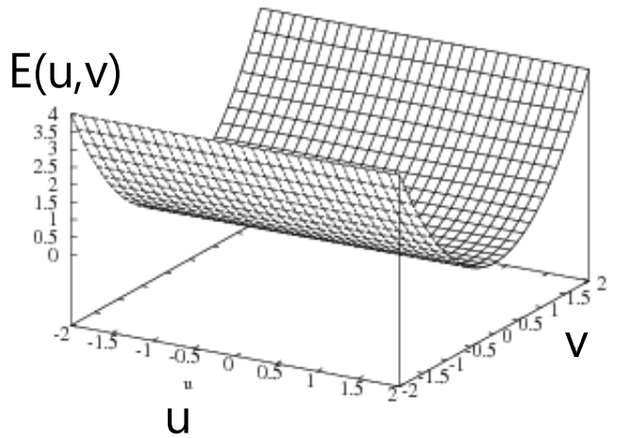
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Horizontal edge: $I_x = 0$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

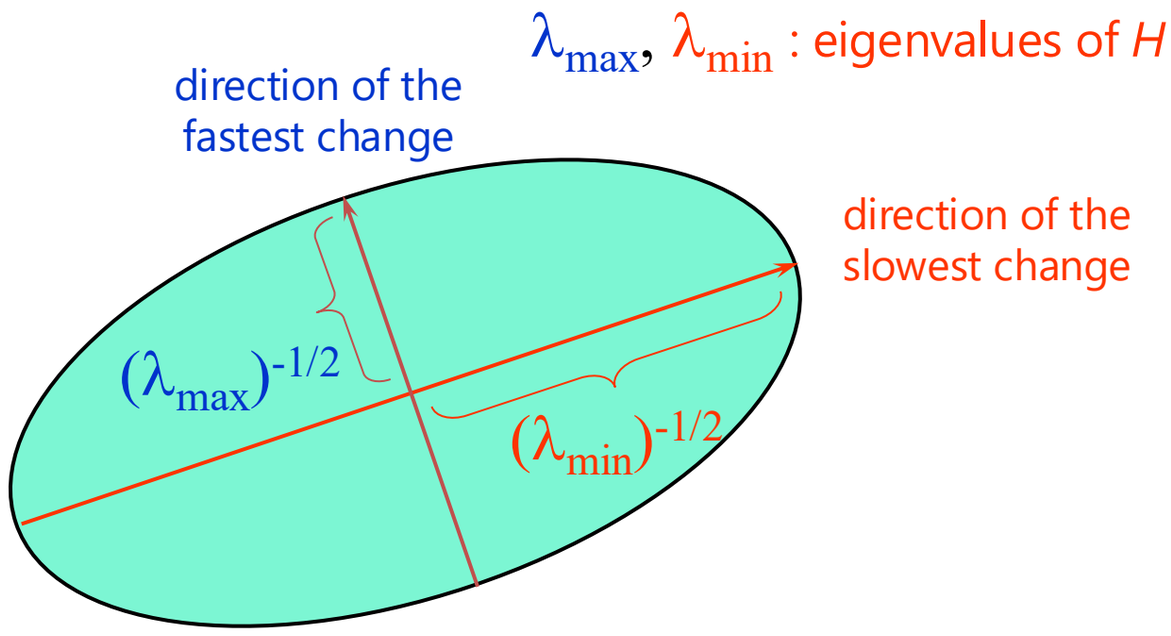


General case

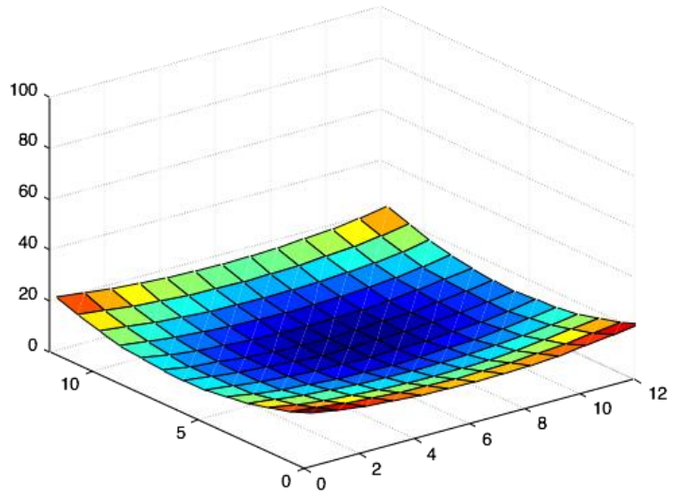
We can visualize *H* as an ellipse with axis lengths determined by the *eigenvalues* of *H* and orientation determined by the *eigenvectors* of *H*

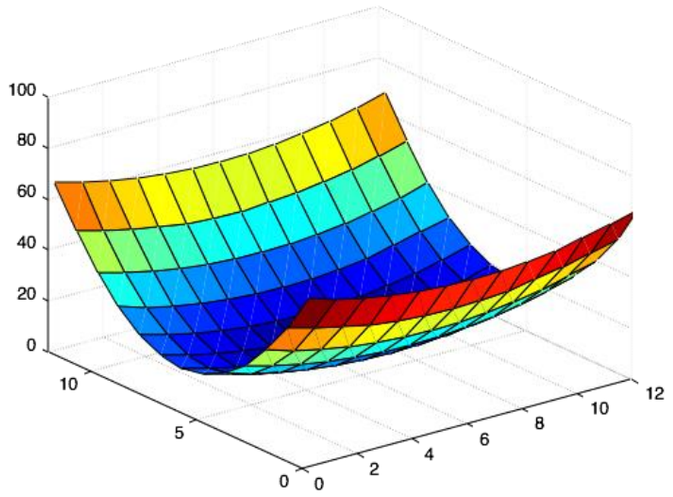
Ellipse equation:

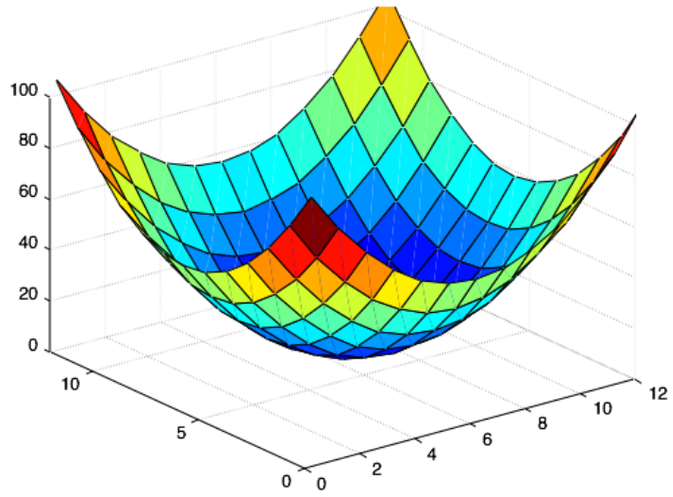
$$[u \quad v] \; H \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Which error surface indicates a good image feature?

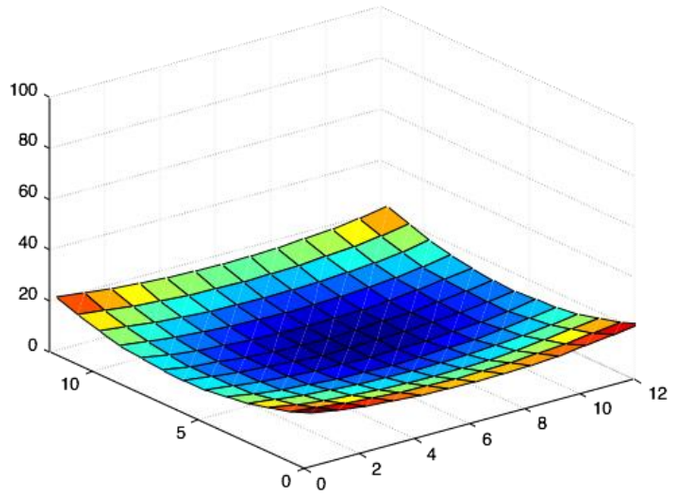




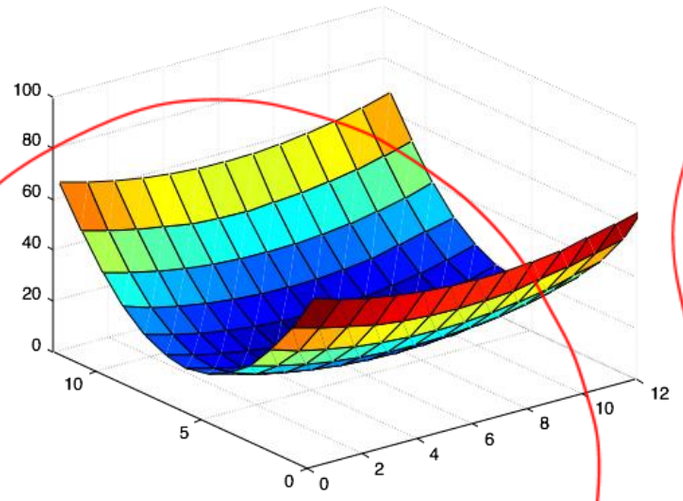


What kind of image patch do these surfaces represent?

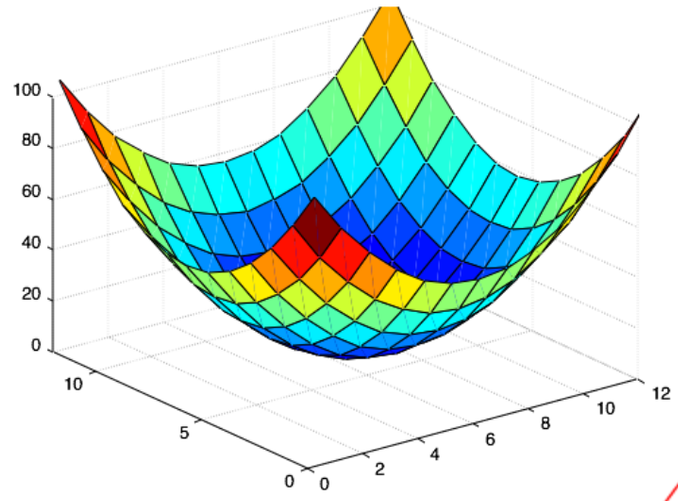
Which error surface indicates a good image feature?



flat

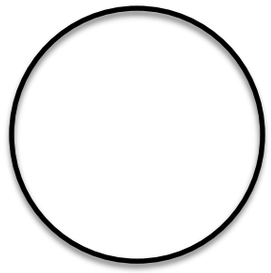


edge
‘line’



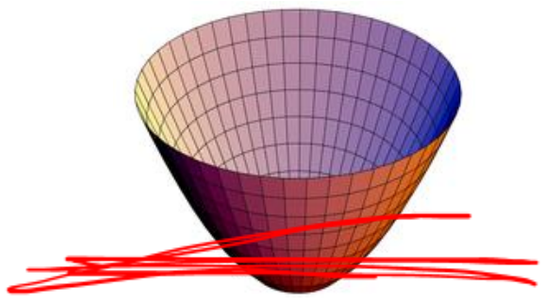
corner
‘dot’

Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



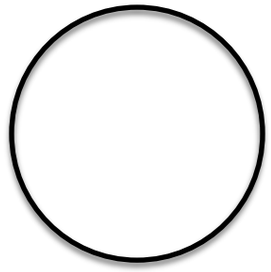
Equation of a ‘bowl’ (paraboloid)

$$f(x, y) = x^2 + y^2$$

If you slice the bowl at

$$f(x, y) = 1$$

what do you get?



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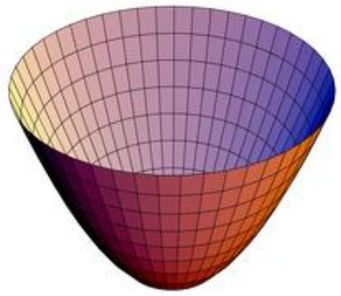

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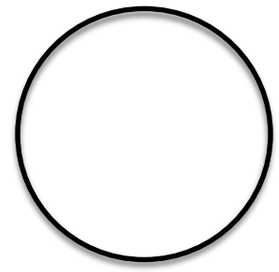


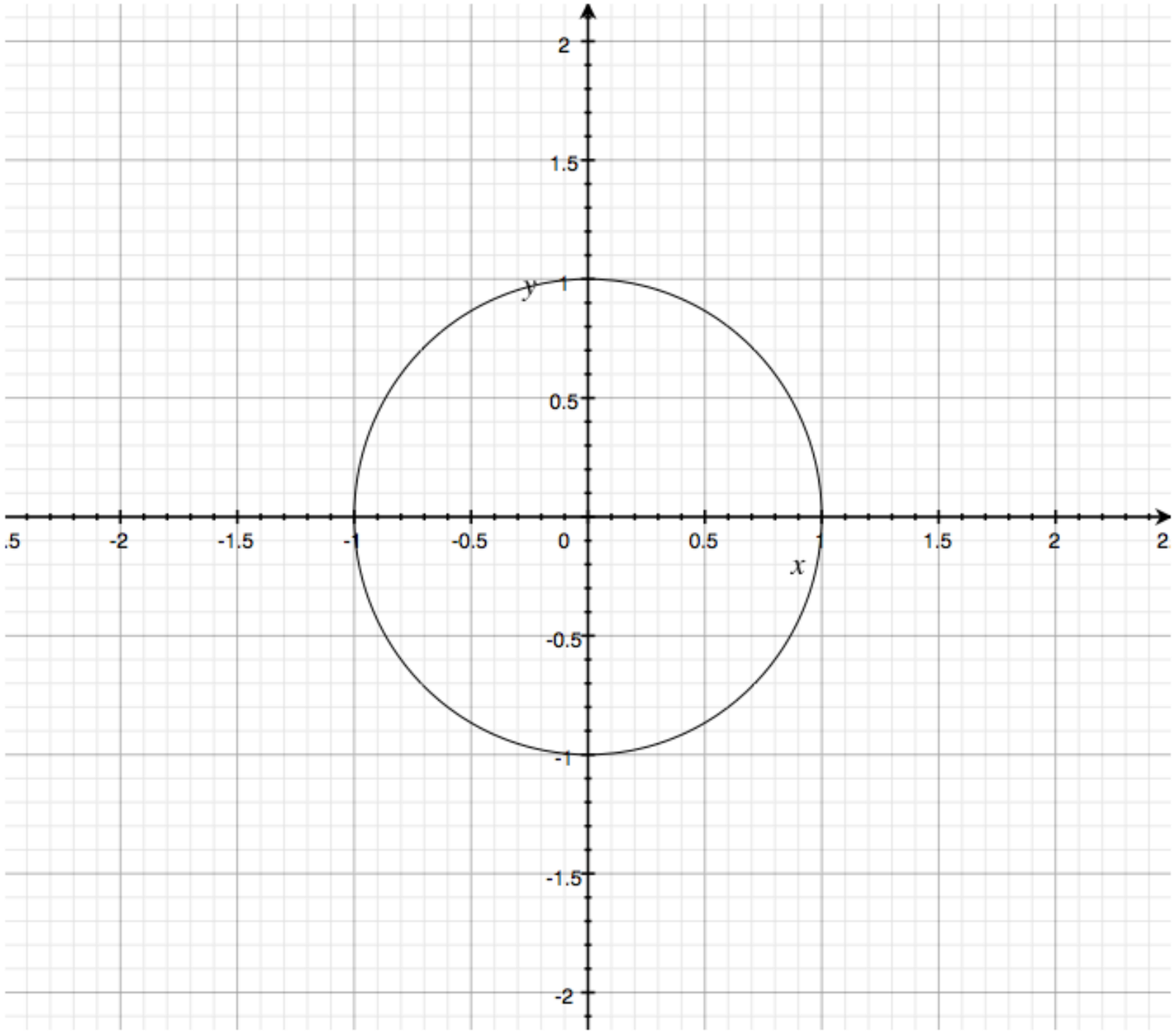
$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

‘sliced at 1’



*What happens if you **increase** coefficient on **x**?*

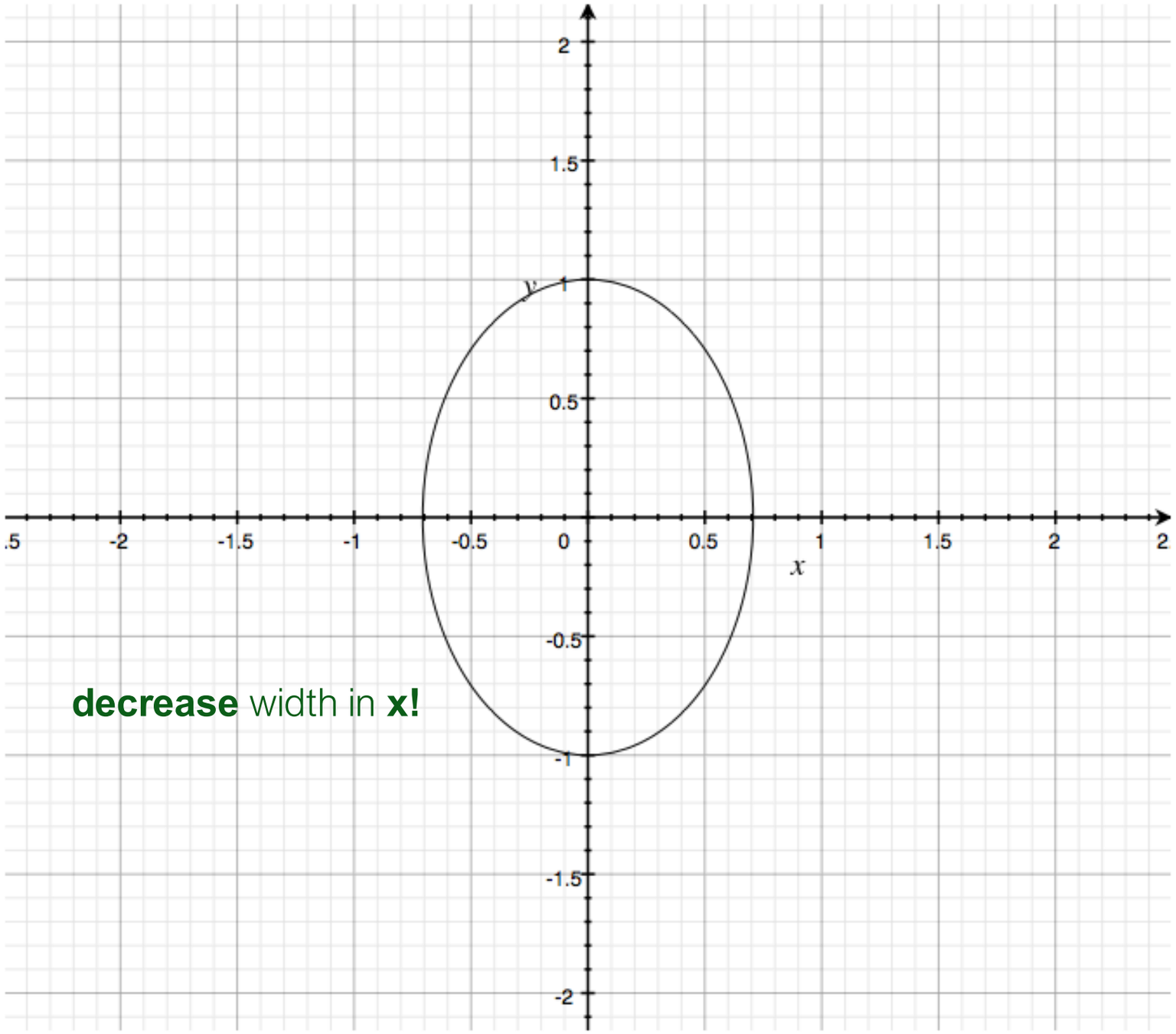
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **x**?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



decrease width in **x!**

*What happens if you **increase** coefficient on **y**?*

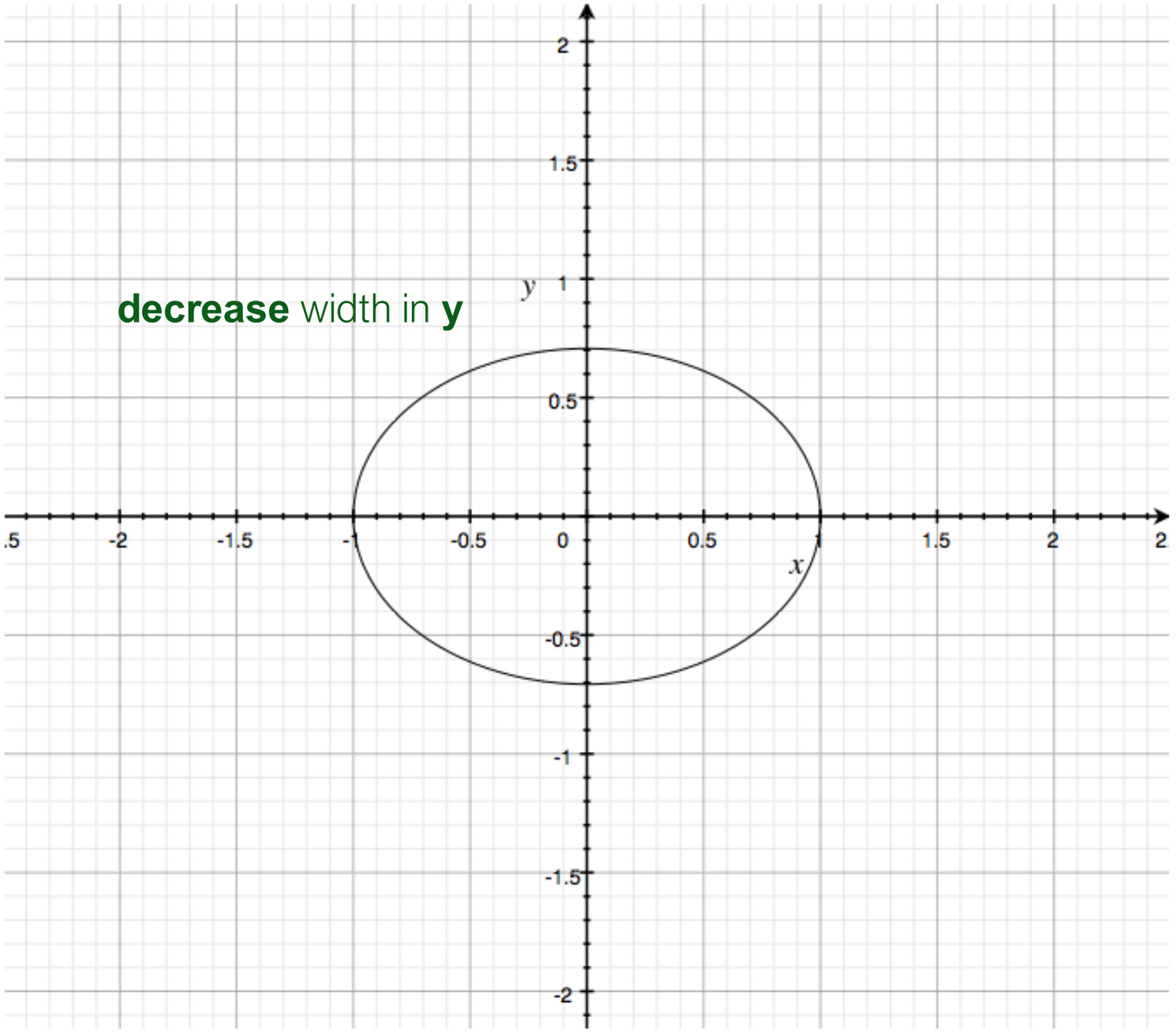
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **y**?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's the shape?
What are the eigenvectors?
What are the eigenvalues?

$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Result of Singular Value Decomposition (SVD)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^\top$$

eigenvectors

axis of the ‘ellipse slice’

eigenvalues along diagonal

Inverse sqr of length of the quadratic along the axis

Note: Eigen value decomposition and SVD should be same for real symmetric matrix.

Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

- In our case, ***A*** = ***H*** is a 2x2 matrix, so we have

$$det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

- The solution:

$$\lambda_{\pm} = \frac{1}{2}\left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}\right]$$

Once you know λ, you find **x** by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

4. Compute eigenvalues and eigenvectors

eigenvalue

$$Me = \lambda e$$

eigenvector

$$(M - \lambda I)e = 0$$

1. Compute the determinant of $M - \lambda I$

(returns a polynomial)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$
(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
(returns eigenvalues)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$Me = \lambda e$$

eigenvector

$$(M - \lambda I)e = 0$$

1. Compute the determinant of $M - \lambda I$
   (returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
   (returns eigenvalues)

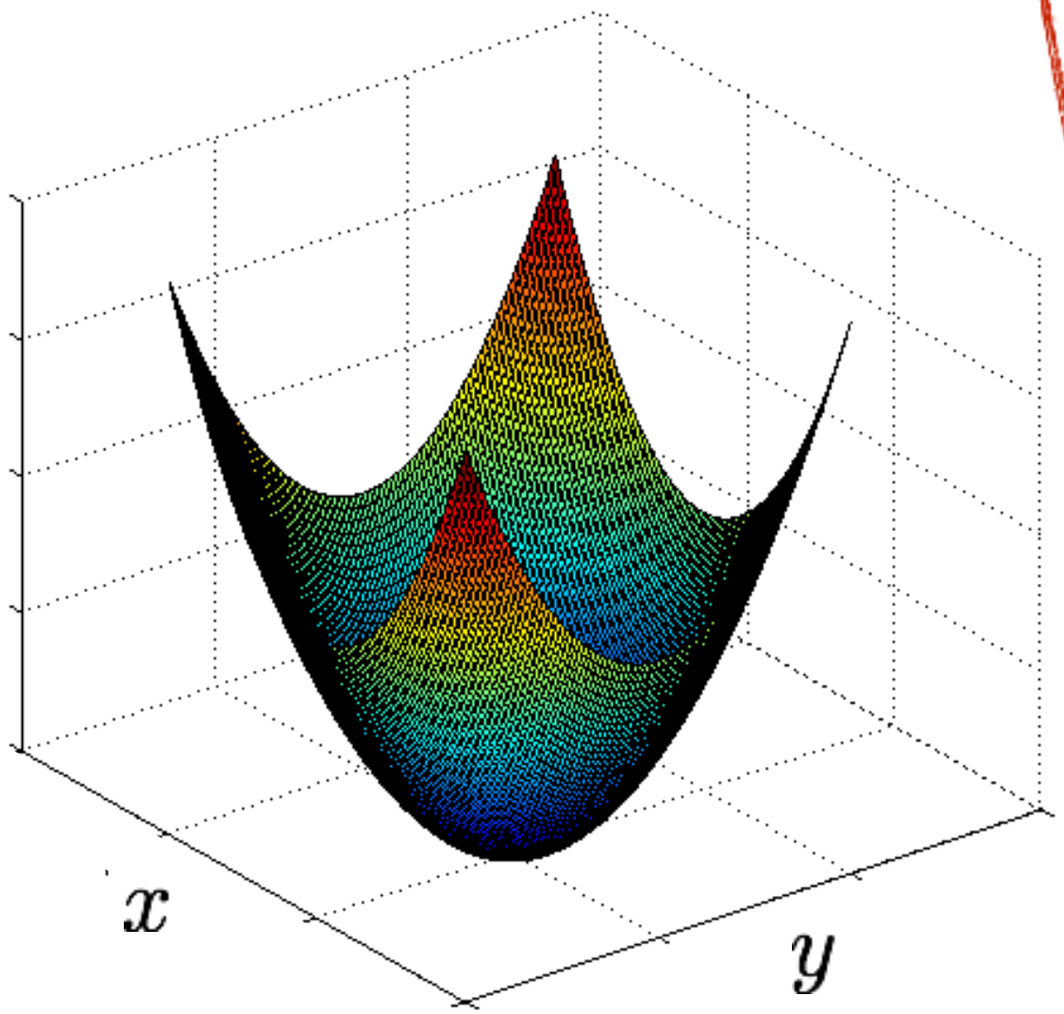
3. For each eigenvalue, solve $(M - \lambda I)e = 0$
   (returns eigenvectors)

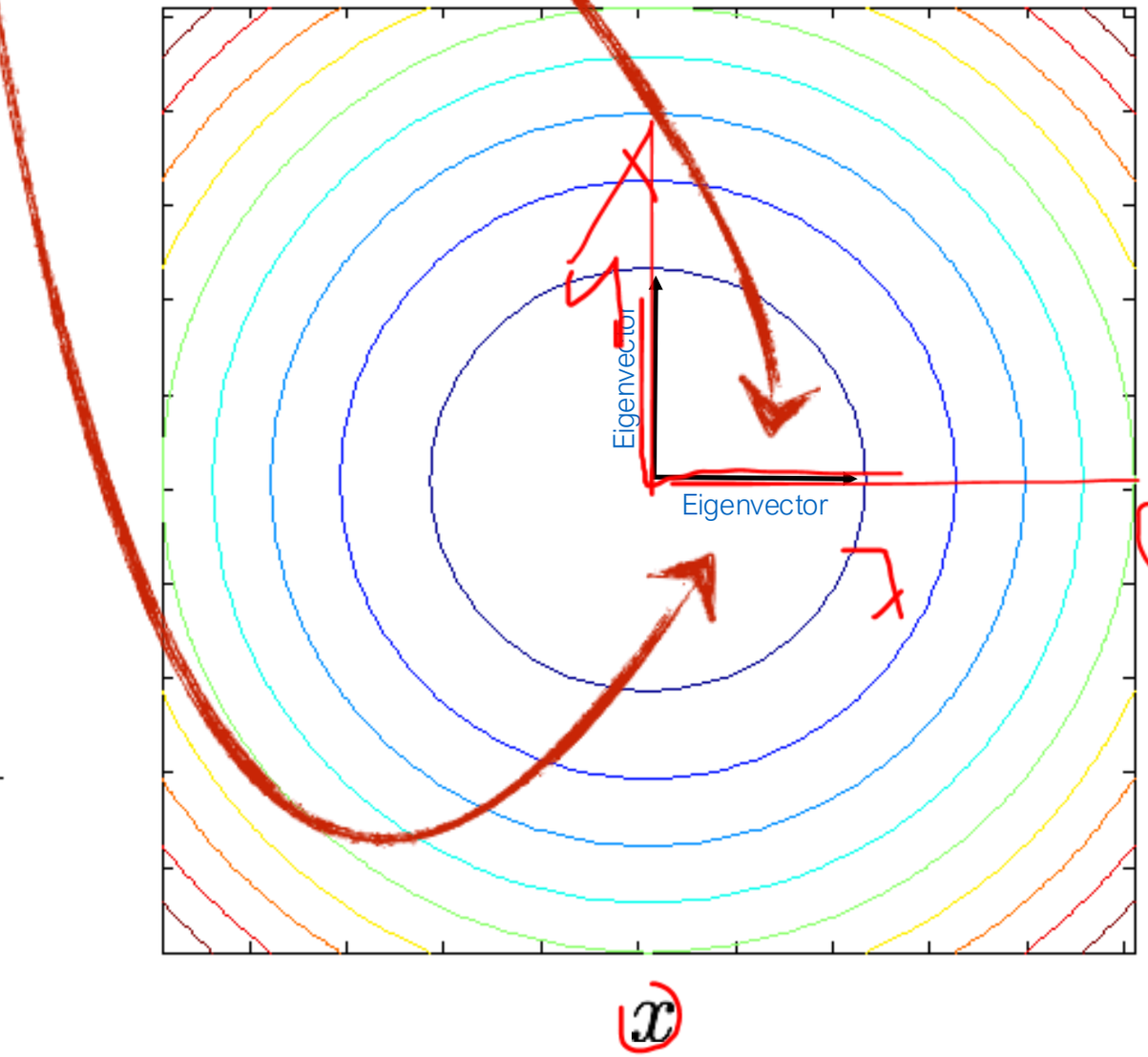
Eigenvectors Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

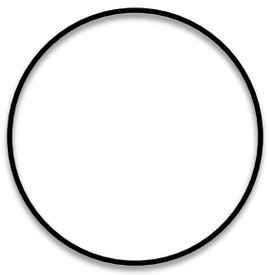
Eigenvectors

Inverse sqr of the size of the axis



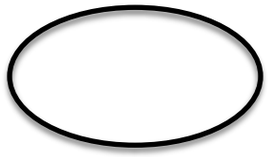


Recall:



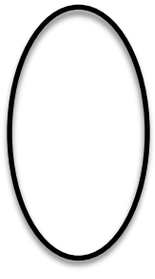
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **y** direction



$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **x** direction



$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

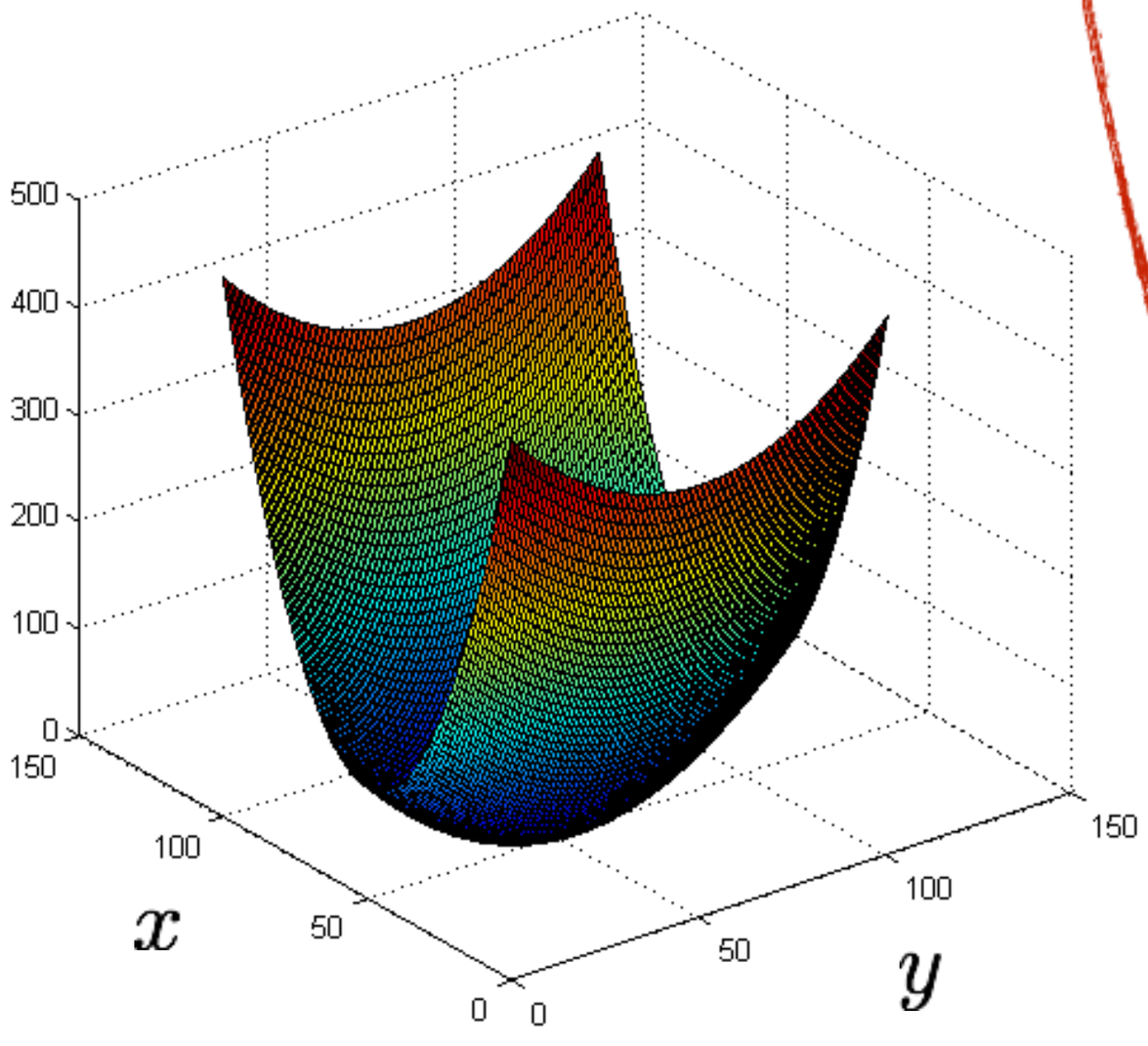
$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

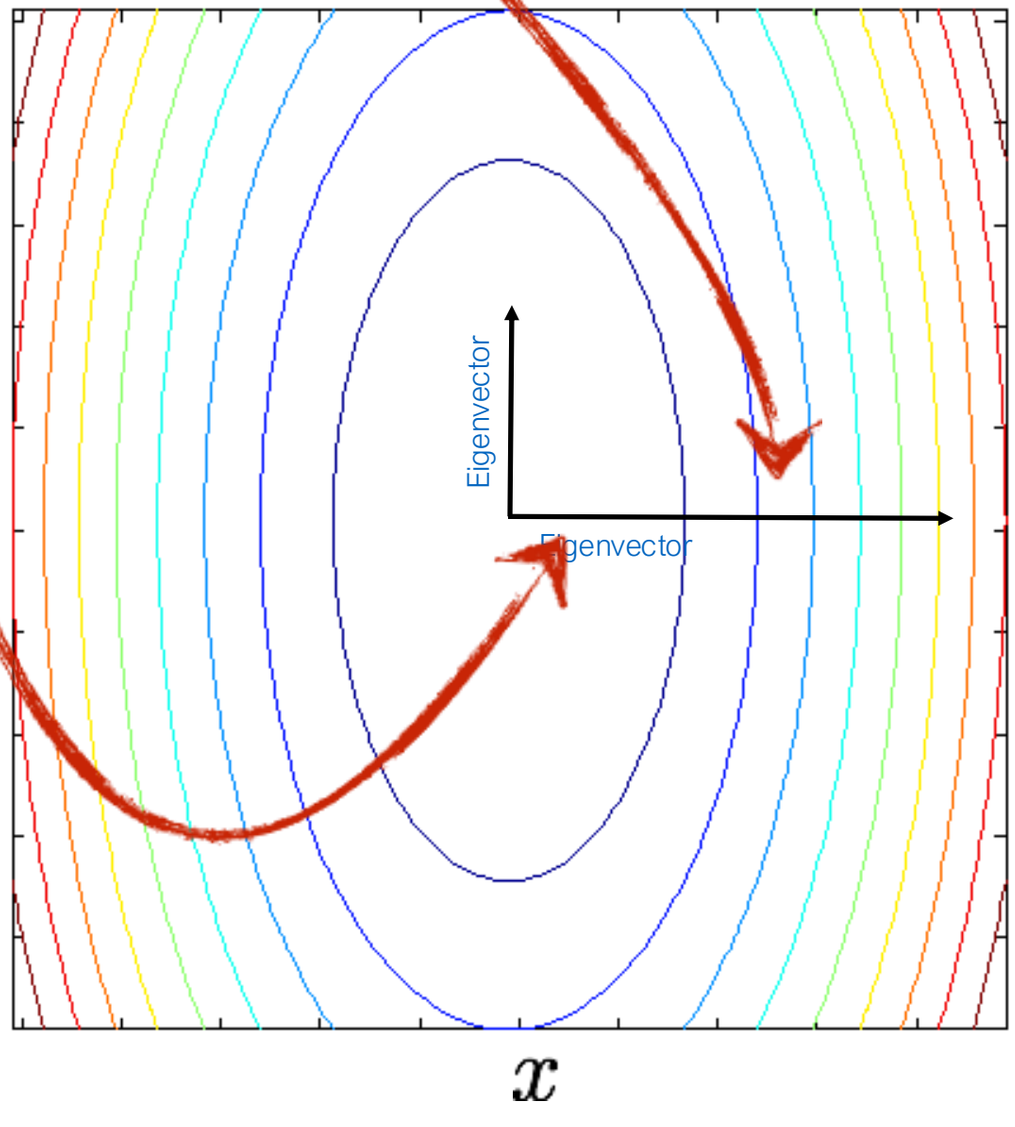
Eigenvalues

Eigenvectors

Eigenvectors

Inverse sqr of the size of the axis



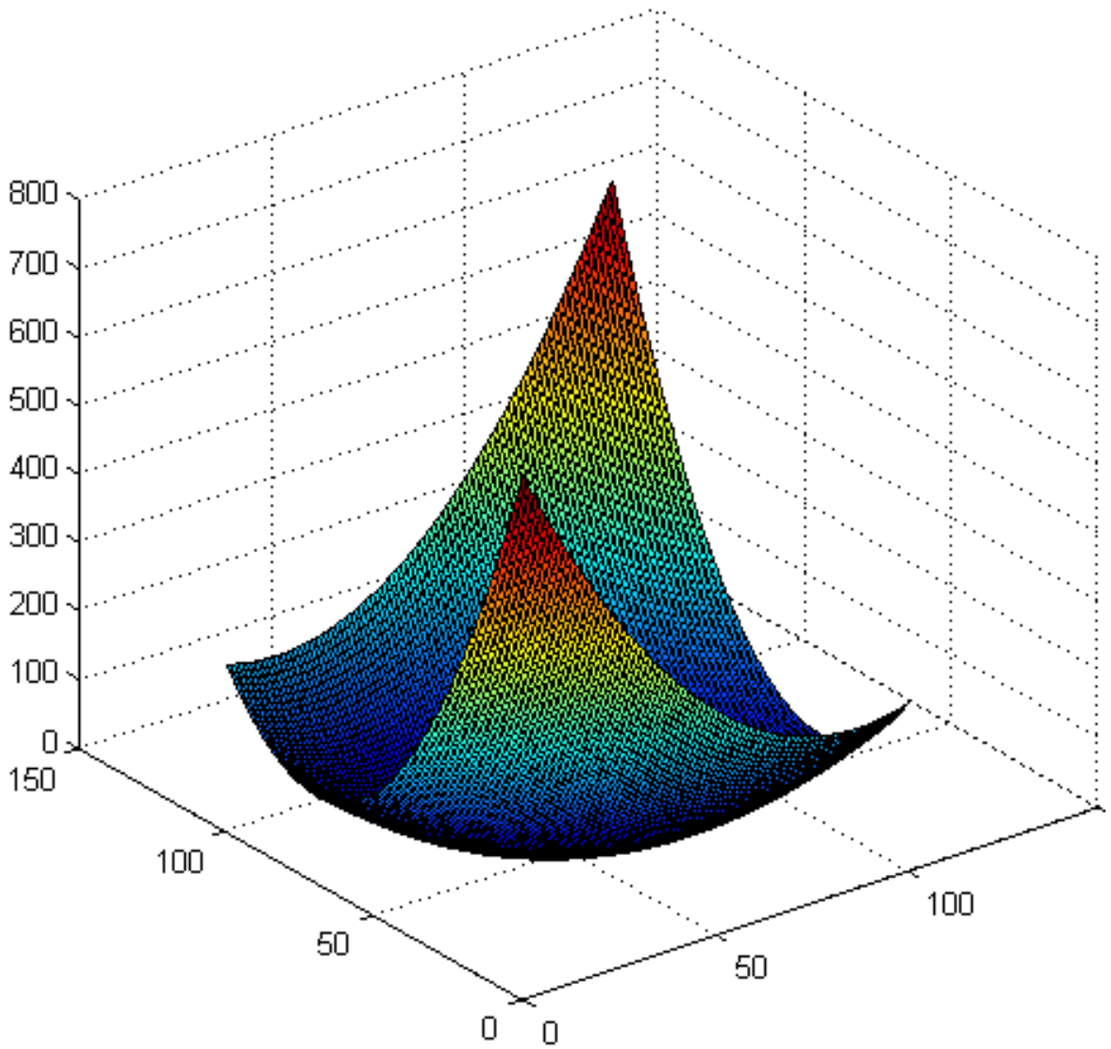


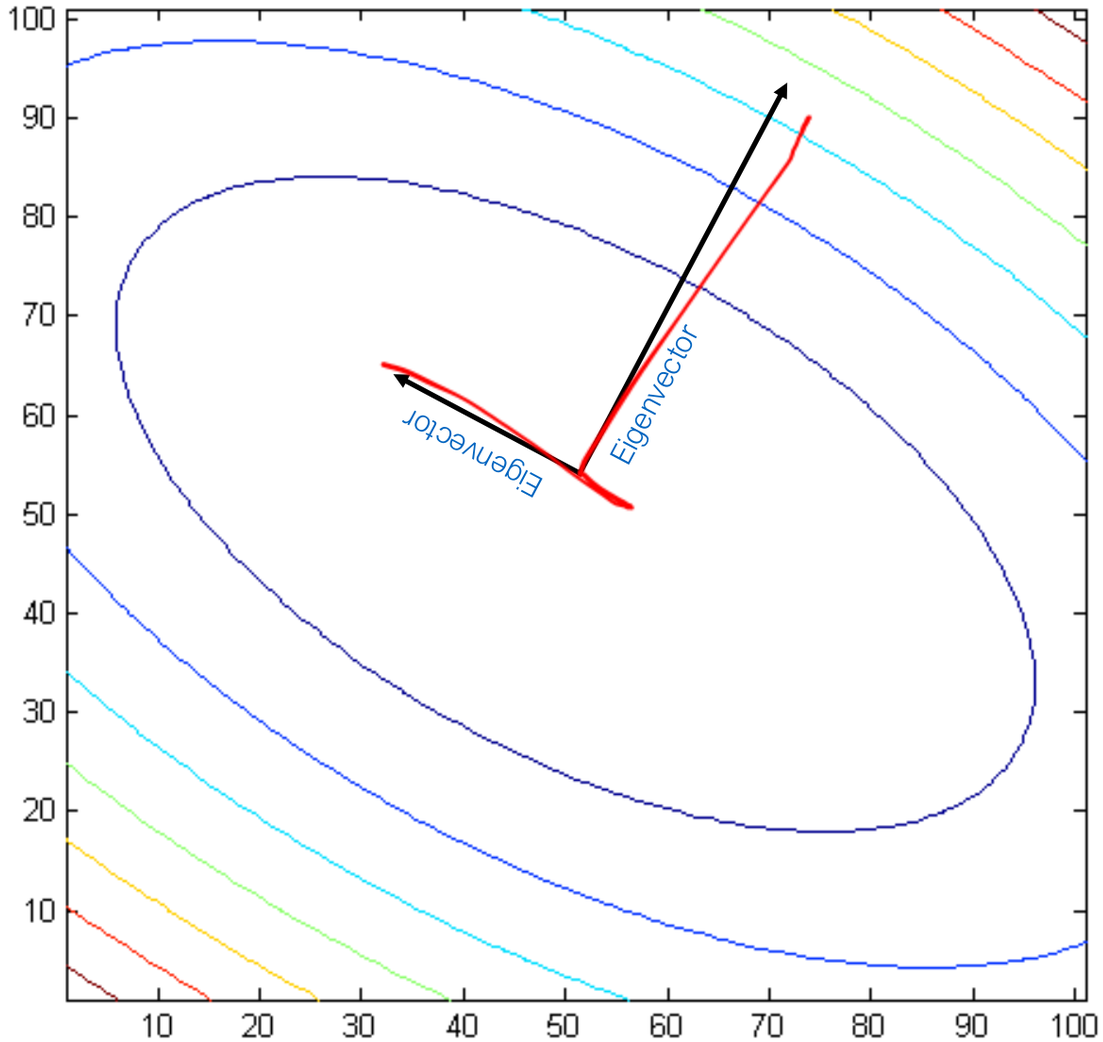
$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvalues

Eigenvectors

Eigenvectors



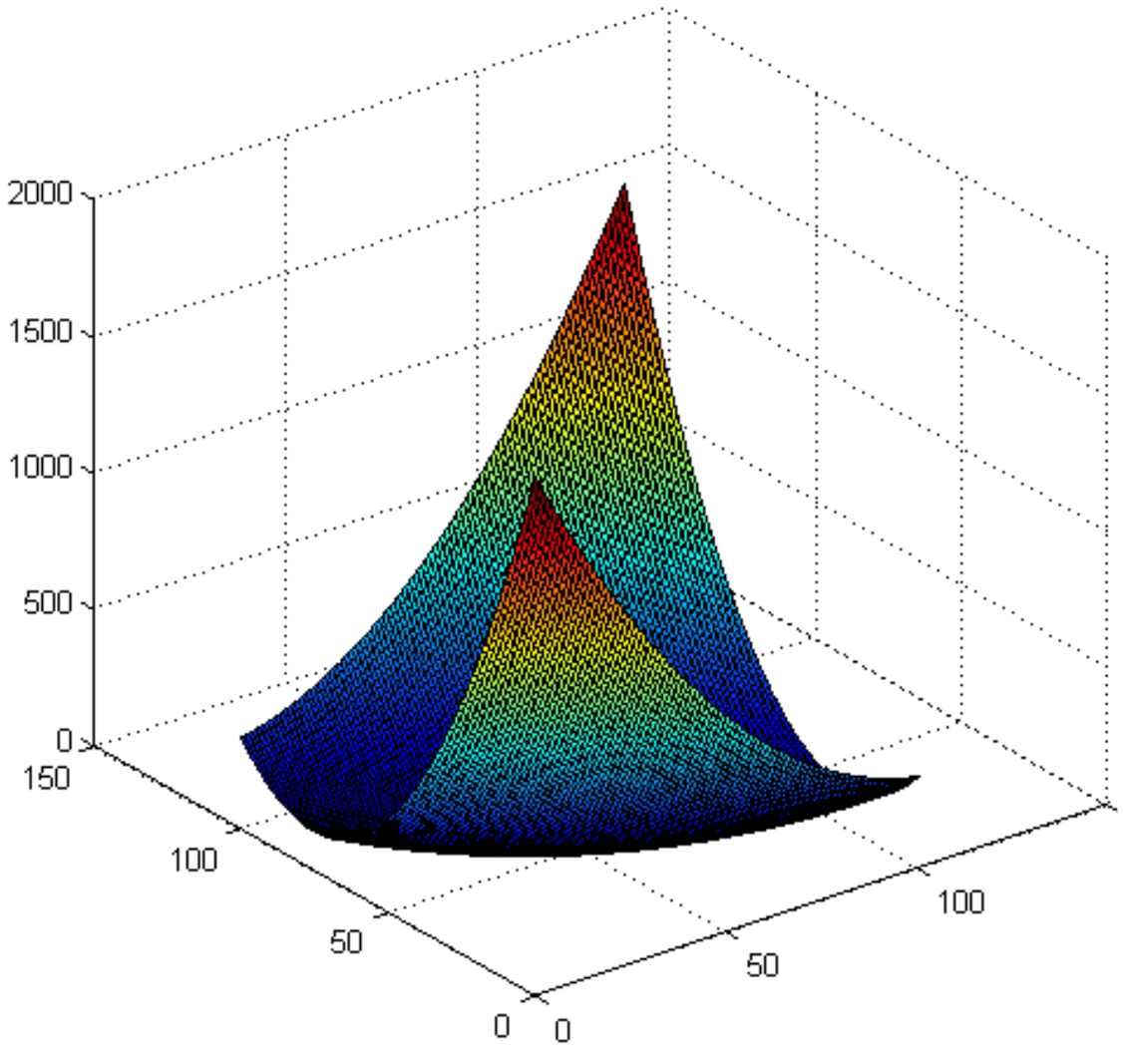


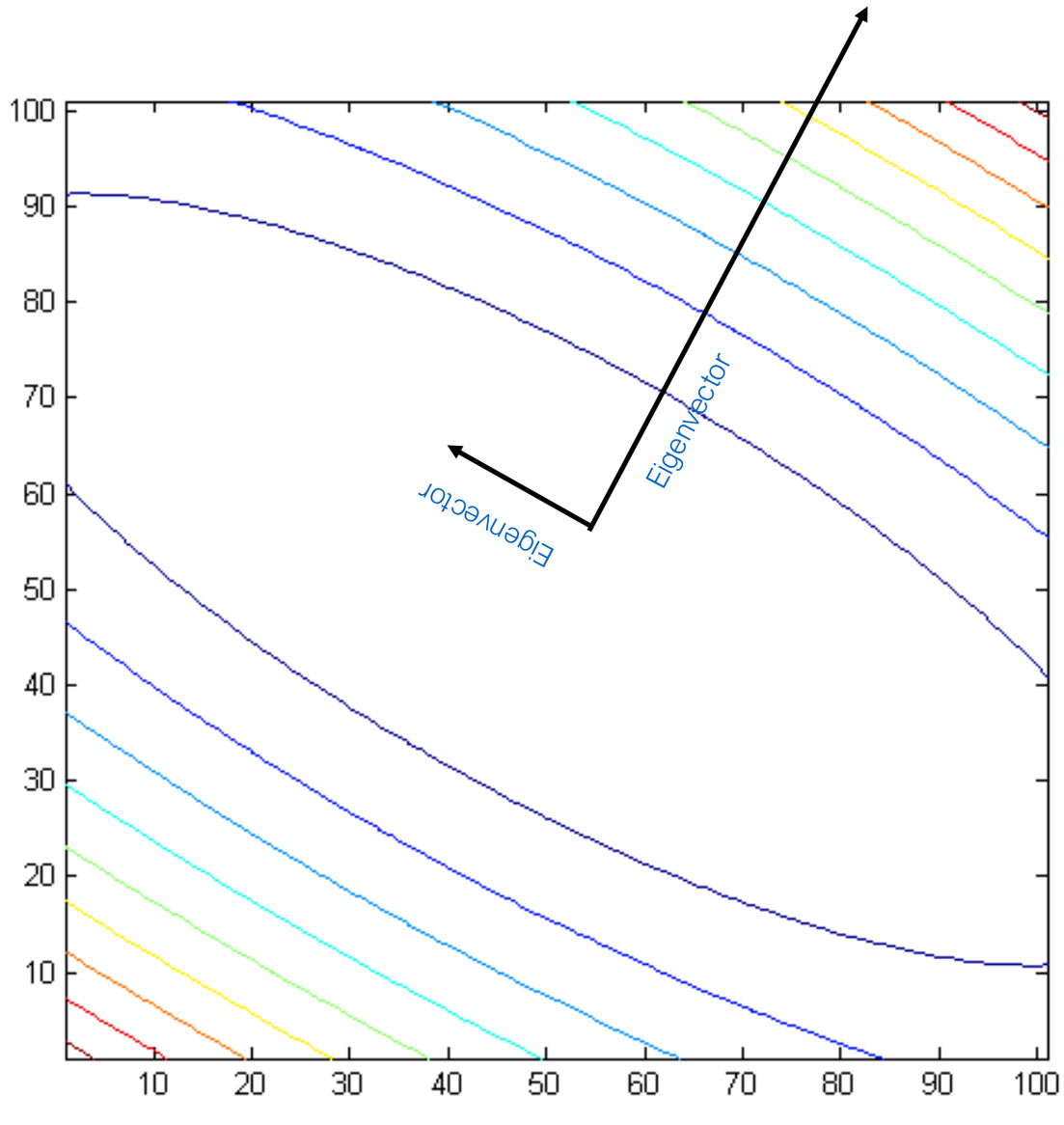
$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvalues

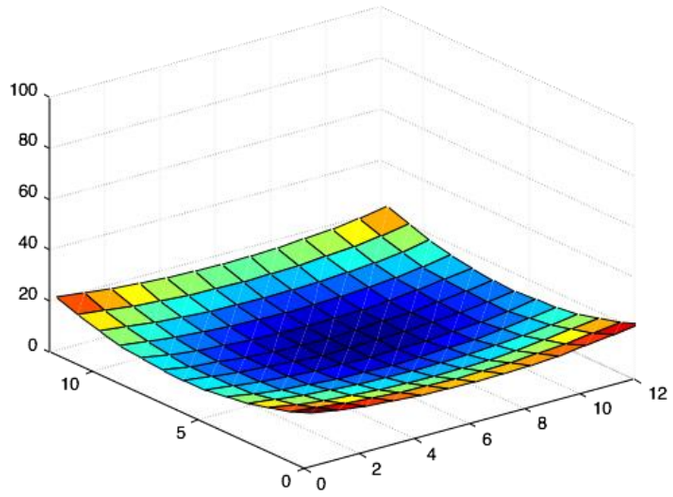
Eigenvectors

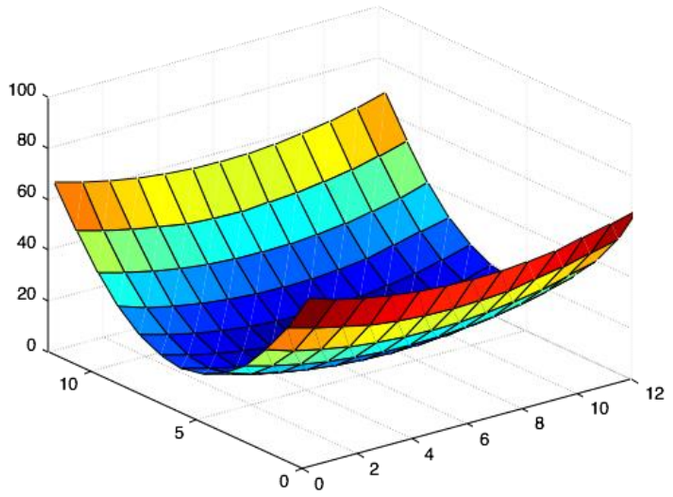
Eigenvectors

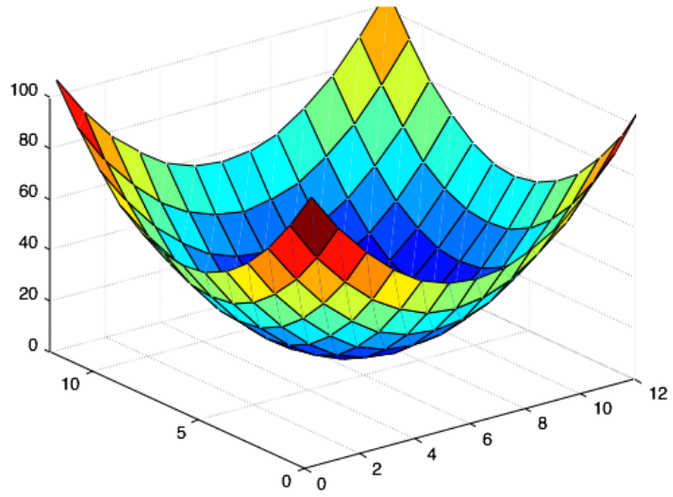




Which error surface indicates a good image feature?

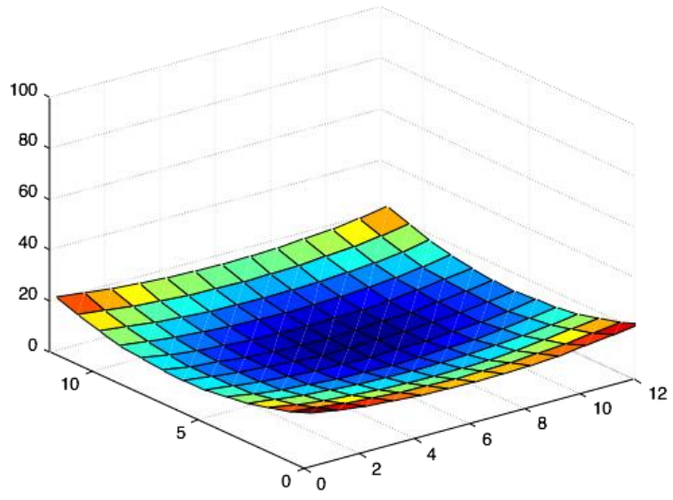




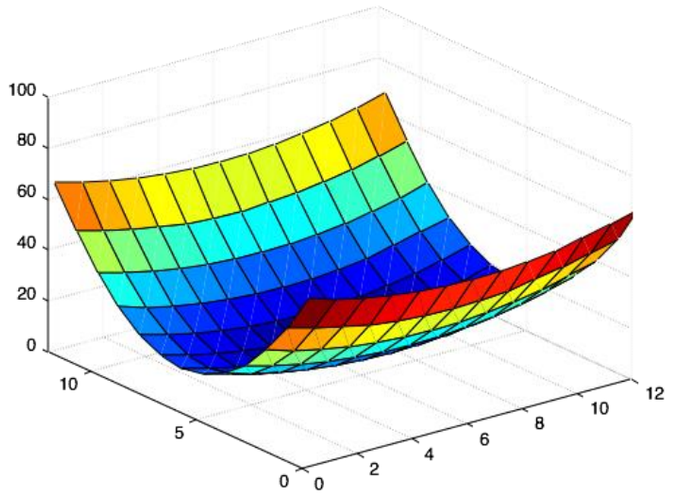


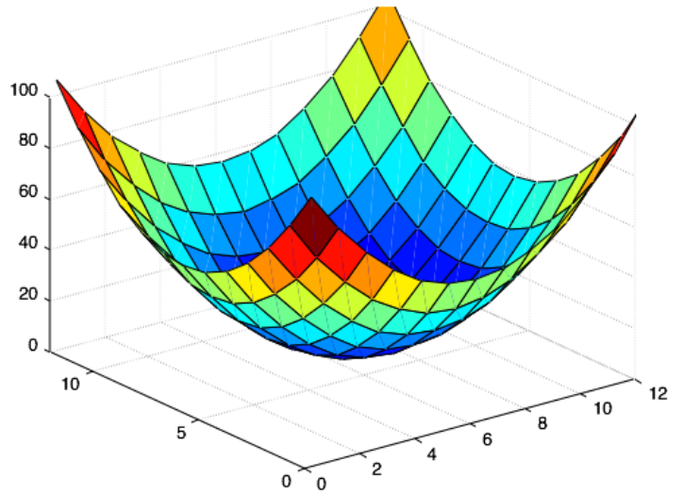
What kind of image patch do these surfaces represent?

Which error surface indicates a good image feature?

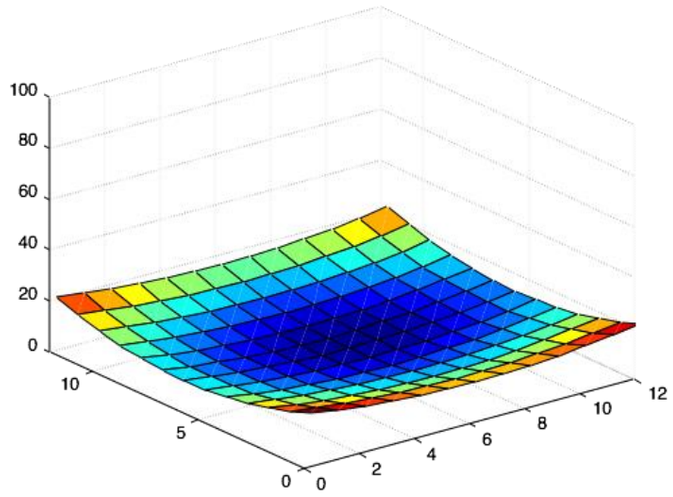


flat

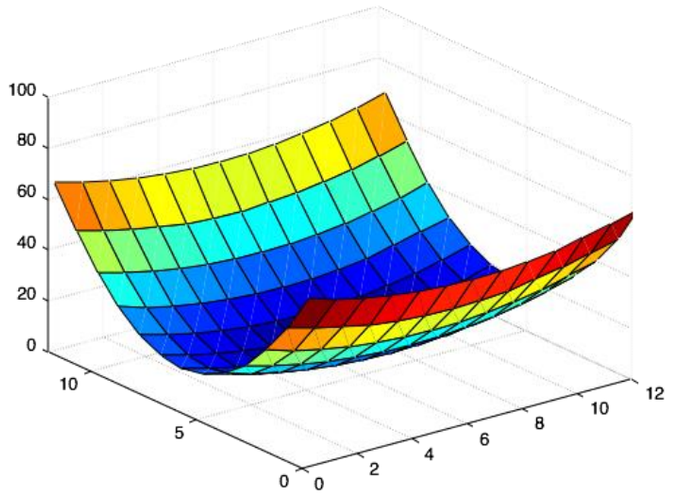




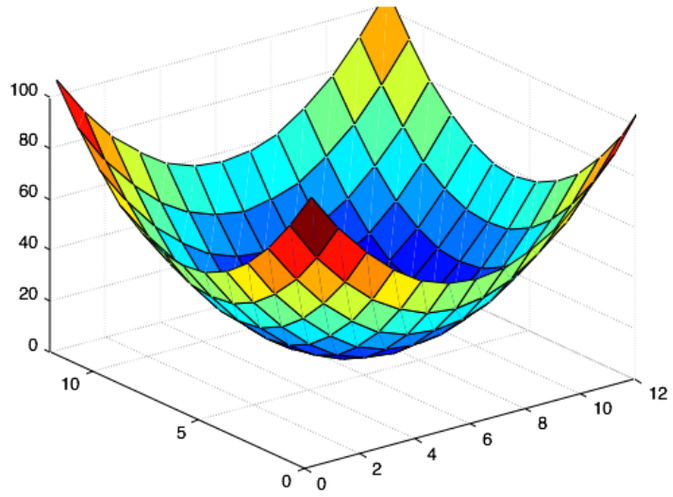
Which error surface indicates a good image feature?



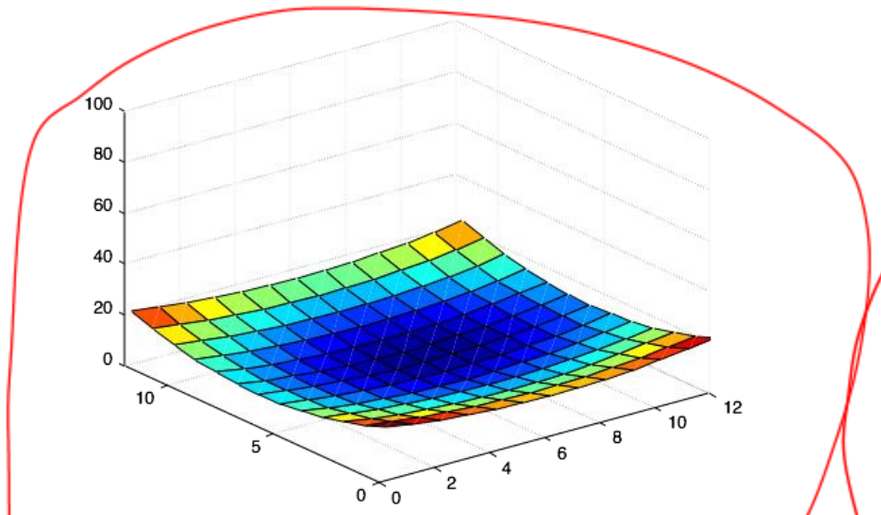
flat



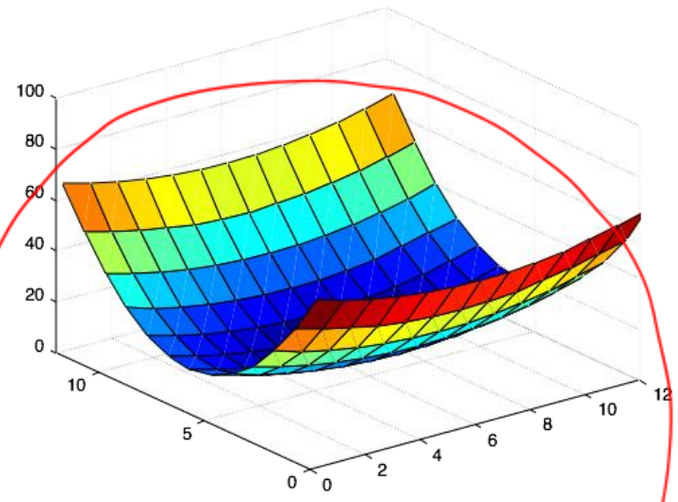
edge
‘line’



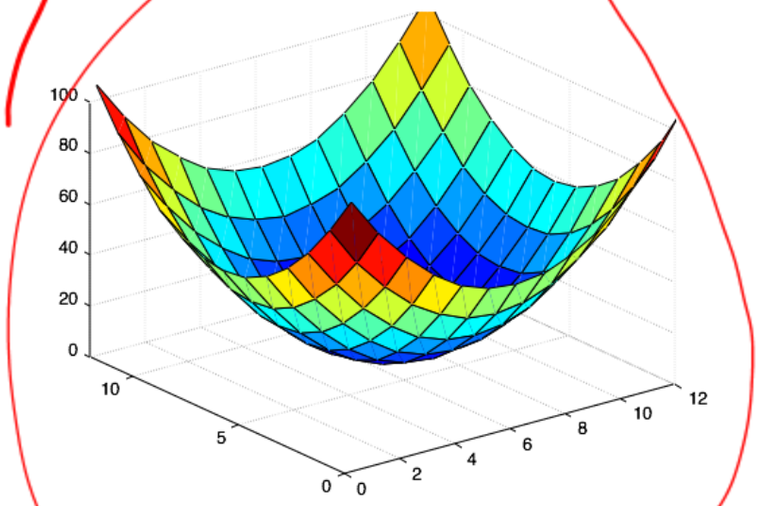
Which error surface indicates a good image feature?



flat



edge
‘line’



corner
‘dot’

Design a program to detect corners
(hint: use image gradients)

Finding corners

1. Compute image gradients over small region

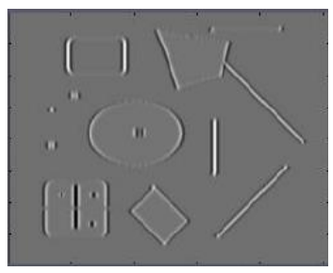
2. Subtract mean from each image gradient

3. Compute the covariance matrix

4. Compute eigenvectors and eigenvalues

5. Use threshold on eigenvalues to detect corners

$$I_x = \frac{\partial I}{\partial x}$$



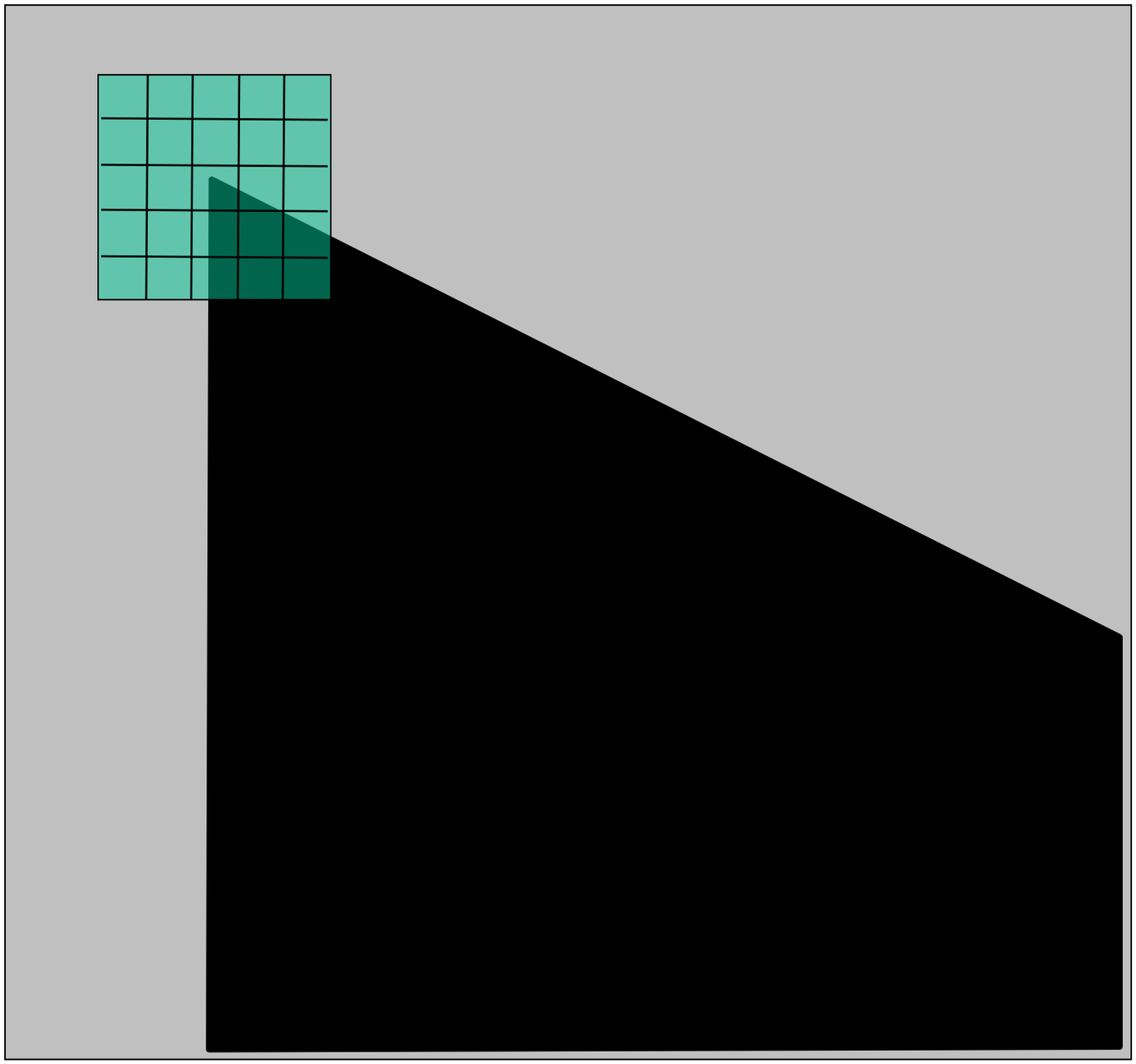
$$I_y = \frac{\partial I}{\partial y}$$

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

1. Compute image gradients over a small region (not just a single pixel)

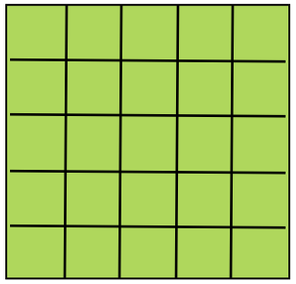
1. Compute image gradients over a small region

(not just a single pixel)



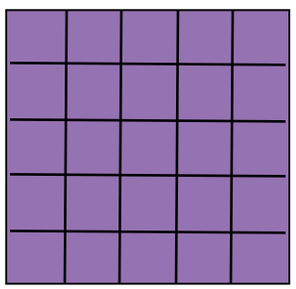
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$

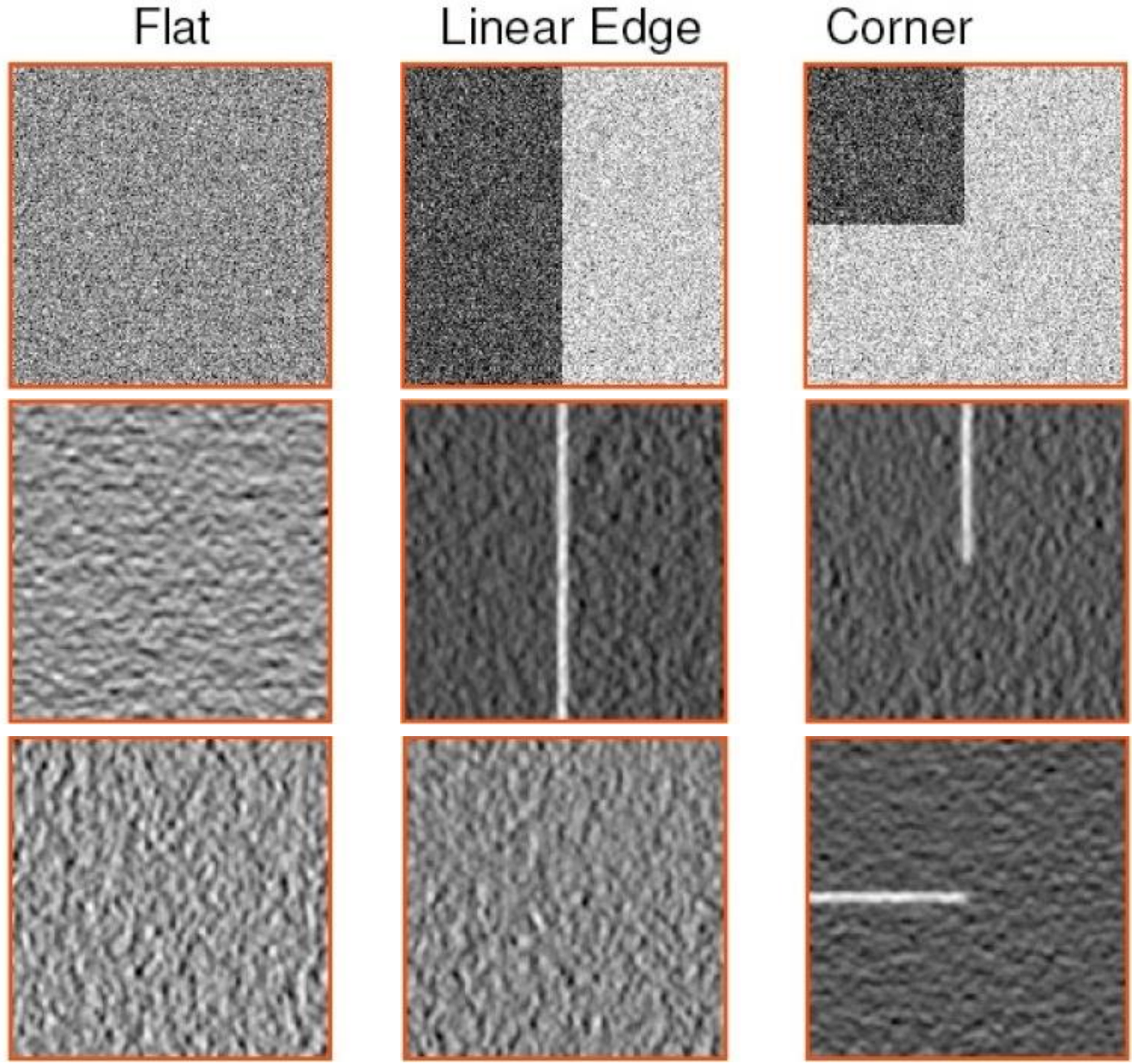


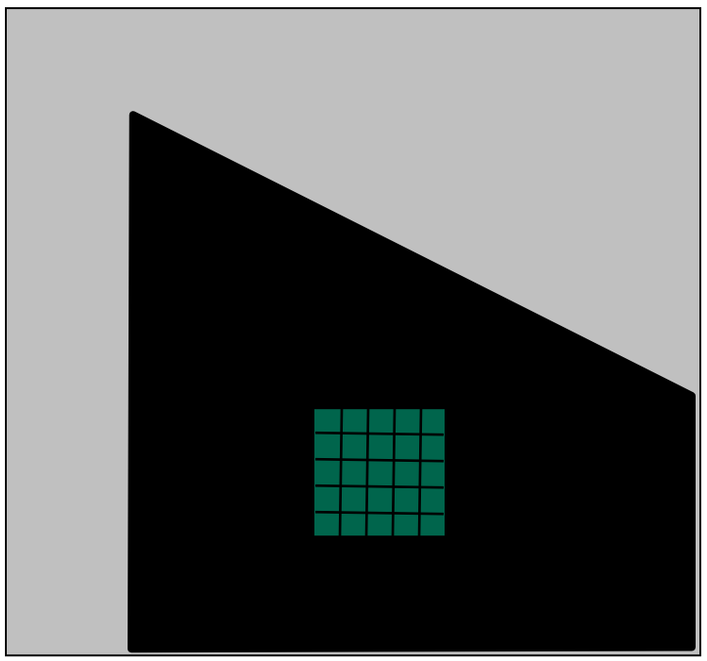
array of y gradients

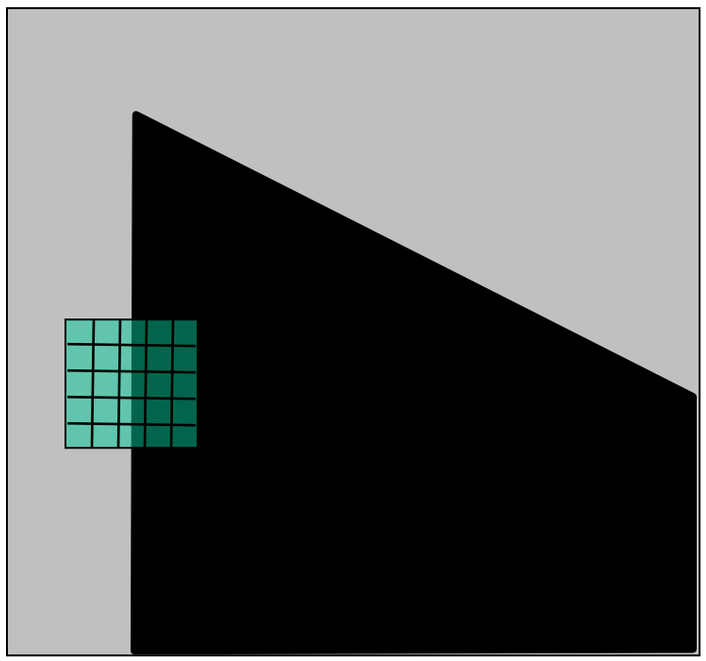
$$I_y = \frac{\partial I}{\partial y}$$

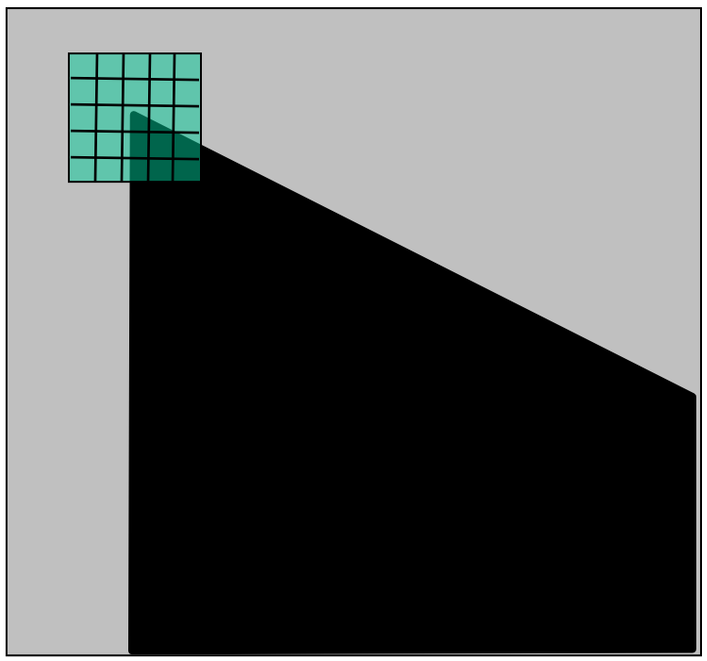


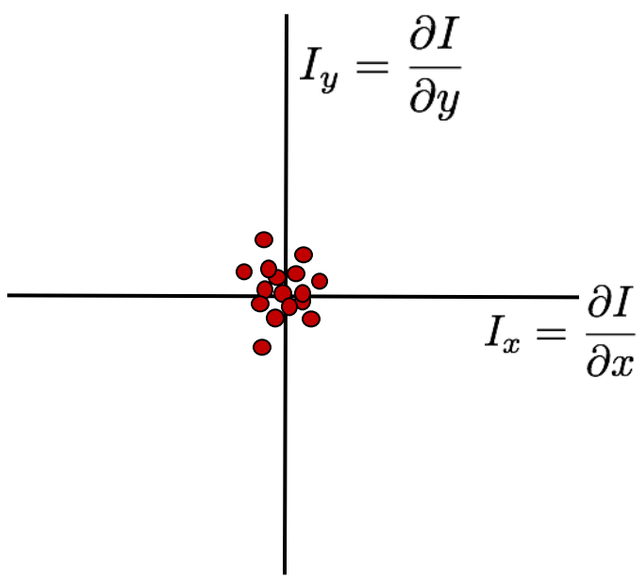
visualization of gradients

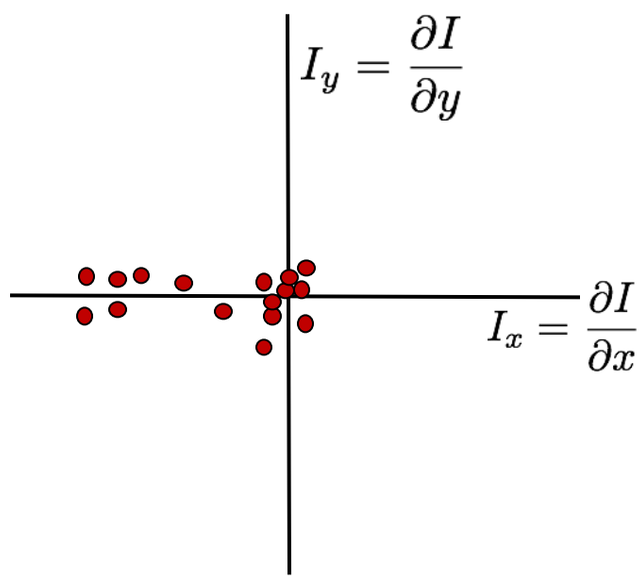


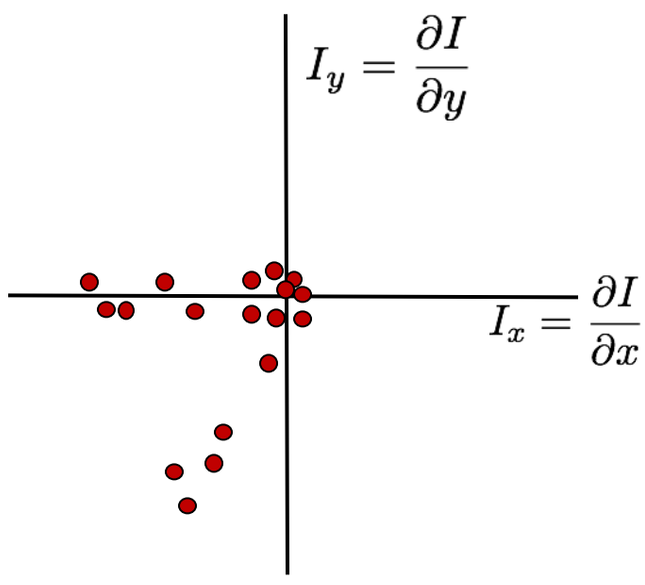




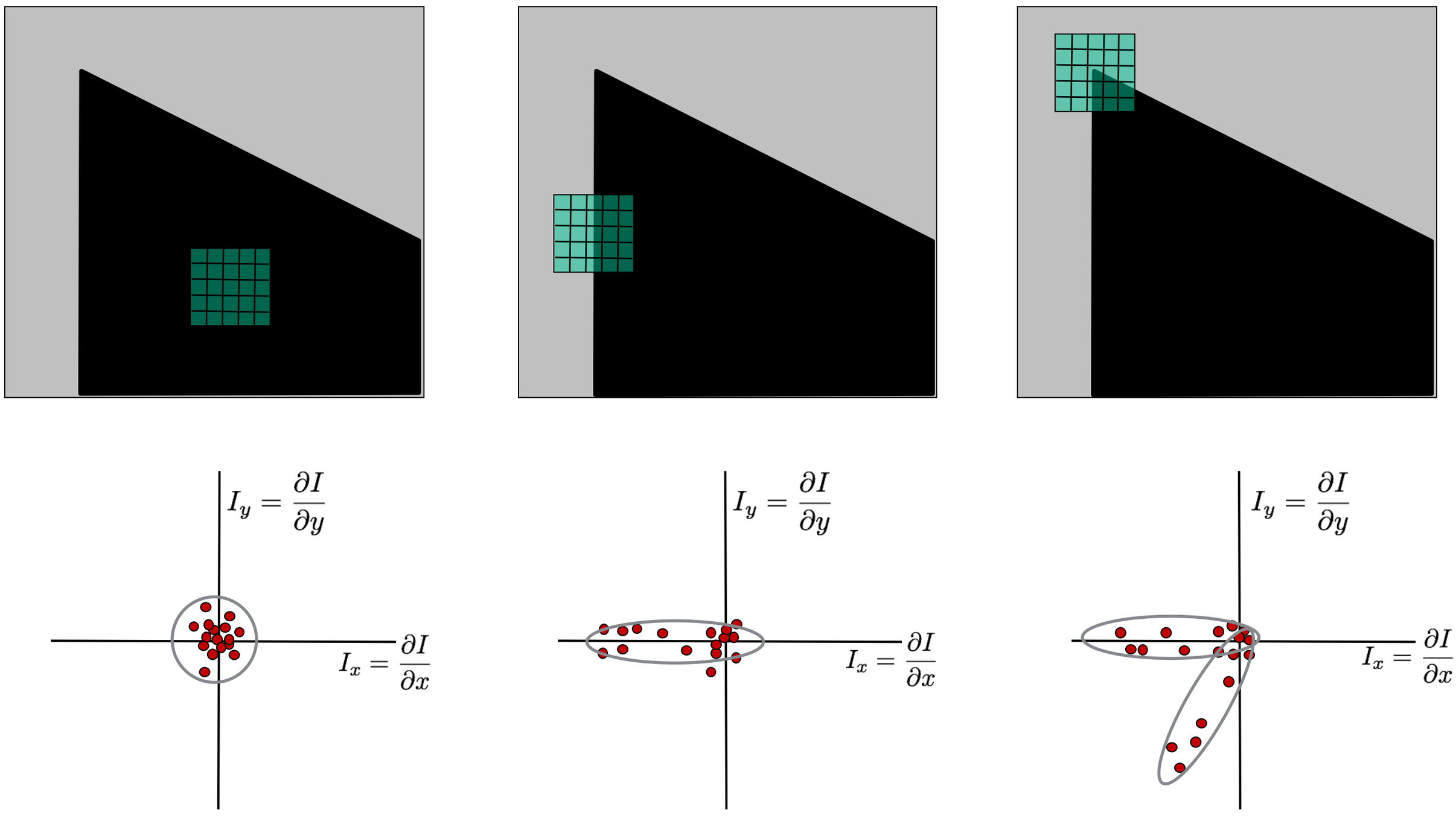




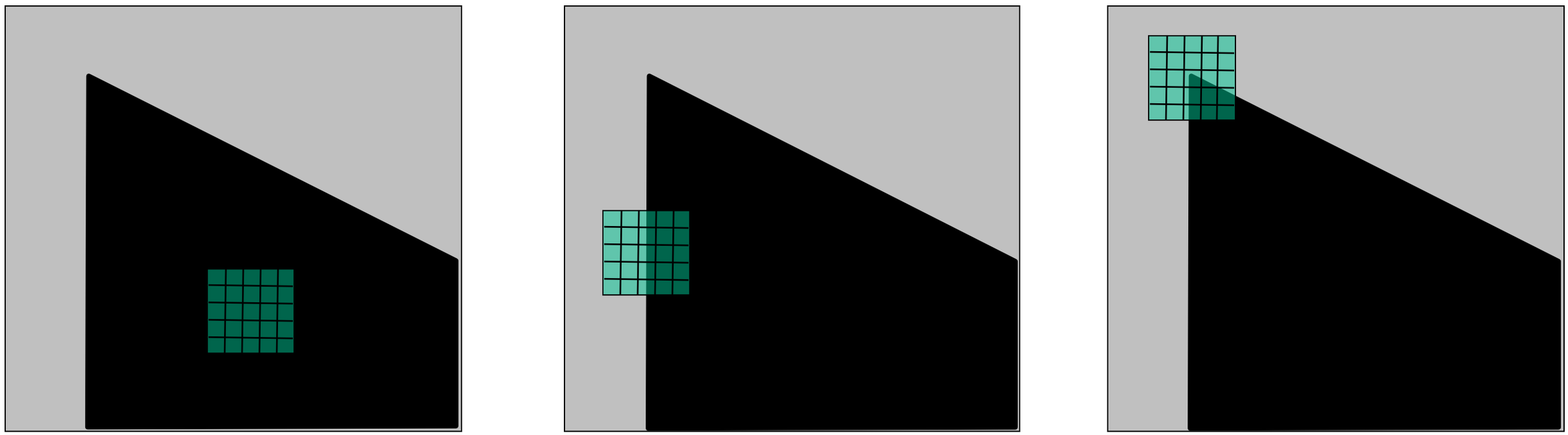


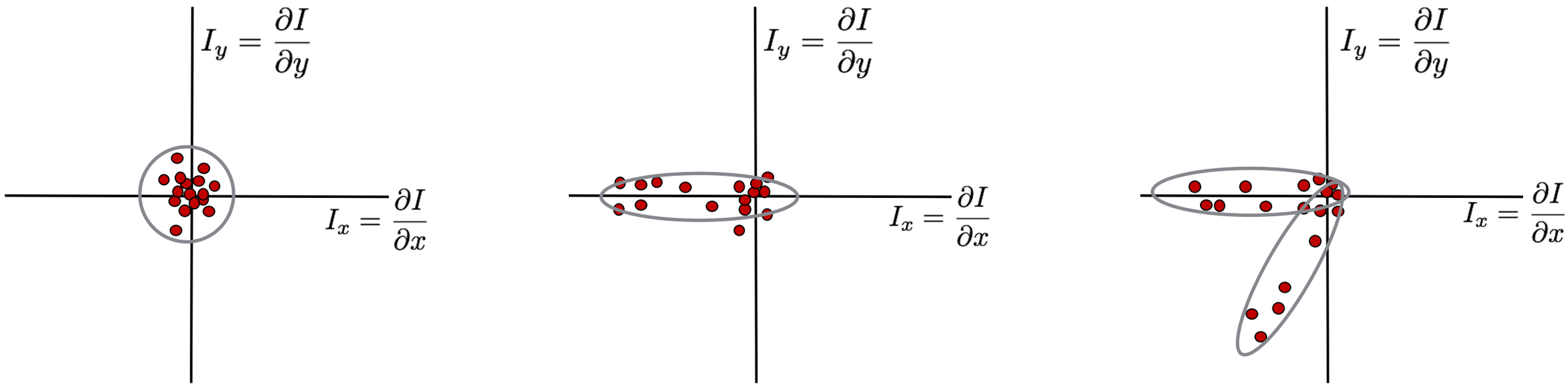


What does the distribution tell you about the region?



distribution reveals edge orientation and magnitude



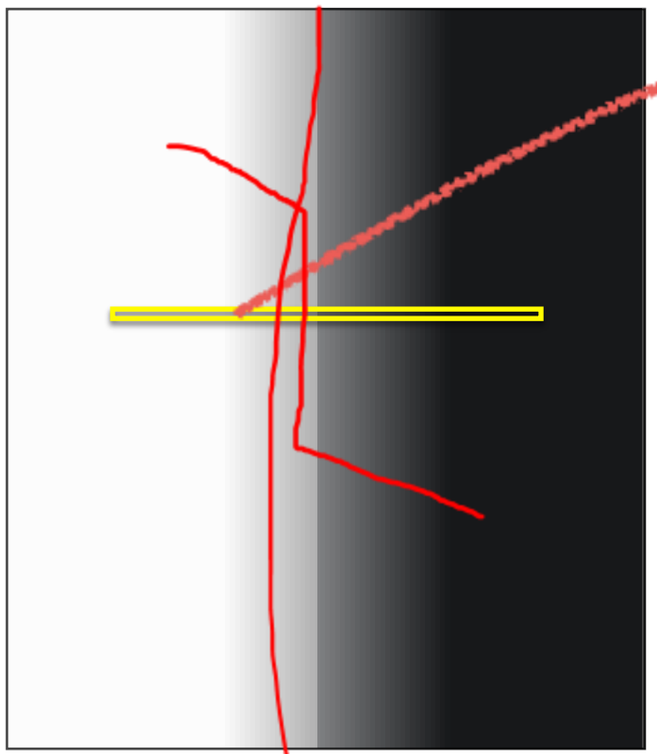


How do you quantify orientation and magnitude?

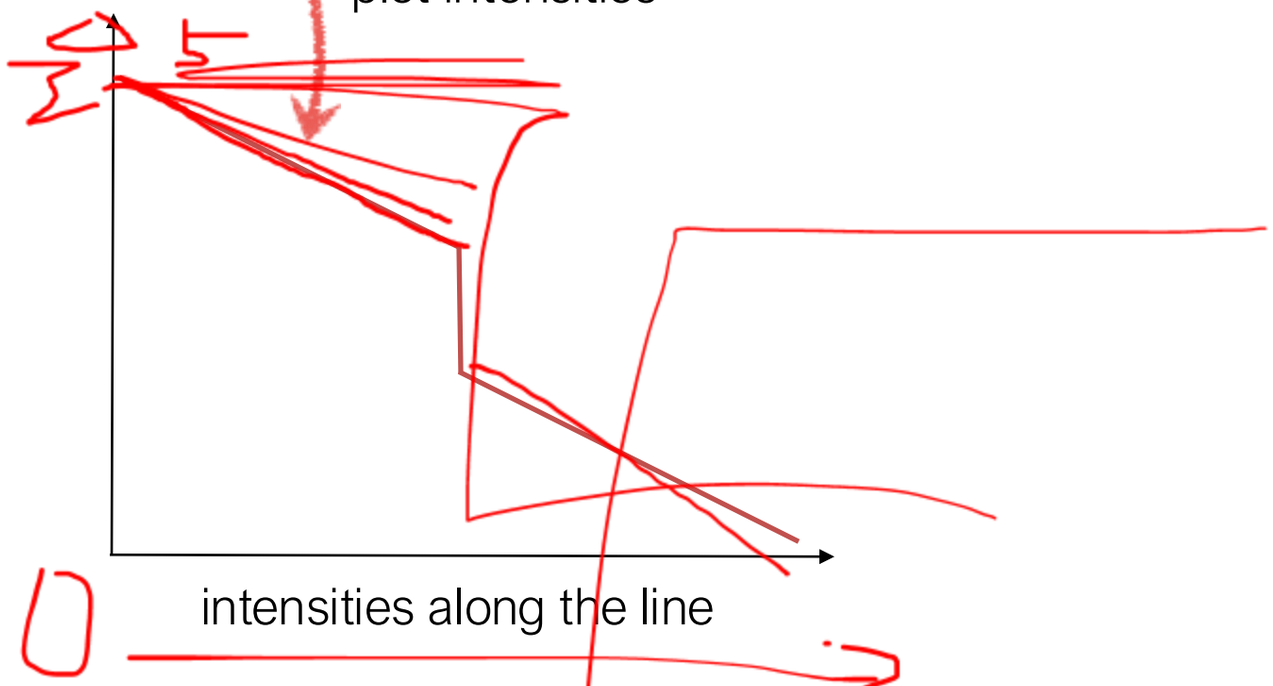
2. Subtract the mean from each image gradient

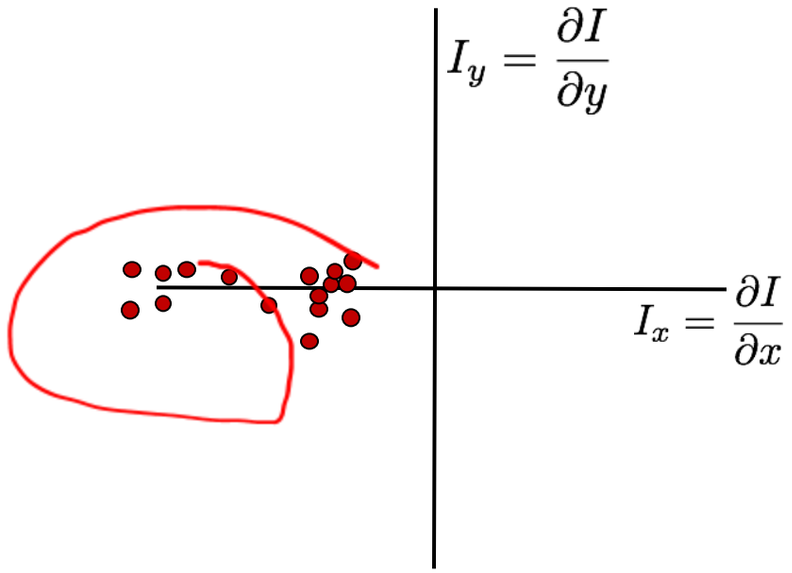
2. Subtract the mean from each image gradient

constant intensity gradient



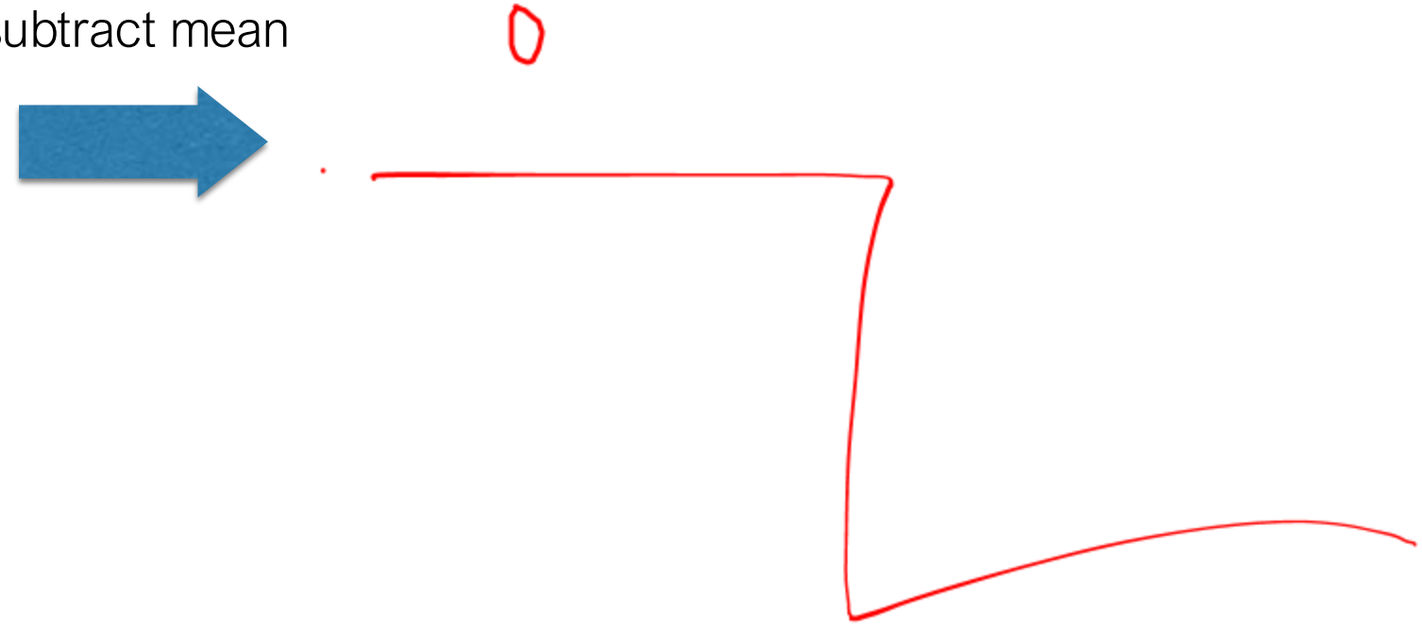
plot intensities





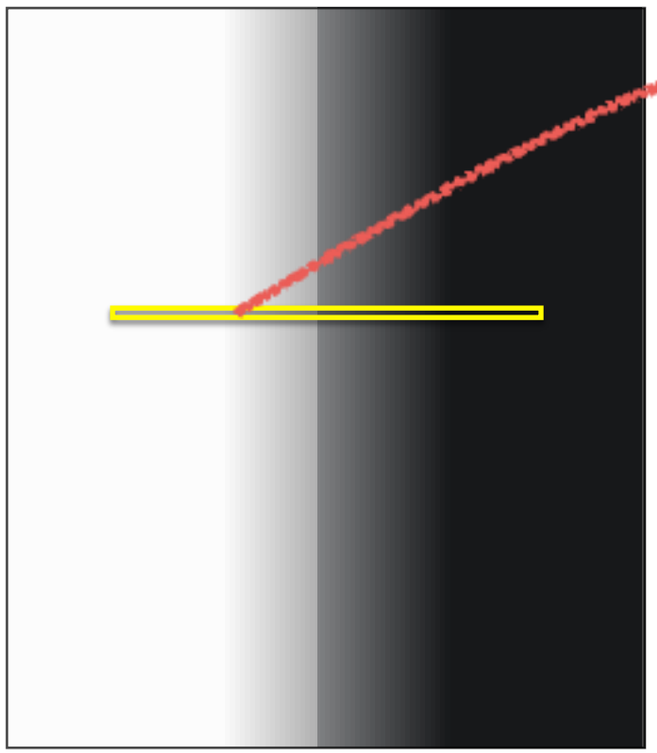
plot of image gradients

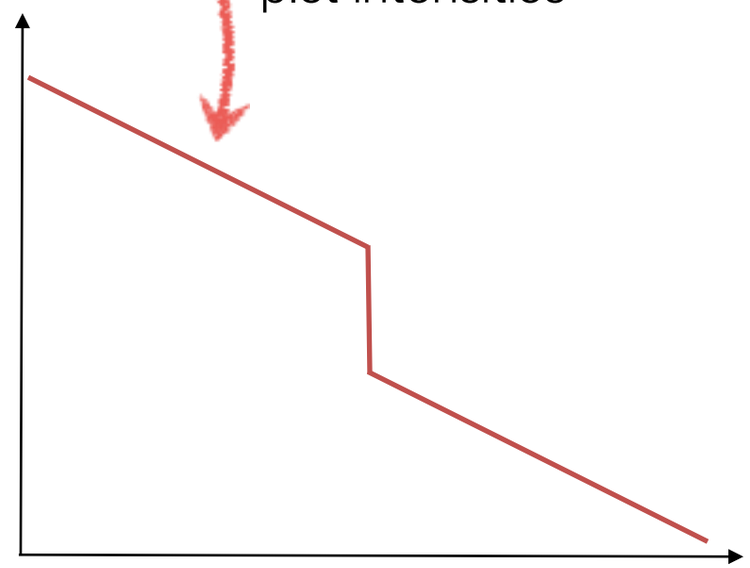
subtract mean

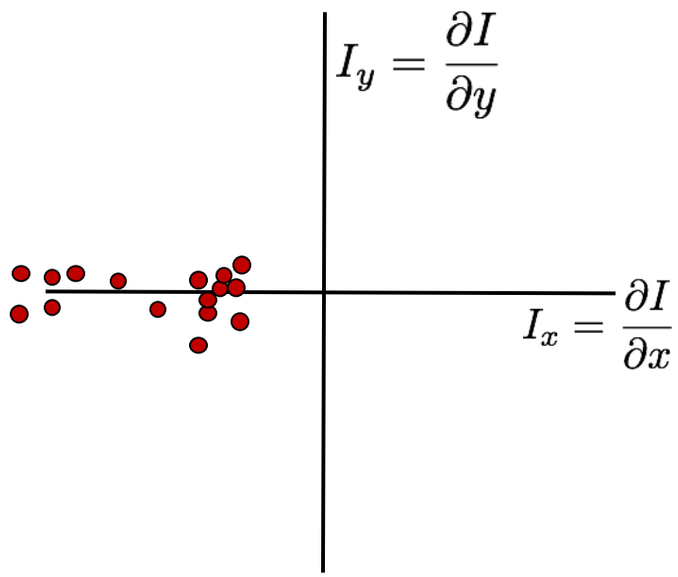


2. Subtract the mean from each image gradient

constant intensity gradient

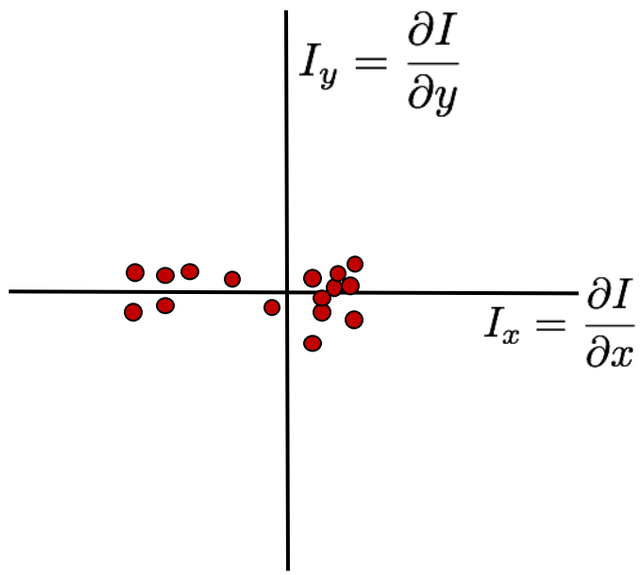






plot of image gradients

subtract mean



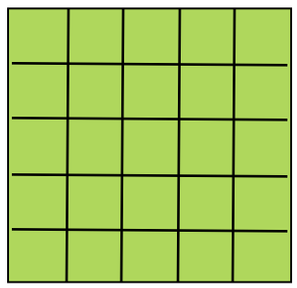
data is centered
('DC' offset is removed)

3. Compute the covariance matrix

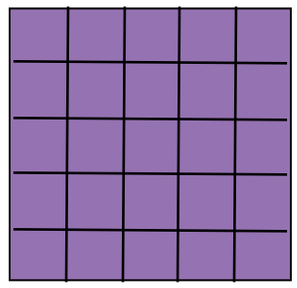
3. Compute the covariance matrix

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

$$\sum_{p \in P} I_x I_y = \text{sum}\Big(\underbrace{\quad}_{I_x = \frac{\partial I}{\partial x}} * \underbrace{\quad}_{I_y = \frac{\partial I}{\partial y}} \Big)$$



array of x gradients

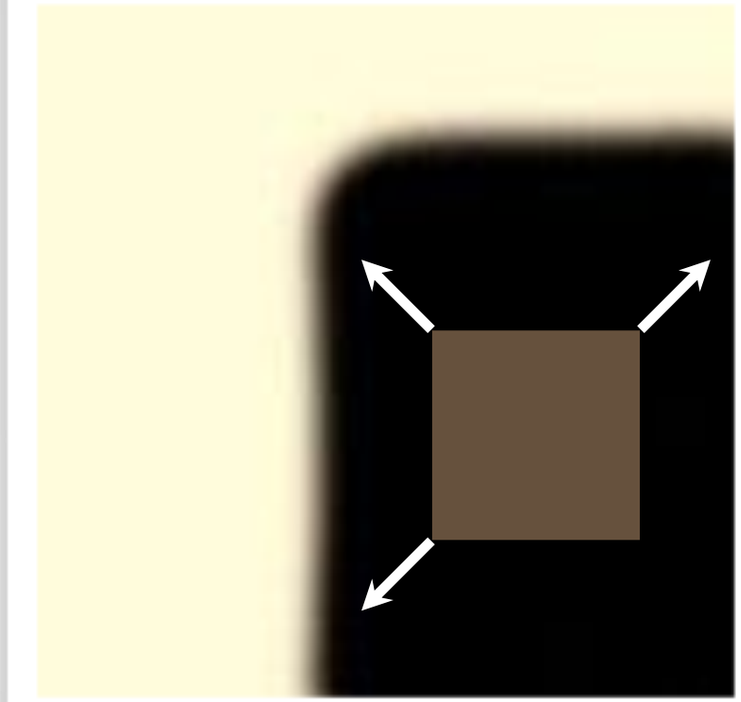


array of y gradients

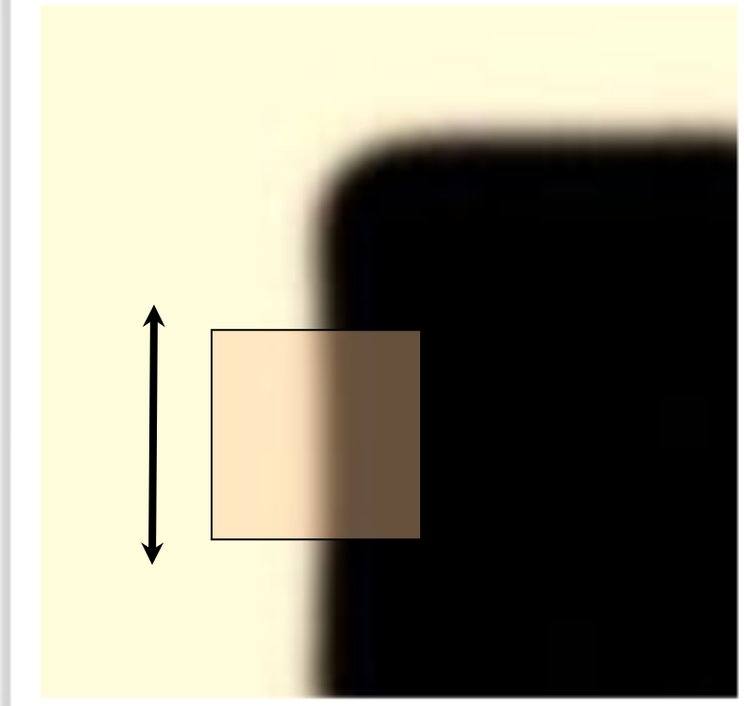
Where does this covariance matrix come from?

Easily recognized by looking through a small window

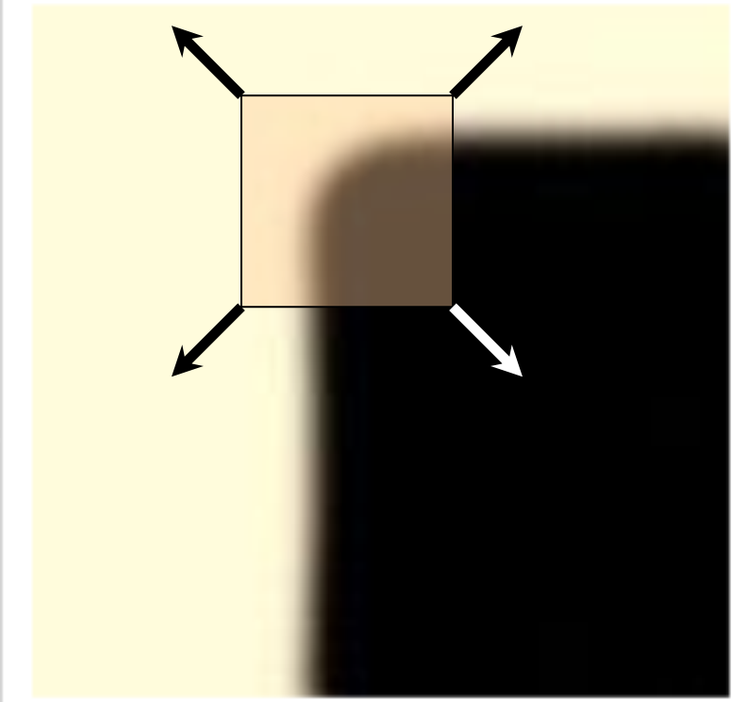
Shifting the window should give large change in intensity



“flat” region:
no change in all directions



“edge”:
no change along the edge direction



“corner”:
significant change in all directions

[Moravec 1980]

Some mathematical background…

Error function

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

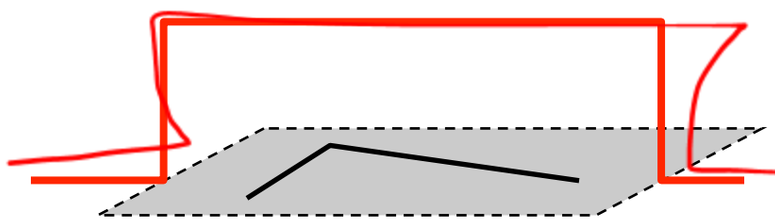
Error function

Window function

Shifted intensity

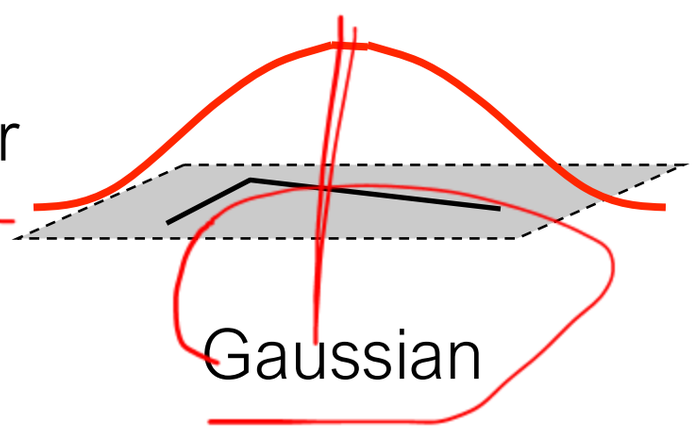
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

Error function approximation

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

First-order Taylor expansion of I(x,y) about (0,0)
(bilinear approximation for small shifts)

Bilinear approximation

For small shifts $[u,v]$ we have a 'bilinear approximation':

Change in appearance for a shift [u,v]

$$E(u,v) \cong \begin{bmatrix} u,v \end{bmatrix} \; M \; \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

'second moment' matrix
'structure tensor'

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

By computing the gradient covariance matrix…

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

we are fitting a quadratic to the gradients over a small image region

4. Compute eigenvalues and eigenvectors

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$

(returns a polynomial)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$
(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
(returns eigenvalues)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$Me = \lambda e$$

eigenvector

$$(M - \lambda I)e = 0$$

1. Compute the determinant of $M - \lambda I$
(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
(returns eigenvalues)

3. For each eigenvalue, solve $(M - \lambda I)e = 0$
(returns eigenvectors)

eig(M)

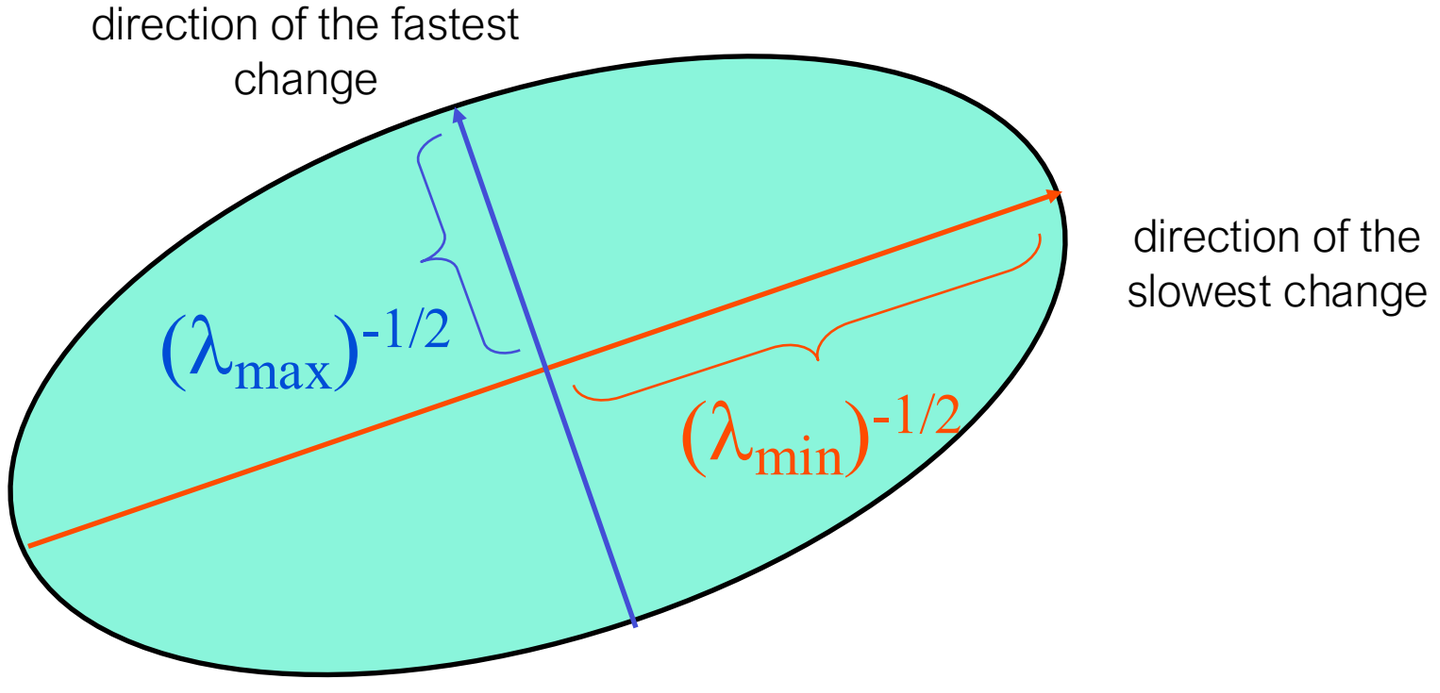
Visualization as an ellipse

Since M is symmetric, we have $M = R^{-1}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}R$

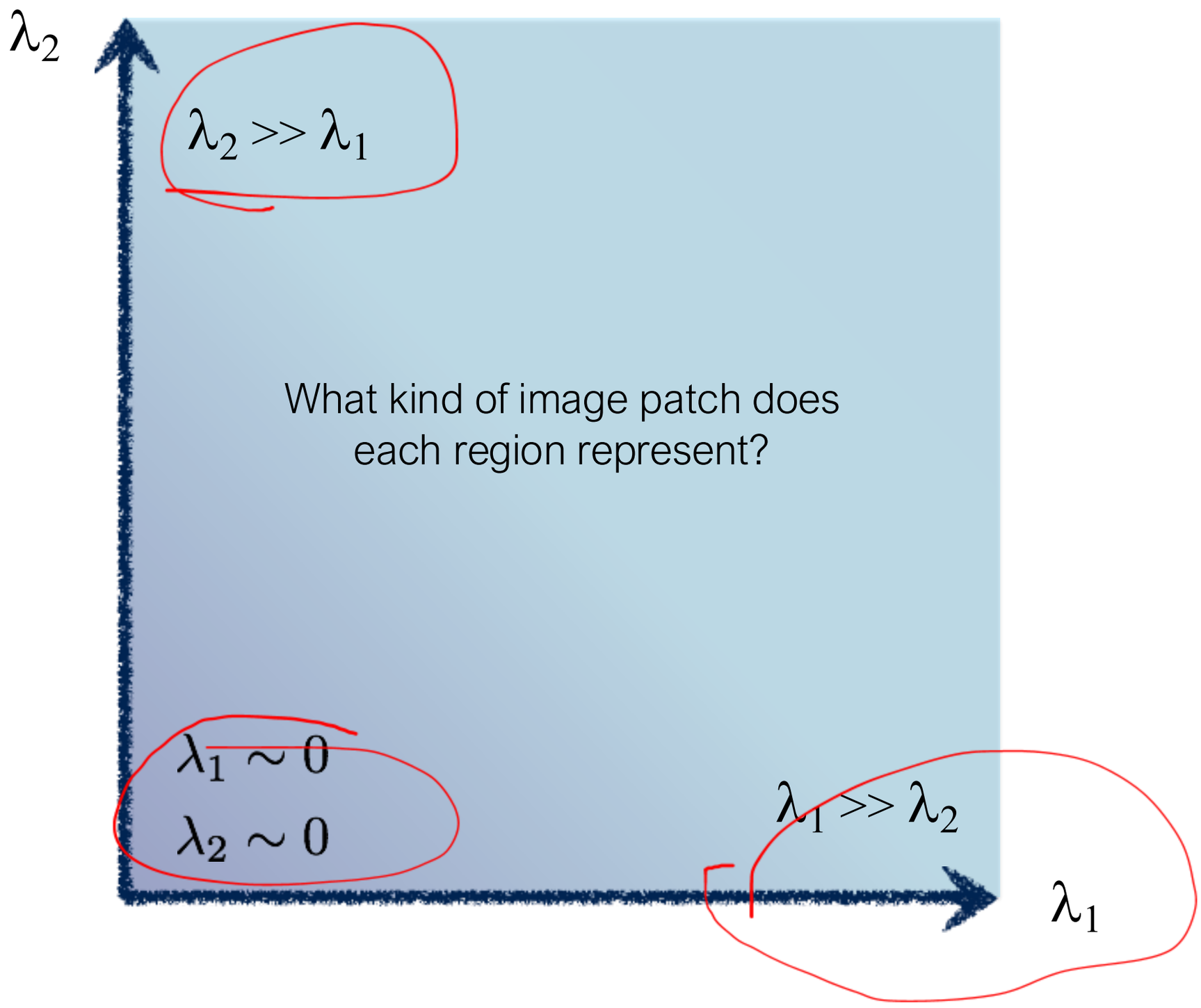
We can visualize ***M*** as an ellipse with axis lengths determined by the eigenvalues and orientation determined by ***R***

Ellipse equation:

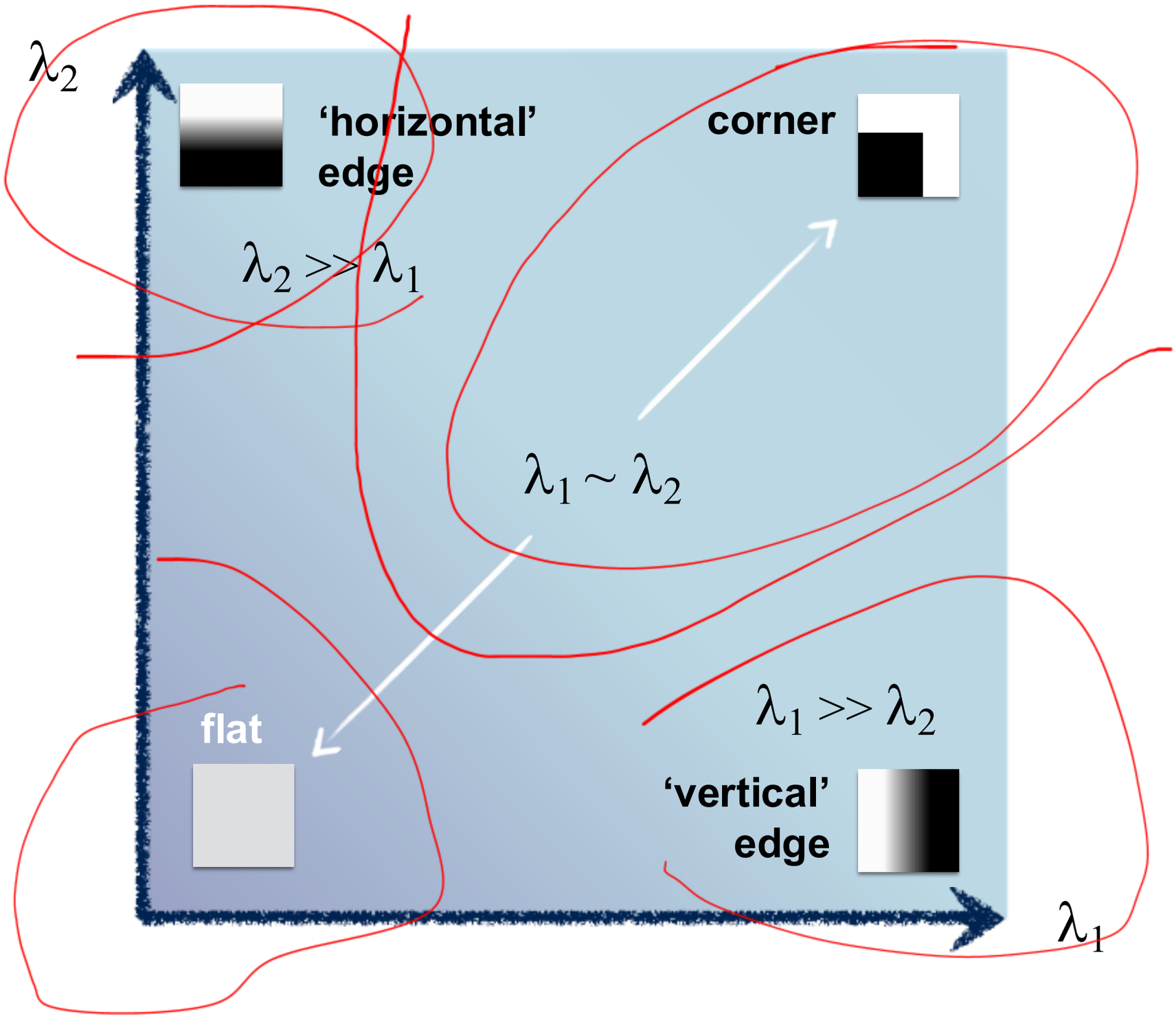
$$[u \quad v]\ M \begin{bmatrix}u\\ v\end{bmatrix} = \text{const}$$



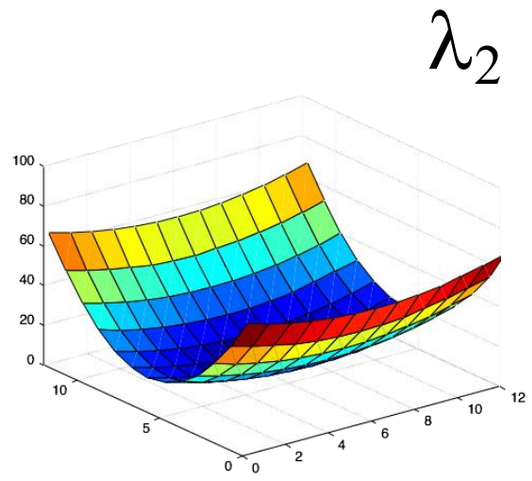
interpreting eigenvalues

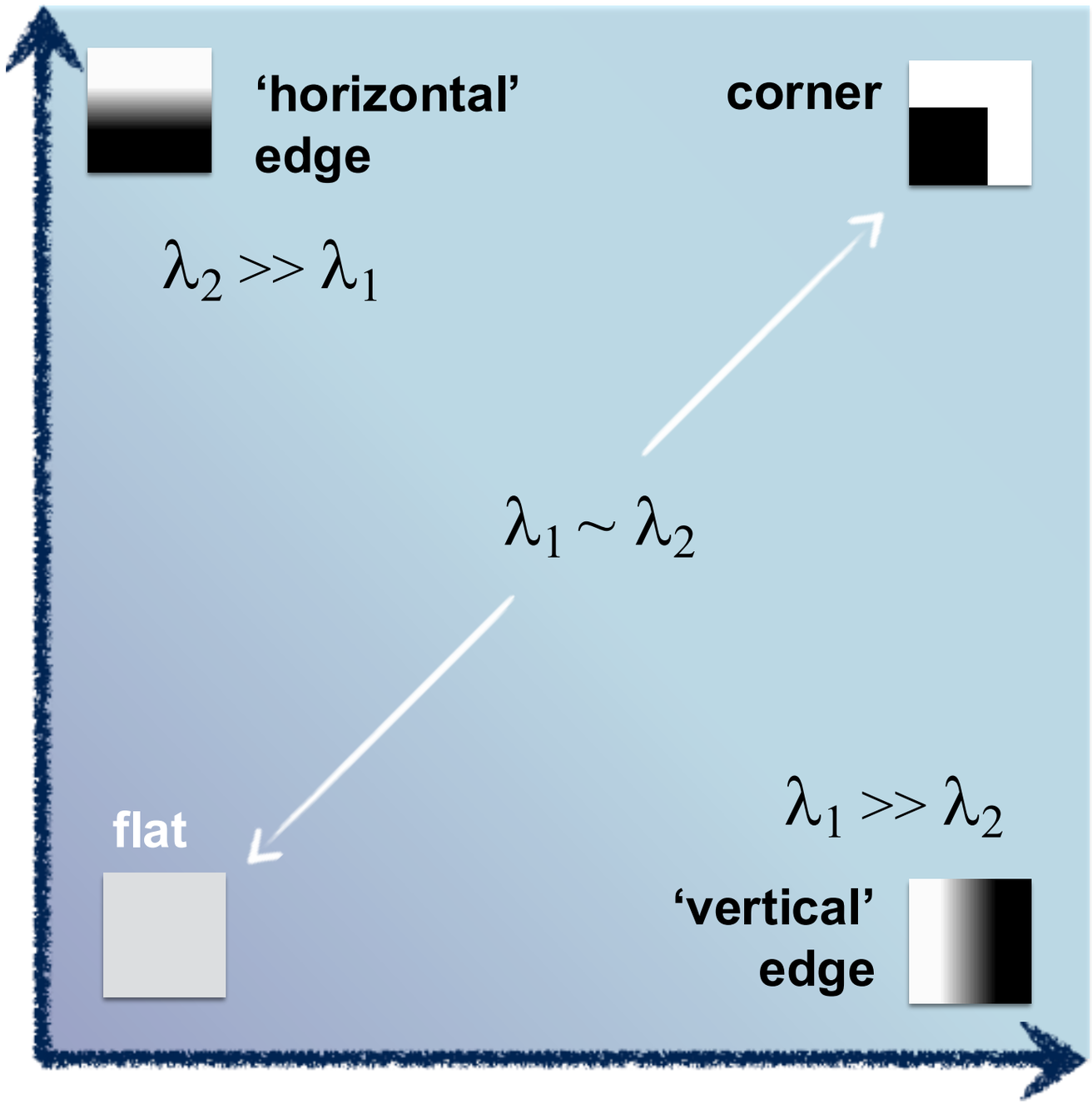


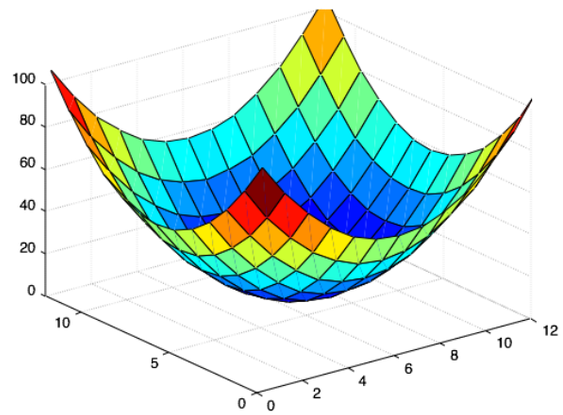
interpreting eigenvalues

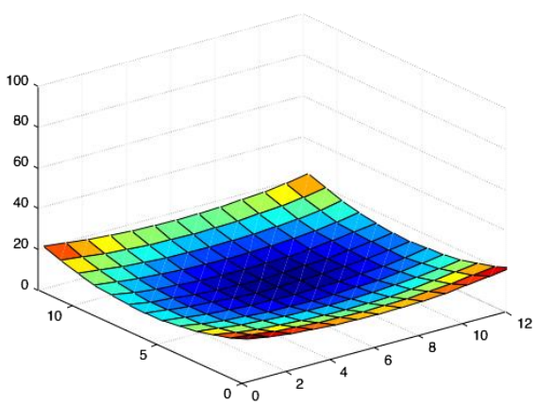


interpreting eigenvalues

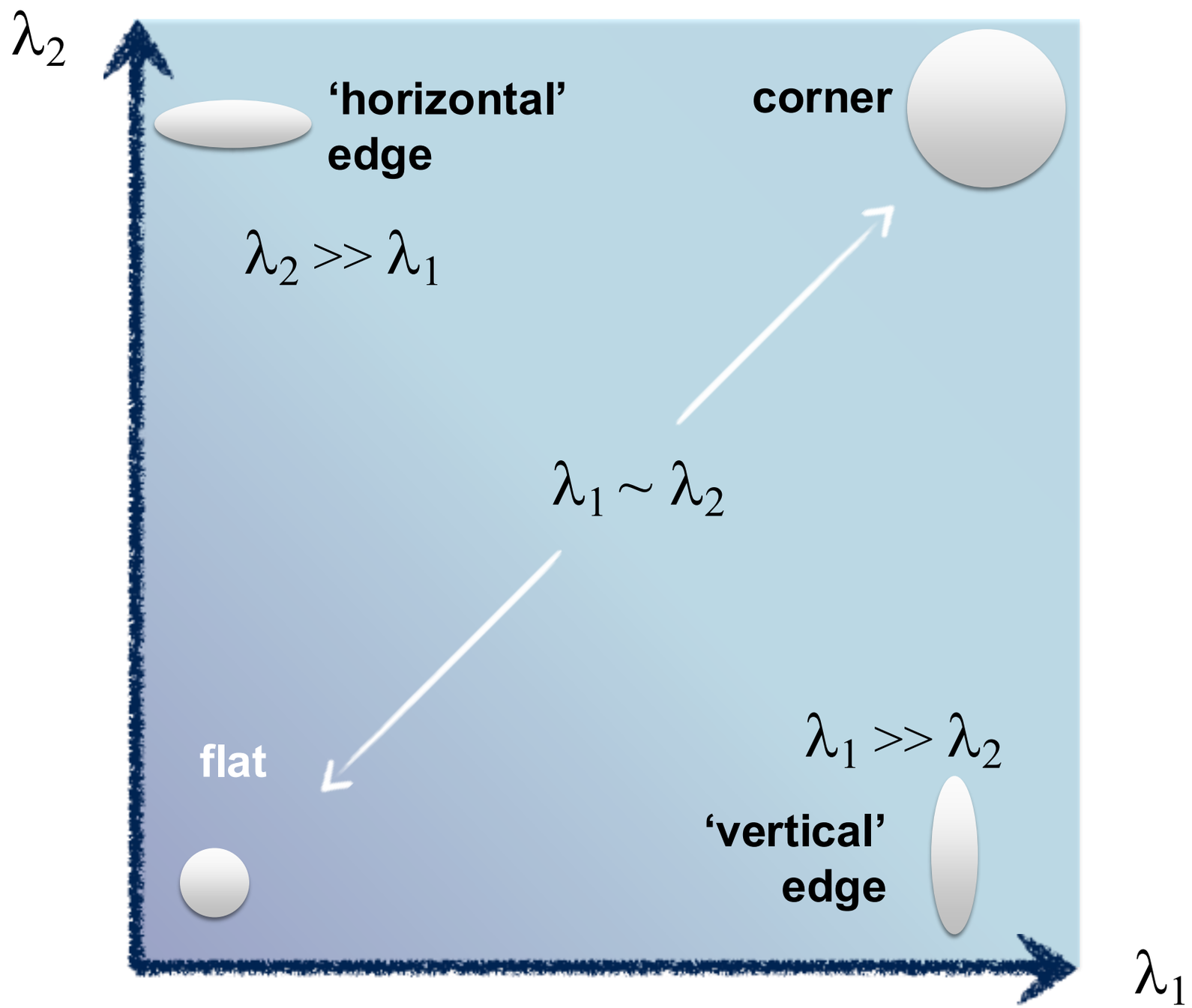






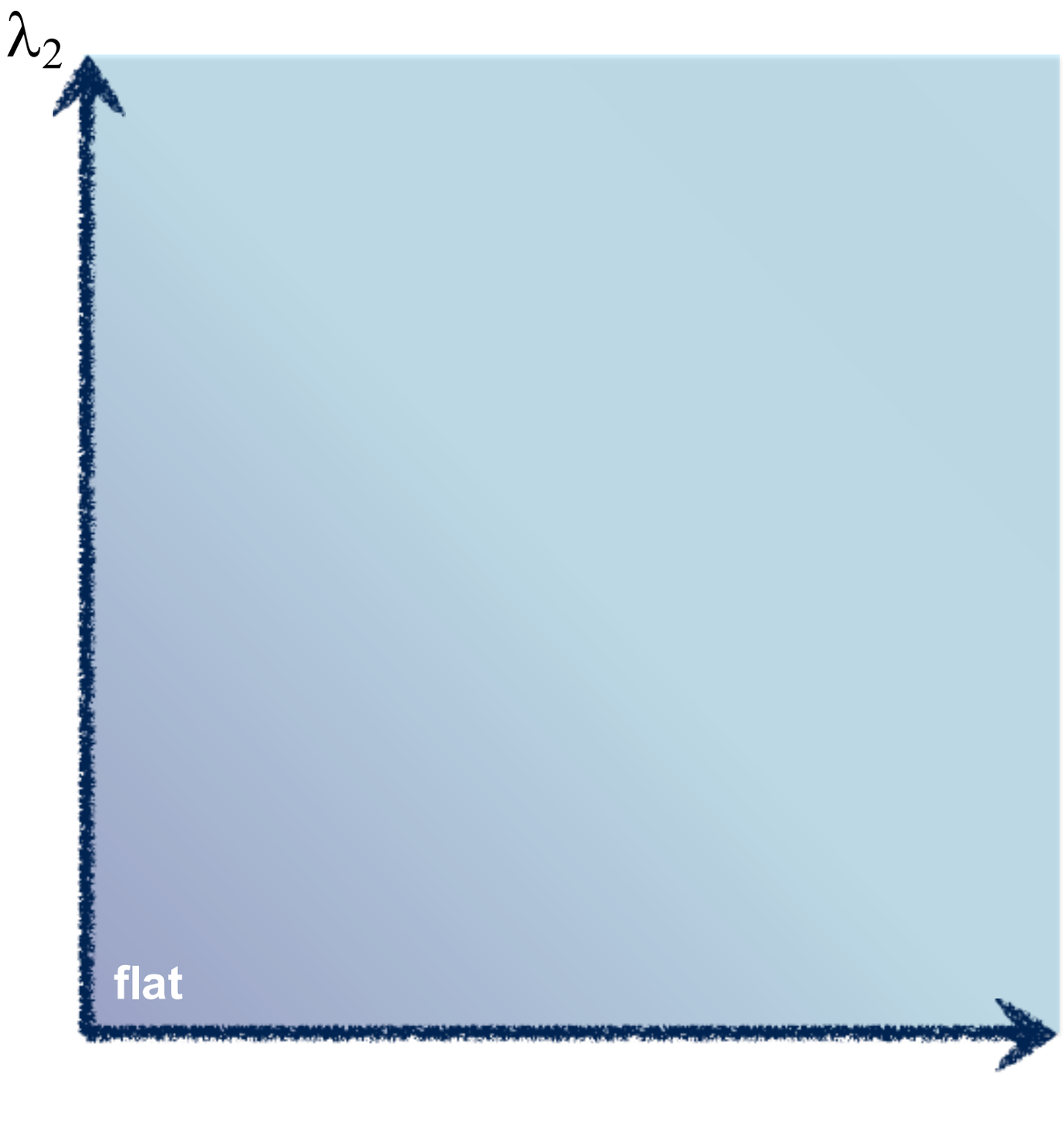


interpreting eigenvalues



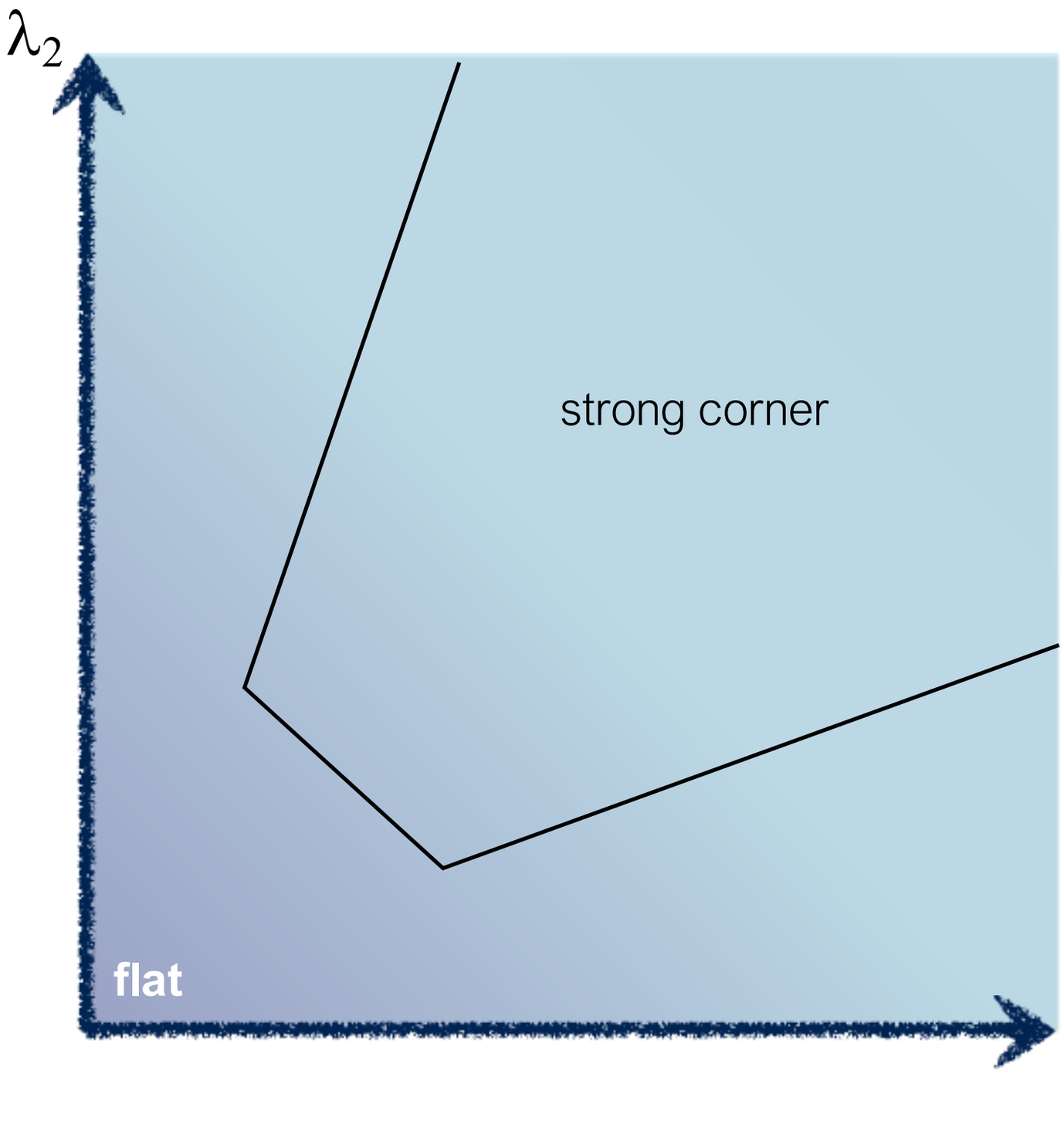
5. Use threshold on eigenvalues to detect corners

5. Use threshold on eigenvalues to detect corners



Think of a function to score ‘cornerness’

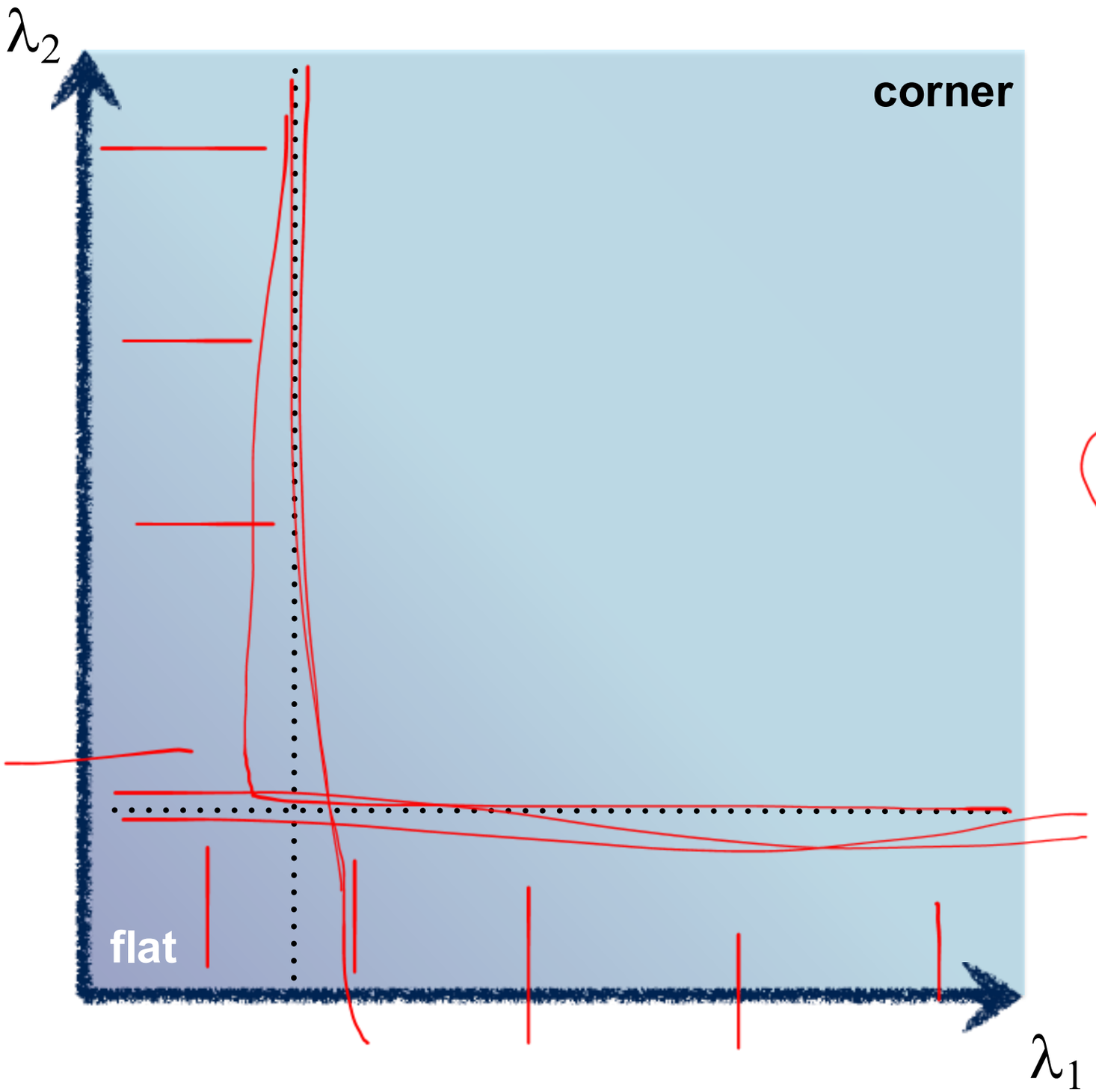
5. Use threshold on eigenvalues to detect corners



Think of a function to score ‘cornerness’

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)

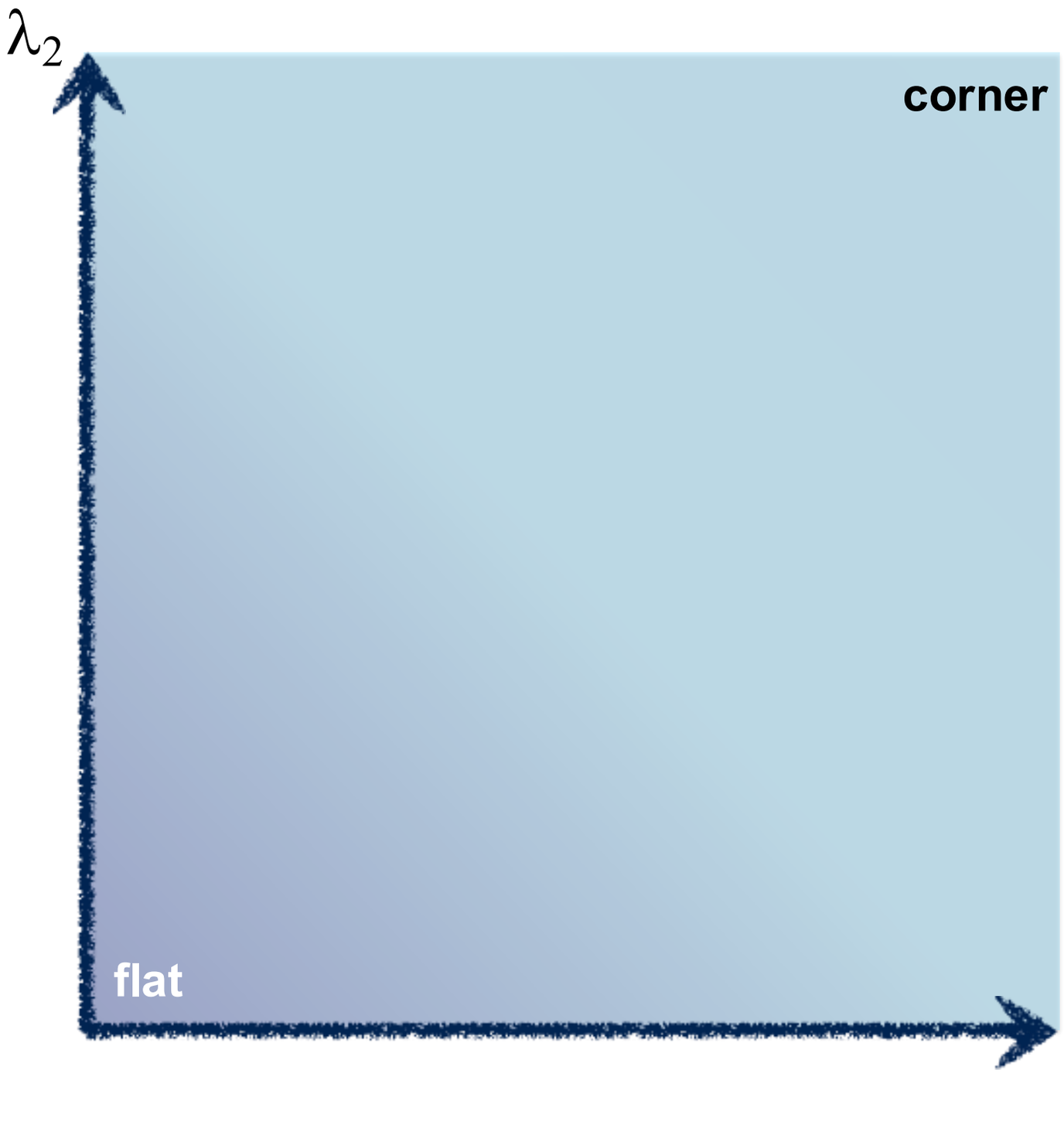


Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)



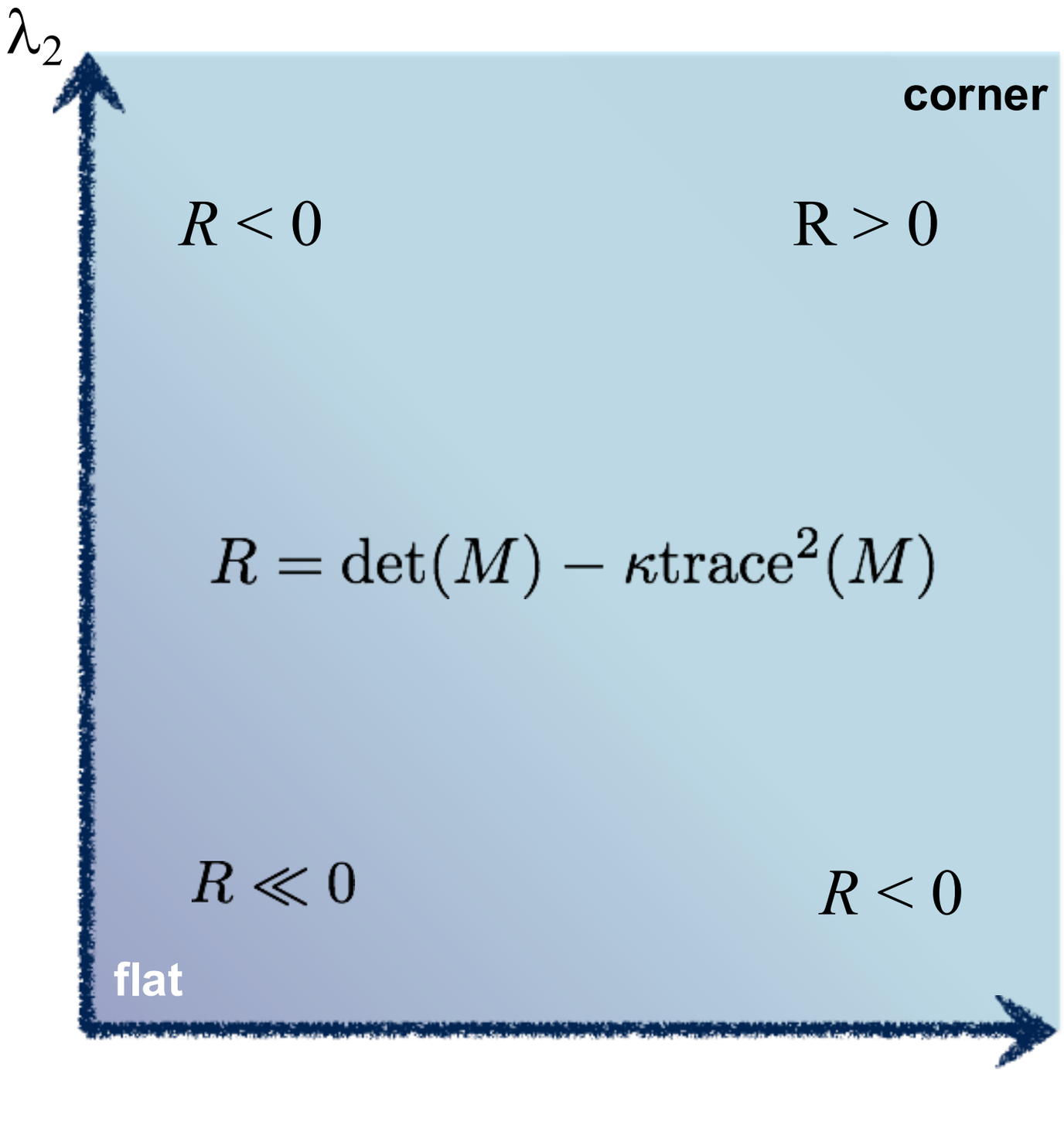
Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently…

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)



$$\det M = \lambda_1 \lambda_2$$

$$\text{trace}\, M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \text{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\text{trace}(M) + \epsilon}$$

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

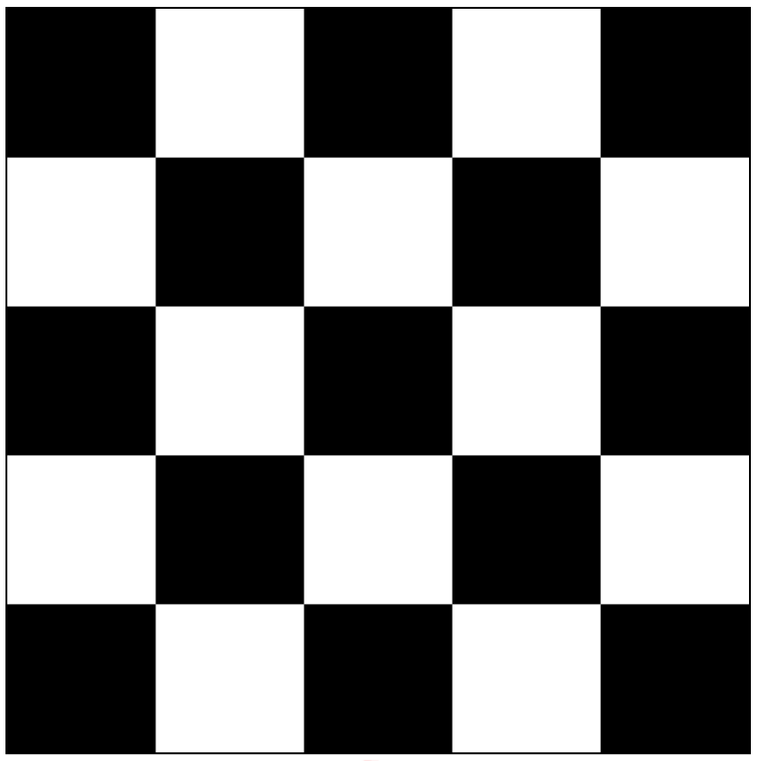
4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

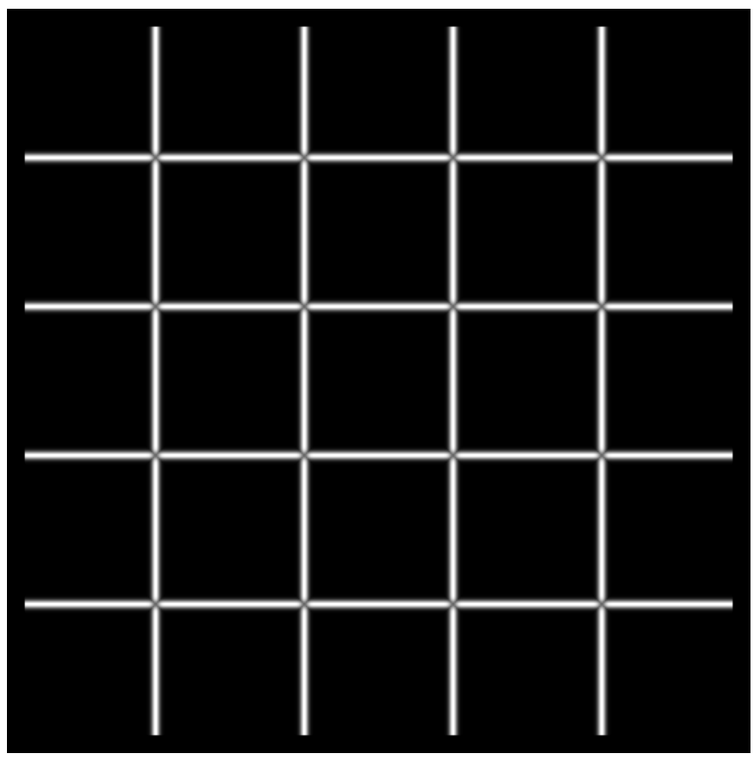
5. Compute the response of the detector at each pixel

$$R = \det M - k\left(\text{trace}M\right)^2$$

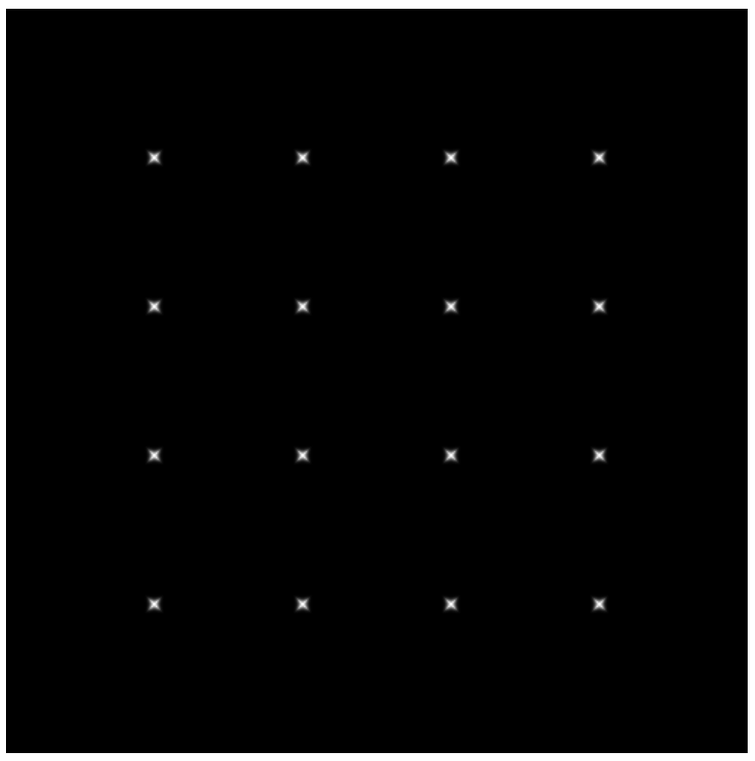
6. Threshold on value of R; compute non-max suppression.



I



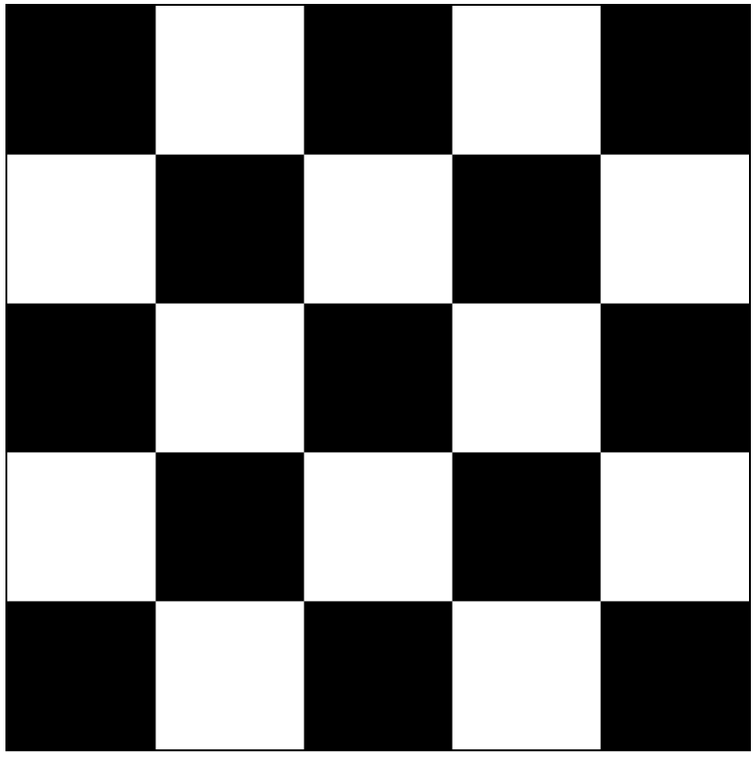
$\lambda_{\max}$



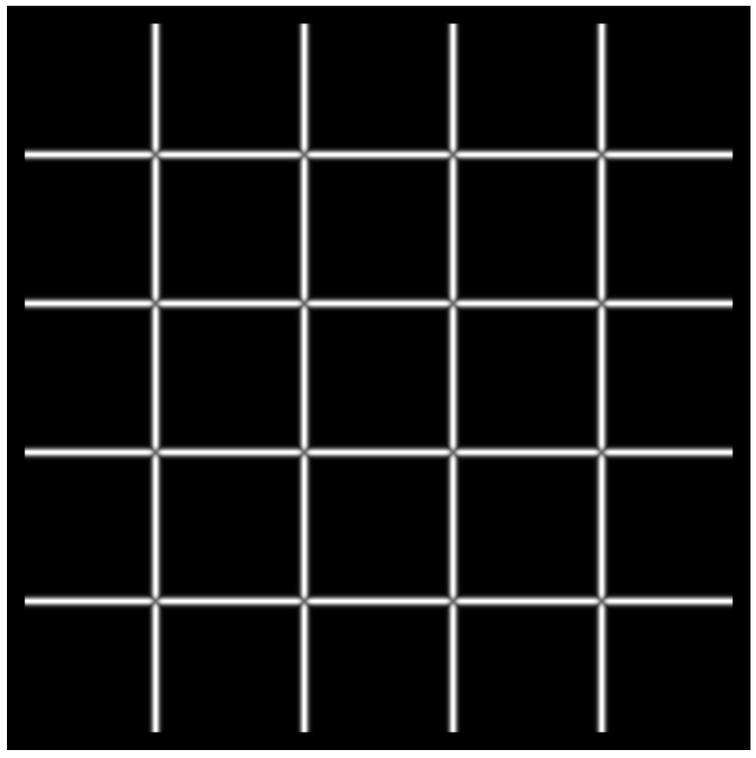
$\lambda_{\min}$

Yet another option: $f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

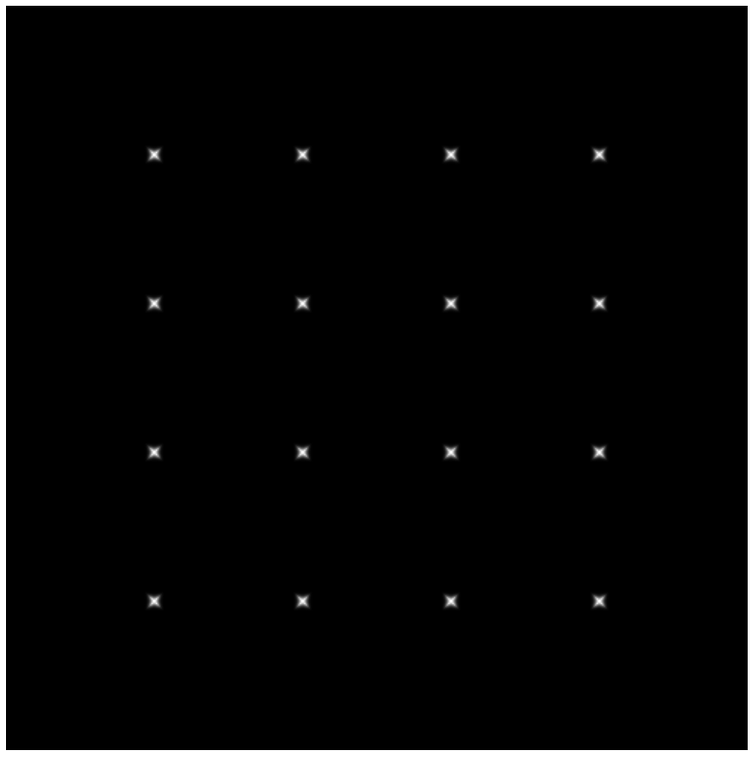
How do you write this equivalently using determinant and trace?



I



$\lambda_{\max}$

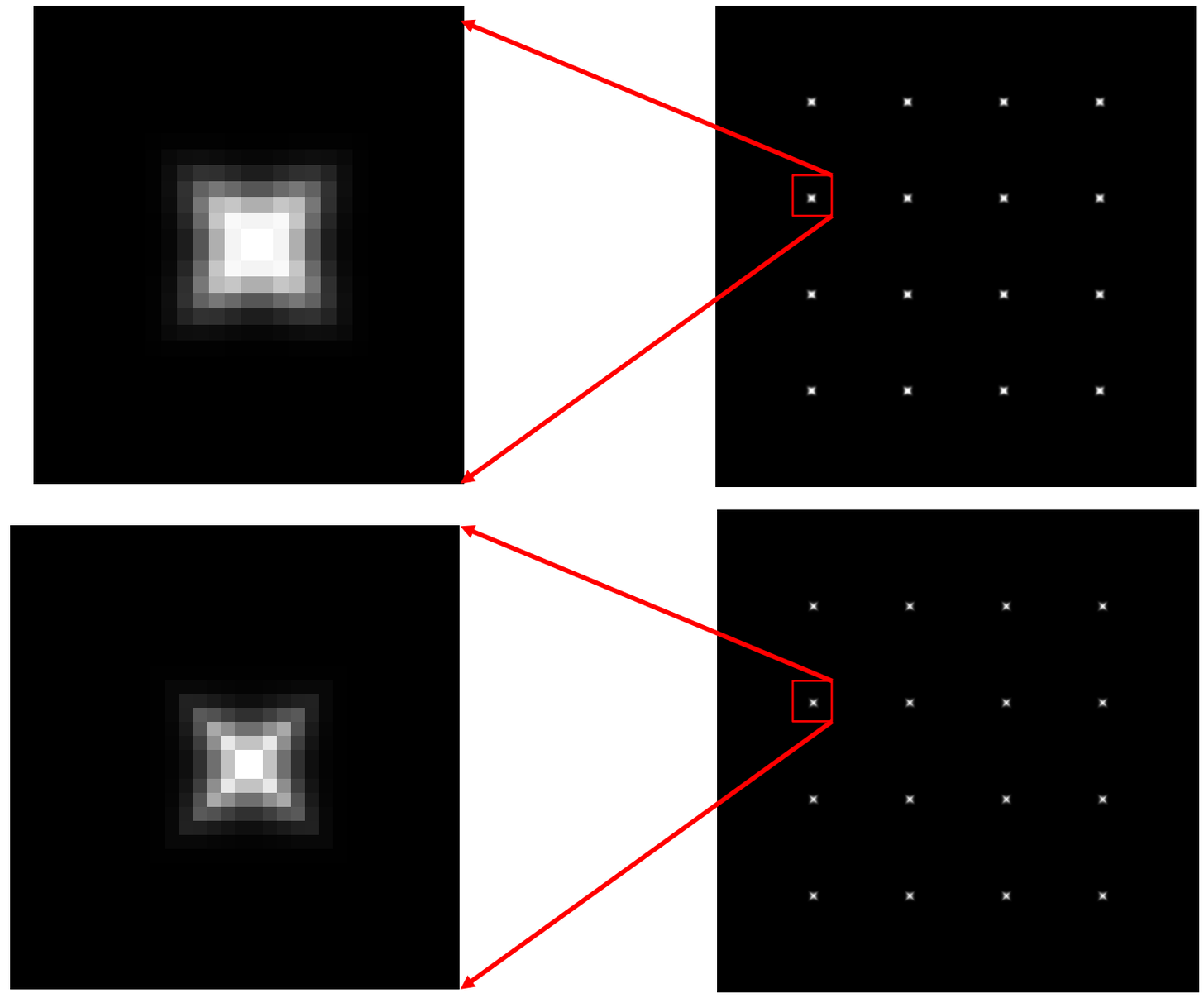


$\lambda_{\min}$

Yet another option:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

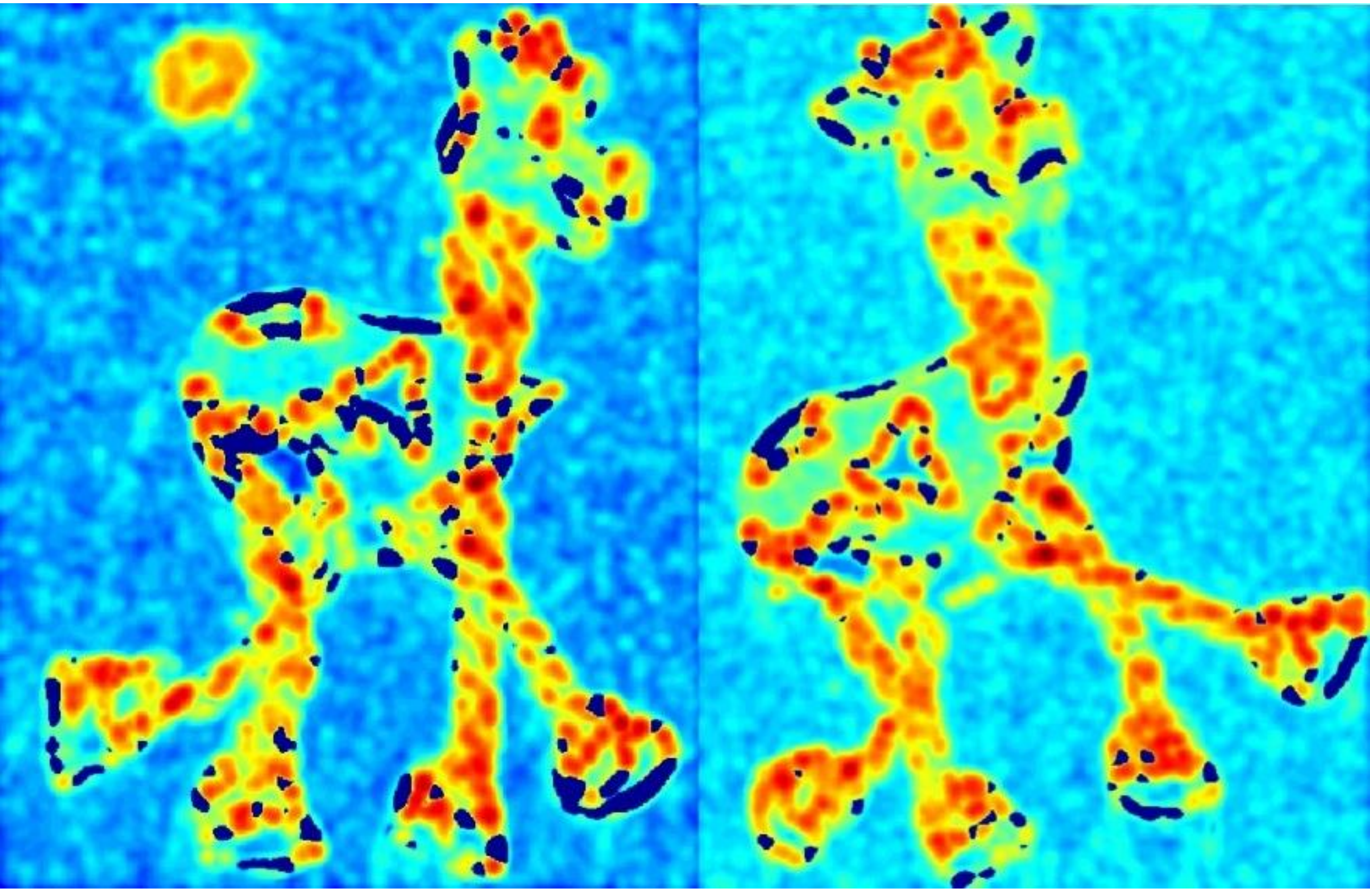
Different criteria



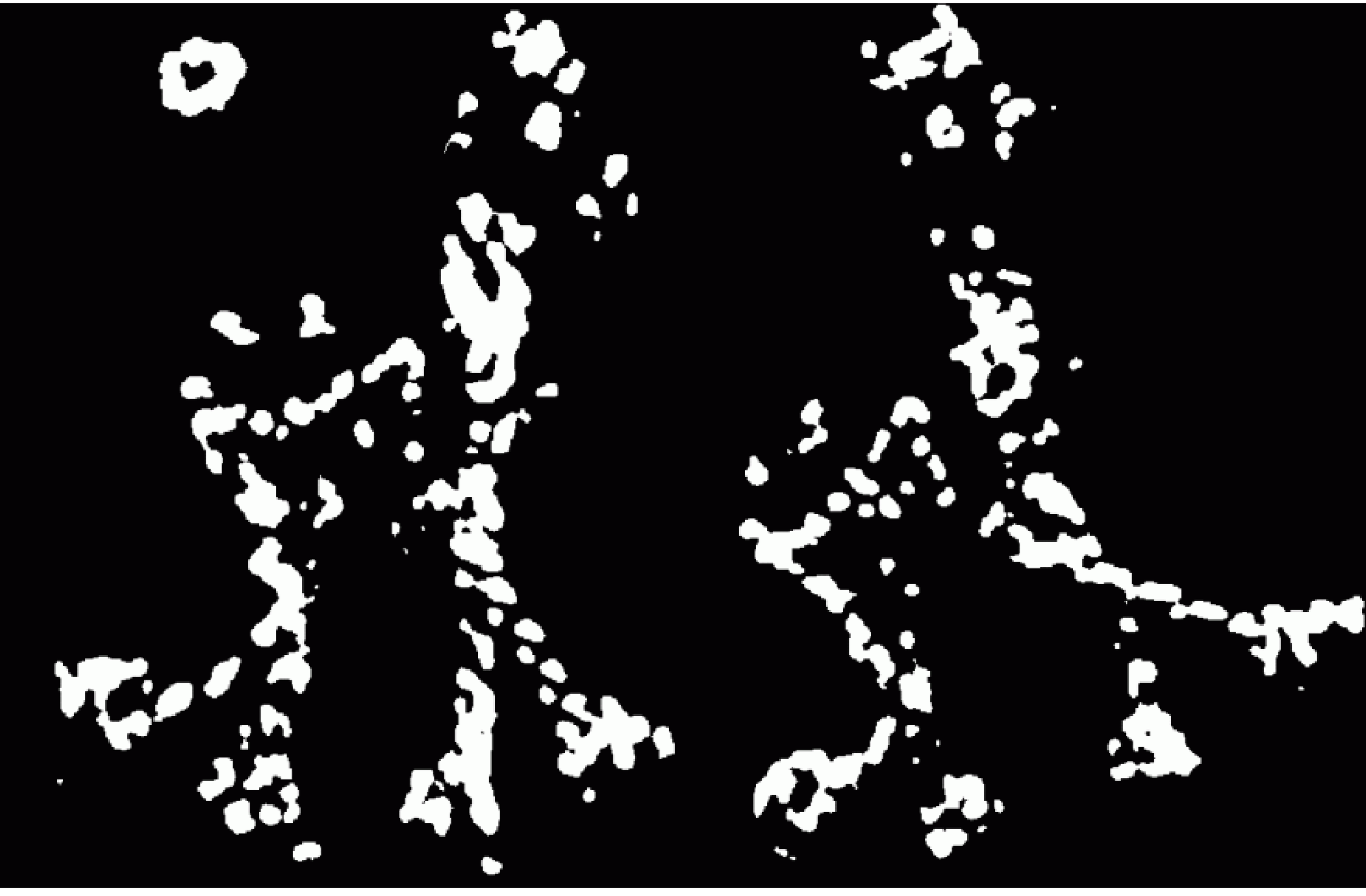
Harris criterion

$\lambda_{\min}$

Corner response

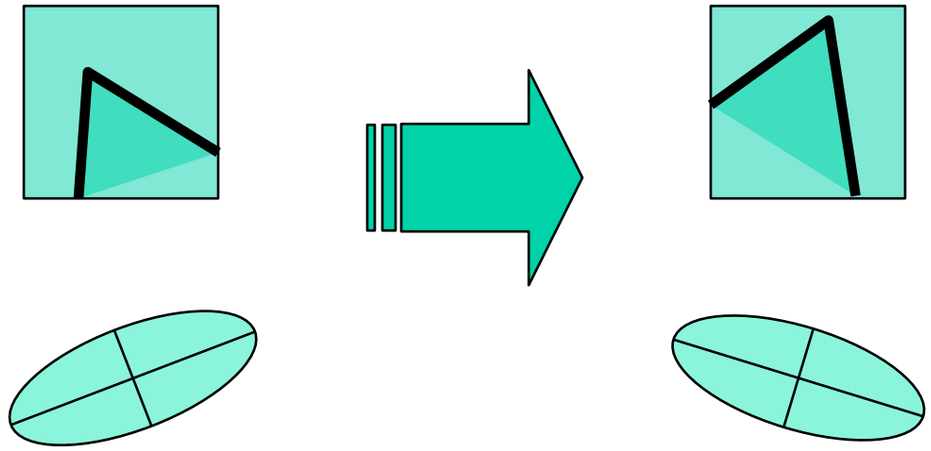


Thresholded corner response



Non-maximal suppression

Harris corner response is invariant to rotation



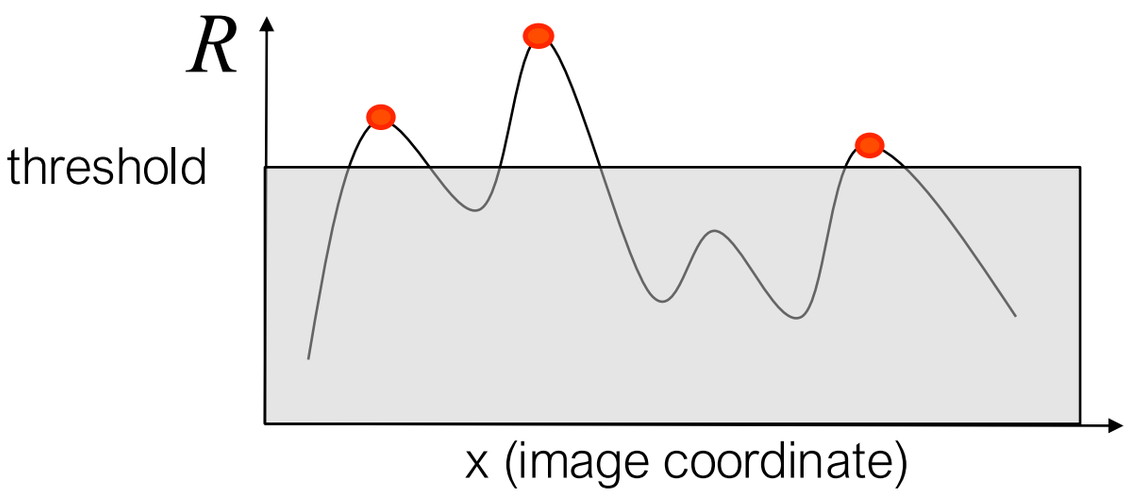
Ellipse rotates but its shape
(**eigenvalues**) remains the same

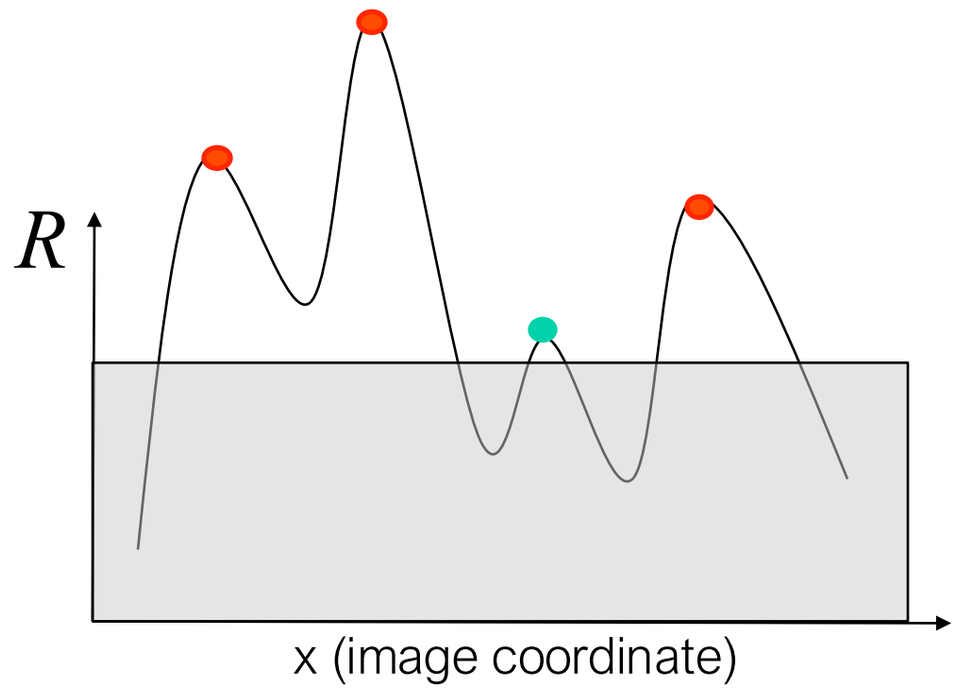
Corner response R is invariant to image rotation

Harris corner response is invariant to intensity changes

Partial invariance to *affine intensity* change

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scale: $I \rightarrow a\, I$





The Harris detector is not invariant to changes in …

The Harris corner detector is <u>not</u> invariant to scale

edge!

corner!

Multi-scale detection

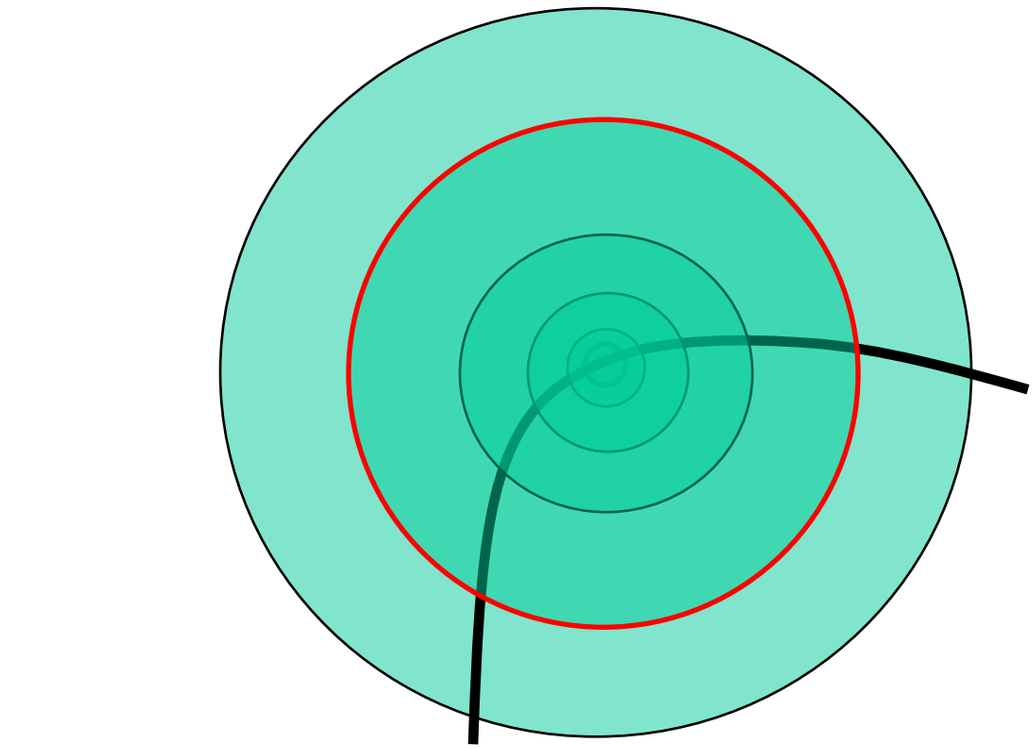
How can we make a feature detector scale-invariant?

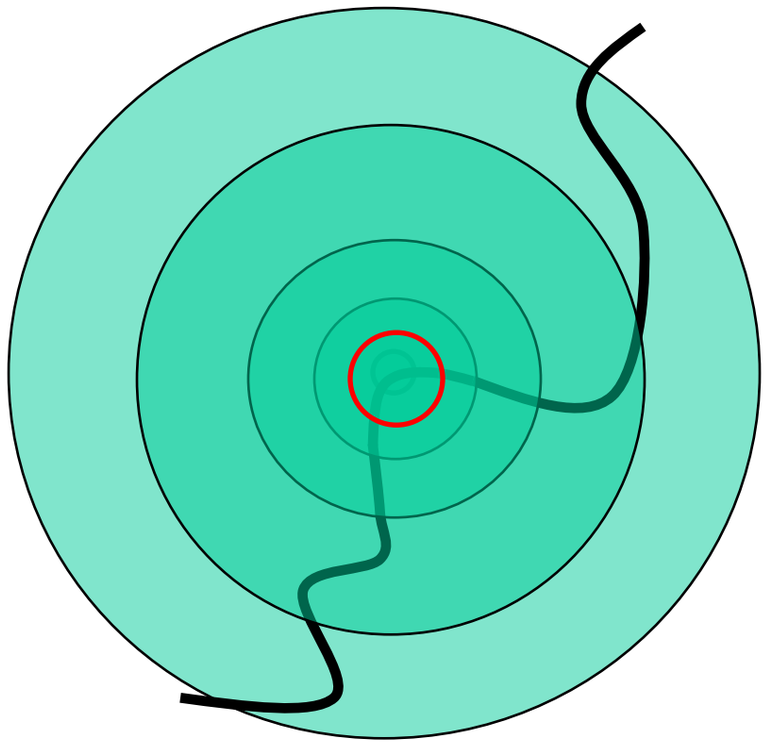
How can we automatically select the scale?

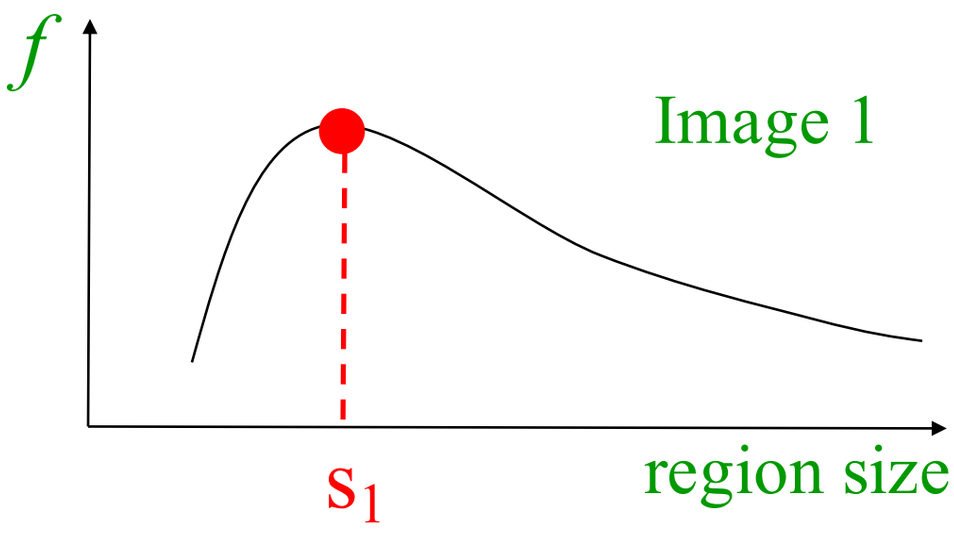
Multi-scale blob detection

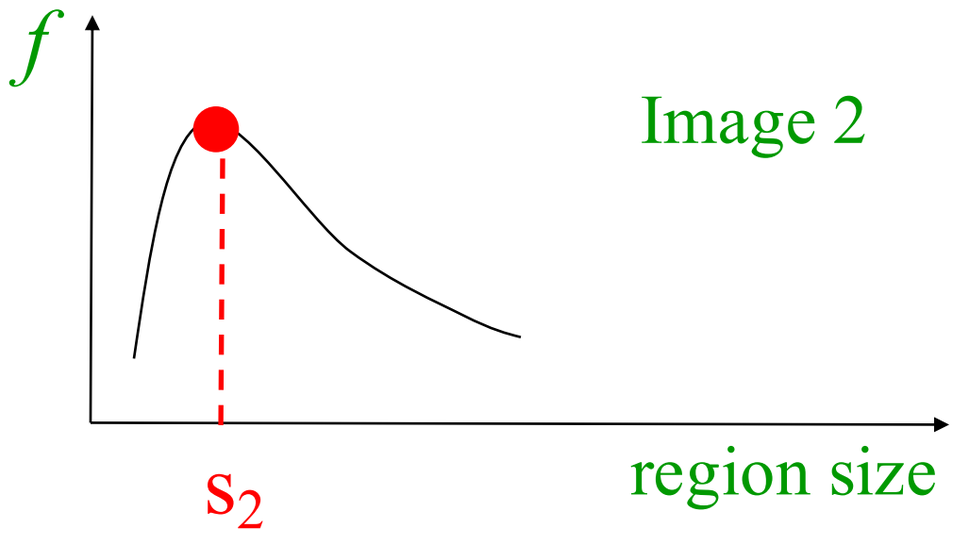
Intuitively…

Find local maxima in both **position** and **scale**



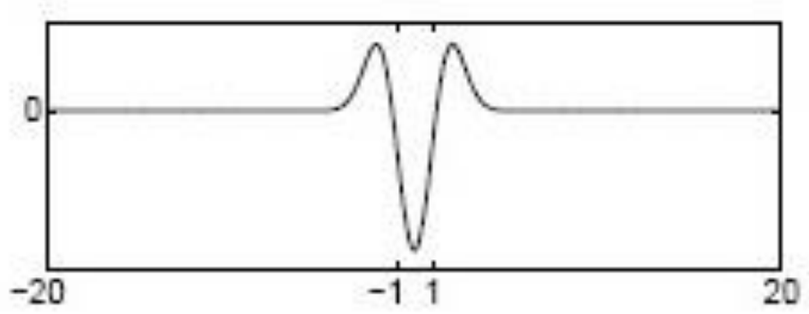


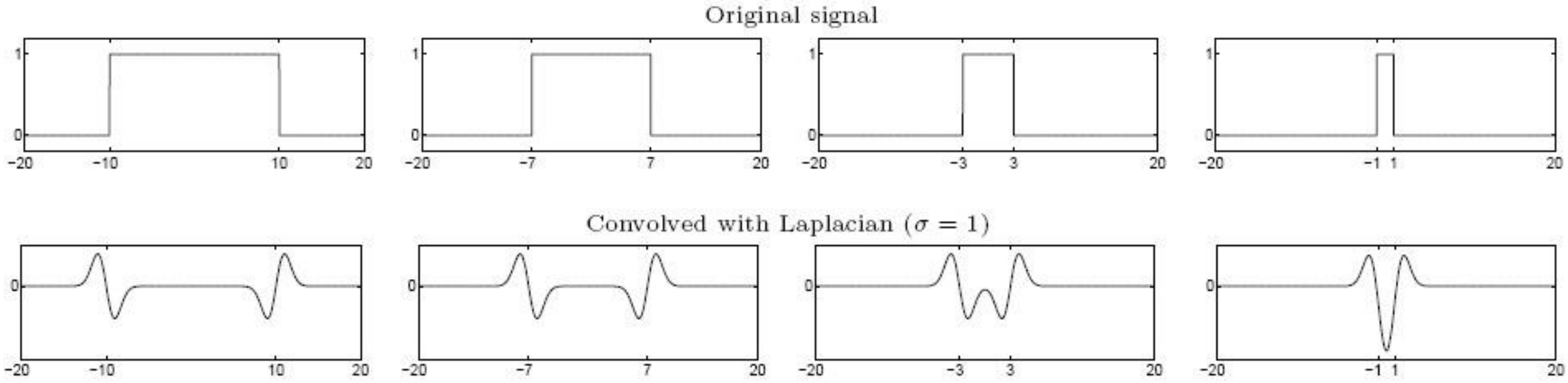




Formally…

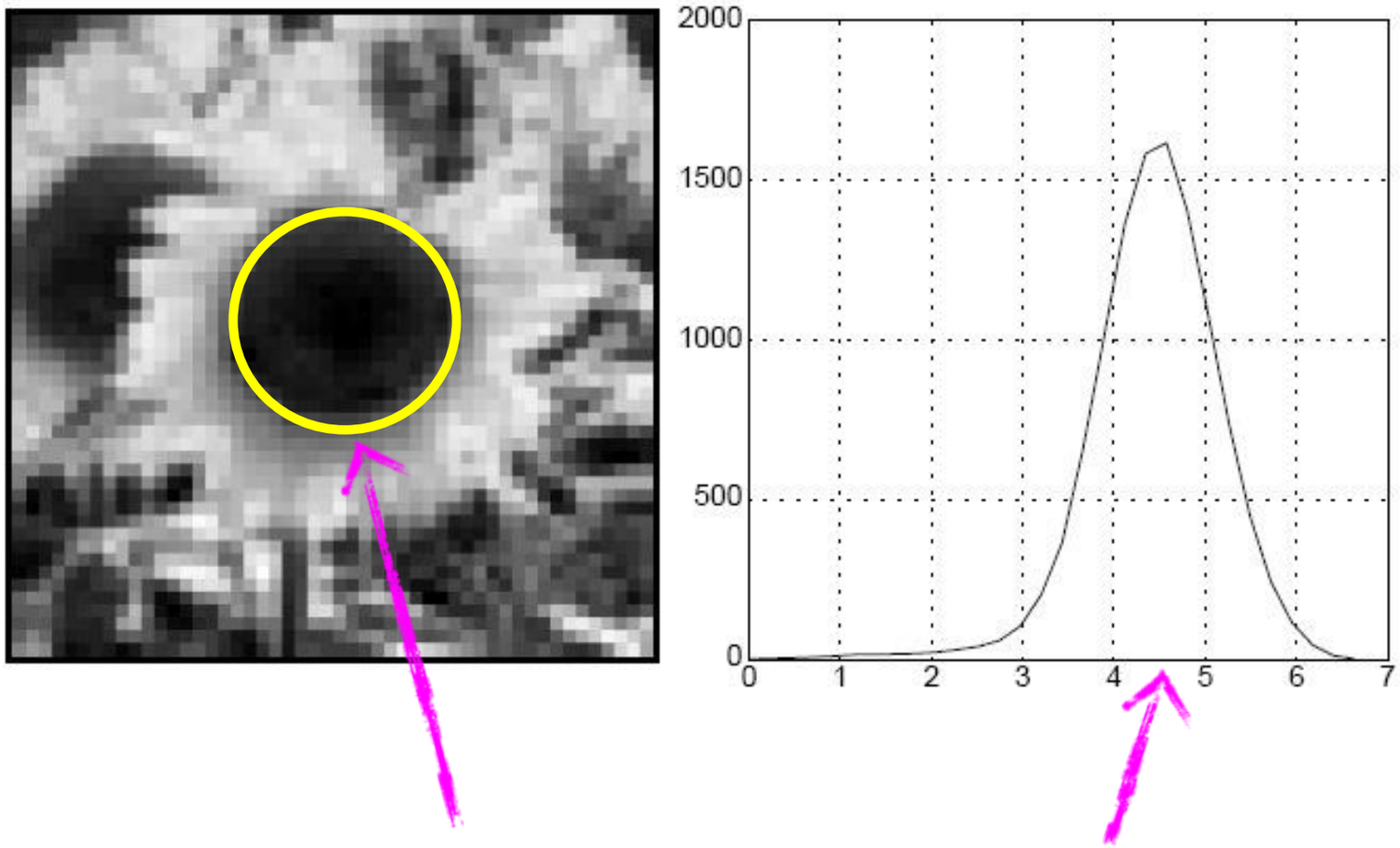
Laplacian filter





Highest response when the signal has the same **characteristic scale** as the filter

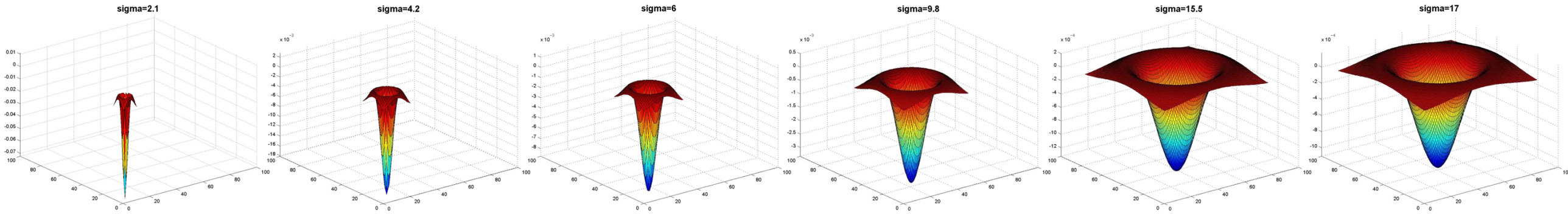
characteristic scale - the scale that produces peak filter response



characteristic scale

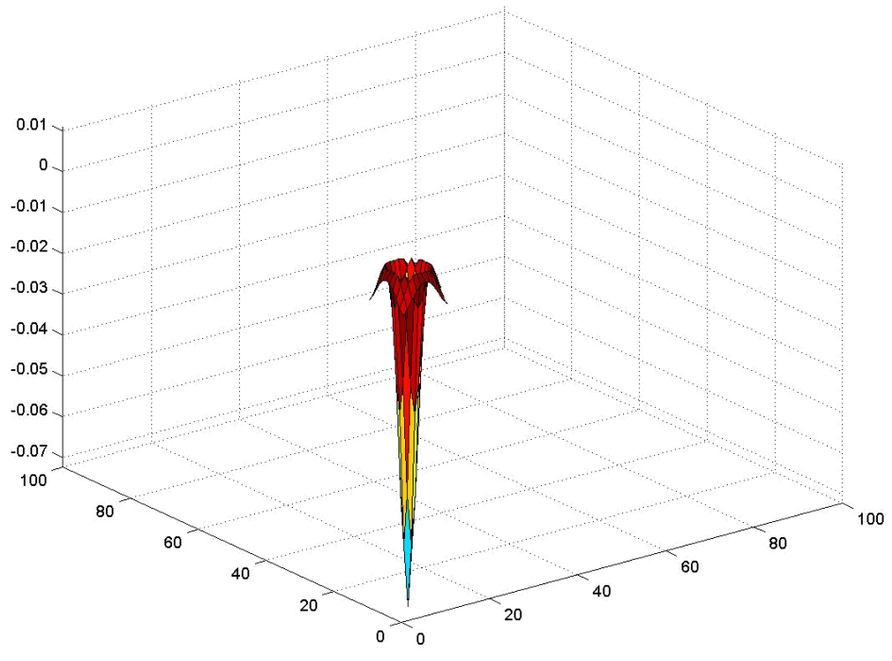
we need to search over characteristic scales

What happens if you apply different Laplacian filters?

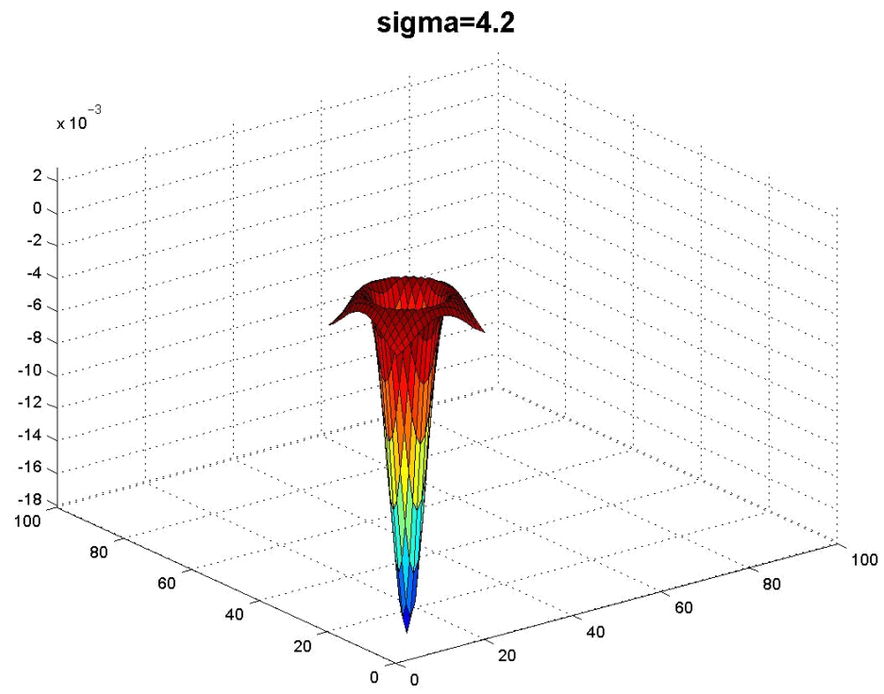


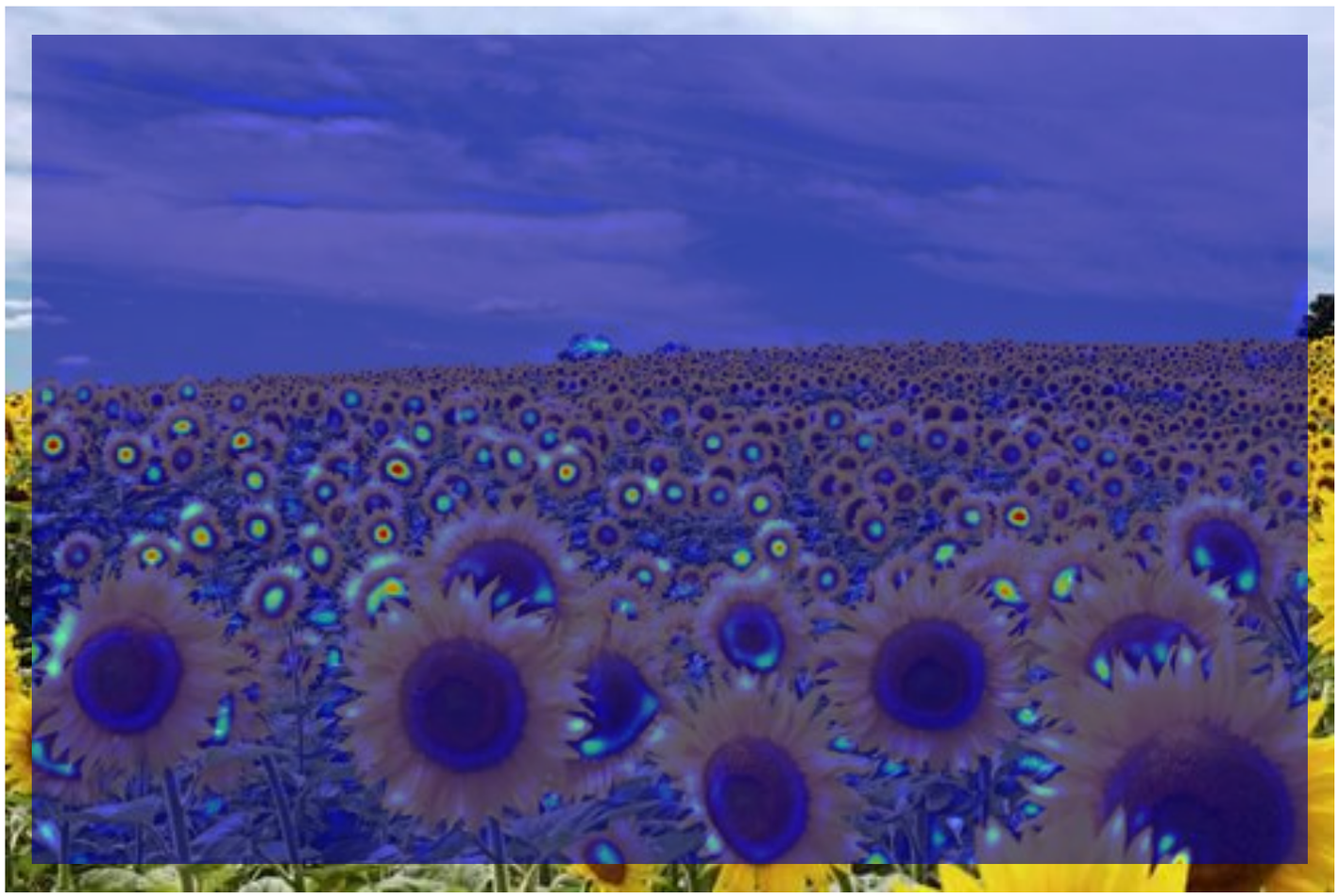
Full size

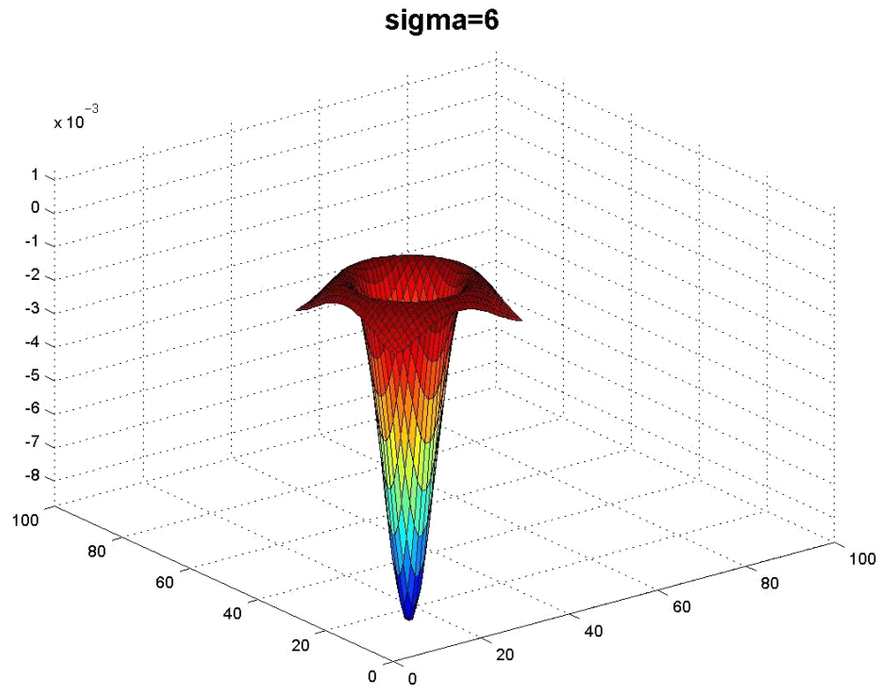
3/4 size

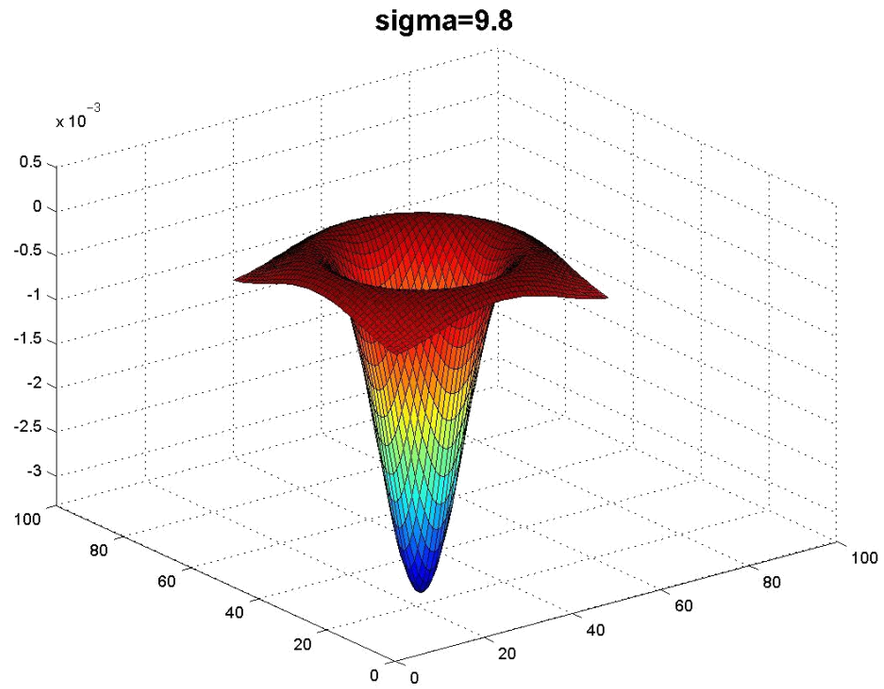


jet color scale
blue: low, red: high


sigma=4.2
x 10^-3
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
100
80
60
40
20
0
0
20
40
60
80
100




sigma=6
x 10^-3
1
0
-1
-2
-3
-4
-5
-6
-7
-8
100
80
60
40
20
0
0
20
40
60
80
100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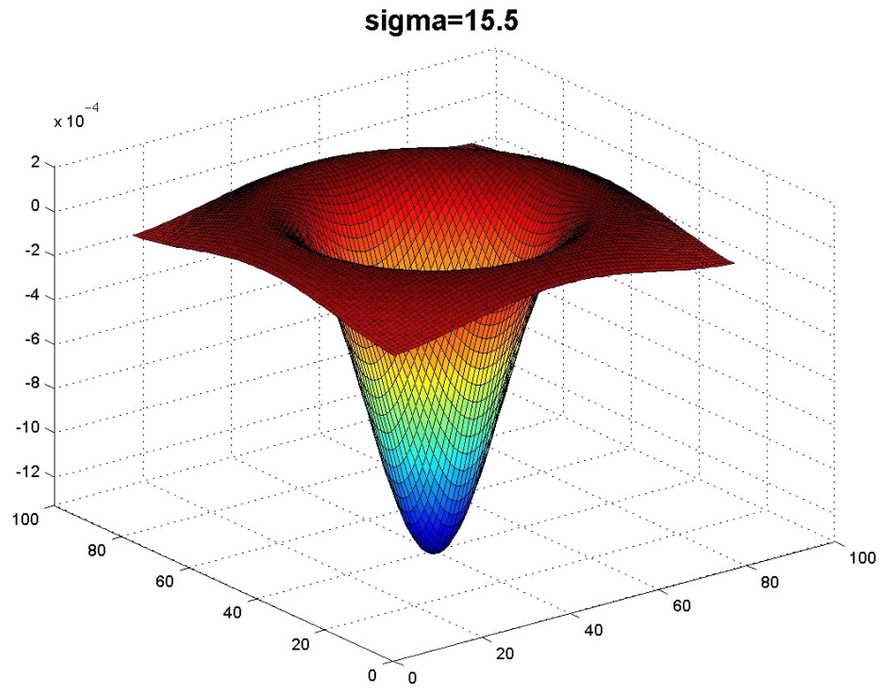
x 10^-3
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
100
80
60
40
20
0
0
20
40
60
80
100


sigma=15.5
x 10^-4
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-2
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-8
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60
80
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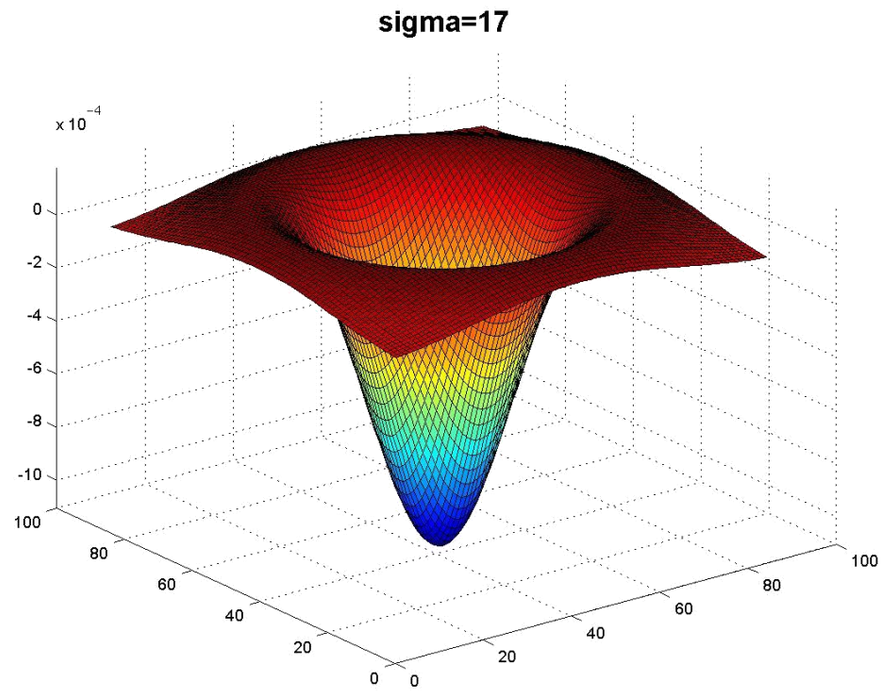
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x 10^-4
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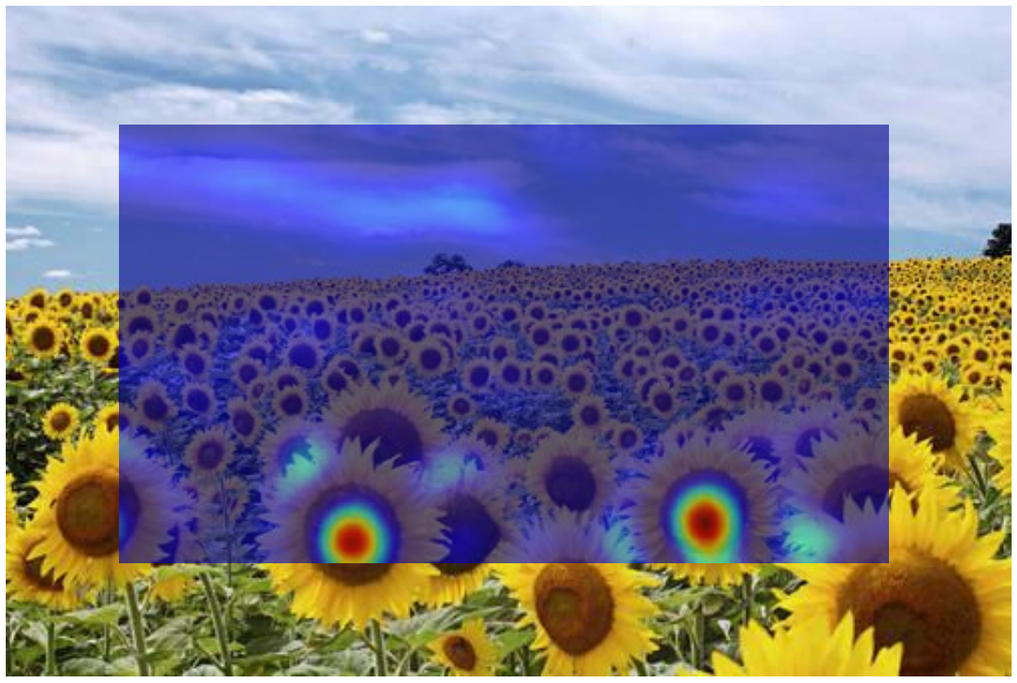
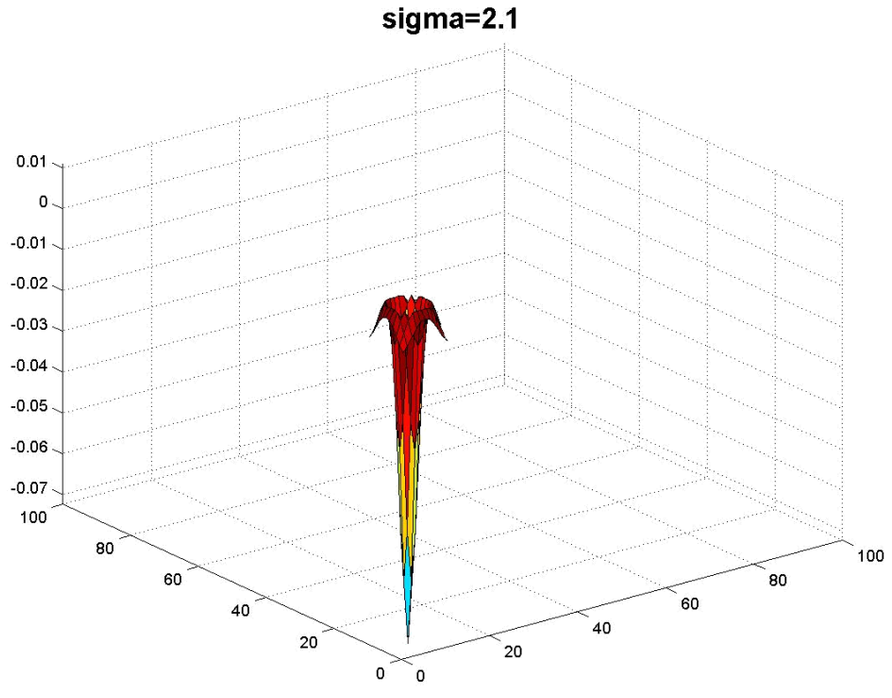
-2
-4
-6
-8
-10
100
80
60
40
20
0
0
20
40
60
80
100



What happened when you applied different Laplacian filters?

Full size

3/4 size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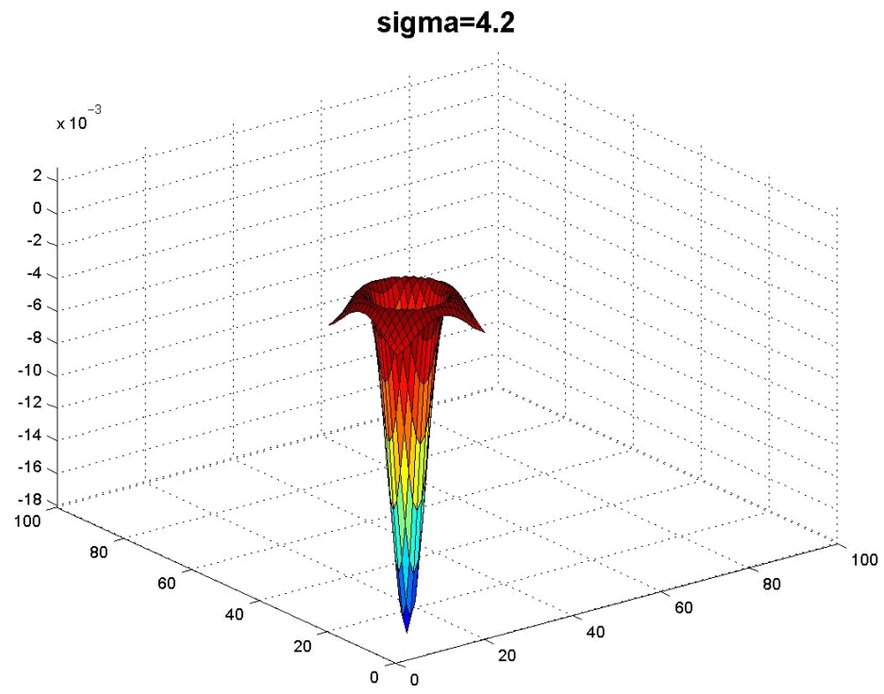
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
100
80
60
40
20
0
0
20
40
60
80
100


sigma=4.2
x 10^-3
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
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20
0
0
20
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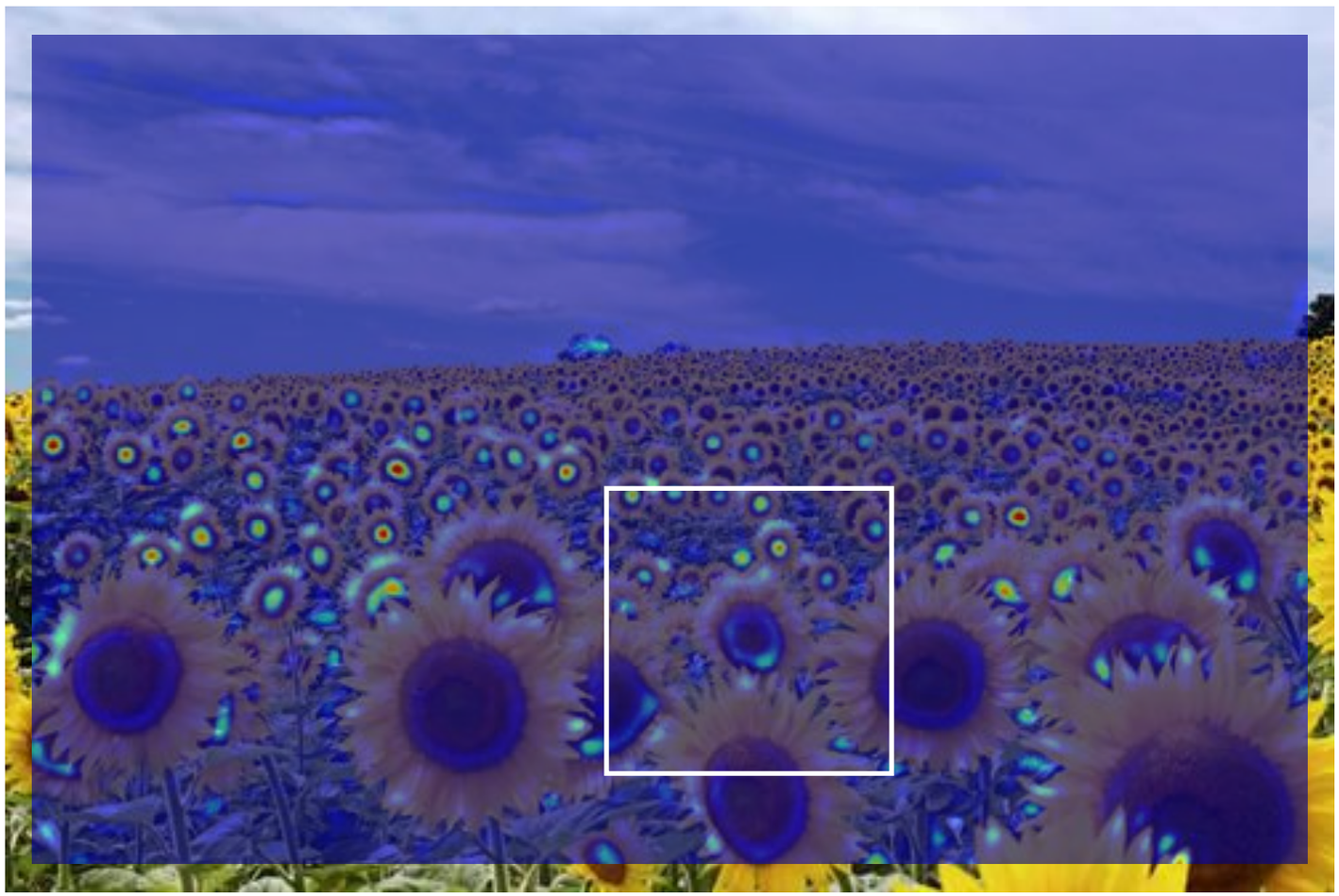
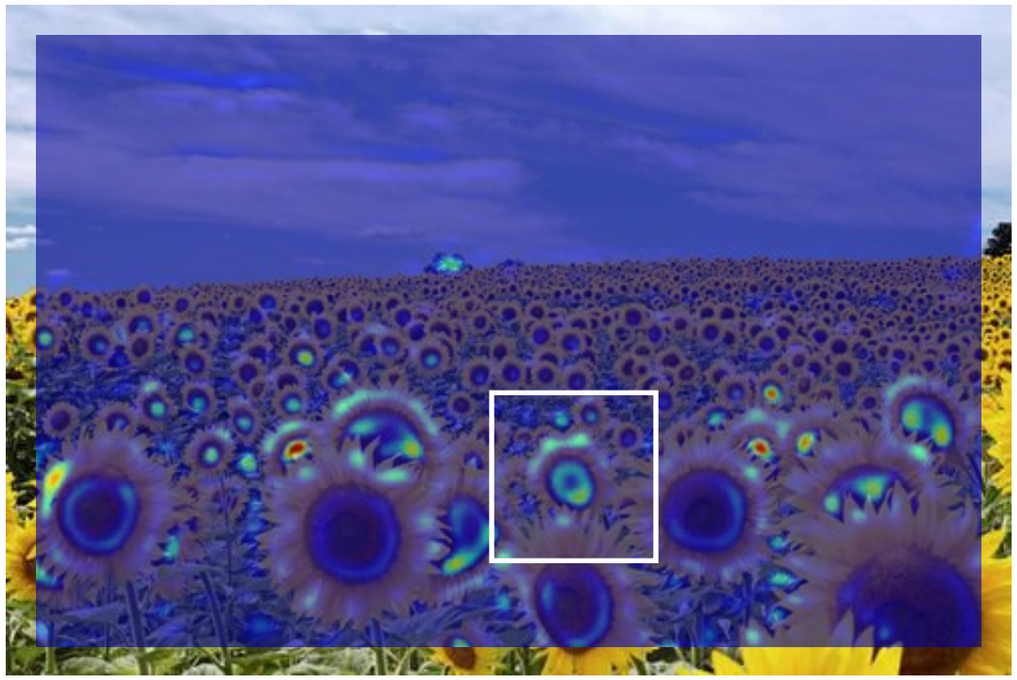
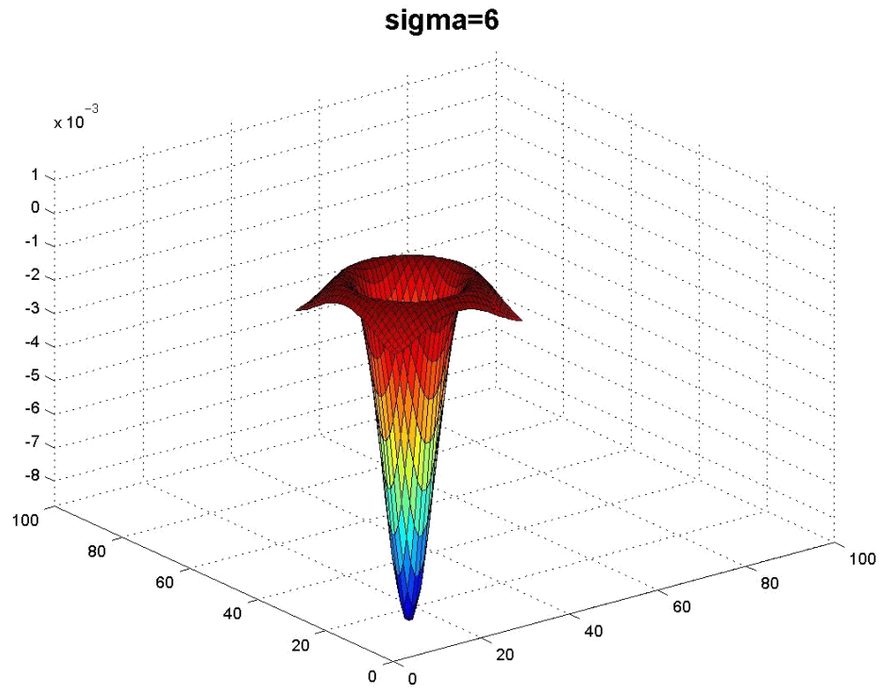
80
100



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-2
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-6
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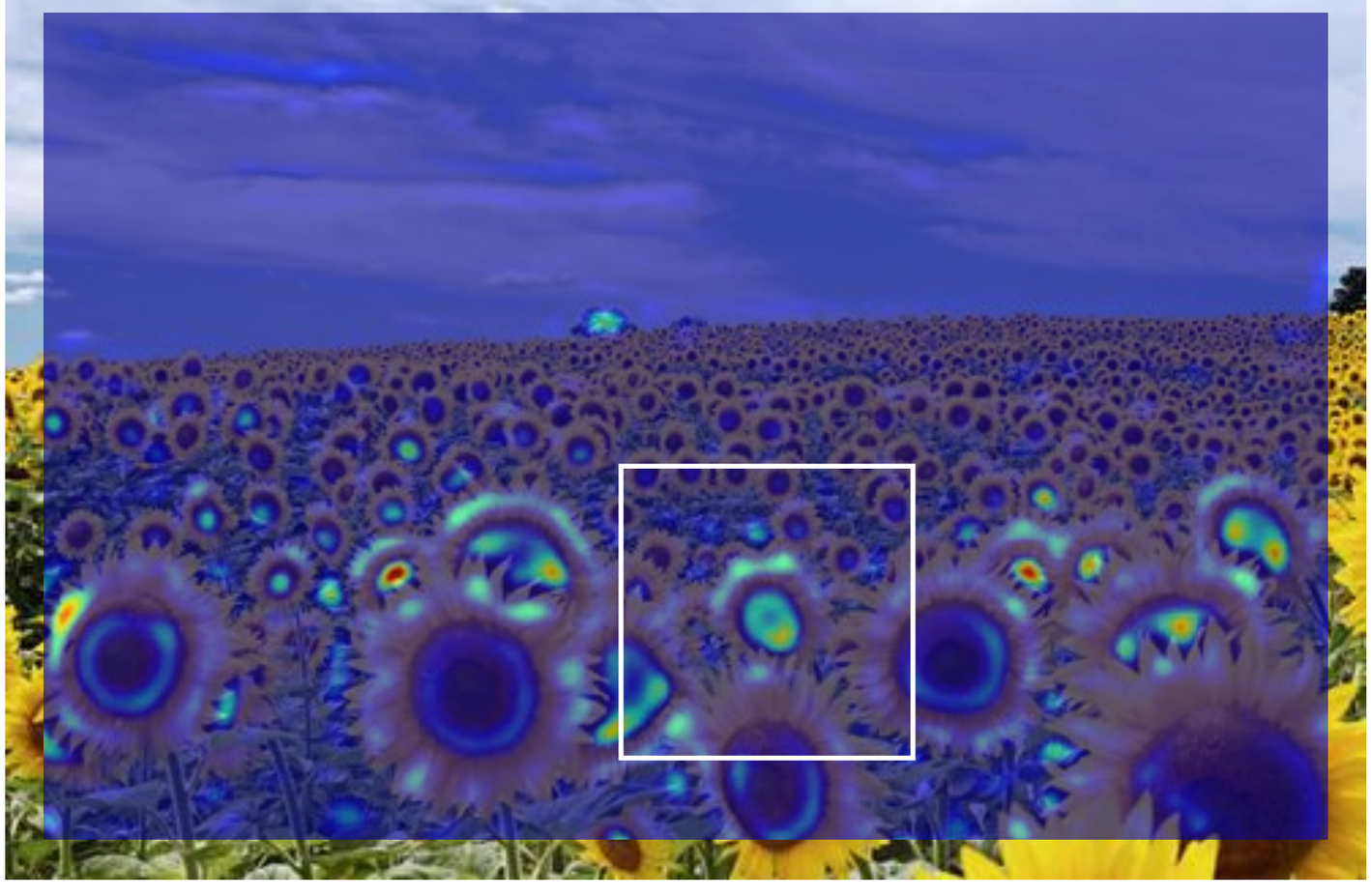
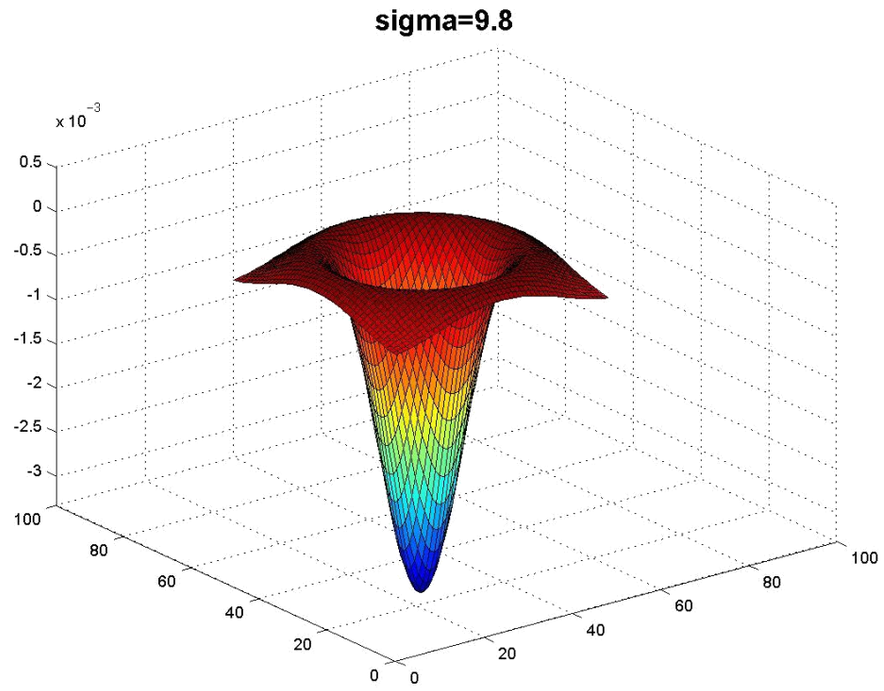
20
0
0
20
40
60
80
100




peak!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-3
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80
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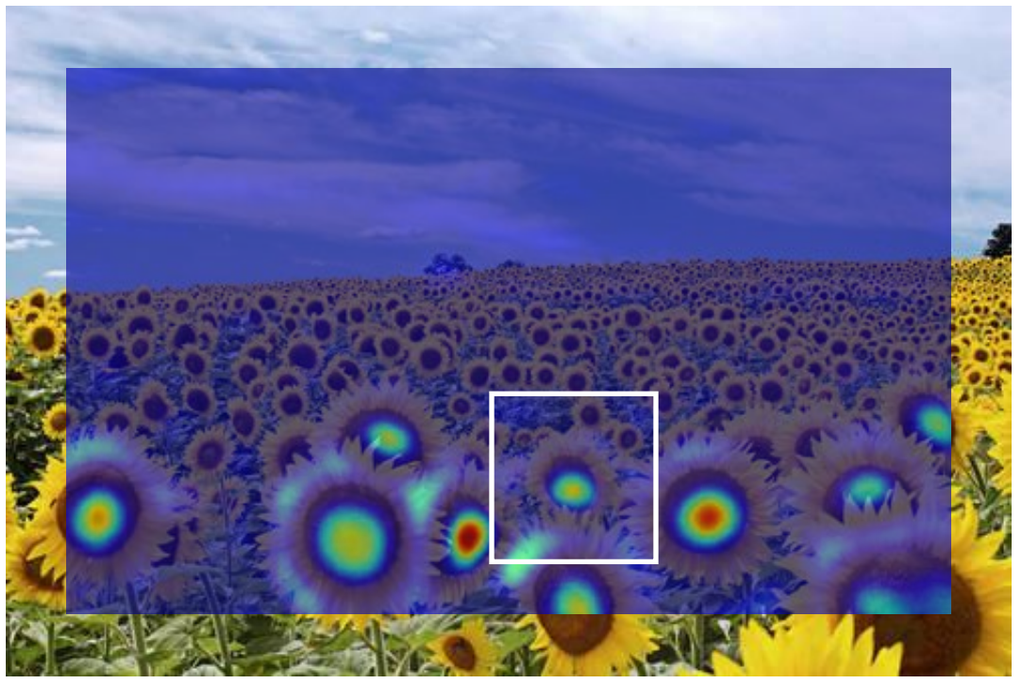
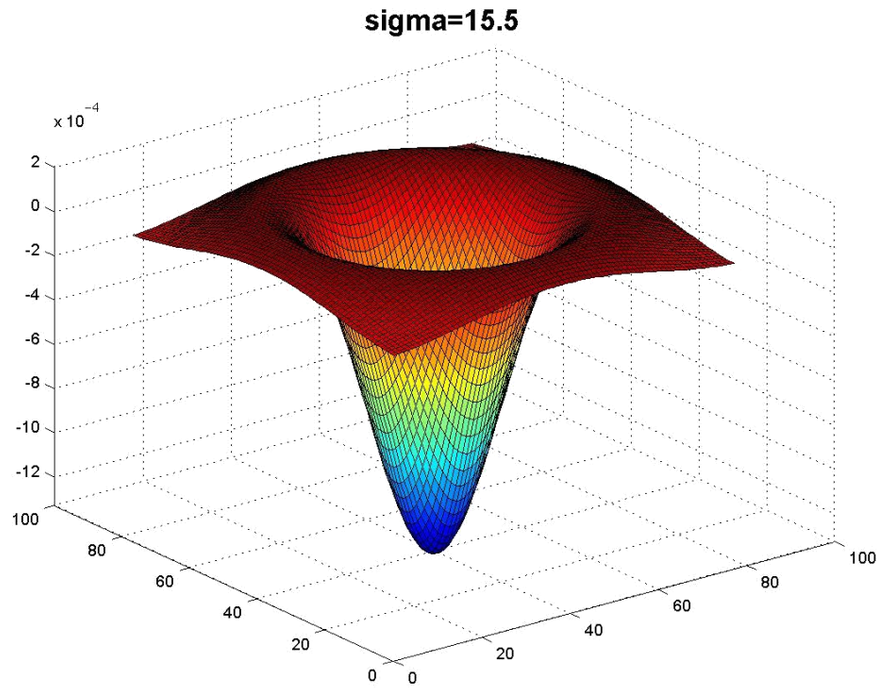
0
20
40
60
80
100


peak!

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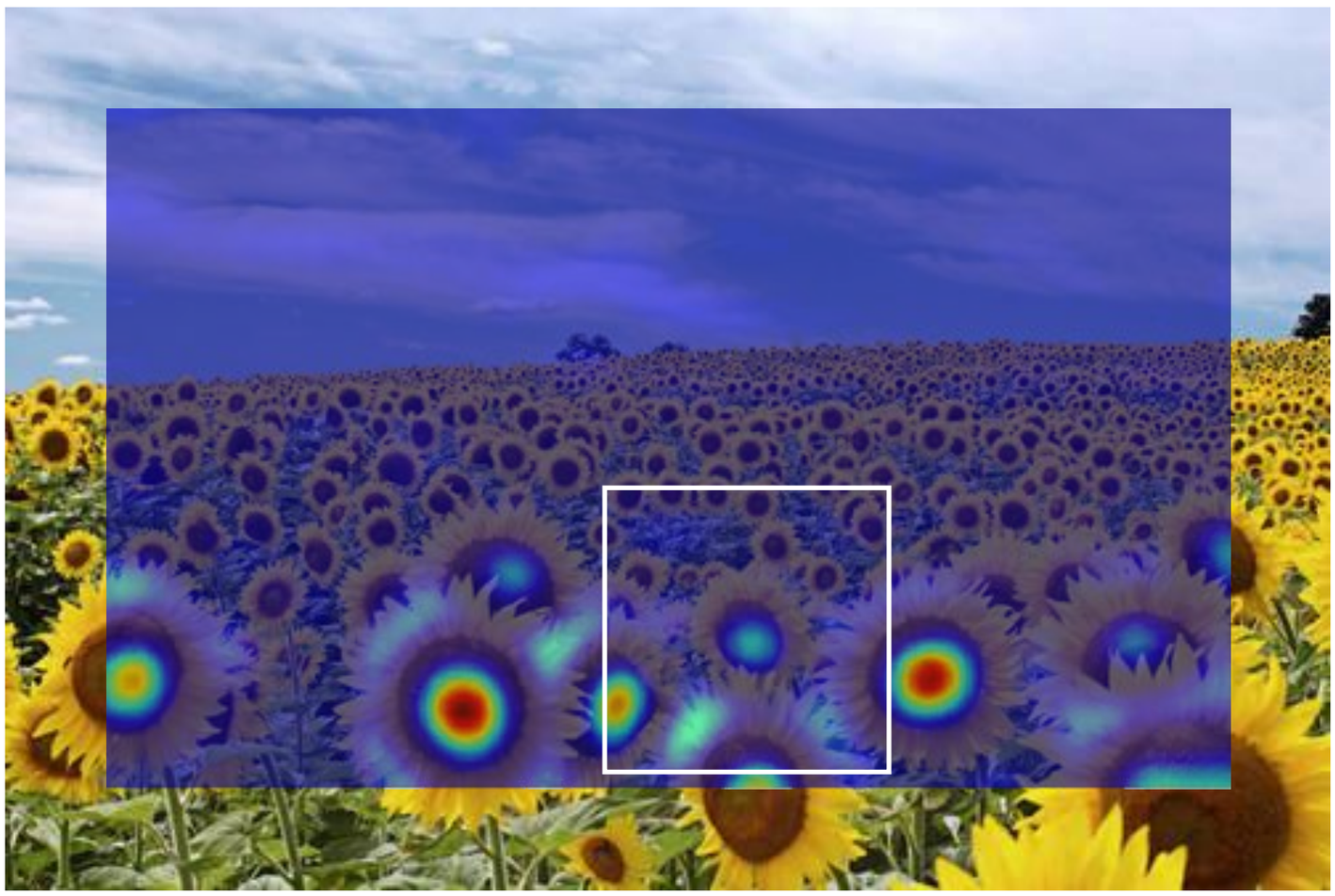
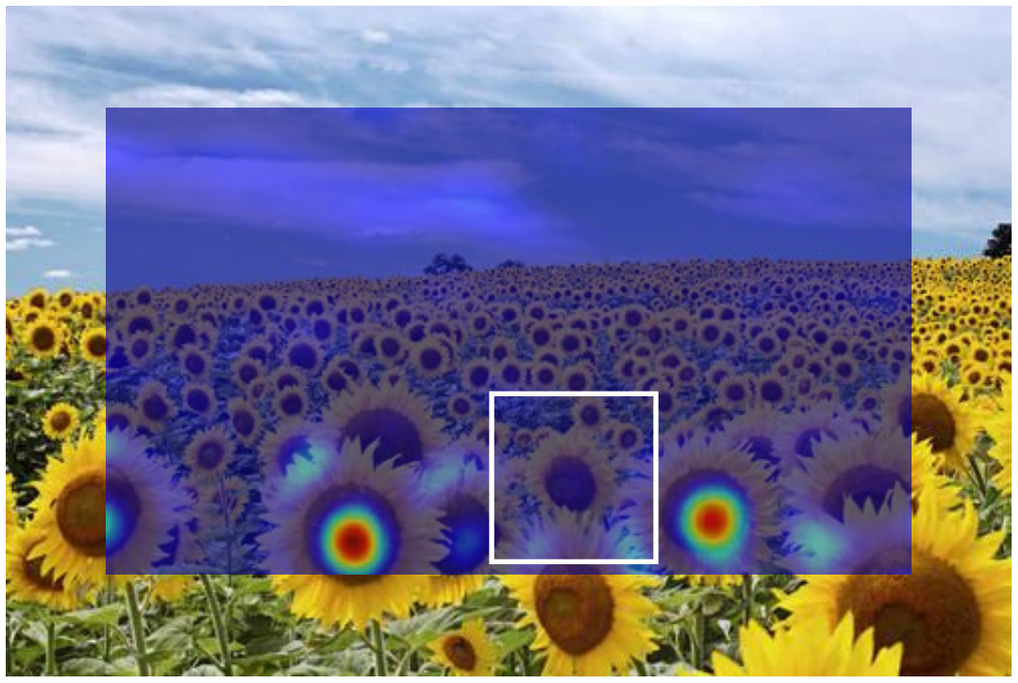
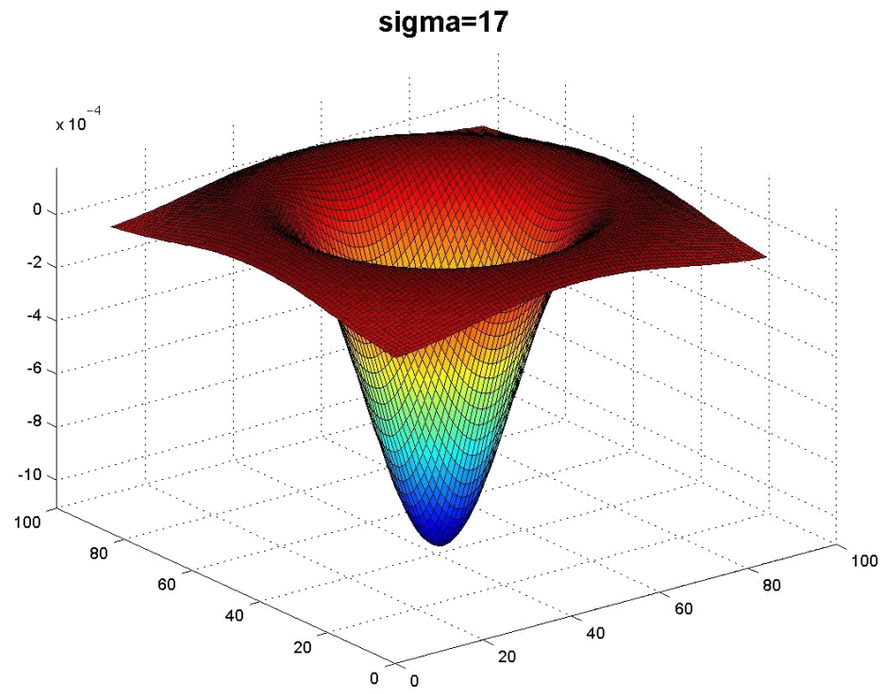
60
80
100



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x 10^-4
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-6
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60
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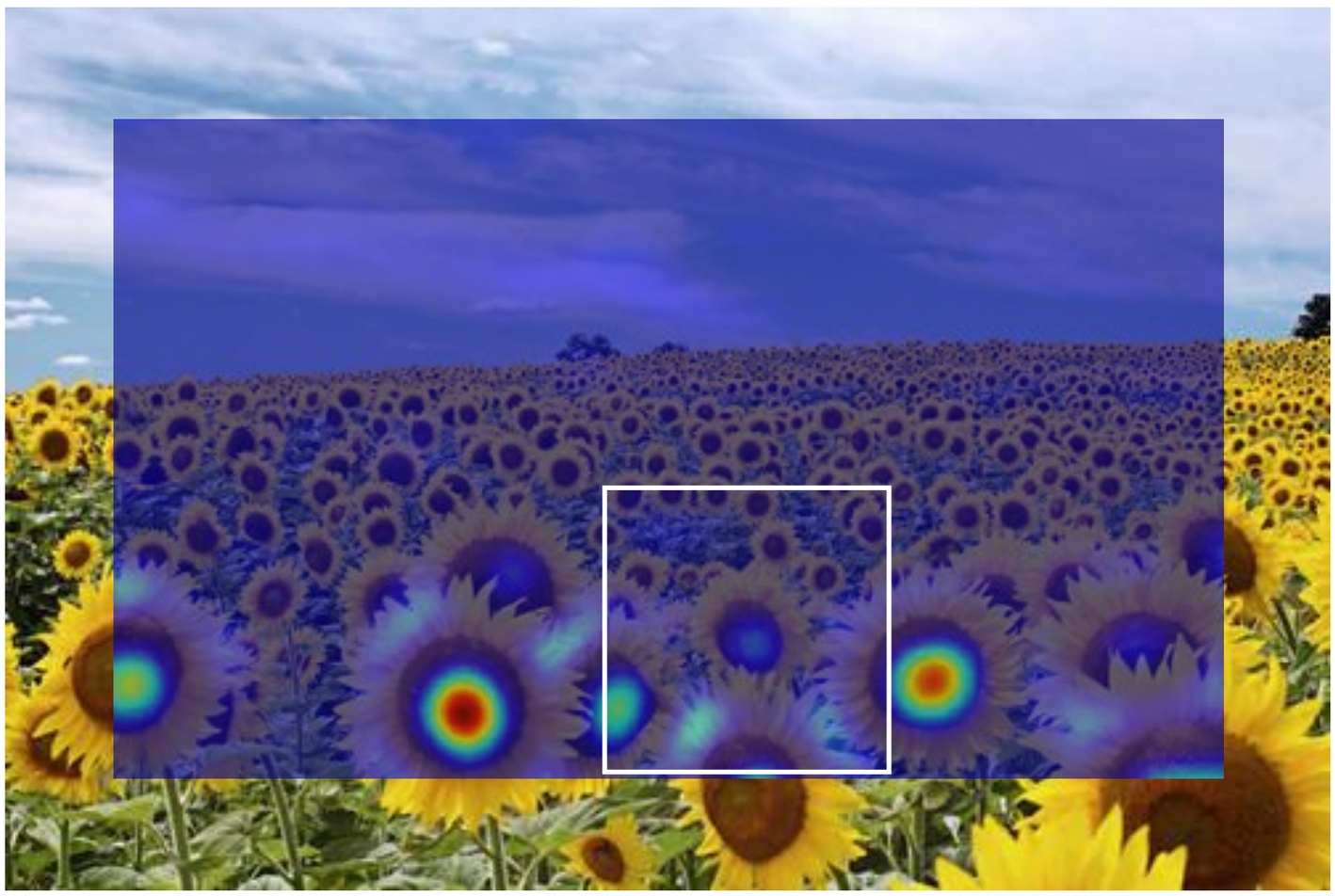
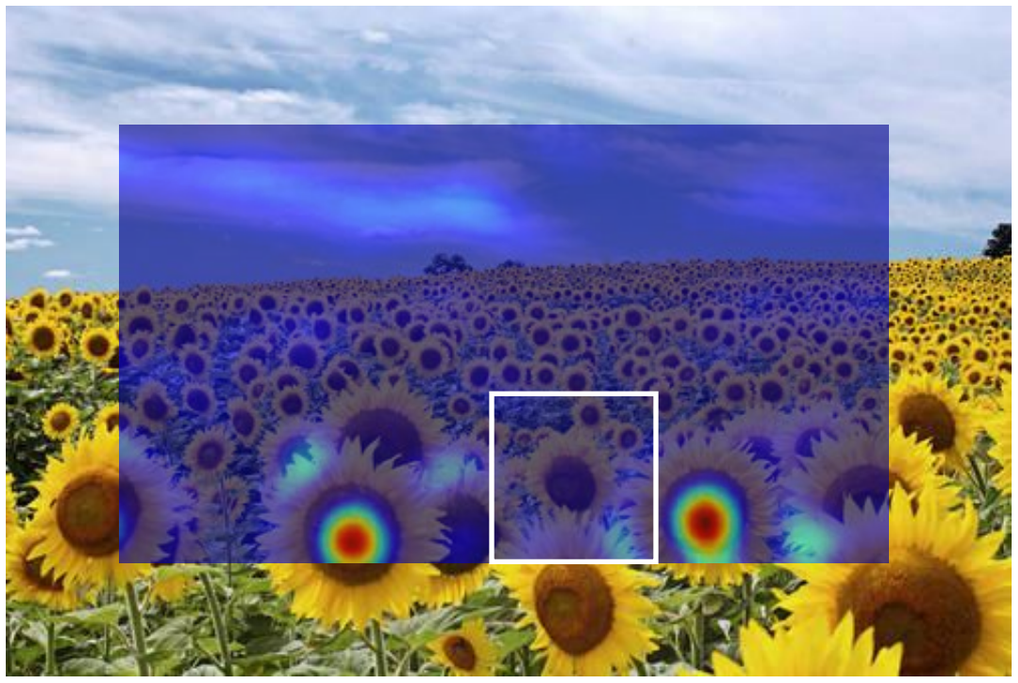
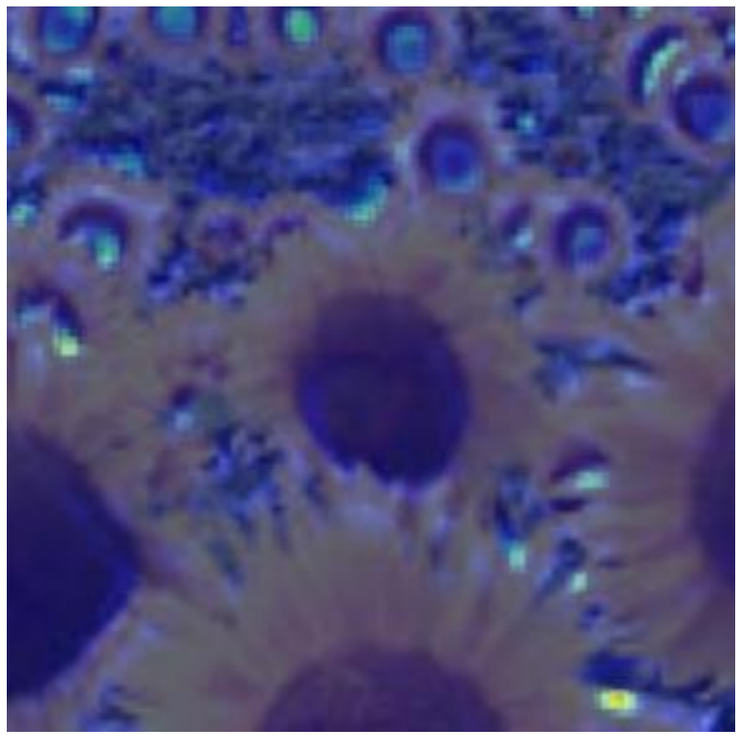
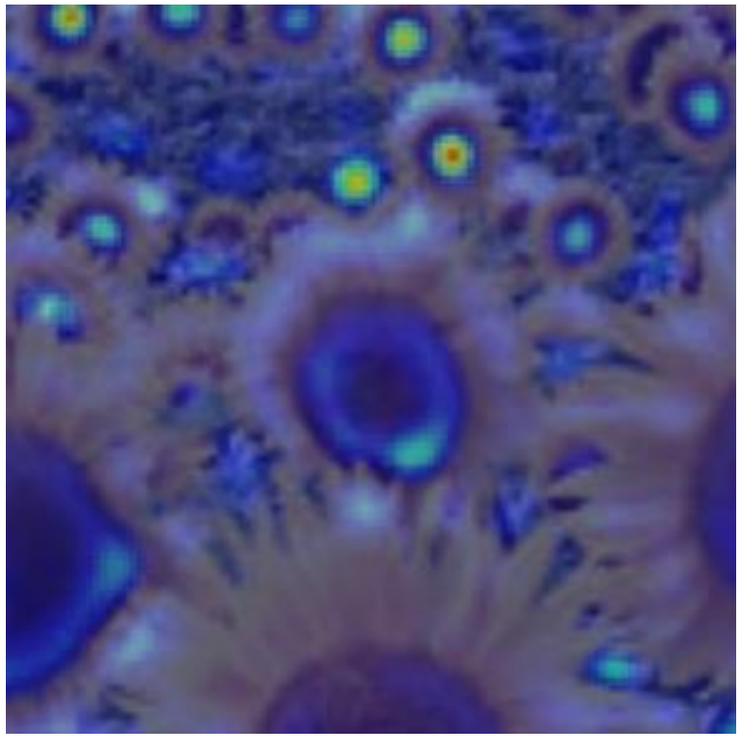




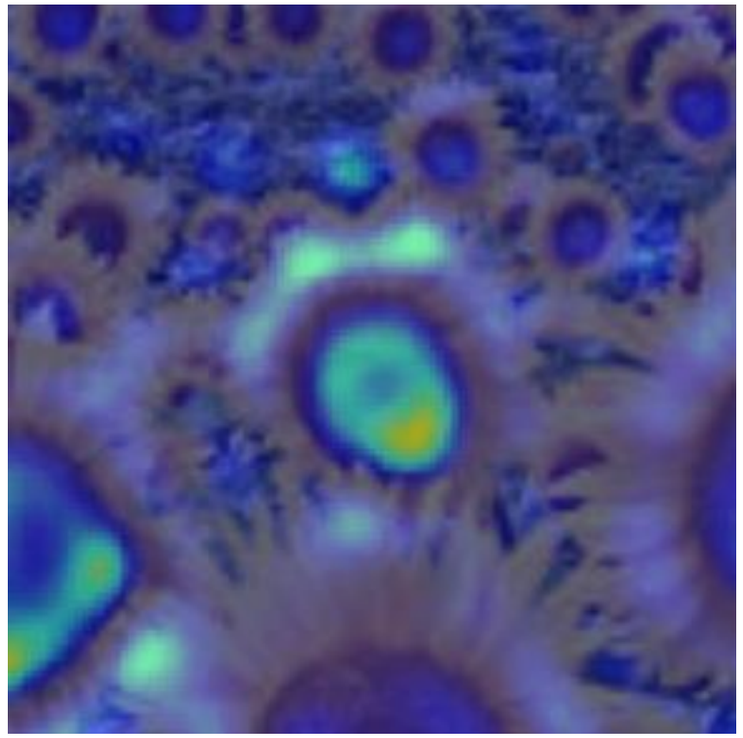
2.1



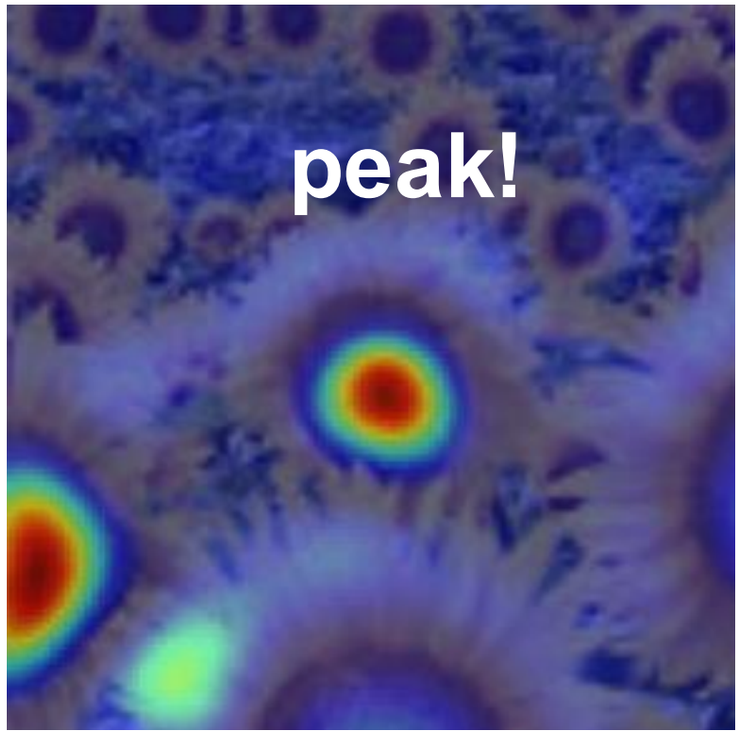
4.2



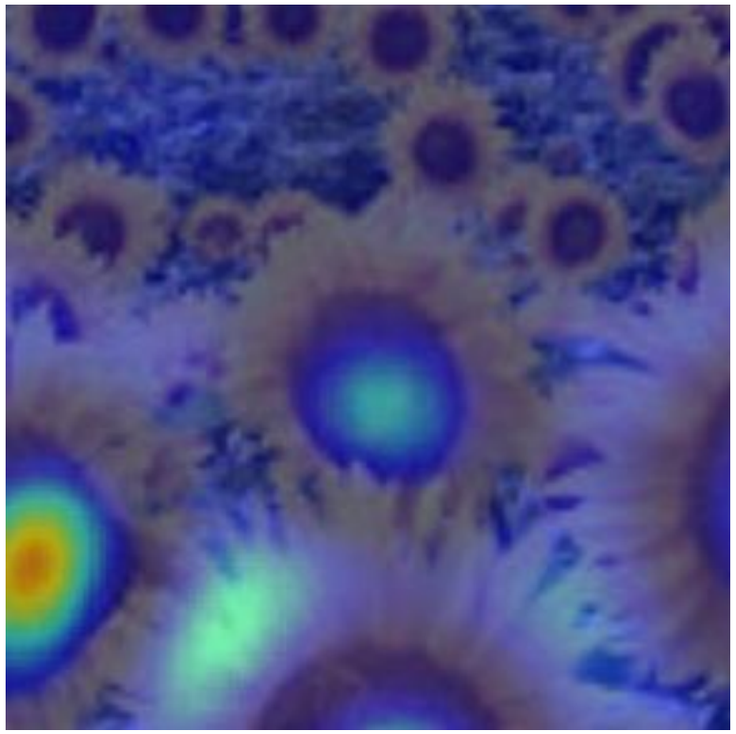
6.0



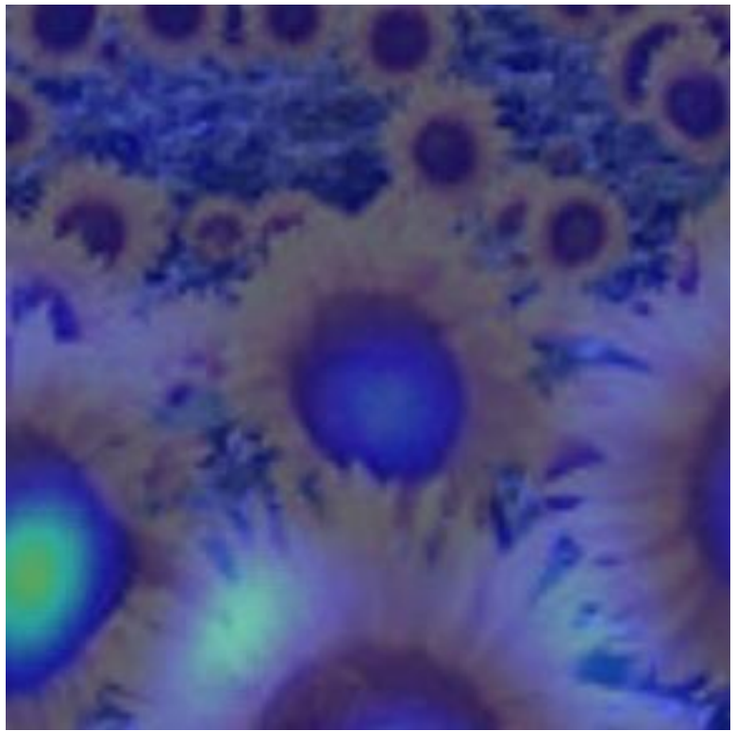
9.8



15.5

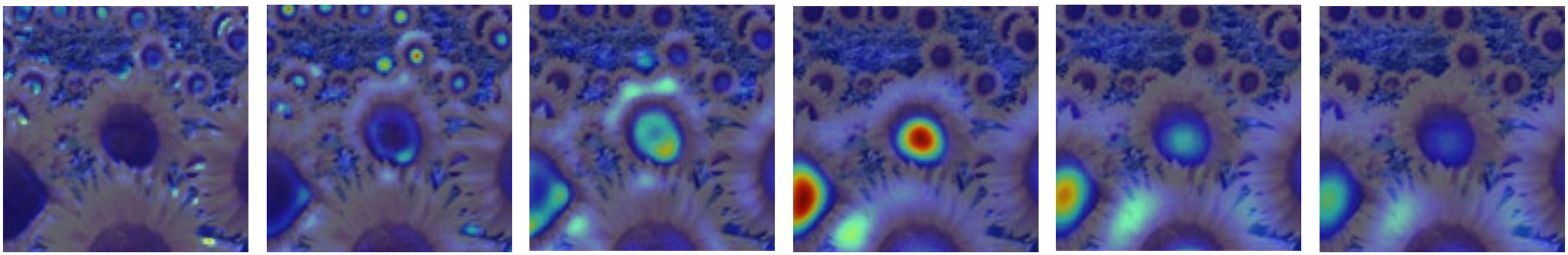


17.0



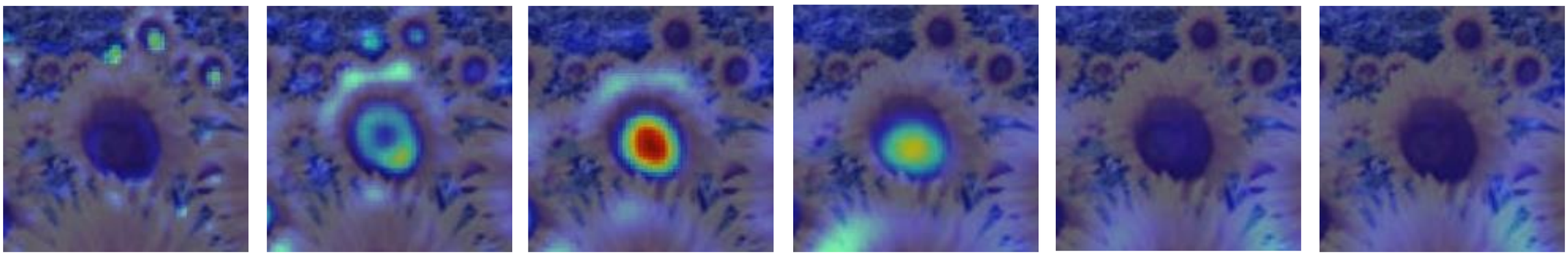
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



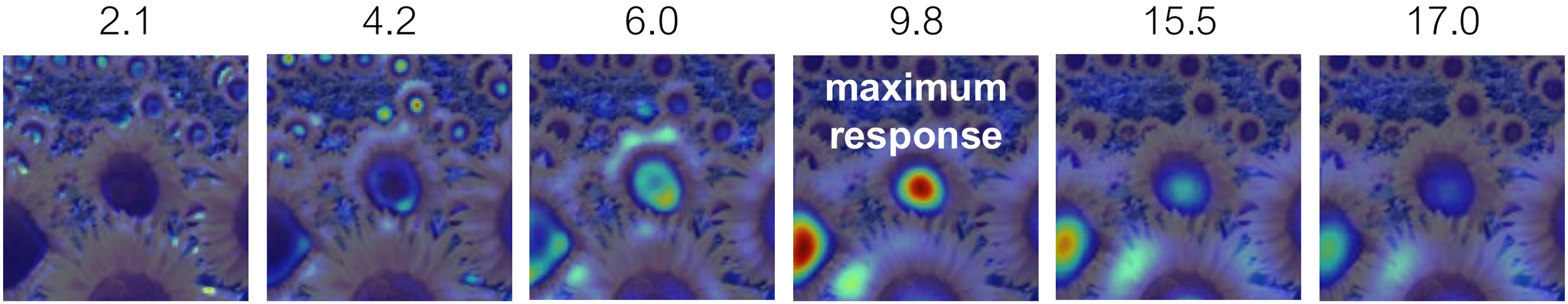
Full size image

2.1 4.2 6.0 9.8 15.5 17.0

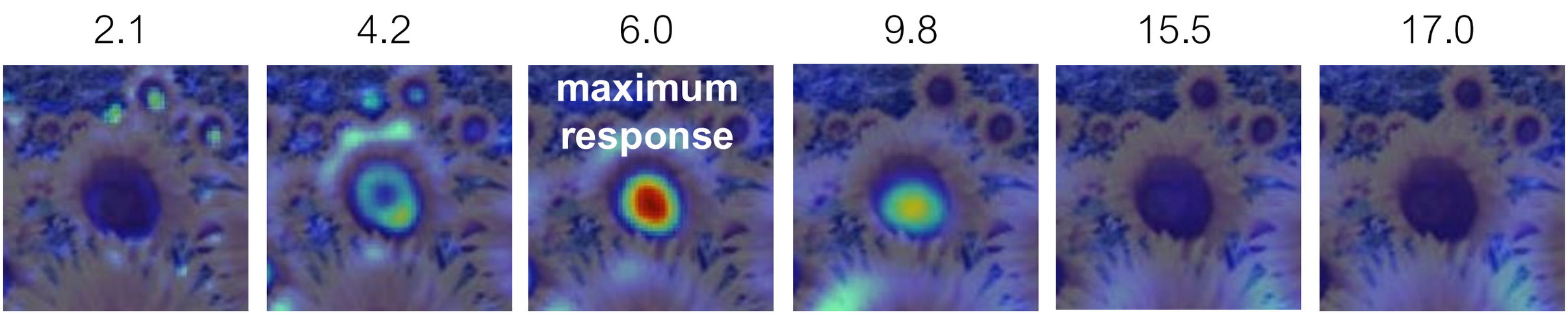


3/4 size image

optimal scale

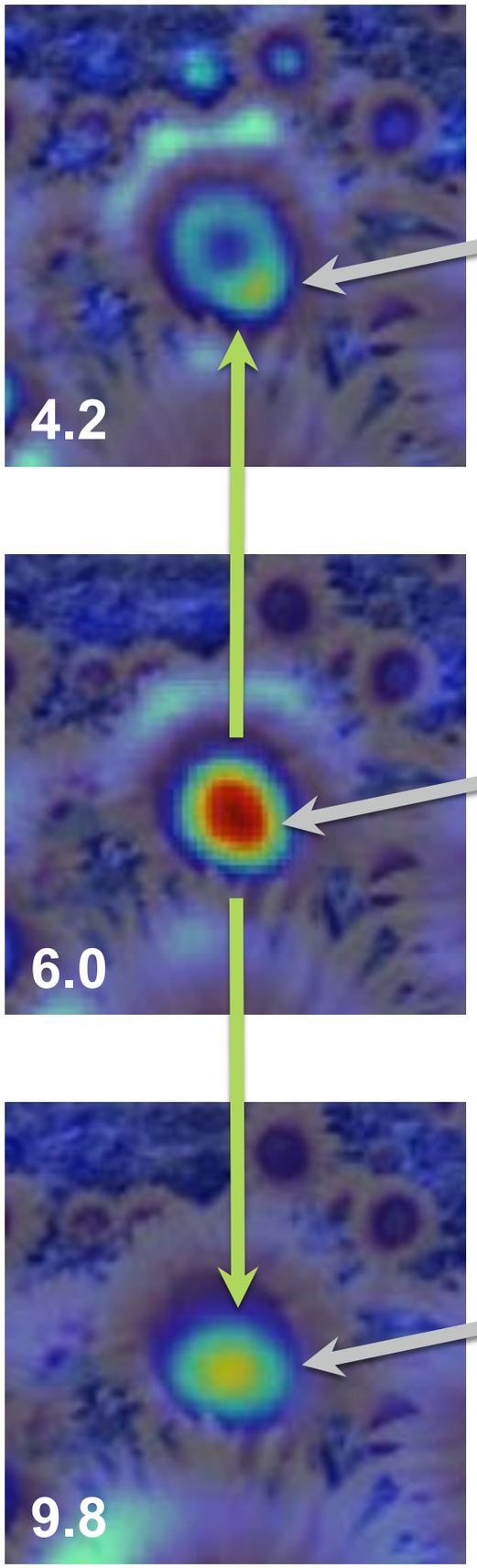


Full size image



3/4 size image

cross-scale maximum



local maximum

local maximum

local maximum

How would you implement scale selection?

Implementation

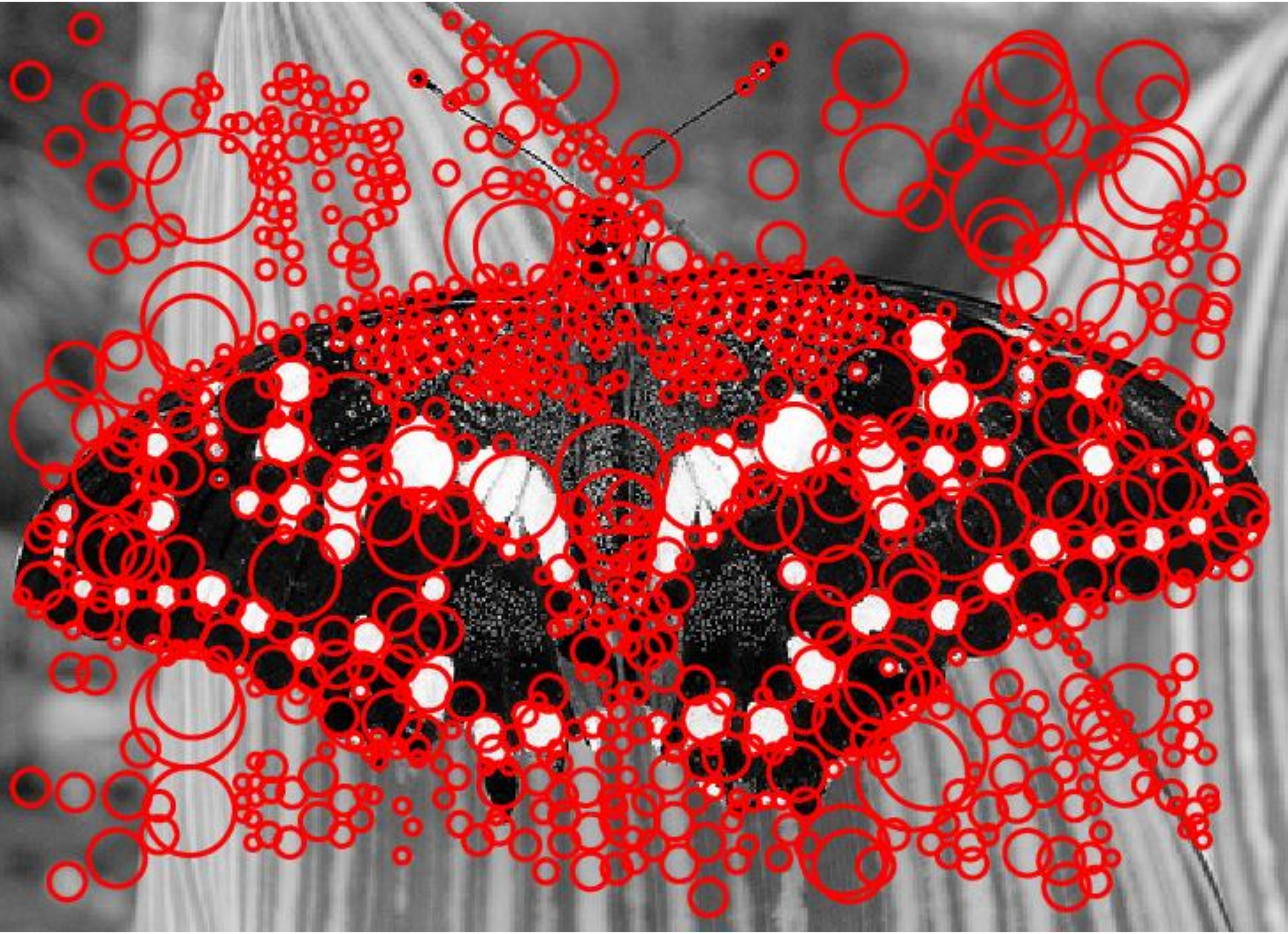
For each level of the Gaussian pyramid

compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid

if local maximum and cross-scale

save scale and location of feature (x, y, s)

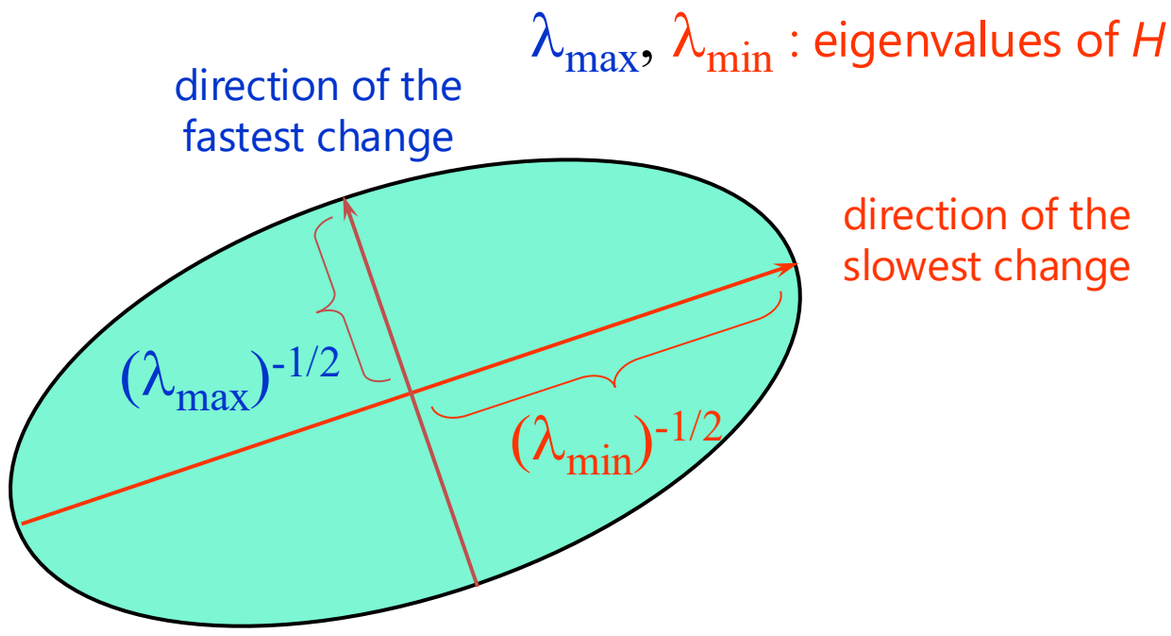


General case

We can visualize *H* as an ellipse with axis lengths determined by the *eigenvalues* of *H* and orientation determined by the *eigenvectors* of *H*

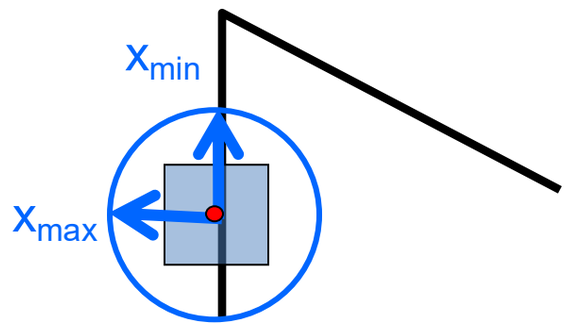
Ellipse equation:

$$[u \quad v] \; H \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Corner detection: the math

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$



$$Hx_{\max} = \lambda_{\max} x_{\max}$$

$$Hx_{\min} = \lambda_{\min} x_{\min}$$

Eigenvalues and eigenvectors of H

- Define shift directions with the smallest and largest change in error
- x_{max} = direction of largest increase in *E*
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in *E*
- λ_{min} = amount of increase in direction x_{min}

Corner detection: the math

How are λ_{max}, x_{max}, λ_{min}, and x_{min} relevant for feature detection?

- What's our feature scoring function?

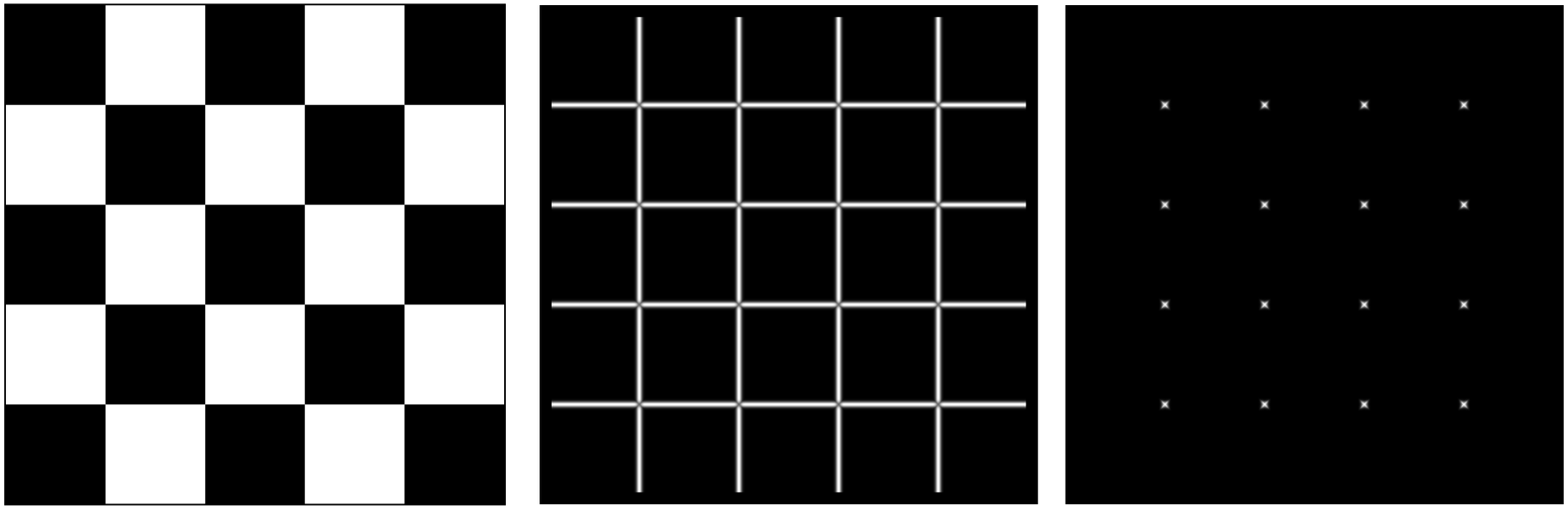
Corner detection: the math

How are λ_{max}, x_{max}, λ_{min}, and x_{min} relevant for feature detection?

- What's our feature scoring function?

Want $E(u,v)$ to be large for small shifts in all directions

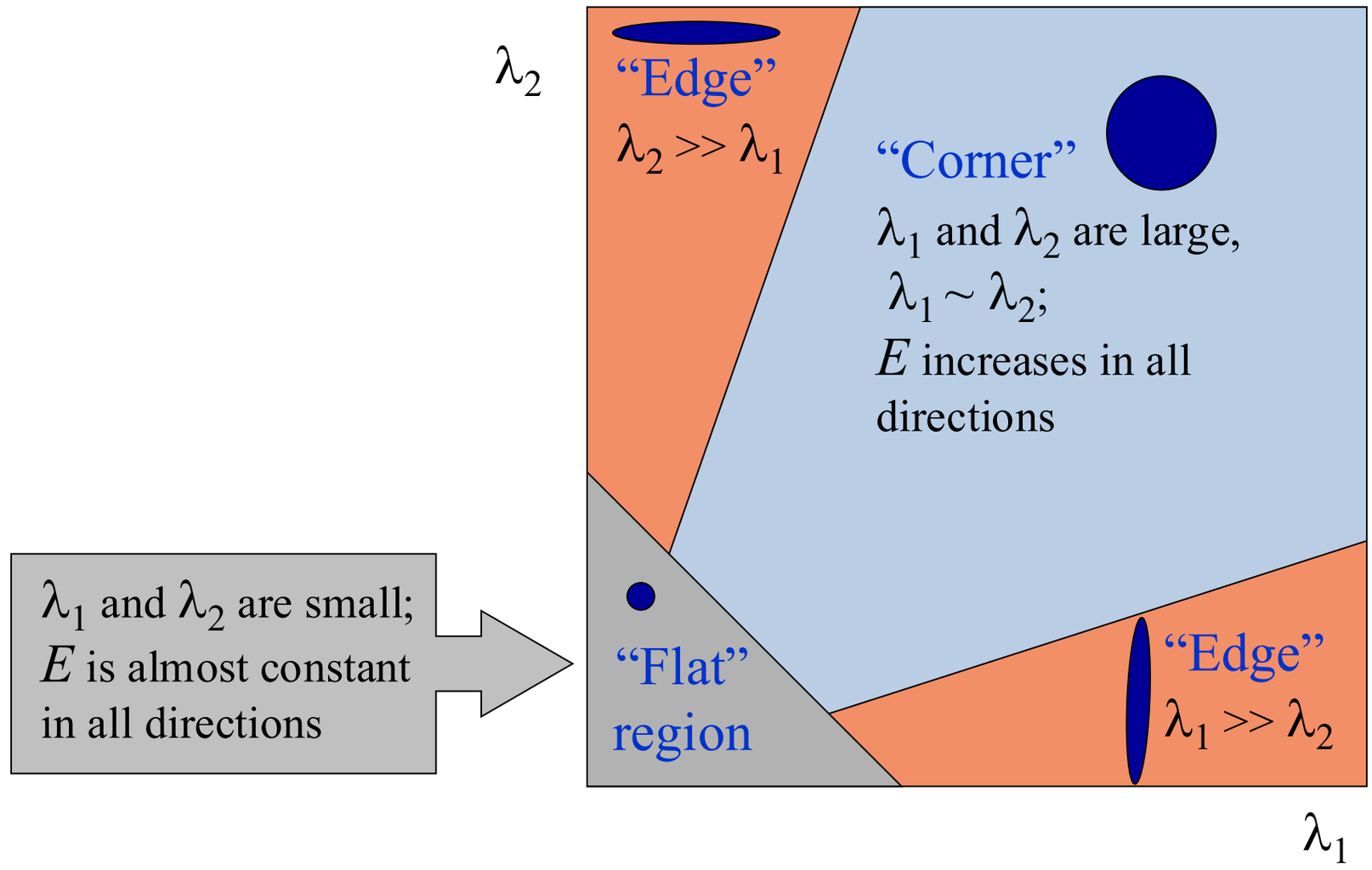
- the minimum of $E(u,v)$ should be large, over all unit vectors $[u\ v]$
- this minimum is given by the smaller eigenvalue (λ_{min}) of H



I $\lambda_{\max}$ $\lambda_{\min}$

Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:

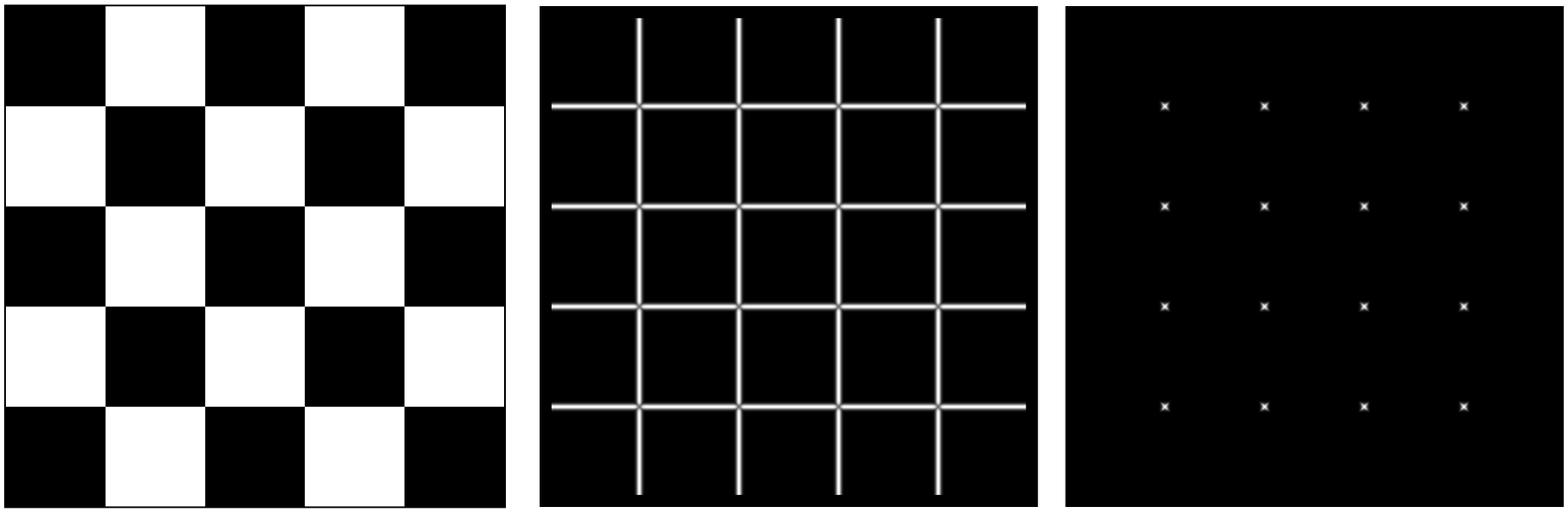


Corner detection summary

Here's what you do:

- Compute the gradient at each point in the image
- For each pixel:
  - Create the *H* matrix from nearby gradient values
  - Compute the eigenvalues.
  - Find points with large response (λ_{min} > threshold)
- Choose those points where λ_{min} is a local maximum as features

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



I $\lambda_{\max}$ $\lambda_{\min}$

Corner detection summary

Here's what you do:

- Compute the gradient at each point in the image
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  - Create the *H* matrix from nearby gradient values
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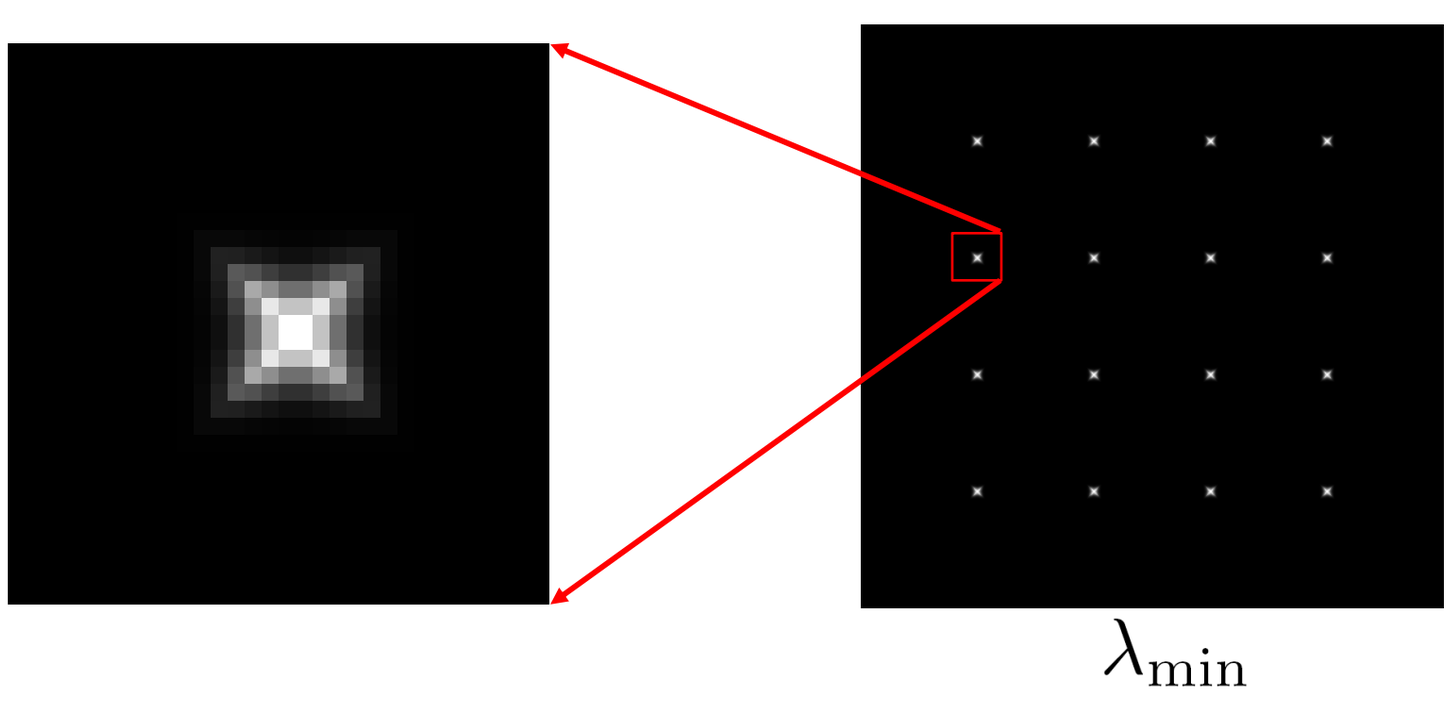


The Harris operator

λ_{min} is a variant of the "Harris operator" for feature detection

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$
$$= \frac{determinant(H)}{trace(H)}$$

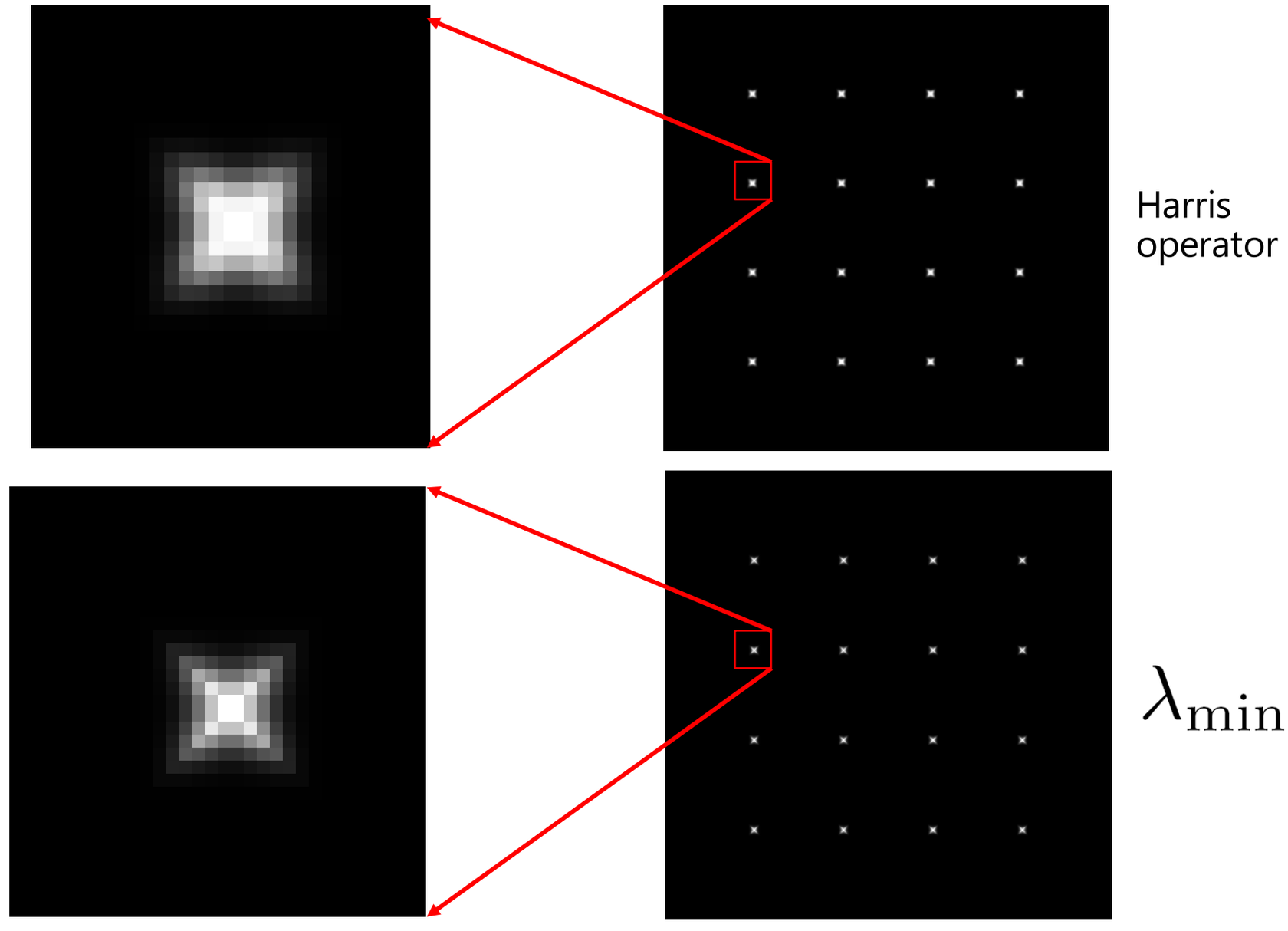
- The *trace* is the sum of the diagonals, i.e., *trace(H) = h_{11} + h_{22}*
- Very similar to λ_{min} but less expensive (no square root)
- Called the *Harris Corner Detector* or *Harris Operator*
- Lots of other detectors, this is one of the most popular

Alternate Harris operator

- For Project 2, you will use an alternate definition of the Harris operator:

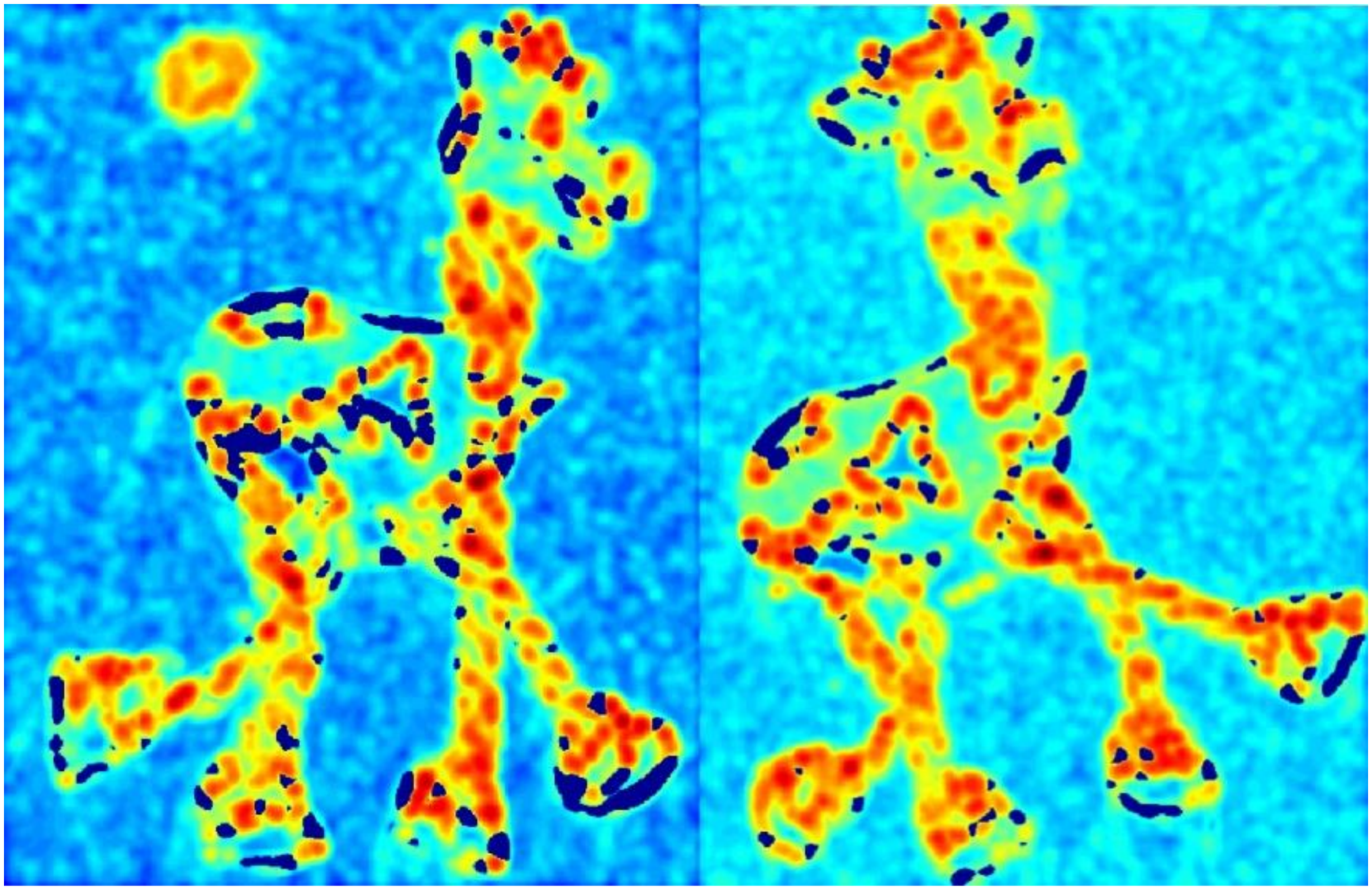
$$R = \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2 = \det(M) - k \cdot \mathrm{tr}(M)^2$$

The Harris operator

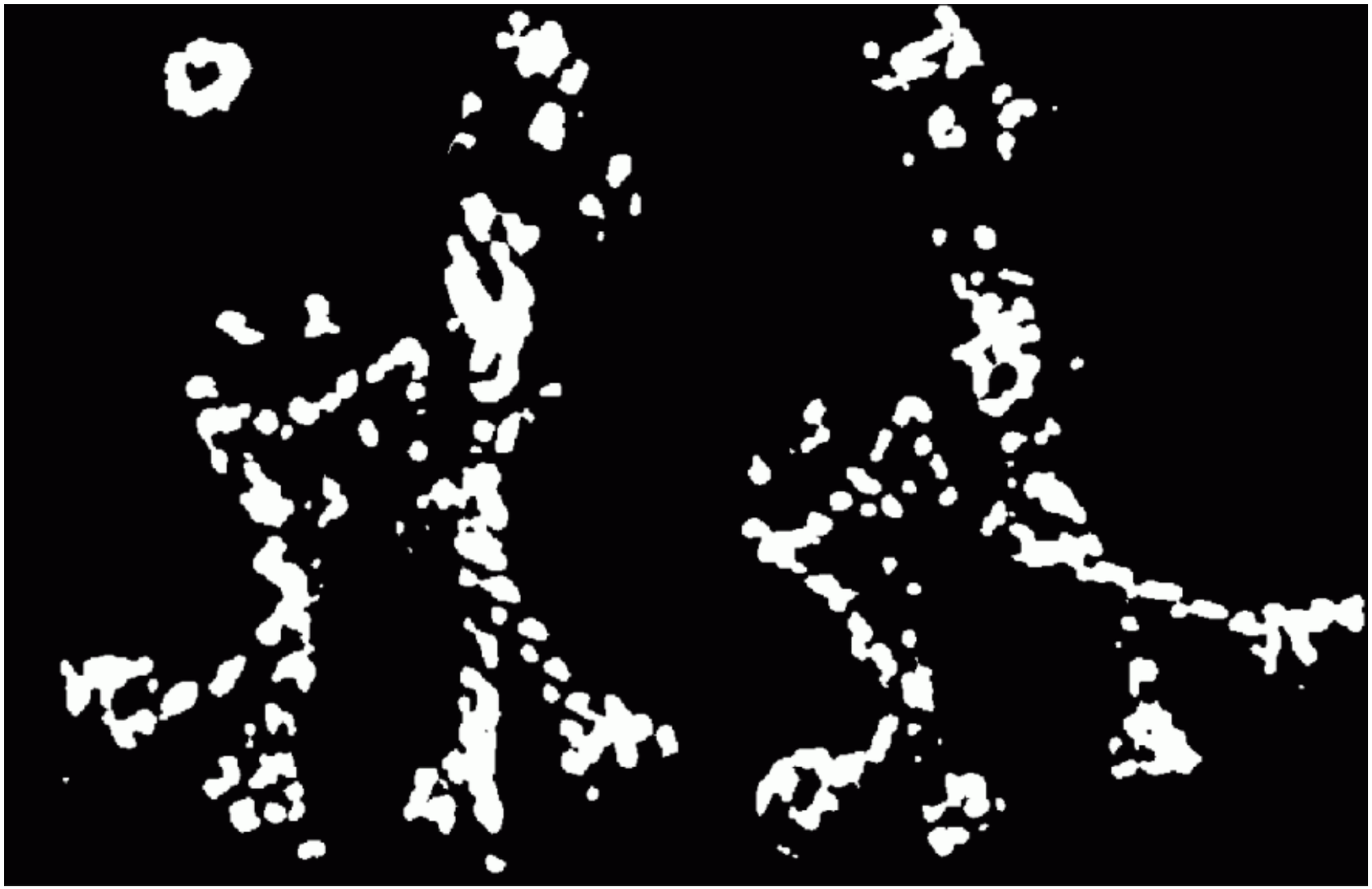


Harris detector example

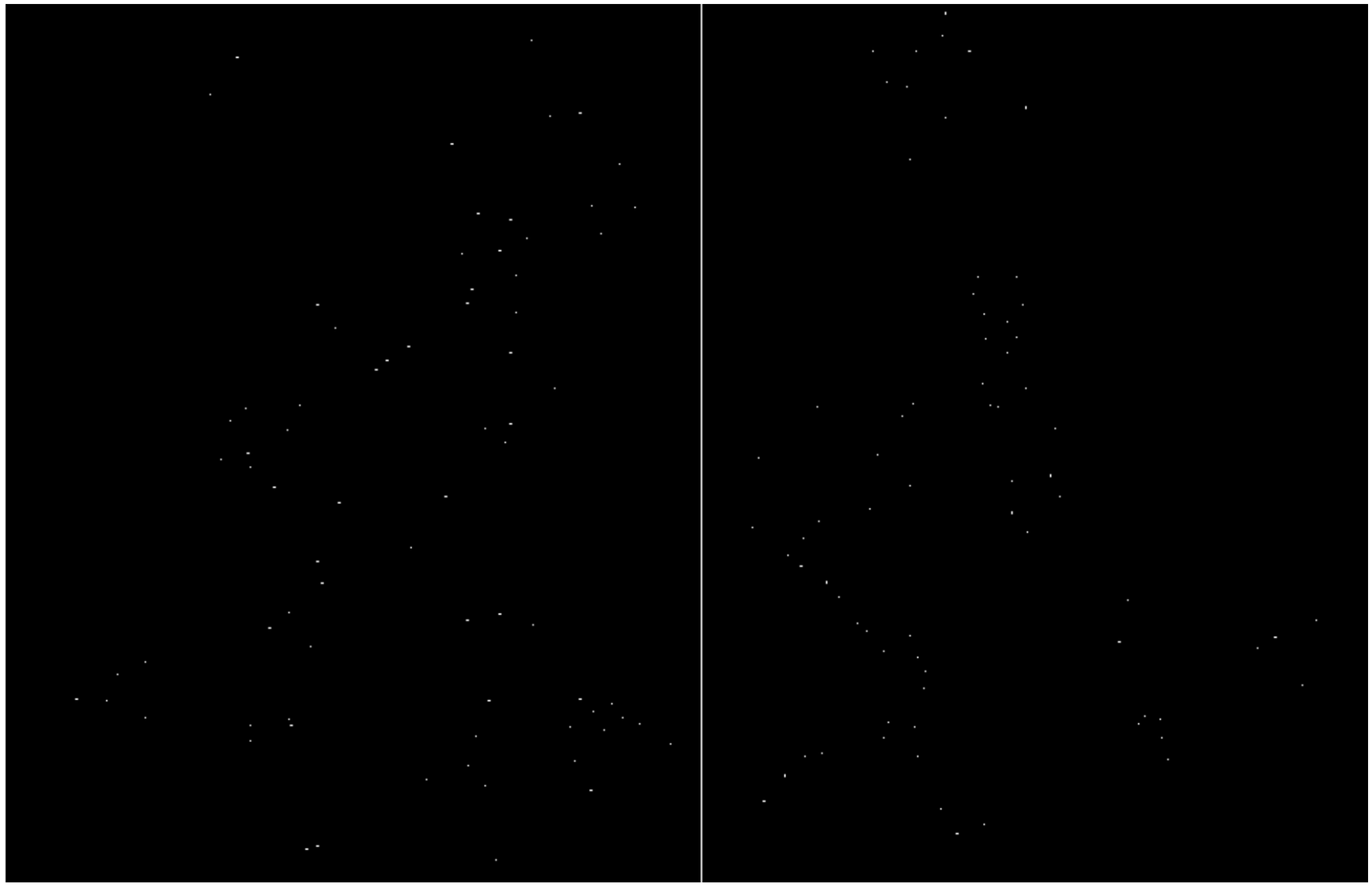
f value (red high, blue low)



Threshold (f > value)



Find local maxima of f (non-max suppression)



Harris features (in red)

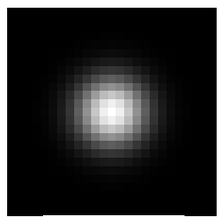
Weighting the derivatives

- In practice, using a simple window *W* doesn't work too well

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Instead, we'll *weight* each derivative value based on its distance from the center pixel

$$H = \sum_{(x,y)\in W} w_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

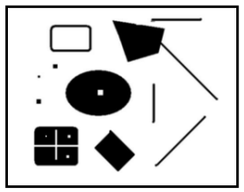


$w_{x,y}$

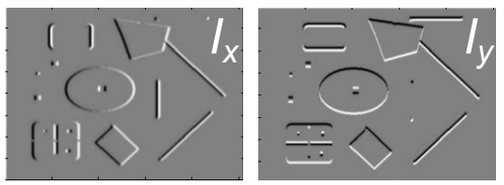
Harris Detector [Harris88]

- Second moment matrix

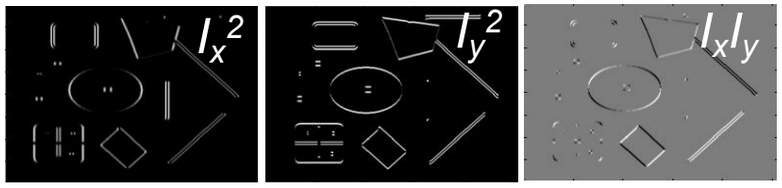
$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$



1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(s_I)$

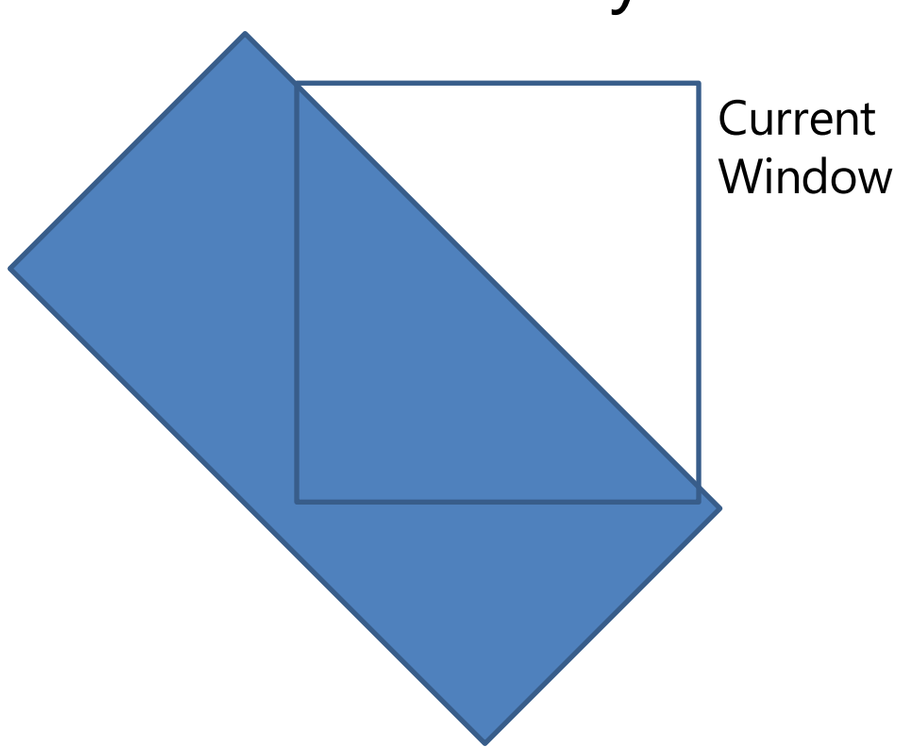
$$\det M = \lambda_1 \lambda_2$$
$$\text{trace}\, M = \lambda_1 + \lambda_2$$

4. Cornerness function – both eigenvalues are strong

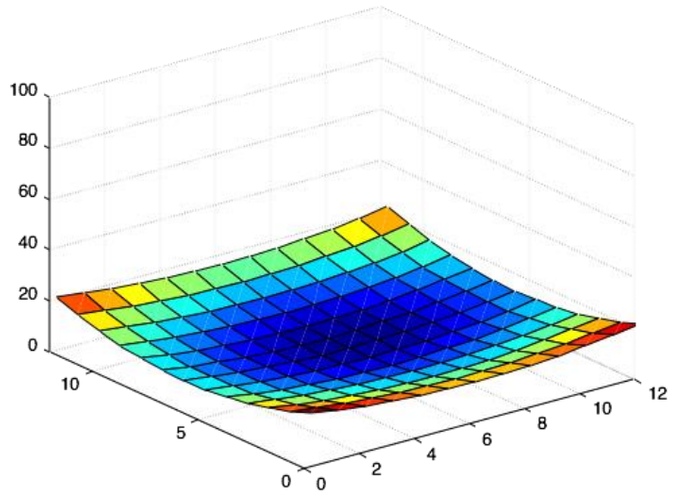
5. Non-maxima suppression

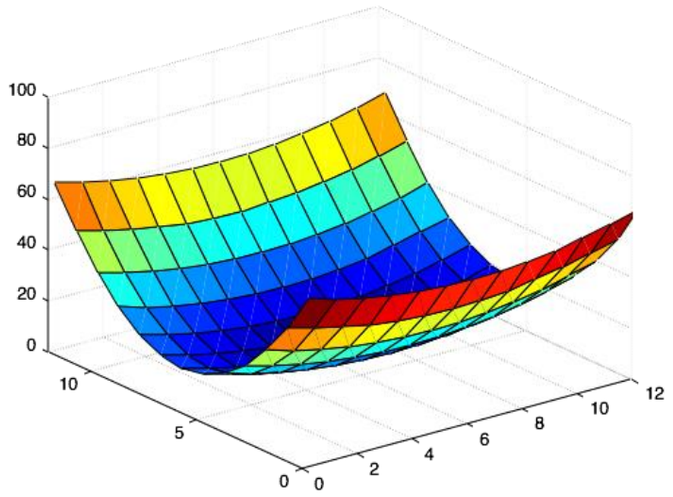
Harris Corners – Why so complicated?

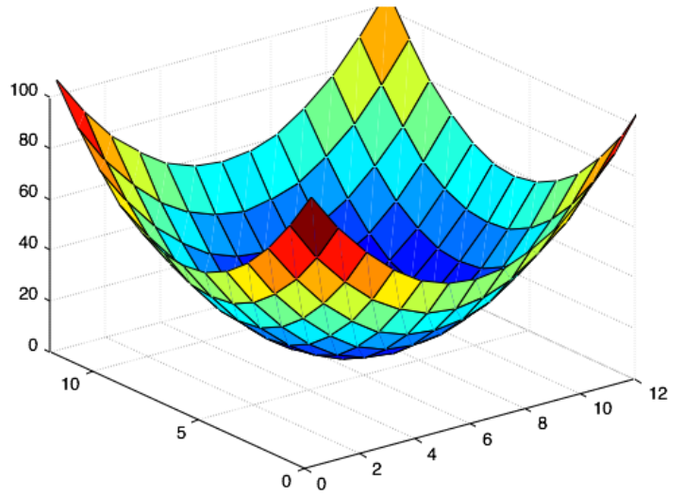
- Can’t we just check for regions with lots of gradients in the x and y directions?
  - No! A diagonal line would satisfy that criteria



Which error surface indicates a good image feature?

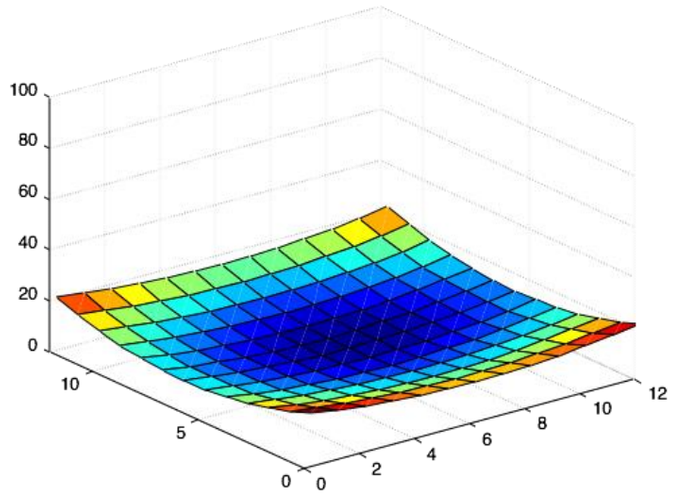




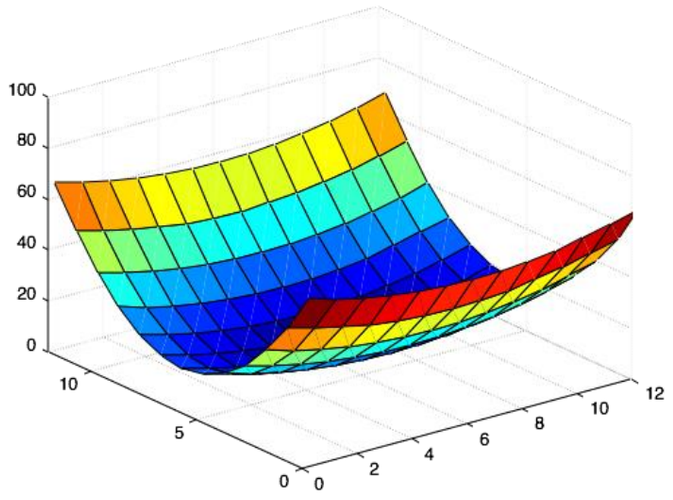


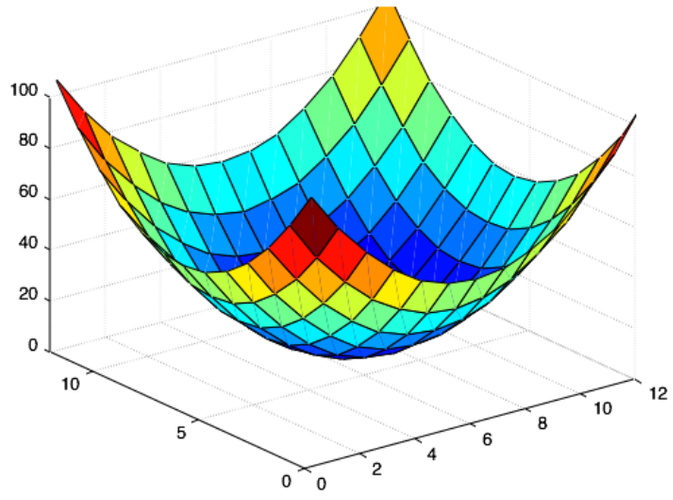
What kind of image patch do these surfaces represent?

Which error surface indicates a good image feature?

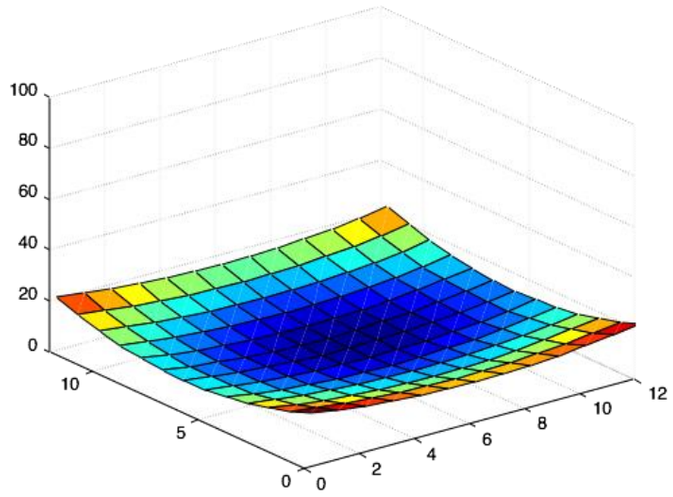


flat

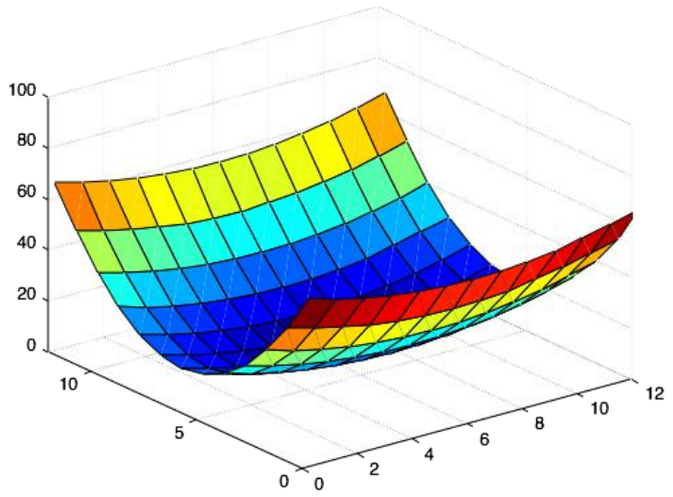




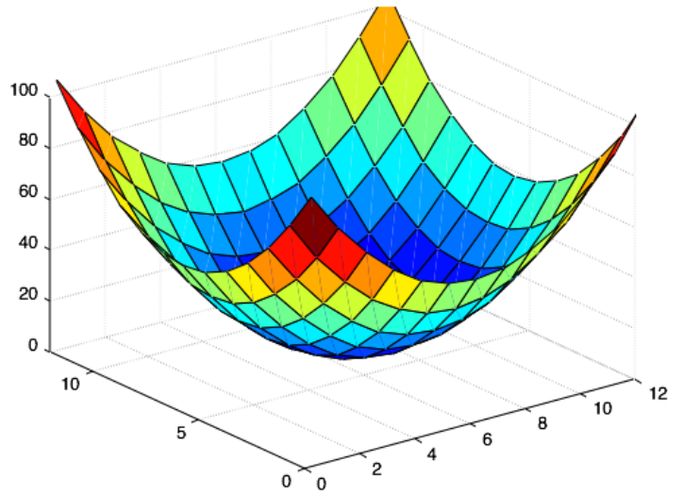
Which error surface indicates a good image feature?



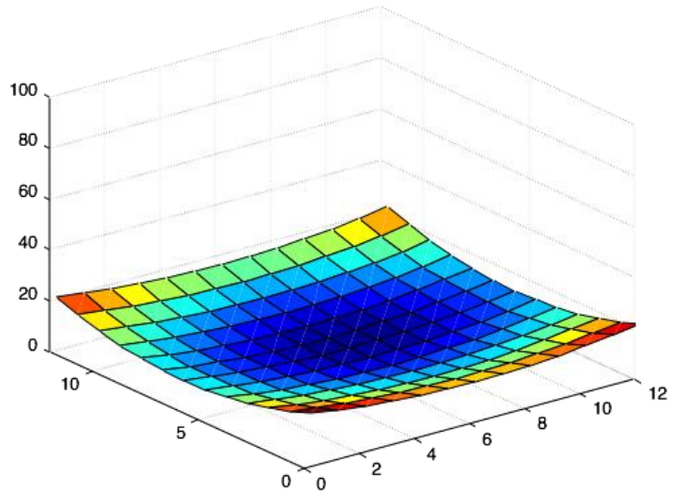
flat



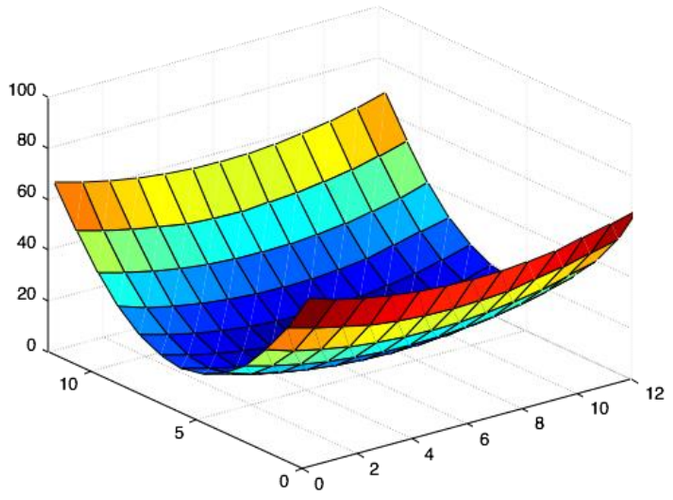
edge
‘line’



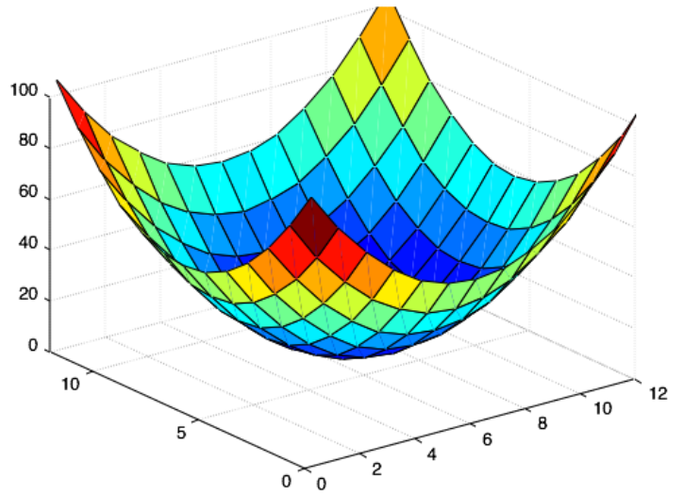
Which error surface indicates a good image feature?



flat



edge
‘line’



corner
‘dot’

Recap

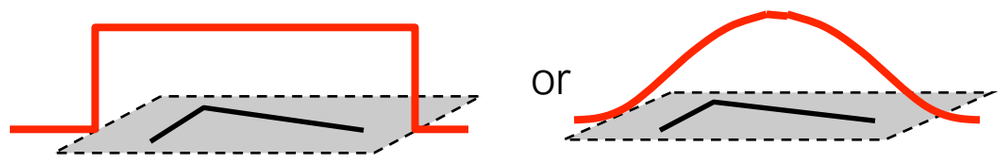
- Why detect corners?
- Visualizing quadratics.
- **Harris corner detector.**
- Multi-scale detection.
- Multi-scale blob detection.

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Error function — Window function — Shifted intensity — Intensity

Window function $w(x,y)$ =

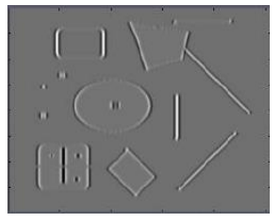


1 in window, 0 outside — or — Gaussian

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Recap

$I_x = \frac{\partial I}{\partial x}$



$I_y = \frac{\partial I}{\partial y}$

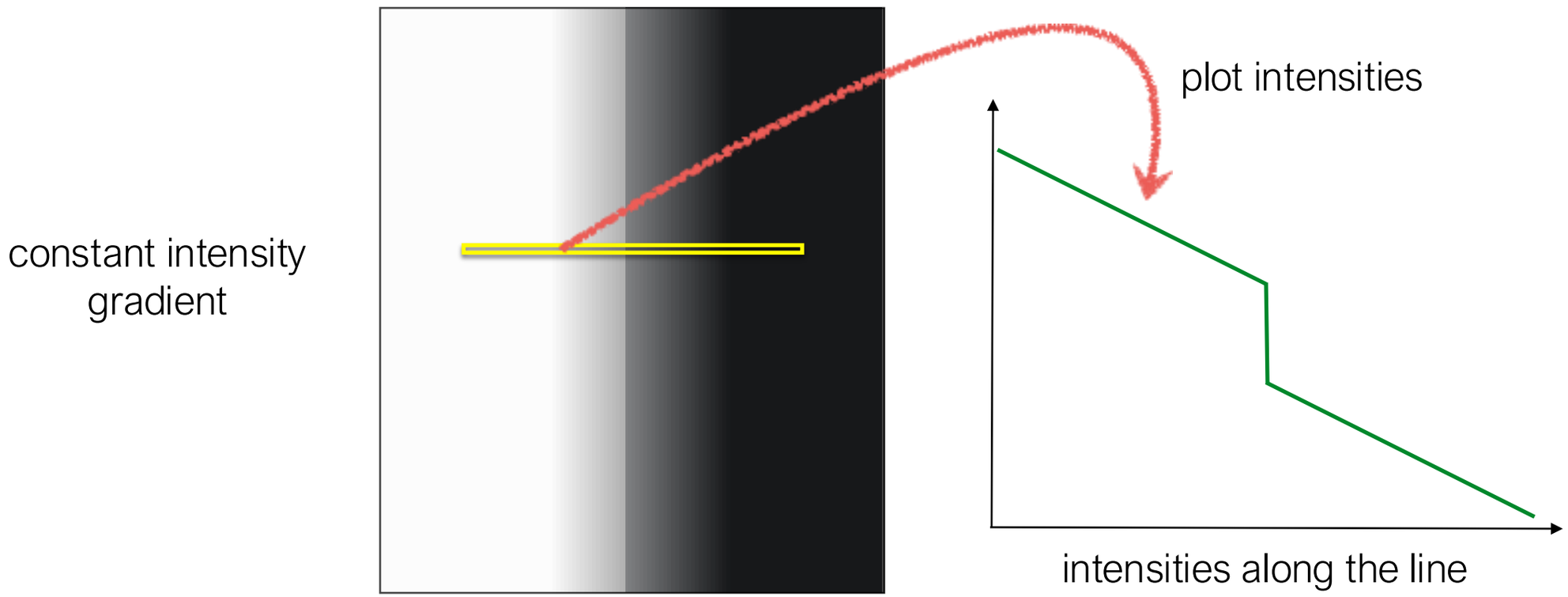
- Why detect corners?
- Visualizing quadratics.
- **Harris corner detector.**
- Multi-scale detection.
- Multi-scale blob detection.

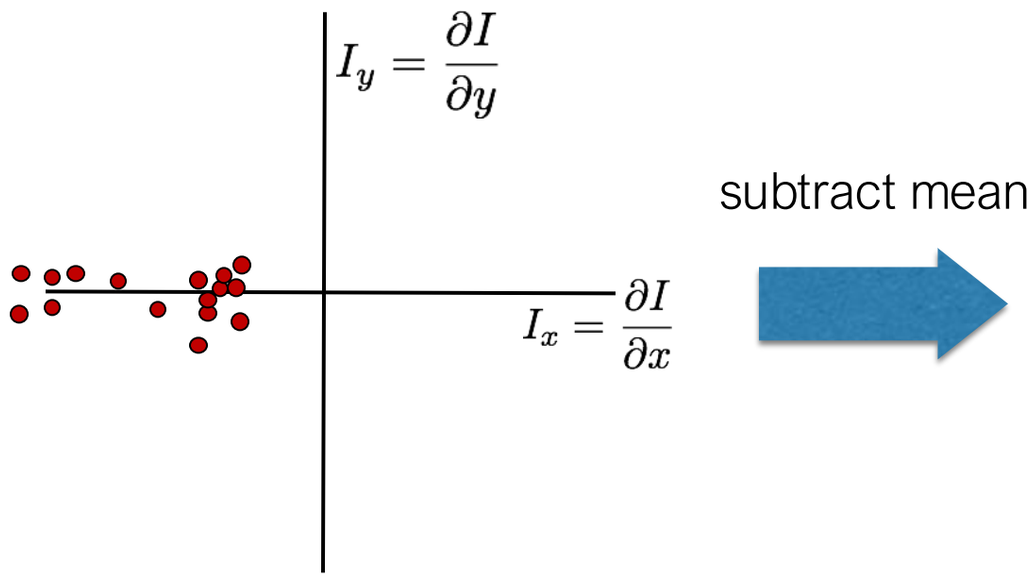
1. Compute image gradients over small region
2. Subtract mean from each image gradient
3. Compute the covariance matrix
4. Compute eigenvectors and eigenvalues

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

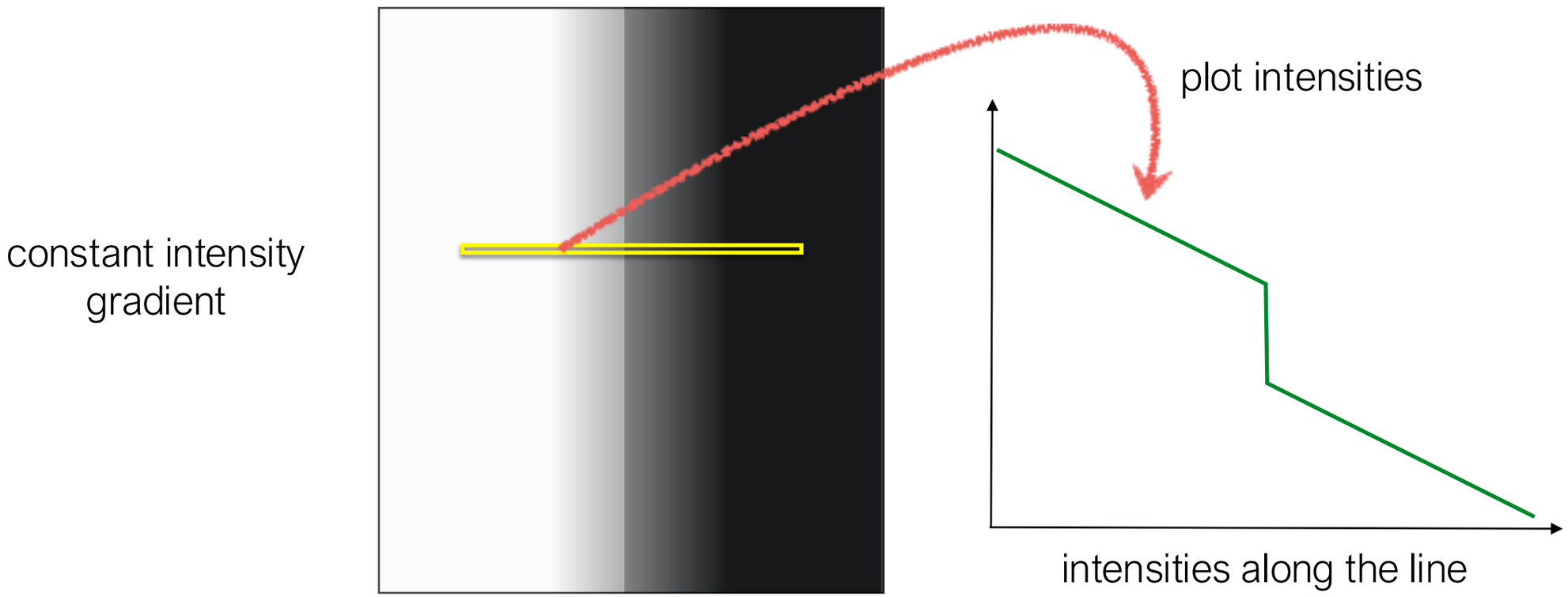
5. Use threshold on eigenvalues to detect corners

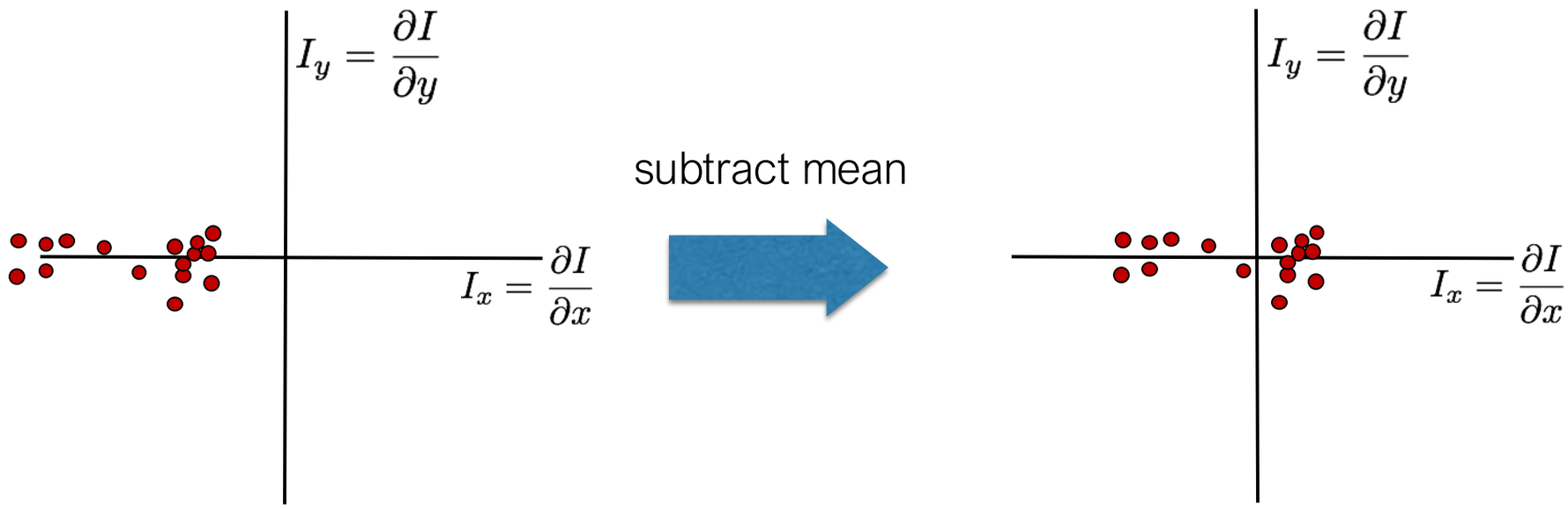
2. Subtract the mean from each image gradient





2. Subtract the mean from each image gradient





plot of image gradients

data is centered
('DC' offset is removed)

eig(M)

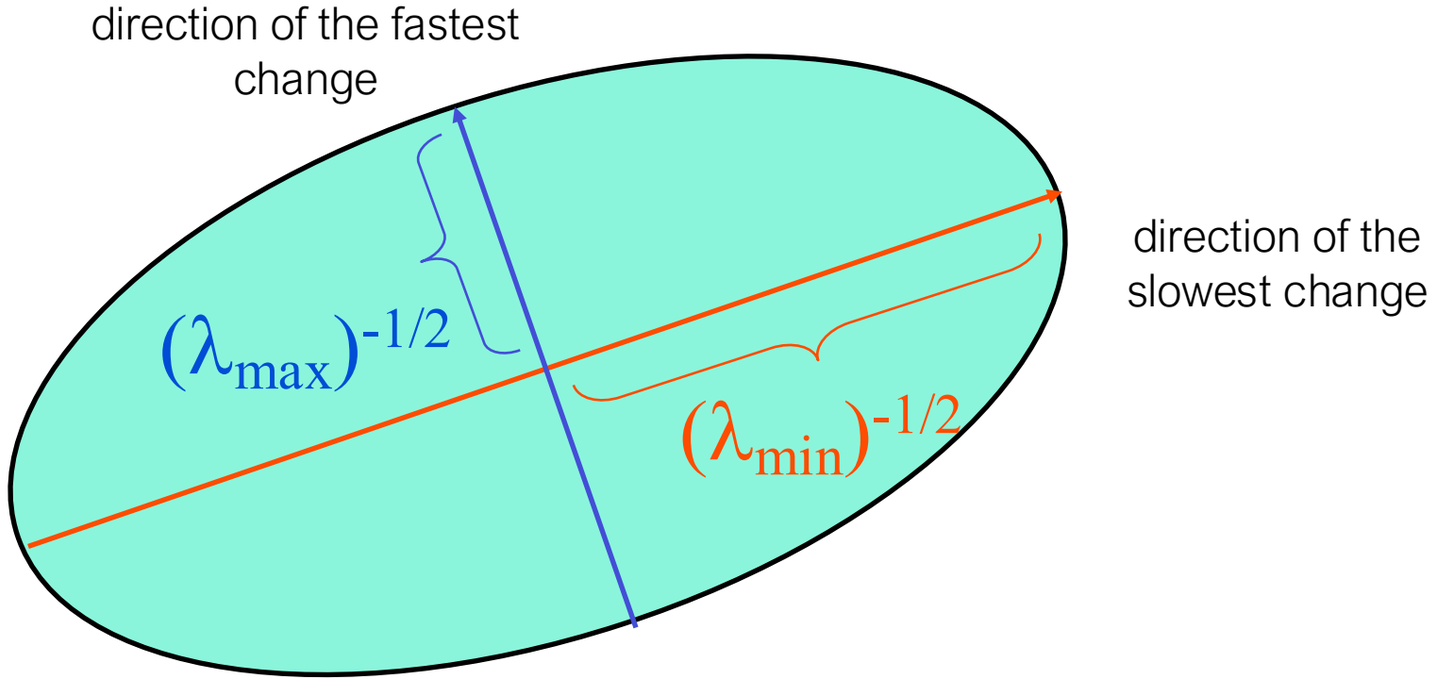
Visualization as an ellipse

Since M is symmetric, we have $M = R^{-1}\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}R$

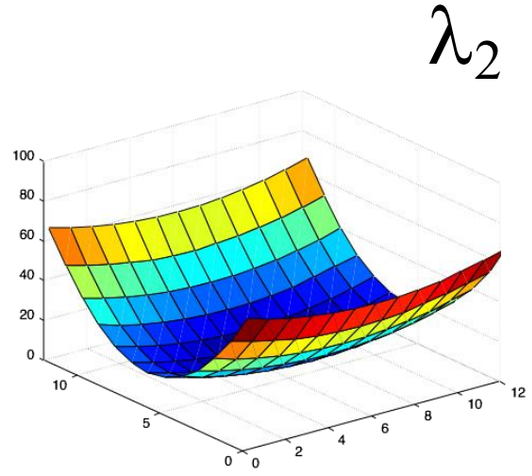
We can visualize ***M*** as an ellipse with axis lengths determined by the eigenvalues and orientation determined by ***R***

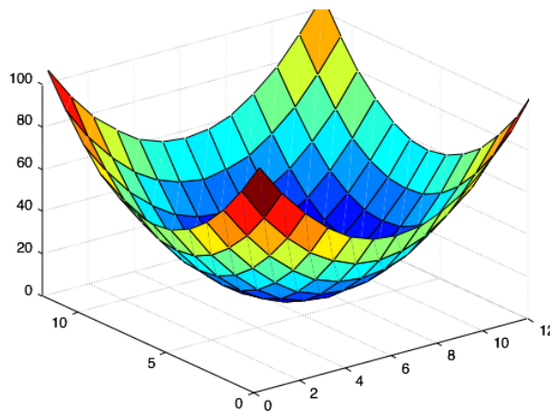
Ellipse equation:

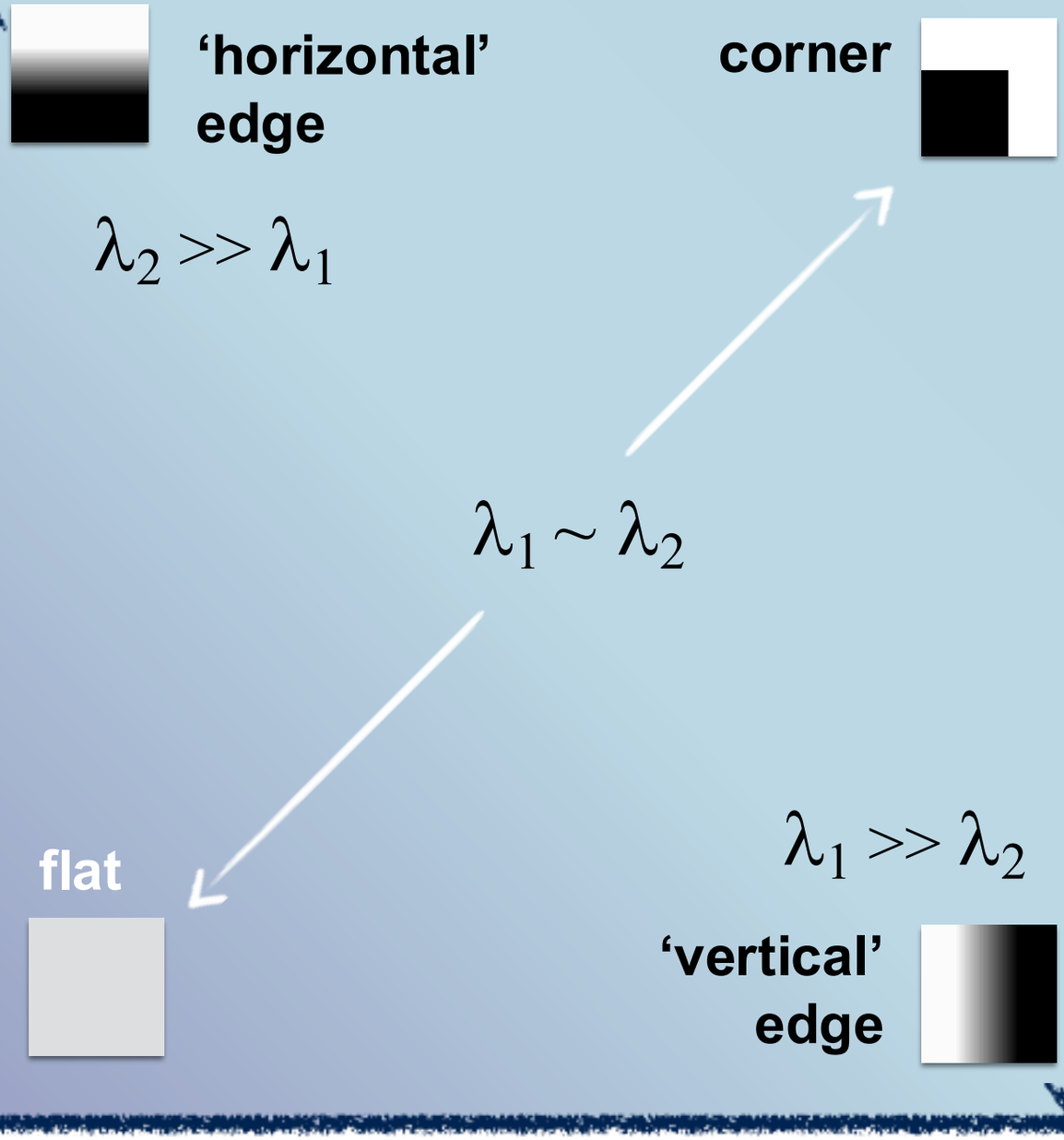
$$[u \quad v]\ M \begin{bmatrix}u\\ v\end{bmatrix} = \text{const}$$

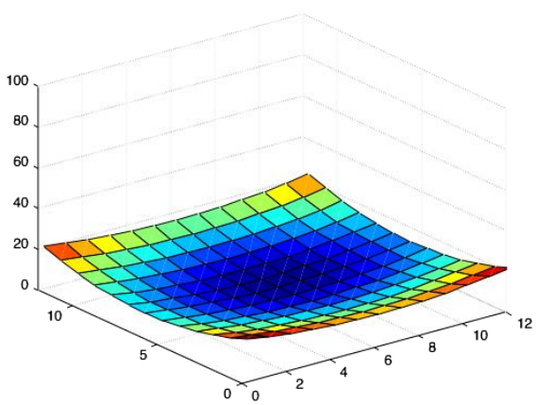


interpreting eigenvalues

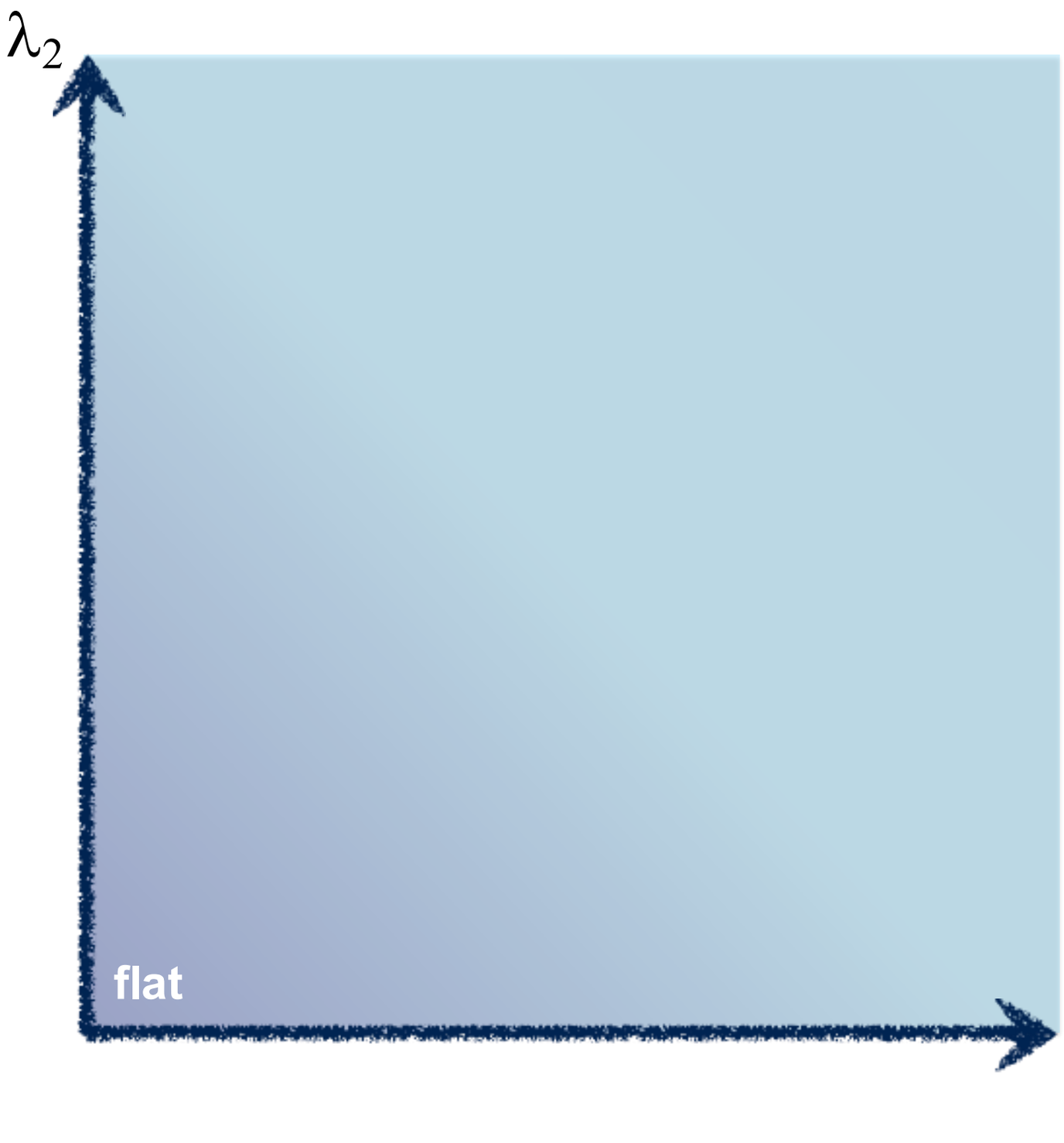






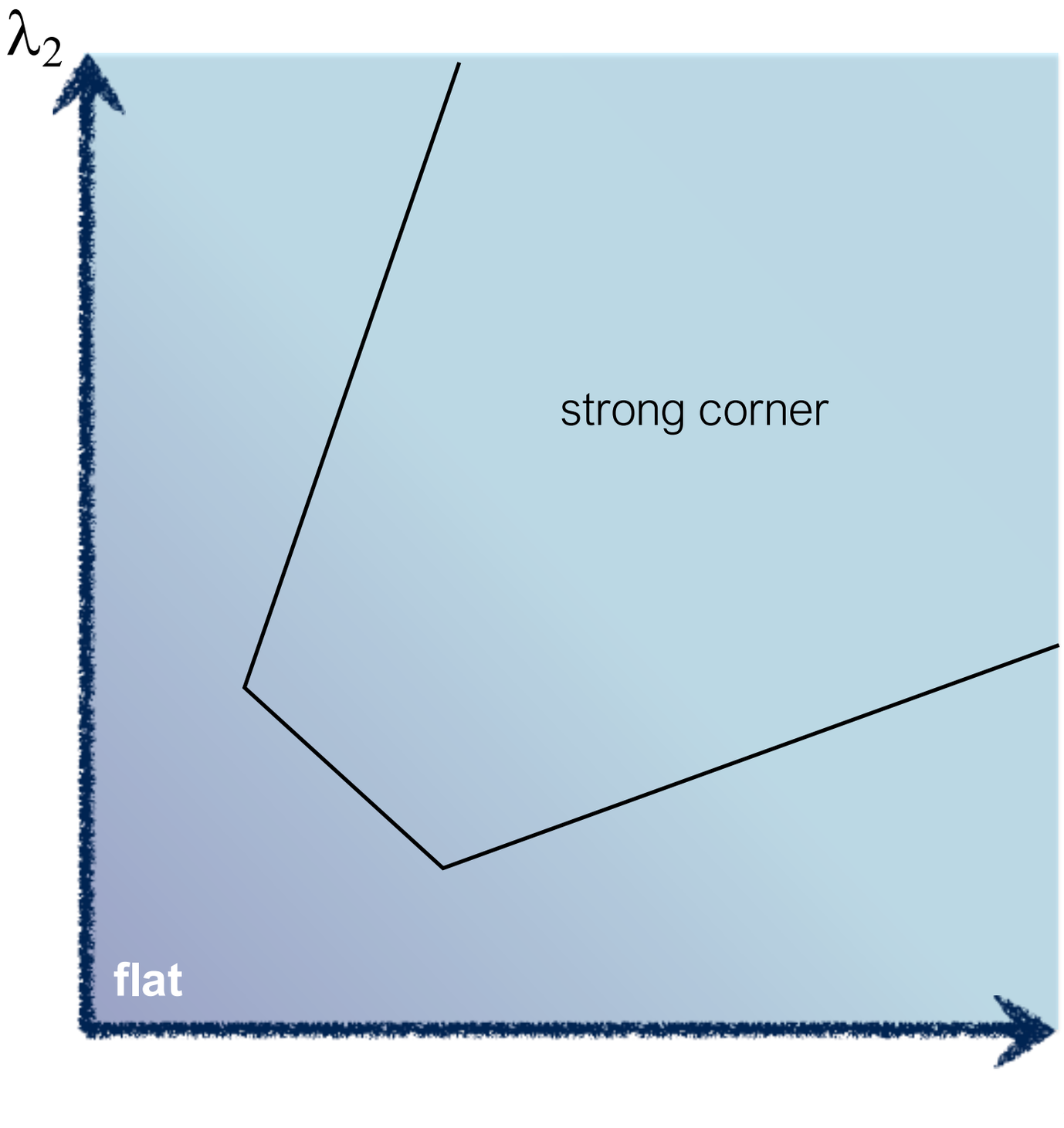


5. Use threshold on eigenvalues to detect corners



Think of a function to score ‘cornerness’

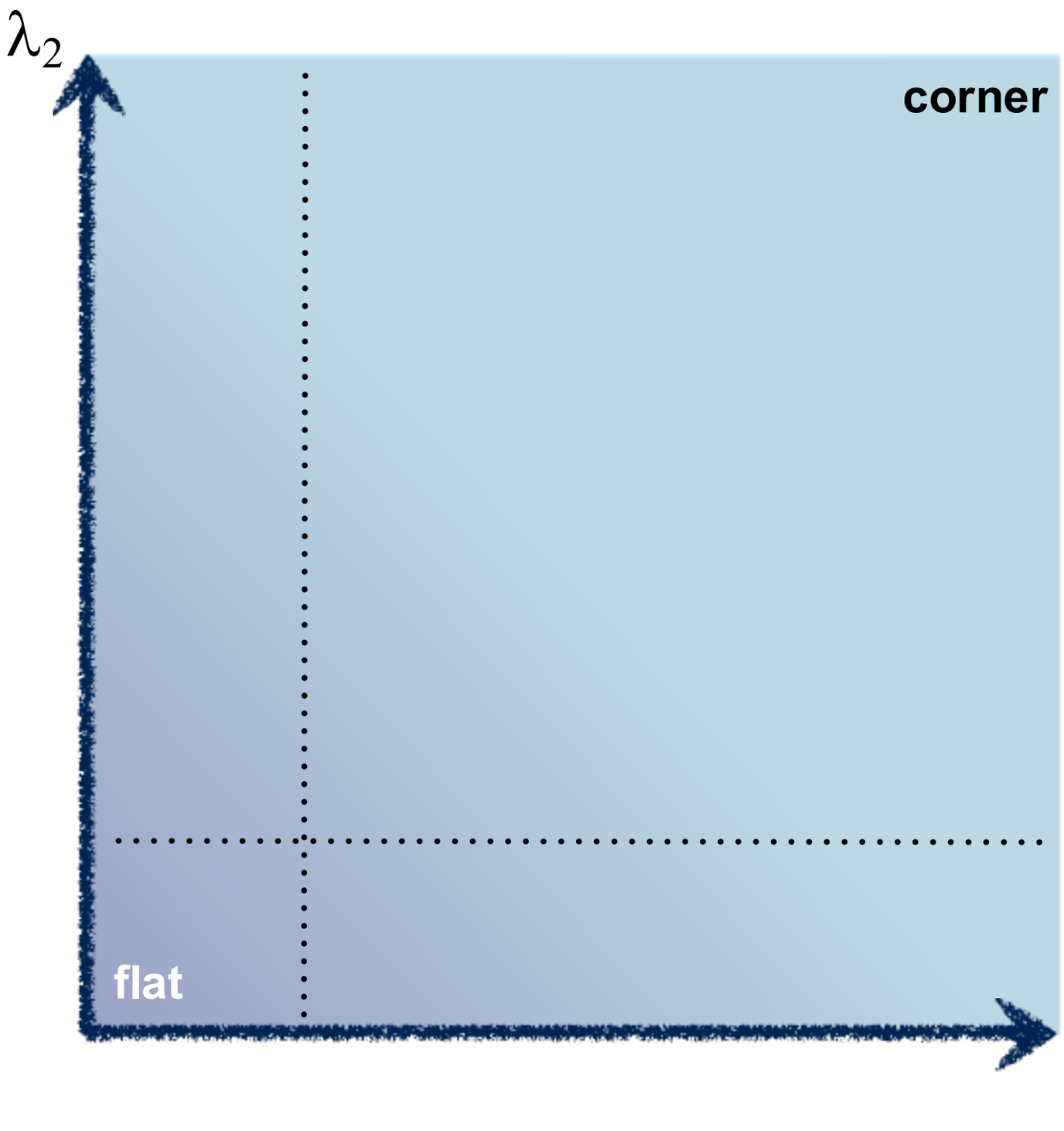
5. Use threshold on eigenvalues to detect corners



Think of a function to score ‘cornerness’

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)

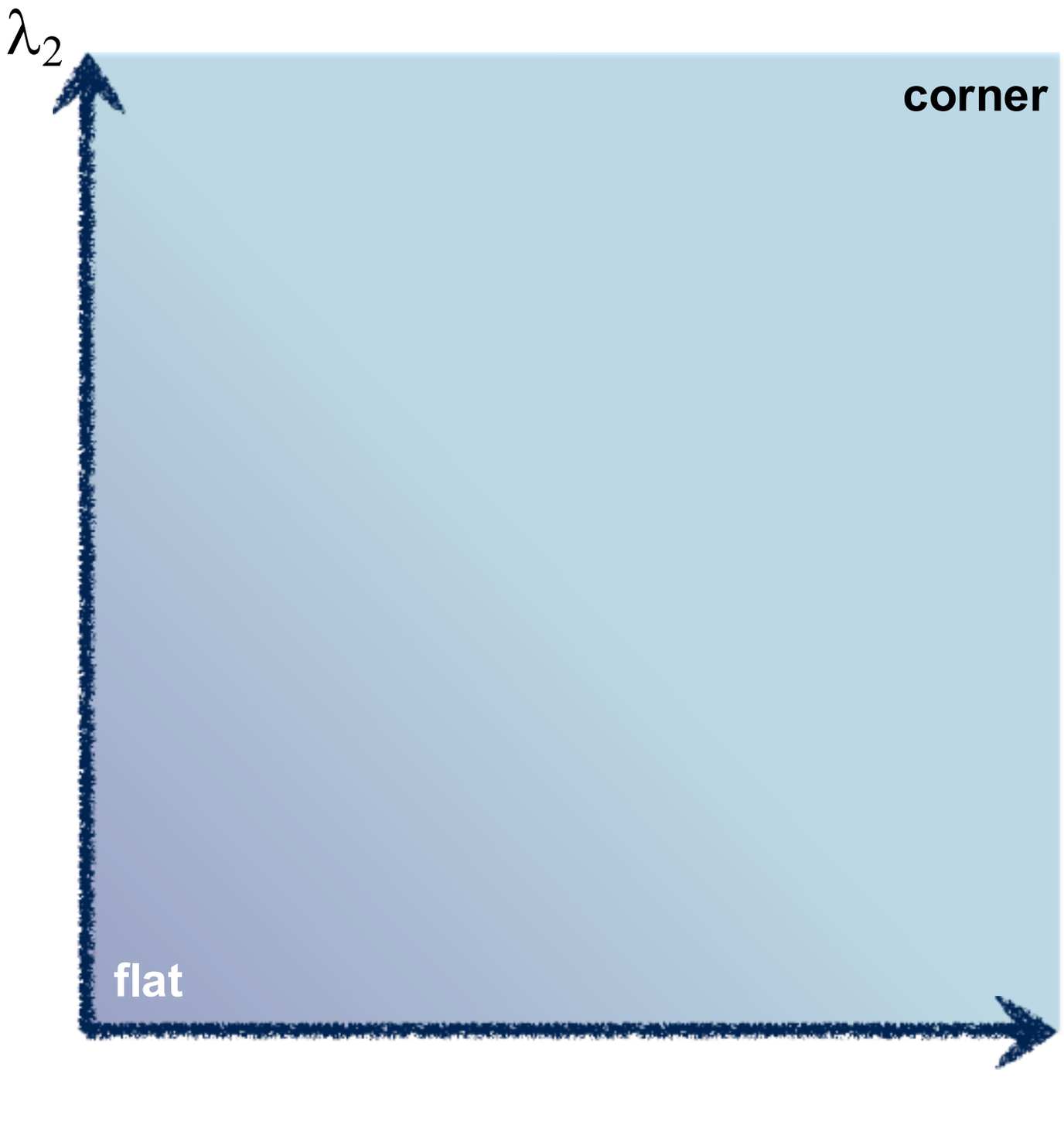


Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)

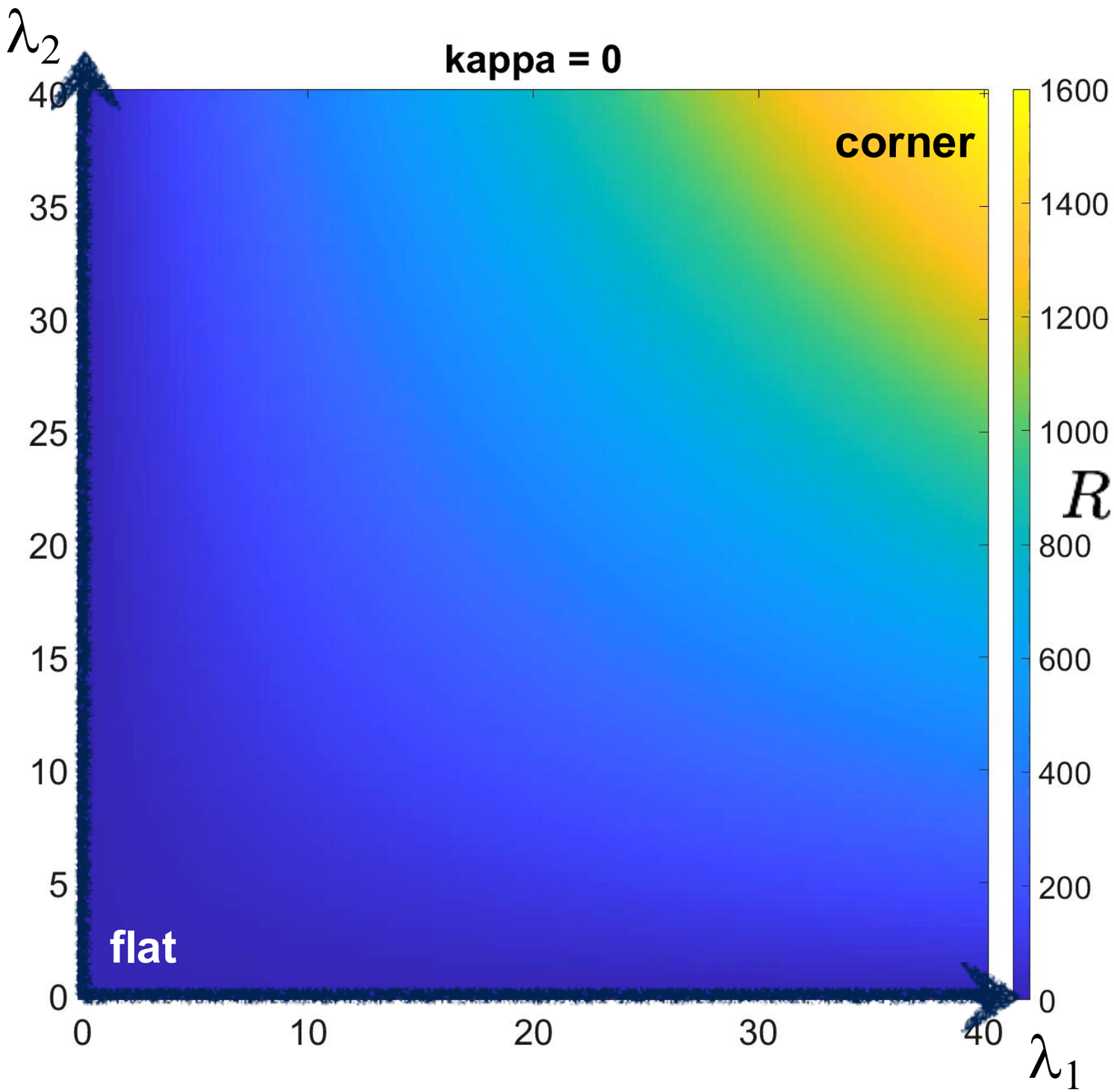


Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)



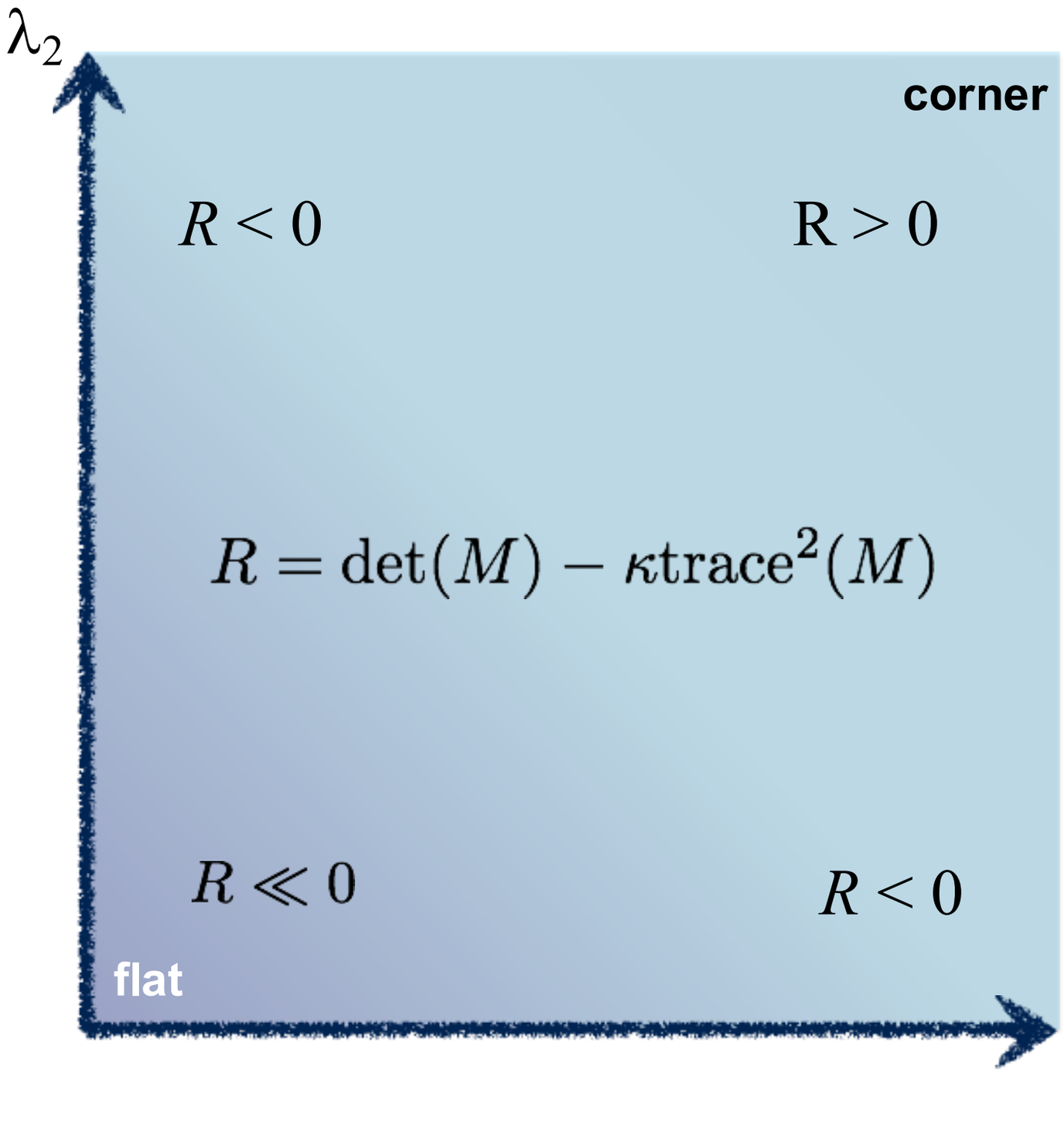
Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently…

5. Use threshold on eigenvalues to detect corners

(a function of $\hat{}$)



$$\det M = \lambda_1 \lambda_2$$

$$\text{trace}\, M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \text{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\text{trace}(M) + \epsilon}$$

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

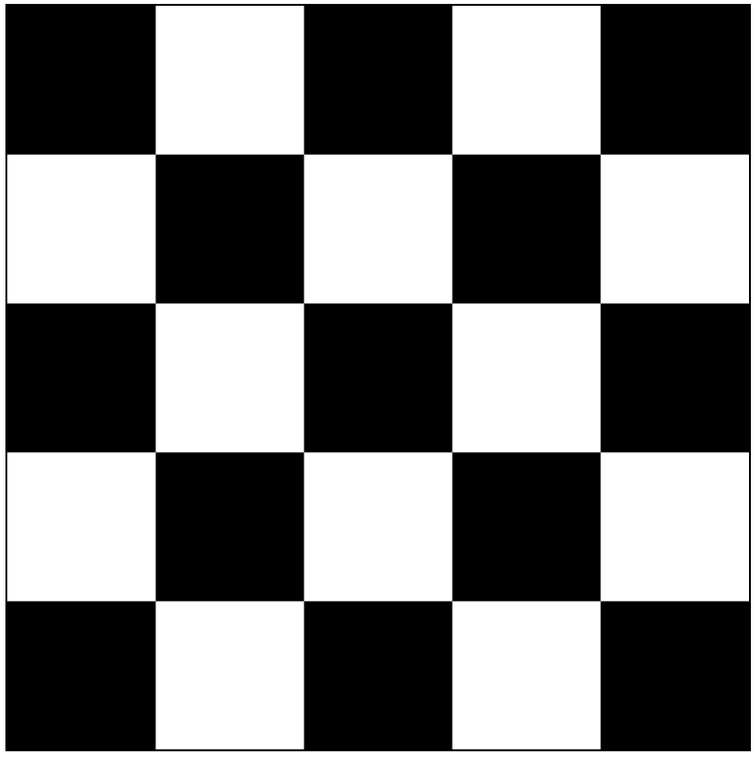
4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

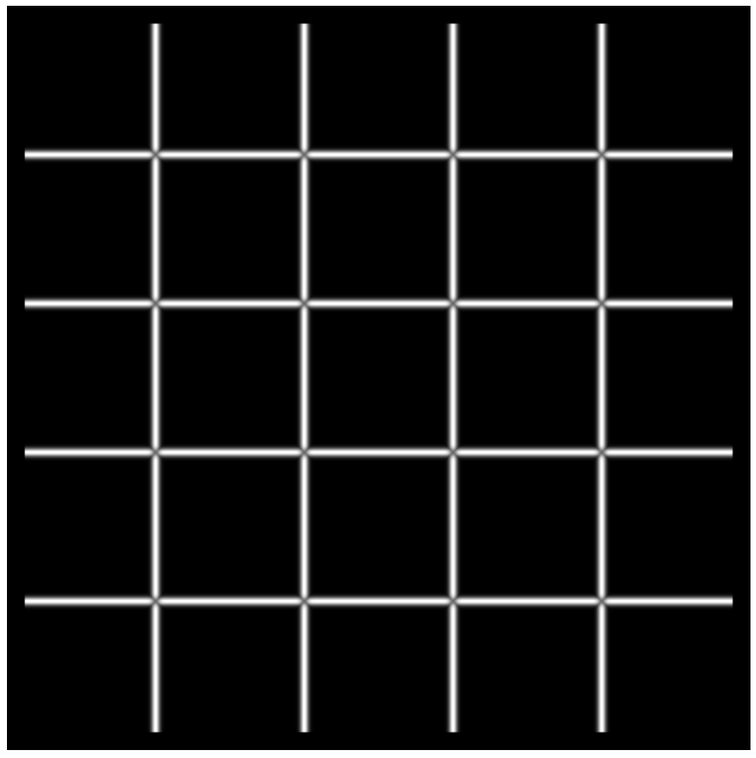
5. Compute the response of the detector at each pixel

$$R = \det M - k\left(\text{trace}M\right)^2$$

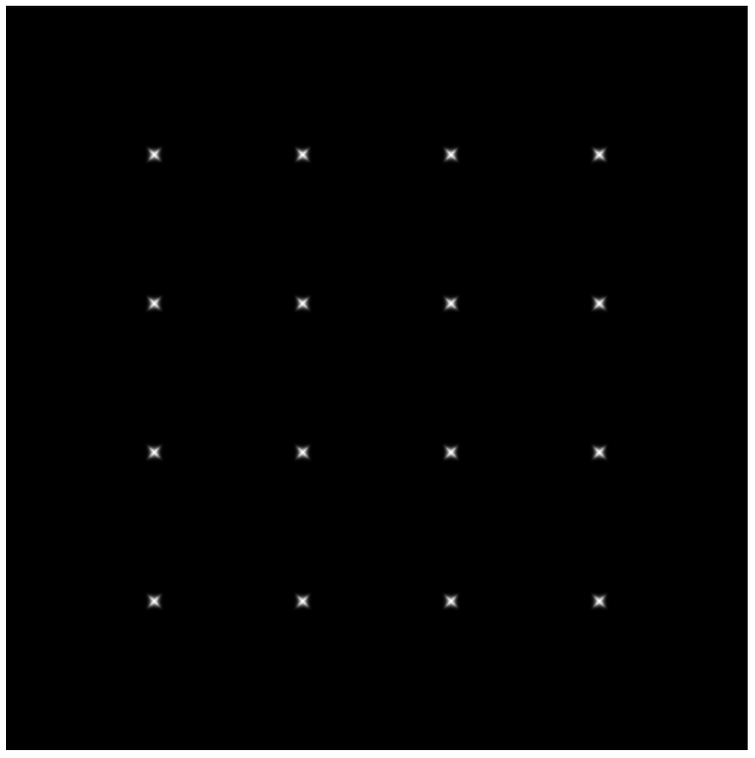
6. Threshold on value of R; compute non-max suppression.



I



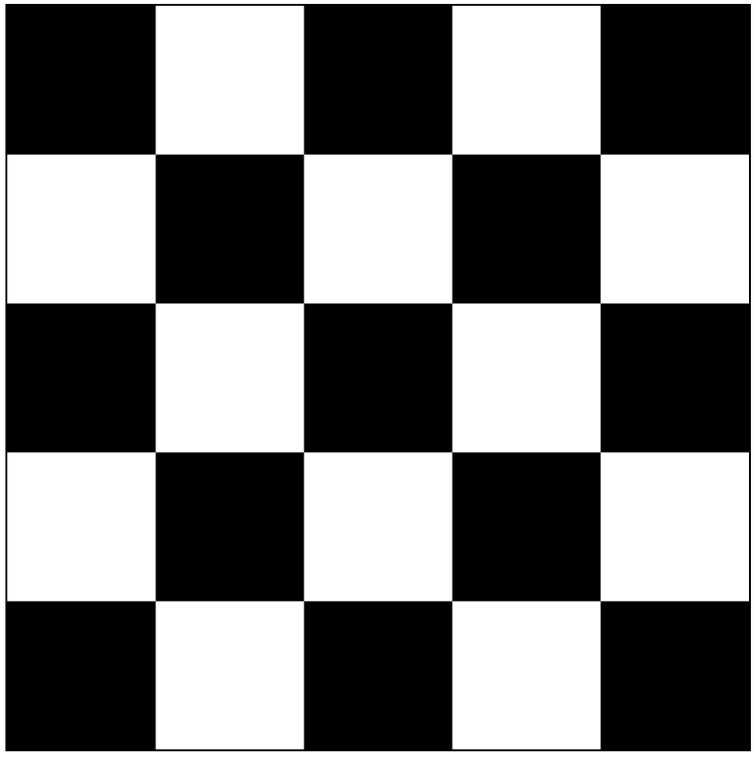
$\lambda_{\max}$



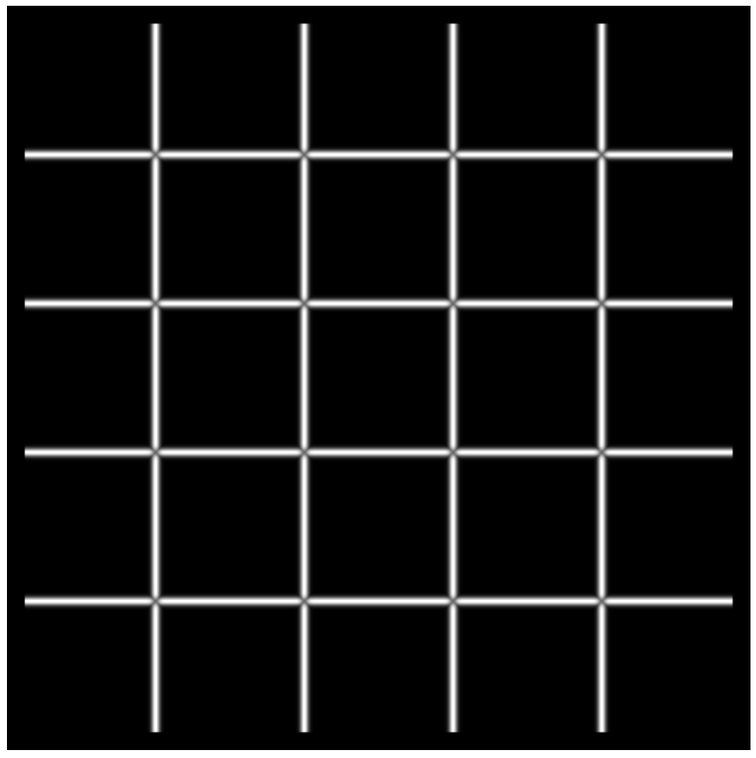
$\lambda_{\min}$

Yet another option: $f = \dfrac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

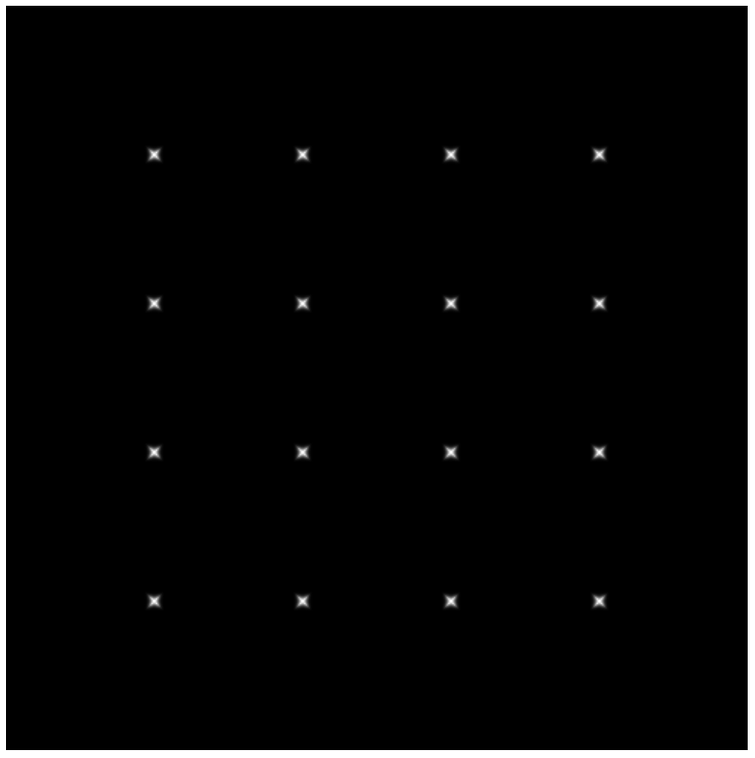
How do you write this equivalently
using determinant and trace?



I



$\lambda_{\max}$

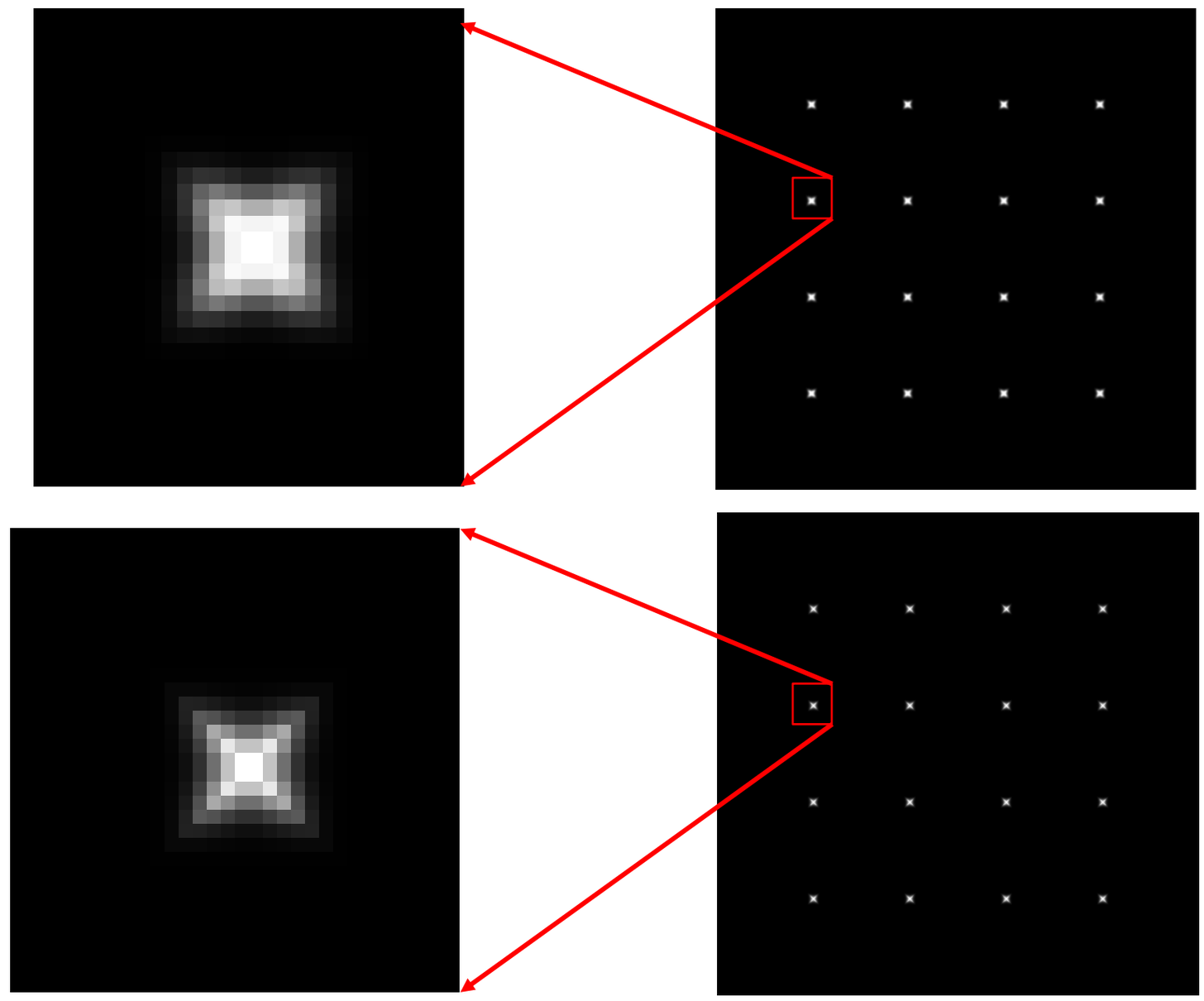


$\lambda_{\min}$

Yet another option:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

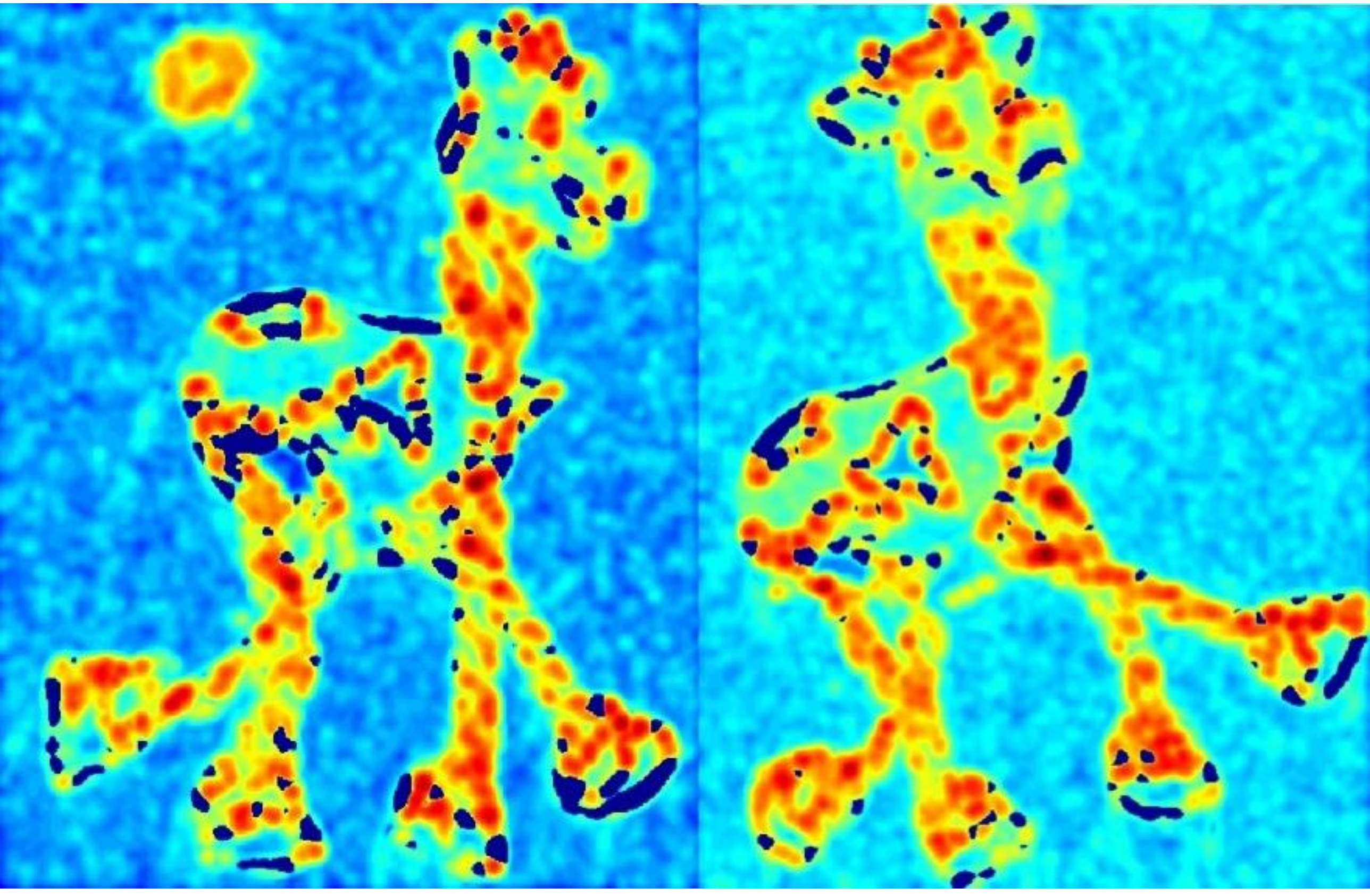
Different criteria



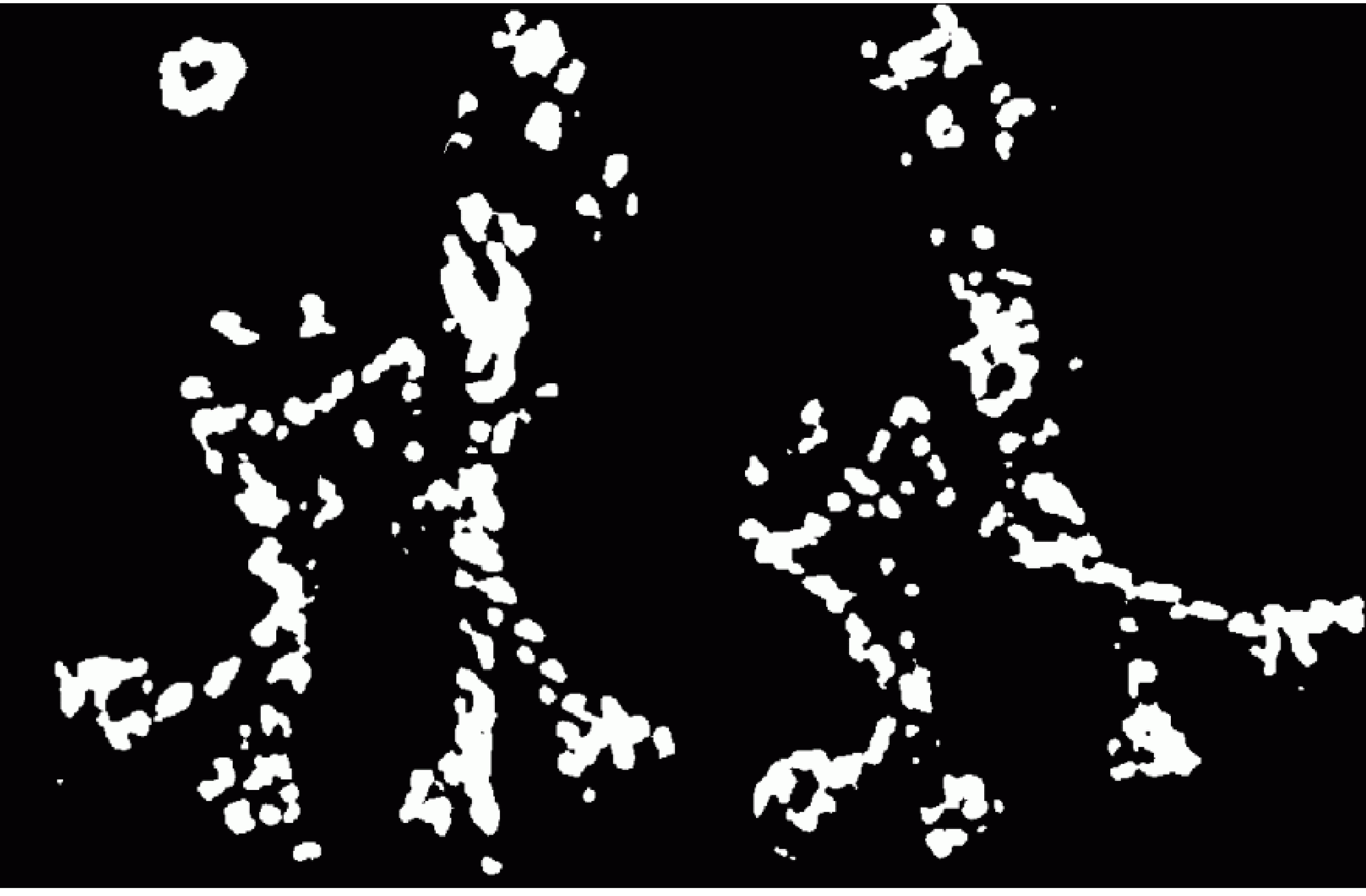
Harris criterion

$\lambda_{\min}$

Corner response



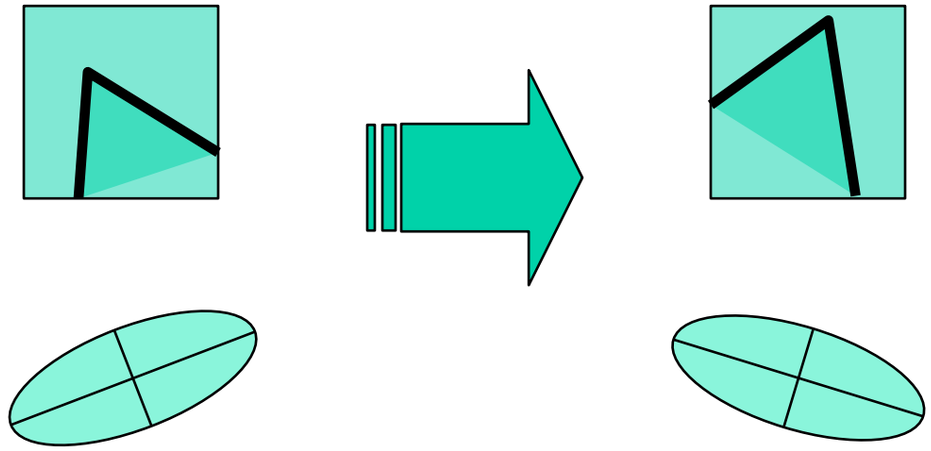
Thresholded corner response



Non-maximal suppression

Is Harris corner response is invariant to rotation?

Harris corner response is invariant to rotation



Ellipse rotates but its shape
(**eigenvalues**) remains the same

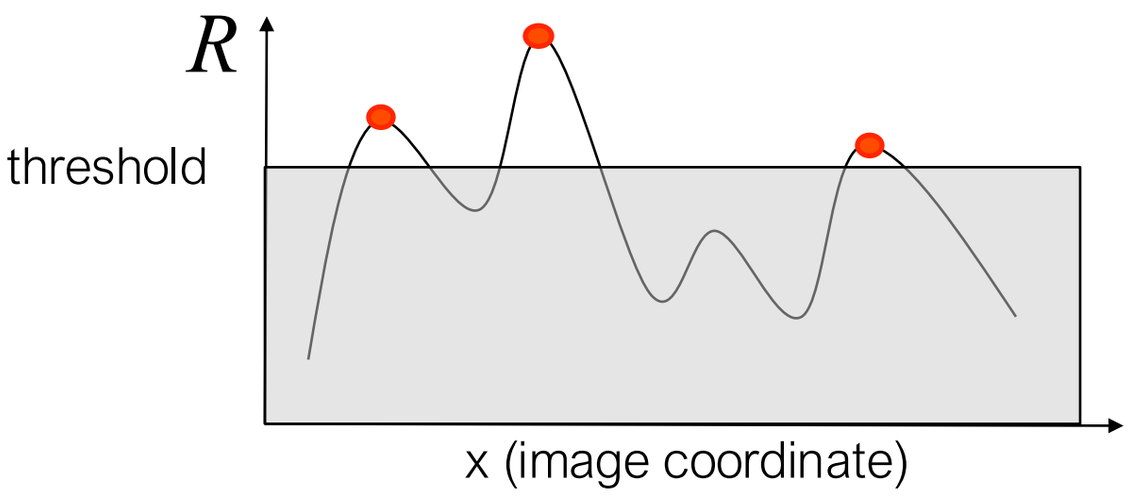
Corner response R is invariant to image rotation

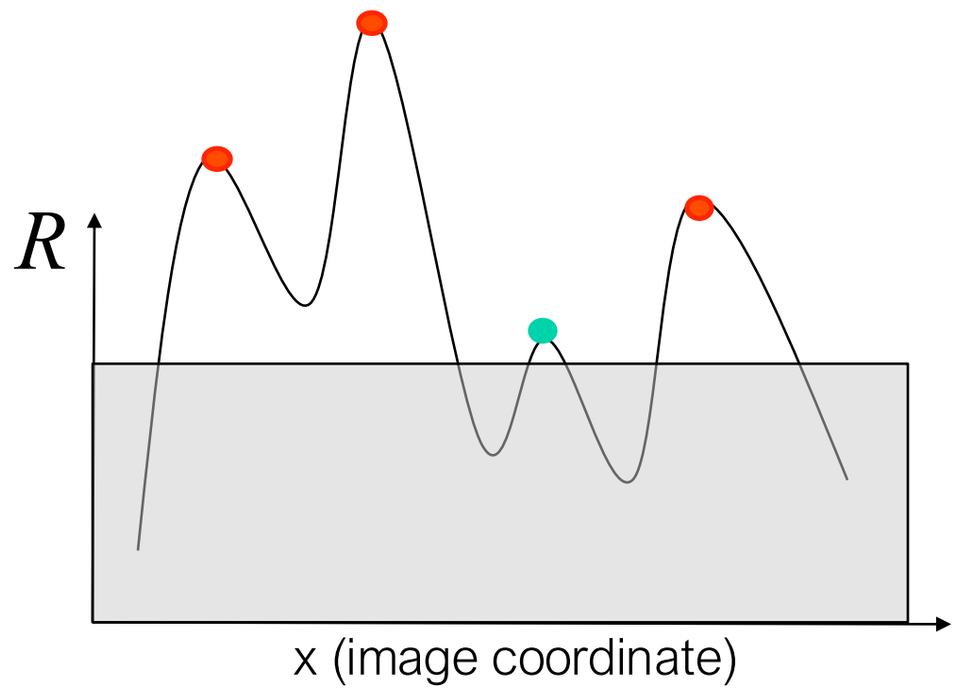
Harris corner response is invariant to intensity changes

Harris corner response is invariant to intensity changes

Partial invariance to ***affine intensity*** change

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scale: $I \rightarrow a\, I$

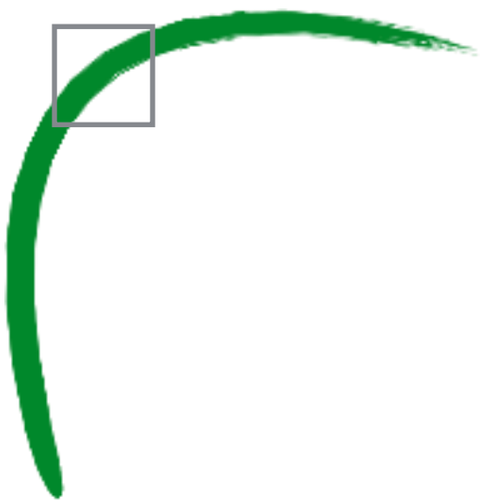




The Harris detector is not invariant to changes in …

The Harris corner detector is <u>not</u> invariant to scale

edge!



corner!

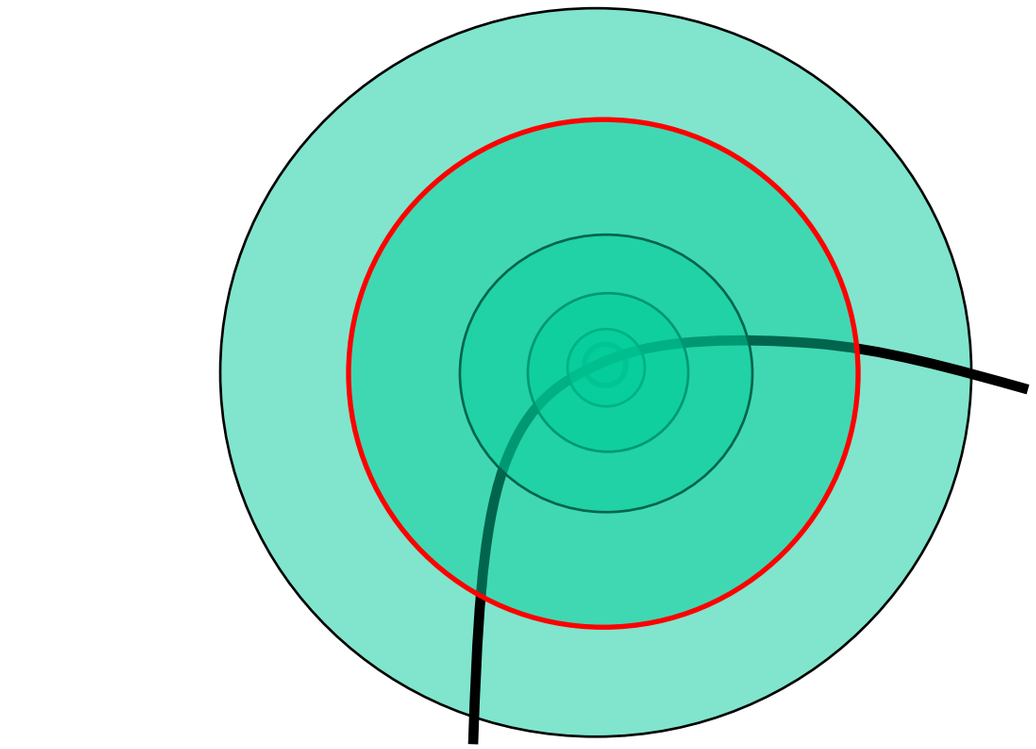
Multi-scale detection

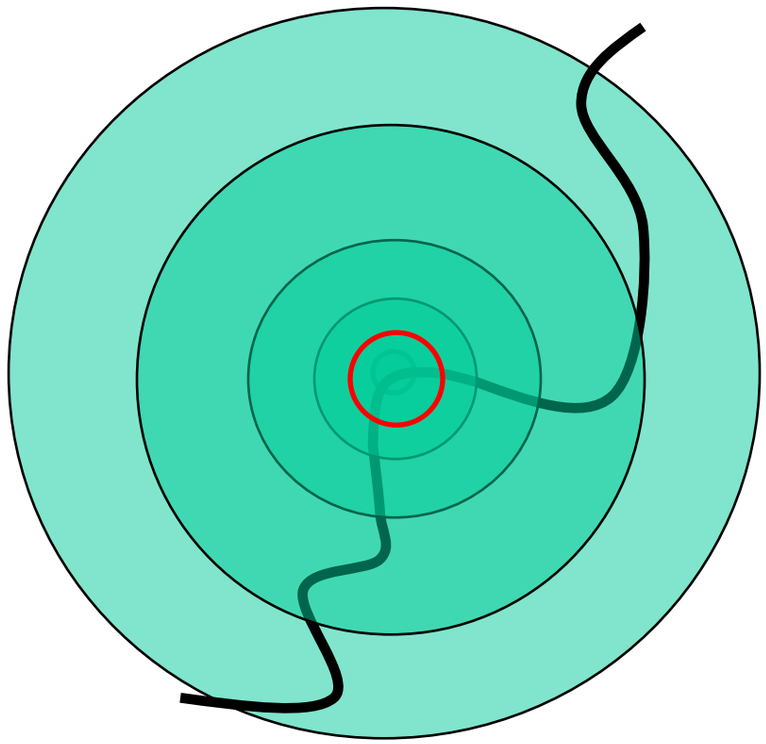
How can we make a feature detector scale-invariant?

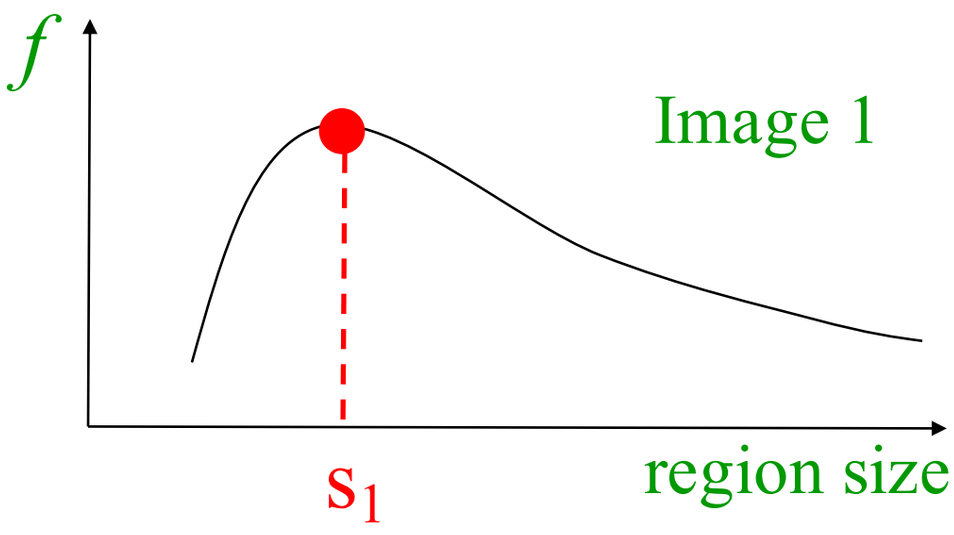
How can we automatically select the scale?

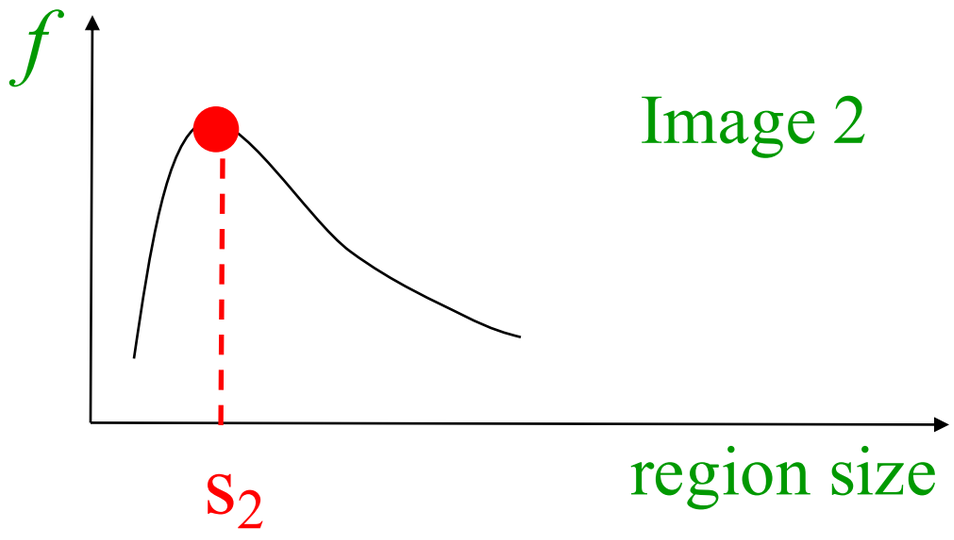
Intuitively…

Find local maxima in both **position** and **scale**





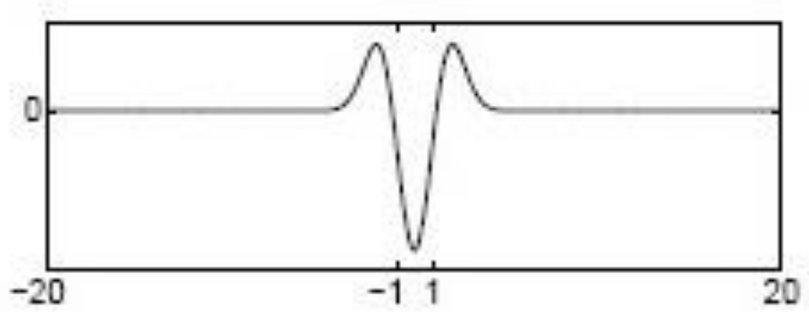


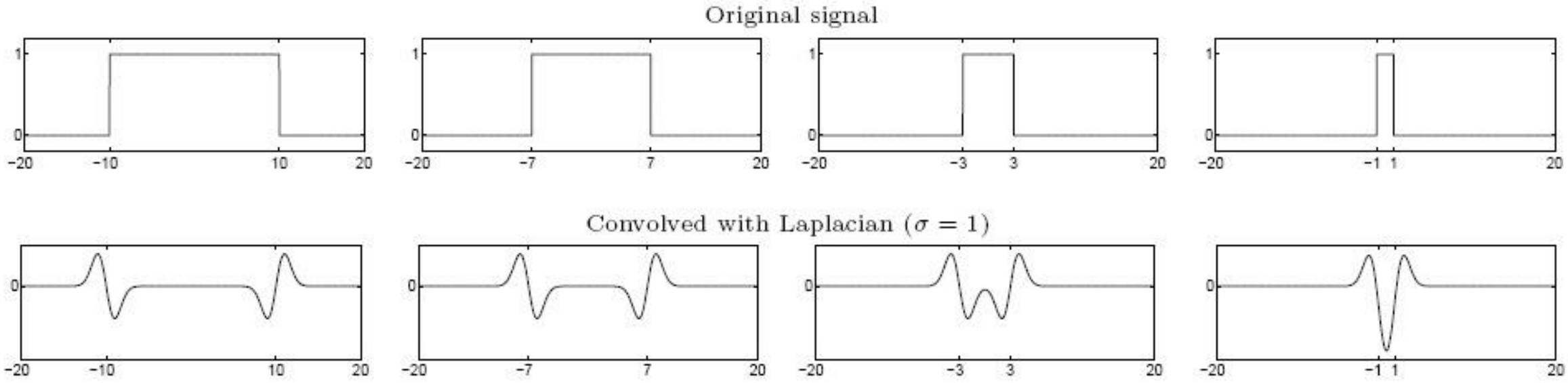


Multi-scale blob detection

Formally…

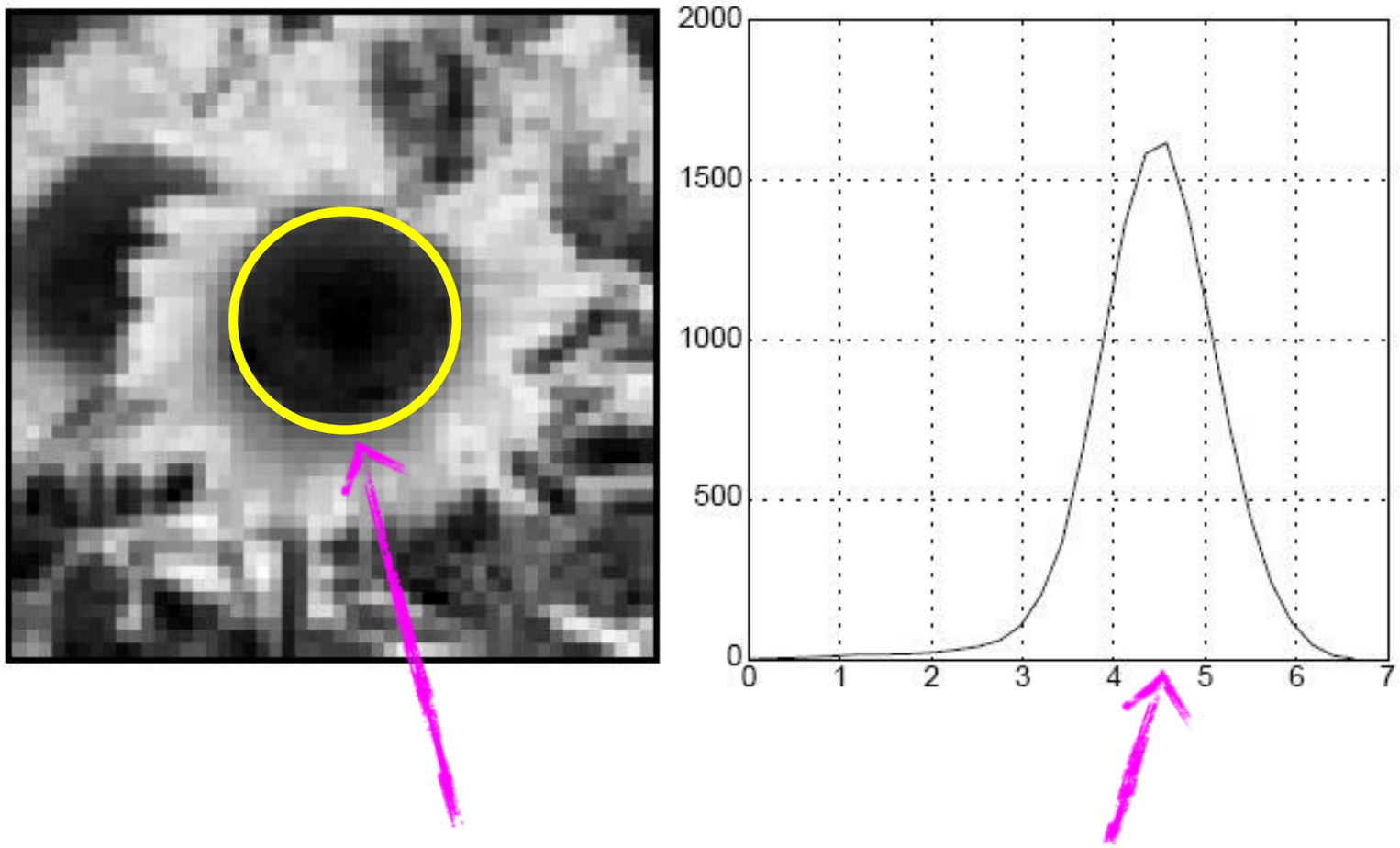
Laplacian filter





Highest response when the signal has the same **characteristic scale** as the filter

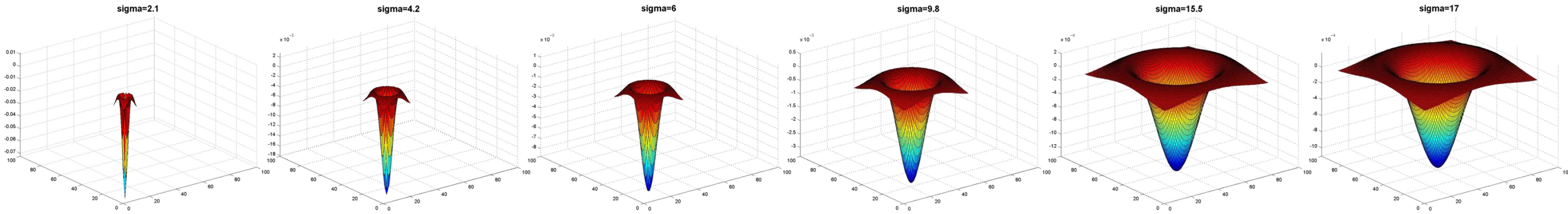
characteristic scale - the scale that produces peak filter response



characteristic scale

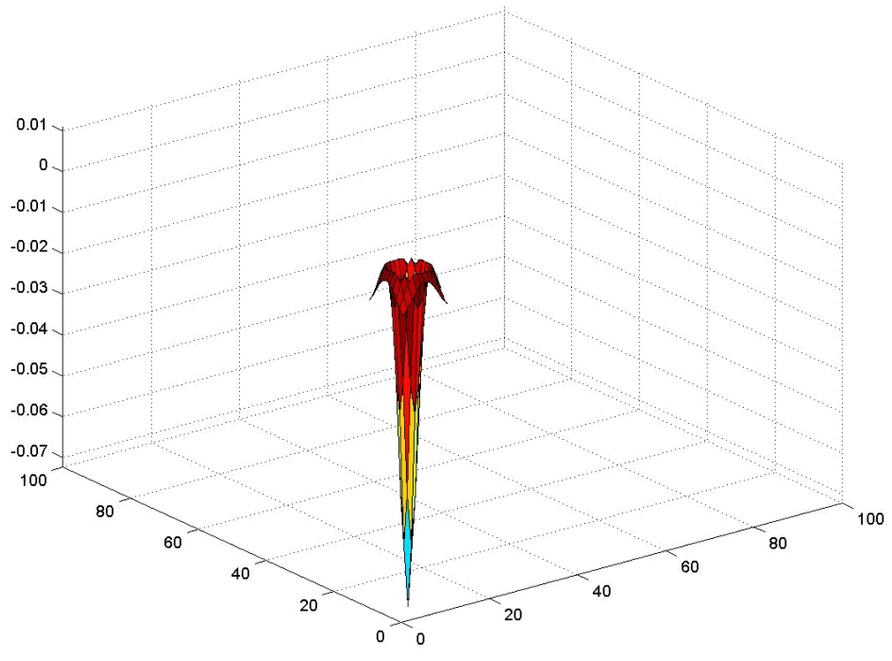
we need to search over characteristic scales

What happens if you apply different Laplacian filters?

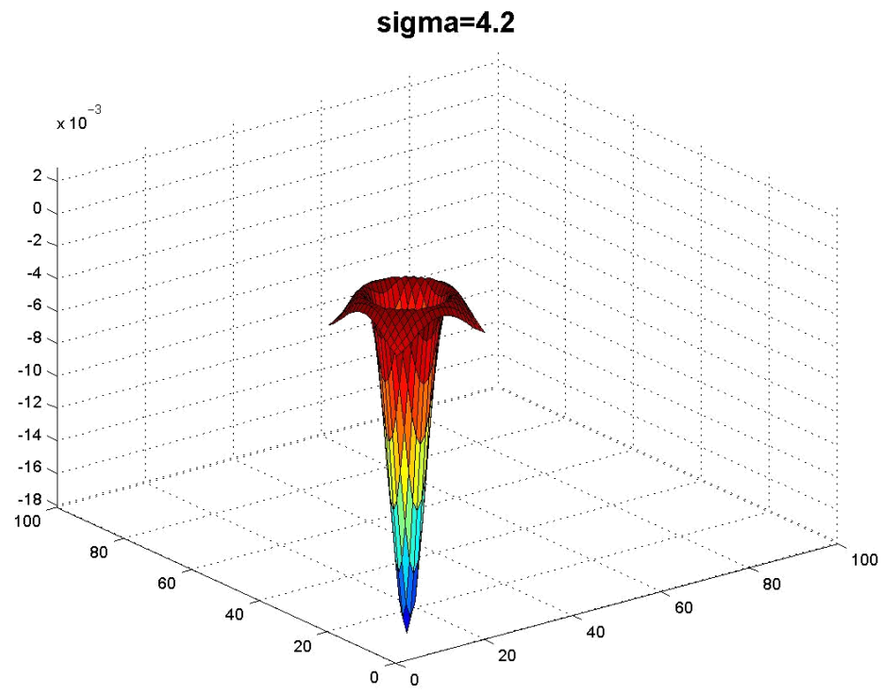


Full size

3/4 size



jet color scale
blue: low, red: high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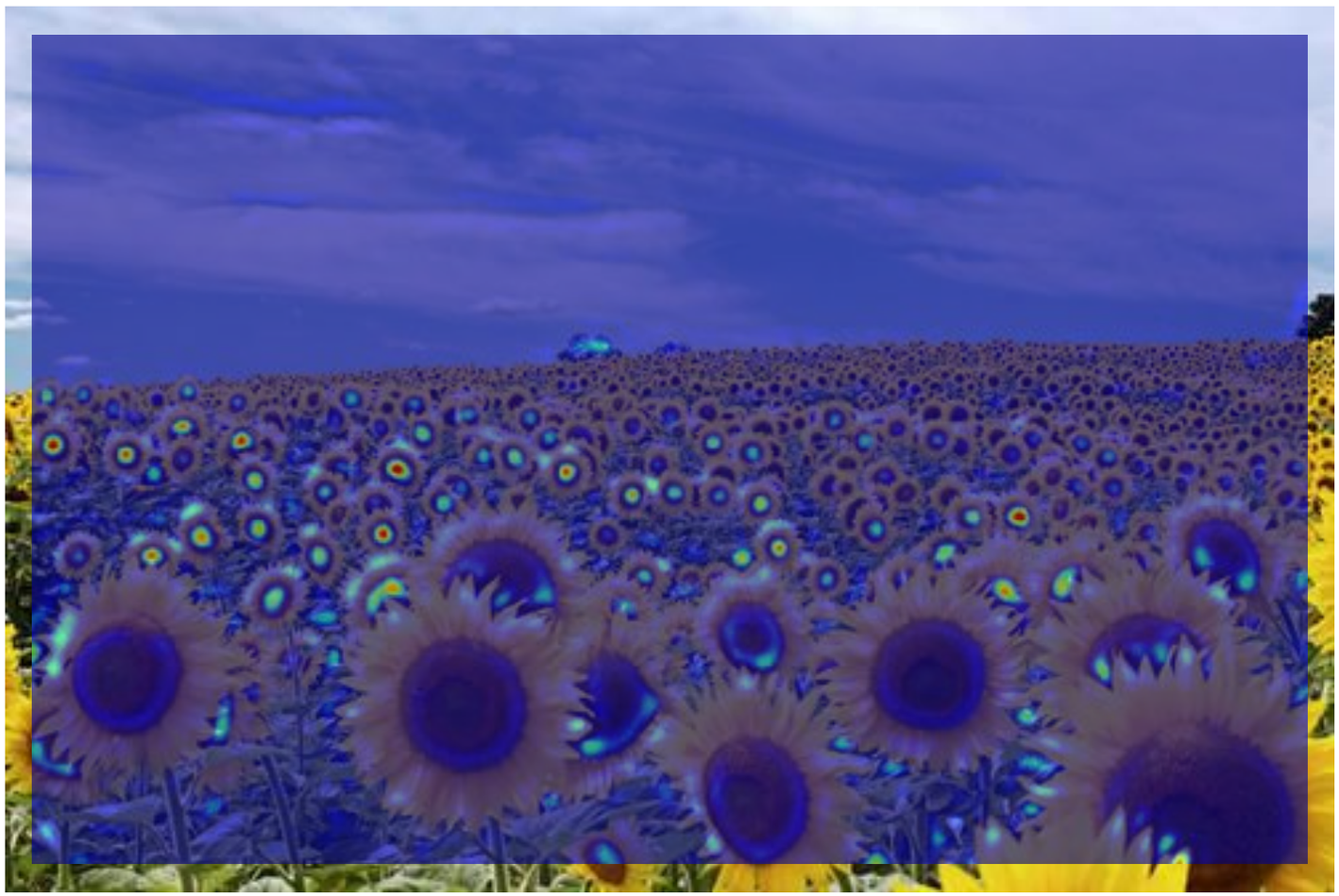

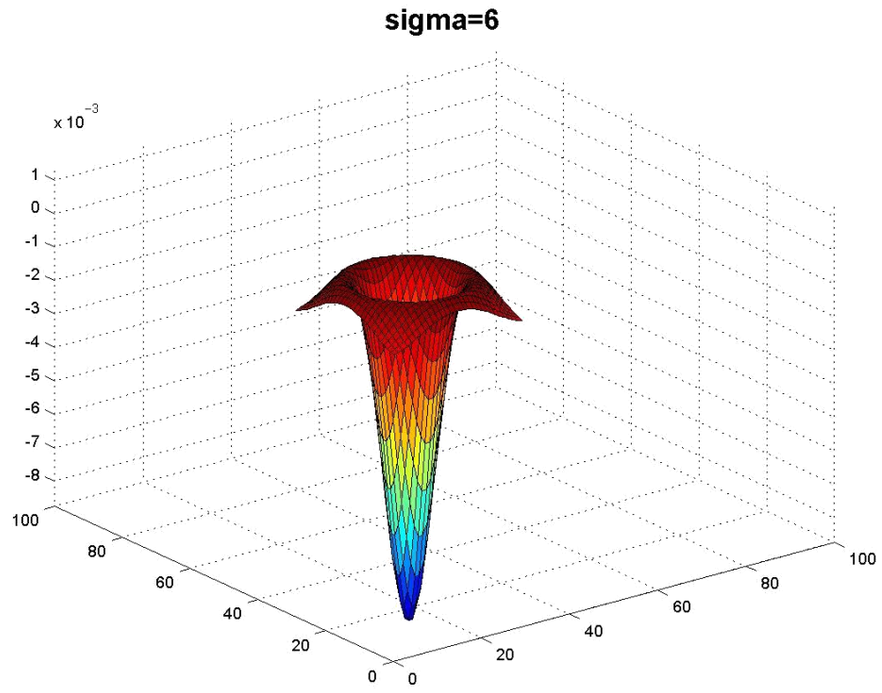
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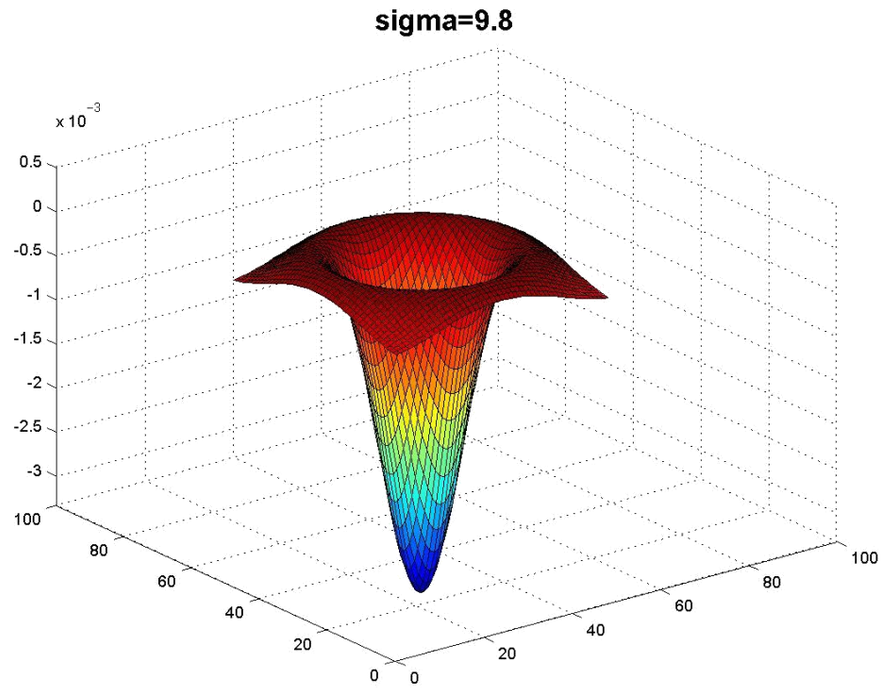
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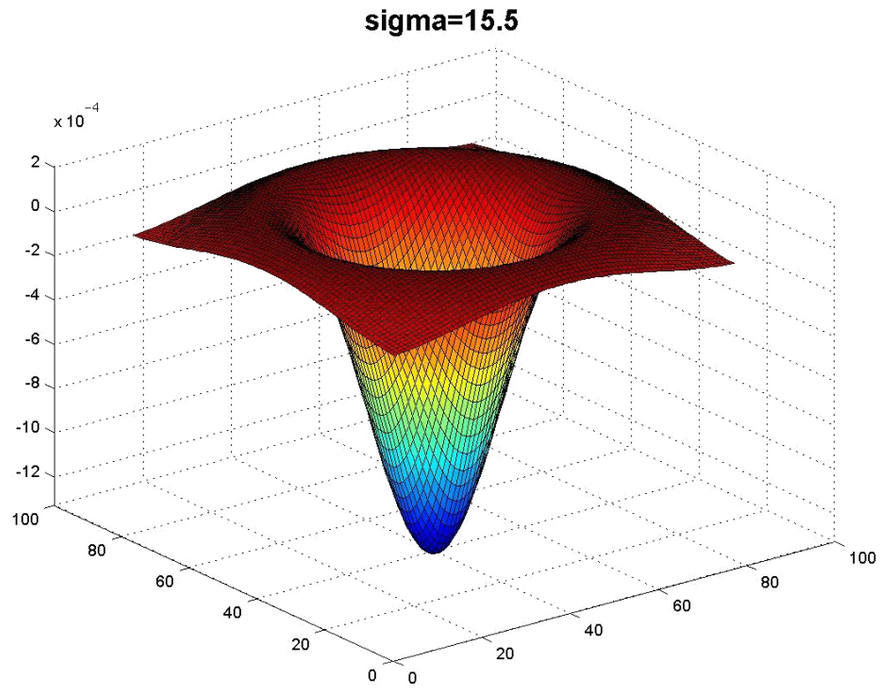
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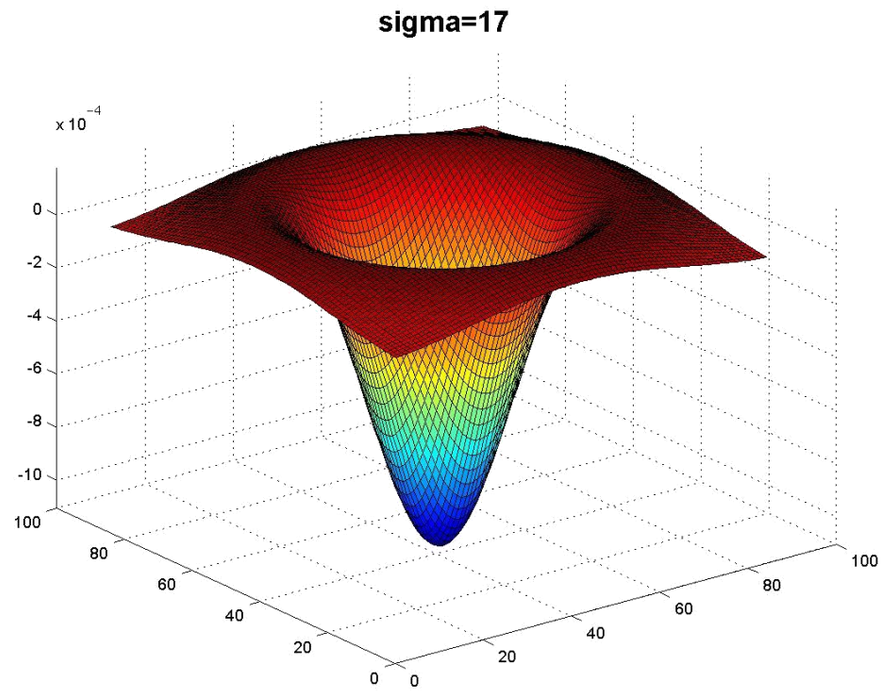
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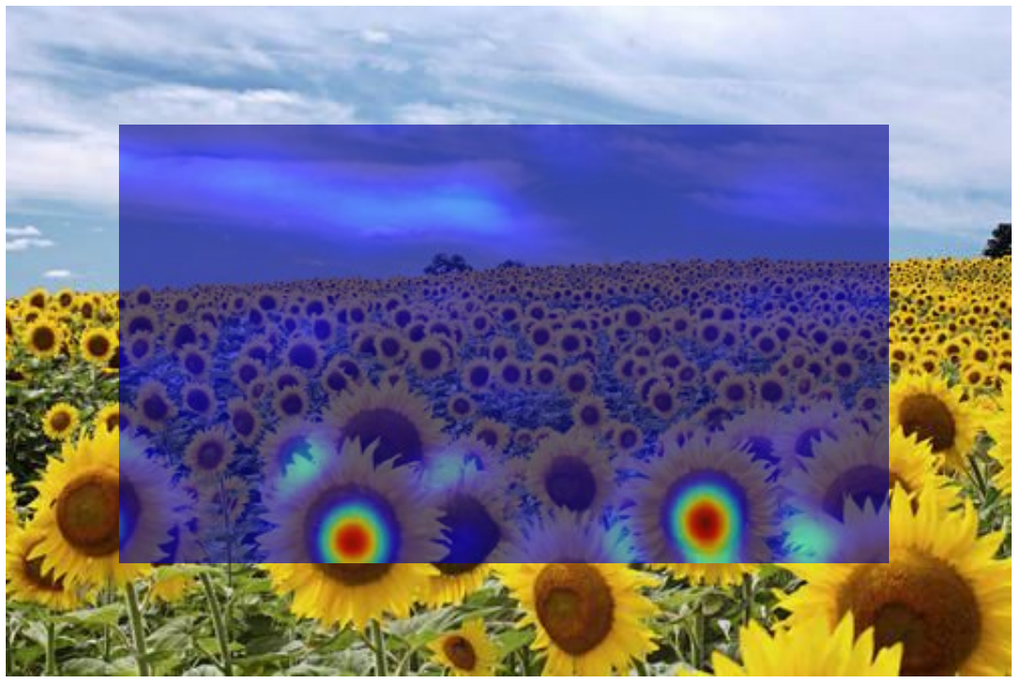
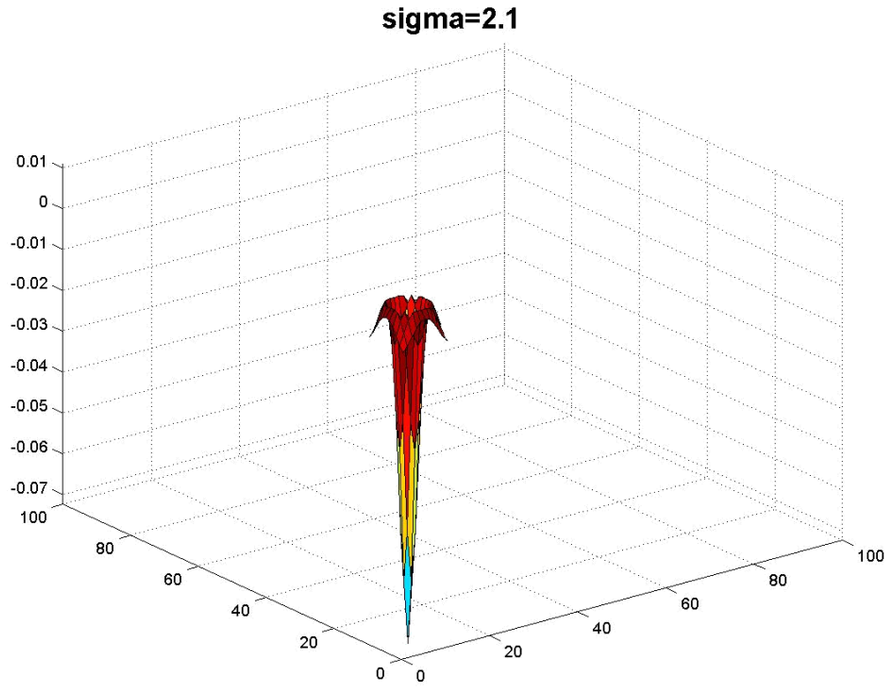


What happened when you applied different Laplacian filters?

Full size

3/4 size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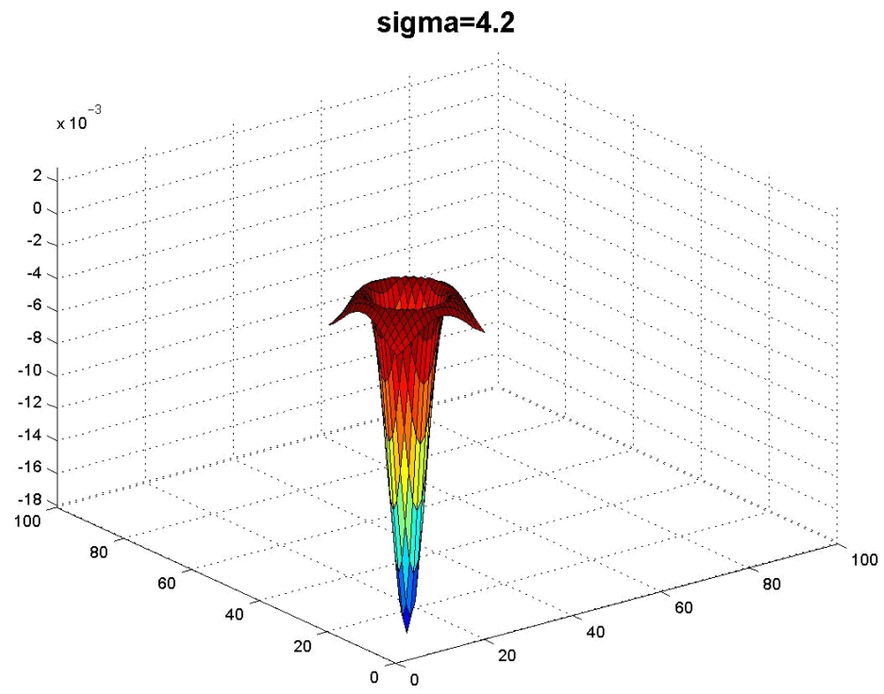
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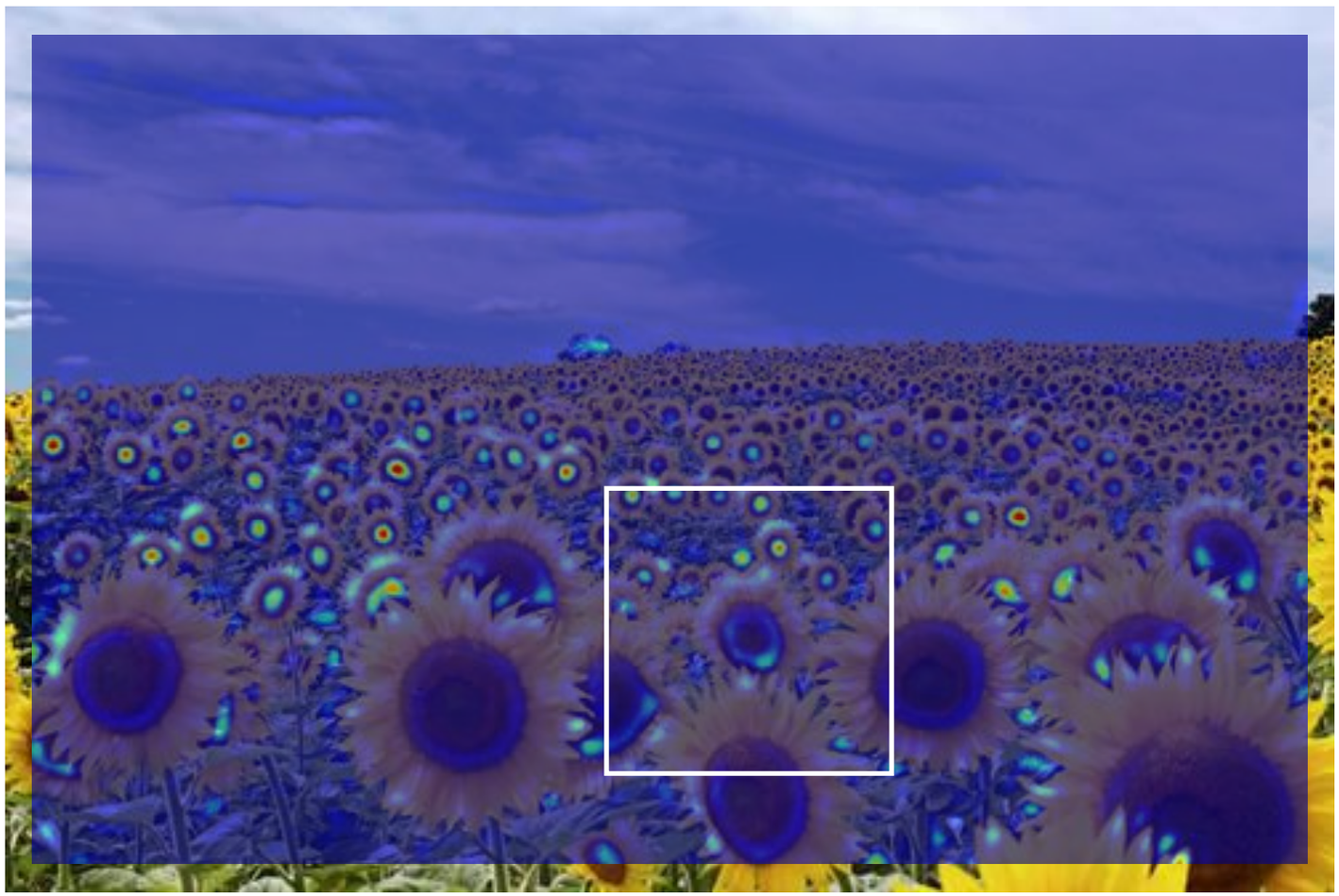

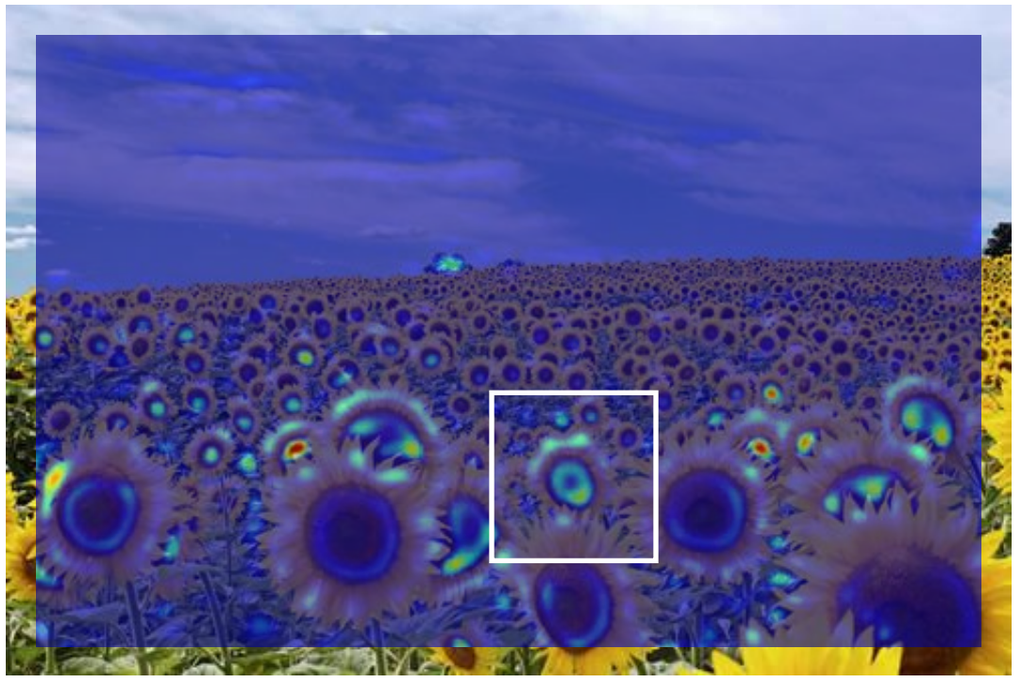

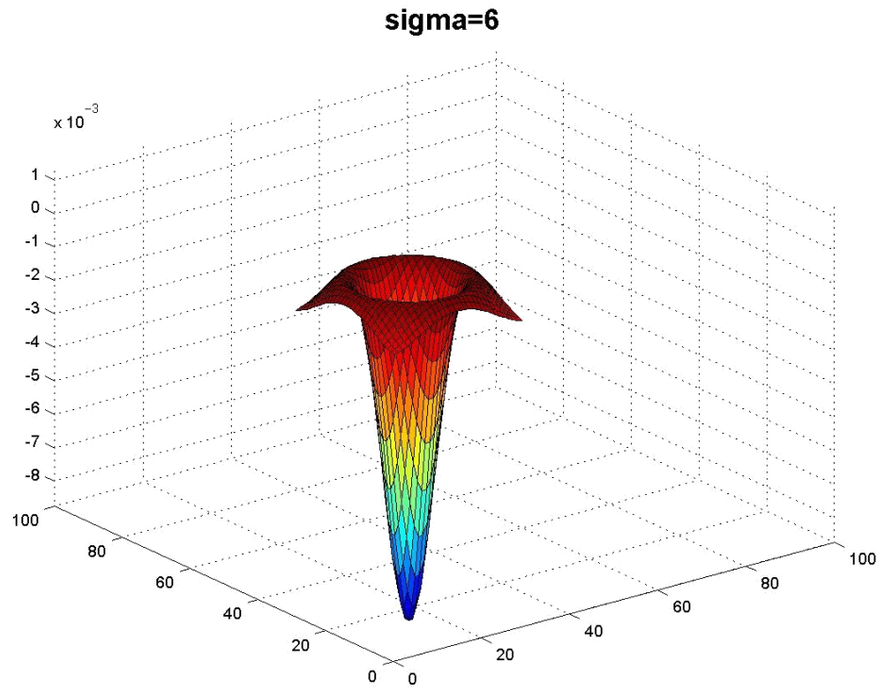
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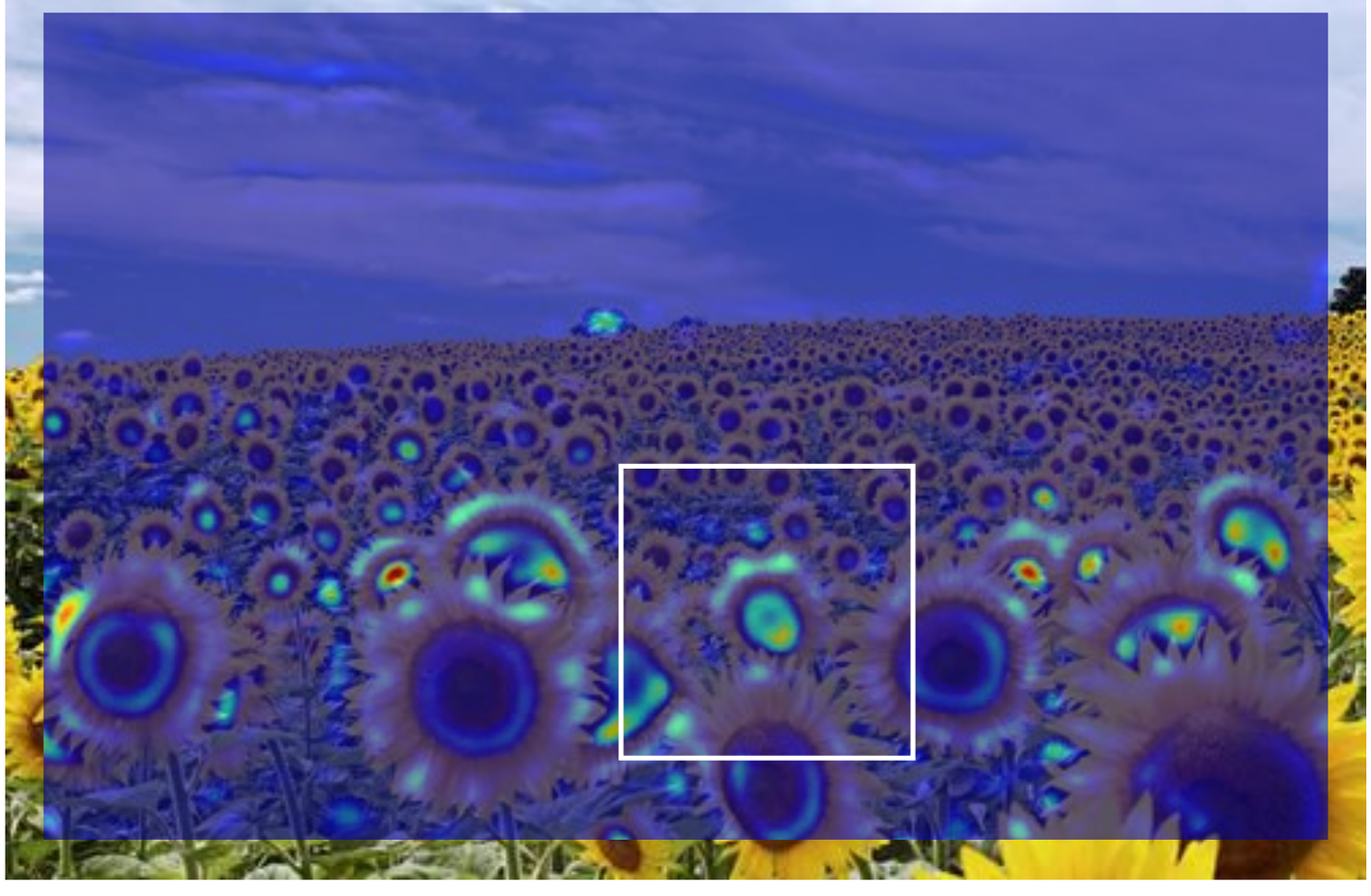
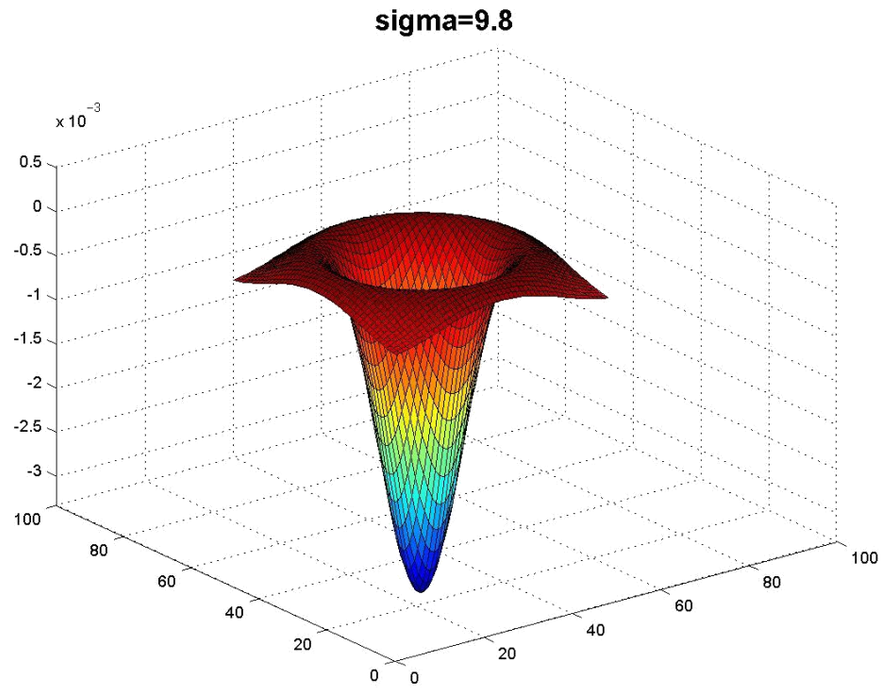



peak!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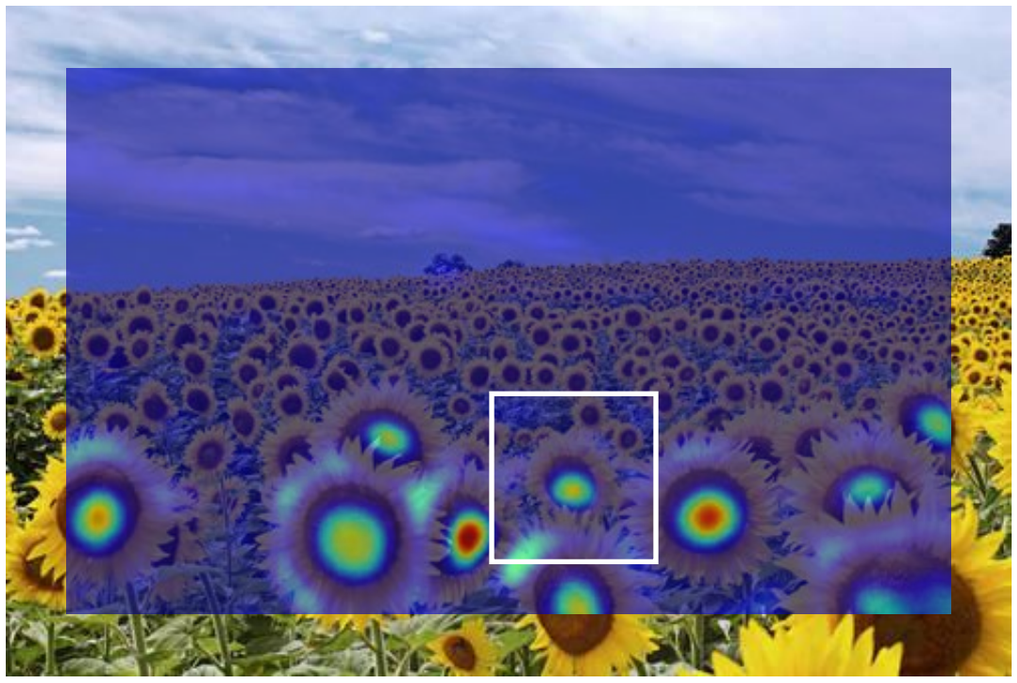
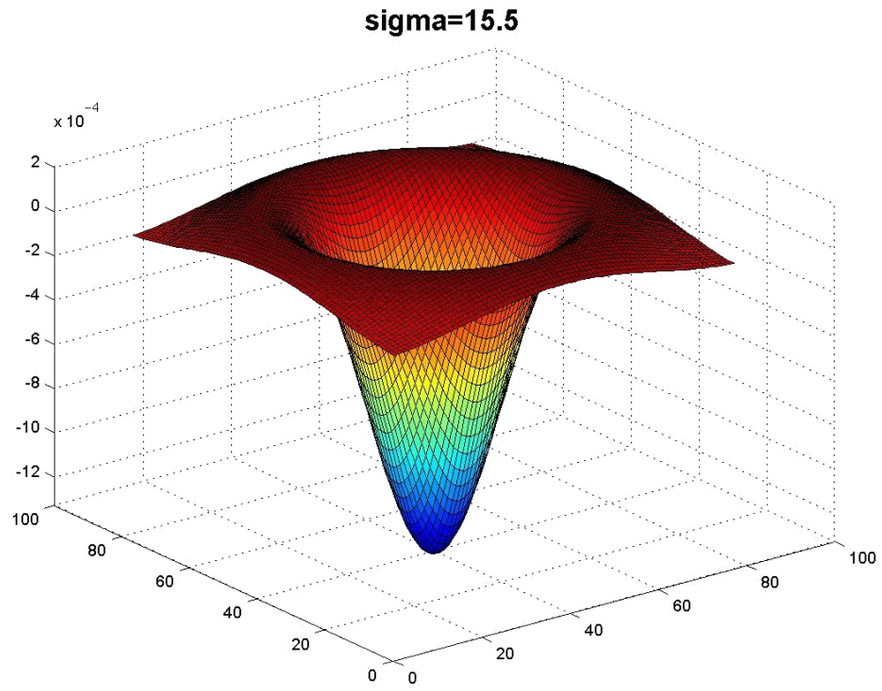
100


peak!

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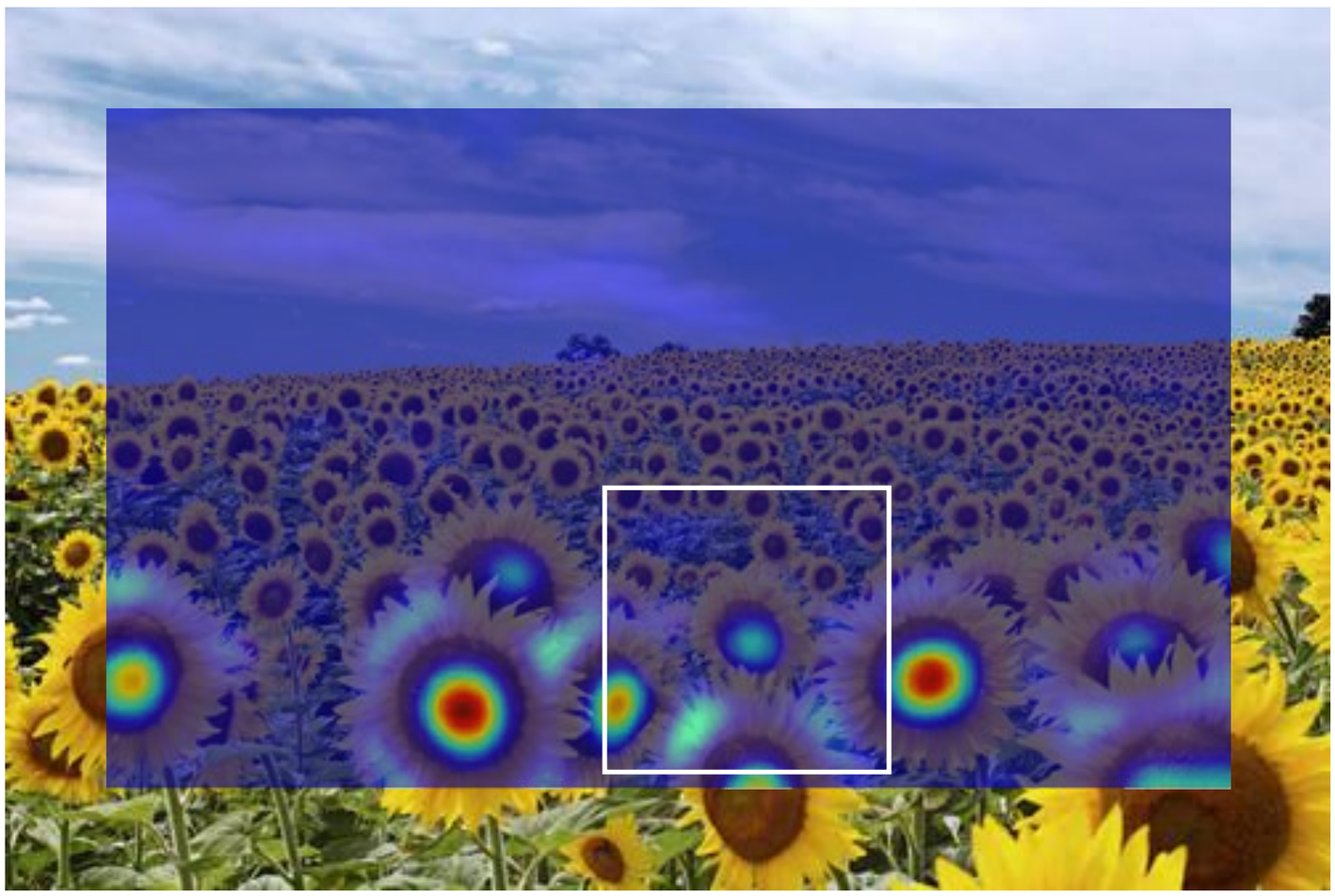

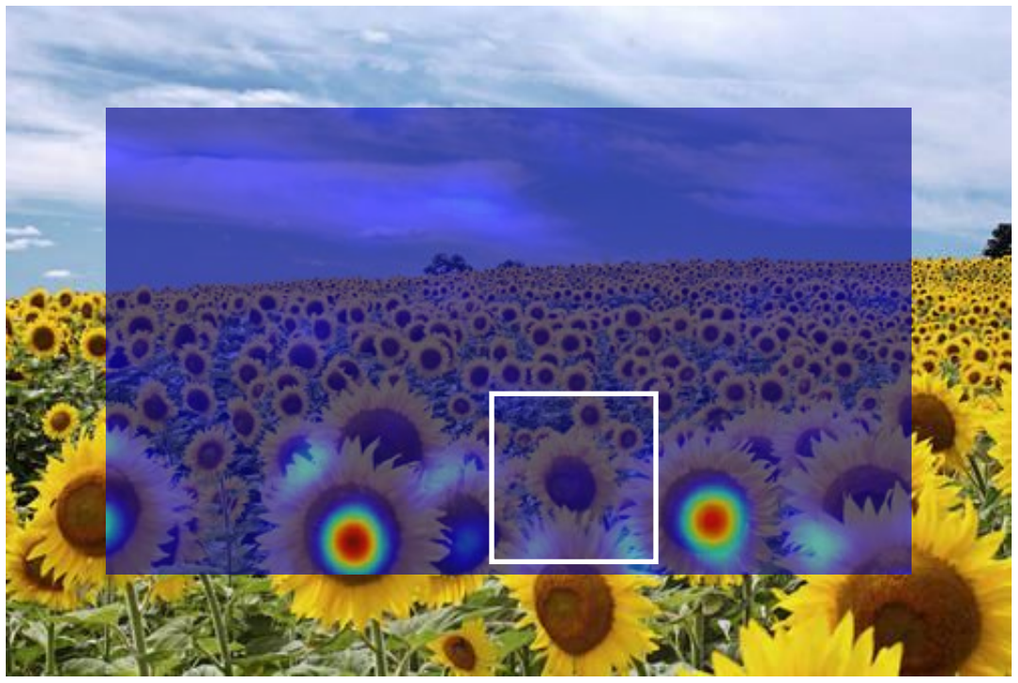

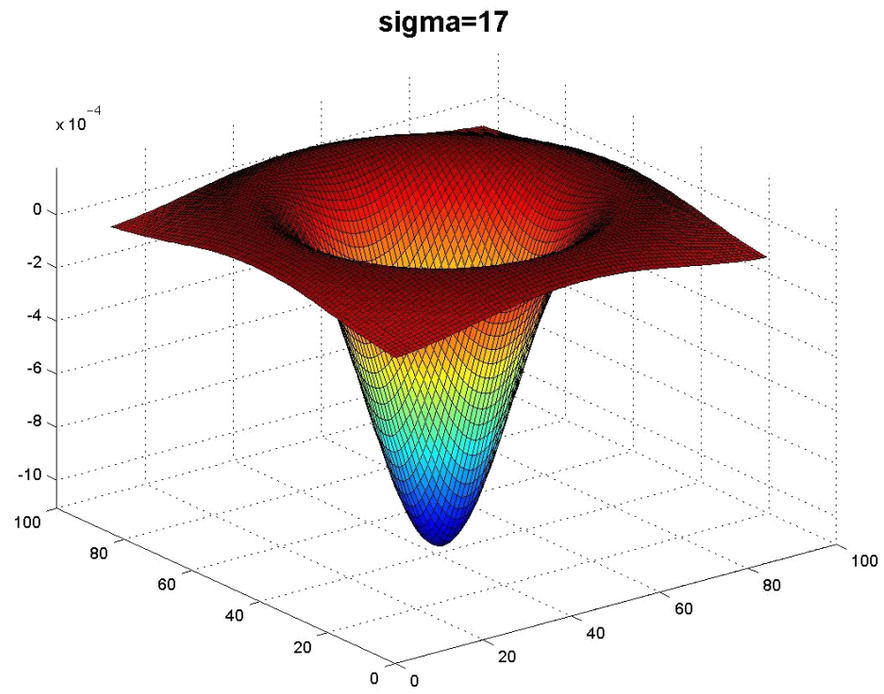
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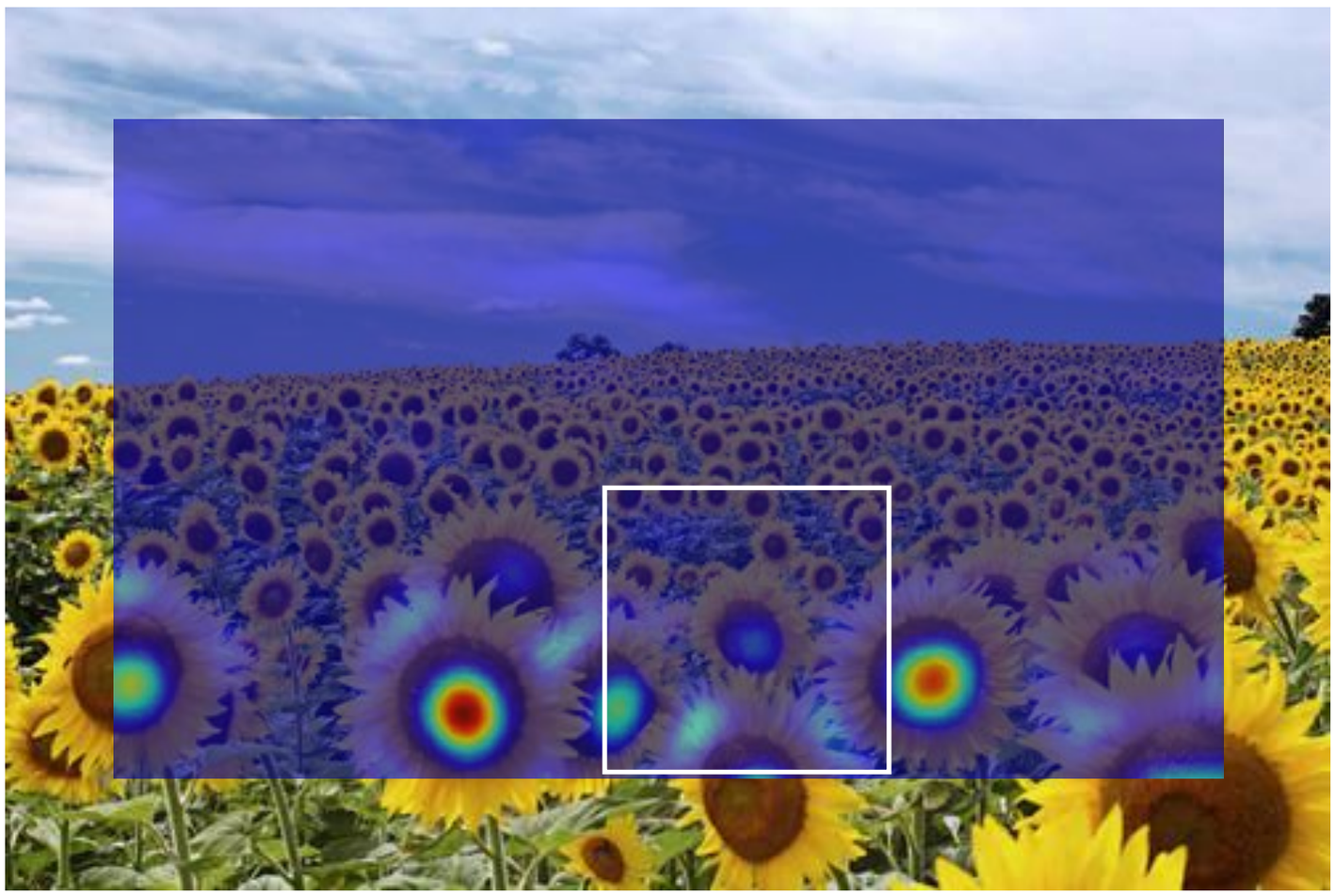

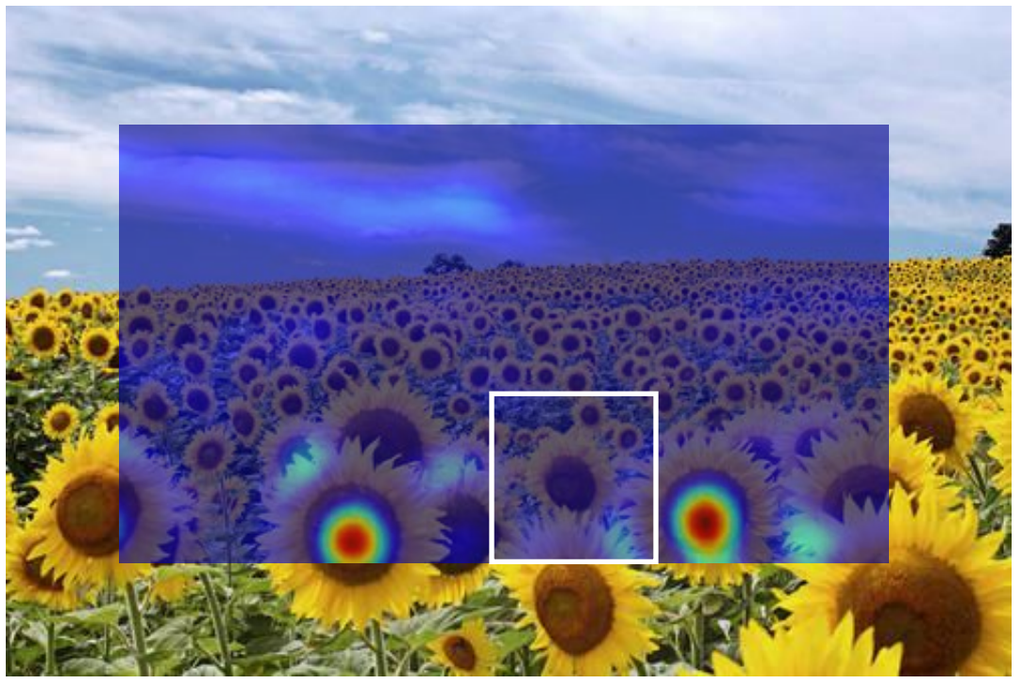

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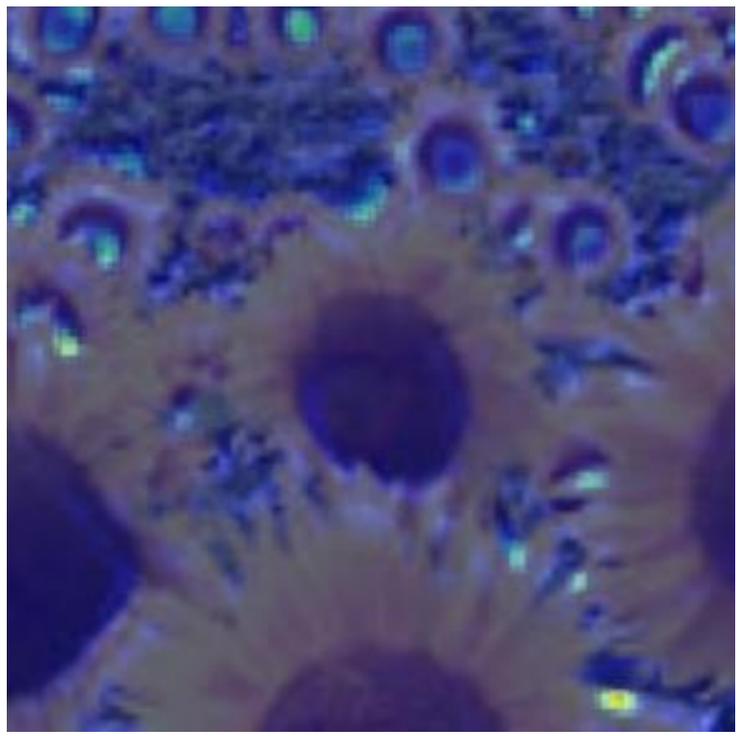

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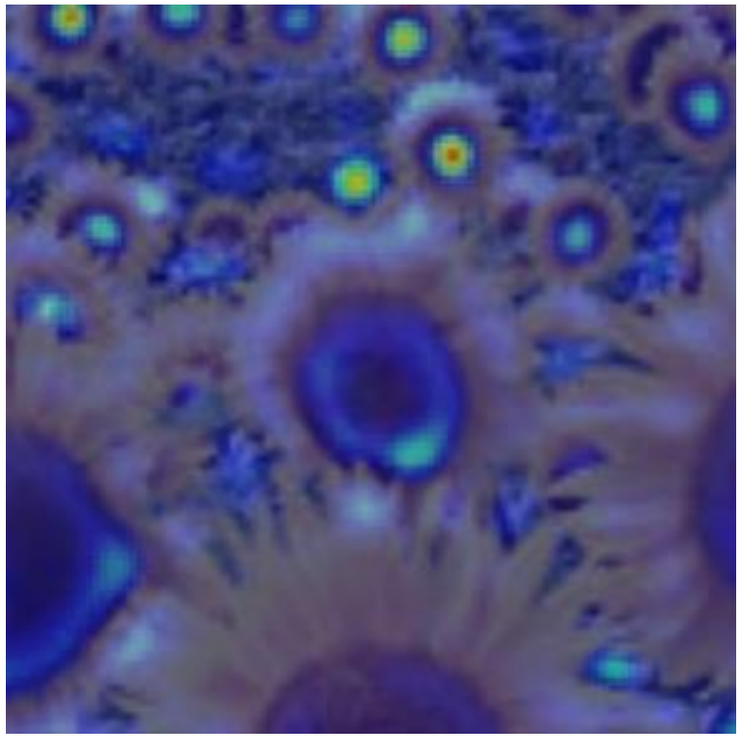
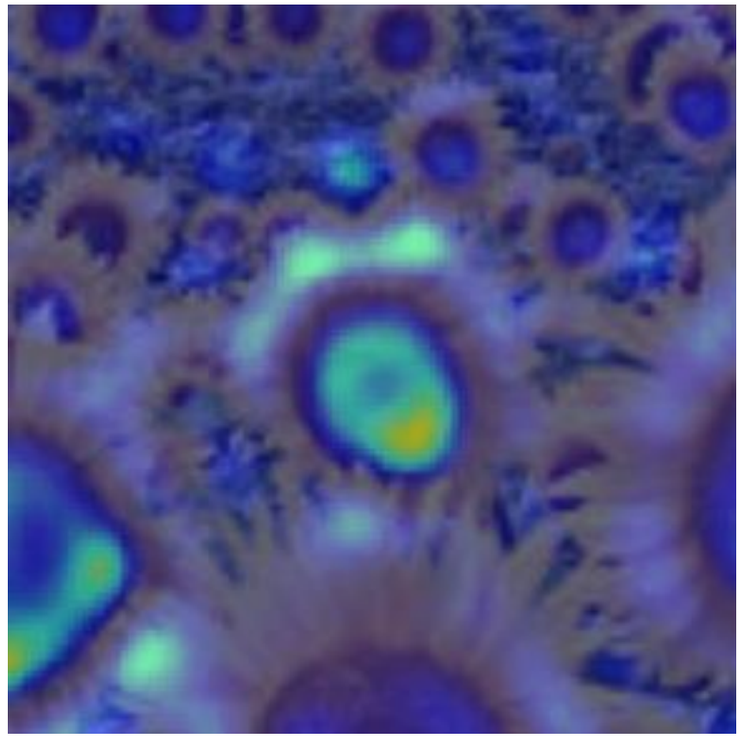


6.0



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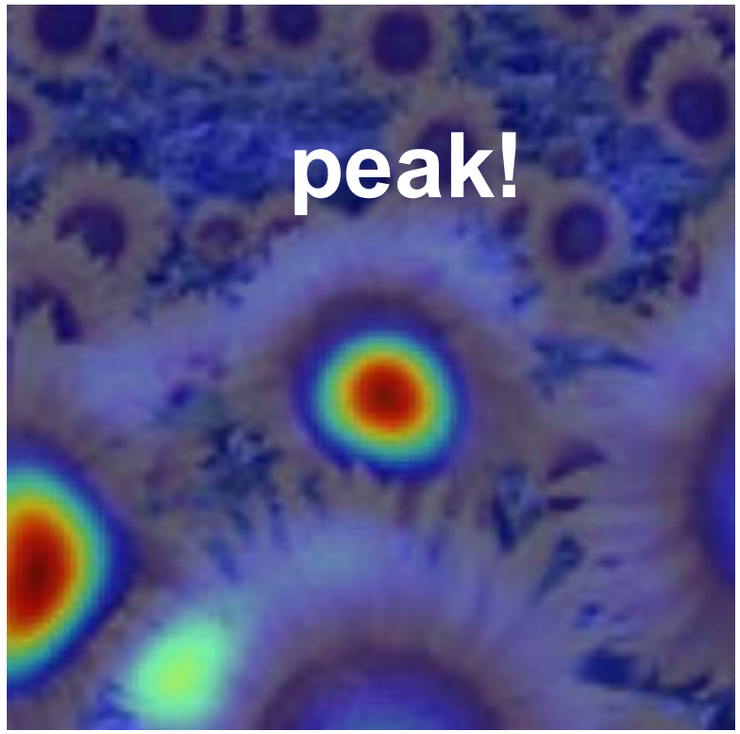

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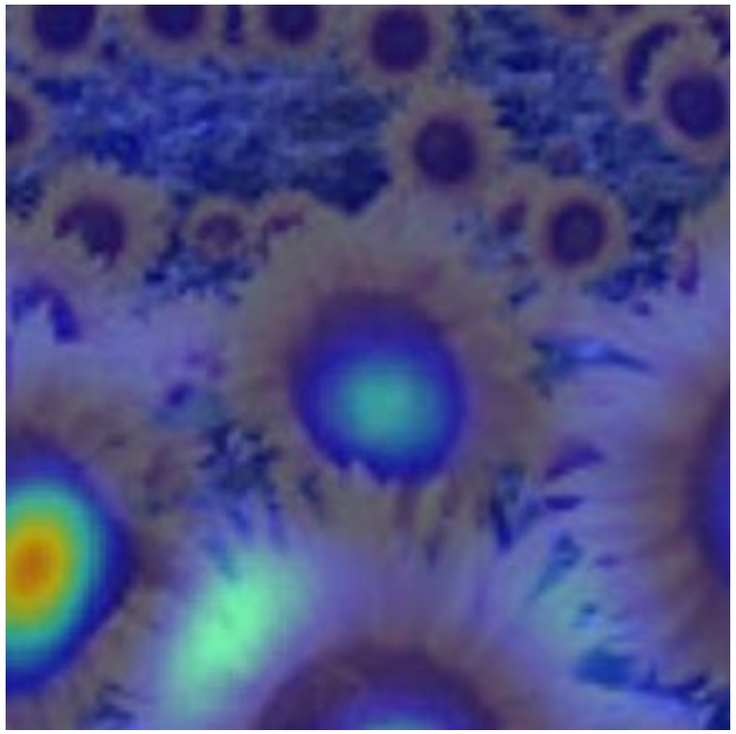

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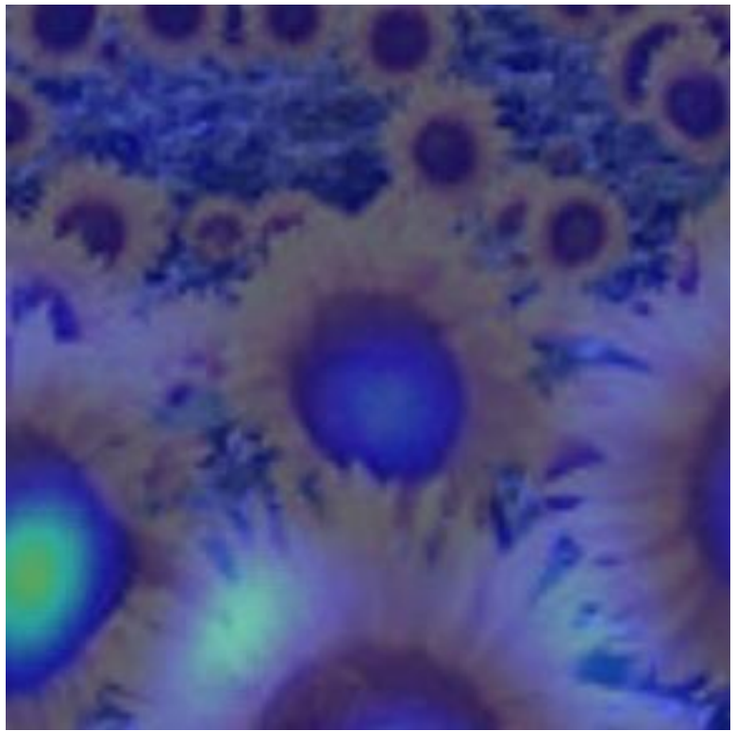
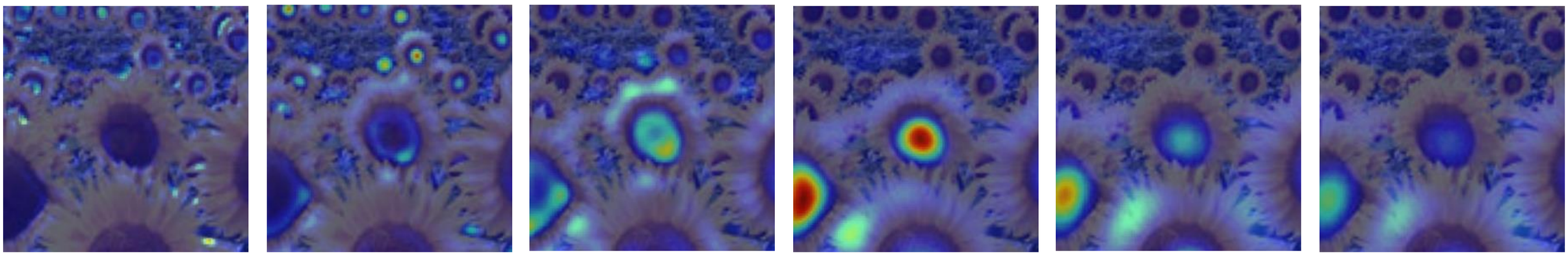


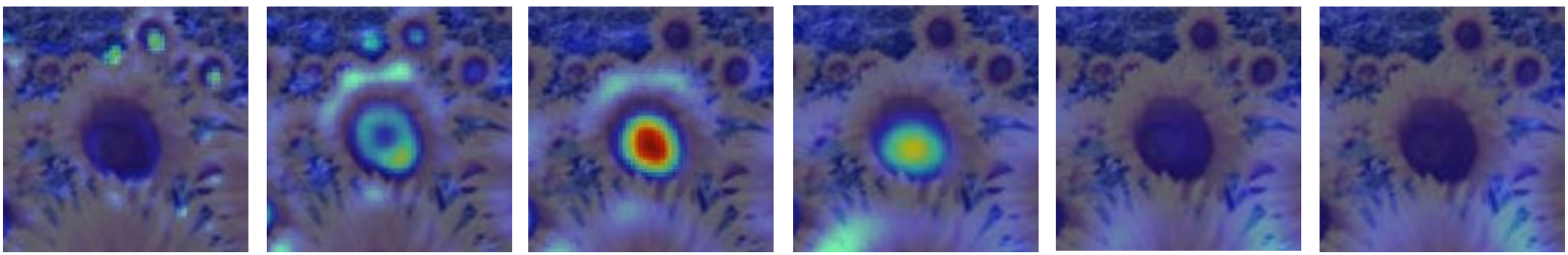
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



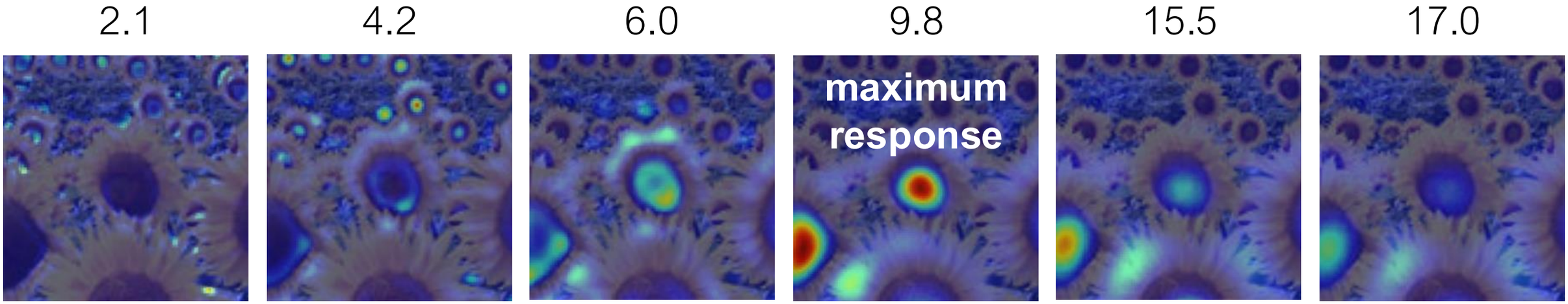
Full size image

2.1 4.2 6.0 9.8 15.5 17.0

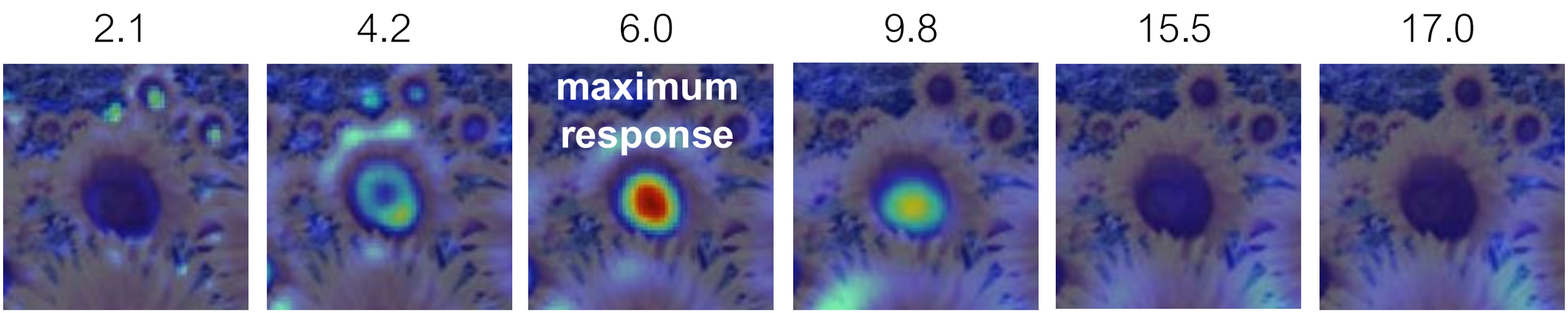


3/4 size image

optimal scale

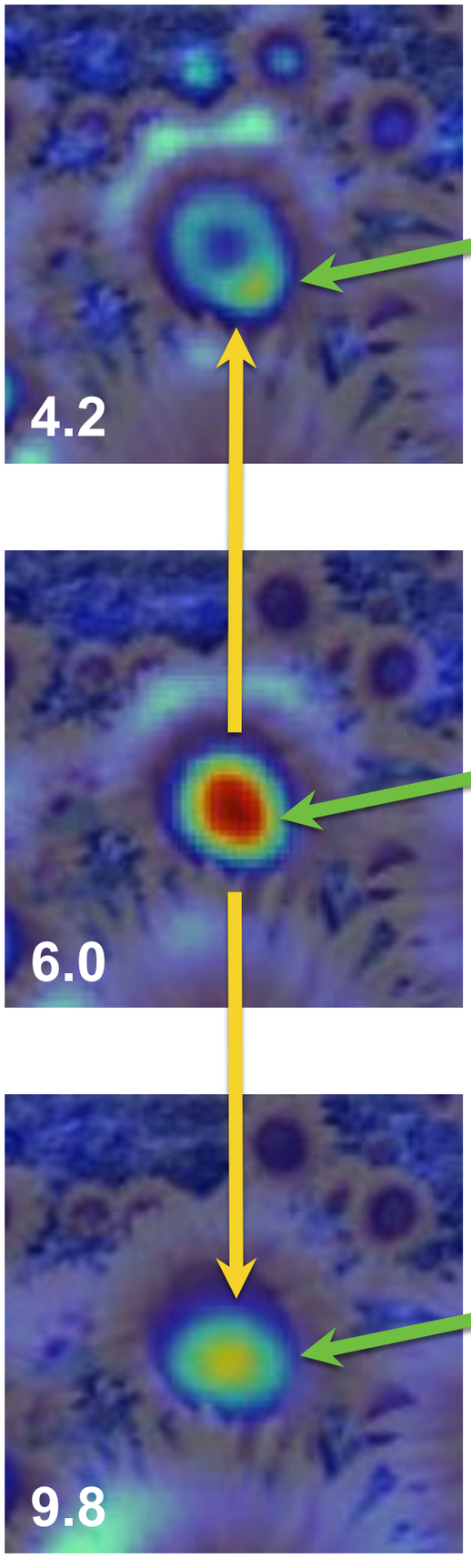


Full size image



3/4 size image

local maximum



local maximum

cross-scale maximum

local maximum

How would you implement scale selection?

Implementation

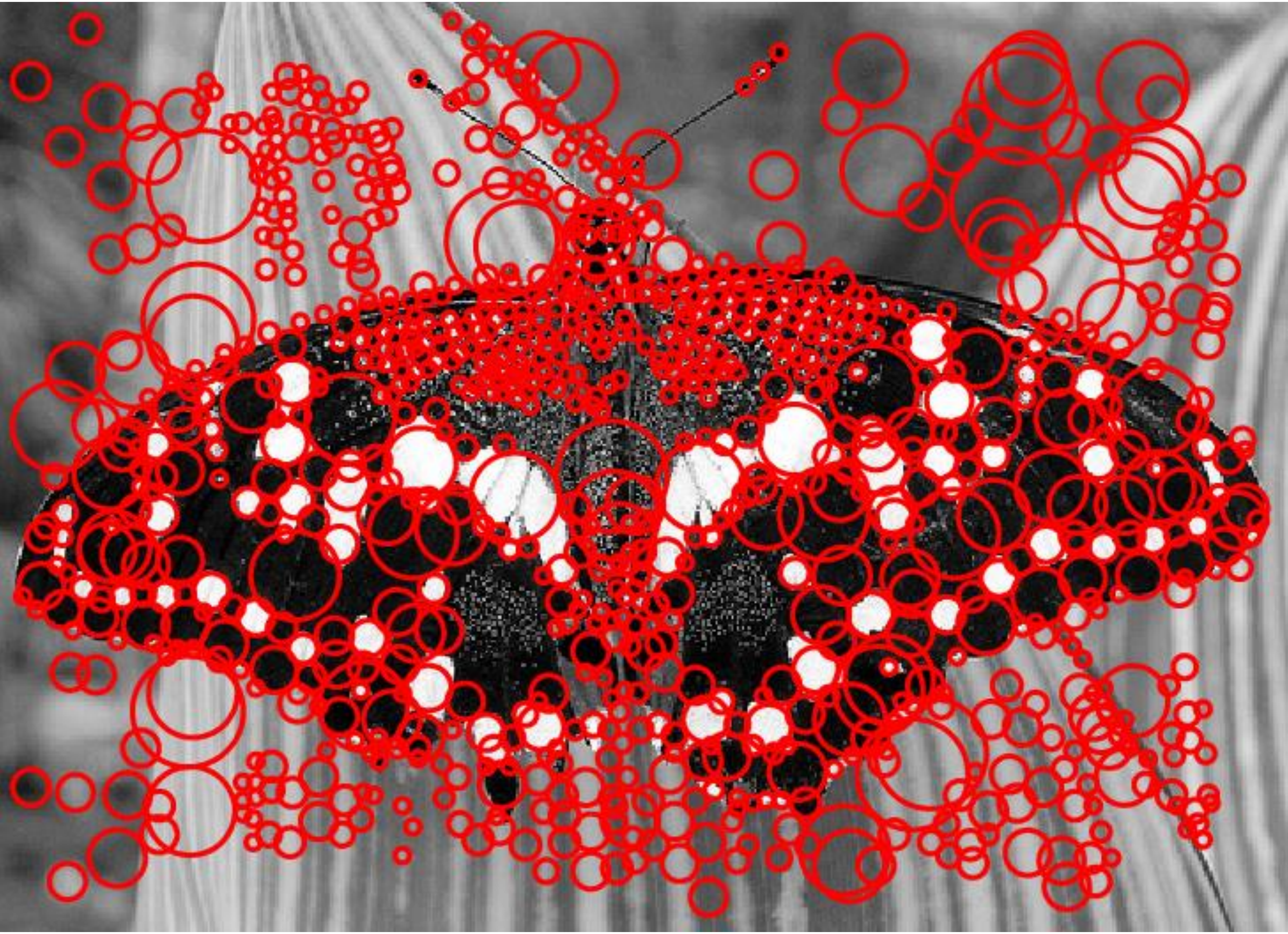
```
For each level of the Gaussian pyramid

        compute feature response (e.g. Harris, Laplacian)
```

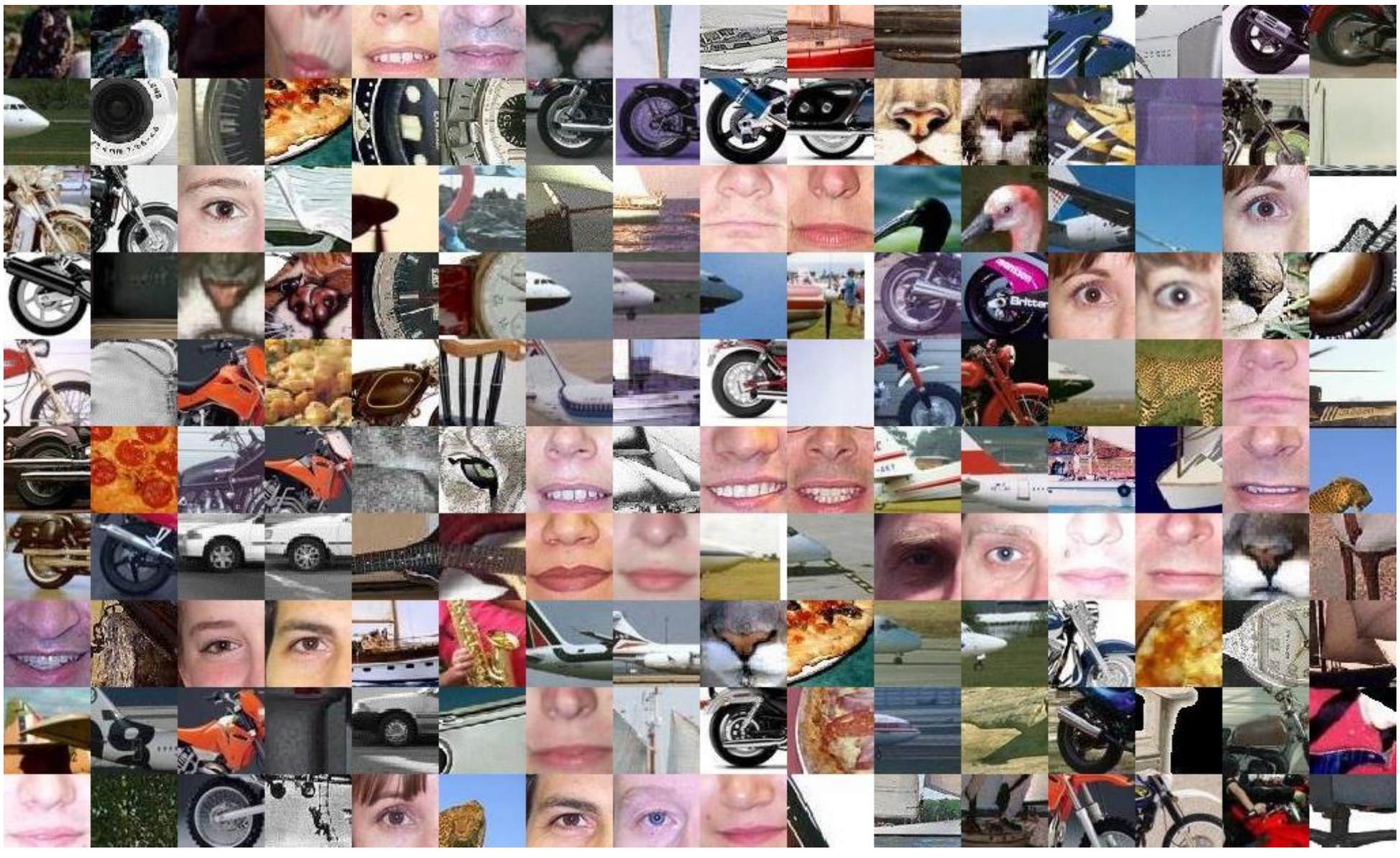
```
For each level of the Gaussian pyramid

        if local maximum and cross-scale

        save scale and location of feature   (x, y, s)
```



Feature detectors and descriptors



ICS 483 Computer Vision
Fall 2024, Lecture 7

Overview of today's lecture

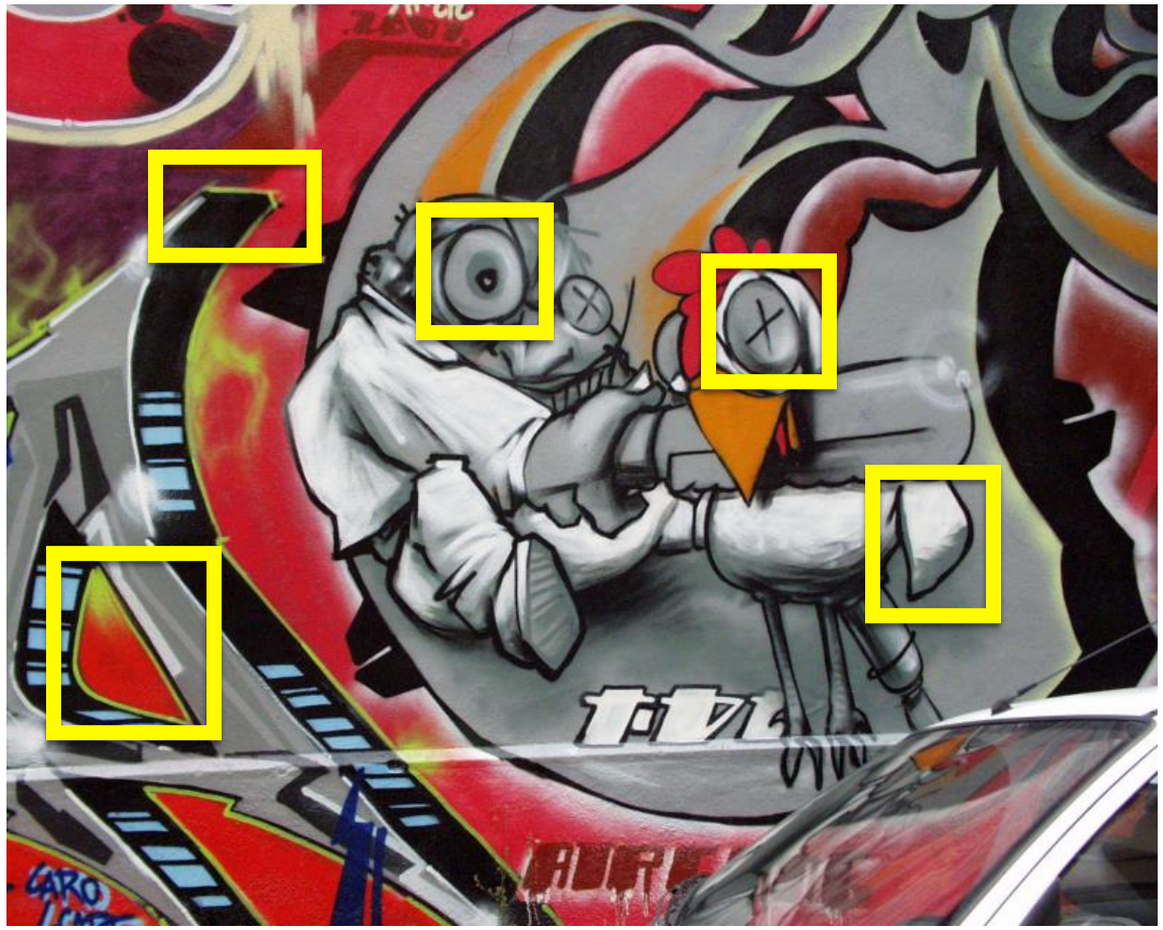
- Why do we need feature descriptors?
- Designing feature descriptors.
- MOPS descriptor.
- GIST descriptor.
- SIFT descriptor.

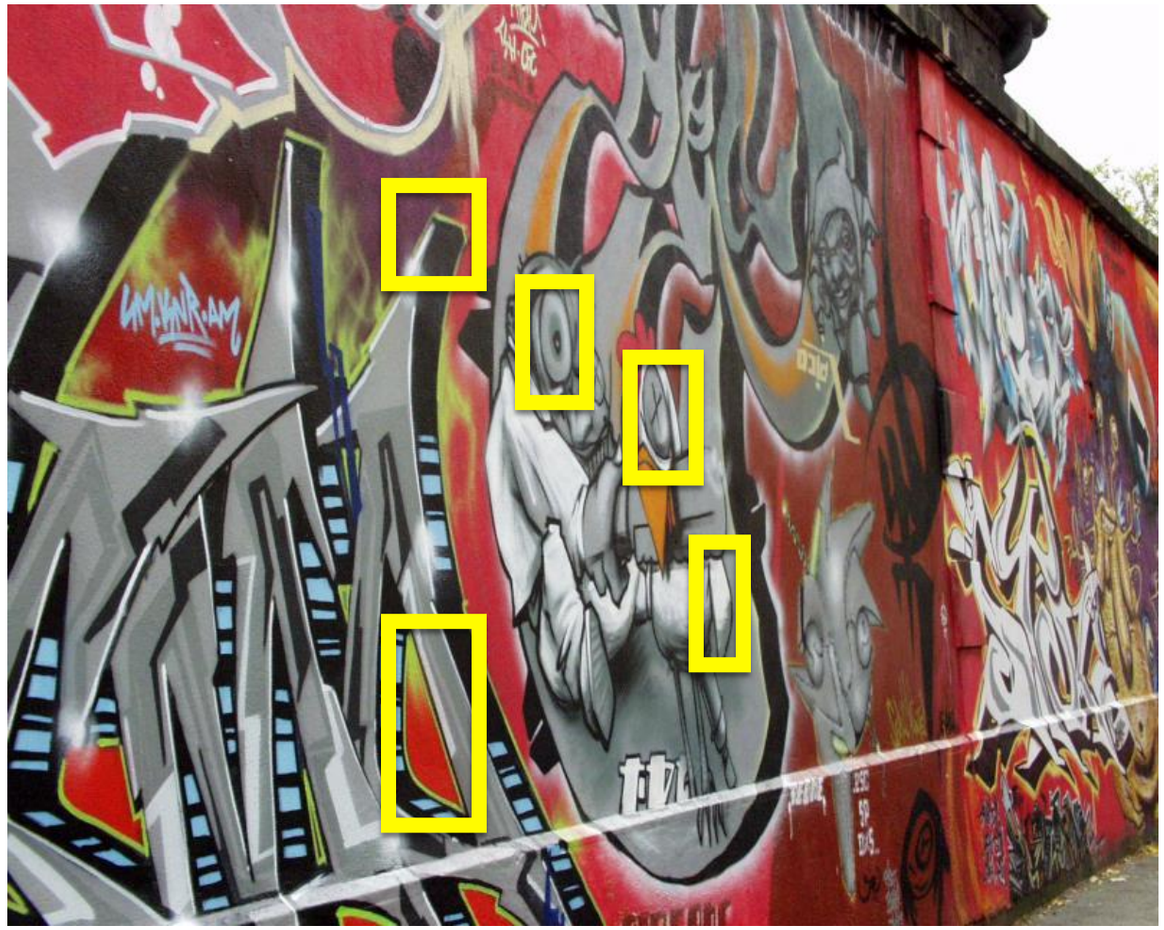
Slide credits

Most of these slides were adapted from:

- Matt O'Toole (CMU, 15-463, Fall 2022).
- Ioannis Gkioulekas (CMU, 15-463, Fall 2020).
- Kris Kitani (CMU, 15-463, Fall 2016).

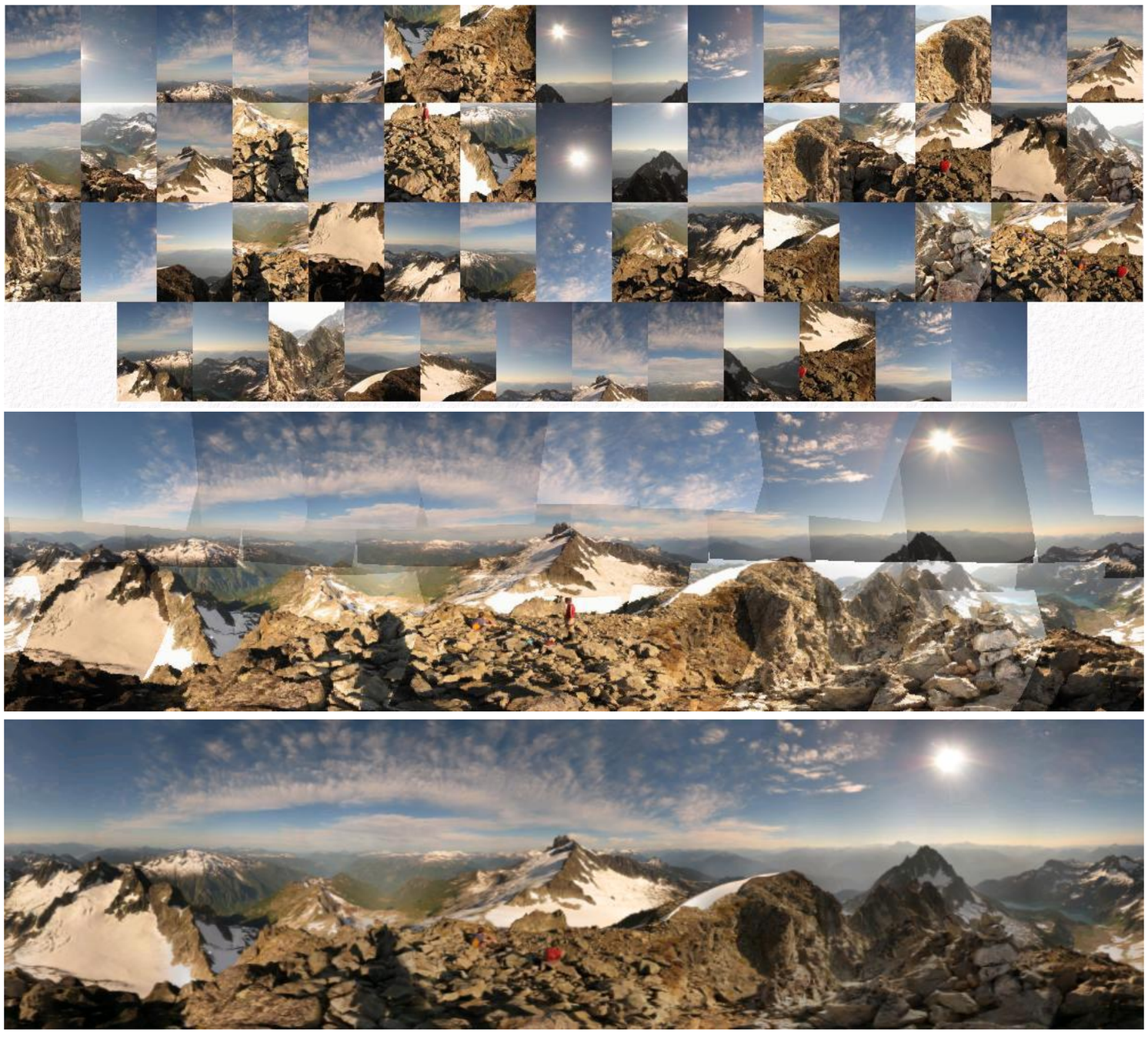
Why do we need feature descriptors?





If we know where the <u>good</u> features are,
how do we <u>match</u> them?

Image mosaicing



How do we describe an image patch?

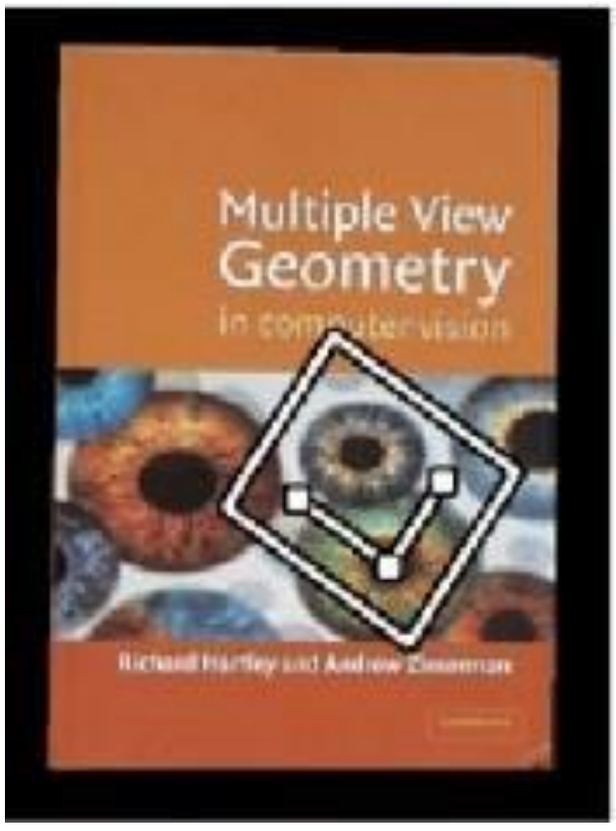
Patches with similar content should have similar descriptors.

Designing feature descriptors

What is the best descriptor for an image feature?

Photometric transformations

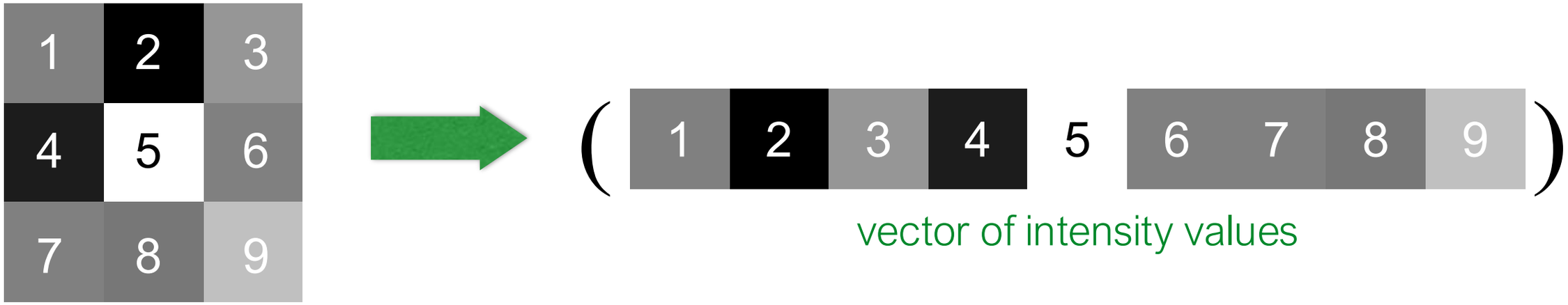
Geometric transformations



objects will appear at different scales, translation and rotation

Image patch

Just use the pixel values of the patch!

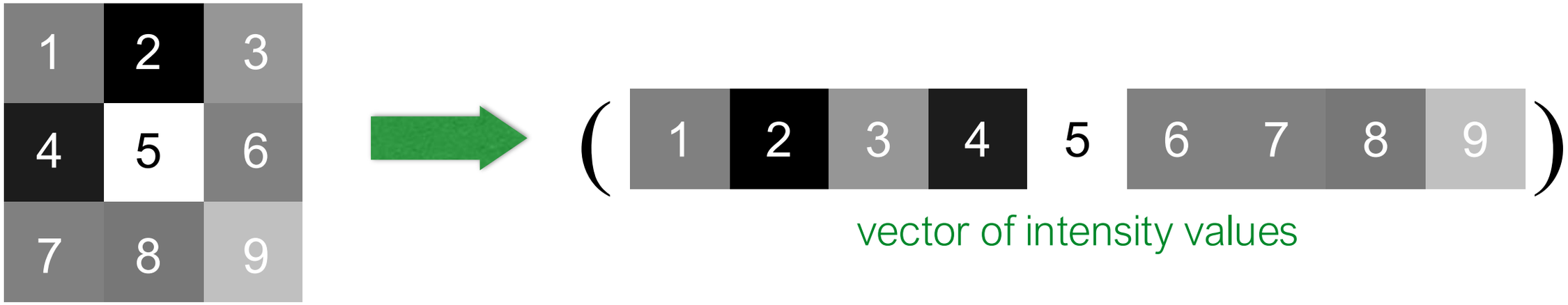


Perfectly fine if geometry and appearance is unchanged

(a.k.a. template matching)

Image patch

Just use the pixel values of the patch!



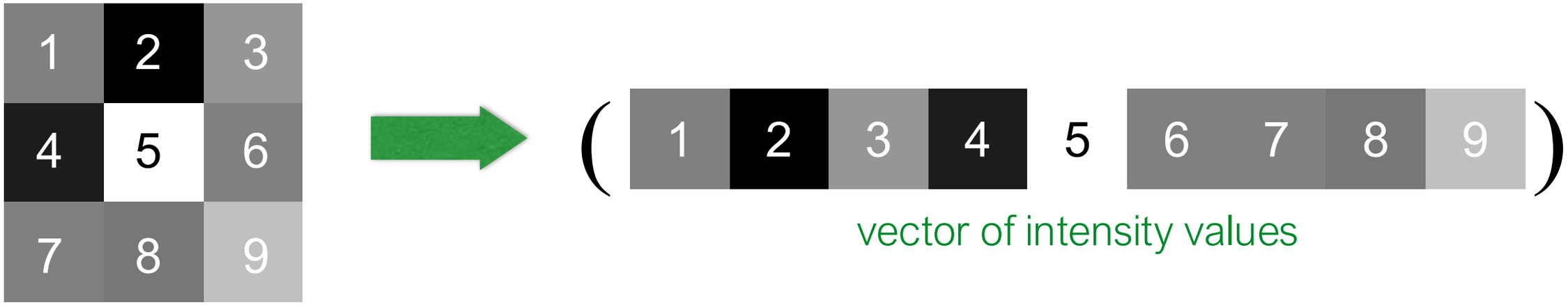
Perfectly fine if geometry and appearance is unchanged

(a.k.a. template matching)

What are the problems?

Image patch

Just use the pixel values of the patch!



Perfectly fine if geometry and appearance is unchanged

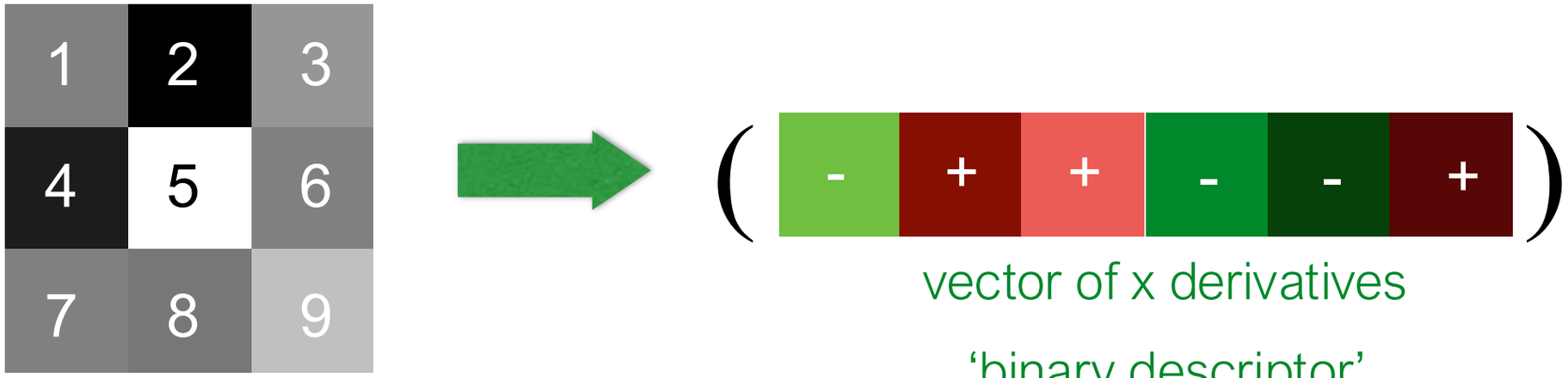
(a.k.a. template matching)

What are the problems?

How can you be less sensitive to absolute intensity values?

Image gradients

Use pixel differences

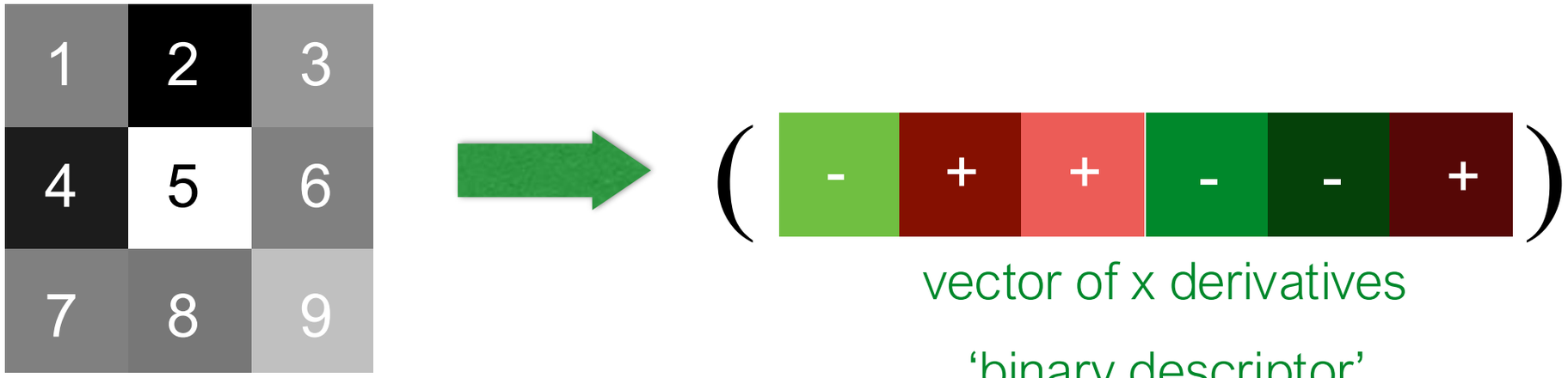


Feature is invariant to absolute intensity values

What are the problems?

Image gradients

Use pixel differences



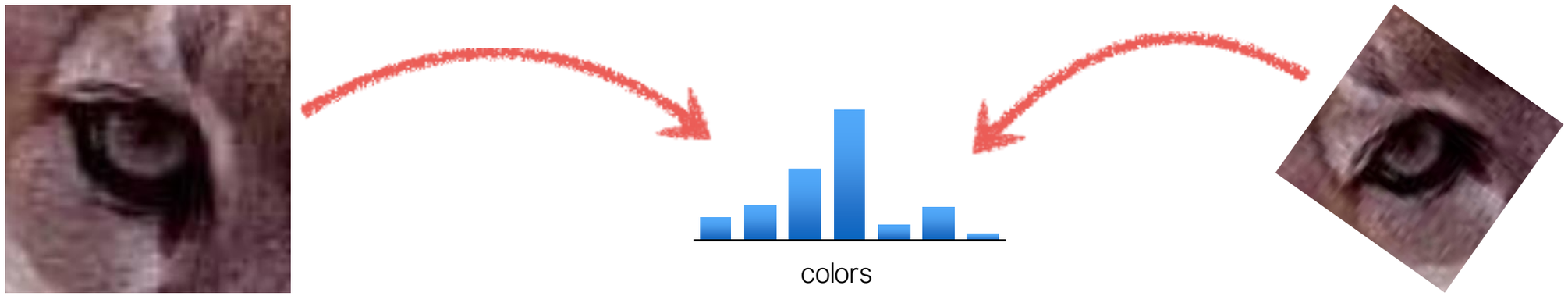
Feature is invariant to absolute intensity values

What are the problems?

How can you be less sensitive to deformations?

Color histogram

Count the colors in the image using a histogram

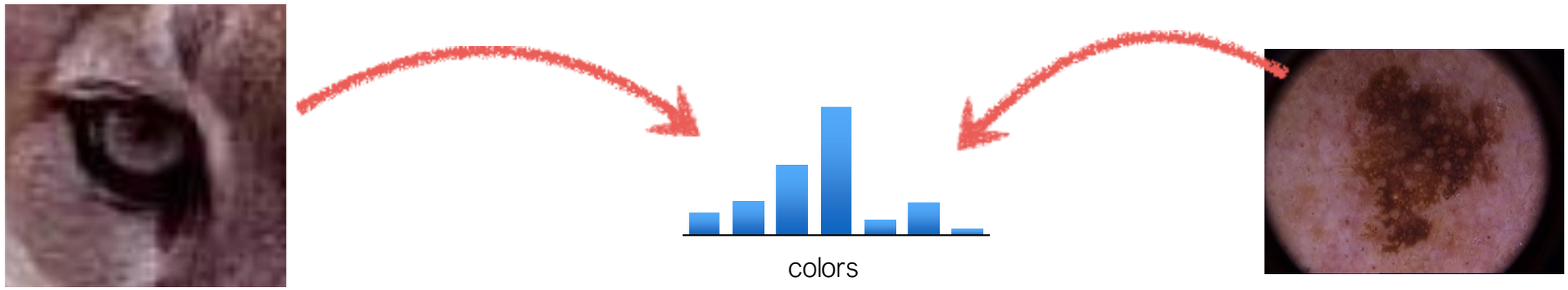


Invariant to changes in scale and rotation

What are the problems?

Color histogram

Count the colors in the image using a histogram

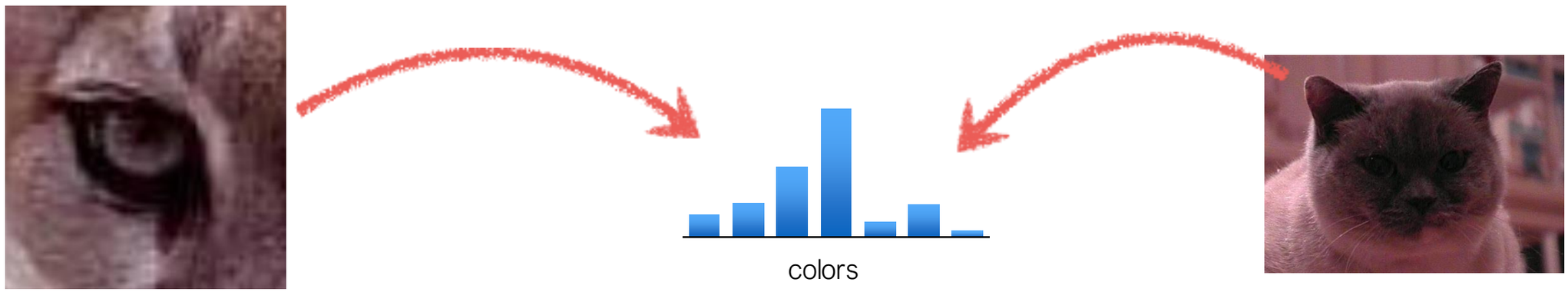


Invariant to changes in scale and rotation

What are the problems?

Color histogram

Count the colors in the image using a histogram



Invariant to changes in scale and rotation

What are the problems?

How can you be more sensitive to spatial layout?