Department of Electrical Engineering

EEE4118F – Process Control & Instrumentation

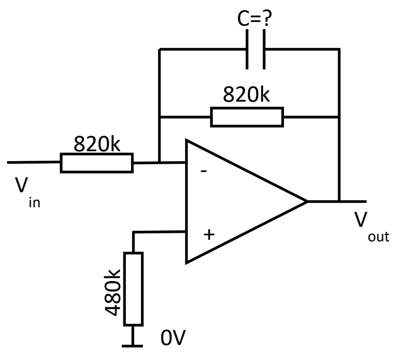
Robust Sensitivity Design

Please note submission requirement for Q3.

1) Consider the plant with uncertain corner frequency, $P = \frac{1}{s/a+1},\ a \in [2,5]$. Choose $P_0 = \frac{1}{s/3+1}$.
   a) Draw plant templates (by hand) for $P(j0.5)$ and $P(j5)$ and hence nominal boundaries for the following specifications
      i) Zero steady state error to a step output disturbance
      ii) $\left|\frac{1}{1+L}\right| \leq -20\text{ dB } \omega \leq 0.5$ (Use $P(j0.5)$ for this specification – why?)
      iii) $\left|\frac{1}{1+L}\right| \leq 3\text{ dB } \forall\ \omega$ (Use $P(j5)$ for this specification but generally check at e.g. 3 and 10 rad/s)
   b) Design a PI controller to meet the specifications (by hand). (First confirm that you need an integrator.)
   c) Check your design with the QFT Toolbox using the following commands. Use the help files to check that each command makes sense to you.

```
% Tutorial_3_Q1
s=tf('s');
P(1,1,1)=1/(s/3+1);
P(1,1,2)=1/(s/2+1);
P(1,1,3)=1/(s/5+1);
bnd=sisobnds(2,[0.5,3,5,10],10.^([-20,3,3,3]/20),P);
lpshape(logspace(-1,2),bnd,P(1,1,1))
```

   d) Check the closed loop Bode plots and step response over the plant set. You are encouraged to use LTI tools in Matlab:




```
G=        % Your design
L=G*P;    % Forms LTI objects for loop transfer functions for all P
Sensitivity=1/(1+L);
subplot(121), bodemag(Sensitivity);
subplot(122), step(Sensitivity);
```

   e) Find the range of capacitor values in the following op-amp circuit that would result in the above plant transfer function (with negative sign).

2) Consider the plant with uncertain corner frequency, $P = \frac{ke^{-sT}}{s/10+1}$, $k \in [2, 5]$ $T \in [0, 0.1]$. Choose $P_0 = \frac{5e^{-0.1s}}{s/10+1}$ (the highest gain worst phase lag plant case).

a) Confirm that the plant templates are rectangles and draw (by hand) the plant templates for $\omega = 0.5, 1$ and 5 rad/s with the smallest number of calculations.

b) Design a controller to achieve the following specifications. Note that although there is no zero steady state error specification, your controller must roll off at low frequency so using an integrator (or a low frequency pole) is useful.

i) $\left|\frac{1}{1+L}\right| \leq -20 \text{ dB } \omega \leq 0.5$

ii) $\left|\frac{1}{1+L}\right| \leq -10 \text{ dB } \omega \leq 1$

iii) $\left|\frac{1}{1+L}\right| \leq 6 \text{ dB } \forall\, \omega$

c) Use Matlab to repeat the design and to check closed loop bode plot and unit step response.

3) Repeat design in Q1 for digital implementation of the controller. As the critical design frequency for the high frequency specification was $\omega = 5$, allocate 15° (design choice) to sampling effects at this frequency. This gives $T = 0.107$, so select $T = 0.1$. (Make sure you follow this!) Add $(1 - wT/2)$ to the plant (or calculate $P_z(w)$ exactly) and then increase the phase lead (of the controller zero) in the design. Do the design "by hand". Use Matlab to check that you meet the specifications in the w-domain and use Simulink to assess your design performance, including looking at the plant output and input and comparing this to the response in Q1. (Submit a design report on your design, one per group.)

4) Consider the underdamped second order plant, $P = \frac{k}{(s/\omega_n)^2 + 2\zeta s/\omega_n + 1}$, $k \in [1, 5], \omega_n \in [3, 6], \zeta \in [0.1, 0.3]$. This could be a flexible robotic arm subjected to input (i.e., torque) disturbances. Design a controller to achieve the following specifications for input disturbance rejection and robust stability. Perform the design using the Matlab CAD toodbox and limit your design to second order. Aim to minimise the controller gain at $\omega = 50$ rad/s. Check your design performance.

i) $\left|\frac{P}{1+L}\right| \leq -20 \text{ dB } \forall\, \omega$ (use `sisobnds(3,…)` for this.)

ii) $\left|\frac{1}{1+L}\right| \leq 3 \text{ dB } \forall\, \omega$ (use `grpbnds` and `sectbnds` for this.)

Notes

a) This design is tricky to do by hand because the T_{Y/D_i} spec is a linear fractional mapping on G but not on the loop gain, $L = GP$. When $L \gg 1$, $\left|\frac{P}{1+L}\right| \approx \left|\frac{1}{G}\right|$ so we can just make $G \geq 10$ (20 dB) but we need to take cognisance of high and low gain plant cases in the intermediate frequency range when P starts to roll off. At high frequency, $L \ll 1$, $\left|\frac{P}{1+L}\right| \approx |P|$ so the controller has no work to do when the plant gain is less than 20 dB.

b) If you look at the plant input, you see that the input signal is very large even where the plant has no gain. This is only possible if the system input can respond to these large input signals. For resonant systems, we are always faced with the challenge of whether to control them in the low frequency (i.e., below the plant corner frequency) or try to control them at high frequency as in this example. The last possibility is to use an under-damped zero (anti-resonance) to cancel the resonant behaviour and insert your own loop dynamics via the controller. This does not work well because of the uncertainty in the plant.