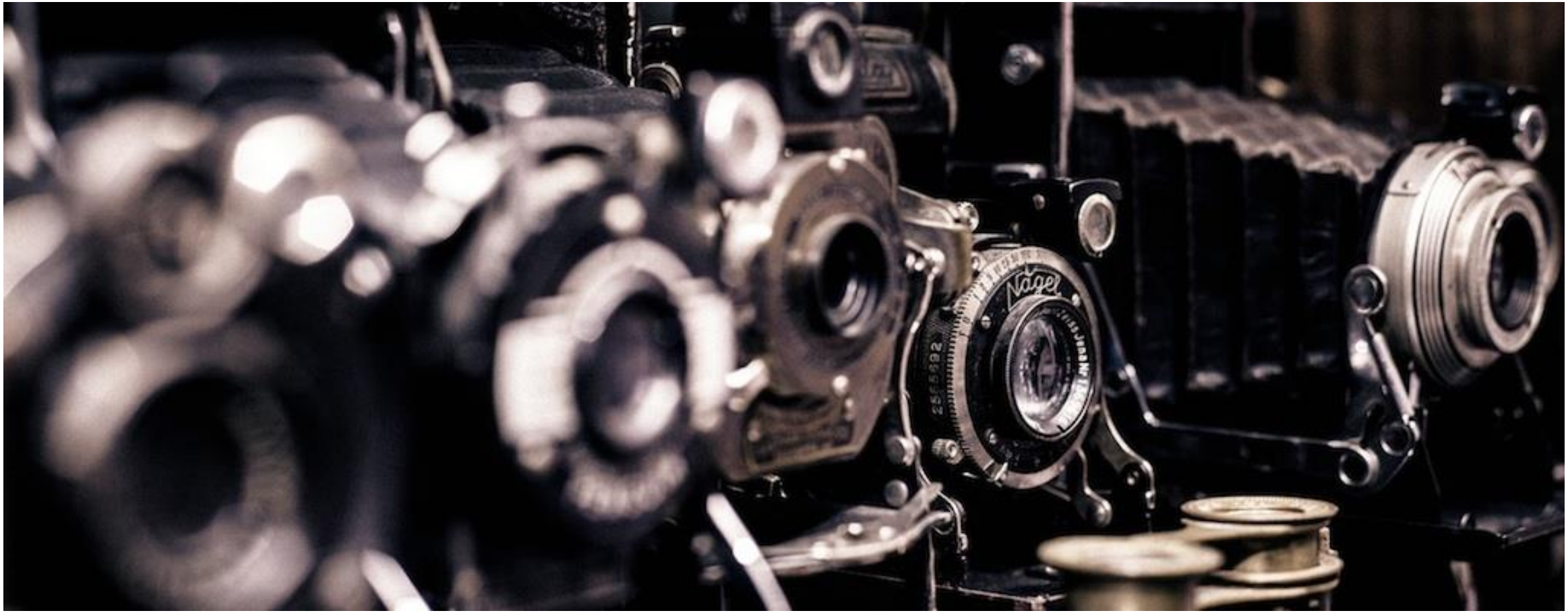
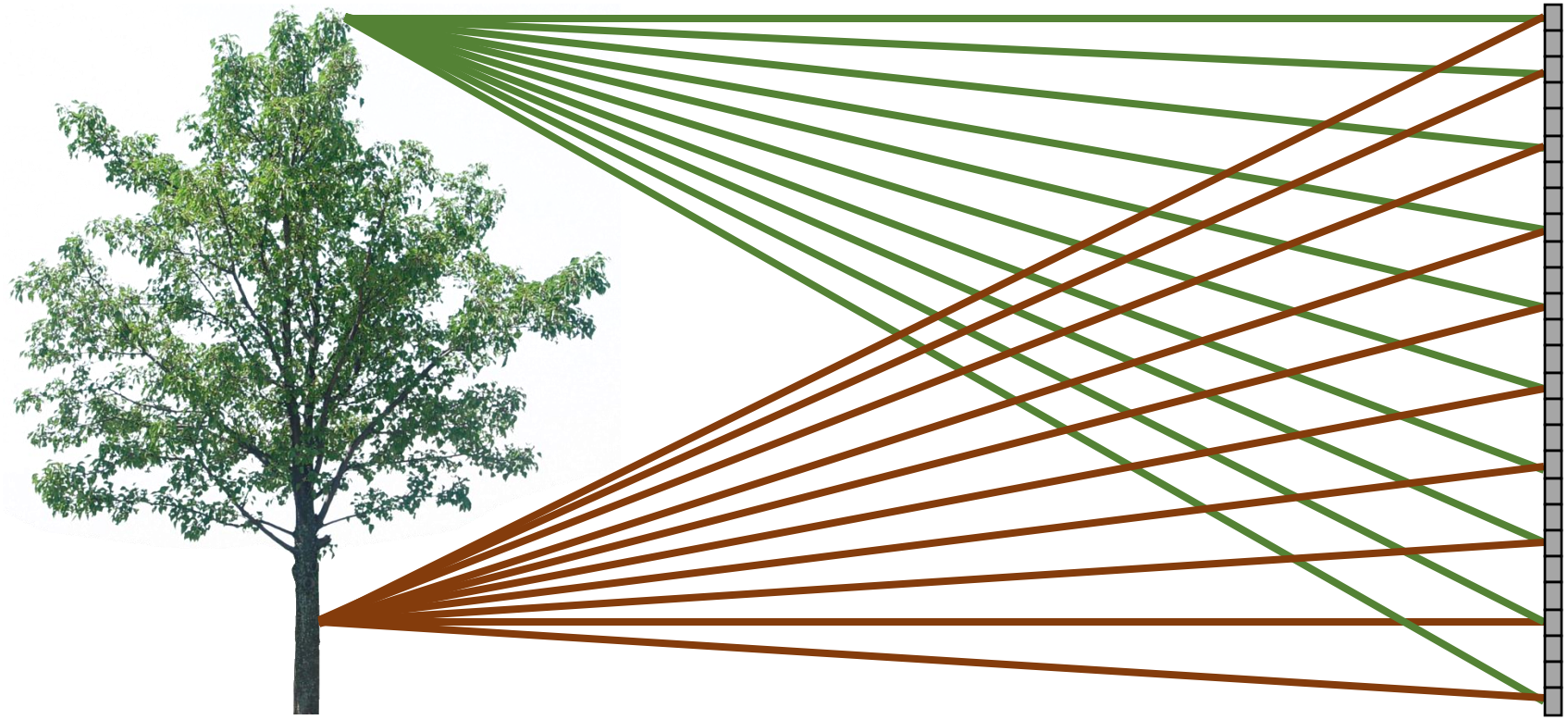
Geometric camera models



ICS 483 Computer Vision
Fall 2025, Lecture 15

Recap: Bare-sensor imaging

real-world object



digital sensor (CCD or CMOS)

All scene points contribute to all sensor pixels

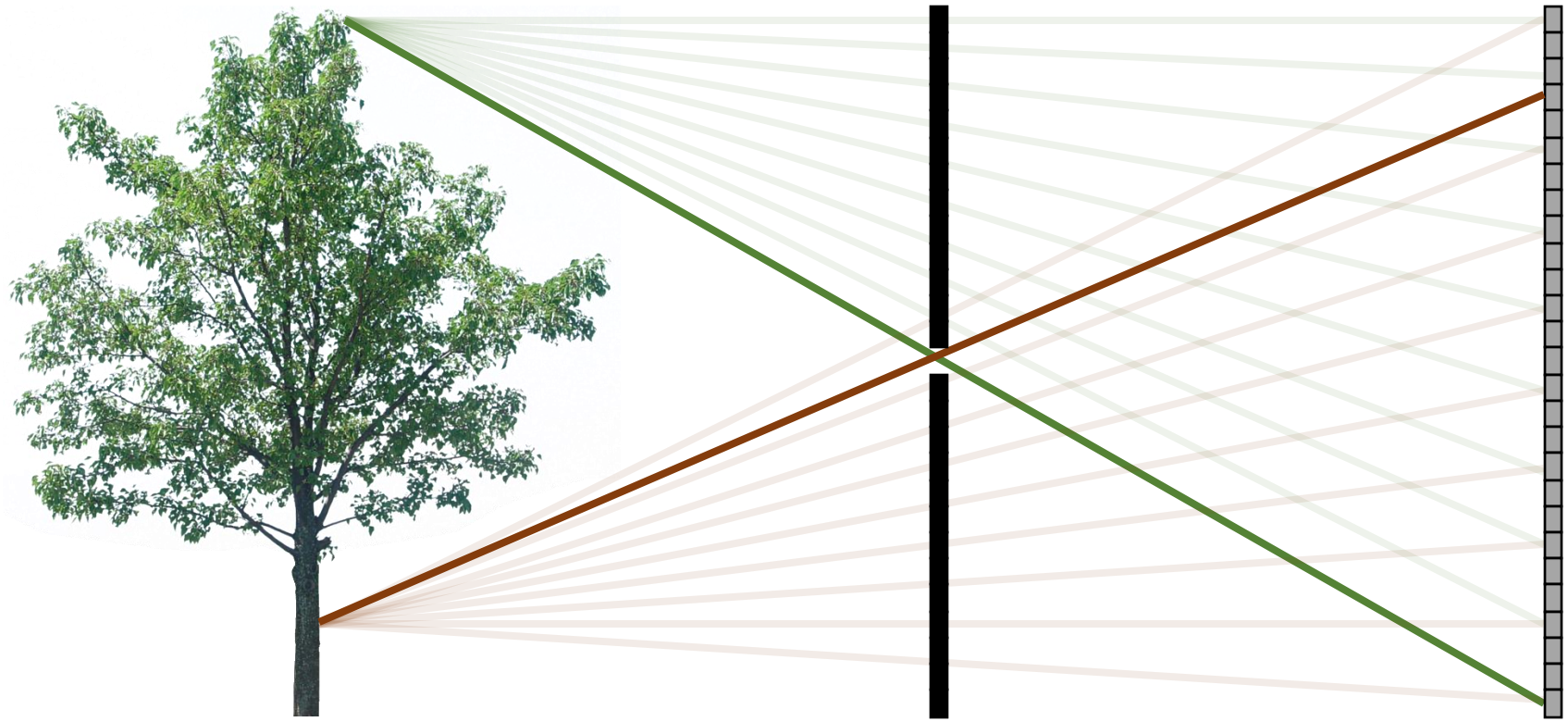
What does the image on the sensor look like?

Recap: Bare-sensor imaging

All scene points contribute to all sensor pixels

Recap: Pinhole imaging

real-world object

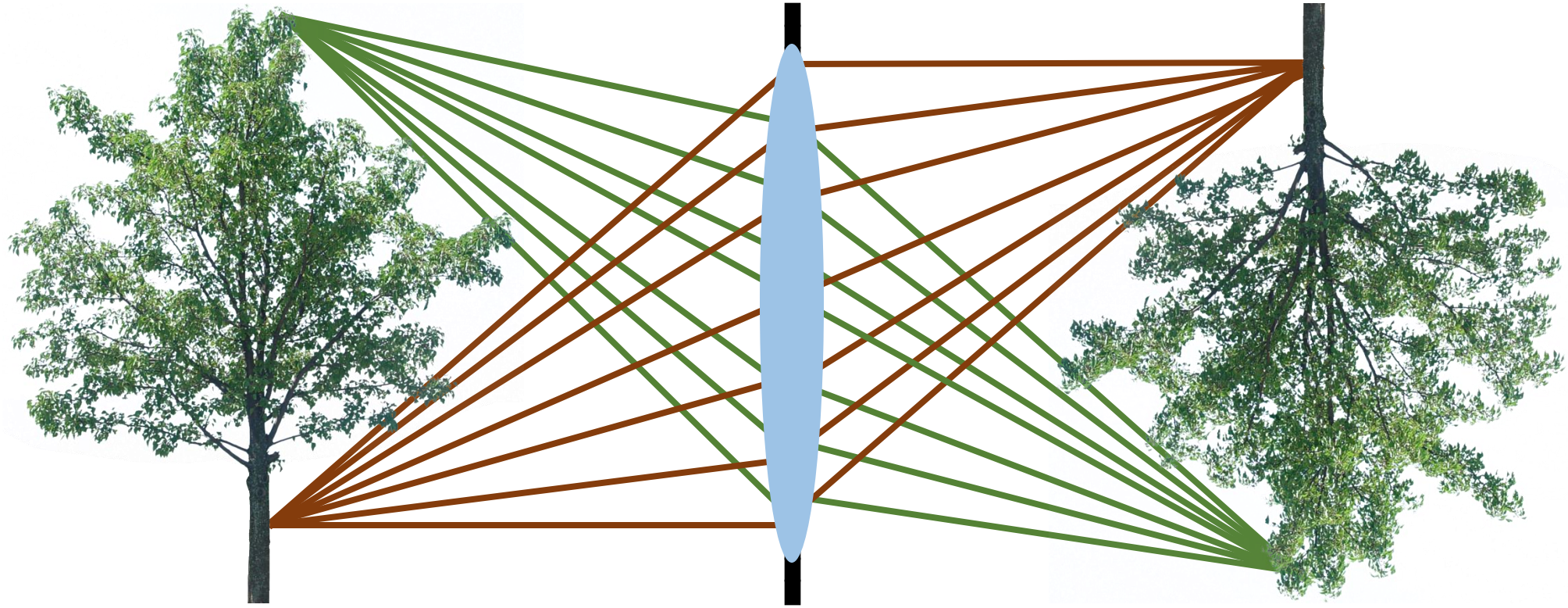


digital sensor (CCD or CMOS)

Each scene point contributes to only one sensor pixel

What does the image on the sensor look like?

Recap: The lens camera

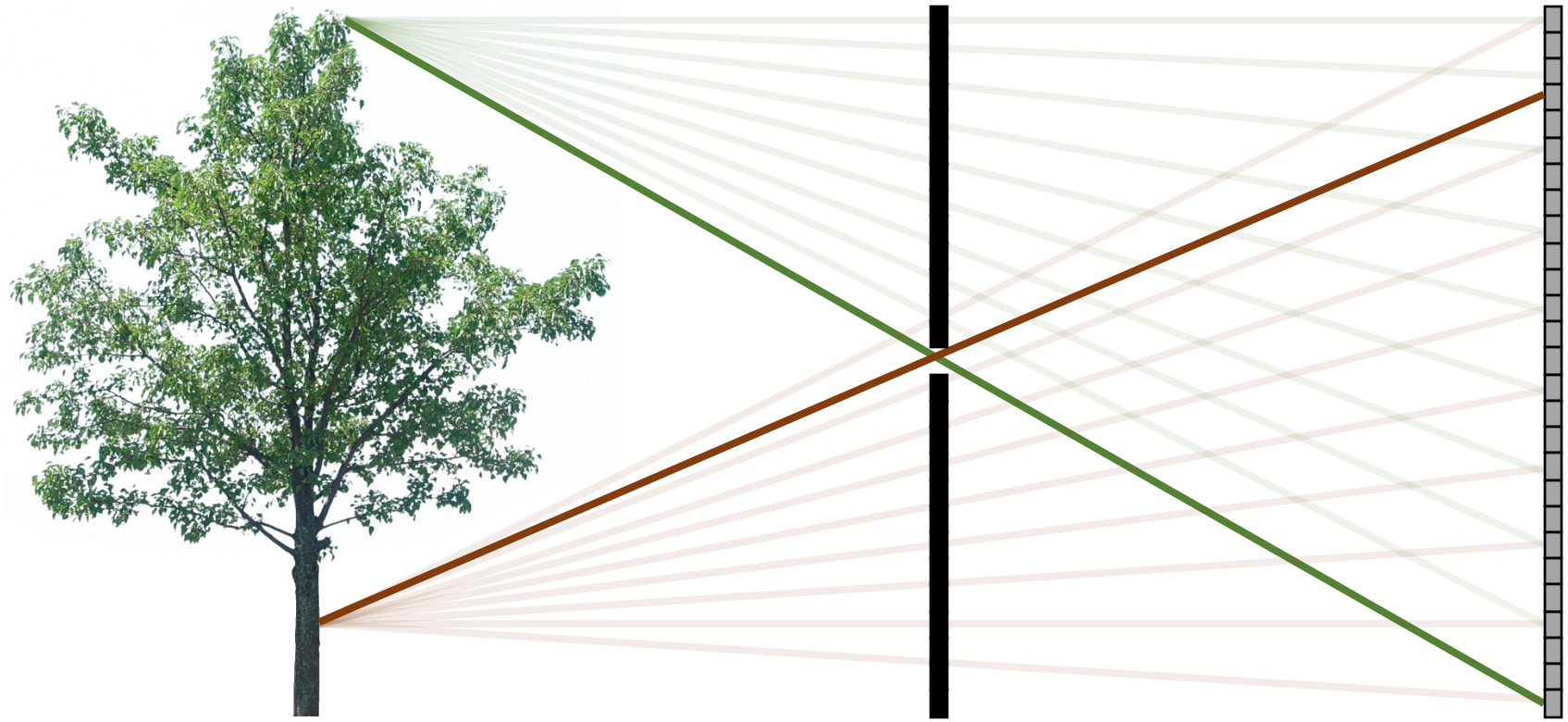


Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?

Recap: Pinhole imaging

real-world object



digital sensor (CCD or CMOS)

Each scene point contributes to only one sensor pixel

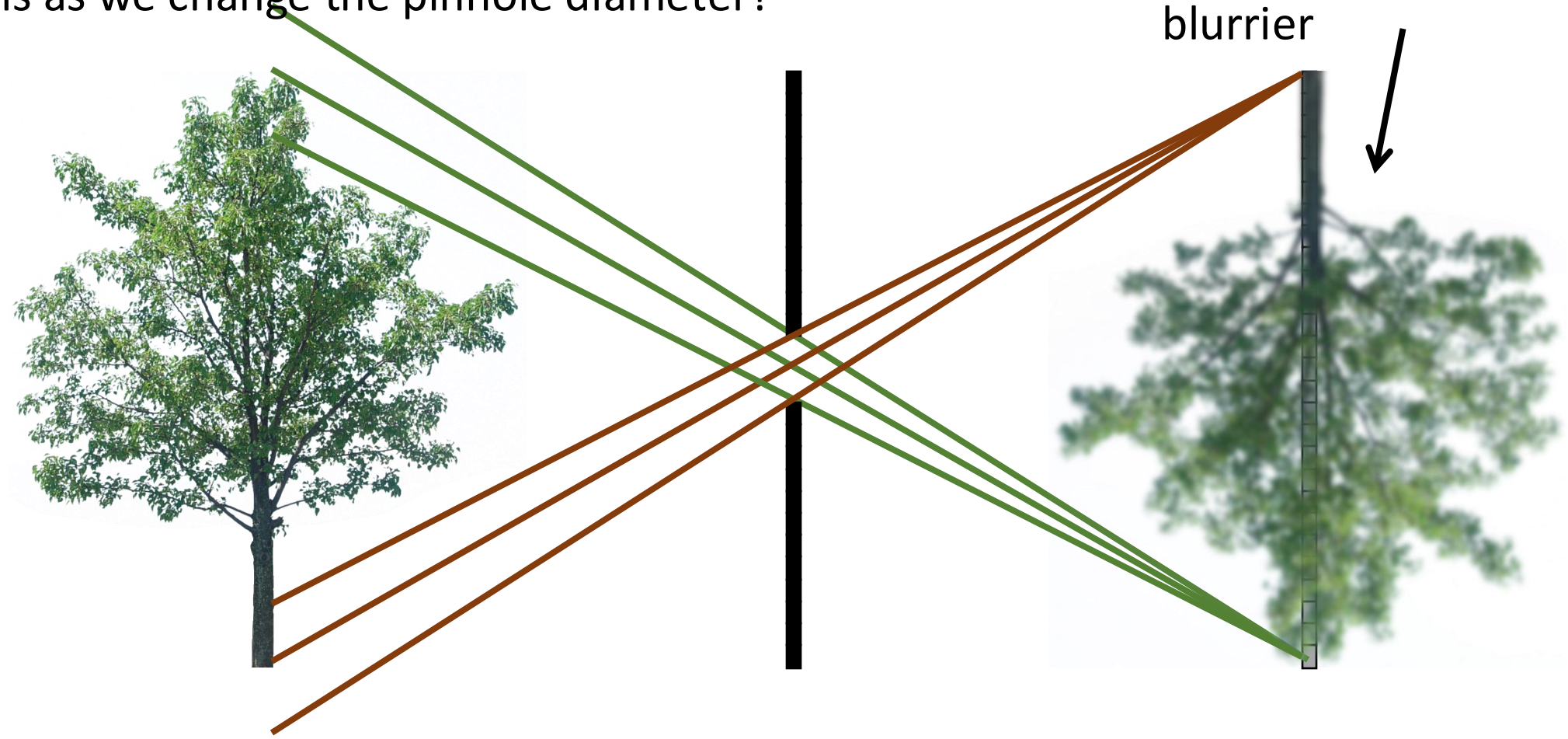
What does the image on the sensor look like?

Pinhole size

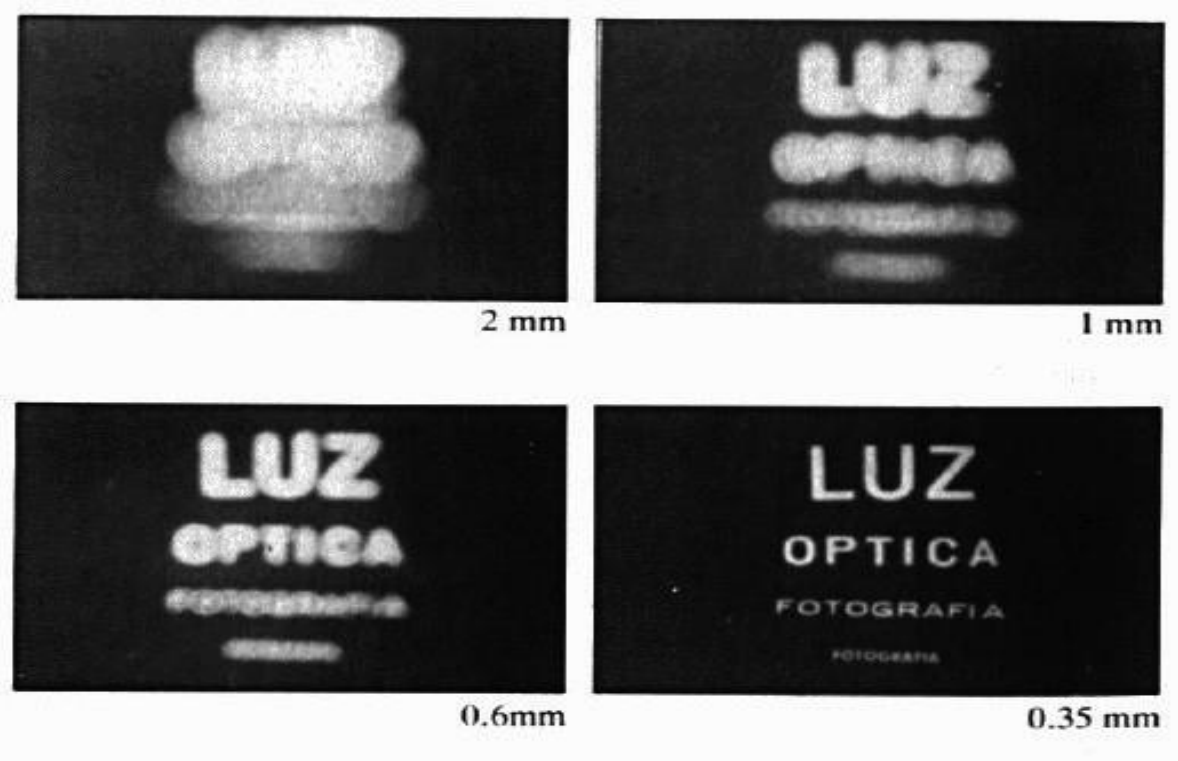
What happens as we change the pinhole diameter?

object projection becomes blurrier

real-world object

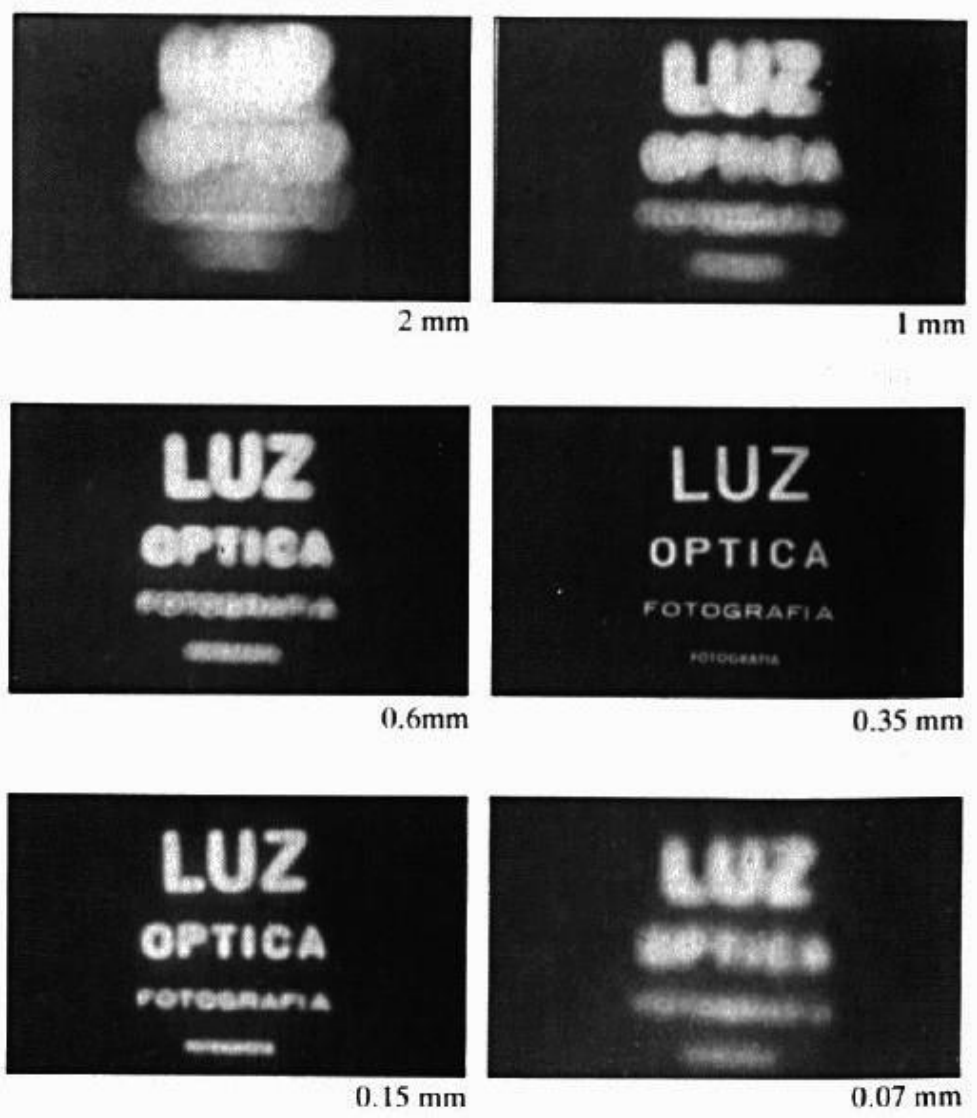


Shrinking the aperture

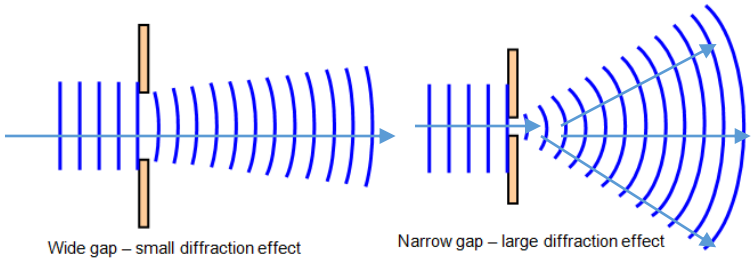


- Why not make the aperture as small as possible?
  - Less light gets through
  - *Diffraction* effects...

Shrinking the aperture



https://physics.highpoint.edu/~jregeste
r/potl/Waves/DiffractionInterference/D
ffraction.html



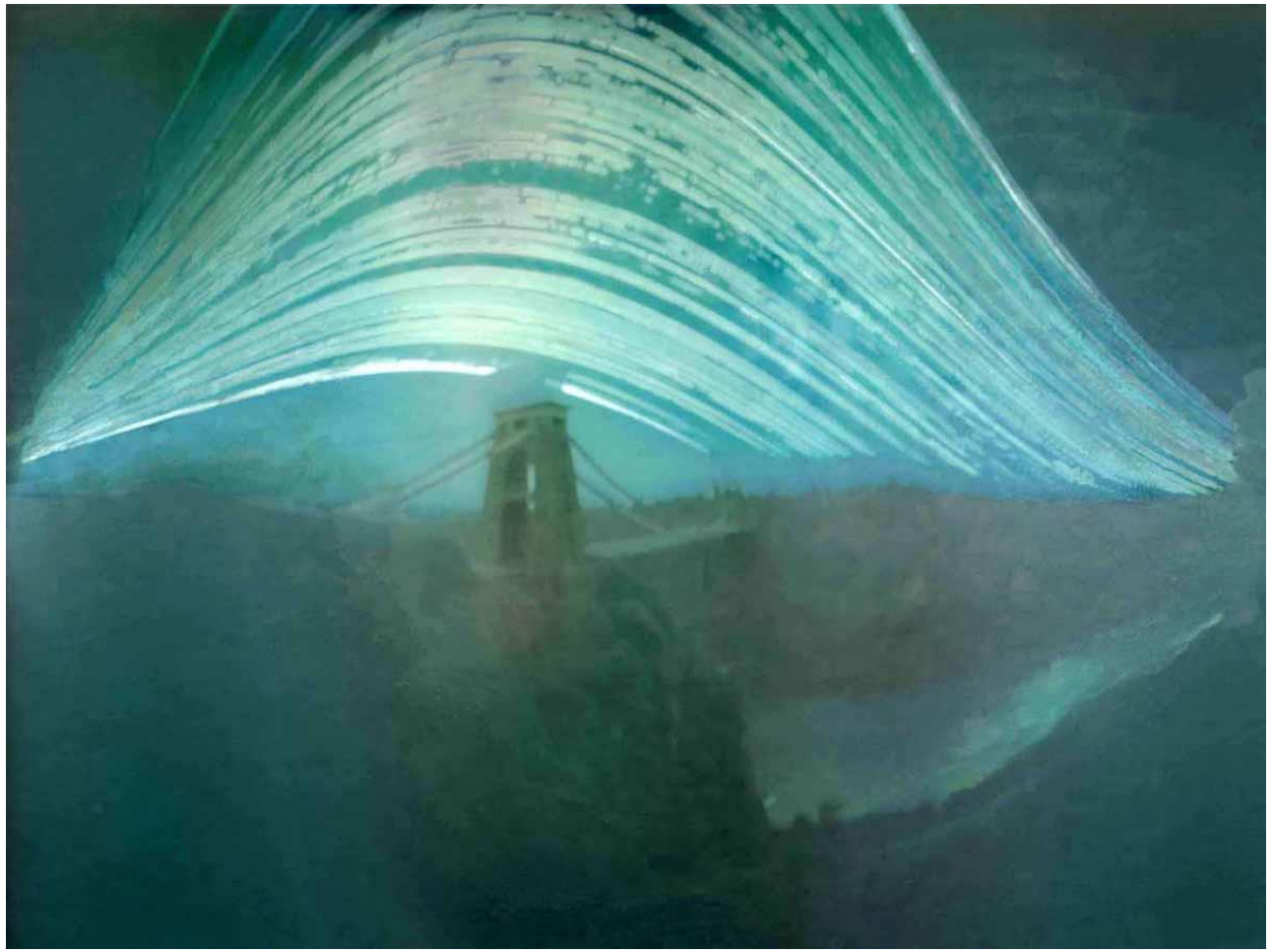
Let’s try the DIY pinhole camera!

Home-made pinhole camera

Slide by A. Efros

http://www.debevec.org/Pinhole/

Pinhole photography for solargraphy



Justin Quinnell, The Clifton Suspension Bridge. December 17th 2007 - June 21st 2008
6-month **exposure**

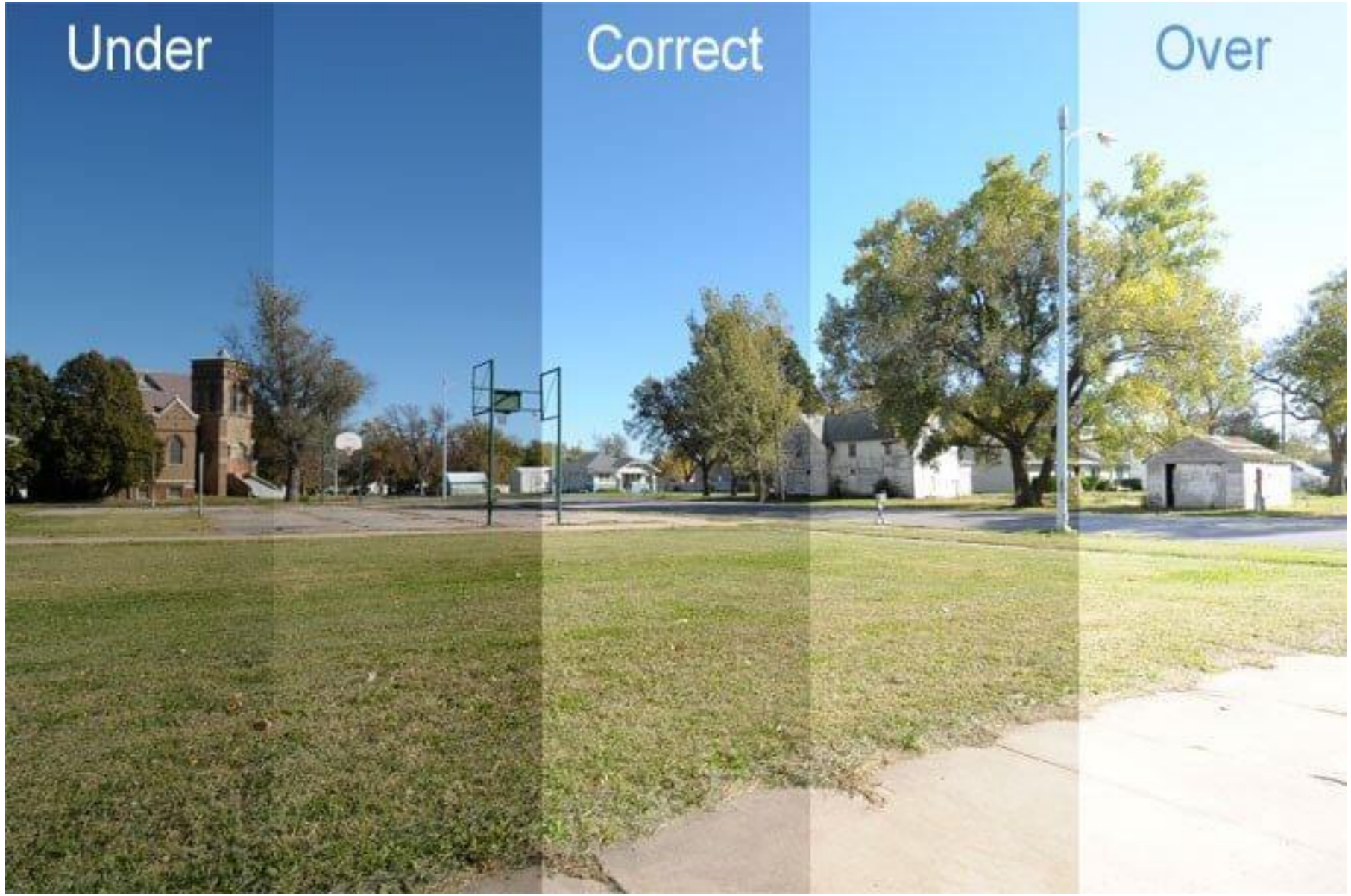
Moving subjects

Shutter speed choice becomes more important when you photograph moving objects. The quicker the subject is moving, the faster the shutter speed you need to freeze the subject. Go for a slower speed and the moving elements will appear blurred – but get the right degree of blur and your shot can look great.

https://mrs-cook.weebly.com/motion.html

Motion blur

Long exposure time



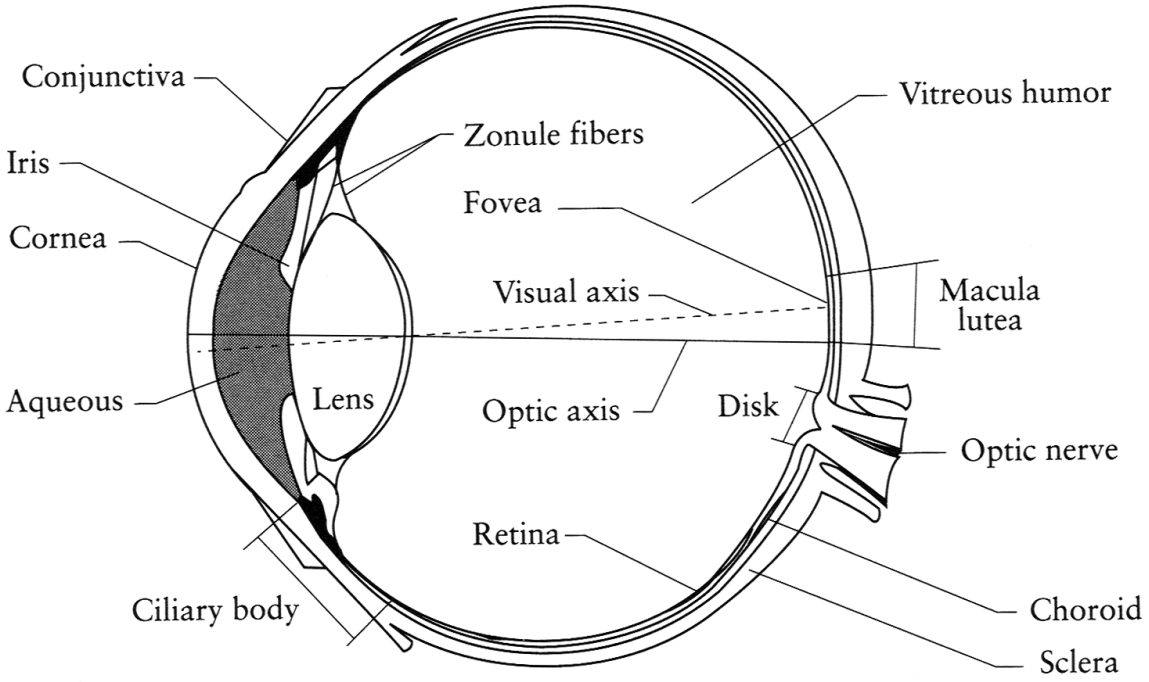
https://www.studiobinder.com/blog/what-is-overexposure-in-photography/

https://petapixel.com/2019/07/17/this-is-the-worlds-first-solargraphy-timelapse/

How’s time-lapse video made?

https://www.huffpost.com/entry/time-lapse-gifs_n_4421810

The eye

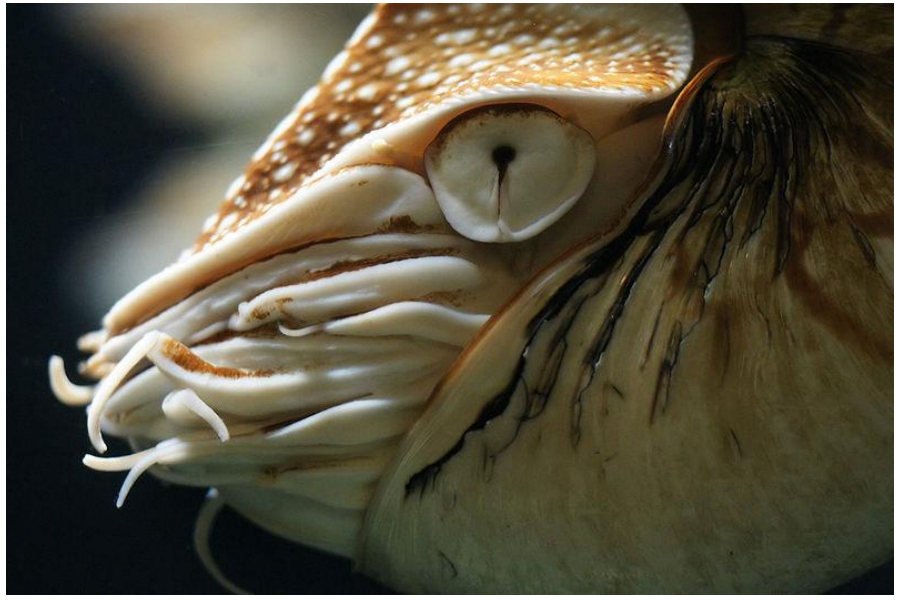


The human eye is a camera

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What's the "film"?
  - photoreceptor cells (rods and cones) in the **retina**

http://www.telegraph.co.uk/news/earth/earthpicturegalleries/7598120/Animal-eyes-quiz-Can-you-work-out-which-creatures-these-are-from-their-eyes.html?image=25

Eyes in nature: eyespots to pinhole camera and compound eyes



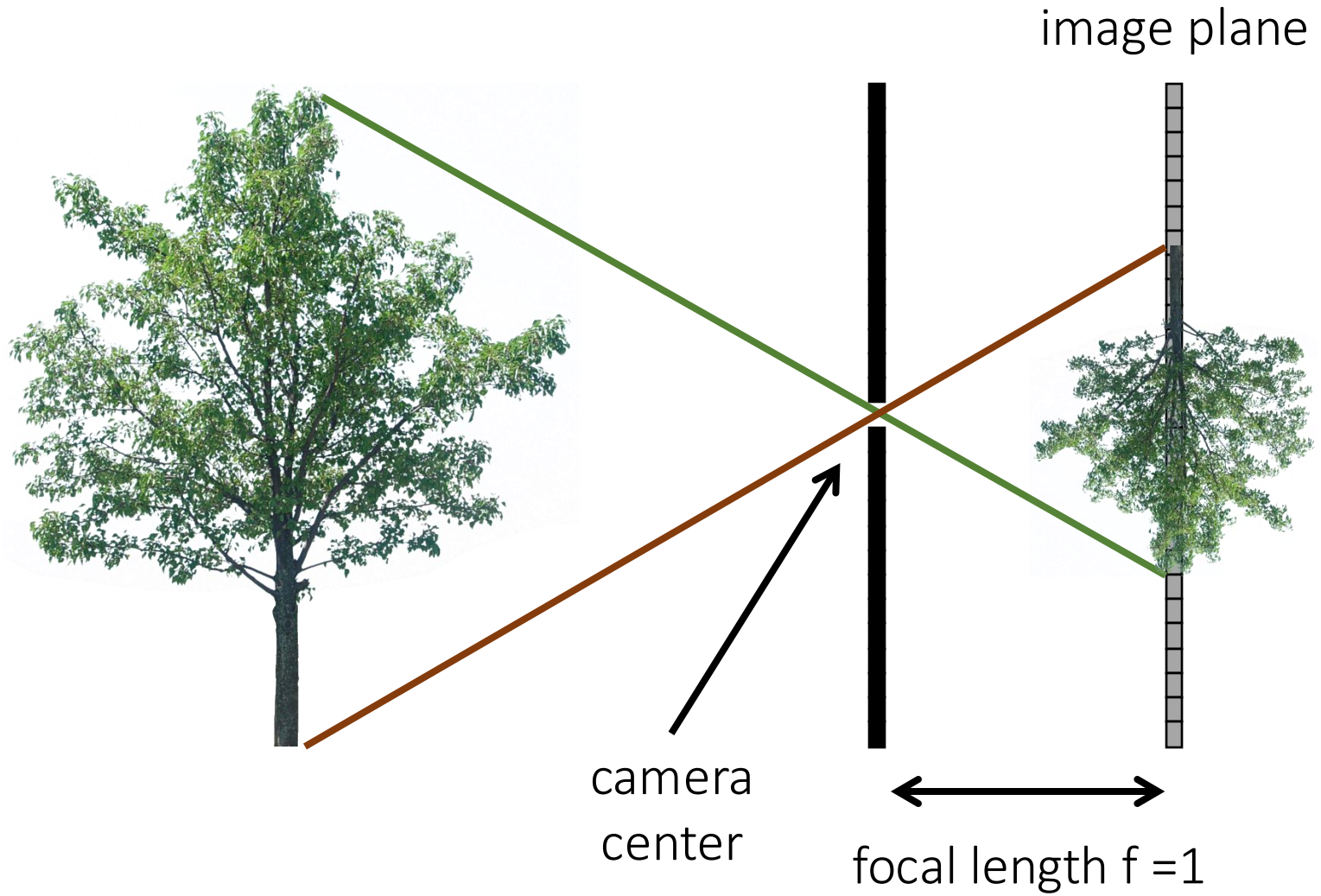
http://upload.wikimedia.org/wikipedia/commons/6/6d/Mantis_shrimp.jpg

Great Camera Demo

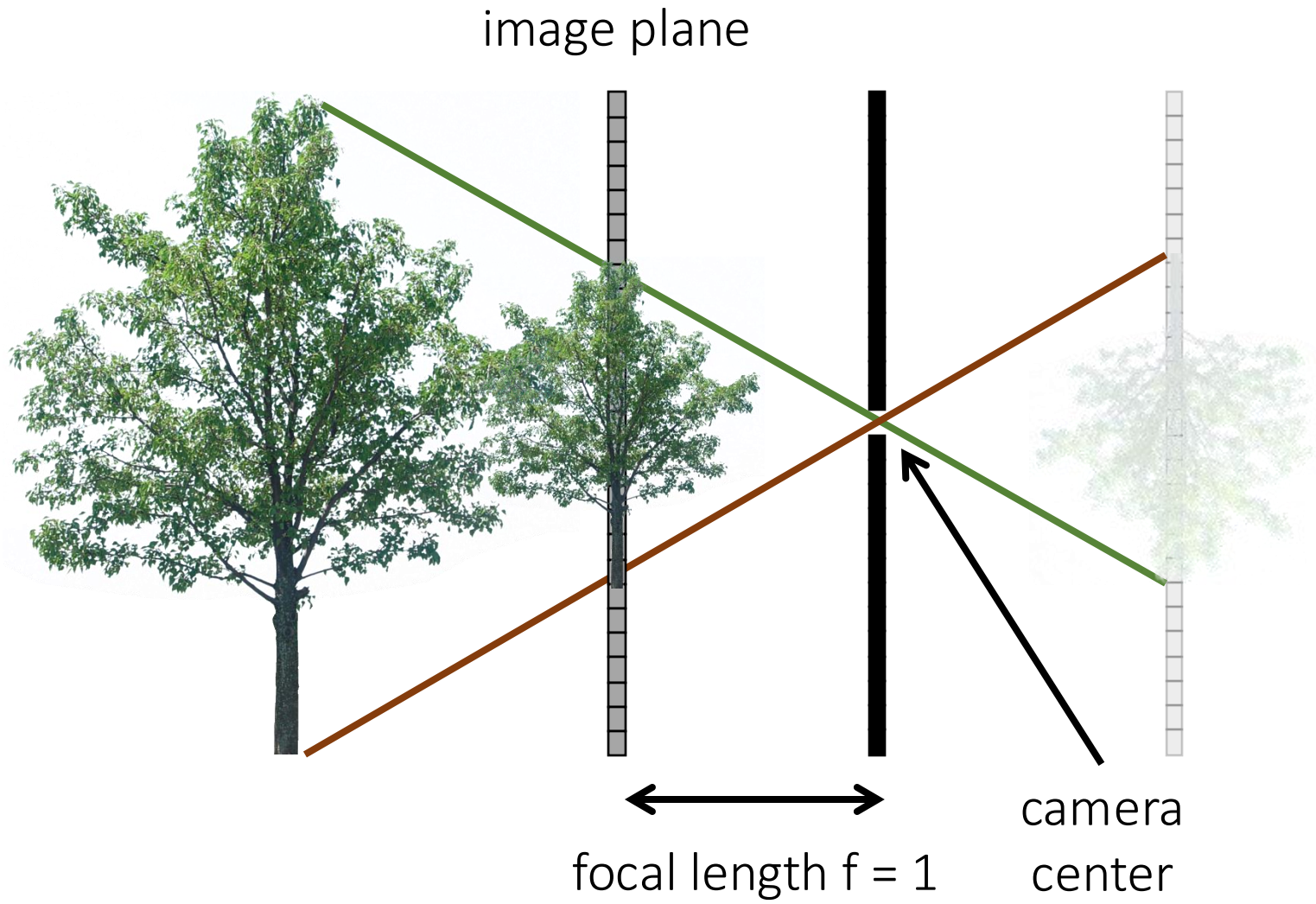
https://ciechanow.ski/cameras-and-lenses/

Make sure you check it out!

The pinhole camera



The (rearranged) pinhole camera



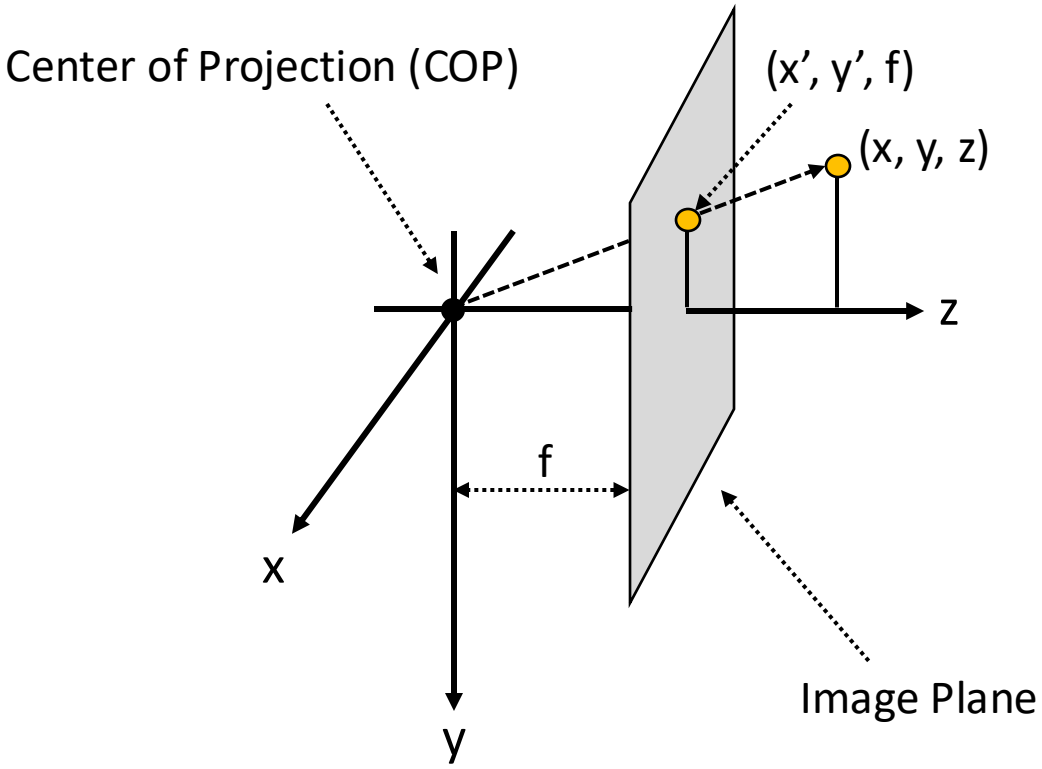
Modeling projection

- Projection equations
  - Compute intersection with PP of ray from (x,y,z) to COP
  - Derived using similar triangles

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, f)$$

  - We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$



Modeling projection

- Is this a linear transformation?
  - no—division by z is nonlinear

Homogeneous coordinates to the rescue—again!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- (Can also represent as a 4x4 matrix – OpenGL does something like this)

Perspective Projection

How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

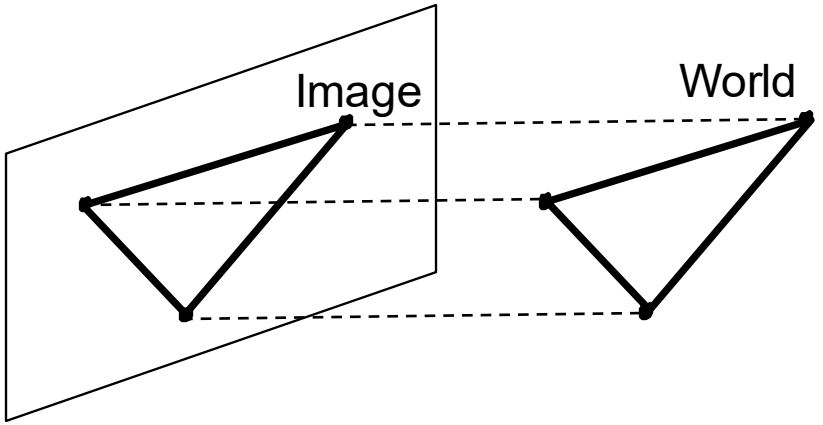
Scale by *f*:

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Scaling a projection matrix produces an equivalent projection matrix!

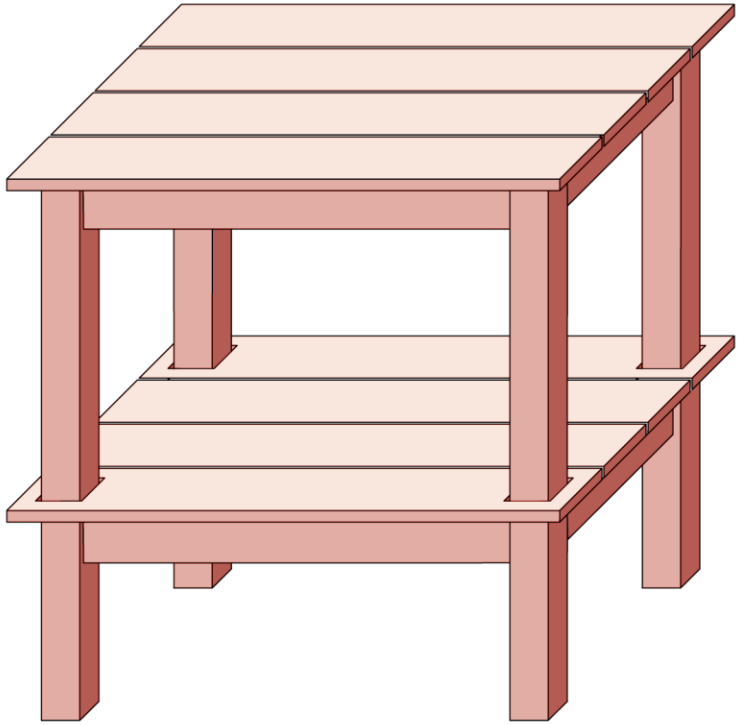
Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



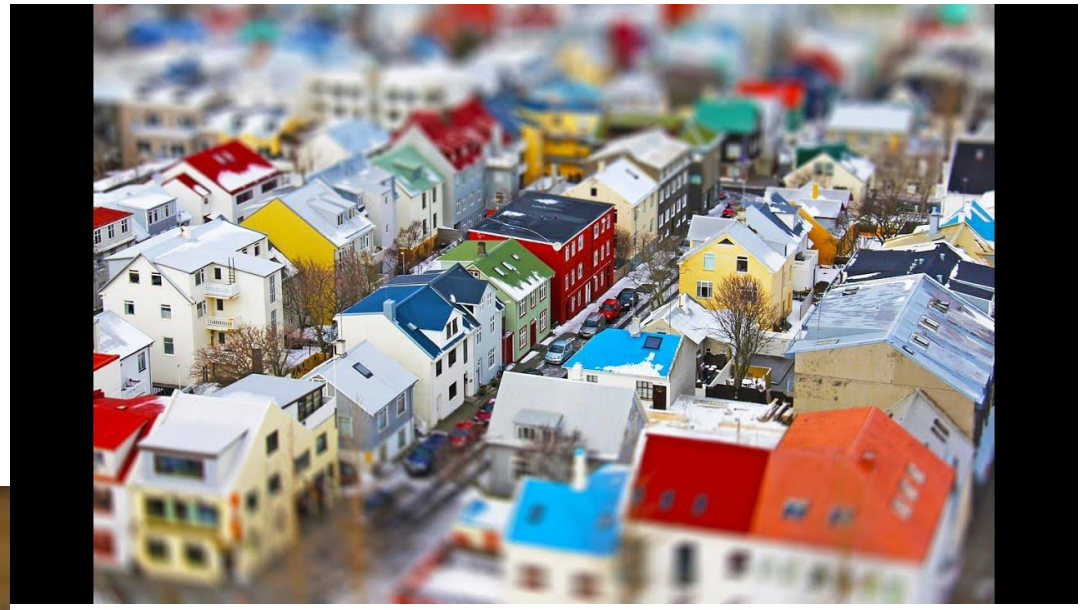
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic projection



How's this related to "miniature" effect by tilt-shift photography?

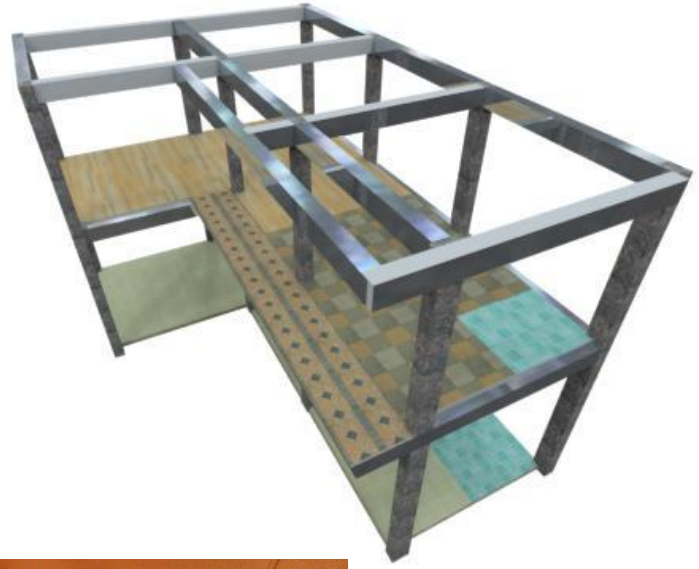

SPITFIRE




WILLMONEYMAKER.COM


CANON LENS TS-E 24mm 1:3.5 L II
Canon
TS-E 24mm
Canon
Av
F3.5
MODE
LOCK
ON OFF
EOS

Perspective projection



Variants of orthographic projection

- Scaled orthographic
  - Also called “weak perspective”

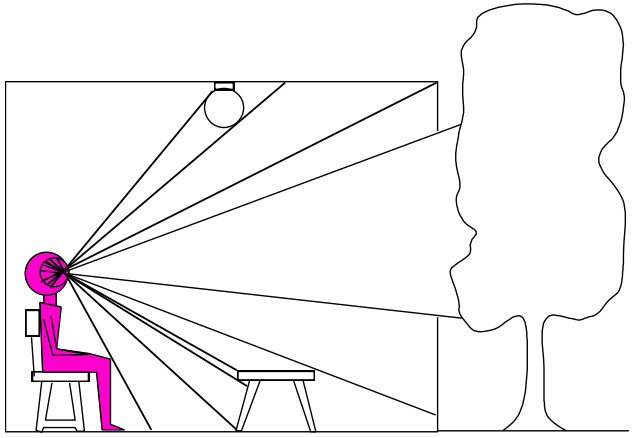
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

- Affine projection
  - Also called “paraperspective”

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

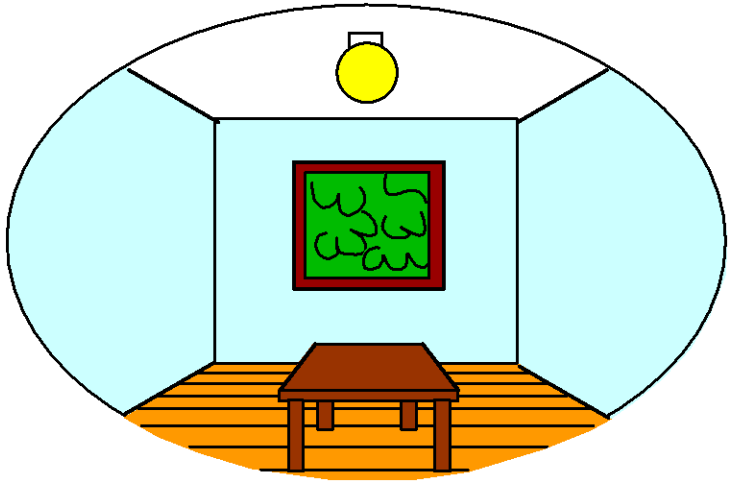
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

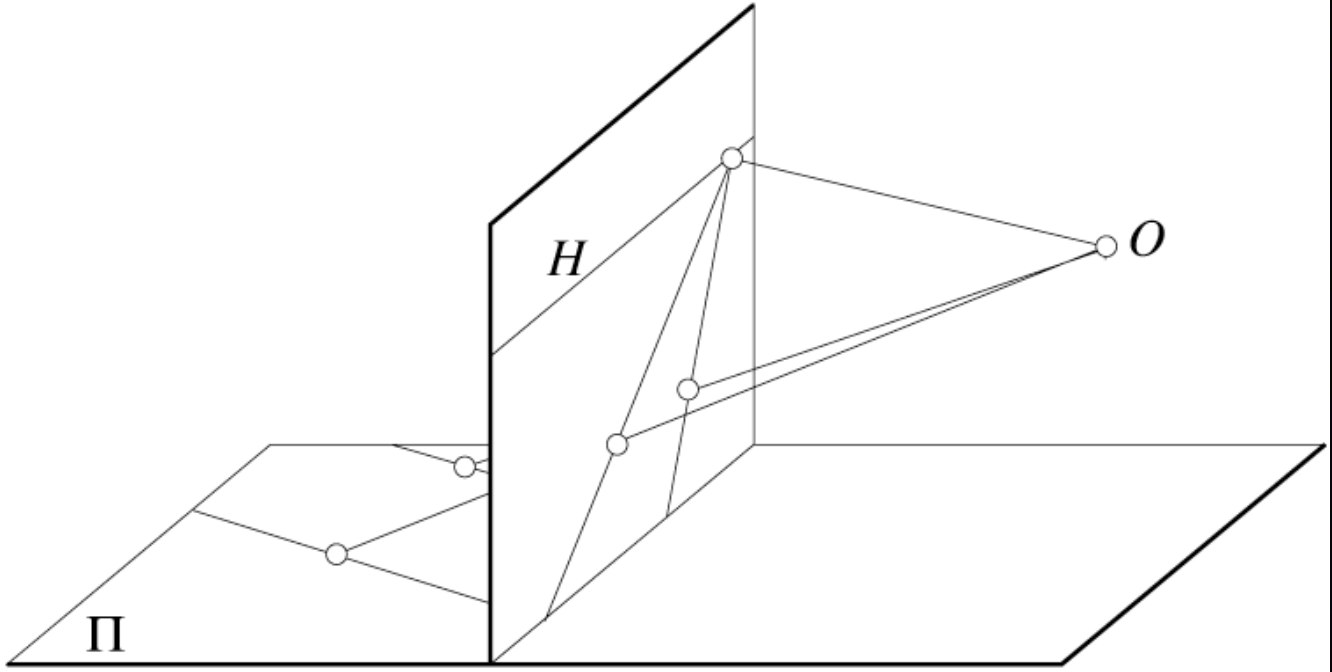
Figures © Stephen E. Palmer, 2002

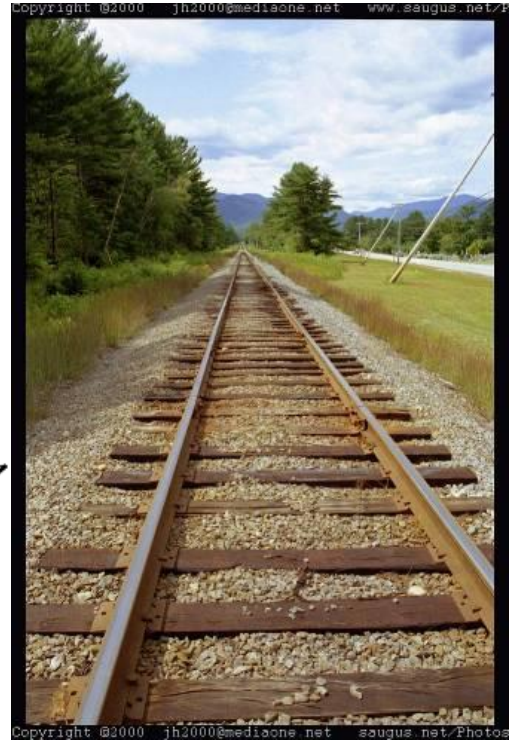
Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity is preserved)
  - But line through focal point projects to a point
- Planes → planes (or half-planes)
  - But plane through focal point projects to line

Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But parallel lines parallel to the image plane remain parallel

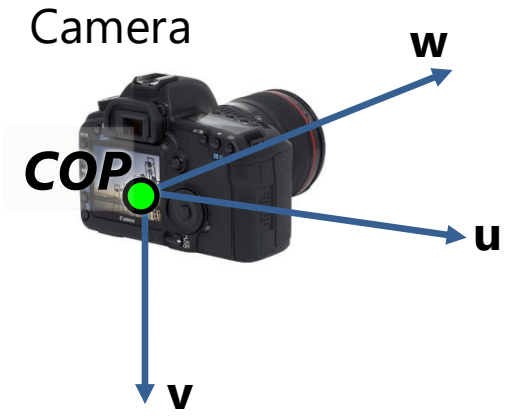


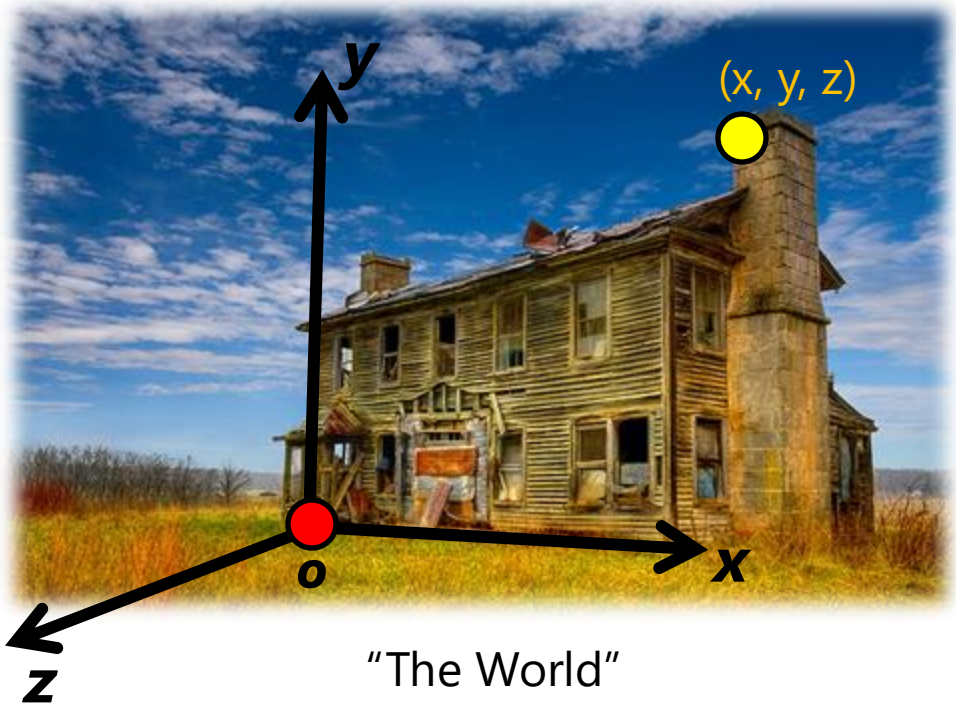


Questions?

Camera parameters

- How can we model the geometry of a camera?





"The World"

Three important coordinate systems:

1. *World* coordinates
2. *Camera* coordinates
3. *Image* coordinates

How do we project a given world point (x, y, z) to an image point?

Coordinate frames

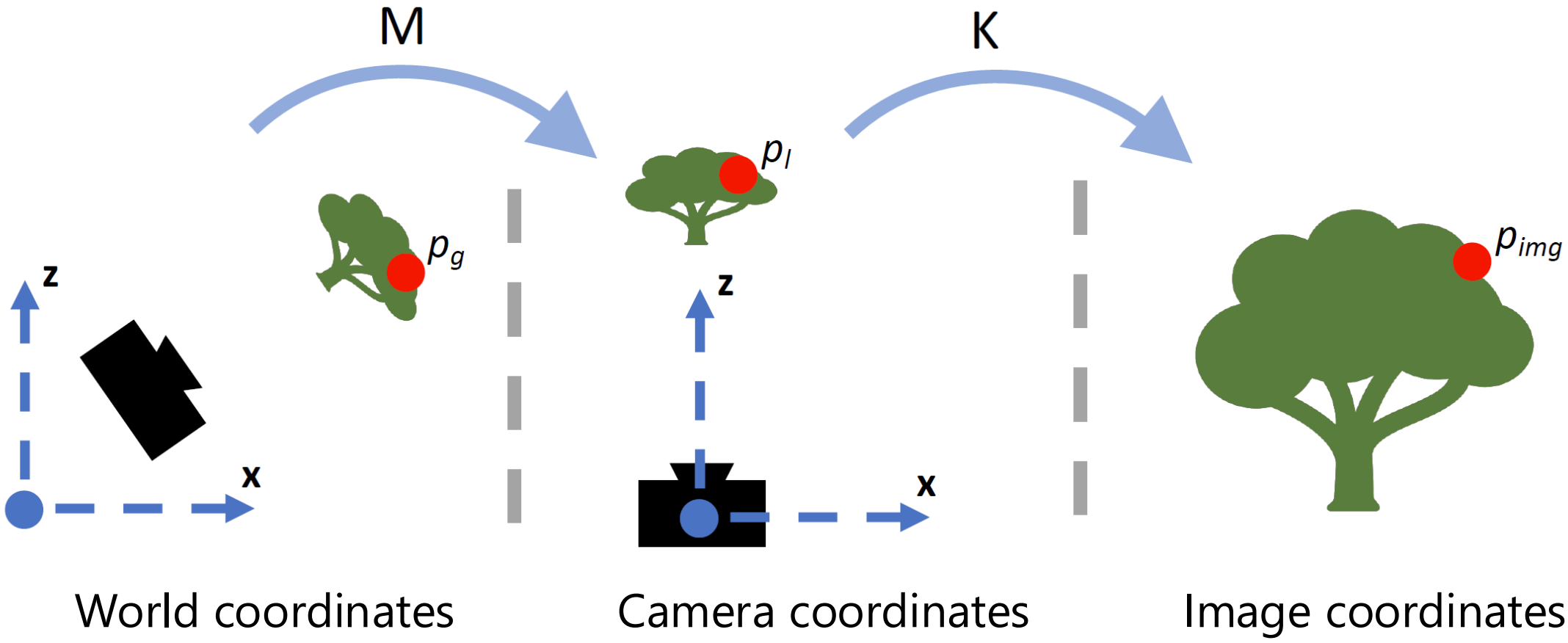


Figure credit: Peter Hedman

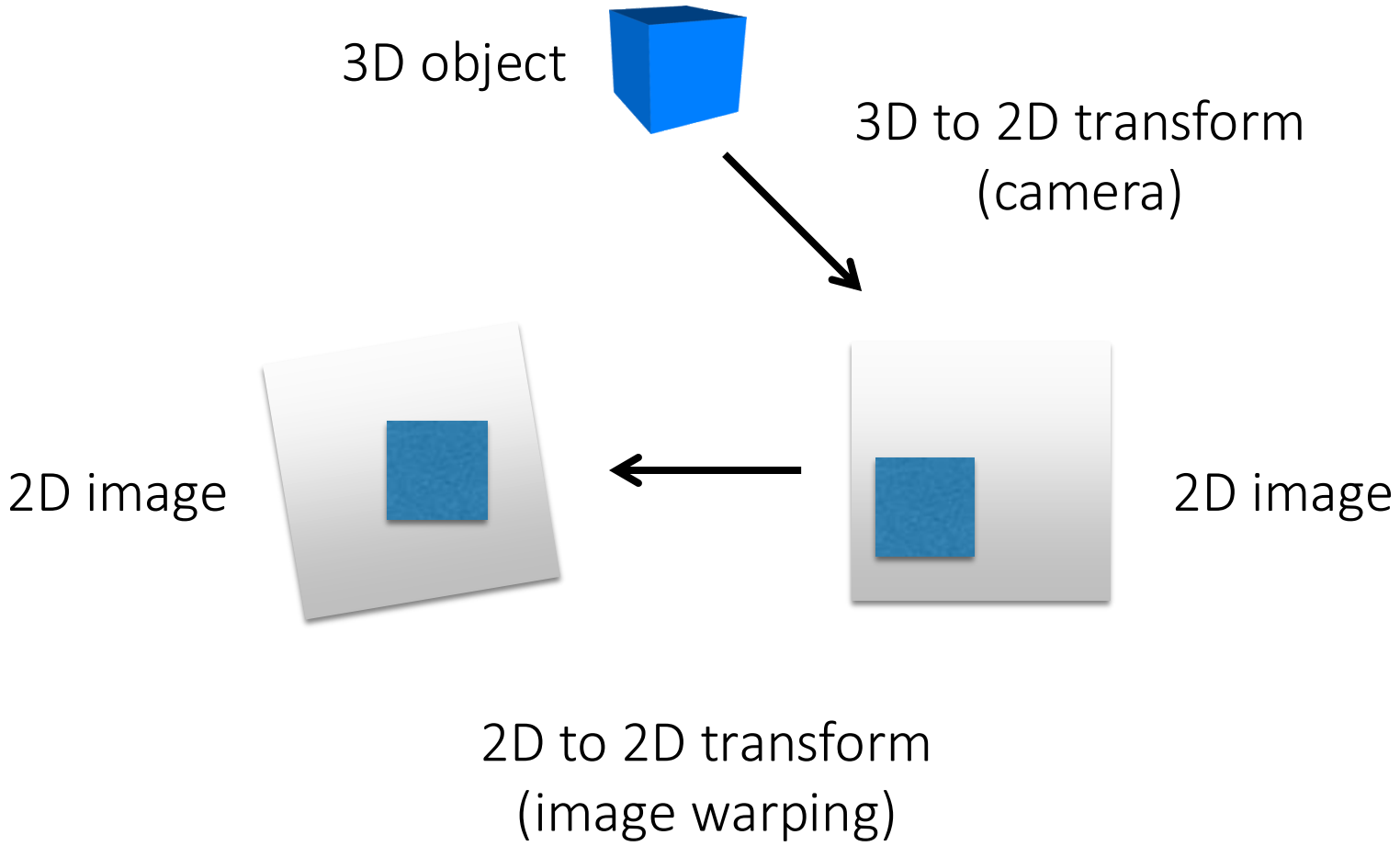
Recap: The camera as a coordinate transformation

A camera is a mapping from:

the 3D world

to:

a 2D image



Recap: The camera as a coordinate transformation

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

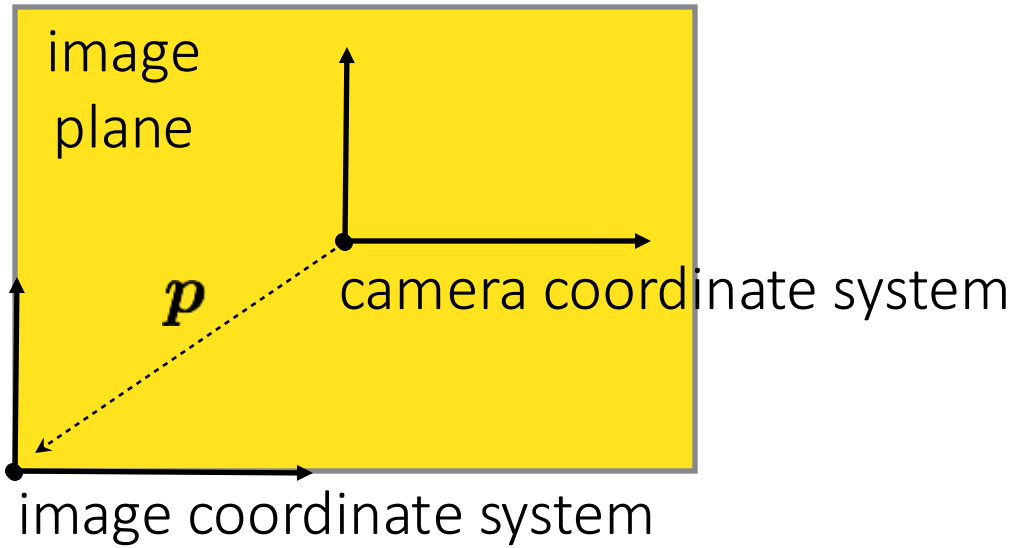
homogeneous image coordinates 3 x 1

camera matrix 3 x 4

homogeneous world coordinates 4 x 1

Recap: Generalizing the camera matrix

In particular, the camera origin and image origin may be different:



How does the camera matrix change?

shift vector transforming camera origin to image origin

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Recap: Camera matrix decomposition

We can decompose the camera matrix like this:

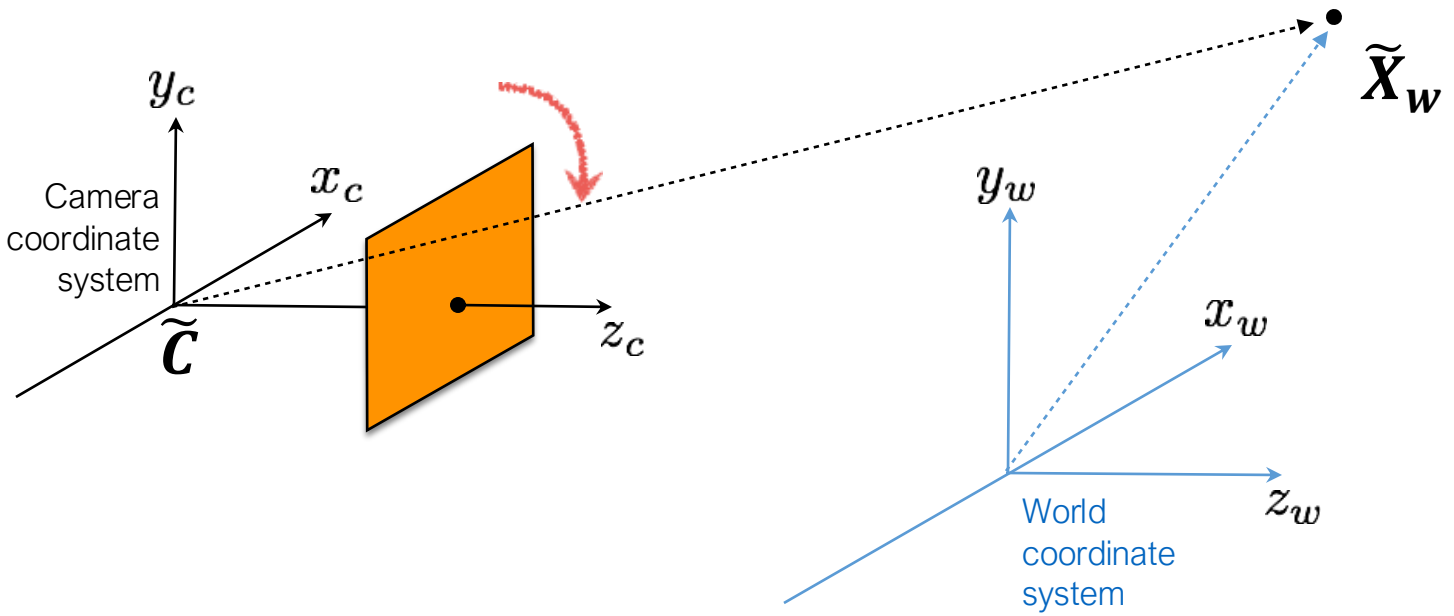
$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(homogeneous) transformation from 2D to 2D, accounting for not unit focal length and origin shift

(homogeneous) perspective projection from 3D to 2D, assuming image plane at z = 1 and shared camera/image origin

Also written as: $\mathbf{P} = \mathbf{K}[\mathbf{I}|\mathbf{0}]$ where $\mathbf{K} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$

Recap: World-to-camera coordinate system



points now coincide

$$\boldsymbol{R} \cdot \left(\widetilde{\boldsymbol{X}}_{\boldsymbol{w}} - \widetilde{\boldsymbol{C}}\right)$$

rotate translate

Recap: Modeling the coordinate system

In heterogeneous coordinates, we have:

$$\widetilde{\mathbf{X}}_{\mathbf{c}} = \mathbf{R} \cdot \left(\widetilde{\mathbf{X}}_{\mathbf{w}} - \widetilde{\mathbf{C}} \right)$$

How do we write this transformation in homogeneous coordinates?

Recap: Modeling the coordinate system

In heterogeneous coordinates, we have:

$$\widetilde{\mathbf{X}}_{\mathbf{c}} = \mathbf{R} \cdot \left(\widetilde{\mathbf{X}}_{\mathbf{w}} - \widetilde{\mathbf{C}}\right)$$

In homogeneous coordinates, we have:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\widetilde{\boldsymbol{C}} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad \text{or} \quad \boldsymbol{X_c} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\widetilde{\boldsymbol{C}} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \boldsymbol{X}_{\mathbf{w}}$$

Recap: Putting it all together

We can write everything into a single projection:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_\mathbf{w}$$

The camera matrix now looks like:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I} \quad | \quad \mathbf{0}] \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix}$$

intrinsic parameters (3 x 3): correspond to camera internals (image-to-image transformation)

perspective projection (3 x 4): maps 3D to 2D points (camera-to-image transformation)

extrinsic parameters (4 x 4): correspond to camera externals (world-to-camera transformation)

Recap: Putting it all together

We can write everything into a single projection:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_\mathbf{w}$$

The camera matrix now looks like:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\vdots\; -\mathbf{RC}\right]$$

intrinsic parameters (3 x 3):
correspond to camera internals
(sensor not at f = 1 and origin shift)

extrinsic parameters (3 x 4):
correspond to camera externals
(world-to-image transformation)

Recap: General pinhole camera matrix

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} r_1 & r_2 & r_3 & t_1 \\ r_4 & r_5 & r_6 & t_2 \\ r_7 & r_8 & r_9 & t_3 \end{array}\right]$$

intrinsic parameters

extrinsic parameters

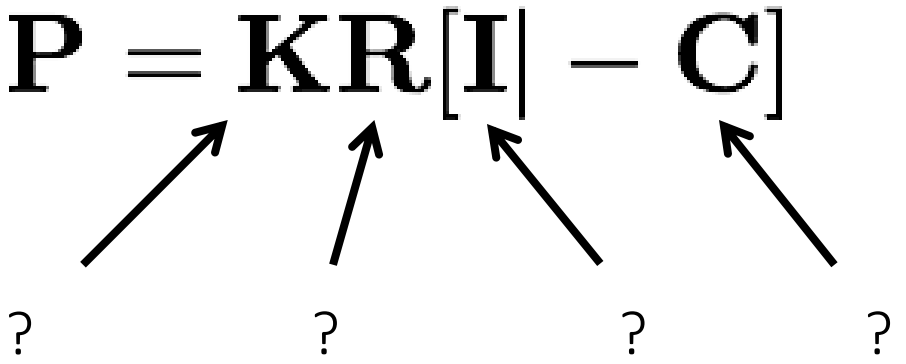
$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \qquad \mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$$

3D rotation

3D translation

Recap

What is the size and meaning of each term in the camera matrix?



Recap

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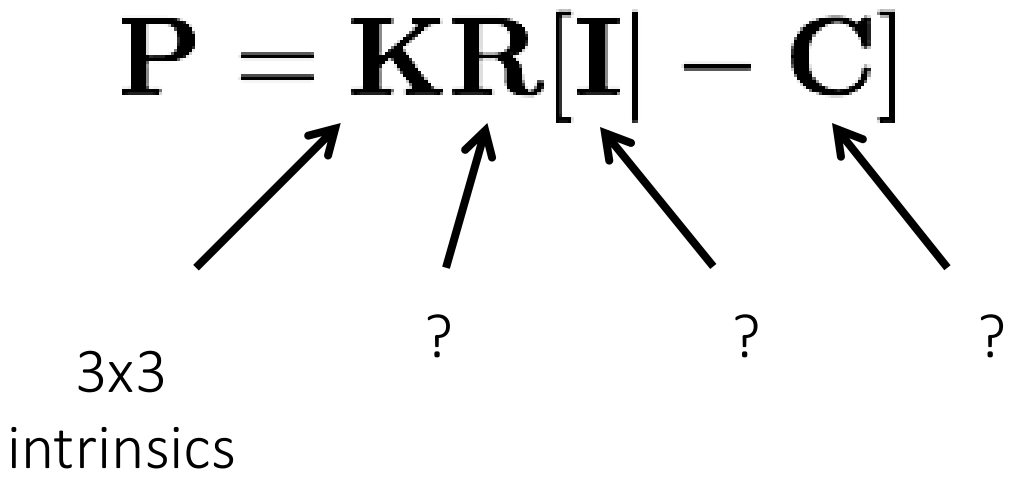

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}|-\mathbf{C}]$$

3x3 intrinsics

3x3 3D rotation

?

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}|-\mathbf{C}]$$

3x3 intrinsics

3x3 3D rotation

3x3 identity

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}|-\mathbf{C}]$$

3x3	3x3	3x3	3x1
intrinsics	3D rotation	identity	3D translation

Quiz

The camera matrix relates what two quantities?

Quiz

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$$\boldsymbol{x} = \mathbf{PX}$$

homogeneous 3D points to 2D image points

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The camera matrix can be decomposed into?

Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

homogeneous 3D points to 2D image points

The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

intrinsic and extrinsic parameters

More general camera matrices

The following is the standard camera matrix we saw.

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \left[\mathbf{R} \;\vdots\; -\mathbf{RC}\right]$$

More general camera matrices

CCD camera: pixels may not be square.
Lens: focal length along x and y may be different

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \vdots & -\mathbf{RC} \end{bmatrix}$$

How many degrees of freedom?

Anamorphic Lens

Lens flares are scattered light that you usually try to avoid having in photos. It's usually caused when a bright light shines into the lens. Paramount Pictures

Abrams loves them. Paramount Pictures

You can see them across his rebooted "Star Trek" movies. Paramount Pictures

More general camera matrices

CCD camera: pixels may not be square.
Lens: focal length along x and y may be different

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \end{bmatrix}$$

How many degrees of freedom?

10 DOF

More general camera matrices

Finite projective camera: sensor be skewed.

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \;\vdots\; -\mathbf{RC} \right]$$

How many degrees of freedom?

More general camera matrices

Finite projective camera: sensor be skewed.

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \end{bmatrix}$$

How many degrees of freedom?

11 DOF

Projection matrix

$$\Pi = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} & \mathbf{R} & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} & & & \\ & \mathbf{I}_{3\times 3} & & -\mathbf{c} \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{translation}}$$

The **K** matrix converts 3D rays in the camera's coordinate system to 2D image points in image (pixel) coordinates.

This part converts 3D points in world coordinates to 3D rays in the camera's coordinate system. There are 6 parameters represented (3 for position/translation, 3 for rotation).

Projection matrix

$$\Pi = \mathbf{K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} \mathbf{R} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\qquad}_{\text{translation}}$$

$$\begin{bmatrix} \mathbf{R} & \underbrace{-\mathbf{R}\mathbf{c}}_{\mathbf{t}} \end{bmatrix}$$

(**t** in book's notation)

$$\Pi = \mathbf{K} \begin{bmatrix} \mathbf{R} & -\mathbf{R}\mathbf{c} \end{bmatrix}$$

Perspective projection

$$\underbrace{\begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics)

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$

(upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y) : **principal point** ((w/2,h/2) unless optical axis doesn't intersect projection plane at image center)

Focal length

- Can think of as “zoom”


24mm

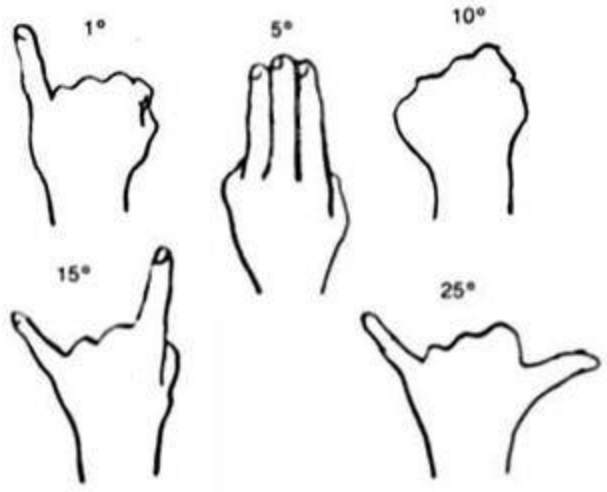

50mm


200mm


800mm

- Also related to *field of view*

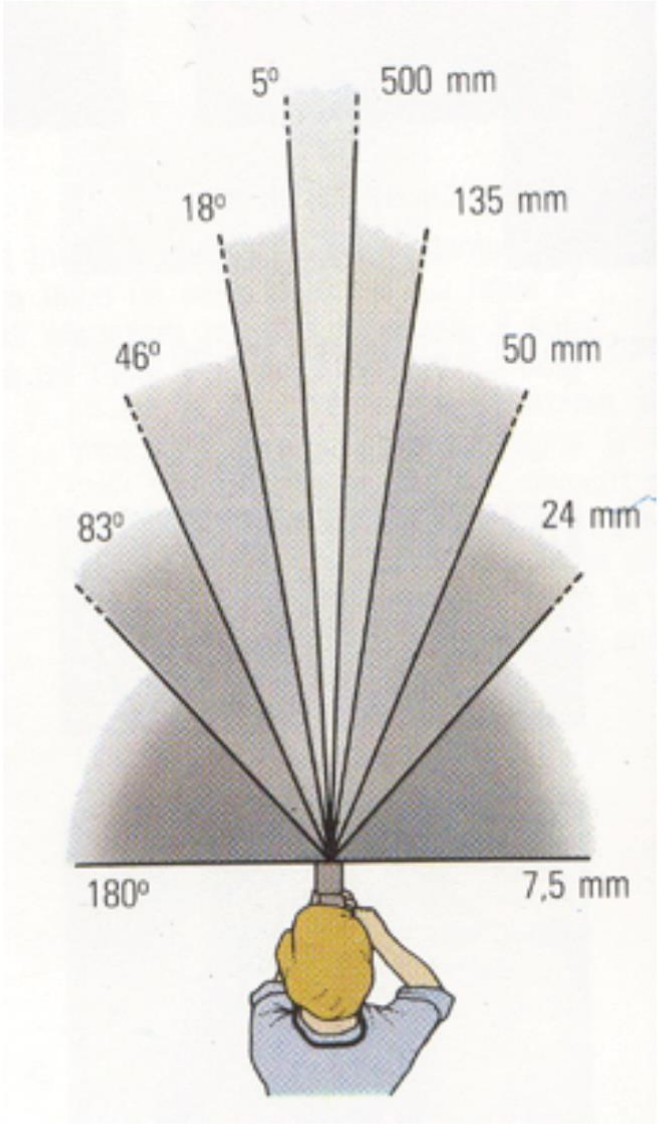
Field of view



APS-C Crop Body Measurement Table

Lens	After 1.62 Multiplier	APS-C Sensor (1.62 lens multiplier) Canon 60D, 7D, 70D, T3i, T4i	Hand Positions
18mm	29.16mm	**Three hands wide** at full arms length.	
28mm	45.36mm	**Slightly less than two hands wide** at full arms length.	
35mm	56.7mm	**One hand + width of one fist** at full arms length.	
50mm	81mm	**One hand wide + width of thumb** at full arms length.	
55mm	89.1mm	**Slightly less than one hand wide** at full arms length.	
85mm	137.7mm	**Inside edge of thumb to tip of forefinger wide** with hand in “L” shape, thumb up.	

Focal length in practice



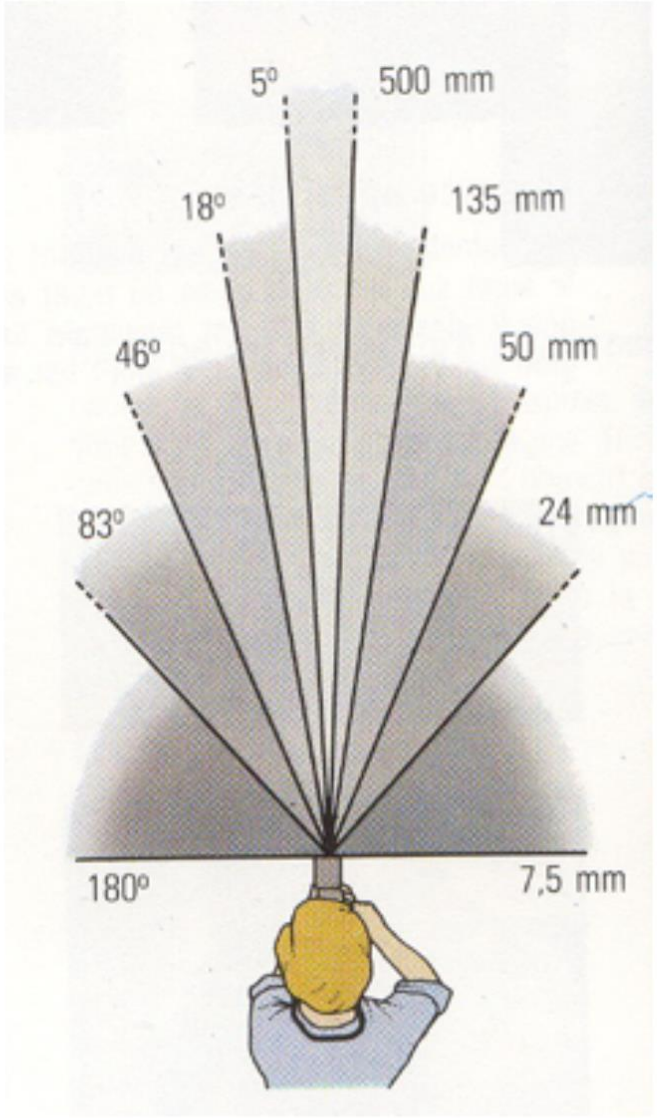
24mm

50mm

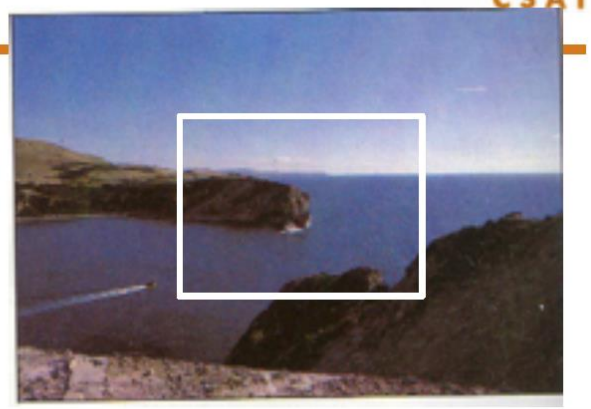
135mm

Fredo Durand

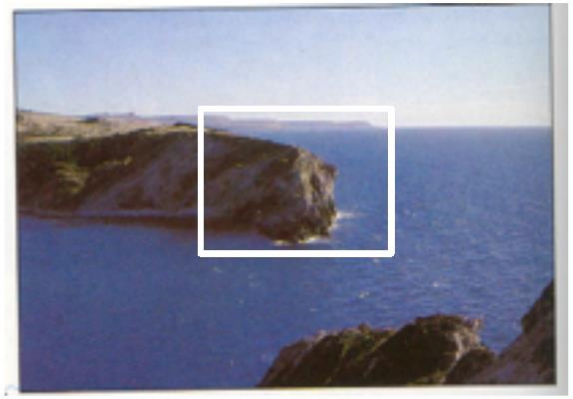
Focal length = cropping



24mm



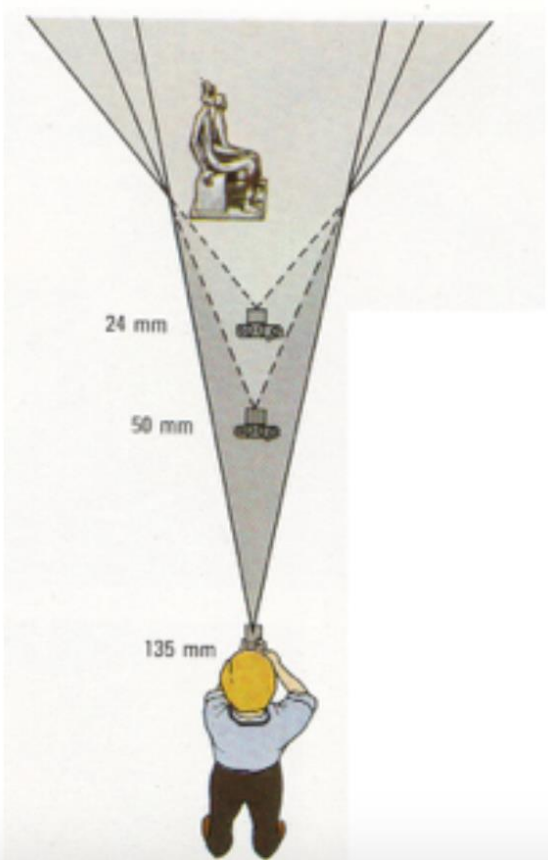
50mm



135mm

Focal length vs. viewpoint

- **Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.**



Grand-angulaire 24 mm

Normal 50 mm

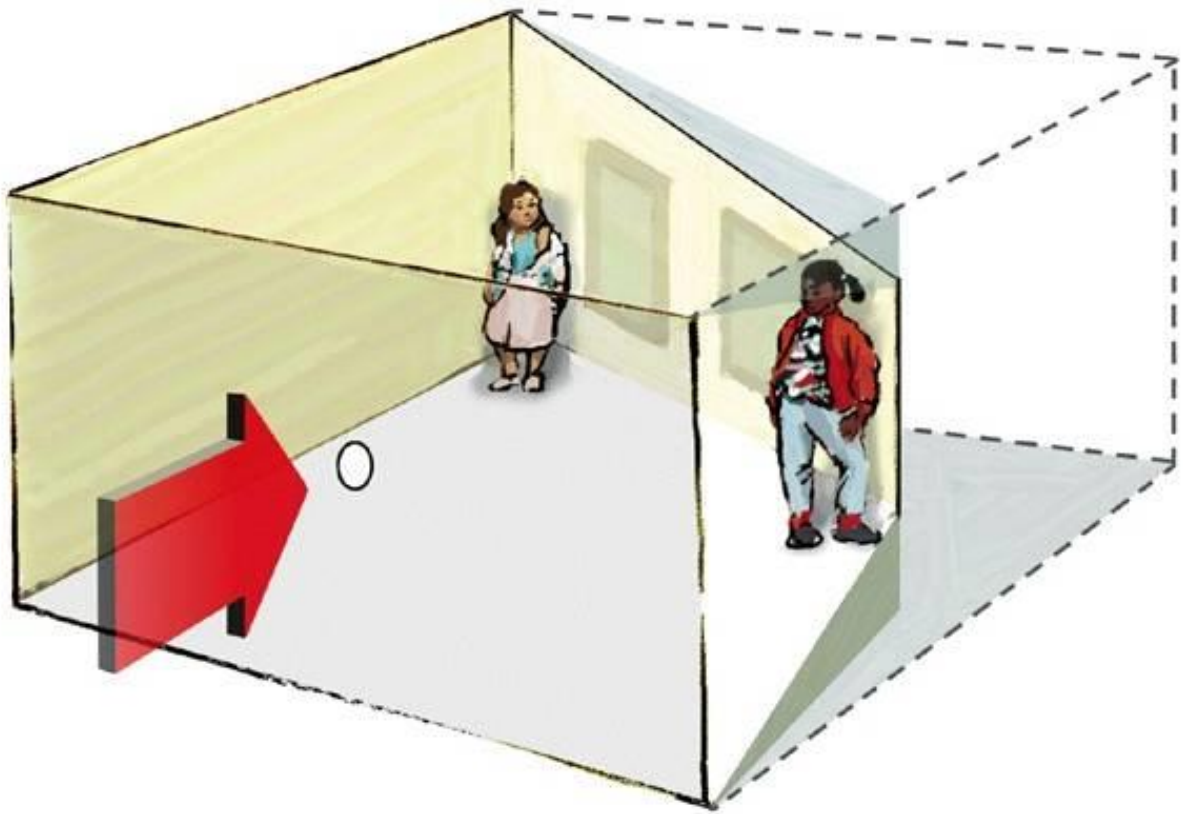
Longue focale 135 mm

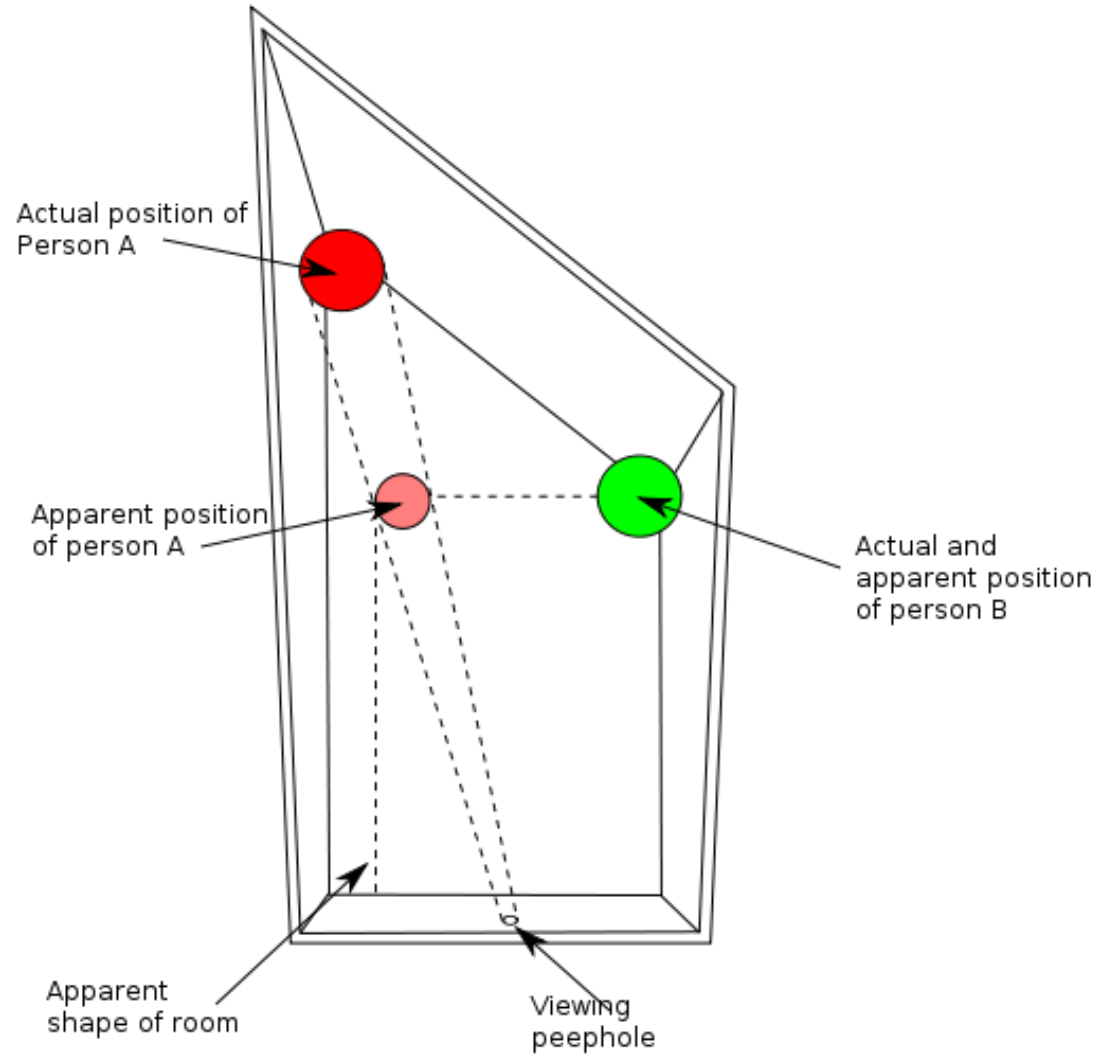
Fredo Durand

Forced perspective

The Ames room illusion

The Ames room illusion





The arrow illusion

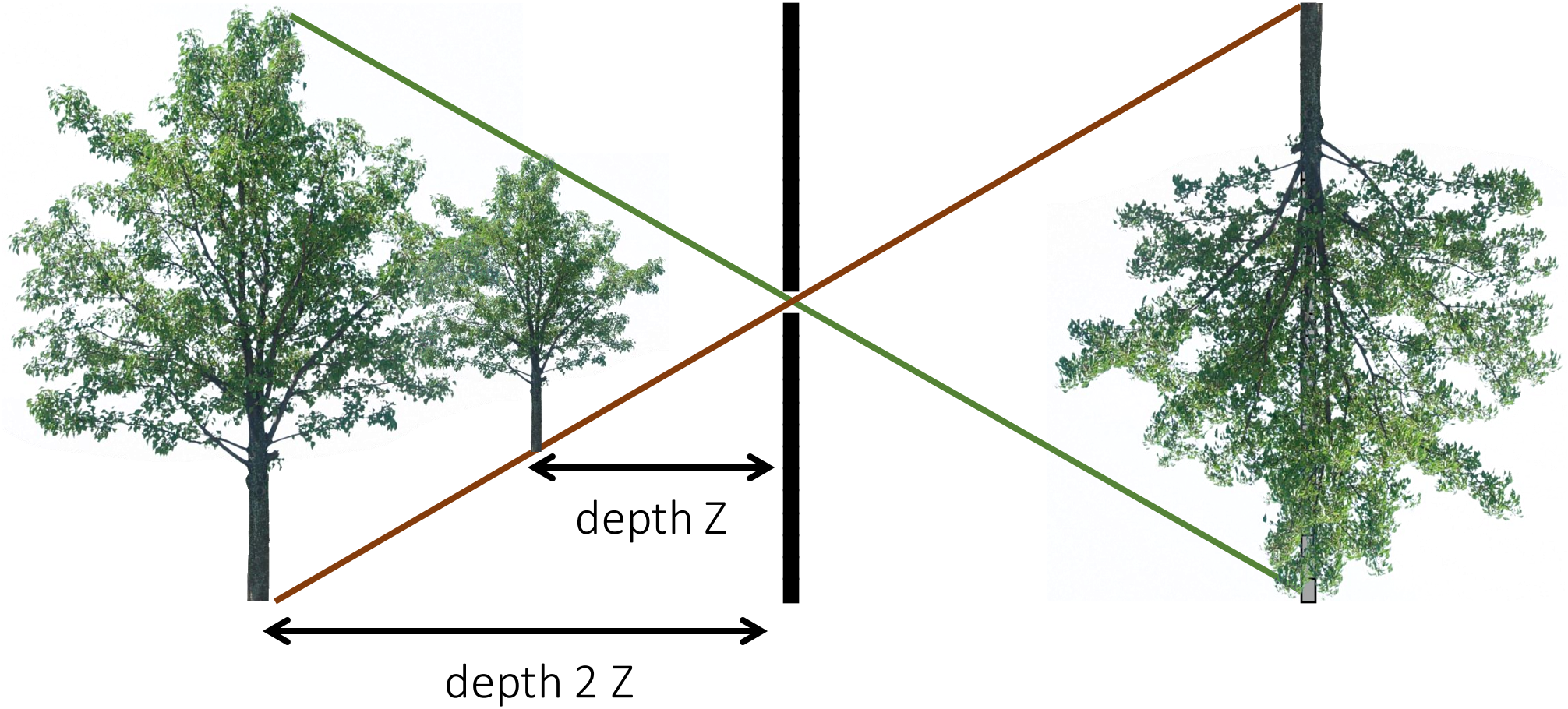
Forced Perspective Display

Forced Perspective Display

Magnification depends on depth

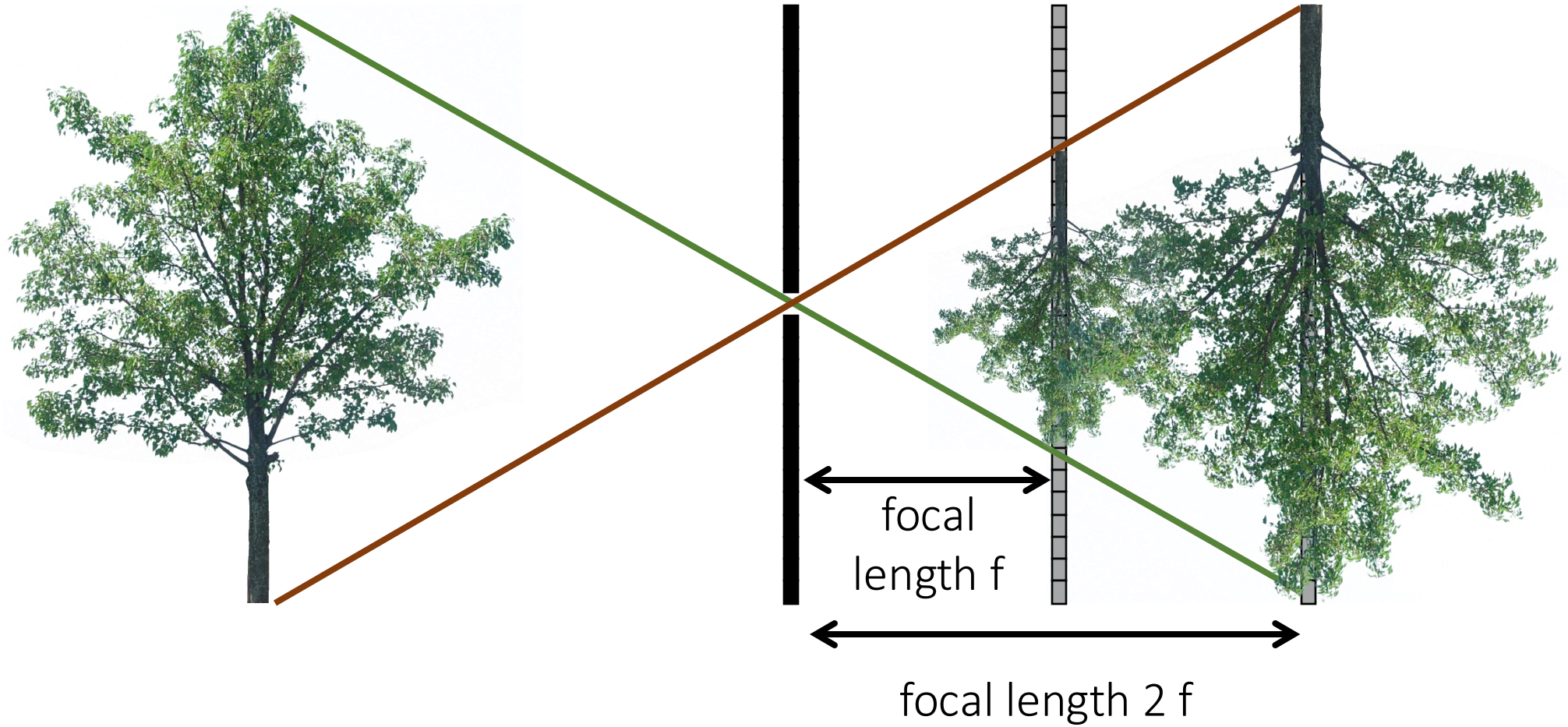
What happens as we change the focal length?

real-world object



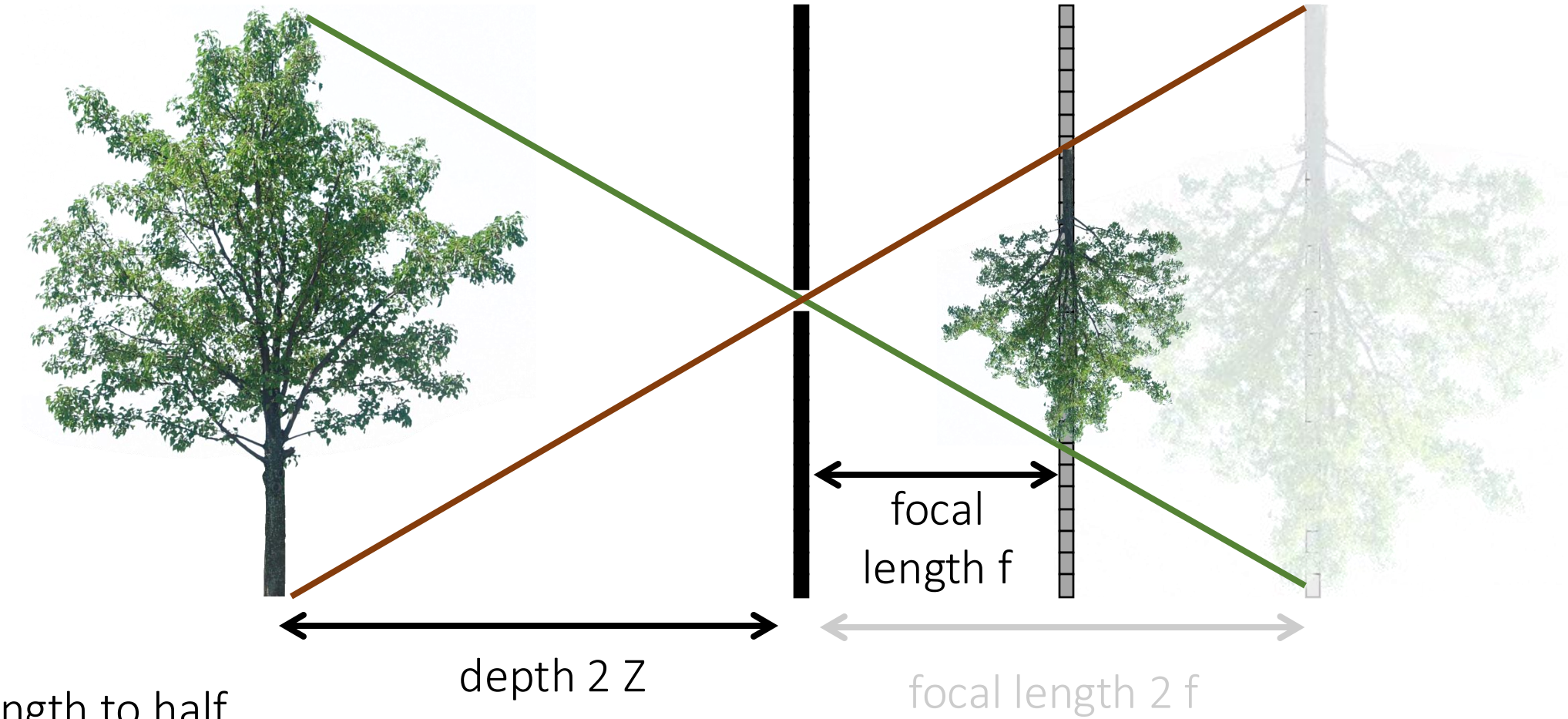
Magnification depends on focal length

real-world object



What if…

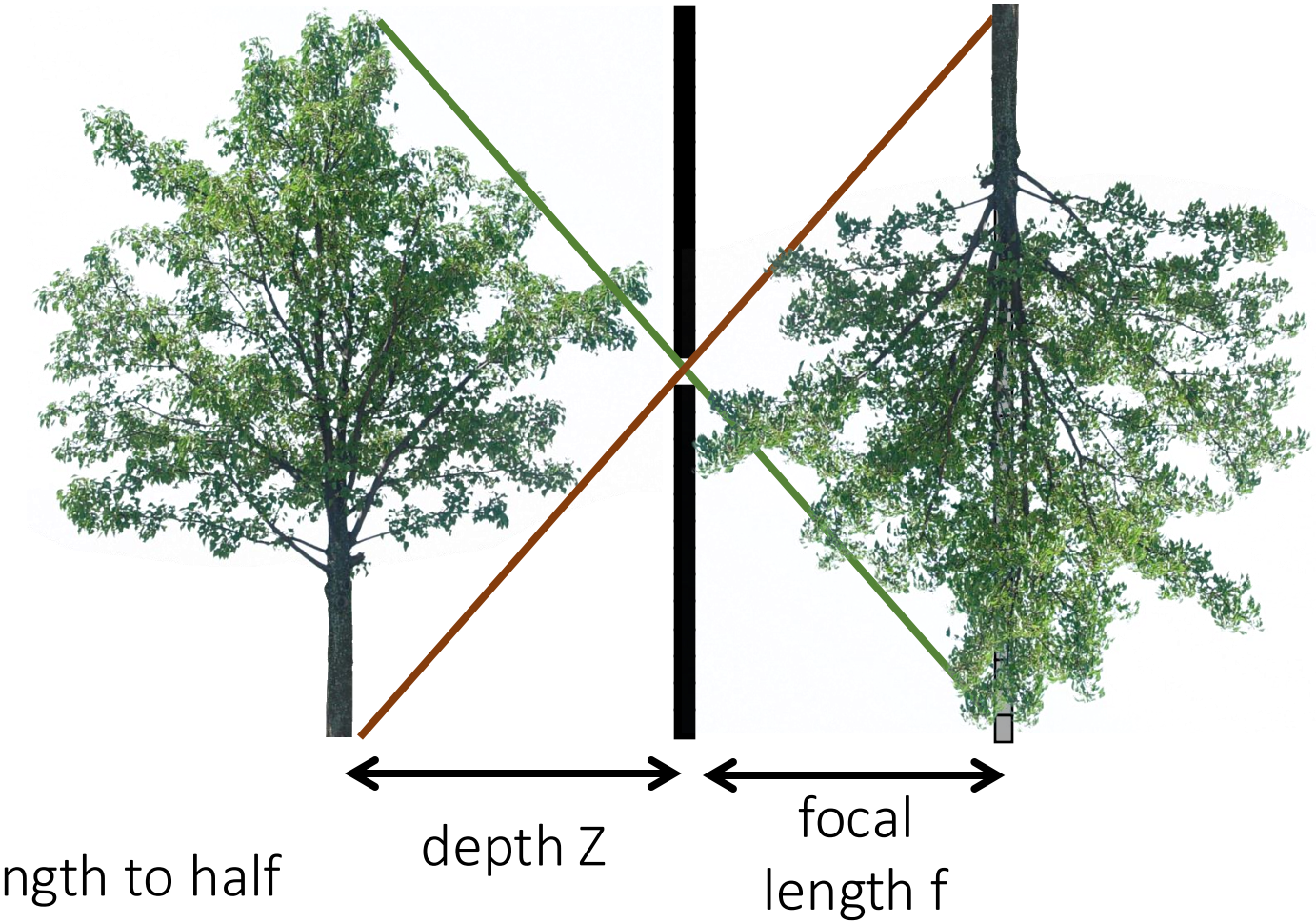
real-world object



1. Set focal length to half

What if…

real-world object



Is this the same image as the one I had at focal length 2f and distance 2Z?

1. Set focal length to half
2. Set depth to half

Vertigo effect

Named after Alfred Hitchcock's movie

- also known as "dolly zoom"

Vertigo effect

How would you create this effect?

Vertigo effect

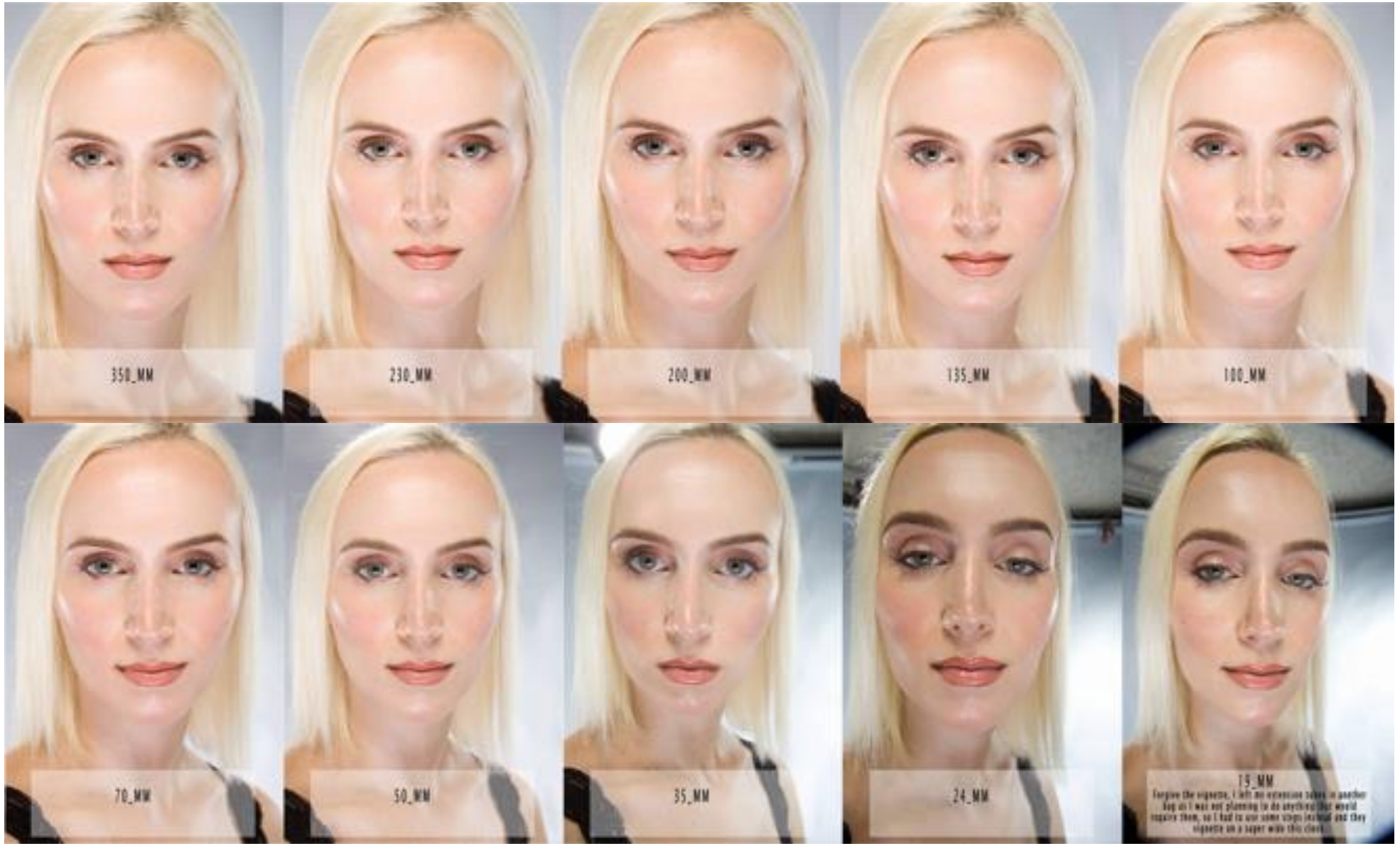
Perspective distortion

long focal length

mid focal length

short focal length

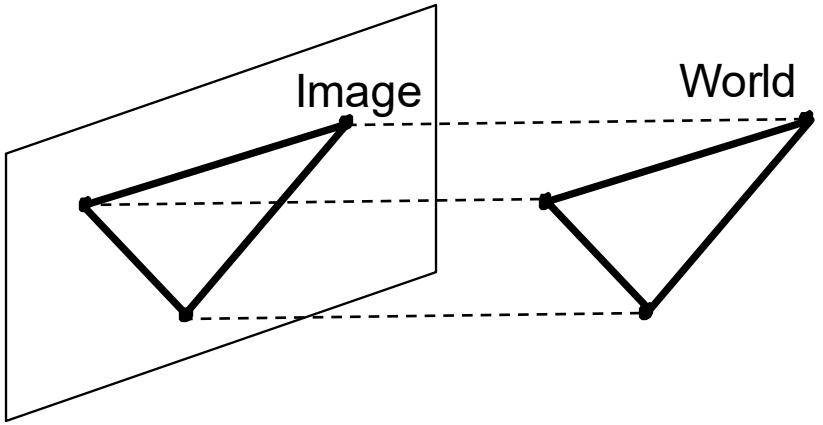
Perspective distortion



Other camera models

Orthographic projection

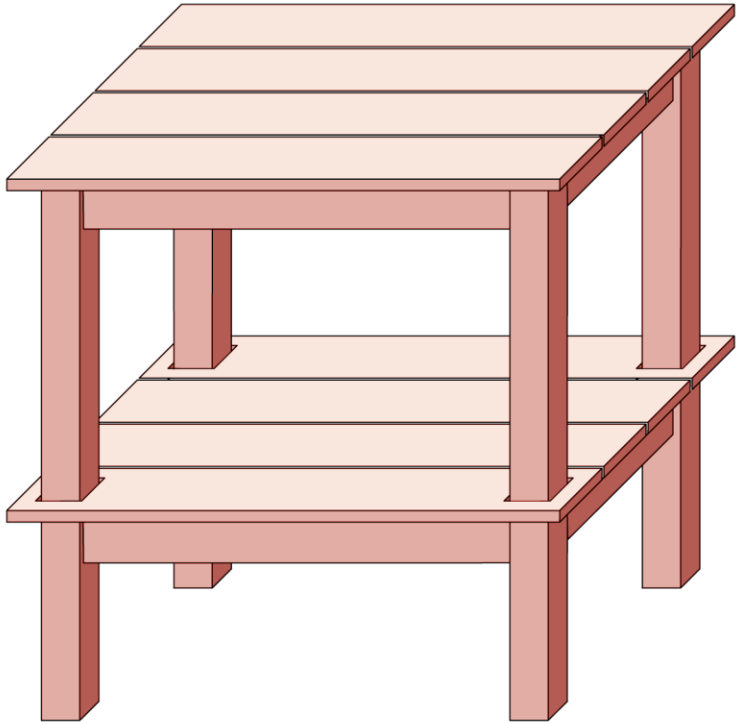
- Special case of perspective projection
  - Distance from the COP to the PP is infinite



- Good approximation for telephoto optics

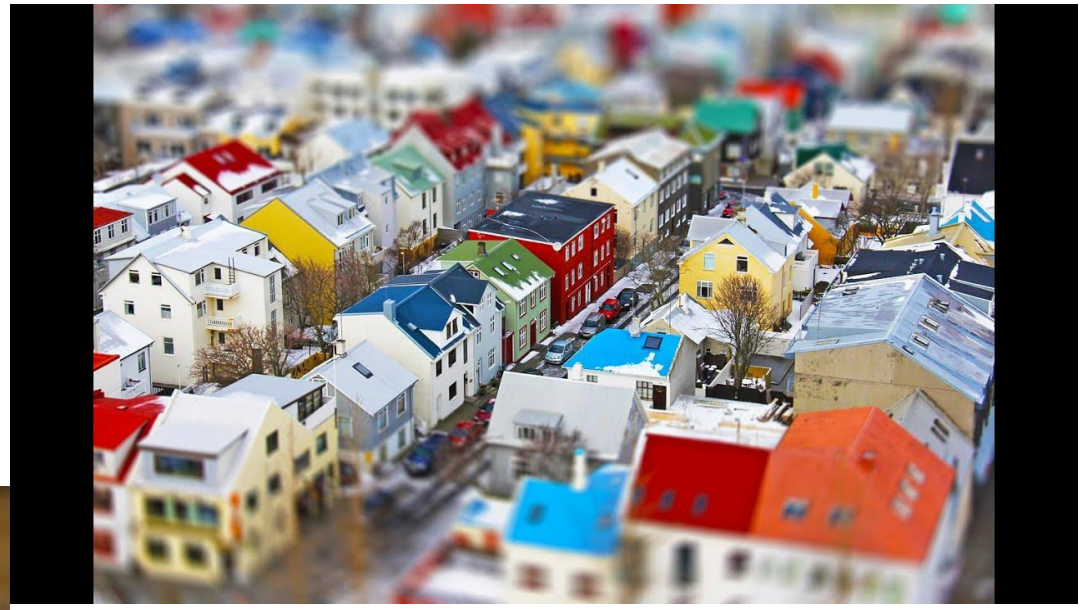
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Orthographic projection



How’s this related to ”miniature” effect by tilt-shift photography?


SPITFIRE

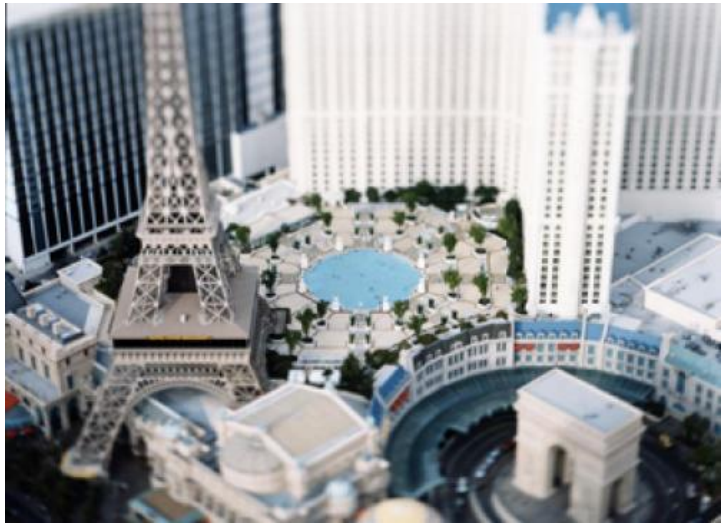



WILLMONEYMAKER.COM


CANON LENS TS-E 24mm 1:3.5 L II
Canon
TS-E 24mm
Canon
Av
F3.5
MODE
LOCK
ON OFF
EOS

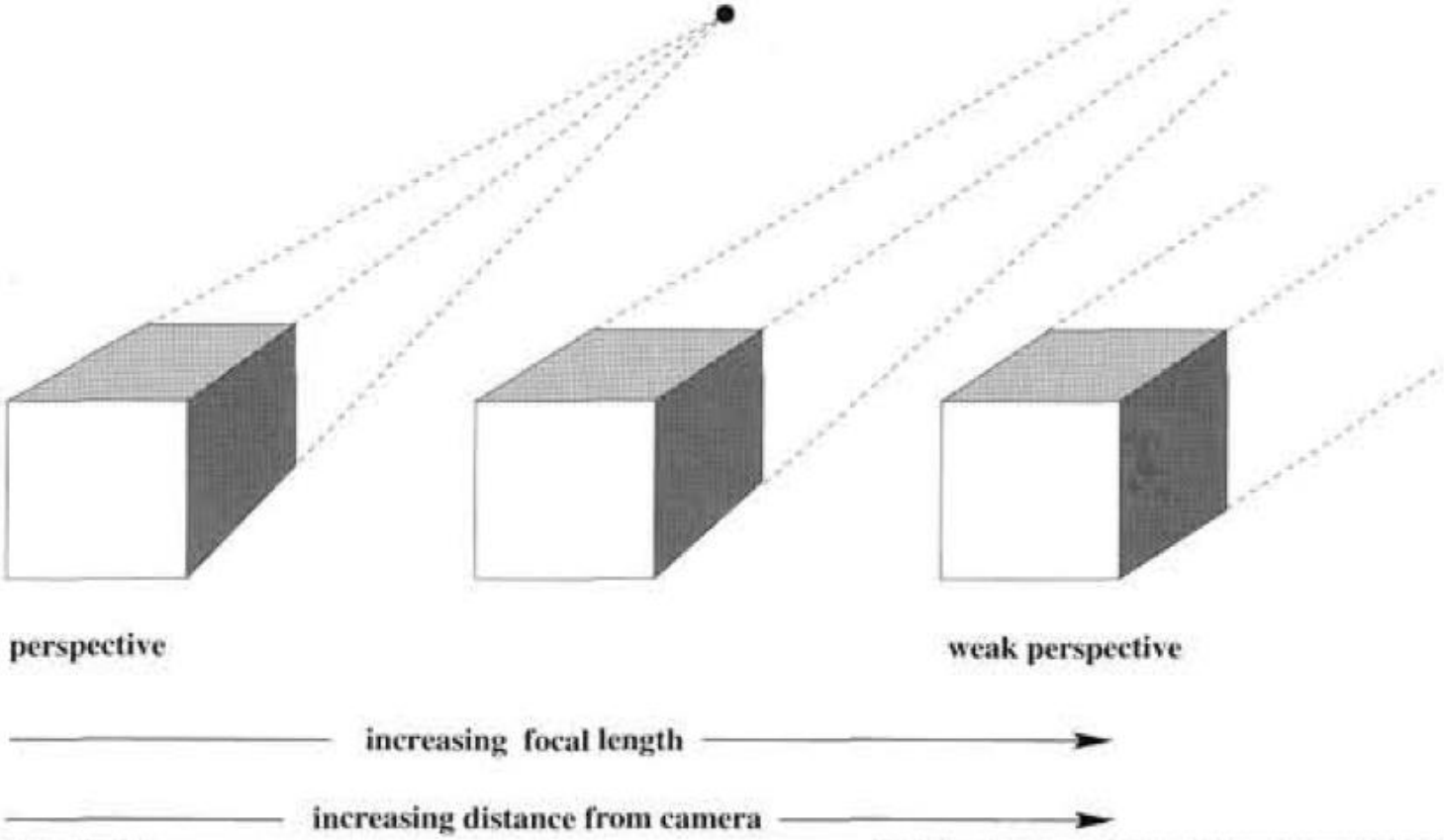
Tilt-shift

http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html

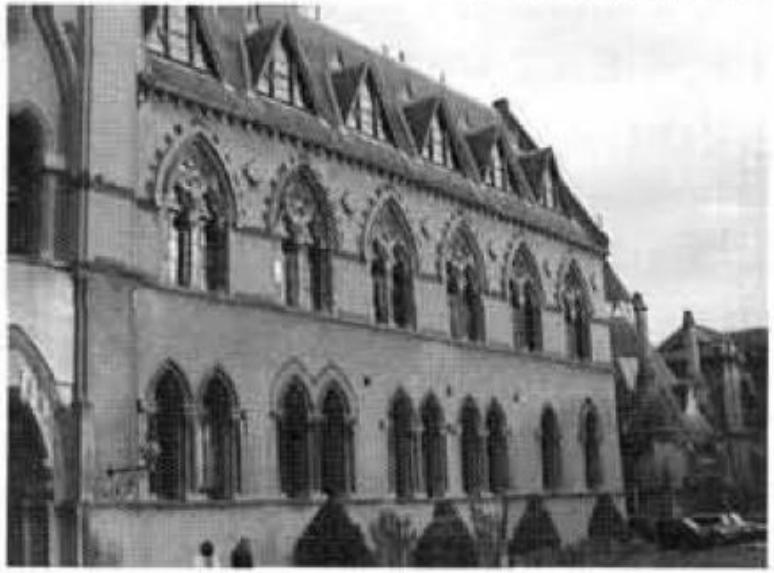


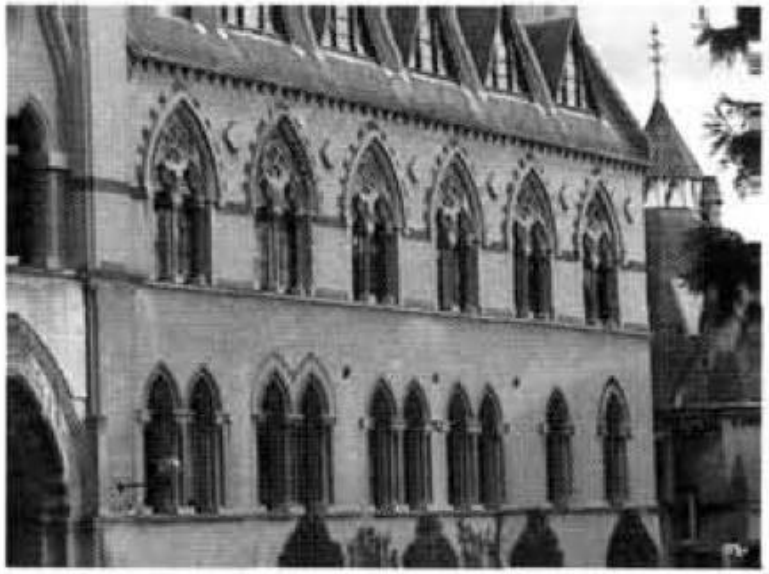
Tilt-shift images from Olivo Barbieri
and Photoshop imitations

camera is *close* to object and has *small* focal length

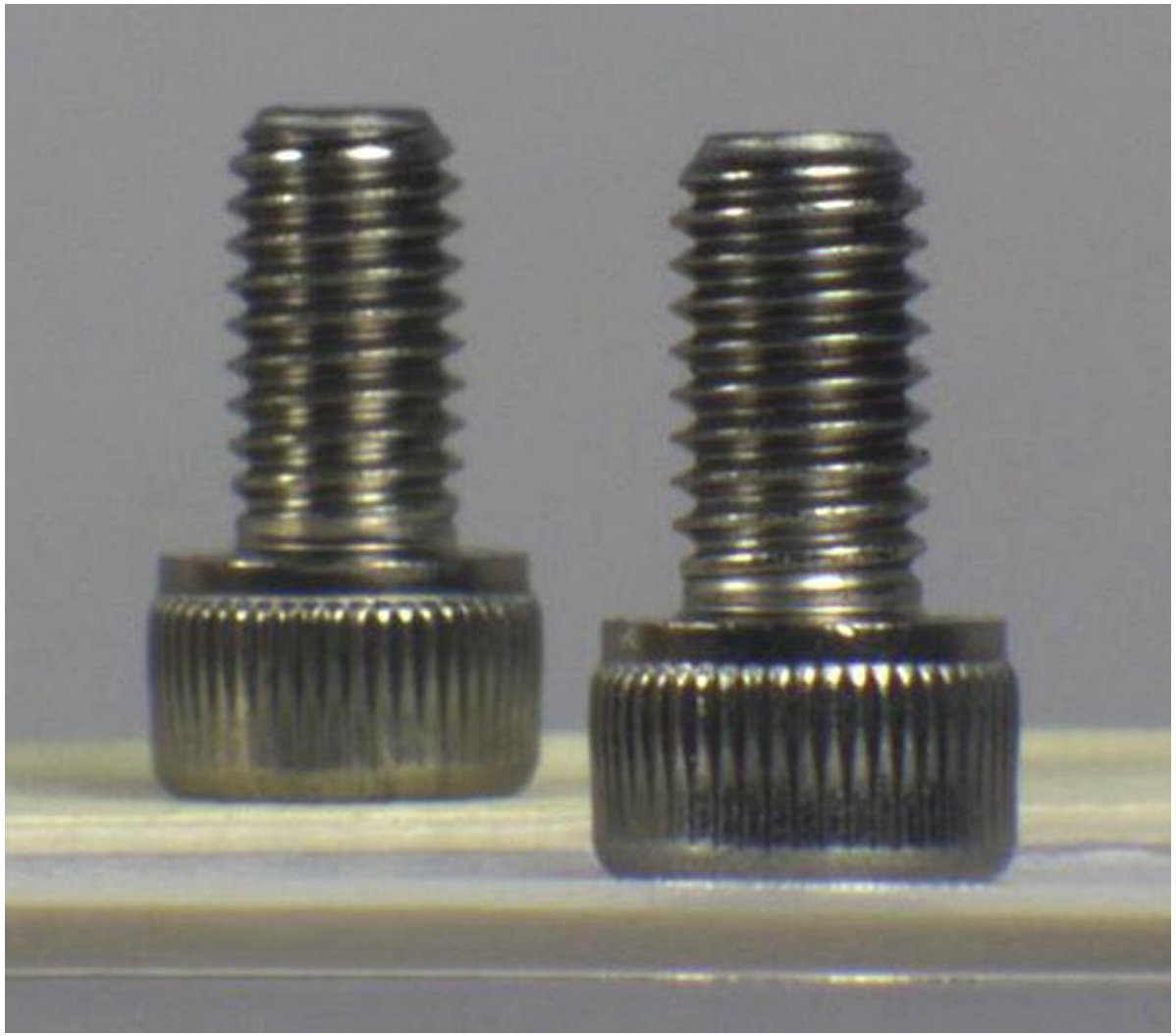


camera is *far* from object and has *large* focal length

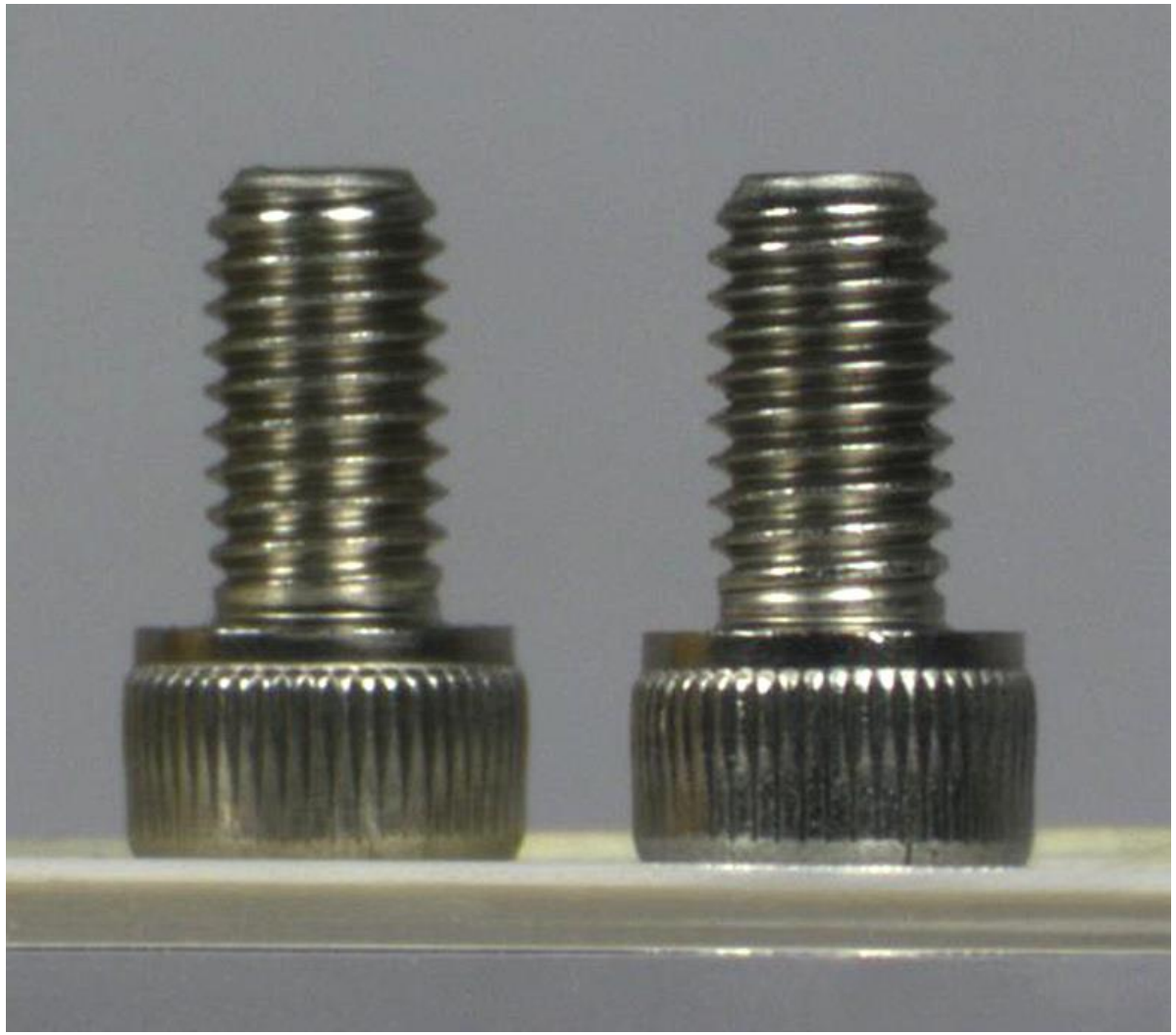




Different cameras



perspective camera

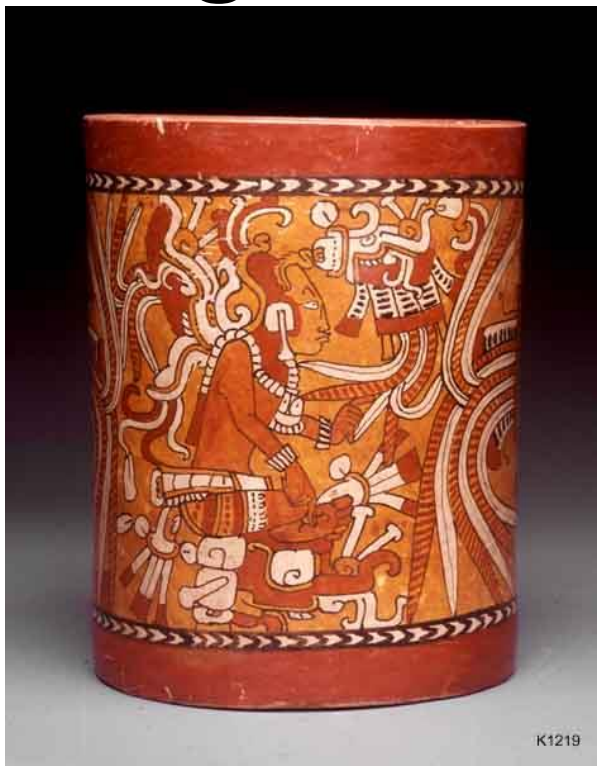


weak perspective camera

360 degree field of view…

- Basic approach
  - Take a photo of a parabolic mirror with an orthographic lens (Nayar)
  - Or buy one a lens from a variety of omnicam manufacturers…
    - See http://www.cis.upenn.edu/~kostas/omni.html

Rotating sensor (or object)

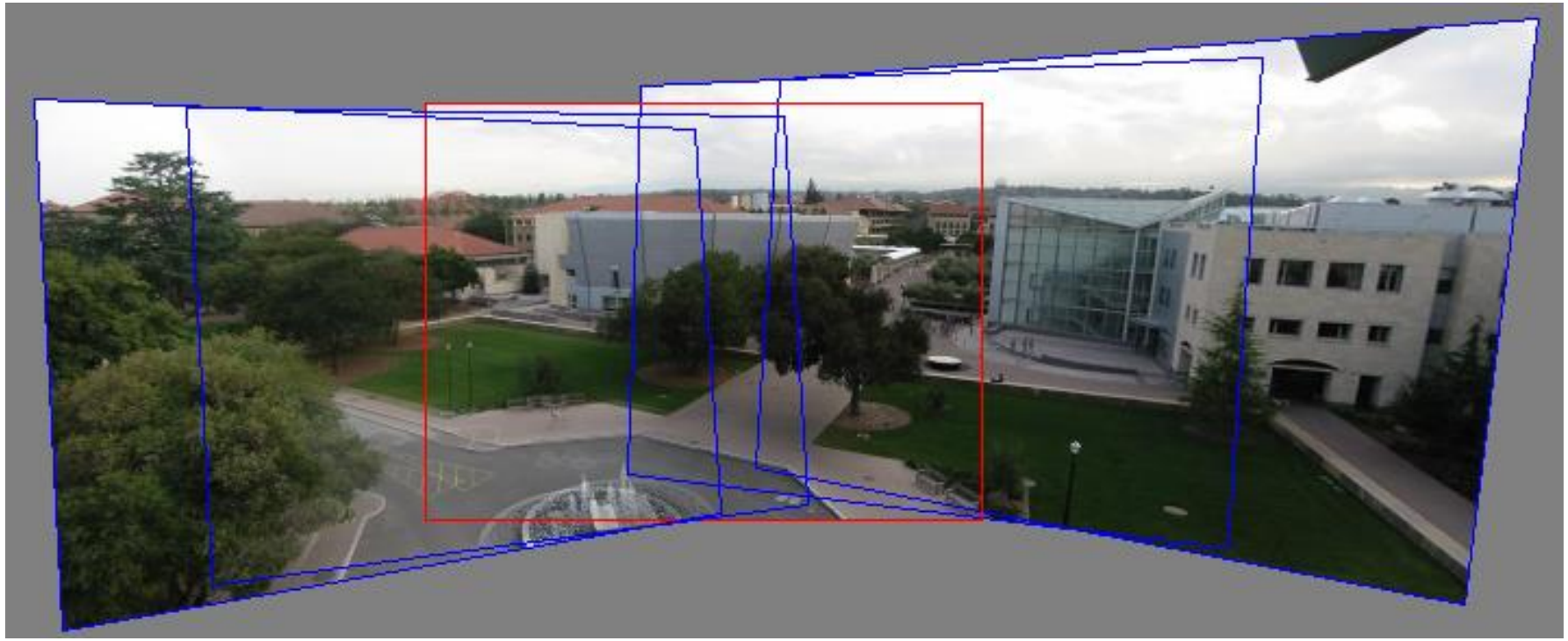


Rollout Photographs © Justin Kerr

http://research.famsi.org/kerrmaya.html

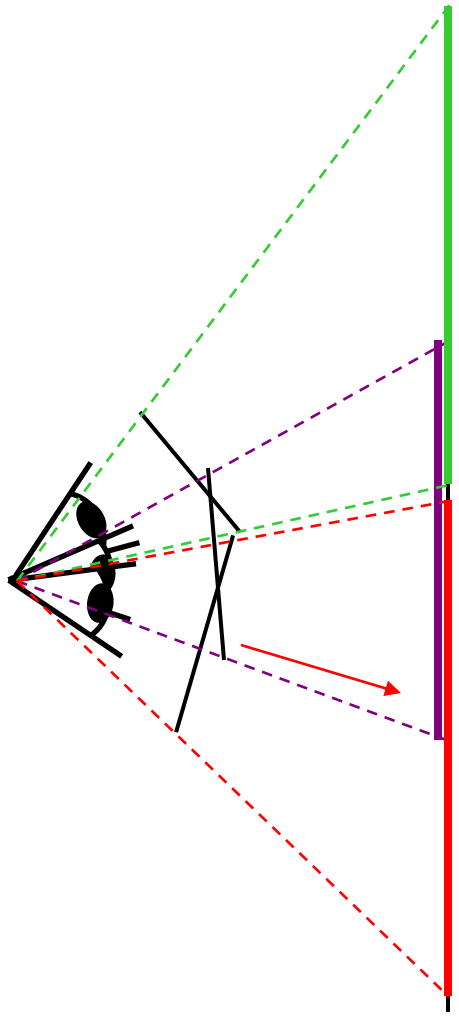
Also known as “cyclographs”, “peripheral images”

Back to panoramas



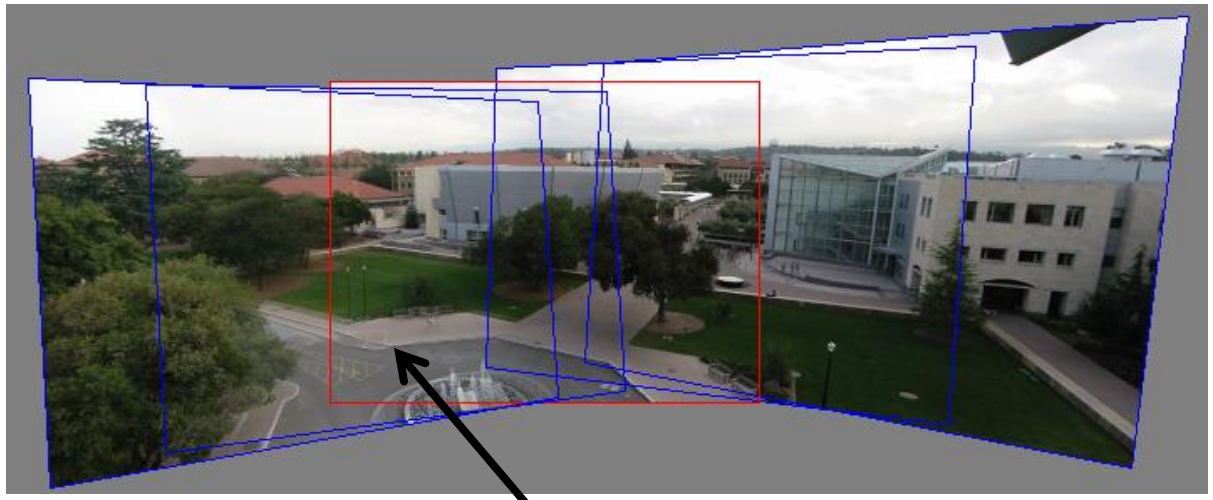
Can we use homographies to create a 360 degree panorama?

Idea: project images onto a common plane



mosaic projection plane

"Mosaic"



each image is warped
with a homography $\mathbf{H}$

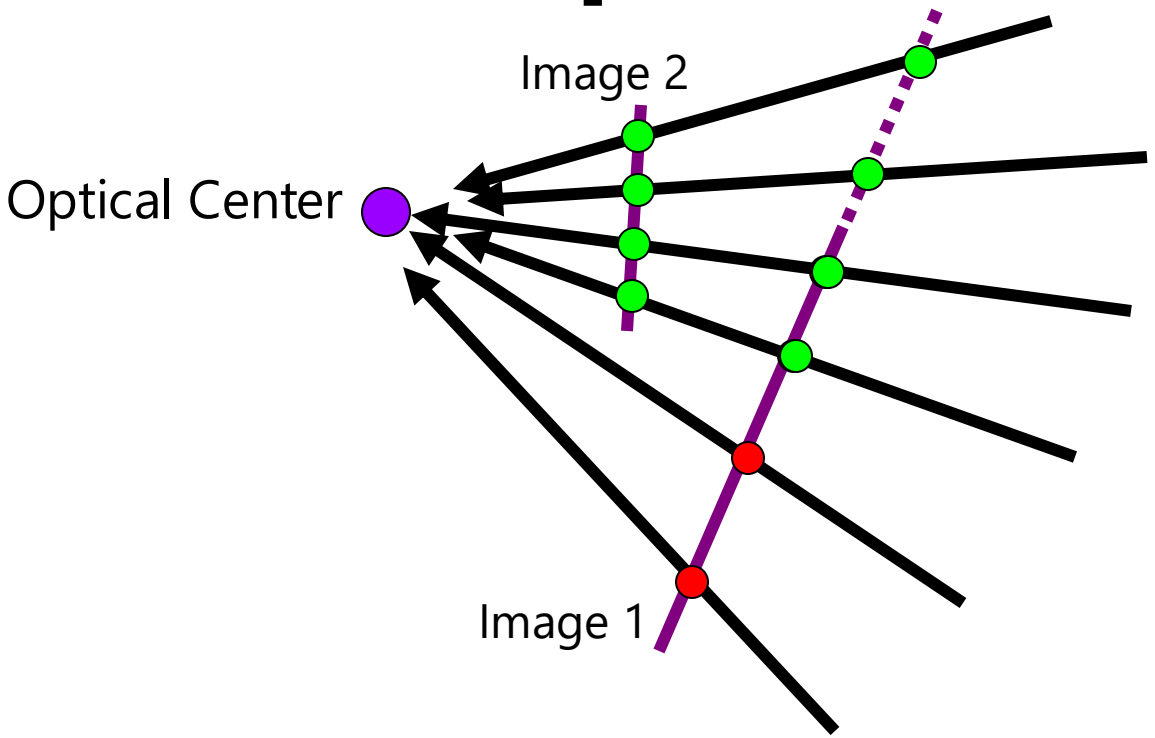
We'll see what this homography means next

Can't create a 360 panorama this way… we'll fix this shortly

Creating a panorama

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - If there are more images, repeat

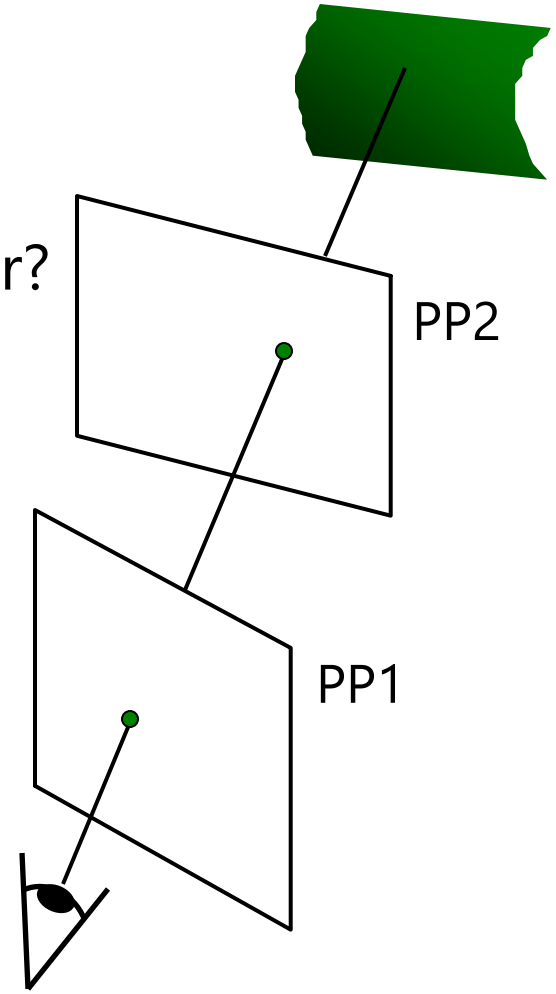
Geometric interpretation of mosaics



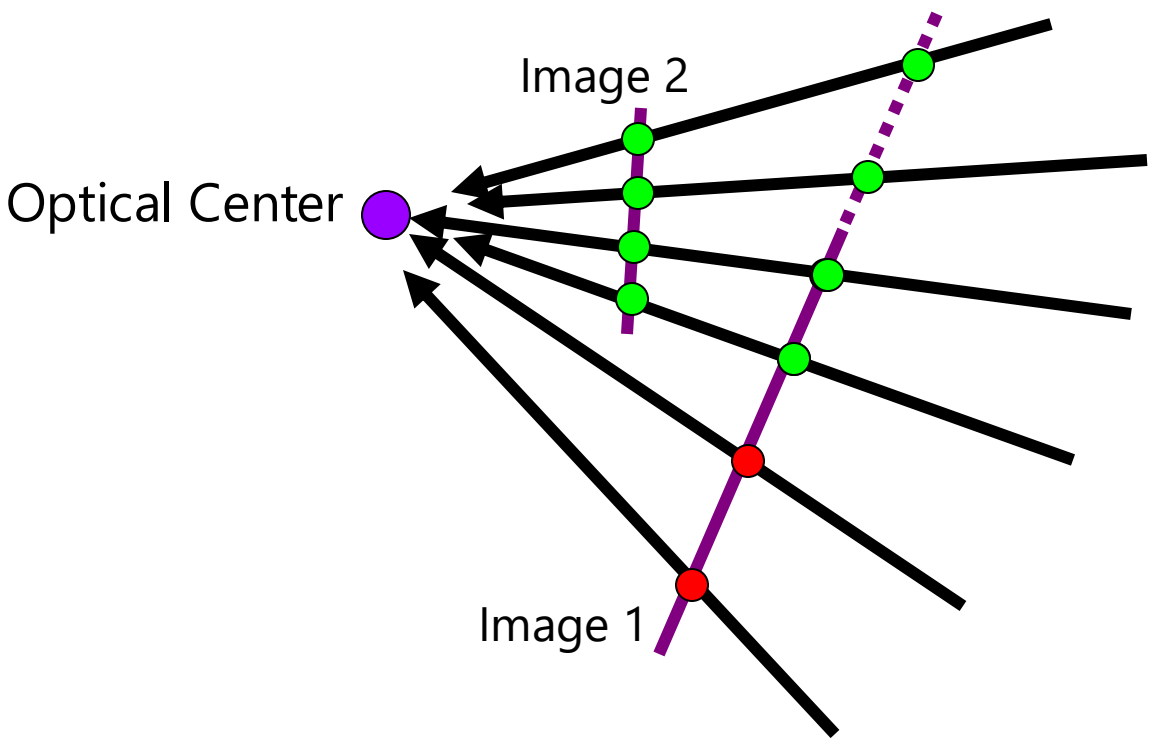
- If we capture all 360º of rays, we can create a 360º panorama
- The basic operation is *projecting* an image from one plane to another
- The projective transformation is scene-INDEPENDENT
  - This depends on all the images having the same optical center

Image reprojection

- Basic question
  - How to relate two images from the same camera center?
    - how to map a pixel from PP1 to PP2
- Answer
  - Cast a ray through each pixel in PP1
  - Draw the pixel where that ray intersects PP2



What is the transformation?



How do we map points in image 2 into image 1?

	image 1	image 2
intrinsics	$\mathbf{K}_1$	$\mathbf{K}_2$
extrinsics (rotation only)	$\mathbf{R}_1 = \mathbf{I}_{3\times 3}$	$\mathbf{R}_2$

Step 1: Convert pixels in image 2 to rays in camera 2's coordinate system.

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Step 2: Convert rays in camera 2's coordinates to rays in camera 1's coordinates.

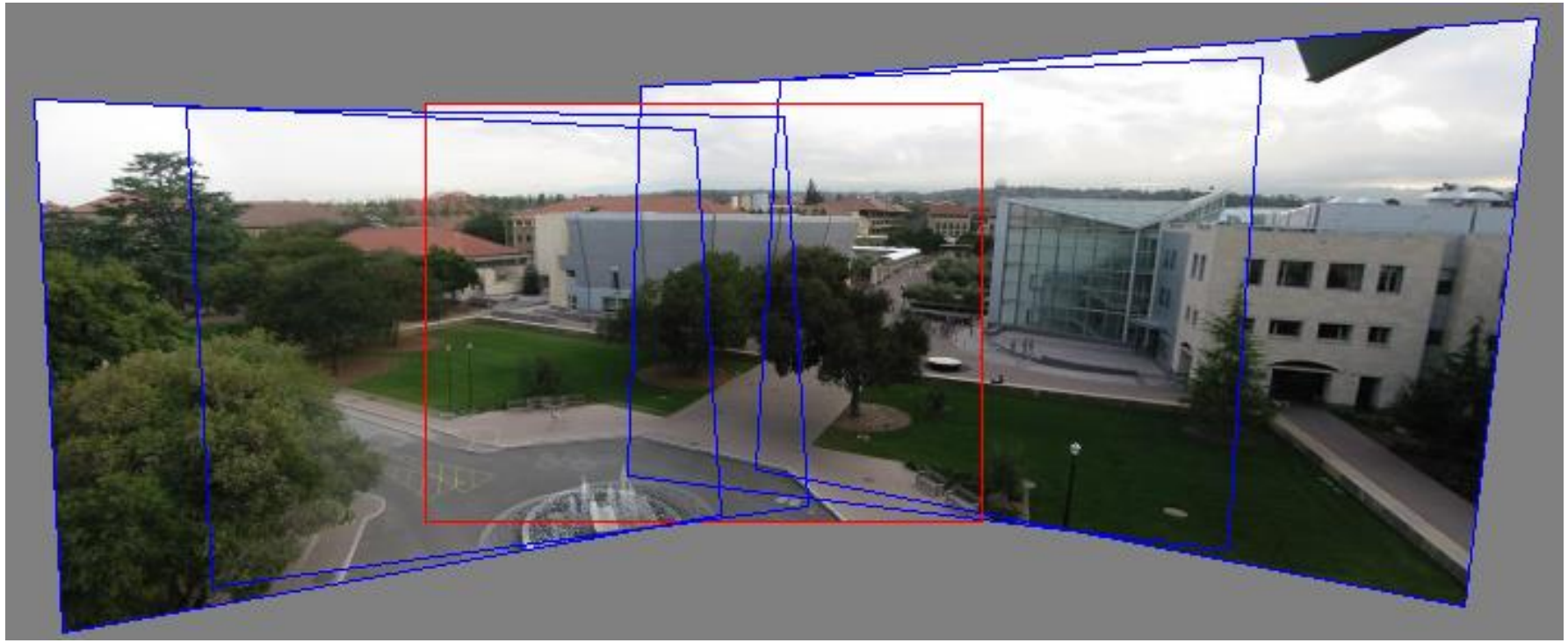
$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \mathbf{R}_2^T \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Step 3: Convert rays in camera 1's coordinates to pixels in image 1's coordinates.

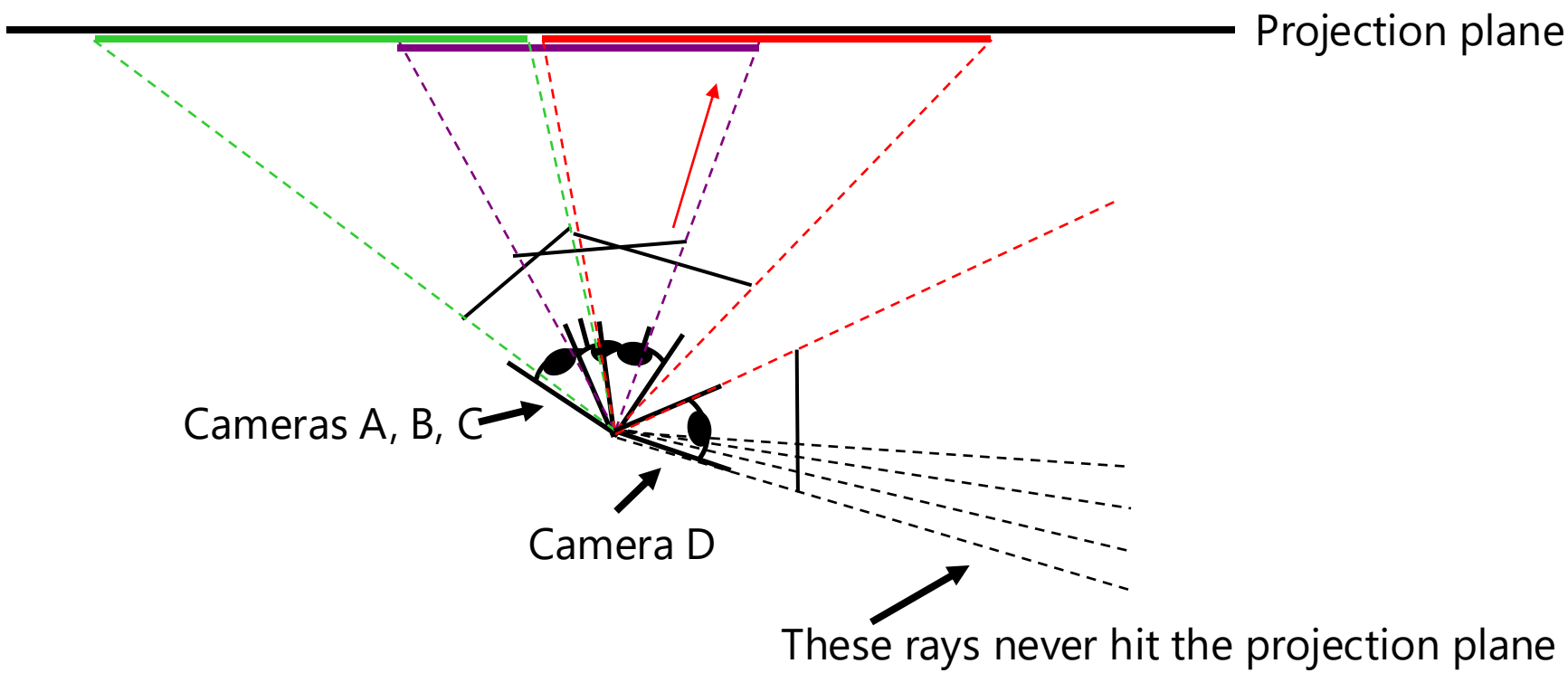
$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \sim \underbrace{\mathbf{K}_1 \mathbf{R}_2^T \mathbf{K}_2^{-1}} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

3x3 homography

Can we use homography to create a 360 panorama?

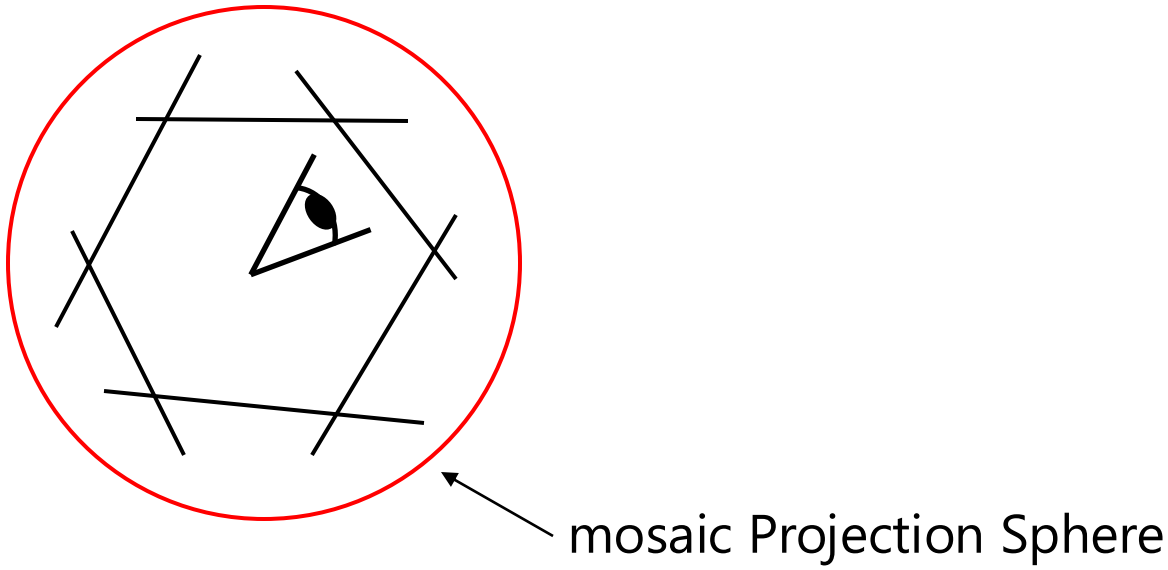


Answer: No

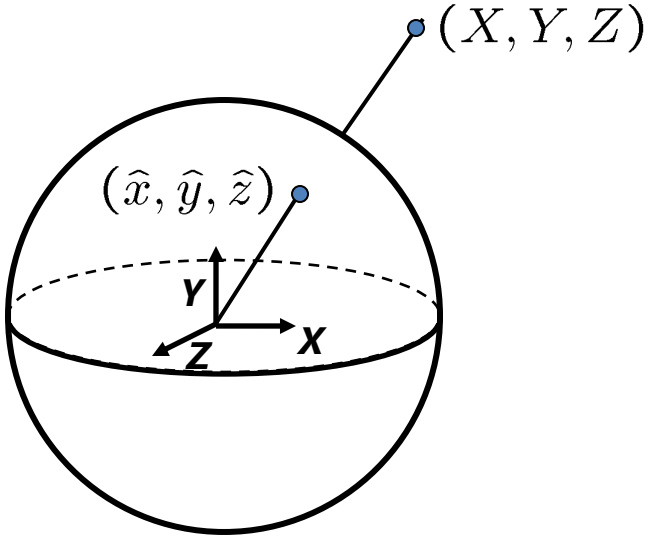

Projection plane
Cameras A, B, C
Camera D
These rays never hit the projection plane

Panoramas

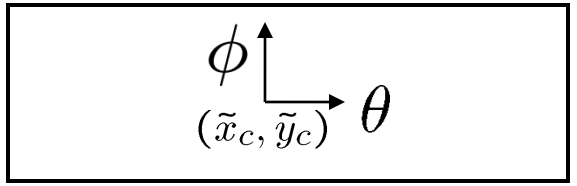
- What if you want a 360° field of view?



Spherical projection



unit sphere



unwrapped sphere

- Map 3D point (X,Y,Z) onto sphere

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2+Y^2+Z^2}}(X, Y, Z)$$

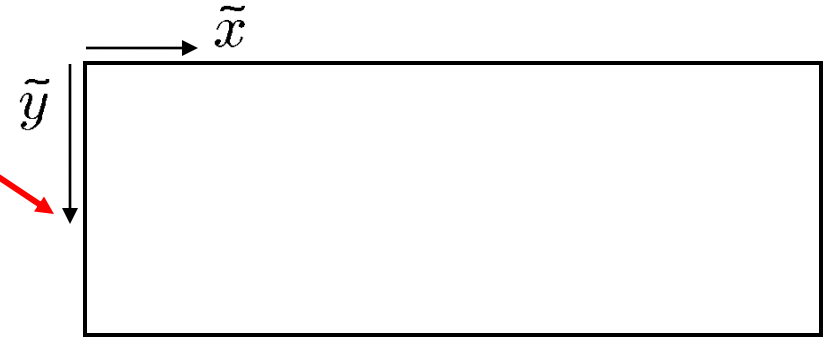
- Convert to spherical coordinates

$$(sin\theta cos\phi, sin\phi, cos\theta cos\phi) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to spherical image coordinates

$$(\tilde{x}, \tilde{y}) = (s\theta, s\phi) + (\tilde{x}_c, \tilde{y}_c)$$

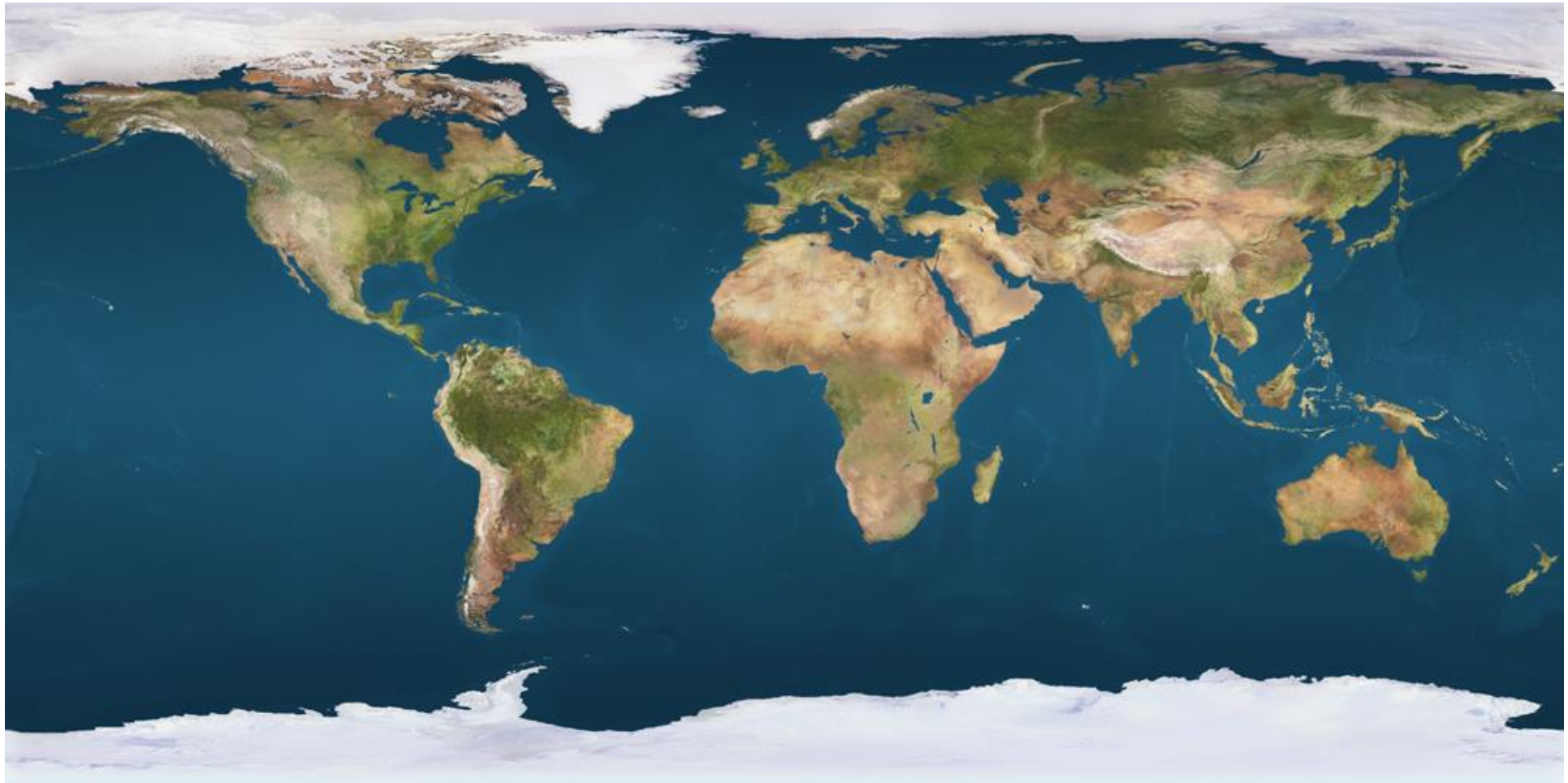
  - s defines size of the final image
    - » often convenient to set s = camera focal length



Spherical image

Unwrapping a sphere

Credit: JHT's Planetary Pixel Emporium

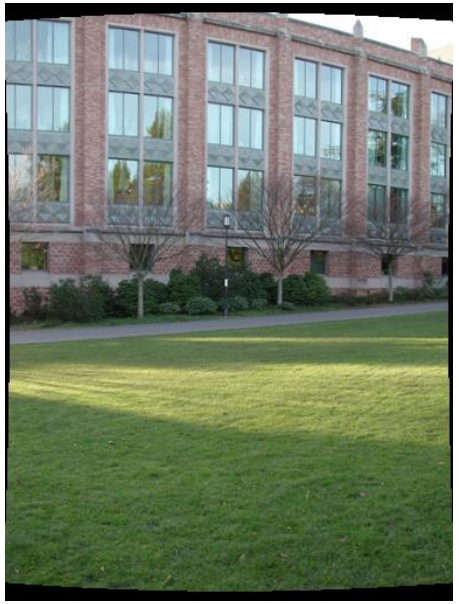


Spherical reprojection

input | f = 200 (pixels) | f = 400 | f = 800

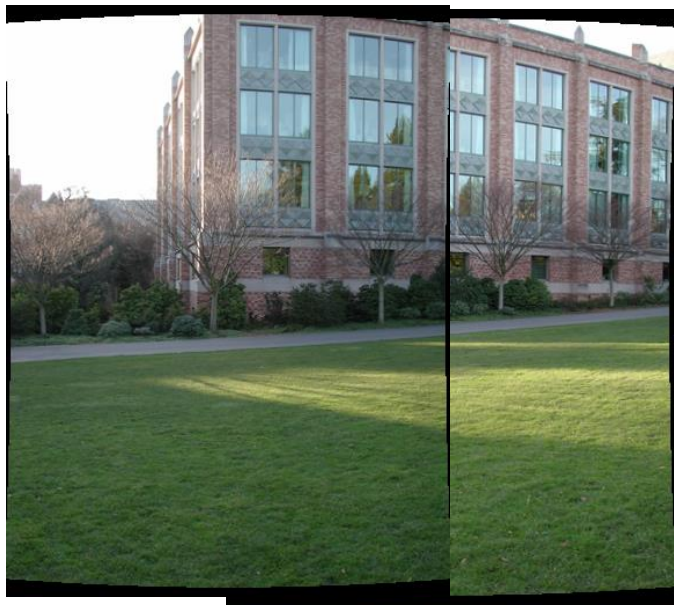
- Map image to spherical coordinates
  - need to know the focal length

Aligning spherical images



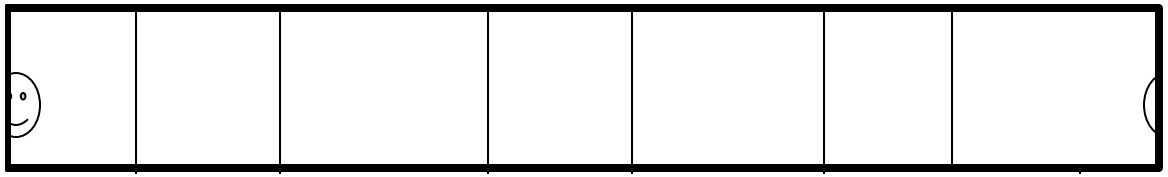
- Suppose we rotate the camera by θ about the vertical axis
  - How does this change the spherical image?

Aligning spherical images



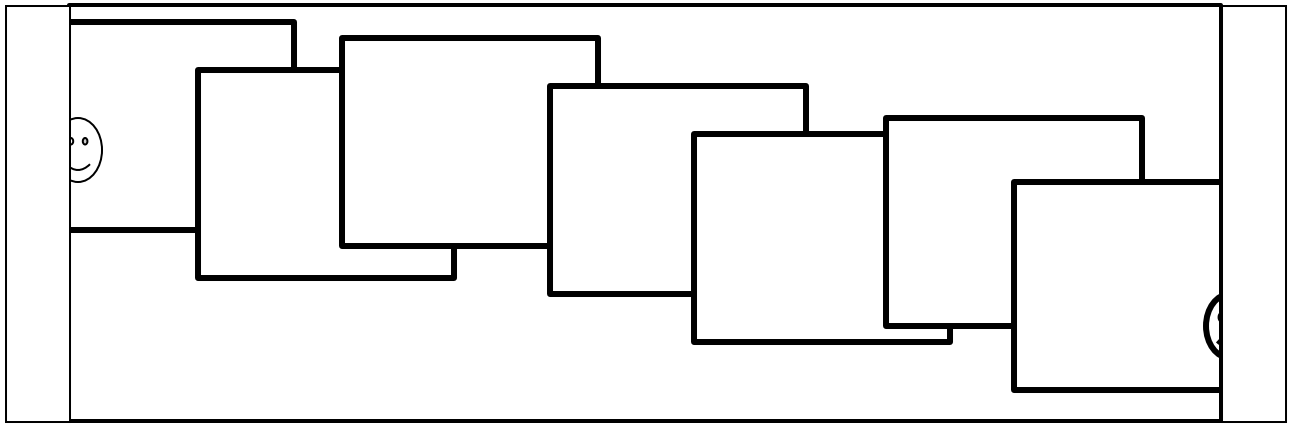
- Suppose we rotate the camera by θ about the vertical axis
  - How does this change the spherical image?
    - Translation by θ
  - This means that we can align spherical images by translation

Assembling the panorama



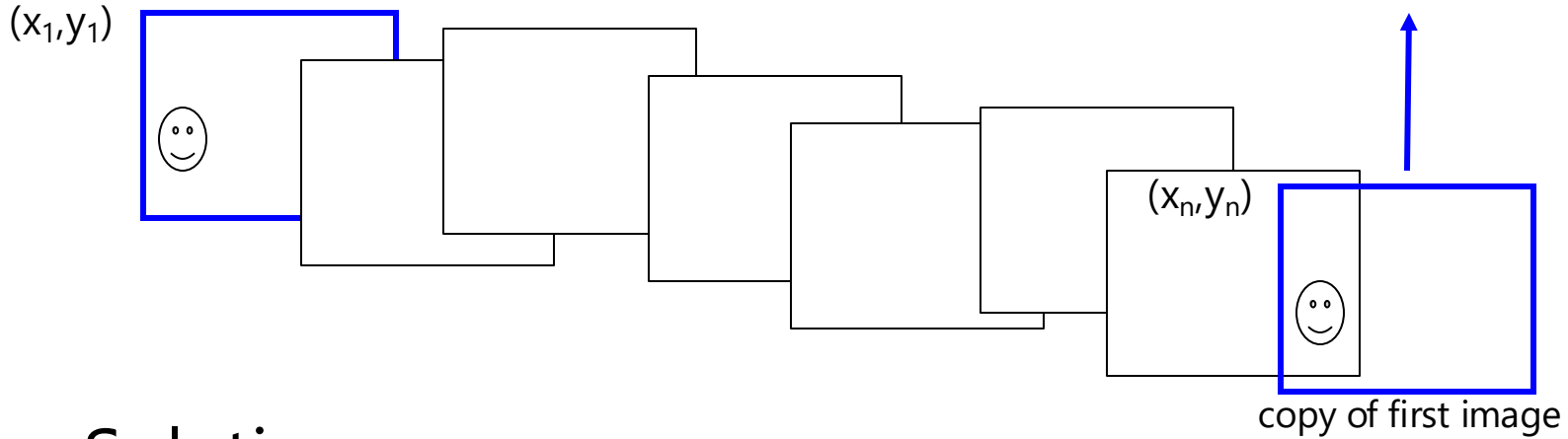
- Stitch pairs together, blend, then crop

Problem: Drift



- Error accumulation
  - small errors accumulate over time

Problem: Drift



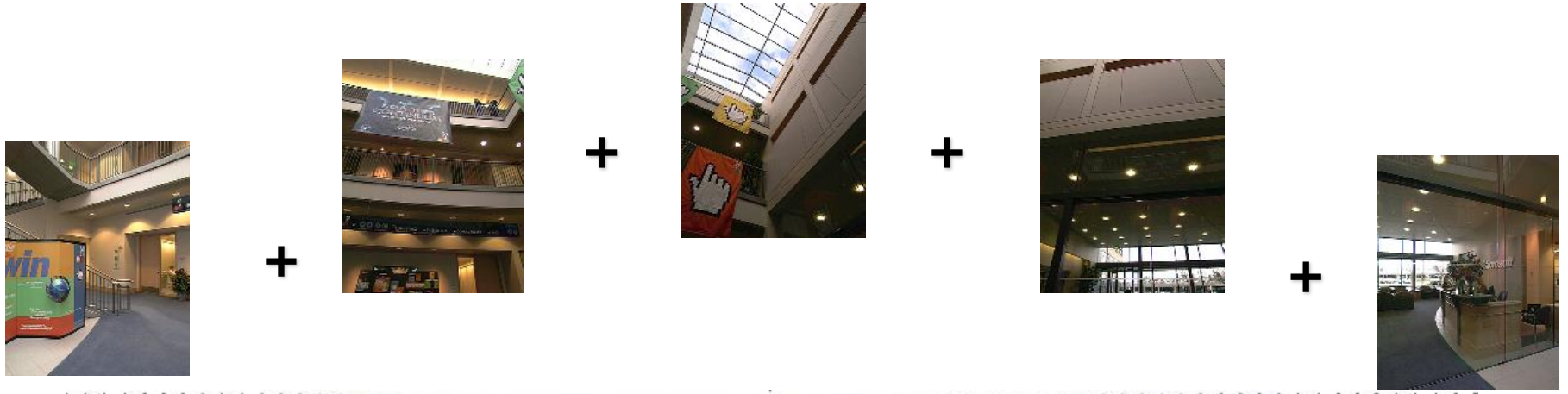
- Solution
  - add another copy of first image at the end
  - this gives a constraint: $y_n = y_1$
  - there are a bunch of ways to solve this problem
    - add displacement of $(y_1 - y_n)/(n-1)$ to each image after the first
    - **apply an affine warp: $y' = y + ax$ [you will implement this for P3]**
    - run a big optimization problem, incorporating this constraint
      - best solution, but more complicated
      - known as "bundle adjustment"

Project 3

1. Take pictures on a tripod (or handheld)
2. Warp to spherical coordinates (not needed if using homographies to align images)
3. Extract features
4. Align neighboring pairs using feature matching + RANSAC
5. Write out list of neighboring translations
6. Correct for drift
7. Read in warped images and blend them
8. Crop the result and import into a viewer

- Roughly based on **Autostitch**
  - By Matthew Brown and David Lowe
  - http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html

Spherical panoramas



Microsoft Lobby: http://www.acm.org/pubs/citations/proceedings/graph/258734/p251-szeliski

Different projections are possible

Cube-map

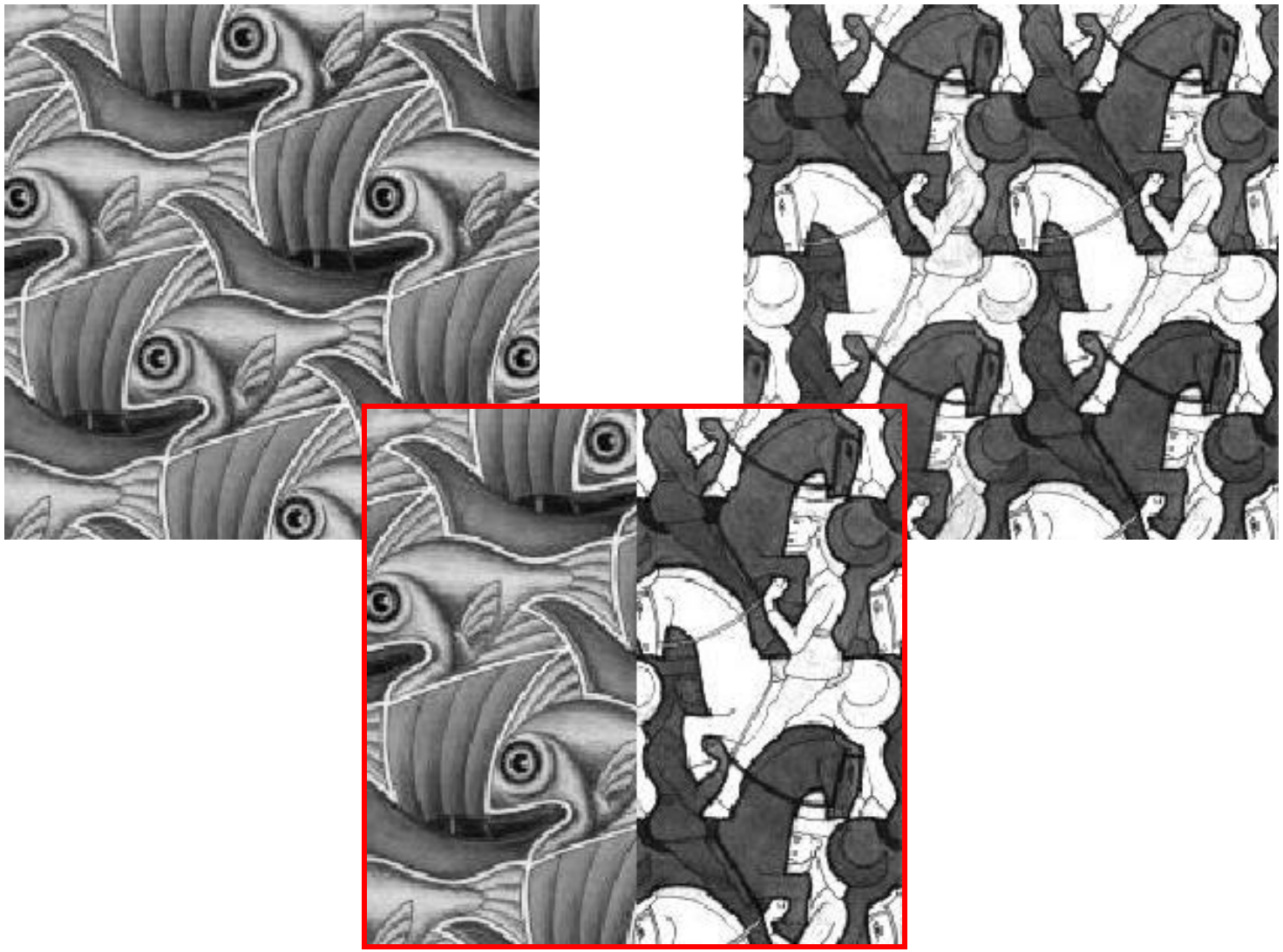
Blending

- We’ve aligned the images – now what?

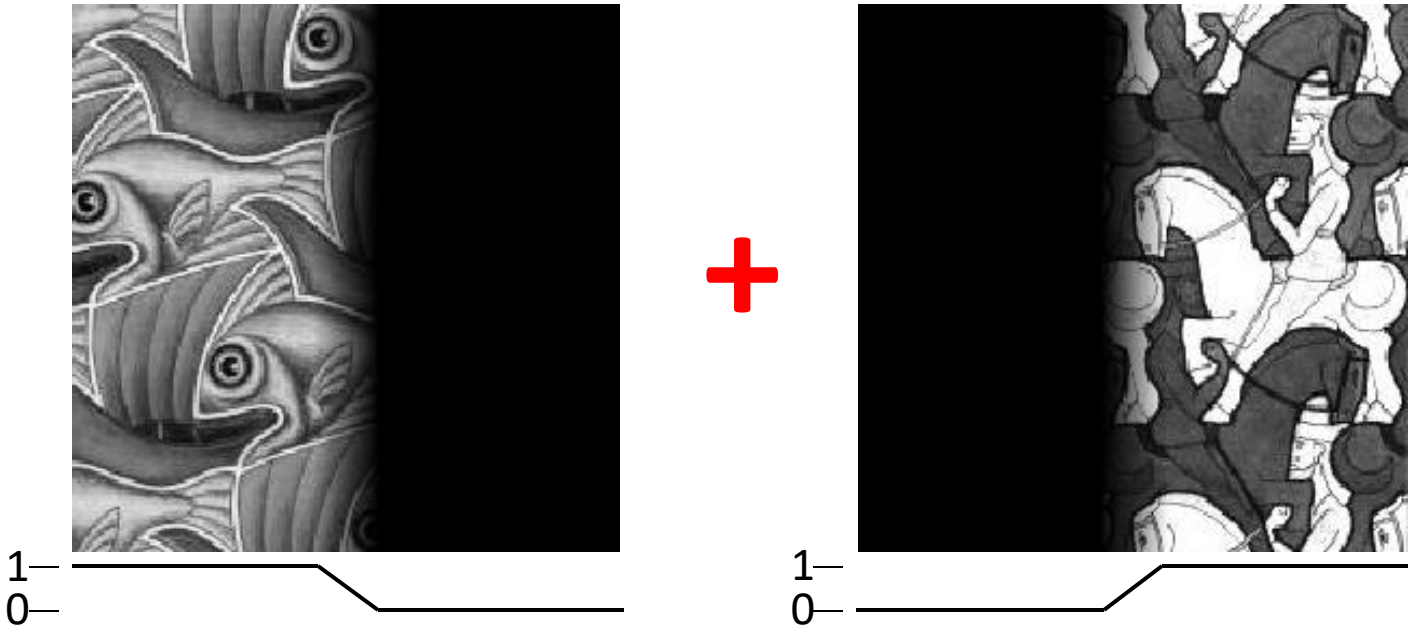
Blending

- Want to seamlessly blend them together

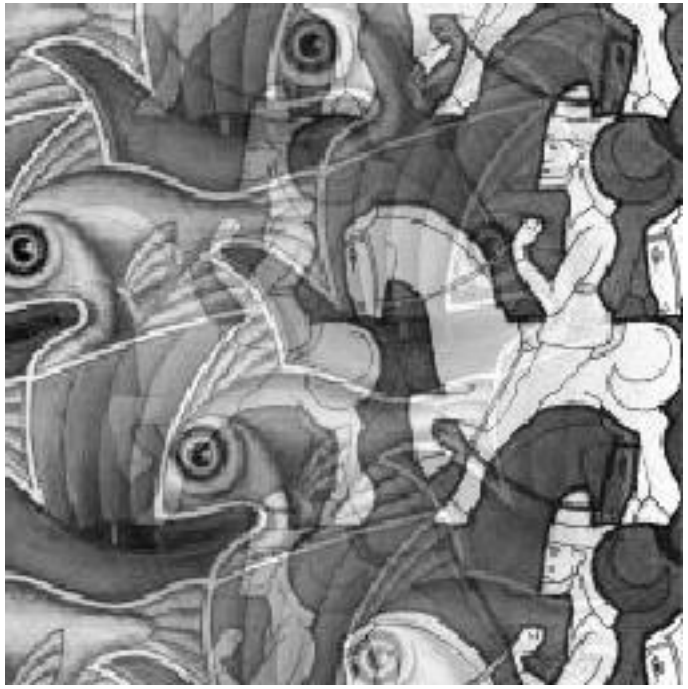
Image Blending

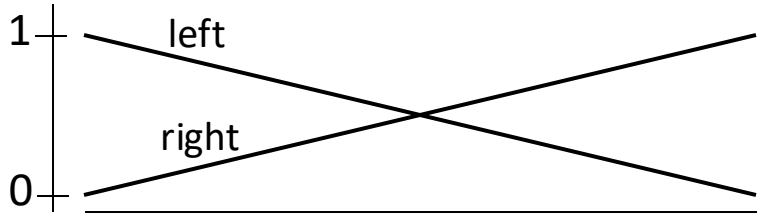


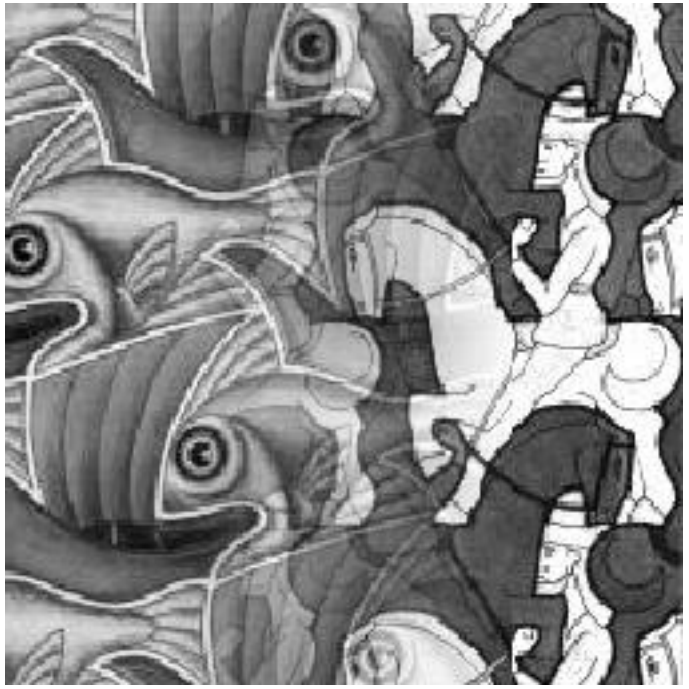
Feathering

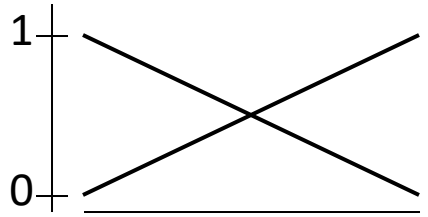


Effect of window size

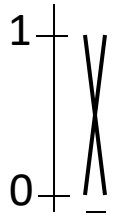


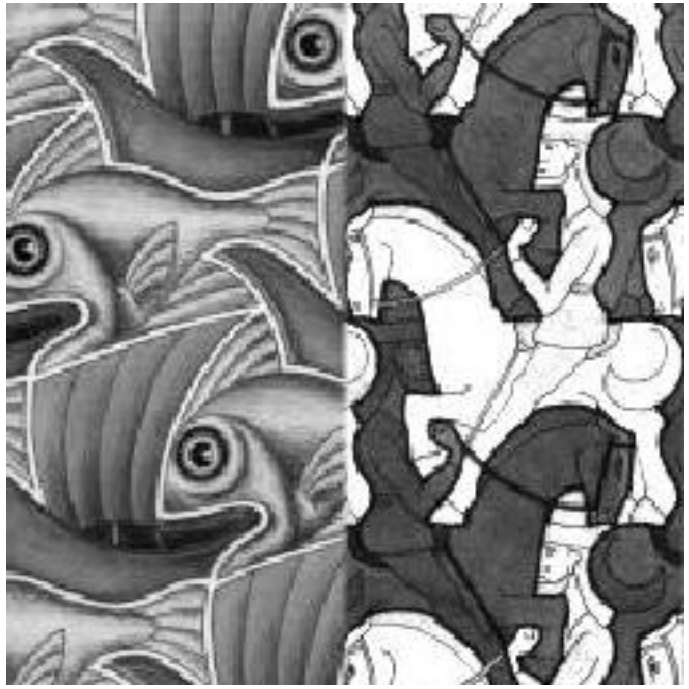


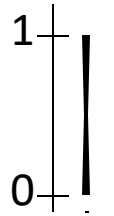




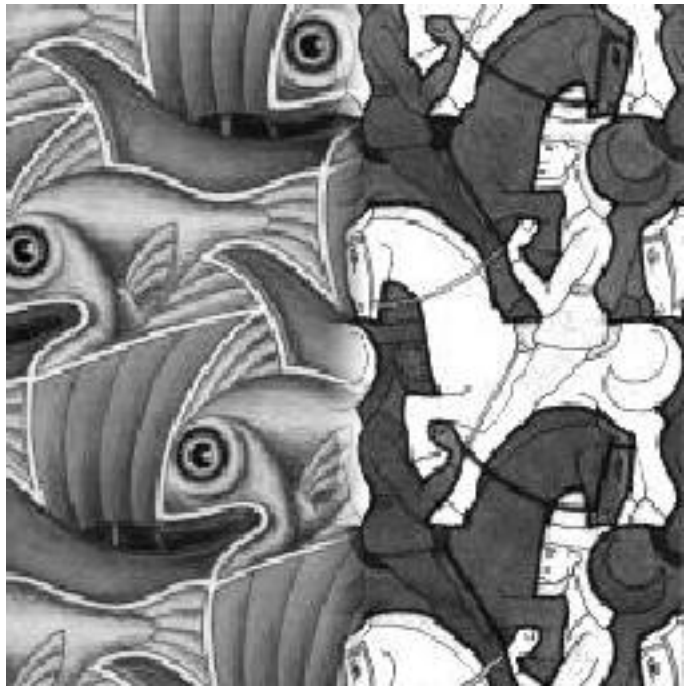
Effect of window size

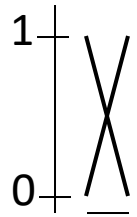






Good window size

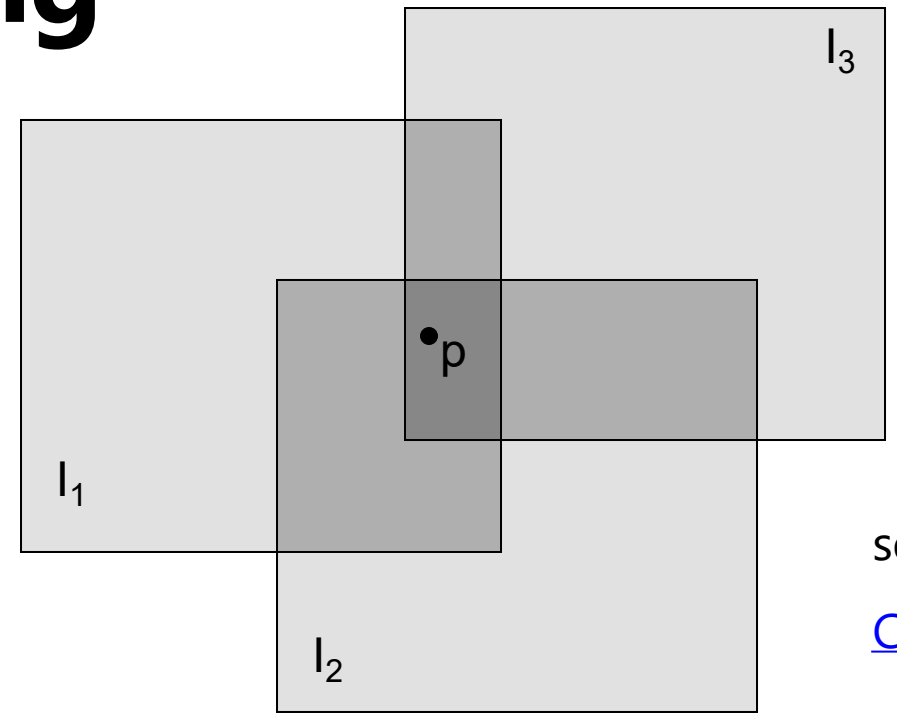




"Optimal" window: smooth but not ghosted

- Doesn't always work...

Alpha Blending



see Blinn (CGA, 1994) for details:

Compositing, Part 1: Theory

Encoding blend weights: I(x,y) = (αR, αG, αB, α)

color at p = $\frac{(\alpha_1 R_1,\ \alpha_1 G_1,\ \alpha_1 B_1) + (\alpha_2 R_2,\ \alpha_2 G_2,\ \alpha_2 B_2) + (\alpha_3 R_3,\ \alpha_3 G_3,\ \alpha_3 B_3)}{\alpha_1 + \alpha_2 + \alpha_3}$

Implement this in two steps:

1. accumulate: add up the (α premultiplied) RGBα values at each pixel
2. normalize: divide each pixel's accumulated RGB by its α value

   Q: what if α = 0?

Poisson Image Editing

For more info: Perez et al, SIGGRAPH 2003

Some panorama examples

“Before SIGGRAPH Deadline” Photo credit: Doug Zongker

Some panorama examples

- Every image on Google Streetview

Magic: ghost removal

M. Uyttendaele, A. Eden, and R. Szeliski.
Eliminating ghosting and exposure artifacts in image mosaics.
ICCV 2001

Magic: ghost removal

M. Uyttendaele, A. Eden, and R. Szeliski.
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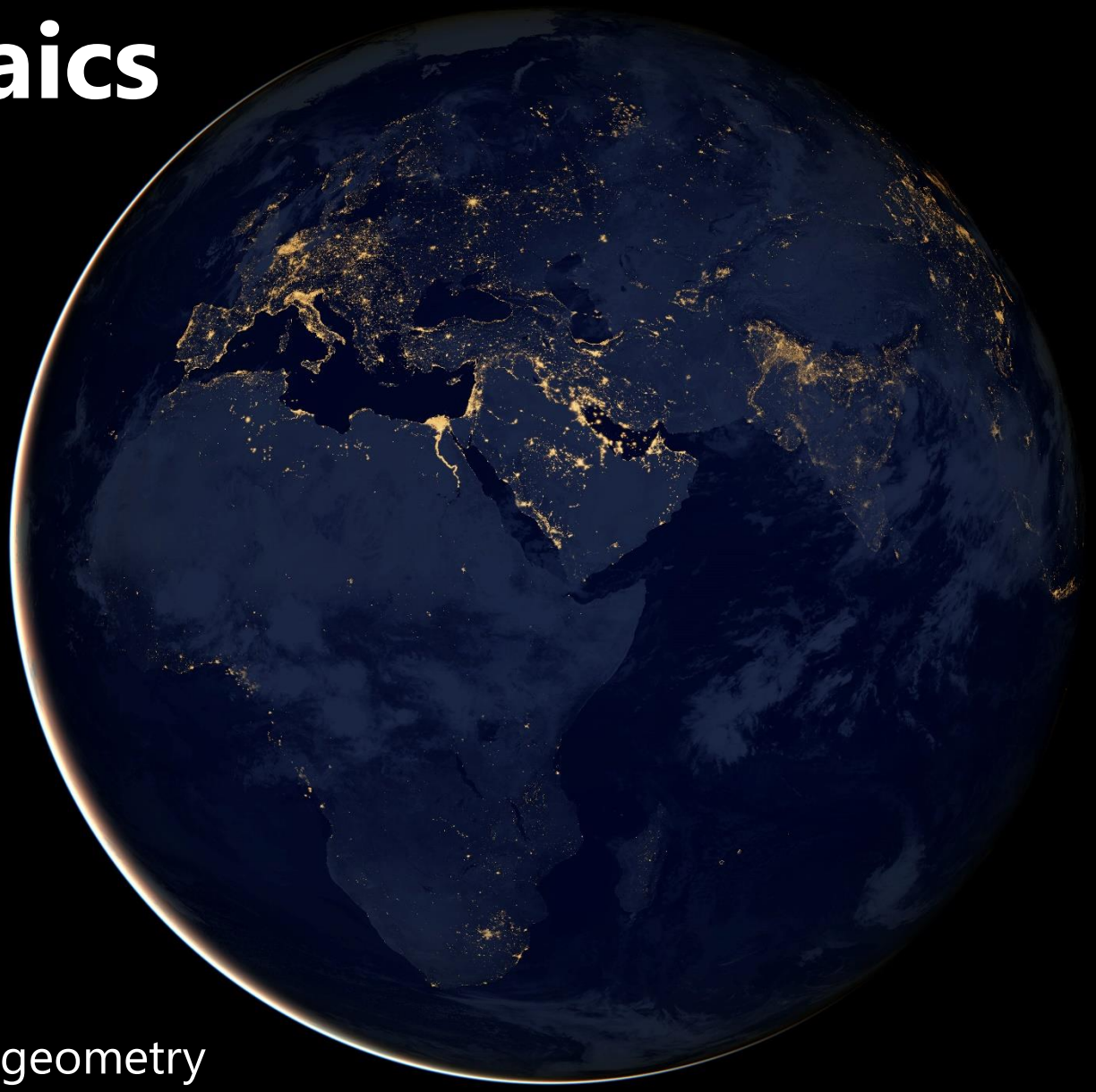
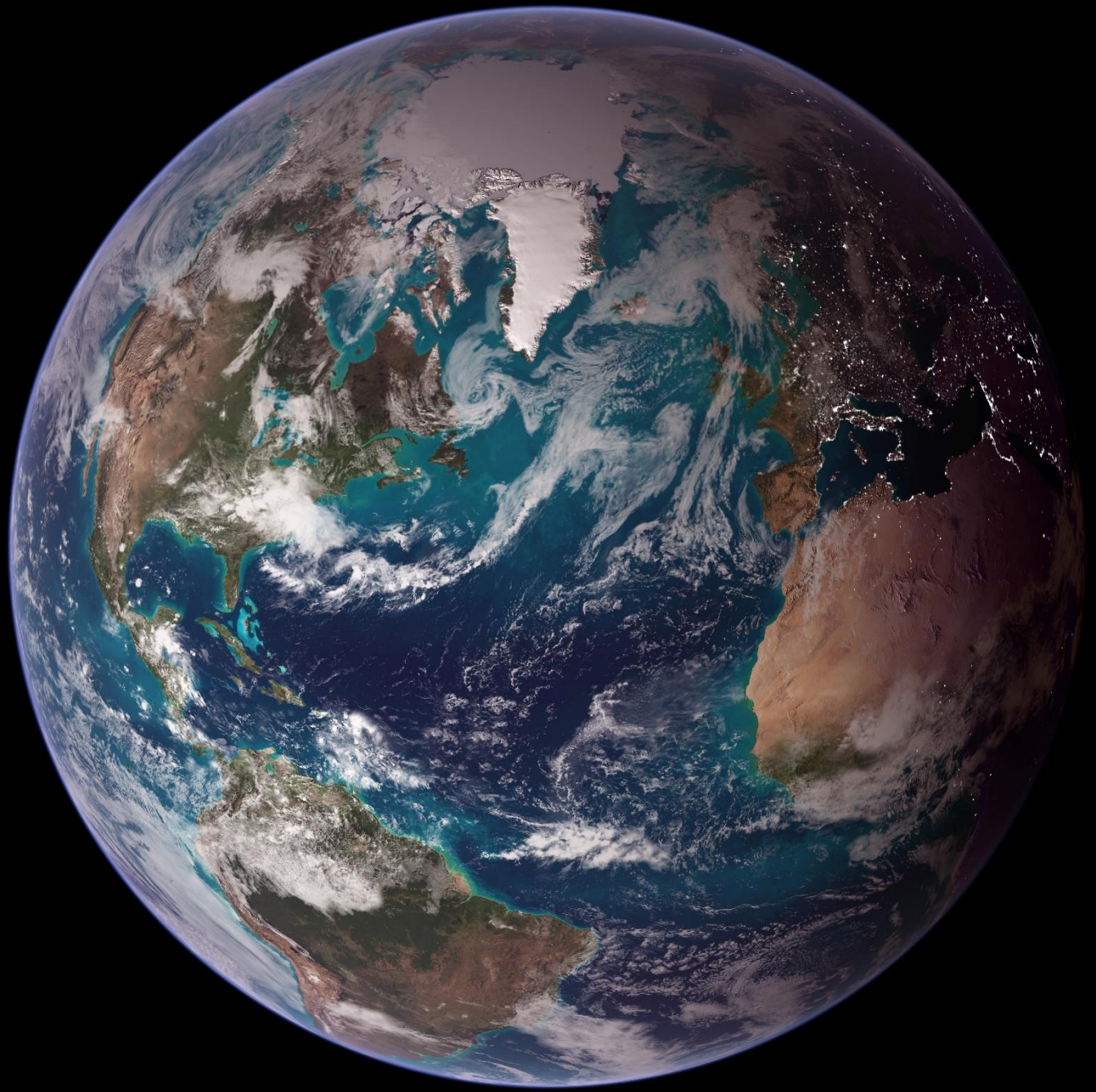
ICCV 2001

Other types of mosaics



- Can mosaic onto *any* surface if you know the geometry
  - See NASA's Visible Earth project for some stunning earth mosaics




https://www.nasa.gov/centers/wallops/news/frozen_sos.ht
ml

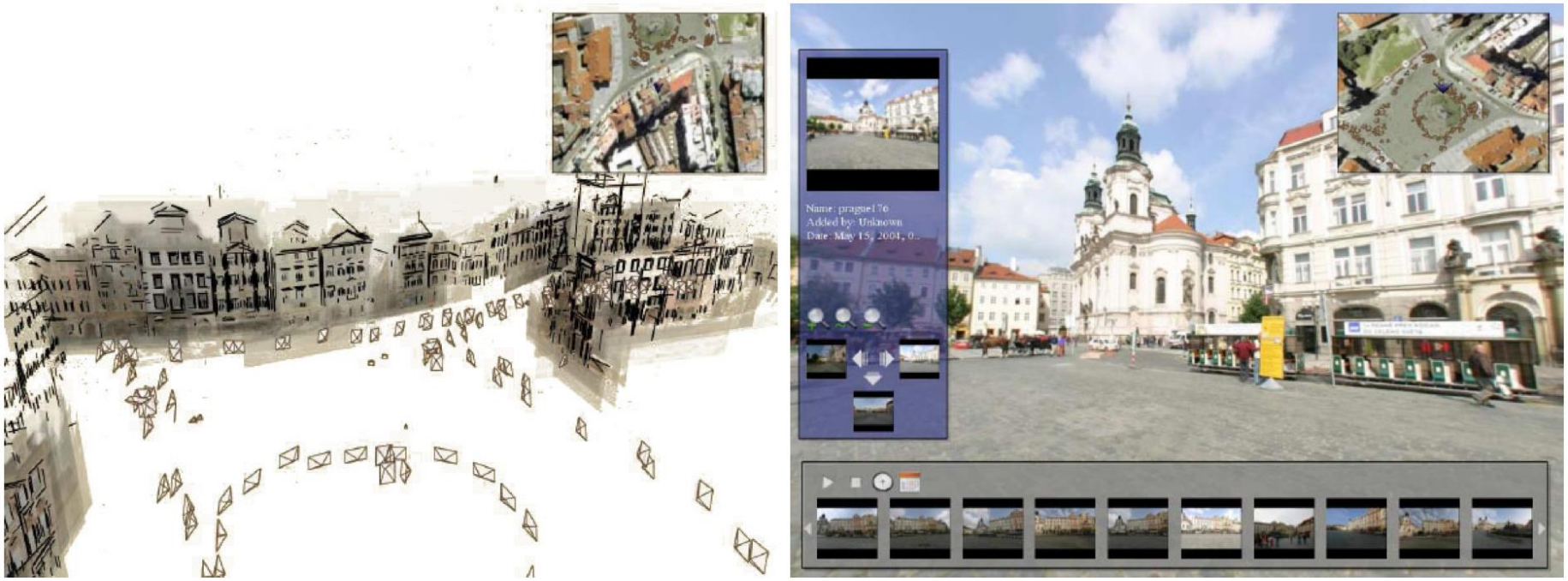
Science on a Sphere

https://news.vcu.edu/article/Science_On_a_Sphere_now_at_VCU_offers_a_world_of_possibilities

Geometric camera calibration

	Structure (scene geometry)	Motion (camera geometry)	**Measurements**
Camera Calibration (a.k.a. Pose Estimation)	known	estimate	3D to 2D correspondences
Triangulation	estimate	known	2D to 2D coorespondences
Reconstruction	estimate	estimate	2D to 2D coorespondences

Pose Estimation



Given a single image,
estimate the exact position of the photographer

Geometric camera calibration

Given a set of matched points

$$\{\mathbf{X}_i, \boldsymbol{x}_i\}$$

point in 3D space

point in the image

and camera model

$$\boldsymbol{x} = \boldsymbol{f}(\mathbf{X}; \boldsymbol{p}) = \mathbf{P}\mathbf{X}$$

projection model

parameters

Camera matrix

Find the (pose) estimate of

$$\mathbf{P}$$

We'll use a **perspective** camera model for pose estimation

Same setup as homography estimation

(slightly different derivation here)

Mapping between 3D point and image points

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What are the unknowns?

Mapping between 3D point and image points

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} — & \boldsymbol{p}_1^\top & — \\ — & \boldsymbol{p}_2^\top & — \\ — & \boldsymbol{p}_3^\top & — \end{bmatrix} \begin{bmatrix} | \\ \boldsymbol{X} \\ | \end{bmatrix}$$

Heterogeneous coordinates

$$x' = \frac{\boldsymbol{p}_1^\top \boldsymbol{X}}{\boldsymbol{p}_3^\top \boldsymbol{X}} \qquad y' = \frac{\boldsymbol{p}_2^\top \boldsymbol{X}}{\boldsymbol{p}_3^\top \boldsymbol{X}}$$

(non-linear relation between coordinates)

How can we make these relations linear?

How can we make these relations linear?

$$x' = \frac{\boldsymbol{p}_1^\top \boldsymbol{X}}{\boldsymbol{p}_3^\top \boldsymbol{X}} \qquad y' = \frac{\boldsymbol{p}_2^\top \boldsymbol{X}}{\boldsymbol{p}_3^\top \boldsymbol{X}}$$

Make them linear with algebraic manipulation…

$$\boldsymbol{p}_2^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} y' = 0$$

$$\boldsymbol{p}_1^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} x' = 0$$

Now we can setup a system of linear equations with multiple point correspondences

$$\boldsymbol{p}_2^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} y' = 0$$

$$\boldsymbol{p}_1^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} x' = 0$$

How do we proceed?

$$\boldsymbol{p}_2^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} y' = 0$$

$$\boldsymbol{p}_1^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} x' = 0$$

In matrix form …

$$\begin{bmatrix} \boldsymbol{X}^\top & \mathbf{0} & -x'\boldsymbol{X}^\top \\ \mathbf{0} & \boldsymbol{X}^\top & -y'\boldsymbol{X}^\top \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix} = \mathbf{0}$$

How do we proceed?

$$\boldsymbol{p}_2^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} y' = 0$$

$$\boldsymbol{p}_1^\top \boldsymbol{X} - \boldsymbol{p}_3^\top \boldsymbol{X} x' = 0$$

In matrix form …

$$\begin{bmatrix} \boldsymbol{X}^\top & \mathbf{0} & -x'\boldsymbol{X}^\top \\ \mathbf{0} & \boldsymbol{X}^\top & -y'\boldsymbol{X}^\top \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix} = \mathbf{0}$$

For N points …

$$\begin{bmatrix} \boldsymbol{X}_1^\top & \mathbf{0} & -x'\boldsymbol{X}_1^\top \\ \mathbf{0} & \boldsymbol{X}_1^\top & -y'\boldsymbol{X}_1^\top \\ \vdots & \vdots & \vdots \\ \boldsymbol{X}_N^\top & \mathbf{0} & -x'\boldsymbol{X}_N^\top \\ \mathbf{0} & \boldsymbol{X}_N^\top & -y'\boldsymbol{X}_N^\top \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix} = \mathbf{0}$$

How do we solve this system?

Solve for camera matrix by

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\mathbf{A}\boldsymbol{x}\|^2 \text{ subject to } \|\boldsymbol{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{X}_1^\top & \mathbf{0} & -x'\boldsymbol{X}_1^\top \\ \mathbf{0} & \boldsymbol{X}_1^\top & -y'\boldsymbol{X}_1^\top \\ \vdots & \vdots & \vdots \\ \boldsymbol{X}_N^\top & \mathbf{0} & -x'\boldsymbol{X}_N^\top \\ \mathbf{0} & \boldsymbol{X}_N^\top & -y'\boldsymbol{X}_N^\top \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix}$$

SVD!

Solve for camera matrix by

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\mathbf{A}\boldsymbol{x}\|^2 \text{ subject to } \|\boldsymbol{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{X}_1^\top & \mathbf{0} & -x'\boldsymbol{X}_1^\top \\ \mathbf{0} & \boldsymbol{X}_1^\top & -y'\boldsymbol{X}_1^\top \\ \vdots & \vdots & \vdots \\ \boldsymbol{X}_N^\top & \mathbf{0} & -x'\boldsymbol{X}_N^\top \\ \mathbf{0} & \boldsymbol{X}_N^\top & -y'\boldsymbol{X}_N^\top \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix}$$

Solution **x** is the column of **V** corresponding to smallest singular value of

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$$

Solve for camera matrix by

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\mathbf{A}\boldsymbol{x}\|^2 \text{ subject to } \|\boldsymbol{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{X}_1^\top & \mathbf{0} & -x'\boldsymbol{X}_1^\top \\ \mathbf{0} & \boldsymbol{X}_1^\top & -y'\boldsymbol{X}_1^\top \\ \vdots & \vdots & \vdots \\ \boldsymbol{X}_N^\top & \mathbf{0} & -x'\boldsymbol{X}_N^\top \\ \mathbf{0} & \boldsymbol{X}_N^\top & -y'\boldsymbol{X}_N^\top \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \boldsymbol{p}_3 \end{bmatrix}$$

Equivalently, solution ***x*** is the Eigenvector corresponding to smallest Eigenvalue of

$$\mathbf{A}^\top\mathbf{A}$$

Now we have:

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix}$$

Are we done?

Almost there …

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix}$$

How do you get the intrinsic and extrinsic parameters from the projection matrix?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R}|-\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M}|-\mathbf{M}\mathbf{c}] \end{aligned}$$

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$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

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Find the camera center **C**

What is the projection of the camera center?

Find intrinsic **K** and rotation **R**

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

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Find the camera center **C**

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

How do we compute the camera center from this?

Find intrinsic **K** and rotation **R**

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

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Find the camera center **C**

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of P!

***c** is the singular vector corresponding to the smallest singular value*

Find intrinsic **K** and rotation **R**

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

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Find the camera center **C**

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of P!

***c** is the singular vector corresponding to the smallest singular value*

Find intrinsic **K** and rotation **R**

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

Any useful properties of K and R we can use?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R}| - \mathbf{R}\mathbf{c}] \\ &= [\mathbf{M}| - \mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center **C**

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of P!

***c** is the singular vector corresponding to the smallest singular value*

Find intrinsic **K** and rotation **R**

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

right upper triangle

orthogonal

How do we find K and R?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array}\right]$$

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Find the camera center **C**

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of P!

***c** is the singular vector corresponding to the smallest singular value*

Find intrinsic **K** and rotation **R**

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

QR decomposition

Geometric camera calibration

Given a set of matched points

$$\{\mathbf{X}_i, \boldsymbol{x}_i\}$$

point in 3D space — point in the image

Where do we get these matched points from?

and camera model

$$\boldsymbol{x} = \boldsymbol{f}(\mathbf{X}; \boldsymbol{p}) = \mathbf{P}\mathbf{X}$$

projection model — parameters — Camera matrix

Find the (pose) estimate of

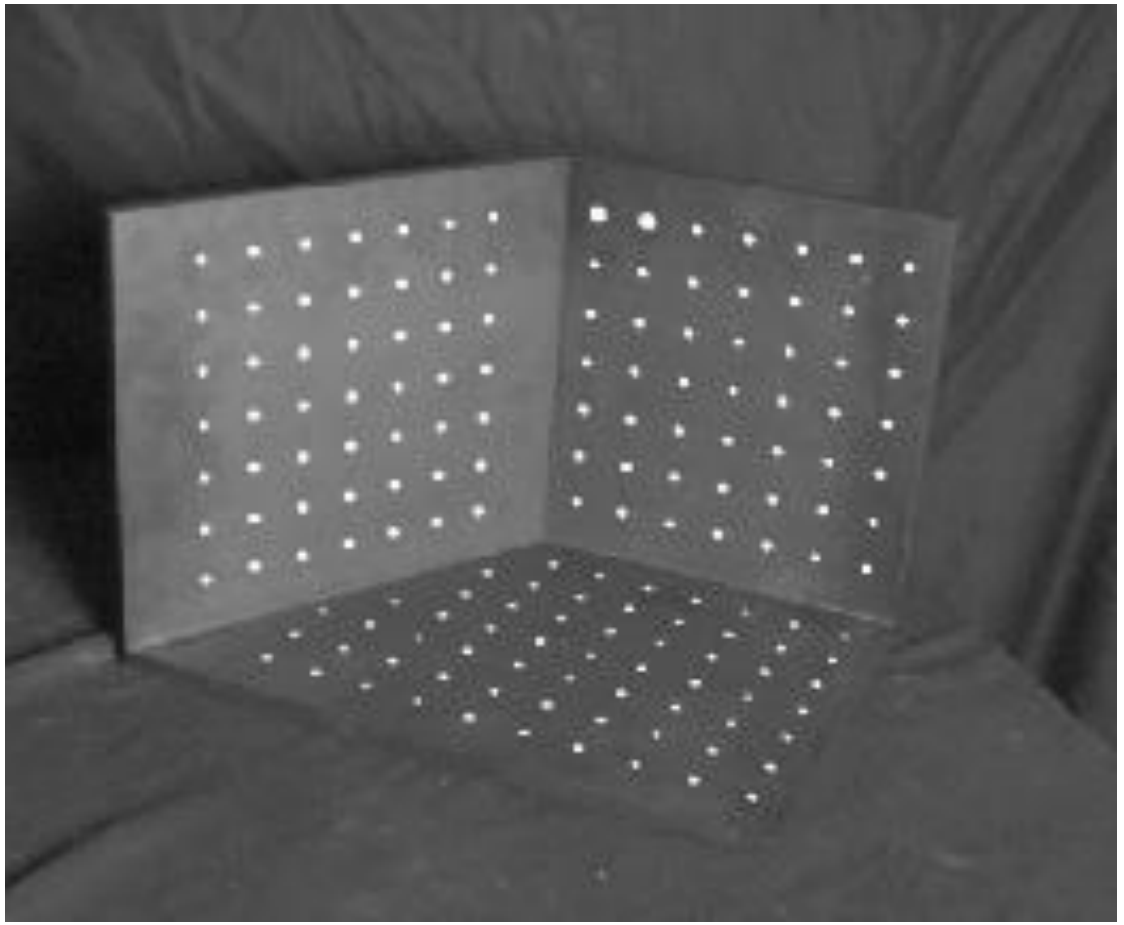
$$\mathbf{P}$$

We'll use a **perspective** camera model for pose estimation

Calibration using a reference object

Place a known object in the scene:

- identify correspondences between image and scene
- compute mapping from scene to image



Issues:

- must know geometry very accurately
- must know 3D->2D correspondence

Geometric camera calibration

Advantages:

- Very simple to formulate.
- Analytical solution.

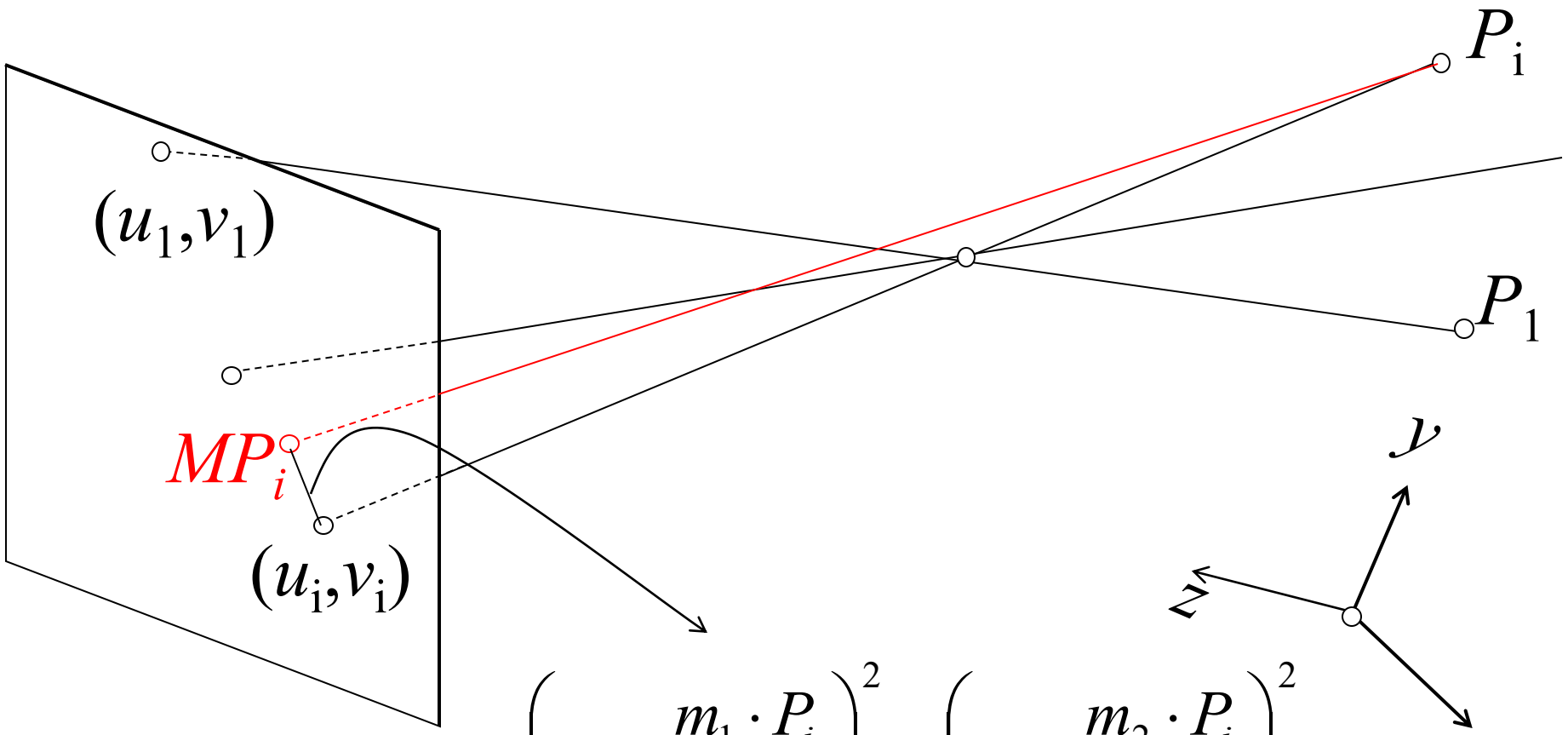
Disadvantages:

- Doesn’t model radial distortion.
- Hard to impose constraints (e.g., known *f*).
- Doesn’t minimize the correct error function.

For these reasons, *nonlinear methods* are preferred

- Define error function E between projected 3D points and image positions
  - E is nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques

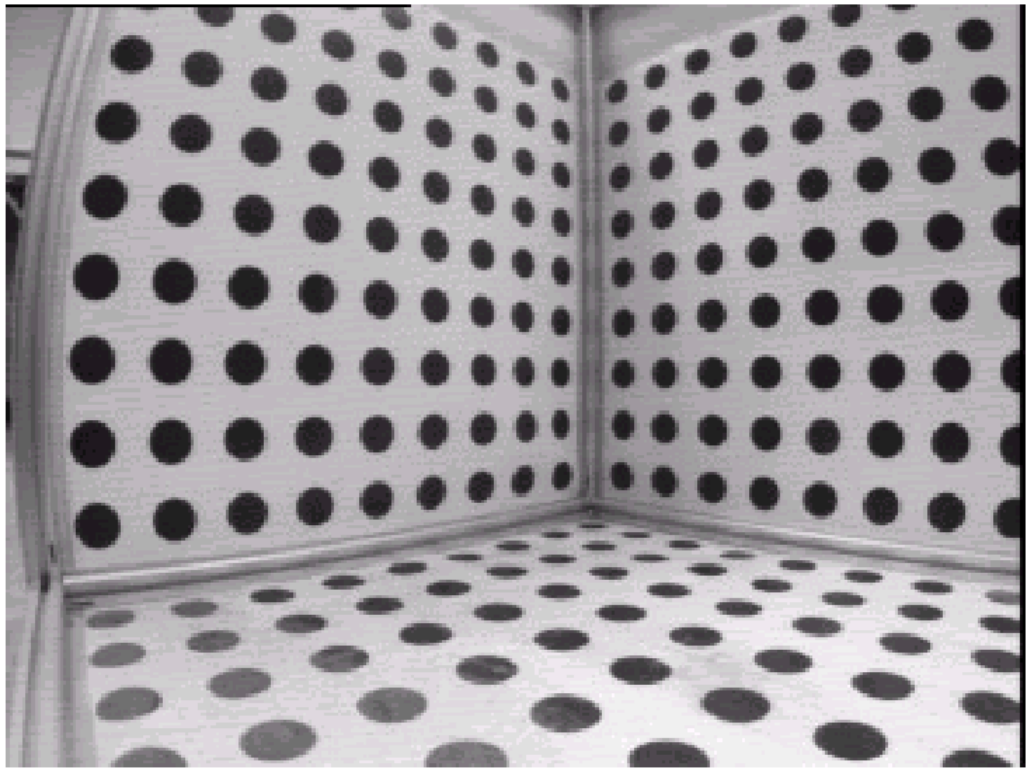
Minimizing reprojection error



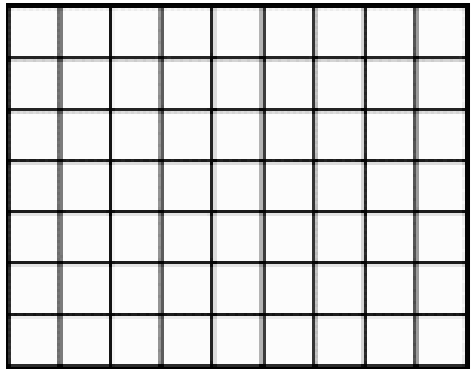
$$\left(u_i - \frac{m_1 \cdot P_i}{m_3 \cdot P_i}\right)^2 + \left(v_i - \frac{m_2 \cdot P_i}{m_3 \cdot P_i}\right)^2$$

Is this equivalent to what we were doing previously?

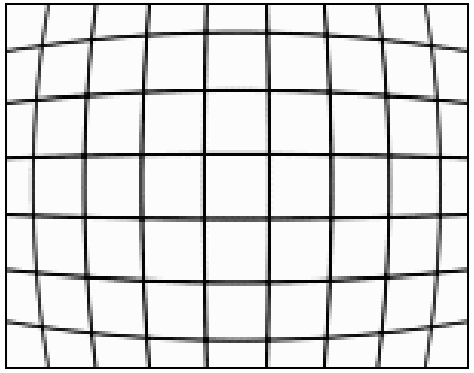
Radial distortion



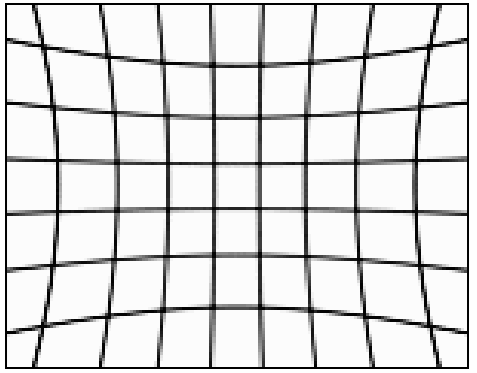
What causes this distortion?



no distortion

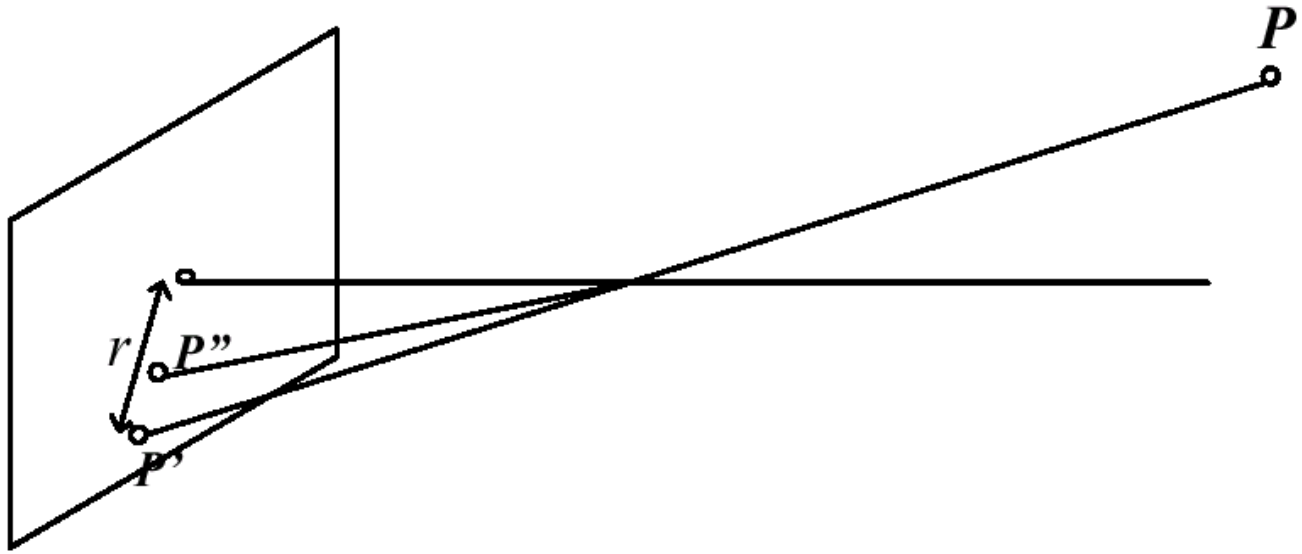


barrel distortion



pincushion distortion

Radial distortion model



Ideal:

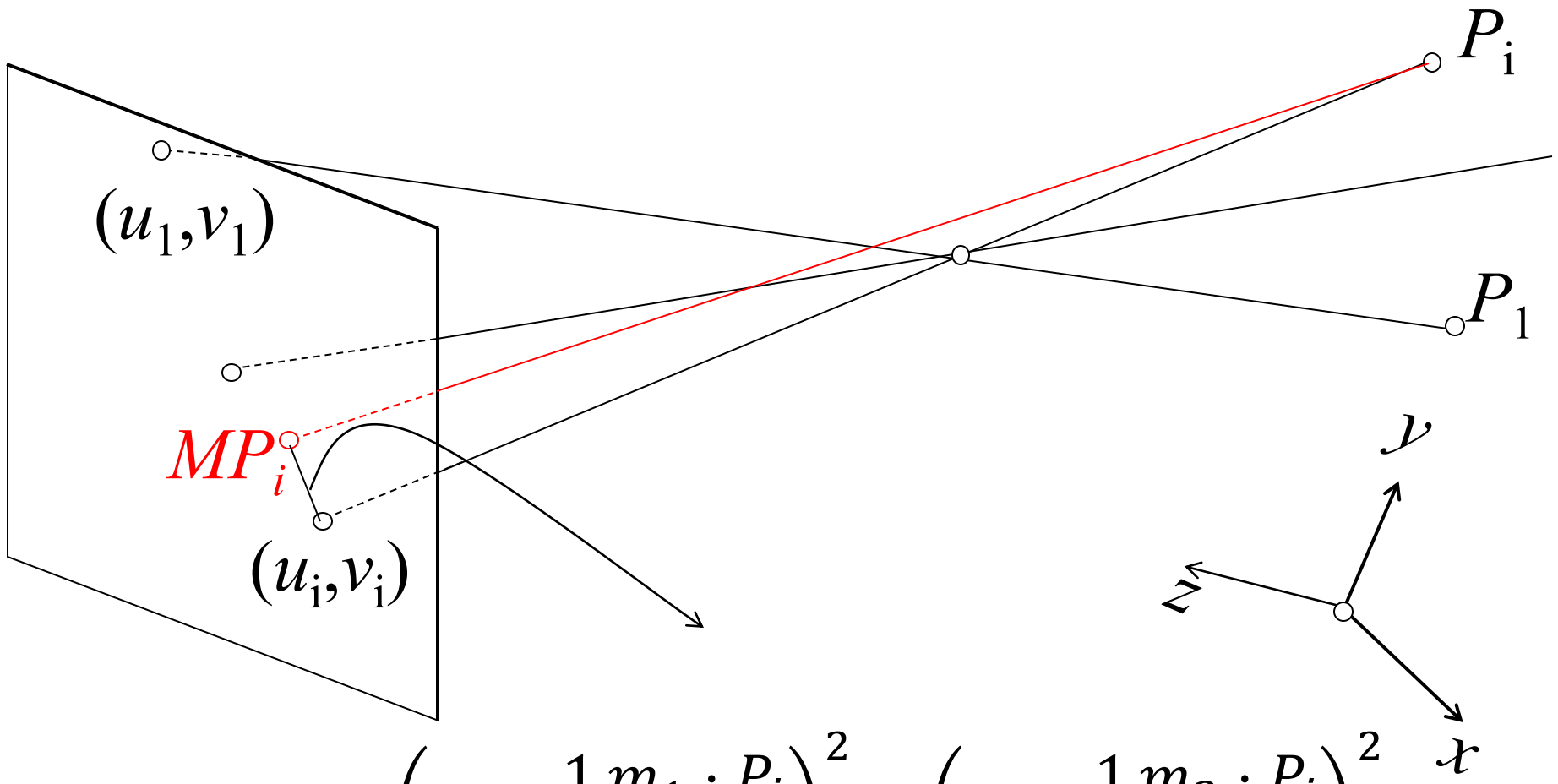
$$x' = f\frac{x}{z} \qquad y' = f\frac{y}{z}$$

Distorted:

$$x'' = \frac{1}{\lambda}x' \qquad y'' = \frac{1}{\lambda}y'$$

$$\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$$

Minimizing reprojection error with radial distortion



Add distortions to reprojection error:

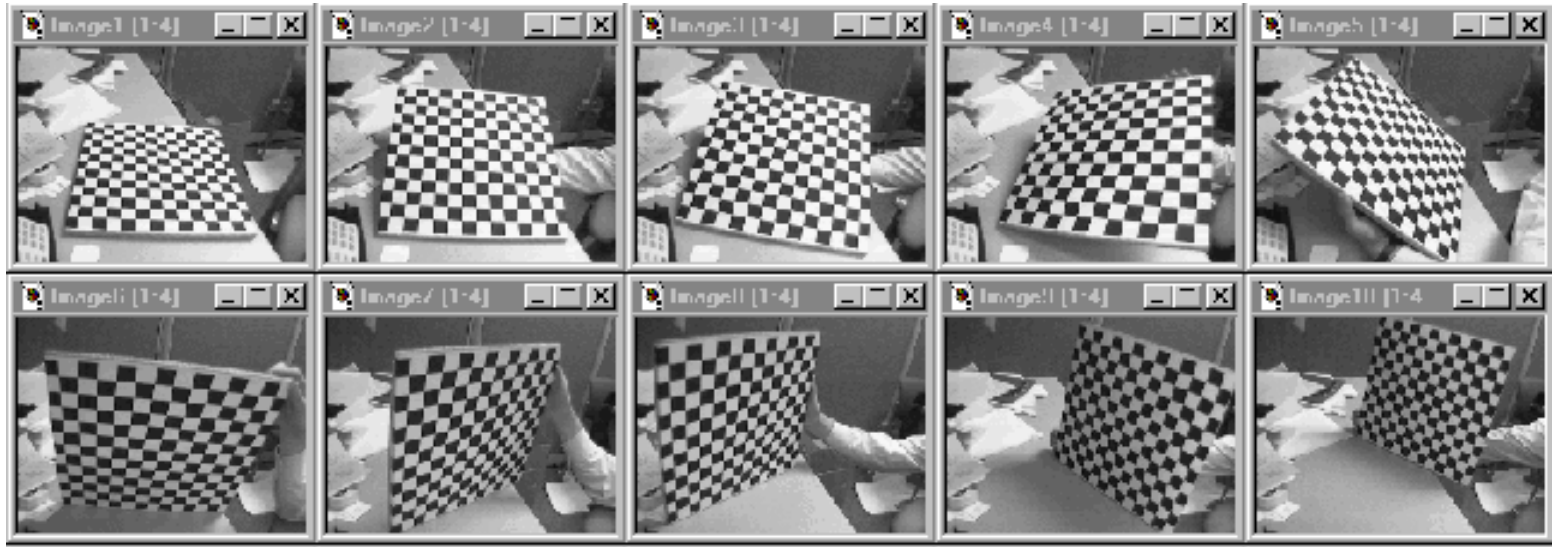
$$\left(u_i - \frac{1}{\lambda}\frac{m_1 \cdot P_i}{m_3 \cdot P_i}\right)^2 + \left(v_i - \frac{1}{\lambda}\frac{m_2 \cdot P_i}{m_3 \cdot P_i}\right)^2$$

Correcting radial distortion

before

after

Alternative: Multi-plane calibration

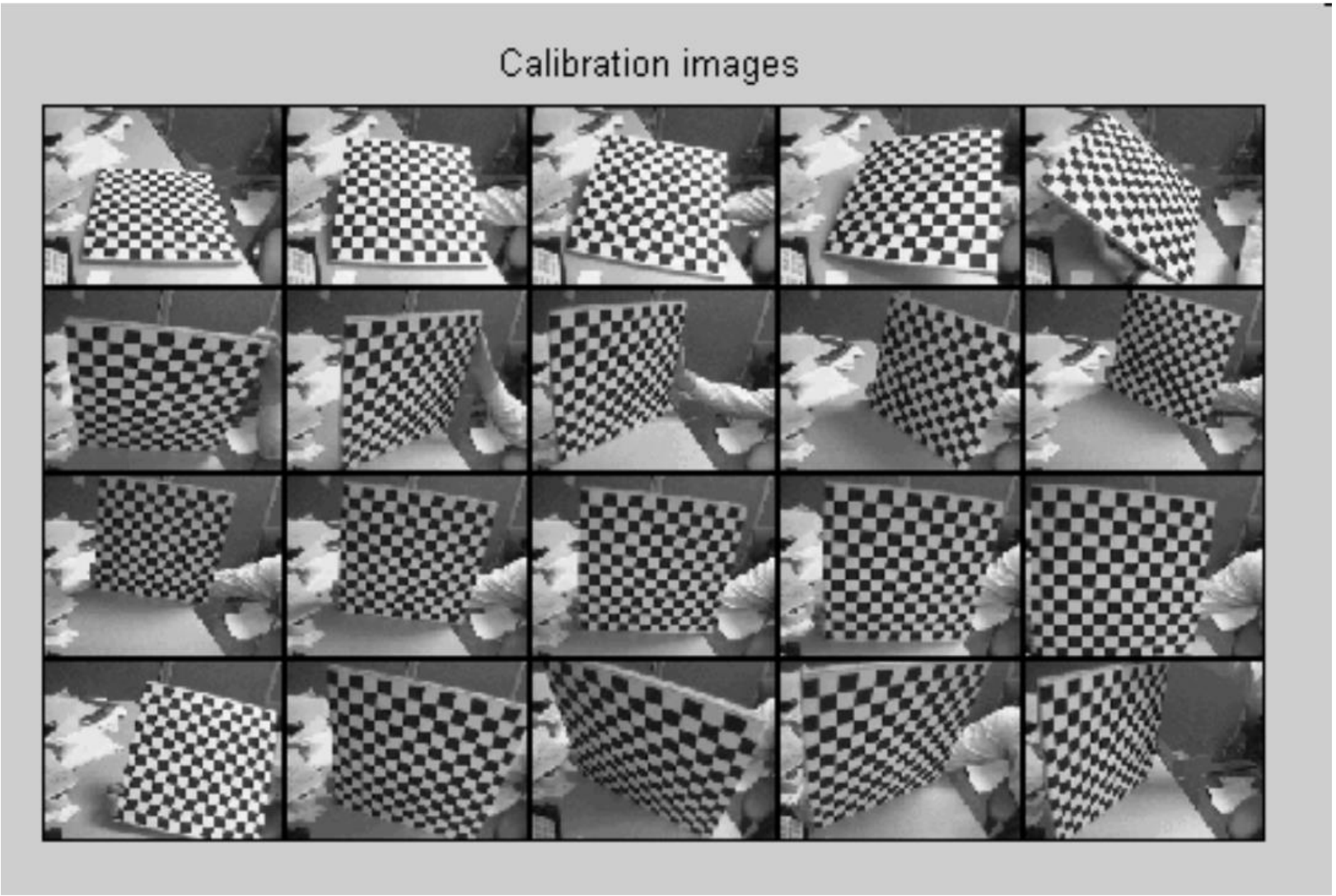


Advantages:

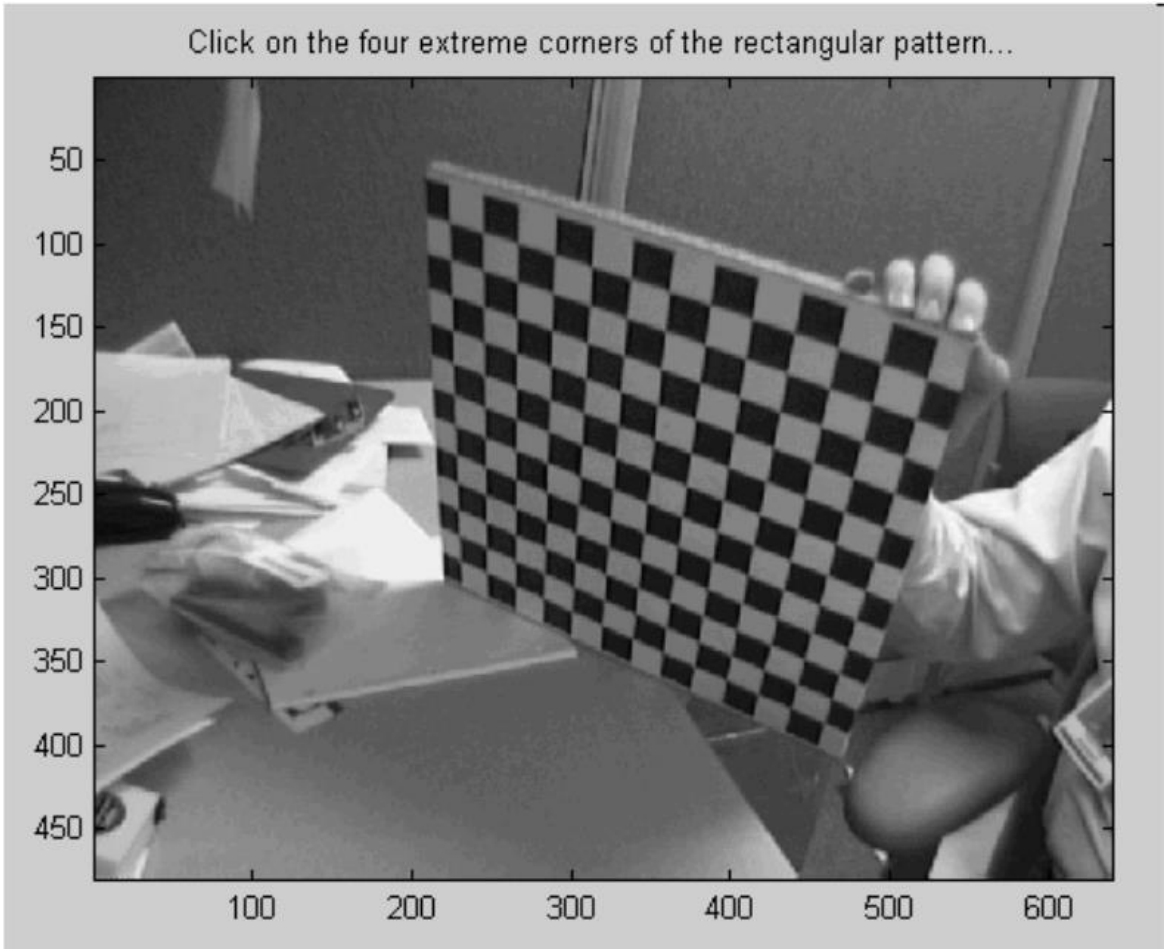
- Only requires a plane
- Don’t have to know positions/orientations
- Great code available online!
  - Matlab version: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Also available on OpenCV.

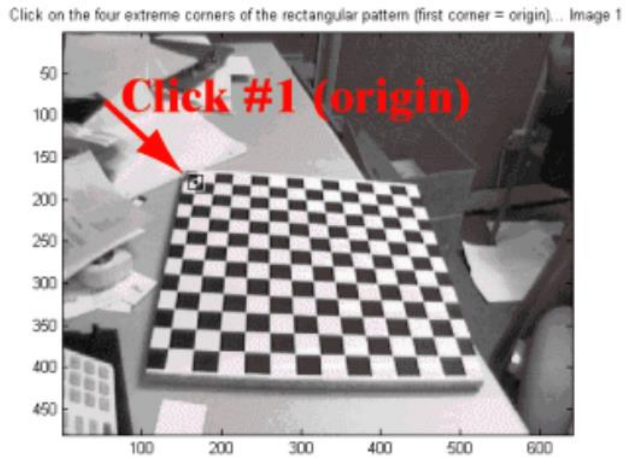
Disadvantage: Need to solve non-linear optimization problem.

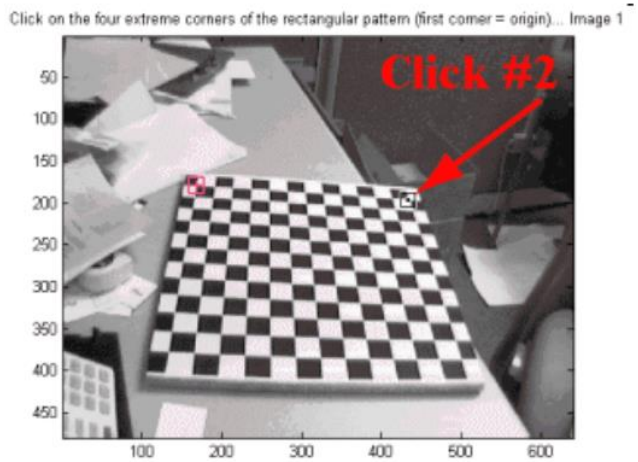
Step-by-step demonstration

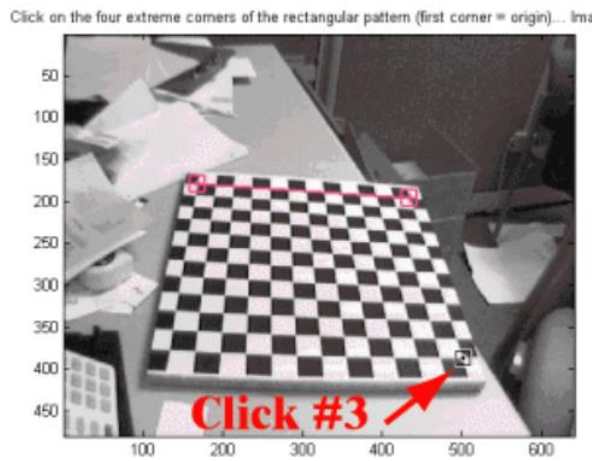


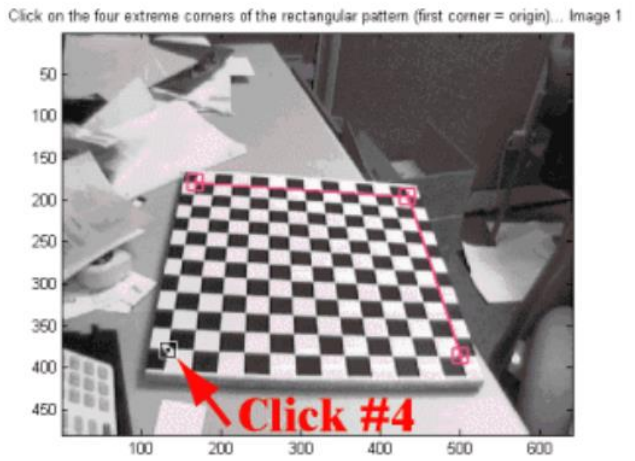
Step-by-step demonstration



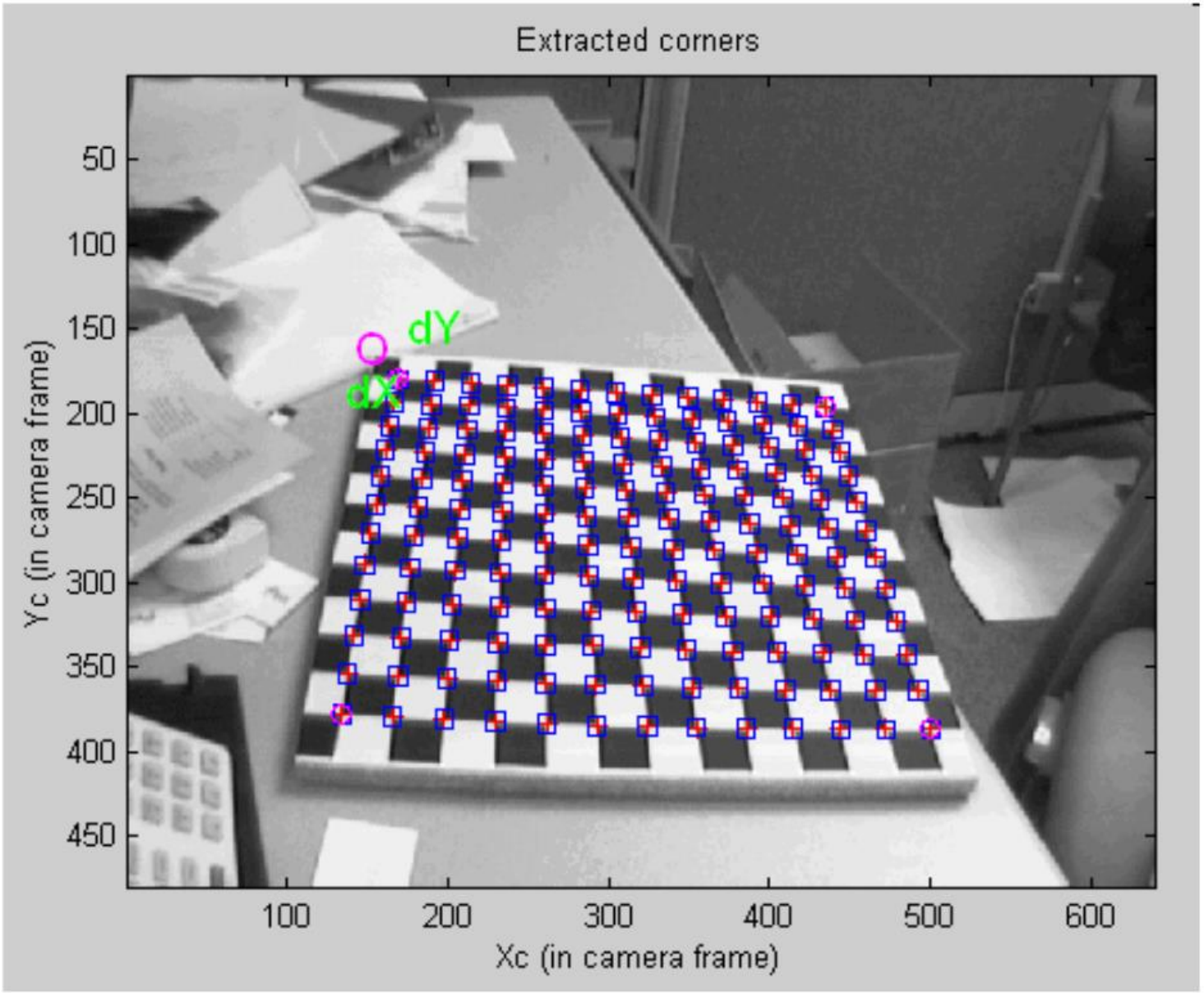




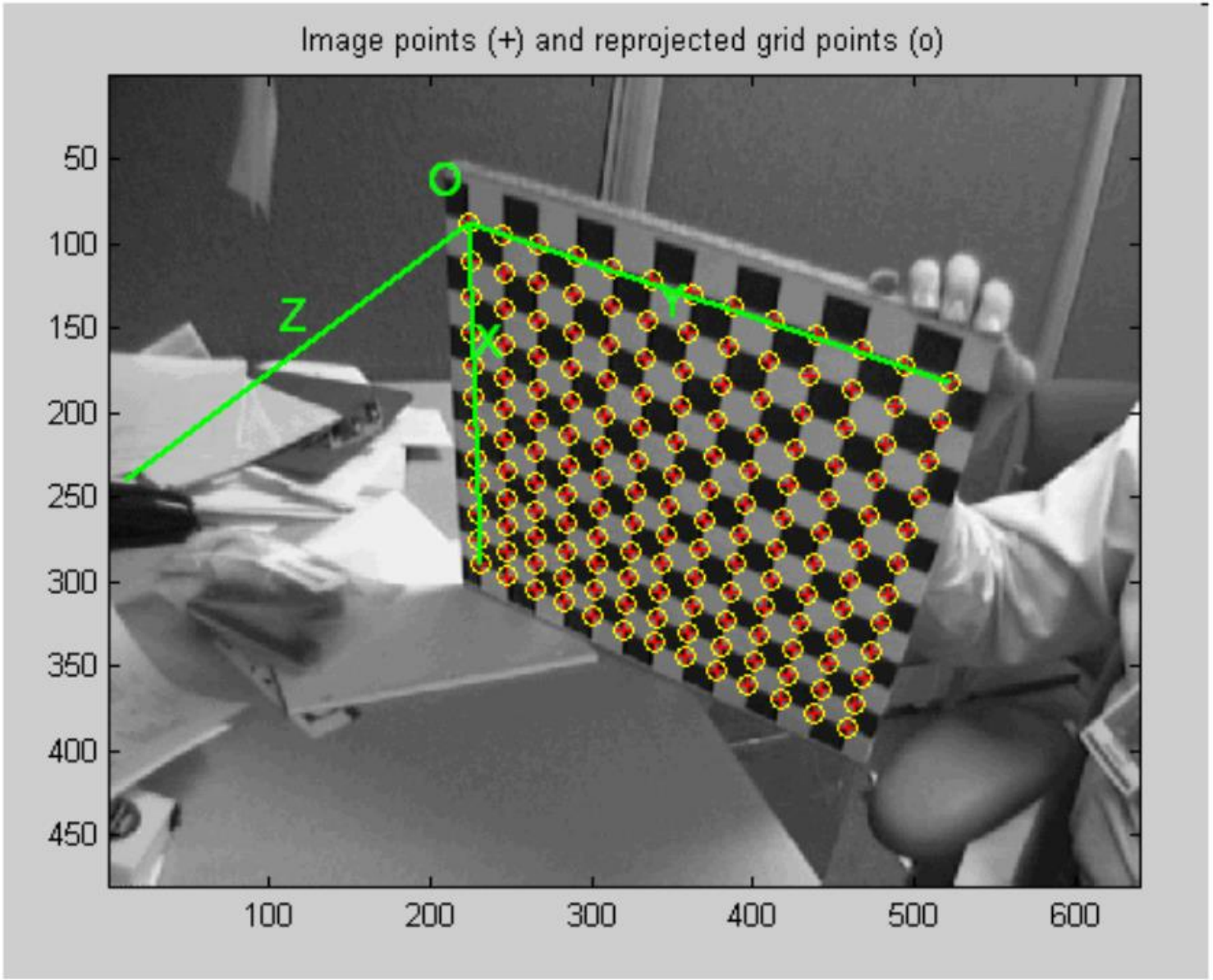




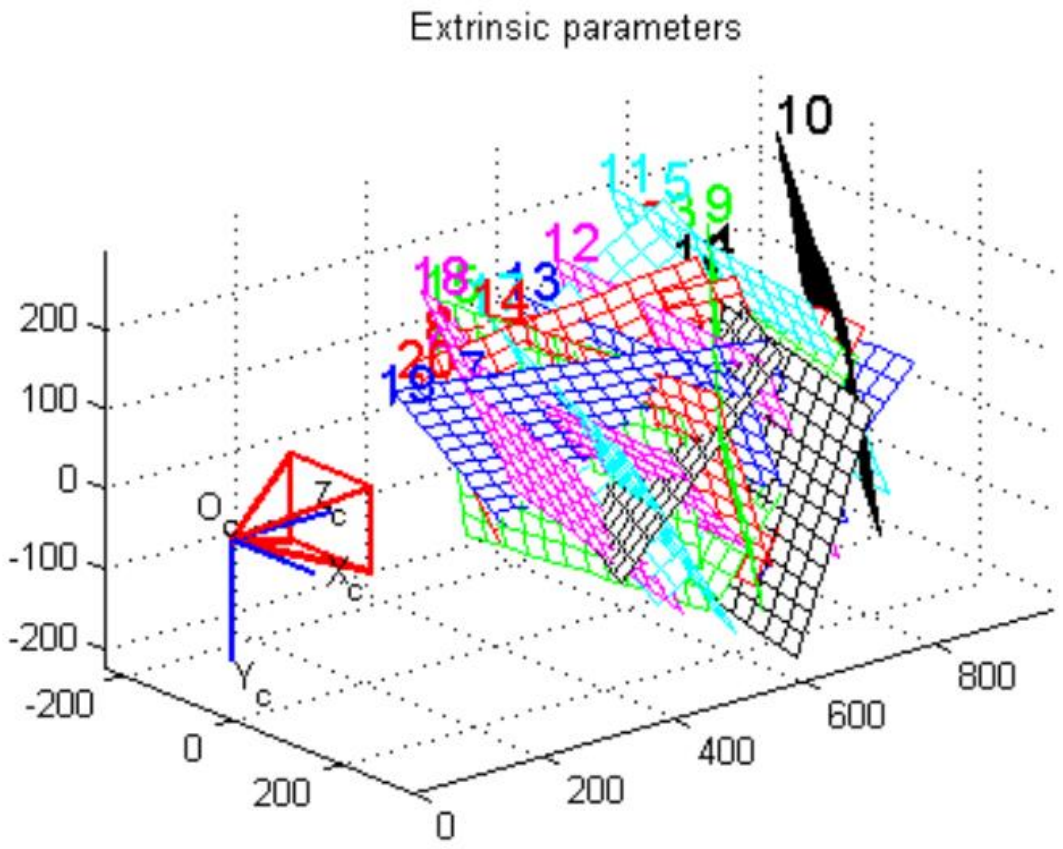
Step-by-step demonstration

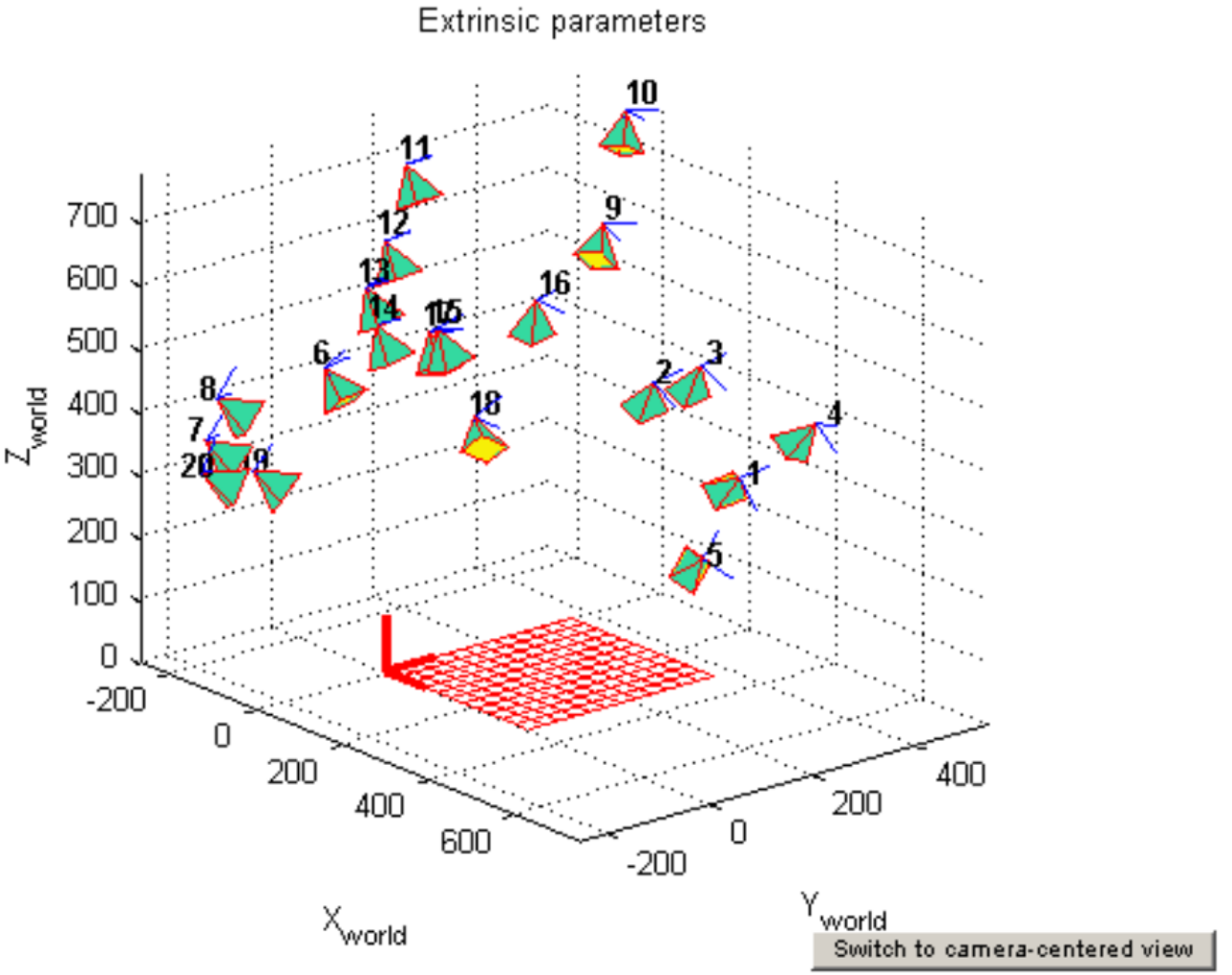


Step-by-step demonstration



Step-by-step demonstration





What does it mean to “calibrate a camera”?

What does it mean to “calibrate a camera”?

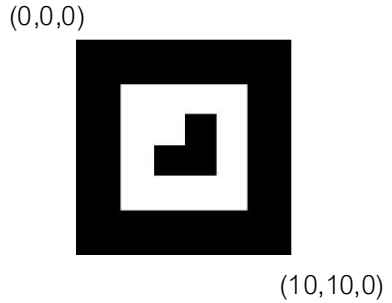
Many different ways to calibrate a camera:

- Radiometric calibration.
- Color calibration.
- Geometric calibration.
- Noise calibration.
- Lens (or aberration) calibration.

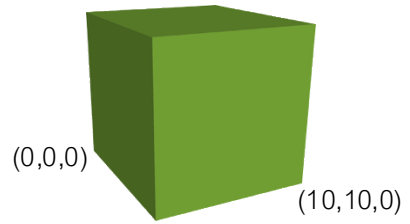
We’ll briefly discuss radiometric and color calibration in later lectures. For the rest, see CS 73 (computational photography).


Tower Health
htc

3D locations of planar marker features are known in advance



3D content prepared in advance



Simple AR program

1. Compute point correspondences (2D and AR tag)
2. Estimate the pose of the camera **P**
3. Project 3D content to image plane using **P**


junaio
SAMSUNG