Advanced Machine Learning Homework 5

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March 24, 2020

1 Kernel Functions for Support Vector Machines

Given input dimension d, input data $\mathbf{x}, \mathbf{y}$ can be expressed as $\vec{\mathbf{x}} = (x_1, ..., x_d), \vec{\mathbf{y}} = (y_1, ..., y_d)$, An SVM kernel function $k(\vec{\mathbf{x}}, \vec{\mathbf{y}})$ can be expressed as the dot product of the transformation of $\vec{\mathbf{x}}, \vec{\mathbf{y}}$ by a transformation function γ, that is to say,

$$k(\vec{\mathbf{x}}, \vec{\mathbf{y}}) = \gamma(\vec{\mathbf{x}}) \cdot \gamma(\vec{\mathbf{y}}) \tag{1}$$

Problems

Derive the transformation function γ for the following SVM kernels, also compute the VC Dimension for the SVM model based on these kernels.

1. $k(\vec{\mathbf{x}}, \vec{\mathbf{y}}) = (\vec{\mathbf{x}} \cdot \vec{\mathbf{y}})^n$
2. $k(\vec{\mathbf{x}}, \vec{\mathbf{y}}) = (\vec{\mathbf{x}} \cdot \vec{\mathbf{y}} + 1)^n$
3. $k(\vec{\mathbf{x}}, \vec{\mathbf{y}}) = e^{-\sigma(||\vec{\mathbf{x}}||^2 + ||\vec{\mathbf{y}}||^2)}$

1. $ k(\vec x, \vec y) = (\vec x\cdot \vec y)^{n}$

展开：

$(x_{1}y_{1} + x_{2}y_{2} + ... + x_{d}y_{d})^{n}$

根据多项式定理：

$\sum \frac {n!}{n_1!n_2!...n_d!} (x_1y_1)^{n_1}(x_2y_2)^{n_2}...(x_dy_d)^{n_d}$

,

其中 $n_1+n_2+ ... + n_d = n$

由此可得：

$\gamma(\vec x) = \sqrt{ \sum \frac {n!}{n_1!n_2!...n_d!}} (x_1)^{n_1}(x_2)^{n_2} ... (x_d)^{n_d}$

VC dimension 取决于多项式的项数，因此为：$C^{n-1}_{n+d-1} +1$

2. $k(\vec{x}, \vec{y}) = (\vec{x} \cdot \vec{y} + 1)^n$

和上面的类似，多一个常数项： 可以令 $\vec{x} = (\vec{x}, 1), \vec{y} = (\vec{y}, 1)$

$\gamma(\vec{x}) = \sqrt{\sum \frac{n!}{n_1!n_2!\ldots n_d!n_{d+1}!}}(x_1)^{n_1}(x_2)^{n_2}\ldots(x_d)^{n_d}1^{n_{d+1}},$

相应的，VC dimension = $C^{n-1}_{n+d} + 1$

3. $k(\vec{x}, \vec{y}) = e^{-\sigma(\|\vec{x}\|^2 + \|\vec{y}\|^2)}$

可得： $\gamma(\vec{x}) = e^{-\sigma(\|\vec{x}\|^2)}$

dimension = 1， 因此对应的SVM VC dimension = 2.