University of Cape Town

EEE3094S

Control Systems

LAB 1

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August 14, 2024

Contents

1 Unit Step Test

1.1 Data Collection

A step function of the amplitude of 1 is applied to the jet suspension system which was logged and plotted as shown below.

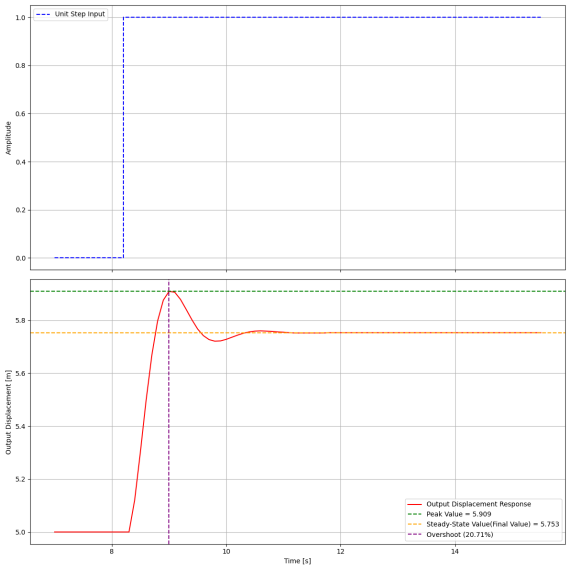

Figure 1: Unit Step Response

1.2 System Parameters

A step response is a widely used method for characterizing a system's behavior. By examining the system's step response, we can determine key parameters that describe its performance. The following sections outline the process and equations used to extract these system parameters from the step response.

1.2.1 System Gain

The ratio of output to input; usually used to describe the amplification in the steady state of the magnitude of sinusoidal inputs, including dc.

$$Gain \ = \frac{Output\ value}{Input\ Value} = \frac{(5.753 - 5)}{1} = 0.753 = -2.464\ dB \tag{1}$$

1.2.2 Peak time, T_P

The time required to reach the first, or maximum, peak.

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{2}$$

1.2.3 Percent overshoot %OS

The amount that the waveform overshoots the steady-state, or final, value at the peak time, expressed as a percentage of the steady-state value.

$$\%OS = \frac{Peak\ Value\ -\ Final\ Value}{Final\ Value} \times 100 \tag{3}$$

1.2.4 Settling time, Ts

The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value

1.2.5 Natural Frequency, ω_n and Damping Ratio, ζ

The natural frequency ω_n, of a second-order system is the frequency of oscillation of the system without damping, when damping is present we define ω_d, oscillation damping frequency. Damping Ratio, ζ, the exponential decay frequency of the envelope to the natural frequency

$$\omega_n = \frac{\pi}{T_P \sqrt{1-\zeta^2}} \tag{4}$$

$$\omega_d = T_P \sqrt{1-\zeta^2}, \tag{5}$$

$$\zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} \tag{6}$$

1.2.6 Transfer Function

$$H(s) = \frac{Gain \times \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{7}$$

Finding % OverShoot

From step response inspection the peak value and final value of the response was observed to be; Peak Value = 5.909, Final Value = 5.753

Using Equation 3, the %OS was calculated:

$\%OS\ = 20.71\%$

Finding Damping ratio

Using Equation 6, the damping ratio(ζ) was calculated:

$\zeta = 0.4481$

Finding Natural Frequency and oscillation damping frequency

The observed peak time was $T_P = 0.8s$

Using Equation 4 and 5, the ω_n *and* ω_d were calculated:

$\omega_n = 4.393\ rad/sec$

$\omega_d = 0.7152\ rad/sec$

System Poles

Equating the denominator expression to zero,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{8}$$

The system poles can be found to be:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \tag{9}$$

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} = -1.969 + j0.8940 \tag{10}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} = -1.969 - j0.8940 \tag{11}$$

Final Transfer Function

From all the system parameter the following transfer function was concluded:

$$H(s) = \frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} \tag{12}$$

A unit step response can be found by convolution in the time domain or multiplication in the s domain.

$$Y(s) = X(s) \times Y(s) \tag{13}$$

$$let\ X(s) = \frac{1}{s} \tag{14}$$

$$Y(s) = \frac{1}{s} \times \frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} \tag{15}$$

The suspension of the jet was already displaced 5m above the landing platform, which introduces an initial condition to the response.

$$c(t) = 5, C(w) = \frac{5}{s} \tag{16}$$

$$H(s) = \frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} + \frac{5}{s} \tag{17}$$

Then the unit step response becomes:

$$Y(s) = \frac{1}{s} \times (\frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} + \frac{5}{s}) \tag{18}$$

2 Frequency Test

2.1 Data Collection

The equations below for gain and phase are used to find data points for the collected data bode plot.

$$\text{Gain (dB)} = 20\log_{10}\left(\frac{\text{Output Amplitude}}{\text{Input Amplitude}}\right) \tag{19}$$

$$\Delta\phi = \frac{\Delta t}{T} \times 360^\circ \tag{20}$$

Δt can be found by finding the time difference when input and output waveforms are at max value.

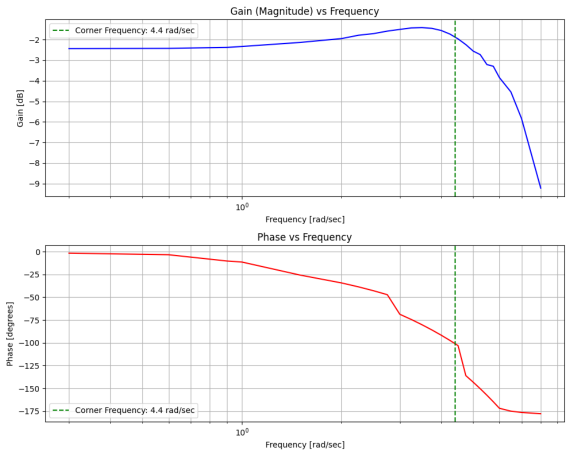

Figure 2: Bode Plot

From figure 2, plot generated from data collected in the lab, the corner frequency was approximated to be w = 4.4 rad/sec

3 System Model

Spring Coefficient

$$k = \omega_n^2 = 4.393^2 = 19.30 \tag{21}$$

Damping Coefficient

$$b = 2\zeta\omega = 2 \times 0.4481 \times 4.393 = 3.937 \tag{22}$$

Transfer Function

Ignoring initial conditions:

$$H(s) = \frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} \tag{23}$$

Including initial conditions:

$$H(s) = \frac{0.753 \times 4.393^2}{s^2 + 2 \times 0.4481 \times 4.393s + 4.393^2} + \frac{5}{s} \tag{24}$$

4 Validation

4.1 Original response vs Model response

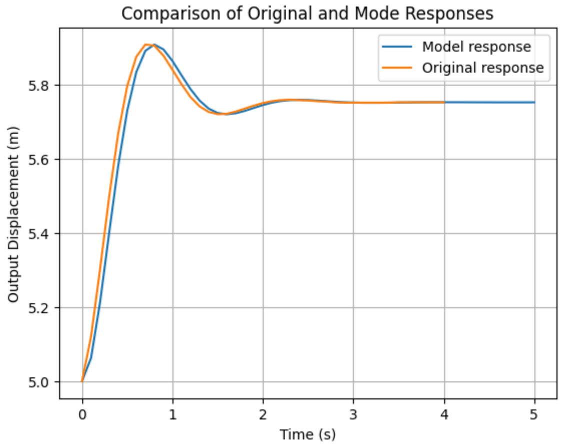

Figure 3: Original response vs Model response

4.2 Simulated response vs Model frequency response (Bode Plot)

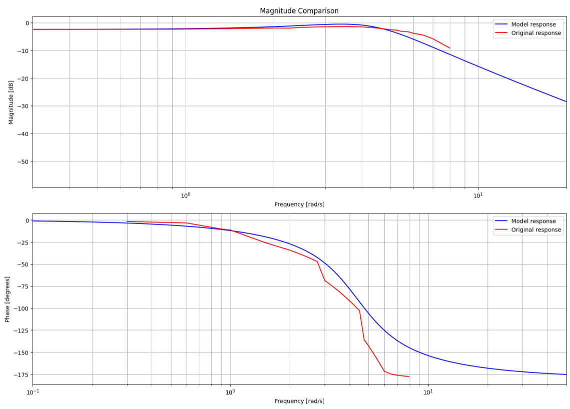

Figure 4: Bode Plot for Simulated response vs Model frequency response

4.3 Validation Analysis

4.3.1 Step Response

In figure 3, the step response from the experiment was matched with the predicted model response with little error observed in the transient response rising, overshoot, and 5% settling. This means the model seems to be accurate based on this test.

4.3.2 Frequency Response (Bode Plot)

Based on figure 4:

Magnitude Plot

The system's low-frequency response closely matched the model predictions. However, there were some errors at higher frequencies, near and beyond the natural frequency.

Phase Plot

The model accurately predicted system behavior at low frequencies. At higher frequencies, while the overall trend was captured, there were inconsistencies in phase response between the model and experimental data. This suggests that the model may require refinement for high-frequency performance

Improvements

Collect more data near points of interest such as corner frequency point. Based on the observed inconsistencies, improve existing model parameters such as natural frequency, and damping ratio.

5 Appendix

link to GitHub repo with the data, code and other things used for the lab resorces `https://github.com/zwivhuyandou23/EEE3094S.git`